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Optimal Multi-User MISO Solution with Application to Multi-User Orthogonal Space Division Multiplexing[§]

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Abstract—**In this paper, we shall show that for a n_T-element base station (BS) communicating with $M(\leq n_T)$ single-element mobile stations (MS) (or multi-user MISO) orthogonally in the spatial domain, the optimization problem is equivalent to the least squares (LS) problem for an underdetermined linear system. We then prove that the optimal BS antenna weights can be expressed as the pseudo-inverse of the multi-user channel matrix. This solution decomposes the multi-user system into many single-user systems with maximal resultant channel responses. The average of the squared channel response (defined as channel gain) and the inverse of the normalized variance of the squared channel response (defined as diversity order) are derived for performance analysis. It is found that every individual user of the resulting system behaves like a single-user system with n_T-M+1 reception diversity. Finally, by applying the solution on a multi-user MIMO antenna system (i.e., with multiple antennas at the MS as well), an iterative approach is proposed to perform multi-user orthogonal space division multiplexing (OSDM) in the downlink.**

I. Introduction

Using multiple antennas for transmission or reception is attractive because of its high energy and spectral efficiency [1]–[9]. Initially, techniques using multiple antennas are only proposed for use at the base station (BS) [1], [2] because mobile station (MS) has to be inexpensive and compact. Inspired by the advances in hardware miniaturization and antenna design, however, it is now also feasible to deploy multiple antennas at a mobile terminal, thereby making multiple-input multiple-output (MIMO) antenna a reality.

Research on MIMO antenna dated back in 1995 by Telatar [3] in which the capacity of MIMO Gaussian channels is derived. Later in [4], Foschini proposed a MIMO processing technique termed BLAST (Bell-Labs Layered Space-Time) to realize the channel capacity and showed that system capacity scales up linearly with the number of antennas at both ends. This finding has proliferated many subsequent studies on more advanced MIMO antenna systems (for example, [5]). Most research into MIMO antenna system has tended to concentrate on single-user, point-to-point transmission. Support of multiple users for downlink, point-to-multipoint communication, using MIMO technology is much less understood.

Conventional approaches on downlink space division multiplexing are mainly based on signal-to-interference plus noise ratio (SINR) balancing [6]–[8]. In other words, the co-channel users are not truly uncoupled.

Orthogonal space division multiplexing (OSDM) in the downlink has not been studied until recently in [9]. In that paper, an iterative approach is developed to perform downlink OSDM when multiple antennas are utilized at the BS and all MS (or multi-user MIMO). However, because the approach relies on a sequence of square transformation updates, the number of BS antennas needs to be equal to the number of co-channel signals in the system or any additional BS antenna will be automatically turned off.

In this paper, we devise a new technique for downlink OSDM that is applicable for a broader problem of OSDM in multi-user MIMO downlink channels. We approach the problem by first considering the OSDM problem for multi-user MISO antenna systems (i.e., when the MS has only one receive antenna). The problem is solved by reformulating it as the least squares (LS) problem for an undetermined linear system. In so doing, the antenna weights performing OSDM can be readily found as the pseudo-inverse of the multi-user channel matrix. This solution is then used to develop an iterative scheme for multi-user MIMO channels.

The remainder of this paper is organized as follows. In Section II, we introduce the multi-user MIMO system model. Section III presents the optimization problem of OSDM. Simulation results are presented in Section IV. Finally, we conclude the paper in Section V.

