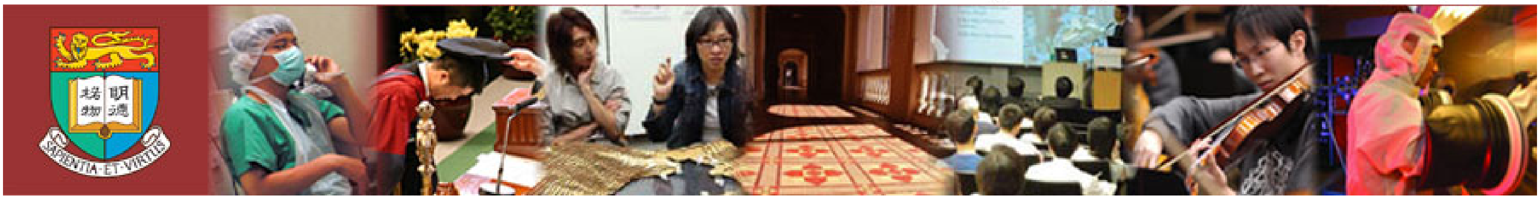

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Author(s)	Yao, Y; Ng, TS
Citation	Ieee Vehicular Technology Conference, 2003, v. 58 n. 1, p. 438-442
Issued Date	2003
URL	http://hdl.handle.net/10722/46446
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Correlation-Based Frequency offset Estimation in MIMO system

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***Abstract*—This paper addresses the problem of frequency offset estimation in the presence of unknown channel information for MIMO system. An efficient pilot symbol-aided algorithm for frequency offsets estimation is proposed. In the paper, taking advantage of orthogonal property of Walsh codes, a second-order estimator, which can substantially mitigate the MAI (Multi-Antenna Interference), is proposed. We also derive the Cramer-Rao Bound for MIMO system, whose numerical results are compared with the simulation results to illustrate the efficiency of the novel algorithm.**

Keywords-MIMO, Frequency Offset, MAI, Correlation

I. INTRODUCTION

In recent years, MIMO (multi-input & multi-output) communication system has become attractive due to its high capacity and spectral efficiency in rich scattering environment. In MIMO system, with multi-element antenna arrays at both transmitter and receiver, independent data streams share the same frequency bands and time slots and thereby to increase the spectral efficiency enormously [1]-[5]. To achieve these advantages, estimation of frequency offsets plays an important role in MIMO system. This is because in wireless communications, the existence of frequency offset due to the mismatch between the transmit and receive oscillators and Doppler effect will directly influence the received signal. In addition, it is much more complicated to estimate frequency offsets in a MIMO system, as the existing algorithms for the SISO (Single-Input & Single-Output) system cannot be applied directly to a MIMO system.

In MIMO systems, the received signal at each receiver is the summation of all the transmitted signals, which makes it difficult to extract the frequency offsets. Therefore, a set of pilot symbols are sent periodically for frequency offset estimation. So far, some data-aided techniques have been investigated for carrier frequency recovery in MIMO systems [3]-[5]. In [3]-[4], the authors assume that only single frequency offset exist between transmit and receive antennas, as delay due to electronics associated with each antenna element varies and is difficult to control, this assumption, in most cases, is unrealistic in practice. Hence, in this paper, without requirement for the channel information, a more general model [5] where multiple frequency offsets exist between the transmit and receive antennas is considered.

The paper is organized as follows. In Section II, system and signal model of MIMO are provided. Section III presents the description of the proposed correlation-based algorithm and the CRB (Cramer-Rao Bound) for MIMO system in flat fading channels. The performance of the proposed algorithm is investigated by computer simulation and compared with the CRB in Section IV. Finally, some conclusions are drawn in Section V.

II. SYSTEM AND SIGNAL MODEL

Fig.1 shows the high-level block diagram of the MIMO system. Here we use $M \times N$ to signify the configuration of the system with M transmitters and N receivers. The data stream is first divided into M substreams, each substream is processed into block format by periodically inserting pilot symbols, then the substreams pass through M transmit antennas and are sent through rich scattering channels. The flat Rayleigh fading channel is assumed in the paper. At the receiver, each receiver collects the signals radiated from all M transmit antennas. Here we assume that the channel is stationary over every block of data, while it changes from block to block.

