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Using 2×2 switching modules to build large 2-D MEMS optical switches

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Abstract—MEMS optical switch technology is one of the key technologies in Wavelength Division Multiplexing (WDM) optical networks. Although the 2-D MEMS optical switch technology is mature, the commonly used crossbar architecture is not amenable to building large switches. In this paper, we propose a design of 2×2 switching modules, and use it to build large 2-D MEMS optical switches with architectures such as Spanke-Beneš and Beneš networks.

I. INTRODUCTION

Wavelength Division Multiplexing (WDM) technology allows us to efficiently utilize the optical network's terabits per second transmission bandwidth by transmitting multiple low bit rate optical signals of different wavelengths in a single optical fiber. This allows us to use electronic devices to process the optical signals and to gradually increase the investment of equipment according to the demand of transmission bandwidth. The cost of electronic processing at intermediate nodes, however, becomes significant when the number of optical signals is large, and/or the bit rate of each optical signal is high. It is desirable to route the optical signals without converting to electronic signals until they arrive at the destination nodes. High volume optical-layer cross connects (OXC) that switch the signal in the optical domain are therefore the key devices for WDM optical networks. Among the reported technologies for building OXC, micro-electro-mechanical systems (MEMS) optical switches have attracted much attention because of their potential of building large OXC [1]–[3].

MEMS optical switches use mirrors to modify the routing paths of the optical signals inside the switches. We can classify MEMS optical switches into two types: those with mirrors which can rotate and stop at multiple positions, and those with mirrors which can only have binary positions. The first kind of MEMS optical switches have smaller optical signal loss when the number of input/output ports N is large, *e.g.*, $N > 32$. In

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general they require only $2N$ mirrors. The number of addressable positions of each mirror, however, is equal to $\mathcal{O}(N)$. The movement precision problem becomes significant if N is large. The multiple position requirement also dramatically increases the system complexity. Reliability, stability, and cost are main concerns [2].

The second kind of MEMS optical switches can use straightforward digital control circuits to drive the movements of mirrors. The mirrors can be fabricated on a single silicon substrate. The optical signal propagation is parallel to the surface that integrates the mirrors. These switches are often called 2-D (two-dimensional) MEMS optical switches. The switching time of 2-D MEMS switches (the required time for establishing a connection between input/output ports) can be within one millisecond because of the usage of simple control algorithms. Currently, 2-D MEMS optical switches use a crossbar architecture that has the advantages of strictly non-blocking and easy fabrication of small size switches. However, crossbars require N^2 mirrors for an $N \times N$ switch. We also have to increase the mirror radius with the switch size to compensate for the divergence of optical signal in free space propagation [5], [6]. Otherwise, the optical signals will encounter significant loss. In general, 2-D MEMS single stage switches larger than 32 ports are seldom used [2], [6]. Although multistage 2-D MEMS optical switches have been proposed [7], the shortcomings are the high optical signal loss and the high interconnection cost between stages [2], [6]. Hence, architectures other than crossbars are desired for 2-D MEMS optical switches.

Apart from using fewer mirrors, the appropriate architectures for 2-D MEMS switches should be able to minimize the optical signal losses including the loss caused by beam divergence. Beam divergence grows with the propagation distance of the optical signals in free space. We can reduce such optical signal loss by dividing the optical signals routing paths into fixed length segments, and refocusing the optical signals on each segment. We can therefore build 2-D MEMS optical switch architectures with long optical signal paths. In this paper, we describe a MEMS 2×2 optical switching module that is suitable for this purpose. We propose to use it to build large 2-D MEMS optical switches. In Section II, we review the model

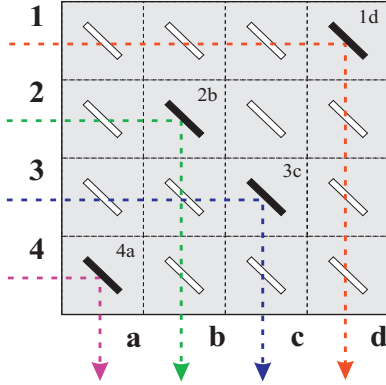


Fig. 1. A 4×4 2-D MEMS optical switch with crossbar architecture. The mirrors $1d$, $2b$, $3c$, and $4a$ are in ON state. Optical signals from input ports 1, 2, 3, and 4 are switched to output ports d , b , c , and a , respectively.

of optical signal losses in 2-D MEMS optical switches with the crossbar, Spanke-Beneš and Beneš architectures. We then discuss the proposed MEMS 2×2 optical switching module in Section III. Some performance evaluations of using the 2×2 switching module are discussed in Section IV. Finally, we conclude in Section V.