II. Multi-User MIMO System Model

The system configuration of the multi-user MIMO antenna system is shown in Figure 1 where signals are transmitted from one BS to M MS, n_T antennas are located at the BS and n_{R_m} antennas are located at the mth MS. In flat fading channels, the received signals at MS m can be conveniently written in matrix form as [8], [9]

$$\mathbf{y}_m = \mathbf{H}_m \left(\sum_{m'=1}^{M} \mathbf{t}_{m'} z_{m'} \right) + \mathbf{n}_m \tag{1}$$

where $\mathbf{H}_m \in \mathbb{C}^{n_{R_m} \times n_T}$ denotes the MIMO channel matrix from the BS to MS m, $\mathbf{t}_{m'} \in \mathbb{C}^{n_T}$ is the transmit antenna weight vector for MS m', $z_{m'}$ denotes the modulated symbol transmitted by MS m', and $\mathbf{n}_m$ is a zero-mean complex additive white Gaussian noise (AWGN) vector with variance of $N_0/2$ per dimension. The channel matrix is assumed zero-mean and complex Gaussian distributed so that whose elements are Rayleigh distributed in amplitude and phases uniformly distributed between 0 and 2π.

At the MS m, signal estimation is done by weighted combining $\mathbf{y}_m$ with the receive antenna weight vector $\mathbf{r}_m \in \mathbb{C}^{n_{R_m}}$, to produce the estimate of z_m

$$\hat{z}_m = \mathbf{r}_m^\dagger \mathbf{y}_m = \mathbf{r}_m^\dagger(\mathbf{H}_m \mathbf{T}\mathbf{z} + \mathbf{n}_m) \quad \forall m \tag{2}$$

where the superscript $\dagger$ denotes complex transposition, and we have defined $\mathbf{T} \triangleq [\mathbf{t}_1 \ \mathbf{t}_2 \ \cdots \ \mathbf{t}_M]$ as the multi-user transmit weight matrix.

III. Orthogonal Space Division Multiplexing

Our objective is to find the antenna weights, $\mathbf{T}, \mathbf{r}_1, \ldots, \mathbf{r}_M$, jointly, in order that co-channel interference (CCI) are entirely eliminated and that the resultant channel responses at the desired MS are maximized. This can be written as

$$(\mathbf{T}, \mathbf{r}_1, \ldots, \mathbf{r}_M)_{\text{opt}} = \arg \max_{\mathbf{T}, \mathbf{r}_1, \ldots, \mathbf{r}_M} |\beta_m|^2 \quad \forall m \tag{3}$$

where β_m is defined by

$$\mathbf{r}_m^\dagger \mathbf{H}_m \mathbf{T} = [0 \ \cdots \ 0 \ \beta_m \ 0 \ \cdots \ 0] \tag{4}$$

and $|\cdot|$ outputs the modulus of the input complex number.

In (4), the entries corresponding to the signals transmitted from the undesired users to MS m are made zero so that the optimized system will ensure interference-free at all MS. Moreover, hereafter, we shall assume that $\|\mathbf{t}_m\| = \|\mathbf{r}_m\| = 1$. In what follows, β_m will represent the resultant channel response of the mth MS irrespective of the transmitted power.

Performing (3) and (4) together involves nullification of many dependent co-channel signals passing through many different channels and this makes the problem untractable. Rather, we shall first reduce the dimensionality of the problem by considering MISO antenna instead and obtain the solution of the reduced problem. Then readdress the problem for MIMO antenna later.

A. Multi-User MISO Antenna System

Using the system model described in Section II, MISO simplification can be realized by assuming $n_{R_m} = 1 \ \forall m$ or the receive weight vectors $\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_M$ are all fixed and known. The latter is more desirable as the result of the reduced problem is more readily used for MIMO antenna systems. Therefore, the latter will be assumed in the following manipulation.