With perfect timing synchronization, the equivalent baseband received signal at the n^{th} receiver sampled at the time for the l^{th} pilot symbol in the b^{th} block is written as

$$r_{n,b}(l) = \sum_{m=1}^{M} a_m(l) e^{j2\pi f_{m,n} lT} h_{m,n,b} + \eta_{n,b}(l) \quad l = 1, \cdots, L \qquad (1)$$

where

$a_m(l)$ l^{th} pilot symbol sent by the m^{th} transmitter, $a_m(l) \in \{+1, -1\}$;

L length of the pilot sequence;

$f_{m,n}$ frequency offset from the m^{th} transmitter to the n^{th} receiver, which is slow varying compared with the observation duration;

T symbol period;

*This work was supported by the Hong Kong Research Grants Council and The University of Hong Kong Research Committee)

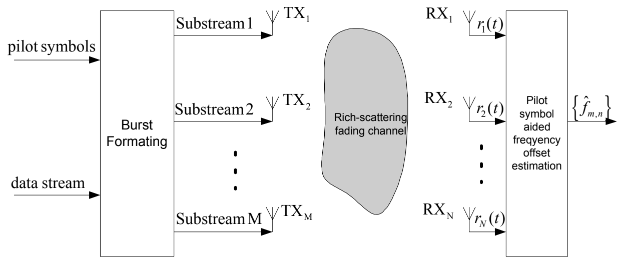

Figure 1. Equivalent baseband block diagram for synchronization of MIMO system

$\{h_{m,n,b}\}$ sampled channel gains, which is an independently identically distributed (i.i.d) complex Gaussian variable with zero mean and unit variance;

$\eta_{n,b}(l)$ complex Gaussian noise with zero mean and variance σ^2, which is statistically independent among receiver antennas and satisfies

$$E\left\{\eta_{n,b}(l)(\eta_{n',b'}(l'))^H\right\} = \sigma^2\delta(n-n')\delta(l-l')\delta(b-b') \tag{2}$$

where $\delta(\cdot)$ is the Kronecker delta.

In this paper, Walsh codes are employed as pilot sequences. In a $M \times N$ MIMO system, the first M rows in the Walsh-Hadamard matrix are assigned to the M individual transmit antennas as pilot sequences. Let Q denote the minimal period of the selected pilot sequences, its orthogonal property is written as

$$\sum_{l=1}^{Q} a_m(l)a_{m'}(l) = \begin{cases} Q & m = m' \\ 0 & otherwise \end{cases} \tag{3}$$

where $Q = 2^{\lceil \log_2 M \rceil}$, $\lceil \log_2 M \rceil$ takes the smallest integer no less than $\log_2 M$.

III. CORRELATION-BASED ESTIMATOR AND CRB

In a MIMO system, the received signal at each receive antenna collects signal from all transmit antennas. When estimating the frequency offset of one transmitter/receiver pair, the MAI needs to be eliminated in order to achieve good performance.

A. Correlation-based Estimator

It is evident that the frequency offset estimation for a MIMO system can be regarded as N independent MISO (Multi-Input & Single Output) system estimation problems, as the received signal of any given receive antenna n only contains the information of a set of M frequency offsets $\{f_{m,n}\}$, where $m = 1, 2..., M$. Therefore, in the rest of the paper, without loss of generality, we shall only consider the 1^{st} MISO sub-system, i.e. $n = 1$.

From (1), the received signal can be separated into three parts as

$$r_{1,b}(l) = h_{m,1,b}a_m(l)e^{j2\pi f_{m,n}lT} + \sum_{\substack{m'=1 \\ m'\neq m}}^{M} h_{m',1,b}a_{m'}(l)e^{j2\pi f_{m',n}lT} + \eta_{1,b}(l) \tag{4}$$

where the first term is the desired signal from the m^{th} transmitter, the second term is MAI and the third term is noise at receiver 1. It is apparent that to accurately estimate the frequency offset associated with the m^{th} transmitter, MAI needs to be eliminated or at least greatly reduced. This can be achieved by taking advantage of the orthogonal property of Walsh codes with the help of correlators.