II. OPTICAL SIGNAL LOSSES IN 2-D MEMS OPTICAL SWITCHES

A. 2-D MEMS crossbar optical switches

Crossbar switches are strictly non-blocking in the sense that any input to an unused output connection can always be made regardless of the connection status of the switches. The number of required mirrors is N^2 for a 2-D MEMS $N \times N$ crossbar switch. Figure 1 illustrates the operation of a 2-D MEMS 4×4 optical switch with the crossbar architecture. Input ports and output ports are labeled from 1 to 4, and a to d , respectively. We label the mirrors according to their row-column position. Each mirror can either be in ON or OFF state. A mirror xy in ON state reflects the optical signal from input port x to the output port y . The mirrors in OFF state have no effect on the switching of optical signals. Only one mirror should be in ON state at each row or column. The mirrors $1d$, $2b$, $3c$, and $4a$ in the Fig. 1 are in ON state. Optical signals from input ports 1, 2, 3, and 4 are switched to output ports d , b , c , and a , respectively. The path length of a connection ($x \rightarrow y$) from an input port x to an output port y can vary from 1 to $2N - 1$ inter-mirror distance units (itches) in a 2-D MEMS $N \times N$ crossbar optical switch. In Fig. 1, the path lengths of connections ($1 \rightarrow d$), ($2 \rightarrow b$), ($3 \rightarrow c$), and ($4 \rightarrow a$) are 7, 4, 4, and 1, respectively.

When an optical signal passes through a MEMS optical switch, it encounters optical power loss because of Gaussian-beam divergence, air absorption, mirror angular misalignment, imperfection of mirror reflection, and the coupling loss between the fibers and the switch [2], [5]. The last two components of the loss are invariant in 2-D MEMS crossbar optical switches

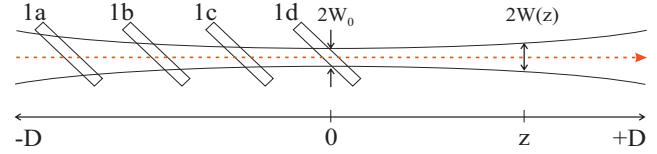


Fig. 2. The Gaussian-beam model for the optical signal from input 1 of the switch in Fig. 1. Mirrors $1a$ to $1d$ are in OFF state. The optical signal is collimated and focused onto the mirror $1d$. $W(z)$ is the Gaussian-beam radius of the optical signal at distance z .

with different sizes. The first three components of loss are related to the optical signal path length. Gaussian-beam divergence becomes the dominant factor when the switch size is large.

Figure 2 shows the Gaussian-beam model of the optical signal from input port 1. For illustration purpose, we assume that mirrors $1a$ to $1d$ are in OFF state. The distance between the input port 1 to mirror $1d$ is D . The coordinate '0' of the propagation distance is set at the location of mirror $1d$. $W(z)$ is called the *Gaussian beam radius* and represents the radius at which the optical signal intensity has dropped to $e^{-2} \approx 0.135$ of its maximum at distance z [8]. In Fig. 2, we assume that *mode matching* has been applied, *i.e.*, the optical signal is collimated and focused onto the midpoint of the optical path to minimize the diffraction [2]. Otherwise, the minimum $W(z)$ of the optical signal will be at the location ' $-D$ ' instead of the location '0'. The optical signal loss due to Gaussian-beam divergence for a mirror of radius R at distance z is