As a consequence, system (2) is now reduced to a multi-user MISO antenna system with the channel matrix

$$\mathbf{H}_e \triangleq \begin{bmatrix} \mathbf{r}_1^\dagger \mathbf{H}_1 \\ \mathbf{r}_2^\dagger \mathbf{H}_2 \\ \vdots \\ \mathbf{r}_M^\dagger \mathbf{H}_M \end{bmatrix} \in \mathbb{C}^{M \times n_T}. \tag{5}$$

In light of (3) and (4), we are now required to find the optimal $\mathbf{t}_m$ so that

$$\mathbf{t}_m|_{\text{opt}} = \arg \max_{\mathbf{t}^{(m)}} |\beta_m|^2 \quad \forall m \tag{6}$$

and

$$\mathbf{H}_e \mathbf{t}_m = [0 \ \cdots \ 0 \ \beta_m \ 0 \ \cdots \ 0]^T \tag{7}$$

where the superscript T denotes the transposition. If we define another set of weight vectors

$$\mathbf{g}_m \triangleq \frac{1}{|\beta_m|} \mathbf{t}_m \quad \forall m, \tag{8}$$

then (6) and (7) can be rewritten as

$$\mathbf{g}_m|_{\text{opt}} = \arg \min_{\mathbf{g}_m} \|\mathbf{g}_m\|^2 \tag{9}$$

and

$$\mathbf{H}_e \mathbf{g}_m = \mathbf{e}_m \triangleq [0 \ \cdots \ 0 \ \underbrace{1}_{\text{the } m\text{th entry}} \ 0 \ \cdots \ 0]^T \quad \forall m \tag{10}$$

respectively. Defining $\mathbf{G} \triangleq [\mathbf{g}_1 \ \mathbf{g}_2 \ \cdots \ \mathbf{g}_M]$, (10) can be concisely expressed as

$$\mathbf{H}_e \mathbf{G} = \mathbf{I} \tag{11}$$

where $\mathbf{I}$ is an identity matrix. In order for (11) to exist, we must have $\text{rank}(\mathbf{H}_e), \text{rank}(\mathbf{G}) \geq \text{rank}(\mathbf{I}) = M$. As a result, OSDM is possible only when $n_T \geq M$.

When $n_T = M$, the optimal $\mathbf{G}$ is simply

$$\mathbf{G}_{\text{opt}} = \mathbf{H}_e^{-1} \tag{12}$$

where the superscript -1 denotes the matrix inversion, and it is the only solution for (11).

When $n_T > M$, in general, there are infinitely many possible solutions for $\mathbf{G}$ that can result (11). Hence, among the possible solutions, we need to select the one that minimizes (9). This problem is recognized as a typical LS problem for an underdetermined linear system [10] and this can be solved by using a singular value decomposition (SVD) approach. Writing $\mathbf{H}_e$ using SVD as $\mathbf{H}_e = \mathbf{U}\mathbf{\Lambda}\mathbf{V}^\dagger$ where $\mathbf{U} = [\mathbf{u}_1 \ \mathbf{u}_2 \cdots]$ is the left singular matrix, $\mathbf{V} = [\mathbf{v}_1 \ \mathbf{v}_2 \cdots]$ is the right singular matrix and $\mathbf{\Lambda} = \text{diag}(\lambda_1, \lambda_2, \ldots) \in \mathbb{R}^{M \times n_T}$, the optimal $\mathbf{g}_m$ [in the sense of (9) and (10)] is given by

$$\mathbf{g}_m|_{\text{opt}} = \sum_{i=1}^{M} \frac{\mathbf{u}_i^\dagger \mathbf{e}_m}{\lambda_i} \mathbf{v}_i \quad \forall m. \tag{13}$$

As such, we have the optimized MISO antenna weights at the transmitter or BS

$$\mathbf{t}_m|_{\text{opt}} = \frac{\mathbf{g}_m|_{\text{opt}}}{\left\| \mathbf{g}_m|_{\text{opt}} \right\|} \quad \forall m. \tag{14}$$

Most importantly, the solution (13) can be rewritten into a more computation-efficient form, so SVD is avoided when obtaining the weights. The details will be discussed in the following definition and theorem.

