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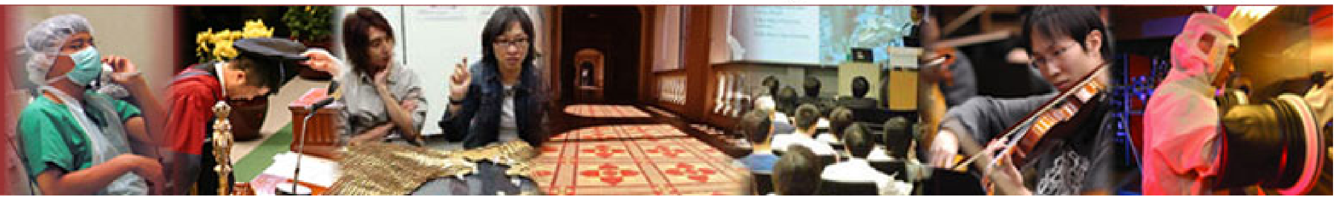

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Design Of Inspection And Maintenance Models Based On The CCC-chart

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Key Words: Inspection, maintenance, statistical process control, economic design.

SUMMARY & CONCLUSION

In this research, six maintenance models are constructed based on whether minor inspection, major inspection, minor maintenance and major maintenance are performed on a system. The system to study is a production process in which items produced can be classified as either conforming or nonconforming, and a statistical process control chart called CCC-chart (cumulative count control chart) can be applied to monitor the process. The maintenance models are analyzed quantitatively, and selection of models can be based on an economic consideration. The total cost can be broken down into inspection cost, maintenance cost, and the cost due to deterioration of the process. From the analytic results obtained, the choice of maintenance plan can be optimized from an economic point of view.

2. INTRODUCTION

In the industry, decisions always have to be made on when to perform maintenance for a system, what type of maintenance work to be performed, or whether to perform maintenance at all. If inspection of the system and minor maintenance are performed at suitable times, further deterioration of the system can be prevented, and sometimes expensive major maintenance can be avoided. However, if inspection and minor maintenance work are performed more frequently than needed, not only will the system be disturbed from time to time and unnecessary costs will be incurred, but additional causes of failure may also be introduced. If maintenance cost is high compared to the failure cost of the system, the system administrator may decide not to perform maintenance at all, but wait until the system fails, and then perform major maintenance on the whole system. Such a practice is not uncommon in the industry.

Statistical process control charts can be used to assist decision making in maintenance ([1]). In this research, geometric random variable is used in the modeling, and the CCC-chart is used to monitor the system. In Section 3, signals that indicate deterioration of the system, forms of inspection for the system, and types of maintenance will be defined. In Section 4, six maintenance models will be established. In Section 5, the six maintenance models will be analyzed quantitatively. In section 6, different costs including penalty cost due to production of nonconforming items, costs for inspection and maintenance of the system are taken into account, and maintenance plans are optimized in terms of the total expected cost. Some examples will be given. Section 7 contains some discussion.

3. STATES OF THE SYSTEM

Suppose that the system under consideration is a production process, in which items produced can be classified as conforming or nonconforming. Assume that the process has three states, in which S_0 is its normal state. The process will deteriorate to state S_1 as time passes. When the process is in state S_1, it will eventually deteriorates to state S_2 if maintenance is not performed. We shall use the abbreviations m_1 and m_2 to denote "minor maintenance" and "major maintenance", respectively. It is assumed that m_1 is less costly to carry out, but is not as thorough as m_2. The following will be assumed.

1. When the process is in state S_0, no maintenance is required, and neither m_1 nor m_2 will change its state.

2. When the process is in S_0, at any instant it will deteriorate to state S_1 with a positive probability π_{01}.

3. When the process is in state S_1, either m_1 or m_2 will restore the process back to state S_0.

4. When the process is in state S_1, at any instant it will deteriorate to state S_2 with a positive probability π_{12}.

5. When the process is in state S_2, m_1 has no effect on it, but m_2 will restore the process back to state S_0.

6. The process will not deteriorate directly from state S_0 to state S_2 without going through state S_1.

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Suppose that two types of signals that indicate deterioration of the process, s_1 and s_2, will appear from time to time. When the process is in state S_0, the probability of occurrence of s_1 or s_2 will be small. When the process is in state S_1, the probability of occurrence of s_1 is large, and when the process is in state S_2, the probability of occurrence of s_2 is large.