First, the received signal $r_{1,b}$ is fed into M correlators, each corresponding to one transmitter. The correlator length P is chosen such that it is at least one minimum period of the pilot sequence, but is a fraction of the pilot length L. The reason for such choice will become apparent later. In the correlator for the m^{th} transmitter, the received signal is first multiplied by the pilot sequence of the m^{th} transmitter $a_m(l)$, and summed over the length of the correlator. The correlation outputs can be written as

$$\begin{aligned} Ra_{m,1,b}(k) &= \sum_{p=1}^{P} a_m(kP+p)r_{1,b}(kP+p) \\ &= h_{m,1,b}\sum_{p=1}^{P} a_m^2(kP+p)e^{j2\pi f_{m,n}(kP+p)T} \\ &+ \sum_{\substack{m'=1 \\ m'\neq m}}^{M}\sum_{p=1}^{P} h_{m',1,b}a_{m'}(kP+p)a_m(kP+p)e^{j2\pi f_{m',1}(kP+p)T} \\ &+ \sum_{p=1}^{P}\eta_{1,b}(kP+p)a_m(kP+p) \\ &= E_{m,1,b}(k) + I_{m,1,b}(k) + N_{m,1,b}(k) \end{aligned} \tag{5}$$

where $k = 0, 1, ...\lfloor L/P \rfloor - 1$, $\lfloor L/P \rfloor$ takes the integer part of L/P. With P at least one minimum period of the pilot sequence, the orthogonal property of Walsh codes is maintained, i.e. $a_m(kP+p) = a_m(p)$.

The correlator output consists of three distinct components: the desired component associated with the m^{th} transmitter $E_{m,1,b}(k)$, the MAI $I_{m,1,b}(k)$, and the background noise component $N_{m,1,b}(k)$. The desired component can be written as

$$E_{m,1,b}(k) = h_{m,1,b}A_{m,1}e^{j2\pi f_{m,1}kPT} \tag{6}$$

where $A_{m,1} = \dfrac{\sin(\pi f_{m,1}TP)e^{j\pi f_{m,1}(P+1)T}}{\sin(\pi f_{m,1}T)}$, It is apparent that $A_{m,1}$ is a complex constant independent of k and b. Therefore, the information of the desired frequency offset can be extracted by exploiting the correlator outputs, which will be shown later.

The MAI term $I_{m,1,b}(k)$ is given by

$$I_{m,1,b}(k)=\sum_{\substack{m'=1\\ m'\neq m}}^{M} h_{m',1,b}\Phi_{m',m}(f_{m',1}T,P)e^{j2\pi f_{m',1}kPT} \quad (7)$$

where $\Phi_{m',m}(f_{m',1}T,P)$ is the spectral cross correlation function defined as

$$\Phi_{m,m'}(f_{m',1}T,P)=\sum_{p=1}^{P} a_m(p)a_{m'}(p)e^{j2\pi f_{m',1}pT} \quad (8)$$

As shown in [6], when the frequency offsets are within a given small range, the MAI is small and can be approximated as a Gaussian variable. Following the discussion in [6], the MAI term $I_{1,m,b}(k)$ can be approximated as a zero-mean Gaussian variable when $|f_{m,n}T|\le 0.05$ in the case of $M=4$.

The term $N_{m,1,b}(k)$ is the AWGN noise component

$$N_{m,1,b}(k)=\sum_{p=1}^{P}\eta_{1,b}(kP+p)a_m(p) \quad (9)$$

Note that the phase of the desired component $E_{m,1,b}(k)$ is the summation of phase introduced by the channel $h_{m,1,b}$, the phase of $A_{m,1}$ and the contribution of the desired frequency offset $e^{j2\pi f_{m,1}kPT}$. To estimate the frequency offset $f_{m,1}$, we need to mitigate the effect of the channel phase and the phase of $A_{1,m}$. Consider a second-order i-lag estimator,

$$C_{m,1,b}(i)=E\left\{Ra_{m,1,b}(k)Ra^{*}_{m,1,b}(k-i)\right\} \quad (10)$$

where $E\{\cdot\}$ is the expectation operation. Furthermore, by Gaussian assumption of the last 2 items in (5), (10) can be rewritten as

$$C_{m,1,b}(i)=\left|h_{m,1,b}A_{m,1}\right|^2 e^{j2\pi i f_{m,1}PT} \qquad i\ge 1 \quad (11)$$

To ensure maximum synchronization range of the frequency offset estimation, i is set to 1 in this paper. The effect of P on frequency offset estimation will be studied later through simulation.

