Market Allocation Between Bilateral Contracts and Spot Market without Financial Transmission Rights

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***Abstract*-- In the electricity market, it is very important for Generation Companies (Gencos) to decide how to sell energy among different transaction markets in order to maximize profits with relatively low risk. In this paper, two energy transaction markets are considered: spot markets and bilateral contract markets. An energy selling allocation approach with network congestion consideration is established based on modern portfolio theory. Analytical solution for the optimal allocation is derived with given bilateral contract prices and statistical characteristics of the spot market prices. The numerical simulation for energy selling allocation is demonstrated based on the actual data of the USA California power market.**

***Index Terms*-- Electricity market, modern portfolio theory, trading decisions**

I. INTRODUCTION

IN the electricity market, the objective of risk-averse Gencos is to maximize profits with relatively low risk. In order to achieve this aim, before making bidding strategies to maximize profits, Genocs should decide how to allocate energy among different transaction markets, i.e., what we call trading decisions here. Generally, for Gencos, there are two approaches to trading energy in the electricity market: one is spot market; the other is bilateral contract market [1]. From risk management point of view, spot market is a risky trading approach because the energy spot price is fluctuating over time and the Gencos cannot confirm the revenue from the spot market when they make their trading decisions. By contraries, the energy price of bilateral contracts is determined a priori. Normally, when there is no congestion in the network, bilateral contract can be considered as a risk-free trading approach and therefore a useful instrument for Gencos to reduce the complete transaction risk. But when the transmission system is congested, whether the bilateral contract can be thought as a risk-free trading approach will depend on the specific congestion management method.

In a zonal pricing system, there will be an only Market Clearing Price (MCP) in the spot market if no congestion occurs. When the transmission network is congested, MCP of the spot market will vary among locations or zones on the network. Prices are higher at locations that are import constrained and lower at locations that are export constrained. The difference between locational prices represents congestion charges that generators at low-priced locations pay to supply power to customers at high-priced locations. Since demand and transmission capacity availability both vary over time, the incidence of network congestion, the differences in locational prices, and congestion charges can also vary widely over time. Although several ISOs in the United States have created and allocated "Financial Transmission Rights – FTRs[1]" to market participants to hedge against the congestion risk [5], it is difficult for Gencos to get requisite FTRs for their short-term (less than one year) bilateral contract transaction. Therefore, the corresponding bilateral contract transaction is risky.

In this paper, we consider the scenario that Gencos should pay corresponding congestion charges if there is congestion in the transmission system and there are no any financial instrument for them to hedge the congestion risk. Under this scenario, local bilateral contract that is signed with local customers is still a risk-free trading approach, but non-local bilateral contract sighed with non-local customers should be considered as a risky trading approach due to the variable congestion charges. What we concern is what the optimal trading decisions will be for different bilateral contracts. Aiming at this question, this paper applies the portfolio theory [6] to energy allocation between risky spot market and risk-free/risky bilateral contract to maximize Gencos' profits with relatively low risk. In what follows, Section II introduces the methodology to energy allocation with network congestion consideration. In Section III, numerical examples are given to demonstrate the described method. Finally, summary is presented in Section IV.

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[1] Financial transmission rights are known by a variety of names. In the Pennsylvania-New Jersey-Maryland Interconnection they are referred to as fixed transmission rights (FTRs); in the New York Power Pool, as transmission congestion rights (TCCs), in California, as firm transmission rights (FTRs), and in the New England Market, as financial congestion rights (FCRs). Generally, FTRs can be defined as either (a) "flowgate" rights (FGRs), i.e., path-based rights on specific network constraints (like orientd network branches, transmission interfaces or flowgates, and nomograms) [2], [3], or (b) "point-to-point" rights (PTP-FTRs) for power transfers between network locations [4].

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II. Energy Allocation Problem

Suppose a Genco can sell electric energy through two markets: spot market, denoted E and bilateral contract market, denoted B. Suppose there is no market power, i.e., no participant's biding strategy can dominate the market price. Therefore the spot market prices can be considered as random variables with known statistical characteristics estimated or forecasted from the current and historical prices, which are generally public information. In the bilateral contract market, Gencos can write bilateral contracts with consumers at negotiated prices. This contract price can be considered as a non-random variable since it is deterministic.

When there is congestion in the system, a Genco will pay corresponding congestion charge for its bilateral transaction. This congestion charge is the product of actually transmitted energy (MW) and zonal price difference between the consumer and the Genco (transmission losses are ignored).