In order that probabilities for occurrence of s_1 and s_2 can be calculated, a statistical model must be assumed. Here we apply a geometric random variable to describe the production process in which items are produced one after another, and the probability for an item produced to be nonconforming is p. The number of items inspected, n, until a nonconforming item is observed, is a geometric random variable ([2]) with the following probability function $f(n)$ and cumulative distribution function $F(n)$ $(n = 1, 2, ...)$:

$$f(n) = p(1-p)^{n-1}, \quad F(n) = 1-(1-p)^n. \tag{3.1}$$

A CCC-chart ([3], [4], [5], [6]) for detecting upward shift of p (deterioration of the process) has a lower control limit n_L for n, which corresponds to a certain probability of false alarm α. It follows from (3.1) that

$$F(n_L) = 1-(1-p)^{n_L} = \alpha, \tag{3.2}$$

$$n_L = \log(1-\alpha)/\log(1-p). \tag{3.3}$$

Divide the set of all positive integers Z^+ into the following three sets:

$$Z_0 = \{n \in Z^+ : n_1 + 1 \le n < \infty\},$$
$$Z_1 = \{n \in Z^+ : n_2 + 1 \le n \le n_1\},$$
$$Z_2 = \{n \in Z^+ : 1 \le n \le n_2\},$$

where $1 \le n_2 < n_1$ are given integers. Occurrence of a nonconforming item at $n \in Z_i$ $(i = 0, 1, 2)$ is defined as a type i signal.

A signal s_0 indicate that the process is in state S_0, and such a signal does not call for any action. When a signal s_1 or s_2 appears, either minor inspection, i_1, or major inspection, i_2 may be carried out. Assume that i_2 is a thorough inspection, while i_1 is a superficial one which is less costly to carry out than i_2. The following three arrangements, I_{1+2}, I_2 and I_0, are alternative inspection strategies:

(1) I_{1+2}: Both i_1 and i_2 are employed.

In this arrangement, an s_1 will trigger i_1, and an s_2 will trigger i_2 for the process. When the process is in state S_0 or S_1, both i_1 and i_2 will correctly reveal the true state of the process. When the process is in state S_2, however, i_1 will incorrectly indicate that the process is in state S_1, and only i_2 will correctly reveal that the process is in state S_2.

(2) I_2: Only i_2 is employed.

In this arrangement, either s_1 or s_2 will trigger i_2 which will always correctly reveal the true state of the process, no matter whether the process is in state S_0, S_1 or S_2.

(3) I_0: No inspection.

In this arrangement, no inspection will be carried out, even when s_1 or s_2 appears.

Whether to adopt strategies I_{1+2} or I_2 depends on how much more difficult or costly it is to carry out i_2 than i_1, and the consequences of wrong indication of the state of the process. The arrangement I_0 of no inspection will be suitable when inspecting the process is either too costly or not feasible.

In the next section, these three arrangements for inspection will be combined with three arrangements for maintenance to form six maintenance models.

4. MAINTENANCE MODELS

A cycle of the process is said to be completed when the process starts at state S_0, deteriorates to state S_1 or state S_2, and finally restored back to state S_0 by m_1 or m_2. In what follows, three maintenance arrangements, M_{1+2}, M_2 and M_0, will be considered. In M_{1+2}, both m_1 and m_2 will be carried out; in M_2, only m_2 will be carried out, but not m_1; and in M_0, no maintenance work will be carried out. Under M_{1+2} or M_2, the process will eventually complete a cycle. Under M_0, no maintenance work will be carried out, and after the process has started it will eventually change to state S_2 and remain in state S_2. Combining I_{1+2}, I_2, I_0 with M_{1+2}, M_2, M_0 in various ways produces the following six maintenance models.

(1) Model (I_{1+2}, M_{1+2}).

In this model, all of i_1, i_2, m_1 and m_2 will be carried out. Three possible scenarios under events A_I, B_I, C_I are depicted in Figure 1. In Figure 1, the events enclosed in each pair of square brackets occur in sequence, (p_i) means that the process is in state S_i $(i = 0, 1, 2)$, and three mutually exclusive events A_I, B_I, C_I are defined by:

A_I: An s_1 appears when the process is in state S_1.

B_I: An s_2 appears when the process is in state S_1.

C_I: The process changes from state S_1 to state S_2 before any s_1 or s_2 appears.

The signals s_0's are not indicated in Figure 1 (nor in all other figures below), since such signals do not call for any action. After event A_I or B_I has occurred, m_1 or m_2 will bring the process back to state S_0. After event C_I has occurred and the process is in state S_2, m_1 will not change the state of the process, but m_2 will bring the process back to state S_0.

$$(p_0) \rightarrow (p_1) \rightarrow$$
$$A_I: [\mathrm{s}_1, \mathrm{i}_1, \mathrm{m}_1] \rightarrow (\mathrm{p}_0)$$
$$B_I: [\mathrm{s}_2, \mathrm{i}_2, \mathrm{m}_1] \rightarrow (\mathrm{p}_0)$$
$$C_I: \rightarrow (p_2) \rightarrow [\mathrm{s}_1, \mathrm{i}_1, \mathrm{m}_1] \rightarrow \cdots \rightarrow [\mathrm{s}_1, \mathrm{i}_1, \mathrm{m}_1] \rightarrow [\mathrm{s}_2, \mathrm{i}_2, \mathrm{m}_2] \rightarrow (\mathrm{p}_0)$$

Figure 1. Scenarios for the process to complete a cycle under model $(\mathrm{I}_{1+2}, \mathrm{M}_{1+2})$.

(2) Model $(\mathrm{I}_2, \mathrm{M}_{1+2})$.