In practice, the computation of $C_{m,1,b}(1)$ is performed by averaging $Ra_{m,1,b}(k)Ra^{*}_{m,1,b}(k-1)$ over the length of pilot symbols L in the block. Clearly the choice of L and P reflects this practical needs. The estimated $C_{m,1,b}(1)$ is

$$\hat{C}_{m,1,b}(1)=\frac{1}{\lfloor L/P\rfloor-1}\sum_{k=1}^{\lfloor L/P\rfloor-1} Ra_{m,1,b}(k)Ra^{*}_{m,1,b}(k-1) \quad (12)$$

Then the estimation of the desired frequency offset $f_{m,1}$ can be obtained by extracting the argument of $\hat{C}_{m,1,b}$, i.e.

$$\hat{f}_{m,1}=\frac{1}{2\pi PT}\arg\left(\hat{C}_{m,1,b}(1)\right) \quad (13)$$

In the situation that frequency offset variation from block to block remains small, the estimated frequency offset accuracy $f_{m,1}$ can be improved by averaging $\hat{C}_{m,1,b}(1)$ over the observed recent B blocks of data, i.e. the improved estimation of $f_{m,1}$ is obtained by

$$\hat{f}_{m,1}=\frac{1}{2\pi PT}\arg\left(\sum_{b=1}^{B}\hat{C}_{m,1,b}(1)\right) \quad (14)$$

By varying m from 1 to M, all the frequency offsets corresponding to the 1^{st} receiver can be obtained.

B. Cramer-Rao Bound (CRB) for MIMO System

Before the derivation of the CRB, here some notations for matrix are list first. $(\cdot)^T,(\cdot)^{-1},(\cdot)^*$ denote transpose, inverse, conjugate transpose respectively, $\bar{\mathbf{A}},\tilde{\mathbf{A}}$ denote real part and imaginary part of $\mathbf{A}$.

In this section, we will derive the deterministic CRB on the covariance matrix of unbiased estimator of $f_{m,n}$. Then, the performance of the proposed algorithm will be compared with that of CRB in next section.

As the estimation of the $f_{m,n}$ is only concerning with the received signal of the n^{th} receive antenna, we will first only derived the CRB for the frequency offset estimation related to the n^{th} receive antenna with B blocks. For convenience, we rewrite the received signal (1) in the following matrix model

$$\mathbf{r}_{n,b}=\mathbf{A}_n(\boldsymbol{\theta}_n)\mathbf{h}_{n,b}+\boldsymbol{\eta}_{n,b} \quad (15)$$

where $\mathbf{r}_{n,b}\in\mathbf{C}^{L\times 1}$ is the received data vector, $\mathbf{h}_{n,b}\in\mathbf{C}^{M\times 1}$ is the channel transfer vector, which is defined as $\mathbf{h}_{n,b}=[h_{1,n,b},h_{2,n,b},\ldots,h_{M,n,b}]^T$, $\boldsymbol{\eta}_{n,b}\in\mathbf{C}^{L\times 1}$ is the Gaussian noise vector, and the matrix $\mathbf{A}_n(\boldsymbol{\theta}_n)\in\mathbf{C}^{L\times M}$ has following structure,

$$\mathbf{A}_n(\boldsymbol{\theta}_n)=\left[\mathbf{a}_1(f_{1,n})\cdots\mathbf{a}_M(f_{M,n})\right] \quad (16)$$

$\boldsymbol{\theta}_n=\left[f_{1,n},\cdots,f_{M,n}\right]^T$, and $\mathbf{a}_m(f_{m,n})\in\mathbf{C}^{L\times 1}$, which is defined as

$$\mathbf{a}_m(f_{m,n})=\left[a_m(1)e^{j2\pi f_{m,n}T},\cdots,a_m(L)e^{j2\pi f_{m,n}LT}\right]^T \quad (17)$$

where $a_m(l)$ is the l^{th} pilot symbol of m^{th} transmitter.