Suppose the Genco is located in Area u, a consumer is located in Area v. This Genco sign a bilateral contract with the consumer for specific trading amount at fixed contract prices with duration of n trading intervals (the trading time of each trading interval could be half an hour, an hour or 24 hours depending on the specific contract). Revenue from the bilateral contract at the i[th]-trading interval is:

$$R_i = p_i t\left[\lambda_{B,i} - \left(\lambda_{v,i} - \lambda_{u,i}\right)\right] \tag{1}$$

Where t is the trading time for each trading interval; p_i is the trading amount at the i[th]-trading interval; $\lambda_{B,i}$, $\lambda_{u,i}$ and $\lambda_{v,i}$ are bilateral contract price, Area u's spot price and Area v's spot price of the i[th] trading interval respectively. Assuming the Genco has a quadratic cost-curve, i.e., $c(p,t,\lambda_F) = (a + bp + cp^2)\cdot t \cdot \lambda_F$, where $c(p,t,\lambda_F)$ is the production cost for each trading interval; p (MW) is the output power of generators; a,b,c are fuel consumption coefficients (MBtu/Hour, MBtu/MW/Hour, MBtu/MW^2/Hour) that depend on the generator's input-output characteristic; λ_F is the fuel price (\$/MBtu). Here we assume the fuel price is certain during the contract period.

A. Return on the bilateral contract

If the Genco allocate all of its energy in the bilateral contract, corresponding return during the contract period is:

$$r_B = \frac{\sum_{i=1}^{n} R_i - \sum_{i=1}^{n} c_i\left(p_i, t, \lambda_{F,i}\right)}{\sum_{i=1}^{n} c_i\left(p_i, t, \lambda_{F,i}\right)} \tag{2}$$

For convenience, denote

$$K = \frac{1}{\sum_{i=1}^{n} c_i\left(p_i, t, \lambda_{F,i}\right)} = \frac{1}{\sum_{i=1}^{n}\left(a + bp_i + cp_i^2\right) t \lambda_{F,i}}$$

Taking the expectation, variance of this return:

$$E(r_B) = K\sum p_i t\left[\lambda_{B,i} - \left(E(\lambda_{v,i}) - E(\lambda_{u,i})\right)\right] - 1 \tag{3}$$

$$Var(r_B) = K^2\left\{\sum (p_i t)^2\begin{bmatrix} Var(\lambda_{u,i}) + Var(\lambda_{v,i}) \\ -2Cov(\lambda_{u,i}, \lambda_{v,i})\end{bmatrix} + 2\sum_{i<j}\sum p_i p_j t^2\begin{bmatrix} Cov(\lambda_{u,i}, \lambda_{u,j}) - Cov(\lambda_{v,i}, \lambda_{u,j}) \\ -Cov(\lambda_{u,i}, \lambda_{v,j}) + Cov(\lambda_{v,i}, \lambda_{v,j})\end{bmatrix}\right\} \tag{4}$$

B. Return on the spot market

If the Genco sell all the energy in the spot market, during the contract period, corresponding return is:

$$r_E = \frac{\sum_{i=1}^{n} \lambda_{u,i}\cdot p_i \cdot t - \sum_{i=1}^{n} c_i\left(p_i, t, \lambda_{F,i}\right)}{\sum_{i=1}^{n} c_i\left(p_i, t, \lambda_{F,i}\right)} \tag{5}$$

Taking the expectation and variance of this return:

$$E(r_E) = K\cdot\sum p_i \cdot t \cdot E(\lambda_{u,i}) - 1 \tag{6}$$

$$Var(r_E) = K^2\sum (p_i t)^2\, Var(\lambda_{u,i}) + 2K^2\sum_{i<j}\sum p_i p_j t^2 Cov(\lambda_{u,i}, \lambda_{u,j}) \tag{7}$$

C. Risky portfolio of one bilateral contract and the spot market

Suppose each trading interval has the same energy allocation ratio, i.e., $w_B p_i$ is allocated to the bilateral contract and $w_E p_i$ is trading through the spot market. Where w_B is the proportion of energy allocated to the bilateral contract; w_E is the percent of energy sold in the spot market ($w_B + w_E = 1$). Then return on this risky portfolio, denoted p, is r_P where

$$r_P = w_B r_B + w_E r_E \tag{8}$$

Corresponding expectation, variance and standard deviation are:

$$E(r_P) = w_B E(r_B) + w_E E(r_E) \tag{9}$$

$$\begin{aligned} Var(r_P) &= w_B^2 Var(r_B) + w_E^2 Var(r_E) + 2w_B w_E Cov(r_B, r_E) \\ &= w_B^2 Var(r_B) + w_E^2 Var(r_E) + 2w_B w_E \sigma(r_B)\sigma(r_E)\rho_{r_B,r_E} \end{aligned} \tag{10}$$

$$\begin{aligned} \sigma(r_P) &= \left[w_B^2 Var(r_B) + w_E^2 Var(r_E) + 2w_B w_E Cov(r_B, r_E)\right]^{1/2} \\ &= \left[w_B^2 Var(r_B) + w_E^2 Var(r_E) + 2w_B w_E \sigma(r_B)\sigma(r_E)\rho_{r_B,r_E}\right]^{1/2} \end{aligned} \tag{11}$$