In this model, i_2, m_1 and m_2 will be carried out, but not i_1. This model will be suitable when the effort required to carry out i_1 is about the same as that for i_2. Here either an s_1 or s_2 will be followed by i_2. Three possible scenarios under events A_I, B_I, C_I defined above are depicted in Figure 2.

$$(p_0) \rightarrow (p_1) \rightarrow$$
$$A_I: [\mathrm{s}_1, \mathrm{i}_2, \mathrm{m}_1] \rightarrow (\mathrm{p}_0)$$
$$B_I: [\mathrm{s}_2, \mathrm{i}_2, \mathrm{m}_1] \rightarrow (\mathrm{p}_0)$$
$$C_I: \rightarrow (p_2) \rightarrow [\mathrm{s}_1 \text{ or } \mathrm{s}_2, \mathrm{i}_2, \mathrm{m}_2] \rightarrow (\mathrm{p}_0)$$

Figure 2. Scenarios for the process to complete a cycle under model $(\mathrm{I}_2, \mathrm{M}_{1+2})$.

(3) Model $(\mathrm{I}_0, \mathrm{M}_{1+2})$.

In this model, no inspection will be carried out. This model may be applied when inspection is either too costly or not feasible. When an s_1 occurs, m_1 will be carried out immediately, and when an s_2 occurs, m_2 will be carried out immediately. In this model, the three possible scenarios under events A_I, B_I, C_I defined above are depicted in Figure 3.

$$(p_0) \rightarrow (p_1) \rightarrow$$
$$A_I: [\mathrm{s}_1, \mathrm{m}_1] \rightarrow (\mathrm{p}_0)$$
$$B_I: [\mathrm{s}_2, \mathrm{m}_2] \rightarrow (\mathrm{p}_0)$$
$$C_I: \rightarrow (p_2) \rightarrow [\mathrm{s}_1, \mathrm{m}_1] \rightarrow \cdots \rightarrow [\mathrm{s}_1, \mathrm{m}_1] \rightarrow [\mathrm{s}_2, \mathrm{m}_2] \rightarrow (\mathrm{p}_0)$$

Figure 3. Scenarios for the process to complete a cycle under model $(\mathrm{I}_0, \mathrm{M}_{1+2})$.

(4) Model $(\mathrm{I}_2, \mathrm{M}_2)$.

In this model, when an s_2 occurs, i_2 will be carried out, and if i_2 reveals that the process is in state S_2, m_2 will be carried out. Here m_1 will not be carried out, all s_1's will be ignored. This model will be suitable when the effort required to carry out i_1 and m_1 is nearly as much as those required for i_2 and m_2. Figure 4 depicts two possible scenarios of this model. Two mutually exclusive events B_{II} and C_{II} are defined by:

B_{II}: An s_2 occurs, or some s_1's occur and then follows by an s_2.

C_{II}: The process changes from state S_1 to state S_2 before any s_2 occurs.

$$(p_0) \rightarrow (p_1) \rightarrow$$
$$B_{II}: [\mathrm{s}_1, ..., \mathrm{s}_1, \mathrm{s}_2, \mathrm{i}_2, \mathrm{m}_2] \rightarrow (\mathrm{p}_0)$$
$$C_{II}: \rightarrow (p_2) \rightarrow [\mathrm{s}_1, ..., \mathrm{s}_1, \mathrm{s}_2, \mathrm{i}_2, \mathrm{m}_2] \rightarrow (\mathrm{p}_0)$$

Figure 4. Scenarios for the process to complete a cycle under model $(\mathrm{I}_2, \mathrm{M}_2)$.

(5) Model $(\mathrm{I}_0, \mathrm{M}_2)$.

In this model, no inspection and no m_1 will be carried out. All s_1's will be ignored. When an s_2 occurs, m_2 will be carried out immediately without any inspection. This model can be represented by Figure 4 with the i_2's removed.

(6) Model $(\mathrm{I}_0, \mathrm{M}_0)$.

In this model, neither inspection nor maintenance will be carried out. This model suits the case when the effort required to carry out inspection and maintenance are costly compared with the penalty cost due to production of nonconforming items. Figure 5 shows the model.

$$(p_0) \rightarrow (p_1) \rightarrow (p_2)$$

Figure 5. Scenarios for the process to complete a cycle under model $(\mathrm{I}_0, \mathrm{M}_0)$.

In the above six models, (I_{1+2}, M_{1+2}) and (I_0, M_{1+2}) depend on both n_1 and n_2, (I_2, M_{1+2}) depends on n_1 but not n_2, models (I_2, M_2) and (I_0, M_2) depend on n_2 but not n_1, and (I_0, M_0) is independent of both n_1 and n_2. Table 1 shows these models; the combinations that are not applicable are indicated by "N.A.". For example, if minor maintenance is not to be carried out, there will no point to carry out minor inspection, and therefore the model (I_{1+2}, M_2) is not applicable, and so is the model (I_{1+2}, M_0), and so on.