Definition 3.1—Moore-Penrose Pseudo-Inverse of a Matrix [10]: For any matrix $\mathbf{A} \in \mathbb{C}^{p\times q}$, assuming that $\mathbf{A} = \mathbf{B}\boldsymbol{\Sigma}\mathbf{C}^\dagger$ is the SVD of $\mathbf{A}$, then there exists a unique matrix $\mathbf{A}^+ = \mathbf{C}\boldsymbol{\Sigma}^+\mathbf{B}^\dagger \in \mathbb{C}^{q\times p}$ where

$$\boldsymbol{\Sigma}^+ = \text{diag}\left(\frac{1}{\sigma_1}, \ldots, \frac{1}{\sigma_r}, 0, \ldots, 0\right) \in \mathbb{R}^{q\times p}, \quad (15)$$

$r = \text{rank}(\mathbf{A})$, and $\{\sigma_i\}_{i=1}^r$ are the singular values of $\mathbf{A}$. $\mathbf{A}^+$ is referred to as the Moore-Penrose pseudo-inverse of $\mathbf{A}$.

If $r = \text{rank}(\mathbf{A}) = q \le p$, then $\mathbf{A}^+$ can be computed by

$$\mathbf{A}^+ = \left(\mathbf{A}^\dagger\mathbf{A}\right)^{-1}\mathbf{A}^\dagger. \quad (16)$$

Theorem 3.1—Pseudo-Inverse Expression for the Solution of the Underdetermined System: If $\mathbf{G} = [\mathbf{g}_1 \cdots \mathbf{g}_M]$ is the solution of the underdetermined linear LS problem [i.e., (9) and (10)], and $\text{rank}(\mathbf{H}_e) = M \le n_T$, then $\mathbf{G}^\dagger$ is the pseudo-inverse of $\mathbf{H}_e^\dagger$. Thus, $\mathbf{G}_{\text{opt}}$ can be expressed as

$$\mathbf{G}_{\text{opt}} = \mathbf{H}_e^\dagger\left(\mathbf{H}_e\mathbf{H}_e^\dagger\right)^{-1}. \quad (17)$$

Proof: With the SVD of $\mathbf{H}_e = \mathbf{U}\boldsymbol{\Lambda}\mathbf{V}^\dagger$ (as previously defined), we have also

$$\mathbf{H}_e^\dagger = \mathbf{V}\boldsymbol{\Lambda}^\dagger\mathbf{U}^\dagger, \quad (18)$$

the SVD of $\mathbf{H}_e^\dagger$. From Definition 3.1, we can write the pseudo-inverse of $\mathbf{H}_e^\dagger$ as

$$\mathbf{X} \triangleq \left(\mathbf{H}_e^\dagger\right)^+ = \mathbf{U}\left(\boldsymbol{\Lambda}^\dagger\right)^+\mathbf{V}^\dagger = \mathbf{U}\left(\boldsymbol{\Lambda}^+\right)^\dagger\mathbf{V}^\dagger. \quad (19)$$

Denoting the row vector of $\mathbf{X}$ as $\mathbf{x}_m$ so that $\mathbf{X} = [\mathbf{x}_1; \cdots; \mathbf{x}_M]$, we get

$$\mathbf{x}_m^\dagger = \mathbf{X}^\dagger\mathbf{e}_m = \mathbf{V}\boldsymbol{\Lambda}^+\mathbf{U}^\dagger\mathbf{e}_m = \sum_{i=1}^{M}\frac{\mathbf{u}_i^\dagger\mathbf{e}_m}{\lambda_i}\mathbf{v}_i = \mathbf{g}_m|_{\text{opt}}. \quad (20)$$

Accordingly, $\mathbf{G}_{\text{opt}} = \mathbf{X}^\dagger$. As $\text{rank}(\mathbf{H}_e) = M < n_T$, we finally end up

$$\mathbf{G}_{\text{opt}} = \mathbf{X}^\dagger = \left[\left(\mathbf{H}_e\mathbf{H}_e^\dagger\right)^{-1}\mathbf{H}_e\right]^\dagger = \mathbf{H}_e^\dagger\left(\mathbf{H}_e\mathbf{H}_e^\dagger\right)^{-1}. \quad (21)$$

■

SVD requires $(4Mn_T^2 + 11n_T^3 + M^2)$ flops (floating point operations) while only $[4M^2n_T + (M^3 + M)/3]$ flops are needed if (17) is used. Much reduction in complexity can be gained when $n_T > M$.