Where ρ_{r_B,r_E} is the correlation coefficient between r_B and r_E.

$$Cov(r_B, r_E) = Cov\left(K\sum p_i t\left[\lambda_{B,i} - (\lambda_{v,i} - \lambda_{u,i})\right] - 1, K\sum p_i t\lambda_{u,i} - 1\right)$$
$$= K^2\left[\sum_i (p_i t)^2 Var(\lambda_{u,i}) + 2\sum_{i<j}\sum p_i p_j t^2 Cov(\lambda_{u,i}, \lambda_{u,j})\right]$$
$$-K^2\left[\sum_i (p_i t)^2 Cov(\lambda_{u,i}, \lambda_{v,i}) + 2\sum_{i<j}\sum p_i p_j t^2 Cov(\lambda_{u,i}, \lambda_{v,j})\right] \quad (12)$$

From formula (9) and (11), we can see that the expected return of this risky portfolio is the weighted average of those two trading approaches' expected returns. But this is not true of the standard deviation. Potential benefits from diversification arise when correlation is less than perfectly positive. Here the question is how to decide the values of w_B and w_E to maximize portfolio expected return ($E(r_P)$) with relatively low risk ($Var(r_P)$ or $\sigma(r_P)$). For all the feasible pairs of allocation proportions w_B and w_E, we can calculate all the combination of portfolio expected return and standard deviation. Those $E(r_P)-\sigma(r_P)$ pairs form a portfolio opportunity set.

Choosing an optimal portfolio from this opportunity set will depend on decision-maker's risk aversion. Less risk-averse decision-maker will choose the portfolio with higher expected return and higher risk. More risk-averse Genco would select the portfolio with lower expected return and lower risk. Those choices can be described with a Utility function, $U = E(r_P) - 0.5A\cdot Var(r_P)$, where A is an index of the individual's risk aversion (the moderate risk-aversion parameter is A=3) [6]. Obviously, more risk-averse Genco will assign a higher value to A (greater than 3) and less risk-averse Genco would select lower value for A (less than 3). Therefore, maximizing expected return with relatively low risk can be achieved by maximizing the Utility function, i.e.,

$$\underset{y}{Max} \quad U = E(r_P) - 0.5A\cdot Var(r_P) \quad (13)$$

$$s.t. \quad w_E + w_B = 1 \quad (14)$$

Forming a Lagrange function:

$$\phi = E(r_P) - 0.5AVar(r_P) + \mu(w_B + w_E - 1) \quad (15)$$

Let $\frac{\partial\phi}{\partial w_E} = 0$, $\frac{\partial\phi}{\partial w_B} = 0$ and $\frac{\partial\phi}{\partial\mu} = 0$, optimal w_E and w_B are obtained as follows:

$$w_E^* = \frac{E(r_E) - E(r_B) + AVar(r_B) - ACov(r_B, r_E)}{AVar(r_B) + AVar(r_E) - 2ACov(r_B, r_E)} \quad (16)$$

$$w_B^* = \frac{E(r_B) - E(r_E) + AVar(r_E) - ACov(r_B, r_E)}{AVar(r_B) + AVar(r_E) - 2ACov(r_B, r_E)} \quad (17)$$

III. Example and Discussion

There is a Genco located in Area 1. Corresponding unit characteristics are showed in table 1. There are three consumers located in Area 1, 2 and 3 respectively. The Genco would like to sell part of its energy though the bilateral contract market and trade residual energy in the spot market. The bilateral contract signed with Area 1's consumer is defined as contract 1; with Area 2's consumer, as contract 2 and with Area 3's consumer, as contract 3. The portfolio that consists of contract 1 and the spot market is defined as portfolio 1. Similarly, portfolio 2 includes contract 2 and the spot market; contract 3 and the spot market form portfolio 3. Suppose the bilateral contract's duration is one month, and the trading interval is one day, i.e., t = 24 hours.