In the next section, analytic expressions for the probabilities of events and average number of items inspected will be established.

Table 1. Maintenance models

		M_{1+2}	M_2	M_0
I_{1+2}	*Inspection:*	1 and 2	N.A.	N.A.
	Maintenance:	1 and 2	N.A.	N.A.
I_2	*Inspection:*	2 only	2 only	N.A.
	Maintenance:	1 and 2	2 only	N.A.
I_0	*Inspection:*	nil	nil	nil
	Maintenance:	1 and 2	2 only	nil

5. QUANTITATIVE ANALYSIS

Let ξ_0 denote the instant of time immediately after the process has started, or immediately after the occurrence of a nonconforming item when the process is in state S_0. Let $Q_{0(i)}$ $(i = 0, 1, 2)$ be the probability for a type i signal to appear when the process is in state S_0. Since the number of items inspected to observe a nonconforming item is a geometric distribution, we have

$$Q_{0(0)} = \sum_{j \in Z_0} p_0(1-p_0)^{j-1} = (1-p_0)^{n_1}, \tag{5.1}$$

$$\begin{aligned} Q_{0(1)} &= \sum_{j \in Z_1} p_0(1-p_0)^{j-1} \\ &= (1-p_0)^{n_2} - (1-p_0)^{n_1}, \end{aligned} \tag{5.2}$$

$$Q_{0(2)} = \sum_{j \in Z_2} p_0(1-p_0)^{j-1} = 1-(1-p_0)^{n_2}. \tag{5.3}$$

Let ξ_1 denote the instant of time immediately after the state of the process has shifted from S_0 to S_1, or immediately after the occurrence of a nonconforming item when the process is in state S_1. Let $Q_{1(i)}$ $(i = 0, 1, 2)$ be the probability of observing the first nonconforming item since ξ_1, and this nonconforming item gives a type i signal $(i = 0, 1, 2)$, while the process remains in state S_1. It can be proved that

$$\begin{aligned} Q_{1(0)} &= \sum_{j \in Z_0} p_1(1-p_1)^{j-1}(1-\pi_{12})^j \\ &= \frac{(1-\pi_{12})p_1\left((1-p_1)(1-\pi_{12})\right)^{n_1}}{1-(1-p_1)(1-\pi_{12})}, \end{aligned} \tag{5.4}$$

$$\begin{aligned} Q_{1(1)} &= \sum_{j \in Z_1} p_1(1-p_1)^{j-1}(1-\pi_{12})^j \\ &= \frac{(1-\pi_{12})p_1\left[(1-p_1)(1-\pi_{12})\right]^{n_2}}{1-(1-p_1)(1-\pi_{12})} \\ &\quad - \frac{(1-\pi_{12})p_1\left[(1-p_1)(1-\pi_{12})\right]^{n_1}}{1-(1-p_1)(1-\pi_{12})}, \end{aligned} \tag{5.5}$$

$$\begin{aligned} Q_{1(2)} &= \sum_{j \in Z_2} p_1(1-p_1)^{j-1}(1-\pi_{12})^j \\ &= \frac{(1-\pi_{12})p_1\left[1-((1-p_1)(1-\pi_{12}))^{n_2}\right]}{1-(1-p_1)(1-\pi_{12})}. \end{aligned} \tag{5.6}$$

The probability of observing a shift of the process from state S_1 to state S_2 since ξ_1 is

$$\begin{aligned} Q_{1,2} &= \sum_{j=1}^{\infty} \pi_{12}(1-p_1)^{j-1}(1-\pi_{12})^{j-1} \\ &= \frac{\pi_{12}}{1-(1-p_1)(1-\pi_{12})}. \end{aligned} \tag{5.7}$$

Let ξ_2 denote the instant of time immediately after the state of the process has shifted from S_1 to S_2, or immediately after the occurrence of a nonconforming item when the process is in state S_2. Let $Q_{2(i)}$ $(i = 0, 1, 2)$ be the probability of observing the first nonconforming item since ξ_2, and this nonconforming item gives a type i signal $(i = 0, 1, 2)$, while the process remains in state S_2. We have

$$Q_{2(0)} = \sum_{j \in Z_0} p_2(1-p_2)^{j-1} = (1-p_2)^{n_1}, \tag{5.8}$$

$$Q_{2(1)} = \sum_{j \in Z_1} p_2(1-p_2)^{j-1} = (1-p_2)^{n_2} - (1-p_2)^{n_1}, \tag{5.9}$$

$$Q_{2(2)} = \sum_{j \in Z_2} p_2(1-p_2)^{j-1} = 1-(1-p_2)^{n_2}. \tag{5.10}$$

The probability of occurrence of events A_I, B_I, C_I defined in models (I_{1+2},M_{1+2}), (I_2,M_{1+2}) and (I_0,M_{1+2}) are

$$P(A_I) = \sum_{i=1}^{\infty} Q_{1(0)}^{i-1} Q_{1(1)} = \frac{Q_{1(1)}}{1-Q_{1(0)}}, \tag{5.11}$$

$$P(B_I) = \sum_{i=1}^{\infty} Q_{1(0)}^{i-1} Q_{1(2)} = \frac{Q_{1(2)}}{1-Q_{1(0)}}, \tag{5.12}$$

$$P(C_I) = \sum_{i=1}^{\infty} Q_{1(0)}^{i-1} Q_{1,2} = \frac{Q_{1,2}}{1-Q_{1(0)}}, \tag{5.13}$$

respectively.