$$L_{Gauss} = \exp\left(-\frac{2R^2}{W(z)^2}\right), \quad (1)$$

We can have $L_{Gauss} < 1.1\%$ using a mirror with radius $R = 1.5W(z)$. If the mirror reflectivity $\eta = 98\%$, the loss due to the mirror is about 3%. $W(z)$ can be calculated from

$$W(z) = W_0 \left[1 + \left(\frac{z}{z_0}\right)^2\right]^{\frac{1}{2}}, \quad (2)$$

where $W_0 = W(0)$ is called the *waist radius*, which is the minimum value of $W(z)$. z_0 is called the *Rayleigh range*, which is the distance z for $W(z) = \sqrt{2}W_0$. W_0 and z_0 are related as

$$W_0 = \left(\frac{\lambda z_0}{\pi}\right)^{\frac{1}{2}},$$

where λ is the wavelength of the optical signal. We must minimize $W(D)$ (or W_0) to reduce the size of the mirrors. With mode matching, we have the minimum value of $W(D)$ that is equal to $W_0\sqrt{2}$ or $(2\lambda D/\pi)^{1/2}$ by setting $z_0 = D$. Otherwise, $W(D)$ can be much larger because inappropriate values may be assigned to W_0 and z_0 . Even if mode matching has been used, the value of D is determined by the fabrication limit apart from the requirement of W_0 . Therefore, Lin *et al.* have used the approximation of $3R + 800\mu\text{m}$ for a pitch between mirrors [5].

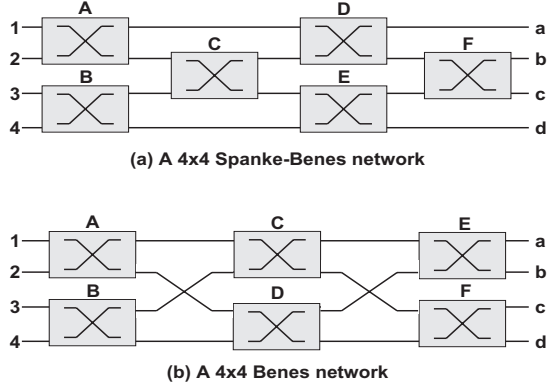


Fig. 3. Two rearrangeably non-blocking switch architectures. (a) A 4x4 Spanke-Beneš network. (b) A 4x4 Beneš network.

In this case, the minimum value of $W(D)$ is around $156 \mu\text{m}$ for a 2-D MEMS 32×32 crossbar optical switch. This causes difficulties in both control and fabrication because large mirrors must be used.

Another problem of the 2-D MEMS crossbar optical switches is that the switch dimension increases rapidly with the number of ports. D is proportional to $W(D)$ if large size mirrors are used. We define $D = N\gamma W(D)$ where γ is a ratio constant. From Eq. (2), we can have

$$W(D) = N\gamma\lambda/2\pi,$$

and

$$D = N^2\gamma^2\lambda/2\pi.$$

This implies that we should double the radius of the mirrors and increase the switch dimension four-fold if we double the number of switch input/output ports. Otherwise, the optical signal loss due to Gaussian-beam divergence increases significantly. Hence, it is difficult to have large 2-D MEMS optical switches with the crossbar architecture. To improve the performance, modified crossbars have been proposed. Gloeckner *et al.* have proposed to construct a large crossbar from several smaller ones that are linked together by recollimating lenses [9]. Yeow *et al.* have proposed to use an L-shape matrix of mirrors to reduce the optical power loss [10]. As the structures of the modified crossbars are not simple anymore, other architectures may be more attractive.

B. 2-D MEMS optical switches using 2×2 switching modules

Many architectures have been proposed for building large switches with 2×2 switching modules [11]. Figure 3 shows two 4×4 switches with different architectures: Spanke-Beneš network [12] and Beneš network [13]. The wires in the figure represent the possible connections between the 2×2 switching modules. For ease of illustration, we also label the inputs and outputs of the switches from 1 to 4, and a to d , respectively. The 2×2 switching modules are labeled from A to F. Spanke-Beneš networks have straightforward wiring between the 2×2

switching modules and are relatively easy to implement. On the other hand, Beneš networks require a minimum number of 2×2 switching modules.