Following numerical simulation of energy allocation is performed based on the actual data of the U.S. California electricity market for the month May [7]. Daily spot price of Area 1 (ZP26), Area 2 (SP15) and Area 3 (NP15) are showed in Fig. 2, 3 and 4 respectively. Based on the historical data, the statistical characteristics of the price of these three Areas could be calculated as:

$E(\lambda_1) = 60.156\ \$/MWh$, $\sigma(\lambda_1) = 38.684\ \$/MWh$ (64.31%);
$E(\lambda_2) = 64.146\ \$/MWh$, $\sigma(\lambda_2) = 42.692\ \$/MWh$ (66.55%)
and $E(\lambda_3) = 62.4\ \$/MWh$, $\sigma(\lambda_3) = 39.3\ \$/MWh$ (62.99%).

Besides, we can calculate the price difference between Area 2 and Area 1 (Average price: 3.9899 Average Std.: 17.0536 (427.4150%)) and the price difference between Area 3 and Area 1 (Average price: 2.23 Average Std.: 8.2619 (370.4829%)) that are showed in Fig. 5 and Fig. 6 respectively. Suppose the Genco's total capacity is available for each trading interval, i.e., $p_i = 250MW$. Fuel price is same for each trading interval and let $\lambda_{F,i} = 3.0\$/MBtu$. Assuming the Genco is moderate risk-averse, i.e., A = 3. Suppose that each trading interval's prices are independent each other, i.e., $Cov(\lambda_{*,i}, \lambda_{*,j}) = 0$ ($i \neq j$, * stands for 1 or 2 or 3).

TABLE 1 UNIT CHARACTERISTICS

P (MW)	a (MBtu/h)	b (MBtu/MW/h)	c (MBtu/MW^2/h)	Average fuel Consumption
250	590.72	2.4435	0.00906	7.07138

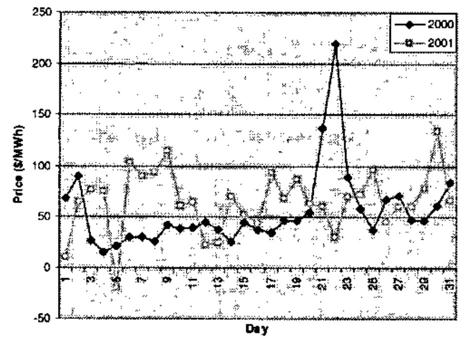

Fig. 2. Daily spot price of Area 1 (ZP26) in May

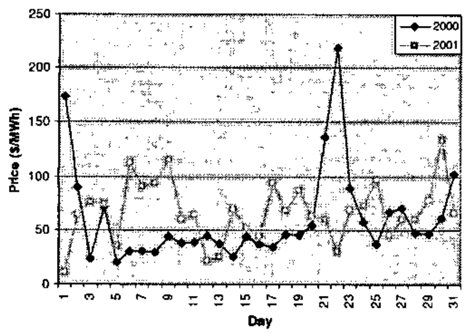

Fig. 3. Daily spot price of Area 2 (SP15) in May

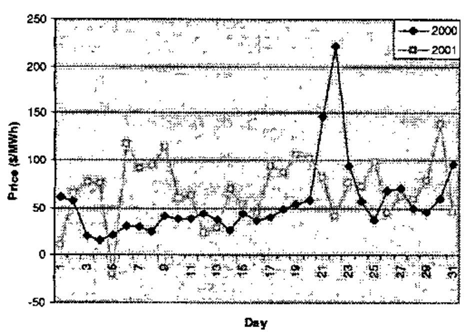

Fig. 4. Daily spot price of Area 3 (NP15) in May

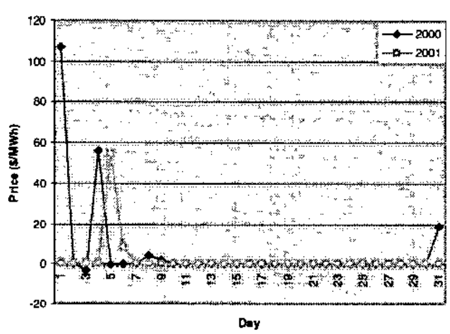

Fig. 5. Daily spot price difference between Area 1 and Area 2

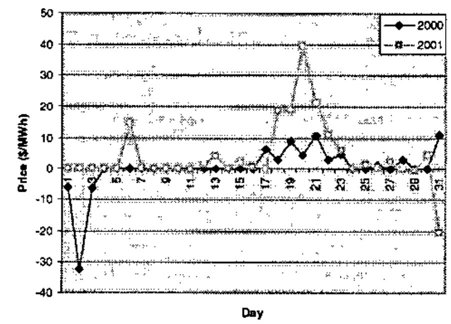

Fig. 6. Daily spot price difference between Area 1 and Area 3

A. *Uniform contract price*

Suppose contract 1, 2 and 3 have the same contract price for each trading interval. Based on (3), (4), (6), (7), (9), (11), (12), (13), (16) and (17) simulation results are showed in table 2.