The probability of occurrence of events B_{II}, C_{II} defined in models (I_2,M_2) and (I_0,M_{1+2}) are

$$\begin{aligned} P(B_{II}) &= \sum_{i=1}^{\infty} (Q_{1(0)} + Q_{1(1)})^{i-1} Q_{1(2)} \\ &= \frac{Q_{1(2)}}{1-Q_{1(0)}-Q_{1(1)}}, \end{aligned} \tag{5.14}$$

$$P(C_{II}) = \sum_{i=1}^{\infty} (Q_{1(0)} + Q_{1(1)})^{i-1} Q_{1,2}$$

$$= \frac{Q_{1,2}}{1 - Q_{1(0)} - Q_{1(1)}}. \quad (5.15)$$

We shall find the expected items inspected for the process to complete a cycle. Let $\delta = (1-p_1)(1-\pi_{12})$, and define

$$L_0 = \sum_{j \in Z_0} j p_1 (1-p_1)^{j-1}(1-\pi_{12})^j = \frac{p_1(1-\pi_{12})\left[\delta^{n_1}(n_1 + 1 - n_1\delta)\right]}{(1-\delta)^2}, \quad (5.16)$$

$$L_1 = \sum_{j \in Z_1} j p_1 (1-p_1)^{j-1}(1-\pi_{12})^j = \frac{p_1(1-\pi_{12})\left[\delta^{n_2}(n_2 + 1 - n_2\delta)\right]}{(1-\delta)^2} - \frac{p_1(1-\pi_{12})\left[\delta^{n_1}(n_1 + 1 - n_1\delta)\right]}{(1-\delta)^2}, \quad (5.17)$$

$$L_2 = \sum_{j \in Z_2} j p_1 (1-p_1)^{j-1}(1-\pi_{12})^j = p_1(1-\pi_{12})\frac{1-\delta^{n_2}(n_2 + 1 - n_2\delta)}{(1-\delta)^2}. \quad (5.18)$$

$$L_3 = \sum_{j=1}^{\infty}(j-1)\pi_{12}(1-p_1)^{j-1}(1-\pi_{12})^{j-1} = \pi_{12}\delta/(1-\delta)^2, \quad (5.19)$$

$$L_4 = \sum_{j \in Z_0 \cup Z_1} j p_2 (1-p_2)^{j-1} = \left[(1-p_2)^{n_2}(n_2 + 1 - n_2(1-p_2))\right]/p_2, \quad (5.20)$$

$$L_5 = \sum_{j \in Z_2} j p_2 (1-p_2)^{j-1} = \frac{1-(1-p_2)^{n_2}(n_2 + 1 - n_2(1-p_2))}{p_2}. \quad (5.21)$$

The following Propositions and Lemmas can be proved. However, their proofs will be omitted here.

Proposition I. For models (I_{1+2},M_{1+2}) or (I_0,M_{1+2}), the average number of items inspected, ANI_I, for the process to complete a cycle which starts from state S_0, deteriorates to state S_1 or state S_2, and finally be restored back to state S_0 is

$$ANI_I = \frac{1}{\pi_{01}} + \frac{1-\pi_{12}}{\pi_{12} + p_1(1-\pi_{12})} \times \frac{1}{1-Q_{1(0)}} + \frac{Q_{1,2}}{1-Q_{1(0)}} \times \frac{1}{p_2\, Q_{2(2)}}.$$

Proposition II. For model (I_2,M_{1+2}), the average number of items inspected, ANI_{II}, for the process to complete a cycle which starts from state S_0, deteriorates to state S_1 or state S_2, and finally be restored back to state S_0 is

$$ANI_{II} = \frac{1}{\pi_{01}} + \frac{1-\pi_{12}}{\pi_{12} + p_1(1-\pi_{12})} \times \frac{1}{1-Q_{1(0)}} + \frac{Q_{1,2}}{1-Q_{1(0)}} \times \frac{1}{p_2(1-Q_{2(0)})}.$$

Proposition III. For models (I_2,M_2) and (I_0,M_2), the average number of items inspected, ANI_{III}, for the process to complete a cycle which starts from state S_0, deteriorates to state S_1 or state S_2, and finally be restored back to state S_0 is

$$ANI_{III} = \frac{1}{\pi_{01}} + \frac{1-\pi_{12}}{\pi_{12} + p_1(1-\pi_{12})} \times \frac{1}{1-Q_{1(0)} - Q_{1(1)}} + \frac{Q_{1,2}}{1-Q_{1(0)} - Q_{1(1)}} \times \frac{1}{p_2\, Q_{2(2)}}.$$

Note that (5.4), (5.5), (5.7), (5.10) show that ANI_I depends on both n_1 and n_2, ANI_{II} depends on n_1 but not n_2, and ANI_{III} depends on n_2 but not n_1. The following Lemmas 1 – 11 are required in the proof of Propositions I – III. All the detailed proofs, however, will be omitted here.