Theorem 3.2—Channel Gain: For an optimized M-user MISO antenna system with $n_T(\ge M)$ antennas at the BS, the channel gain or the average of the squared channel response is given by

$$\Omega = \text{E}[\beta^2] = n_T - M + 1. \quad (22)$$

Proof: See Appendix I. ■

Theorem 3.3—Diversity Order: For an optimized M-user MISO antenna system with $n_T(\ge M)$ antennas at the BS, the diversity order or the inverse of the normalized variance of the squared channel response is given by

$$\Psi = \frac{\Omega^2}{\text{E}[(\beta^2 - \Omega)^2]} = n_T - M + 1. \quad (23)$$

Proof: See Appendix I. ■

B. Multi-User MIMO Antenna System

The normalized solution of (17) can be readily used for OSDM in multi-user MIMO channels when maximum ratio combining (MRC) is used iteratively to update the receiver weights. The details of the algorithm are given as follows.

Step 1) Initialize $\mathbf{r}_m = [1\ 1\ \cdots\ 1]^T\ \forall m$.

Step 2) Obtain $\mathbf{H}_e$ using (5).

Step 3) Find $\mathbf{T}$ by (14) and (17).

Step 4) For all MS m, update $\mathbf{r}_m = \mathbf{H}_m\mathbf{t}_m$.

Step 5) Compute

$$\mathbf{r}_m^\dagger\mathbf{H}_m\mathbf{T} = [\epsilon_1\ \cdots\ \epsilon_{m-1}\ \beta_m\ \epsilon_{m+1}\ \cdots\ \epsilon_M] \quad (24)$$

to check if $|\epsilon_i|$ is less than a preset threshold, ϵ_{Th} $(= 10^{-6})$. If $|\epsilon_i| < \epsilon_{Th}\ \forall i$, the convergence is said to be achieved. Otherwise, go back to Step 2.

IV. Simulation Results

Monte Carlo simulations have been carried out to assess the system performance of the proposed OSDM method. Results on average bit error rate (BER) for various signal-to-noise ratio (SNR) will be presented. Perfect channel state information is assumed to be available at the transmitter (or BS) so that OSDM can be achieved. The channel model used in the simulations is a quasi-static flat Rayleigh fading channel and will be fixed during one frame and change independently between frames. The frame length is 128 symbols and 4-QAM will be assumed. For each simulation, 100,000 independent channel realizations are used to obtain the numerical results. For convenience, we shall use $\{n_T, [n_{R_1}, \ldots, n_{R_M}]\}$ to denote a M-user MIMO system which has n_T transmit antennas and MS m has n_{R_m} receive antennas.

In Figure 2, the average BER results for the proposed method (referred to as P-INV) and the approach in [9] are plotted against the average E_b/N_0 per branch-to-branch. The system configurations are: a) $\{2, [3, 3]\}$ where each MS has the same number of receive antennas, and b) $\{2, [2, 4]\}$ where one MS has more antennas than another MS. Results in this figure show that the two approaches perform nearly the same. Also, results of $\{2, [3, 3]\}$ reveal that the two users have the same BER performance, but in contrast for $\{2, [2, 4]\}$, the performance of user 1 is not as good as that of user 2.

To study the diversity gained by incorporating more BS antennas, the average BER results for the proposed method are plotted for various number of BS antennas in Figure 3. In this figure, the number of users is fixed to 2. Mobile users with 2 or 3 receiving antennas are also considered. As can be seen, for a given number of MS antennas, the BER decreases significantly as the number of BS antennas increases. The change in the slopes of the BER curves indicates the enhanced diversity of the systems.