Both Spanke-Beneš and Beneš networks are rearrangeably non-blocking, *i.e.*, we may be required to re-arrange the paths of existing connections in order to accept new connections. Rearrangeably non-blocking switches are not preferred in traditional telecommunication applications because adding a new connection may disturb the existing connections. It may not be easy to re-arrange the paths of existing connections within the connection setup time (typically seconds) if the switch size is large, *e.g.*, $N > 10^4$ for general telephone systems. In WDM optical networks, the setup of connections is based on the demands of multiple gigabits per second bandwidth. The duration of a wavelength connection is relatively long compared to telephone connections. Since we can tolerate a longer connection setup time, it is possible to minimize the disturbance to existing connections even if rearrangeably non-blocking switches are used. We may use strictly non-blocking architectures but it requires more switching elements [11].

Most architectures with 2×2 switching modules require fewer switching elements in contrast to the crossbars. The number of required 2×2 switching modules in Spanke-Beneš and Beneš networks are $N(N-1)/2$ and $N \log_2 N - N/2$, respectively, for 2-D MEMS $N \times N$ optical switches. Since the function of a 2×2 switching module is to interchange or pass through the two input optical signals, one movable mirror is sufficient in each module. Hence, the required number of movable mirrors in the 2-D MEMS 4×4 optical switches with the crossbar, Spanke-Beneš and Beneš architectures are 16, 6 and 6, respectively.

The number of required switching elements is not the only factor for consideration in the switches design. When architectures other than crossbars are used, an optical signal may encounter multiple reflections before going to the desired output port. The loss caused from mirror angular misalignment and mirror imperfect reflection increases with the number of reflections. The maximum loss due to the multiple reflections is given by

$$L_{refls} = K_{stages} \times K_{sw} \times \sigma, \quad (3)$$

where K_{stages} is the number of stages of 2×2 switching modules in the switches and K_{sw} is the maximum number of reflections in each switching module. σ is the loss due to a mirror reflection. K_{stages} for Spanke-Beneš and Beneš networks are N and $2 \log_2 N - 1$ stages, respectively. In general, $\sigma < 3\%$ for commercially available gold-coated mirrors. It can be largely reduced with improved fabrication process [2]. We only consider L_{refls} if K_{stages} is really large.

A factor that we must consider is the path lengths of the optical signals inside the switches because the Gaussian-beam divergence of optical signals depends on the path length. The maximum path length depends on the number of stages K_{stages} as well as the optical signal propagation distance between the stages of 2×2 switching modules. Although a Beneš net-

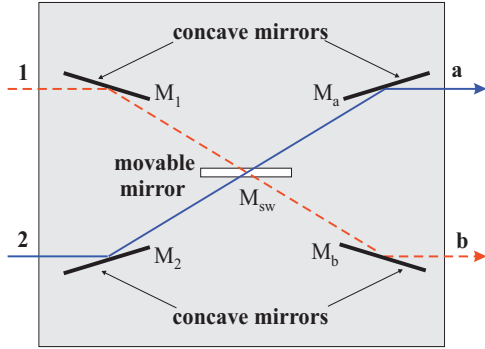


Fig. 4. The proposed 2×2 switching module in cross state. The two optical signals from inputs 1 and 2 will be switched to the outputs b and a , respectively, if the movable switch is in OFF state. Otherwise, optical signals from inputs 1 and 2 are switched to outputs a and b , respectively.

work has smaller K_{stages} , its maximum path length may not be smaller than that of a Spanke-Beneš network because of the longer inter-module optical signal propagation distance. In general, the approximated longest path length for Spanke-Beneš and Beneš networks are $N(1+D_{sw})$ and $N+K_{stages} \times D_{sw}-2$ inter-module distance units, respectively, where D_{sw} is the optical path length inside each 2×2 switching module. Recall that the maximum path length of a crossbar is $2N-1$ pitches. Intuitively, we cannot have significant performance improvement with these architectures. However, the following feature of the architecture allows us to build large 2-D MEMS optical switches regardless of the large path length.