Lemma 1. The expected number of items inspected since the process starts at state S_0 until it shifts to state S_1, is

$$E_1 = \sum_{j=1}^{\infty} j\pi_{01}(1-\pi_{01})^{j-1} = \frac{1}{\pi_{01}}.$$

Lemma 2. Under event A_I, the expected number of items inspected from the instant of time immediately after the process has shifted from state S_0 to state S_1, until a type 1 signal appears while the process still remains in state S_1, without any type 2 signal appearing before this type 1 signal, is

$$E_2 = \frac{Q_{1(1)}L_0}{(1-Q_{1(0)})^2} + \frac{L_1}{1-Q_{1(0)}}.$$

Lemma 3. Under event B_I, the expected number of items inspected from the instant of time immediately after the process has shifted from state S_0 to state S_1, until a type 2 signal appears while the process still remains in state S_1, without any type 1 signal appearing before this type 2 signal, is

$$E_3 = \frac{Q_{1(2)}L_0}{(1-Q_{1(0)})^2} + \frac{L_2}{1-Q_{1(0)}}.$$

Lemma 4. Under event C_I, the expected number of items inspected from the instant of time immediately after the process has shifted from state S_0 to state S_1, until it shifts to state S_2, without any type 1 or type 2 signal appearing during this period of time, is

$$E_4 = \frac{Q_{1,2} L_0}{(1 - Q_{1(0)})^2} + \frac{L_3}{1 - Q_{1(0)}}.$$

Lemma 5. Under event B_{II}, the expected number of items inspected from the instant of time immediately after the process has shifted from state S_0 to state S_1, until a type 2 signal appears, while the process still remains in state S_1, is

$$E_5 = \frac{Q_{1(2)}(L_0 + L_1)}{(1 - Q_{1(0)} - Q_{1(1)})^2} + \frac{L_2}{1 - Q_{1(0)} - Q_{1(1)}}.$$

Here some type 0 or type 1 signal may appear before this type 2 signal.

Lemma 6. Under event C_{II}, the expected number of items inspected from the instant of time immediately after the process has shifted from state S_0 to state S_1, until it shifts to state S_2, without any type 2 signal appearing during this period of time, is

$$E_6 = \frac{Q_{1,2}(L_0 + L_1)}{(1 - Q_{1(0)} - Q_{1(1)})^2} + \frac{L_3}{1 - Q_{1(0)} - Q_{1(1)}}.$$

Lemma 7. The expected number of items inspected from the instant of time immediately after the process has shifted from state S_1 to state S_2, until a type 2 signal occurs, is

$$E_7 = \frac{Q_{2(2)} L_4}{(1 - Q_{2(0)} + Q_{2(1)})^2} + \frac{L_5}{1 - Q_{2(0)} + Q_{2(1)}} = \frac{1}{p_2\, Q_{2(2)}}.$$

Lemma 8. The expected number of type i $(i = 0, 1, 2)$ signals that appear after the process has started, until just before the process shifts from state S_0 state S_1, is

$$E_{8,i} = \frac{p_0\, Q_{0(i)}}{\pi_{01}} \quad (i = 0, 1, 2).$$

Lemma 9. The expected number of type 1 signals that appear immediately after the process has shifted from state S_1 to state S_2, until just before a type 2 signal occurs, is

$$E_9 = \frac{Q_{2(1)}}{Q_{2(2)}}.$$

Lemma 10. The expected number of items inspected immediately after the process has shifted from state S_1 to state S_2, until either a type 1 or type 2 signal appears, is

$$E_{10} = \frac{(Q_{2(1)} + Q_{2(2)}) L_4}{(1 - Q_{2(0)})^2} + \frac{L_5}{1 - Q_{2(0)}} = \frac{1}{p_2(1 - Q_{2(0)})}.$$

Lemma 11. The expected number of items inspected from the instant of time immediately after the process has shifted from state S_0 to state S_1, until it shifts to state S_2, is

$$E_{11} = \sum_{j=1}^{\infty} j \pi_{12} (1 - \pi_{12})^{j-1} = \frac{1}{\pi_{12}}.$$

6. ECONOMIC DESIGN

In reality, various factors such as availability of resources, operational convenience, loss due to process deterioration, and others, determine whether or not to perform maintenance on the process and how frequently should maintenance be performed. Generally speaking, inspection and maintenance keep the process in good shape and prevents unexpected increase of fraction of nonconforming items produced. Therefore, from a economic point of view, inspection and maintenance should be carried out if the benefit achieved is more than the loss due to the nonconforming items produced. In what follows, different costs incurred for the different maintenance models in Table 1 will be calculated.