With the assumption of complex Gaussian noise with power σ^2, the n^{th} log likelihood function is obtained as

$$\ln(LK_n)=const-LB\ln(\sigma^2)-\frac{1}{\sigma^2}\sum_{b=1}^{B}\left[\mathbf{r}^{*}_{n,b}-\mathbf{h}^{*}_{n,b}\mathbf{A}^{*}_{n}\right]\left[\mathbf{r}_{n,b}-\mathbf{A}_{n}\mathbf{h}_{n,b}\right] \quad (18)$$

then the CRB covariance matrix can be obtained by

$$\Omega = (E\psi\psi^T)^{-1} \quad (19)$$

where

$$\psi^T \equiv \partial \ln LK_n \Big/ \partial \left[\sigma^2 \quad \bar{\mathbf{h}}_{n,1}^{\mathbf{T}} \quad \tilde{\mathbf{h}}_{n,1}^{\mathbf{T}} \quad \cdots \bar{\mathbf{h}}_{n,B}^{\mathbf{T}} \; \tilde{\mathbf{h}}_{n,B}^{\mathbf{T}} \; \boldsymbol{\theta}_n^{\mathbf{T}} \right] \quad (20)$$

$\bar{\mathbf{h}}_{n,b}$ and $\tilde{\mathbf{h}}_{n,b}$ are the real part and imaginary part of $\mathbf{h}_{n,b}$, respectively. Therefore, based on the CRB covariance matrix (19) and (20), Similar to the appendix E of [7], we can obtain the CRB for the frequency offset estimation in the n^{th} MISO system

$$CRB(\boldsymbol{\theta}_{\mathbf{n}}) = \frac{\sigma^2}{2} \left\{ Re\left\{ \sum_{b=1}^{B} \mathbf{H}_{\mathbf{n},b}^{*} \mathbf{T}_{\mathbf{n}}^{*} \left(\mathbf{I} - \mathbf{A}_{\mathbf{n}} (\mathbf{A}_{\mathbf{n}}^{*}\mathbf{A})^{-1} \mathbf{A}_{\mathbf{n}}^{*} \right) \mathbf{T}\mathbf{H}_{n,b} \right\} \right\}^{-1} \quad (21)$$

where

$$\mathbf{H}_{n,b} = diag(\left[h_{1,n,b}, \cdots h_{M,n,b} \right]$$
$$\mathbf{T}_n = \left[\frac{\partial \mathbf{a}_1(f_{1,n})}{\partial f_{1,n}}, \cdots \frac{\partial \mathbf{a}_{\mathbf{M}}(f_{M,n})}{\partial f_{M,n}} \right] \quad (22)$$

By varying n from 1 to N, the CRB for the frequency offset estimation in the overall MIMO system can be obtained

IV. Simulation Results

In this section, We evaluate the MIMO system for several different pairs of transmit and receive antennas in a flat fading environment, mainly for the number of transmitter antenna M=2 and 4. for clear illustration, only results associated with the 1st transmitter and the 1st receiver pair are displayed. In our simulation, when M equals to 2, the normalized frequency offset ($f_{m,1}T$) is 0.01 for antenna one ($m=1$) and 0.015 for antenna two ($m=2$), when M equals to 4, the 4 normalized frequency offsets are $\{0.01, 0.015, 0.02, 0.025\}$ for $\{m=1,2,3,4\}$, respectively. With SNR vary from 0-30dB, 1000 Monte Carlo trials were conducted for each SNR value. During the evaluation, without denoted otherwise, the length of pilot sequence L is 32, the length of correlator P is 8. B is the number of blocks used in estimation.

Fig.2 and Fig.3 are the MSE of the estimated frequency offset $f_{1,1}$ when transmit antenna M=2 and 4. It can be observed that the MSE decreases with the increase of the number of block B. The performance approaches the CRB with increase of B. Meanwhile, It can be seen that the degradation of the performance becomes more serious when the transmit antenna is increased from 2 to 4. And the performance reaches the error floor in high SNR cases due to the residual multi-antenna interference. Although the CRB for the system with $(M,B)=(4,8)$ is near to that with $(M,B)=(2,4)$, the performance of the former system is much more poor than that of the later one, because the multi-antenna interference is larger for the system with larger transmit antenna.