We observe that the optical signal paths are divided by the 2×2 switching modules into a set of short segments even if the path length is long. For example, the connection of $(3 \rightarrow c)$ in Fig. 3(a) may use path $3 \rightarrow B \rightarrow C \rightarrow E \rightarrow F \rightarrow c$, or path $3 \rightarrow B \rightarrow E \rightarrow F \rightarrow c$. The wiring between the 2×2 switching modules in neighboring stages are fixed. The Gaussian-beam divergence problem can be solved if we can refocus the optical signals in each segment. Let D_{path} be the maximum path length of the optical signals and D_{seg} be the maximum optical signal propagation distance between two 2×2 switching modules in any pair of neighboring stages. The mirror sizes then depend on D_{seg} instead of D_{path} . This is useful for reducing the loss due to Gaussian-beam divergence. Traditionally, MEMS optical switches use flat mirrors to modify the routing paths and use lenses to collimate the optical signals. For reliability and performance consideration, we have to minimize the required mirrors and lenses in the design of the 2×2 switching modules.

III. THE PROPOSED 2×2 SWITCHING MODULE

Figure 4 shows our design of 2×2 switching modules. There are two input mirrors and two output mirrors that are labeled as M_1 , M_2 and M_a , M_b , respectively. In the center of the 2×2 switching module, there is a binary position movable mirror M_{sw} . The two optical signals from inputs 1 and 2 will be

switched to the outputs b and a if the movable mirror M_{sw} is in OFF state. Otherwise, optical signals from inputs 1 and 2 are switched to outputs a and b . The ON/OFF status of the movable mirror M_{sw} will not affect the beam sizes of the optical signals as we use a flat mirror for M_{sw} . No path length differences to the two input optical signals should be caused by the ON/OFF status of the movable mirror M_{sw} . Hence, the distances between the mirror pairs (M_1, M_a) , (M_2, M_b) , (M_1, M_2) , and (M_a, M_b) are ℓ_w , ℓ_w , ℓ_h and ℓ_h , respectively. In general, we would like to set both ℓ_w and ℓ_h as small as possible to minimize the Gaussian-beam divergence of optical signals inside the switching modules. For a better tolerance to the angular misalignment of the mirrors, ℓ_w should be smaller than ℓ_h .

Mirrors M_1 , M_2 , M_a , and M_b are fixed position concave mirrors whose parameters are determined by the architecture and/or the size of the 2-D MEMS optical switches. The output and input mirrors of a pair of connected 2×2 switching modules in neighboring stages must be matched so that the optical signals can be appropriately refocused. For Spanke-Beneš networks, apart from the 2×2 switching modules at the top and bottom rows, all have the same set of parameters because of the straightforward wiring between modules in neighboring stages [12]. For Beneš networks, the wiring between the modules in neighboring stages change with the location of the stages and the switch size [13]. There are multiple sets of parameters for the 2×2 switching modules. We can compute the desired parameters of each mirror in each 2×2 switching module with the help of lens/mirror design software for specified architecture and switch size [14]. This may result in tailor-made mirrors that are difficult for fabrication with MEMS technology. Therefore, simple spherical mirrors would be used instead. This may cause additional power loss to the optical signals but can significantly reduce the fabrication cost. In this paper, we assume that all desired concave mirrors are available. We also discuss the worst cases in Section IV if flat mirrors are used for M_1 , M_2 , M_a , and M_b .

In general, the number of fixed position concave mirrors in a switch is four times the number of 2×2 switching modules. In some switches, like switches with architecture of Spanke-Beneš networks, we may use fewer mirrors because the input output mirrors of the two 2×2 switching modules in neighboring stages can be combined. This complicates the design procedure of the mirrors but can significantly reduce the production cost. For some architectures such as Beneš networks, however, we may add more mirrors to further divide a long segment so that small mirrors can be used.