The following three types of cost will be considered ([7], [8]): (1) the penalty cost due to the nonconforming items produced, (2) the cost spent in inspecting the process, and (3) the cost spent in carrying out maintenance work. Let

c_{nc} = the penalty cost incurred when a nonconforming item is produced,
$c_{inv,1}$ = cost of carrying out i_1 each time,
$c_{inv,2}$ = cost of carrying out i_2 each time ($c_{inv,1} \leq c_{inv,2}$),
$c_{m,1}$ = cost of carrying out m_1 each time,
$c_{m,2}$ = cost of carrying out m_2 each time ($c_{m,1} \leq c_{m,2}$).

Let $\bar{c}$ by the average total cost per item produced. For the maintenance models (I_{1+2},M_{1+2}) and (I_0,M_{1+2}), $\bar{c}$ will the sum of all the costs divided ANI_I in Proposition I. For model (I_2,M_{1+2}), $\bar{c}$ will the sum of all the expected costs divided ANI_{II} given in Proposition II. For models (I_2,M_2) and (I_0,M_2), $\bar{c}$ will the sum of all the expected costs divided ANI_{III} in Proposition III. As for model (I_0, M_0), since the process will change to state S_2 and remain there indefinitely, the cycle time is infinite and therefore

$$\bar{c} = \lim_{j \to \infty} \frac{p_0 E_1 c_{nc} + p_1 E_{11} c_{nc} + j p_2 c_{nc}}{E_1 + E_{11} + j} = p_2 c_{nc}.$$

For each maintenance models, $\bar{c}$ can be minimized with respect to n_1 and n_2. Then the minimum of $\bar{c}$ for the six maintenance models can be compared, and the most cost effective model with the minimum $\bar{c}$, say $\bar{c}_{min}$, can be selected. The numerical results in Table 8(a)–(e) show that for different values of p_0, p_1, p_2, π_{01}, π_{12}, c_{nc}, $c_{inv,1}$, $c_{inv,2}$, $c_{m,1}$ and $c_{m,2}$, the minimum average cost $\bar{c}_{min}$ can be attained under each of the five models (I_{1+2},M_{1+2}), (I_2,M_{1+2}), (I_0,M_{1+2}), (I_0,M_2) and (I_0,M_0).

As for the model (I_2, M_2), the fact is that given any set of values p_0, p_1, p_2, π_{01}, π_{12}, c_{nc}, $c_{inv,1}$, $c_{inv,2}$, $c_{m,1}$, $c_{m,2}$ and given any maintenance plan under this model, there always exists a less costly maintenance plan under model (I_2, M_{1+2}), provided that $c_{m,1} < c_{m,2}$. To see this, suppose that there is a maintenance plan under model (I_2, M_2), called Plan 2, in which $n_2 = n_0 > 1$. Consider a maintenance plan under model (I_2, M_{1+2}) with $n_1 = n_0$ and n_2 equal to any positive integer less than n_0, which will be called Plan 1. In Plan 2, m_2 will be carried out even though only m_1 is needed, but in Plan 1 both m_1 and m_2 are available and thus the unnecessary m_2 can be avoided. Since $c_{m,1} < c_{m,2}$, Plan 2 is more costly than Plan 1. This can be proved quantitatively by noting that $Z_1 \cup Z_2$ and $Q_{\ell(1)} + Q_{\ell(2)}$ of Plan 1 are identical to Z_2 and $Q_{\ell(2)}$ of Plan 2 ($\ell = 0, 1, 2$). Thus, $P(A_I) + P(B_I)$ and $P(C_I)$ for Plan 1 are identical to $P(B_{II})$ and $P(C_{II})$ for Plan 2, and so are the corresponding costs. Since $c_{m,1} < c_{m,2}$, maintenance cost for Plan 1 is less than that of Plan 2, and so is the total cost.

The argument in last paragraph is based on the assumption that $c_{m,2}$ is the same for both Plan 1 and Plan 2. However, in some situations, an additional cost Δ per maintenance task is required in order to maintain two maintenance procedures (minor and major), rather than just one (major). If Δ is to be absorbed in the maintenance cost, then the cost of carrying out m_2 each time under model (I_2, M_{1+2}) may be $c_{m,2} + \Delta$, which is larger than the cost $c_{m,2}$ required under model (I_2, M_2). In this case, it is possible that $\bar{c}_{min}$ under model (I_2, M_2) will be smaller than that under model (I_2, M_{1+2}), as illustrated in the numerical example in Table 2(f). This idea can be extended to the situation when additional cost is incurred in order to maintain two inspection procedures (minor and major), instead of just one (major), but the details will not be elaborated here.

It is therefore possible that $\bar{c}_{min}$ can be achieved by any of the six maintenance models (I_{1+2}, M_{1+2}), (I_2, M_{1+2}), (I_0, M_{1+2}), (I_2, M_2), (I_0, M_2) and (I_0, M_0). This is illustrated by the numerical examples in Table 2, in which the $\bar{c}_{min}$'s are calculated based on $(p_0, p_1, p_2, \pi_{01}, \pi_{12}) = (0.015, 0.019, 0.05, 0.0004, 0.0035)$. For each data set, the minimum average cost $\bar{c}_{min}$ among the six maintenance models is enclosed in brackets in Table 2.