Next, the effect of length of correlator P on the performance of our algorithm is shown in Fig.4. Here, the system with $(M,B)=(4,8)$ is considered. It can be seen that the MSE of the estimated frequency offset improves with increasing of P. The reason is that at low SNR, the AWGN noise component $N_{m,1,b}(k)$ at the correlator output is relatively large compared with the desired component $E_{m,1,b}$. The improvement becomes insignificant beyond 15dB as the effect of AWGN component becomes minor. Note that as the estimation range of the frequency offset is determined by $-\frac{1}{2iPT} < f_{m,1} < \frac{1}{2iPT}$, it is apparent that longer correlator length P will decrease the synchronization range. In this case

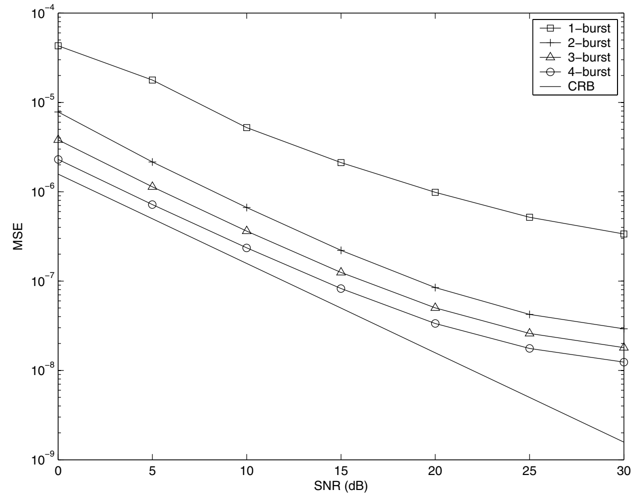

Figure 2. MSE of estimated $f_{1,1}$ vs. SNR with 2 transmit antennas

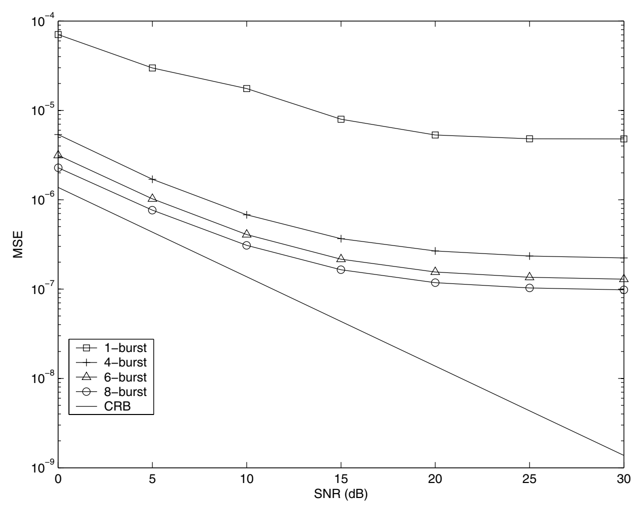

Figure 3. MSE of estimated $f_{1,1}$ vs. SNR with 4 transmit antennas

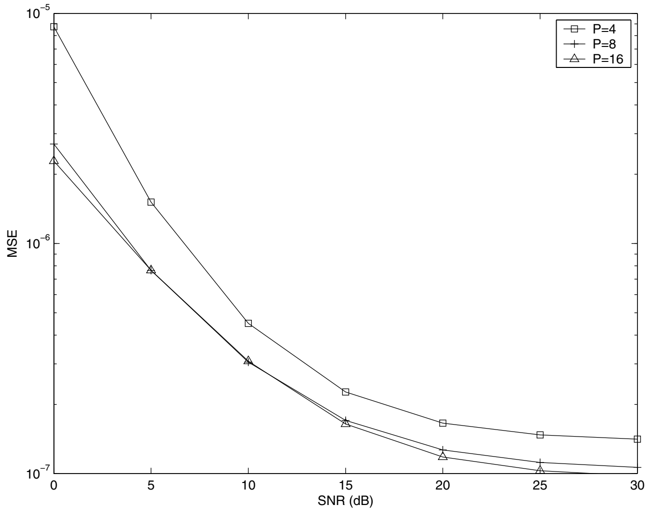

Figure 4. MSE of estimated $f_{1,1}$ with different length of P (M=4,B=8)

$P = 8$ would be a good tradeoff between the synchronization range and the performance.

V. Conclusion

In this paper, we have studied the frequency synchronization problem in MIMO system without the requirement for the knowledge of fading channel information. A general model that multiple frequency offsets exist between the transmit and receive antennas is considered. Using the orthogonal property of the Walsh codes, a correlation-based estimator has been derived. Meanwhile, a close form of CRB is also presented to show the efficiency of the proposed algorithm.

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