IV. PERFORMANCE EVALUATION

In this section, we consider the reduction in loss due to Gaussian-beam divergence with the proposed 2×2 switching modules. With mode matching as described in Section II-A, the maximum optical signal loss due to Gaussian-beam divergence is determined by the ratio of the mirror radius and the waist radius W_0 [2]. Hence, we fix the ratio to 1.5 and compare

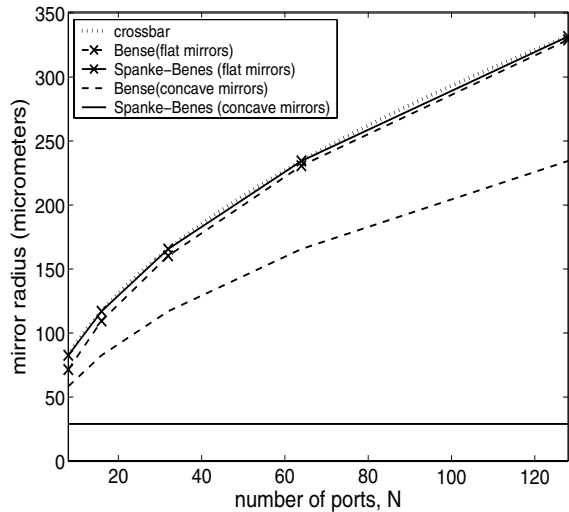


Fig. 5. The required radius of mirrors for 2-D MEMS optical switches with the crossbar, Beneš, and Spanke-Beneš architectures.

the required mirror radii of the switches with different architectures. Those architectures that require smaller mirrors have a larger reduction in loss due to Gaussian-beam divergence. For a fair comparison, we assume that mode matching has been used for switches that use flat mirrors even if they are Spanke-Beneš and Beneš networks. The pitch size between mirrors is set to $3R + 800\mu\text{m}$ for the crossbars, where R is the mirror radius. For the switches using the proposed 2×2 switching modules, the inter-module distance unit is set to $1600\mu\text{m}$ which is nearly twice the pitch size used in crossbar switches. Both ℓ_h and ℓ_w are set to $3R + 800\mu\text{m}$. Figure 5 shows the required mirror radii in micrometers for the MEMS switches with architectures of crossbars, Beneš, and Spanke-Beneš are plotted with the dotted, dashed, and solid lines, respectively. Those mirror radii of switches using both the 2×2 switching modules and flat mirrors are marked with cross marks in the figure. From Fig. 5, we know that the mirror radius of the crossbar switches is the largest, and grows rapidly with the switch size. The mirror radii of those switches using both 2×2 switching modules and flat mirrors are close to that of the crossbar switches. If tailor-made concave mirrors are available, switches with Spanke-Beneš architecture have the smallest mirror radius that is independent of the switches size. The mirror radius of Beneš networks is in between crossbar switches and switches with Spanke-Beneš networks.

This demonstrates that architectures using the proposed 2×2 switching modules with tailor-made concave mirrors can indeed reduce the required mirror radius, or the optical signal loss due to Gaussian-beam divergence. We can build large switches with the proposed 2×2 switching modules if tailor-made concave mirrors are available. Even if flat mirrors are used in the 2×2 switching modules, the maximum loss due to Gaussian-beam divergence is close to that of crossbars. Although Beneš networks require larger mirrors than that of Spanke-Beneš net-

works, the former may be preferred in large switches because they require fewest mirrors and reflections. When the number of ports is large, the loss due to reflections must be considered.

V. CONCLUSION

2-D MEMS optical switch technology is one of the key technologies in WDM optical networks. However, crossbars switches require a rather large number of switching elements in comparison to other switch architectures, *i.e.*, N^2 mirrors for an 2-D MEMS $N \times N$ optical switch. Crossbars also suffer from Gaussian-beam divergence that plays the major role in limiting the switch size. Gaussian-beam divergence grows with the path length of the optical signals. We observe that the 2×2 switching modules used in architectures such as Spanke-Beneš and Beneš networks divide the optical signal routing paths into fixed length segments. Gaussian-beam divergence can be reduced if we can refocus the optical signals in each segment. We have proposed a MEMS 2×2 optical switching module and use it to build large 2-D MEMS optical switches with the Spanke-Beneš and Beneš architectures. A large reduction in Gaussian-beam divergence can be obtained if tailor-made concave mirrors are available for the proposed 2×2 switching modules.

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