Table 2. Numerical examples.

(a) $(c_{nc}, c_{inv,1}, c_{inv,2}, c_{m,1}, c_{m,2}) = (120, 3, 18, 11, 22)$.

	I_{1+2} M_{1+2}	I_2 M_{1+2}	I_0 M_{1+2}	I_2 M_2	I_0 M_2	I_0 M_0
$\bar{c}_{min} =$	(1.891)	1.908	1.913	1.910	1.913	6
$n_2 =$	7	–	11	13	12	–
$n_1 =$	49	13	12	–	–	–

(b) $(c_{nc}, c_{inv,1}, c_{inv,2}, c_{m,1}, c_{m,2}) = (120, 100, 108, 40, 220)$.

	I_{1+2} M_{1+2}	I_2 M_{1+2}	I_0 M_{1+2}	I_2 M_2	I_0 M_2	I_0 M_0
$\bar{c}_{min} =$	2.137	(2.121)	2.185	2.145	2.198	6
$n_2 =$	5	–	3	5	3	–
$n_1 =$	6	5	8	–	–	–

(c) $(c_{nc}, c_{inv,1}, c_{inv,2}, c_{m,1}, c_{m,2}) = (120, 50, 108, 11, 22)$.

	I_{1+2} M_{1+2}	I_2 M_{1+2}	I_0 M_{1+2}	I_2 M_2	I_0 M_2	I_0 M_0
$\bar{c}_{min} =$	2.076	2.071	(1.913)	2.072	1.913	6
$n_2 =$	5	–	11	5	12	–
$n_1 =$	6	5	12	–	–	–

(d) $(c_{nc}, c_{inv,1}, c_{inv,2}, c_{m,1}, c_{m,2}) = (120, 110, 120, 110, 120)$.

	I_{1+2} M_{1+2}	I_2 M_{1+2}	I_0 M_{1+2}	I_2 M_2	I_0 M_2	I_0 M_0
$\bar{c}_{min} =$	2.147	2.124	2.100	2.125	(2.081)	6
$n_2 =$	5	–	5	5	12	–
$n_1 =$	6	5	6	–	–	–

(e) $(c_{nc}, c_{inv,1}, c_{inv,2}, c_{m,1}, c_{m,2}) = (1, 55, 65, 60, 70)$.

	I_{1+2} M_{1+2}	I_2 M_{1+2}	I_0 M_{1+2}	I_2 M_2	I_0 M_2	I_0 M_0
$\bar{c}_{min} =$	1.127	0.741	0.805	0.745	0.545	(0.5)
$n_2 =$	2	–	1	1	1	–
$n_1 =$	3	1	2	–	–	–

(f) $(c_{nc}, c_{inv,1}, c_{inv,2}, c_{m,1}, c_{m,2}) = (120, 5, 10, 11, 22)$. Here $\Delta = 350$ is introduced.

	I_{1+2} M_{1+2}	I_2 M_{1+2}	I_0 M_{1+2}	I_2 M_2	I_0 M_2	I_0 M_0
$\bar{c}_{min} =$	2.0149	2.0151	2.428	(1.888)	1.913	6
$n_2 =$	15	–	3	12	12	–
$n_1 =$	19	17	4	–	–	–

7. DISCUSSION

This research is concerned with design of maintenance strategy, for the case when minor and major signals of deterioration may appear from the system, and minor inspection, major inspection, minor maintenance and major maintenance may be performed on the system. It is assumed that major inspection can better reveal the state of the system but is more expensive than minor inspection, and major maintenance is more thorough but also more expensive than minor maintenance. Adequate inspection and maintenance is necessary, in order to maintain the reliability of the system. However, excess inspection and maintenance will not only disturb the process unnecessarily and but will also introduce additional chance of system failure. From an operational or economic point of view, there is an optimal maintenance strategy. In the optimal maintenance strategy, either both minor and major inspection and maintenance will be performed, only major inspection or major maintenance will be performed, or none of minor or major inspection, minor or major maintenance will be performed. In this research, this concept is applied on a production process in which the number of nonconforming items produced follows a geometric distribution, and a statistical control chart called CCC-chart is used to provide signals that indicate deterioration of the process. Optimal choice of maintenance strategy is based on an economic consideration.

In this research, it is assumed that the state of the process is discrete, and there are two types of inspection and two types of maintenance. Classifying a process into discrete states and assuming change of state of the process follows a memoryless distribution have been justified and commonly used in the literature (see for example, [9], [10], [11], [12], [13], [14]). However, the rationale in this research can be applied to a process whose states is measured on a continuous scale, and with more than two types of inspection and maintenance. If transition of state of the process is time dependent, more complicated statistical modeling is necessary, and distributions with changing failure rate (such as Weibull or nonhomogeneous Poisson distributions) may be appropriate tools.

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