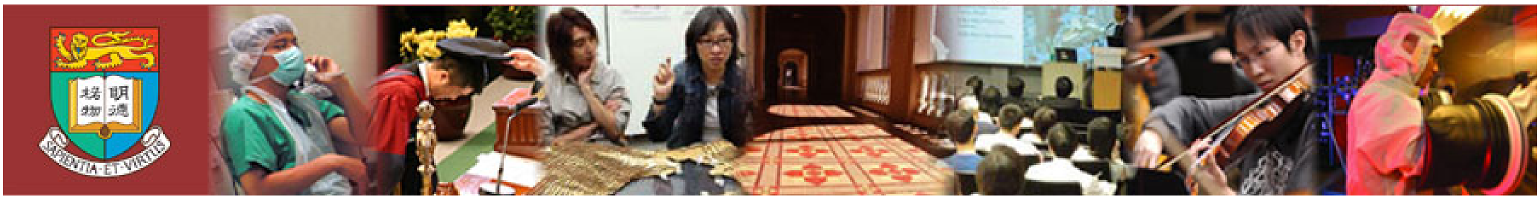

GENERATION OF NOISE SEQUENCES WITH DESIRED NON-GAUSSIAN DISTRIBUTION AND COVARIANCE

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Abstract: It is necessary to generate a noise sequence in the simulation of the communication system and signal processing. In the design of some practical system such as radar system, we must generate a stationary noise sequence with a specified non-Gaussian probability density function and a desired power spectrum to test the performance of the system. For this purpose, this paper presents a method by which such a clutter sequence can be generated. The examples are also demonstrated to show the effectiveness of the proposed method.

Key words: Random sequences, generation, radar simulation.

1. Introduction

In the simulations of the communication systems and the application of radar, communication as well as the signal processing, it is necessary to generate a noise sequence with certain non-Gaussian probability density function and the desired power spectrum in order to improve the performance of the systems and the signal detection. In general, some methods have been introduced to transform a random sequence with a uniform distribution on the interval [0,1] into a random sequence which has a desired probability density function [1-2]. Under certain circumstances, however, it is needed to generate a stationary random sequence with not only a specified probability density function, but also a desired power spectrum in many practical applications such as in the radar system design. Therefore, it is very significant to produce a random sequence with a specified distribution function and desired power spectrum in the practical situation. For this purpose, one procedure is developed in this paper to generate a noise sequence which has a specified probability density function and the desired approximate power spectrum. The simulated results are also demonstrated to prove the effectiveness of the presented method.

2. Basic Scheme

The basic scheme for generating such a random sequence with specified distribution function and approximate autocorrelation function is shown in the following block diagram:

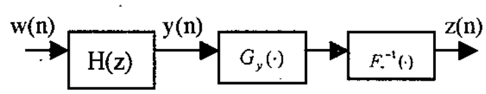

Figure 1. The block diagram for transforming a white Gaussian noise into a desired sequence

In the Fig. 1, $w(n)$ represents the zero-mean unit-variance Gaussian white noise sequence. $H(z)$ is a linear digital filter which is designed to transform a Gaussian white noise sequence $w(n)$ into a Gaussian sequence with certain auto-covariance based on the fact that a linear transformation of a Gaussian process produces another Gaussian process [3]. It is possible to assume that the variance of $y(n)$ can be scaled to unit-variance by normalizing the coefficients of linear digital filter [1]. The nonlinear system includes the normalized Gaussian distribution function $G_y(\cdot)$ and the inverse of the distribution function $F_z^{-1}(\cdot)$ of the

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desired random sequences z(n). $G_y(\cdot)$, the first part of the nonlinear system, is used for transforming the Gaussian sequences $y(n)$ into a random sequence $u(n)$ which is normally distributed on interval [0,1] [7], and because $F_z^{-1}(\cdot)$ is equal to the inverse of the distribution function of desired sequences $z(n)$, the output sequences $z(n)$ will have a specified probability distribution function. Besides, the desired power spectrum of the random sequences $z(n)$ must be determined. The function of the nonlinear system can be written as

$$g(x) = F_z^{-1}[G_y(\cdot)] \tag{1}$$

If the autocorrelation function or the spectrum of the desired sequences $z(n)$ is required, the autocorrelation function $R_y(m)$ can be decided through the transformation of the nonlinear system $G(\cdot)$. Once the $R_y(m)$ is obtained, all the coefficients of the linear system $H(z)$ will be conveniently calculated. In other words, the proposed system in Fig. 1 has been described. Therefore, when a Gaussian white noise sequence is used as an input driving the system, the output sequences $z(n)$ will be a stationary random sequence with specified probability distribution function and desired power spectrum by the transformation of both the linear and the nonlinear systems.