TABLE 2

Type	λ_B	w_E^*	w_B^*	$E(r_p)$	$\sigma(r_p)$	U
1	58	0.32	0.68	1.7661	0.1034	1.7501
	59	0.17	0.83	1.7904	0.0555	1.7858
2	58	0.91	0.19	1.8091	0.2975	1.6764
	59	0.79	0.21	1.7843	0.2591	1.6836
3	58	0.66	0.54	1.7654	0.2183	1.6939
	59	0.52	0.48	1.7588	0.1741	1.7133

Above simulation results indicate that:

(a) For all the portfolios, w_B^*, the optimal proportion to be allocated to the bilateral contract, is increasing with the increase of bilateral contract price.

(b) For different portfolio that consists of different bilateral contract (i.e., contract 1 or 2 or 3), same Genco would make different trading decisions. Generally, Genco will allocate more energy to the bilateral contract that is signed with local-consumers and can be considered as a risk-free trading approach (e.g. portfolio 1) compared with the situation that the bilateral contract is signed with non-local consumers and considered as risky trading approach (e.g. portfolio 2 and 3).

(c) When these three portfolios have the same bilateral contract price, the Genco prefer portfolio 1 because it has the highest utility value. That is, the Genco prefer to write bilateral contract with local consumers.

Discussion:

Actually, in this example, non-local area's customers may give a higher contract price than local consumers because expected spot price of the non-local area is higher than that of local area. Those non-local bilateral contracts will dominate the local bilateral contracts only when their contract price is higher enough than that of the local bilateral contracts. Next, we'll discuss the break-even price[6] for non-local bilateral contract given the price of local bilateral contract.

B. Break-even price for non-local bilateral contract

The objective of the Genco is to find the portfolio that has the maximum Utility value. Therefore for the non-local bilateral contract, the break-even price is the price with which the corresponding portfolio's maximized utility value equals to the given value that is the maximized utility value of a portfolio consisting of local bilateral contracts. In this example, if the price of local bilateral contract (i.e. contract 1) is 58 or 59, break-even prices of non-local contracts (i.e. contract 2 and 3) are shown in table 3. Obviously, for the same utility value that makes the local bilateral contract and non-local bilateral contract indifferent, non-local consumers who are located in energy import areas should pay more than the local bilateral consumers who are located in the energy export areas due to congestion charges.

TABLE 3

Local bilateral contract price	Break-even price for non-local bilateral contract	
	Contract 2	Contract 3
58	62.4	60.36
59	63.5	61.37

IV. CONCLUSION

This paper applies the modern portfolio theory to energy allocation between risky spot market and risk-free/risky bilateral contracts without FTRs. Based on that, analysis and numerical simulation results show that Gencos prefer to sign contract with local-consumers except that non-local consumers would pay a price higher than the break-even price.

V. REFERENCE

[1] Notice of Proposed Rulemaking, United States of America Federal Energy Regulatory Commission Docket No.RM01-12-000, 2002.

[2] Hung-po Chao and Stephen C. Peck, "A Market Mechanism for Electric Power Transmission," *Journal of Regulatory Economics*, pp. 25-59, July 1996.

[3] Hung-po Chao, Stephen C. Peck, Shmuel S. Oren and Robert B. Wilson, "Flow-Based Transmission Rights and Congestion Management," *the Electricity Journal*, pp. 38-58, October 2000.

[4] William Hogan, "Contract Networks for Electric Power Transmission," *Journal of Regulatory Economics*, pp. 211-242, Dec. 1992.

[5] Karen Lyons, Hamish Fraser and Hethie Parmesano, "An introduction to financial transmission rights," *The Electricity Journal*, vol. 13, Issue 10, pp. 31-37, Dec. 2000.

[6] Zvi Bodie, Alex Kane, and Alan J. Marcus, *Investments*, Irwin/McGraw-Hill, 1999.

[7] University of California Energy Institute website: http://www.ucei.berkeley.edu/ucei/

[6] So-called break-even price is the price that make non-local bilateral contract indifferent from local bilateral contract given the price of local bilateral contract.

VI. BIOGRAPHIES

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