3. Procedures

Based on the Fig. 1, it can be seen that the nonlinear function $G(\cdot)$ is described provided the probability distribution function $F_z(\cdot)$ has been definitely given. It is further assumed that the nonlinear function $G(\cdot)$ can be expanded to the Hermite polynomials [2], which is given such that

$$g(X) = \sum_{k=0}^{\infty} b_K H_K(X) \tag{2}$$

where $H_K(x)$ represents the K th Hermite polynomial which can be expressed as

$$H(X) = (-1)^K e^{x^2} \frac{d^K}{dX^K} e^{-X^2} \tag{3}$$

and the following relation is established

$$H_{K+1} = 2xH_K(X) - 2KH_{K-1}(X) \tag{4}$$

the coefficient of Hermite polynomial b_K are given by

$$b_K = (K-1)^{-1} \int_{-\infty}^{\infty} g(X) H_K(X) f(X) dX \tag{5}$$

where $f(X)$ is the unit-normal density function. Considering $y(n)$ is a normalized Gaussian sequence, the relationship between $R_z(m)$ and $R_y(m)$ can be described as

$$R_z(m) = \sum_{k=1}^{\infty} C_K^2 R_y^K(m) \tag{6}$$

where the series coefficients C_K are given by

$$C_K = (K-1)^{-1/2} \int_{-\infty}^{\infty} g(X) H_K(X) f(X) dX \tag{7}$$

Once the C_K is determined and $R_z(m)$ is specified, the $R_y(m)$ can be found by solving the equation (6). In practical simulation, it is possible and applicable to choose the order of equation (6) to be finite because the C_K has the property

$$\sum_{k=1}^{\infty} C_K = 1$$

and, further, for many probability distribution, there exists the following relation [1]

$$\sum_{k=1}^{P} C_K = 1$$

Then the equation (6) can be expressed as

$$R_z(m) = \sum_{k=1}^{P} C_K^2 R_y^K(m) \tag{8}$$

In order to determine the $R_y(m)$ from the given $R_z(m)$, a simple iterative equation for calculating $R_y(m)$ from $R_z(m)$ derived from equation (8) is given as

$$R_y(m+1) = R_z(m) + \frac{uA(m)}{B(m)} \tag{9}$$

where

$$A(m) = R_z(m) - \sum_{K=1}^{P} C_K^2 R_y^K(m) \tag{10}$$

$$B(m) = \sum_{K=1}^{P} K C_K^2 R_y^{K-1}(m) \tag{11}$$

where u is a convergent constant to be decided.

After $R_y(m)$ has been decided, there are several methods for estimating the coefficients of the linear system $H(z)$ [2]. One effective method employed in this paper is to choose an AR model with order N as a linear digital filter. This model is equivalent to the maximum entropy spectral estimation and makes the calculation of the autoregressive coefficients more simple and convenient.

Since the input of the AR model is a Gaussian white noise sequence $w(n)$, the AR model can be written by

$$y(n) = -\sum_{k=1}^{N} a_K y(n-K) + w(n) \tag{12}$$

where a_K is the model parameters to be decided. From the AR model, without loss of generality, it is reasonably assumed that $y(n)$ is a stationary ergodic sequence, thus the autocorrelation function $R_y(m)$ can be described as

$$R_y(m) = \begin{cases} -\sum_{K=1}^{N} a_K R_y(m-K), & m > 0 \\ -\sum_{K=1}^{N} a_K R_y(m-K) + c, & m = 0 \end{cases}$$

which is the well-known expression of Yule-Walker equation, and the model parameters can be obtained by solving the Yule-Walker equation after selecting the appropriate order N of the AR model [4-5]. So far, the whole system in Fig. 1 has been described. Hence, if a Gaussian white noise sequence $w(n)$ is driven to the linear system followed by a nonlinear system which includes the normalized Gaussian distribution function $G_y(\cdot)$ and the inverse of specified distribution function $F^{-1}{}_z(\cdot)$, the output of the system will be such a random sequence with a specified non-Gaussian probability distribution function $F_z(z)$ and an approximated autocorrelation function or spectrum, respectively.

4. Simulation Results

A random sequence with an exponential probability density function and an exponential autocorrelation function is generated by using the proposed method to test the behavior of the presented method. The simulated results are shown in Fig. 2. The results regarding to both specified probability distribution and desired power spectral density function indicate that the numerical behaviors provided a satisfied performance based on the proposed scheme.

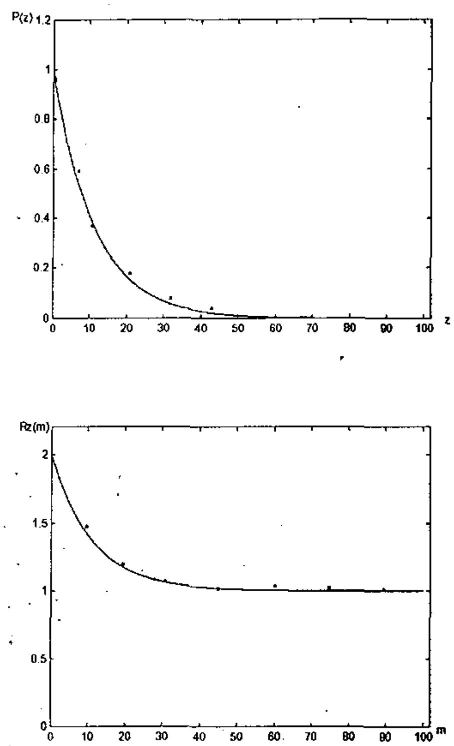

Fig. 2 Example of a random sequence with an specified exponential probability density function (top) and an exponential autocorrelation function (bottom). The continuous curves denote the specified PDF and the autocorrelation, respectively. The star symbols represent the simulation results.

5. Conclusion

This paper is devoted to describing a method for generating a random sequence with the specified non-Gaussian probability distribution function and approximated power spectrum. The proposed algorithm consists of an AR model with a Gaussian white noise sequence input followed by a nonlinear system, which transforms the output sequences of the linear system into such a random sequence with a specified probability density function, and an approximated autocorrelation function, respectively. The method presented in this paper may have the advantages of simplicity and applicability for many applications of communications and radar system analysis.

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