In Figures 4 and 5, we study the impact on the performance of one user by varying the number of antennas at other mobile users. We simplify our study by considering systems with only two users, one antenna at MS 1, and two antennas at the BS. In Figure 4, user 1's BER performance is provided with various number of antennas at MS 2 (from 1 to 4). Note that as the number of antennas at MS 2 increases, user 1's BER decreases and tends to approach the performance of a single-user system with second-order diversity at the expense of little performance degradation for user 2. The reason is that with sufficiently large number of antennas at MS 2, little is done at the BS for suppressing CCI from user 1 to MS 2. Consequently, the signal strength of user 1 will be optimized as if MS 2 does not exist and user 1 will converge to a $\{2, [1, 1]\}$ system.

V. Conclusions

We have shown that the OSDM problem can be viewed as the LS problem for an underdetermined linear system when MS has only one receive antenna. The optimal antenna weights have been shown to be the pseudo-inverse of the channel. The optimized system reduces a M-user system with n_T-element array to many single-user systems with $(n_T - M + 1)$-order diversity. The solution has then been used to propose an iterative algorithm for OSDM in multi-user MIMO downlink channels.

Appendix I
Proof of Channel Gain and Diversity Order

In the optimized MISO system, $\mathbf{g}_m = \mathbf{H}^\dagger(\mathbf{H}\mathbf{H}^\dagger)^{-1}\mathbf{e}_m$ where the subscript e is omitted for conciseness. The square of the resultant channel response can be elegantly expressed as

$$\beta_m^2 = \frac{1}{\mathbf{e}_m^\dagger(\mathbf{H}\mathbf{H}^\dagger)^{-1}\mathbf{e}_m} = \frac{\det(\mathbf{H}\mathbf{H}^\dagger)}{\det(\tilde{\mathbf{H}}_m\tilde{\mathbf{H}}_m^\dagger)} \tag{25}$$

where $\tilde{\mathbf{H}}_m = [\mathbf{h}_1; \cdots; \mathbf{h}_{m-1}; \mathbf{h}_{m+1}; \cdots; \mathbf{h}_M]$ and $\mathbf{h}_i$ corresponds to $\mathbf{H}_i$ in (2). The use of lower case is to highlight that $\mathbf{h}_i$ is now a row vector instead of a matrix. WLOG, we consider $m = 1$ in the following for convenience.

Expanding $\det(\mathbf{H}\mathbf{H}^\dagger)$ gives

$$\det(\mathbf{H}\mathbf{H}^\dagger) = \sum_{m=1}^{M}(-1)^{m-1}\mathbf{h}_1\mathbf{h}_m^\dagger \times \det(\tilde{\mathbf{H}}_1\tilde{\mathbf{H}}_m^\dagger). \tag{26}$$

Therefore,

$$\beta_1^2 = \mathbf{h}_1\mathbf{h}_1^\dagger - \sum_{m=2}^{M}(-1)^m\mathbf{h}_1\mathbf{h}_m^\dagger \times \frac{\det(\tilde{\mathbf{H}}_1\tilde{\mathbf{H}}_m^\dagger)}{\det(\tilde{\mathbf{H}}_1\tilde{\mathbf{H}}_1^\dagger)}. \tag{27}$$

In addition, note that

$$\frac{\det(\tilde{\mathbf{H}}_1\tilde{\mathbf{H}}_m^\dagger)}{\det(\tilde{\mathbf{H}}_1\tilde{\mathbf{H}}_1^\dagger)} = \frac{\sum_{i=2}^{M}(-1)^{i-1}\mathbf{h}_i\mathbf{h}_1^\dagger\det(\mathbf{M}_i)}{\sum_{j=2}^{M}(-1)^{m+j-1}\mathbf{h}_j\mathbf{h}_m^\dagger\det(\mathbf{M}_j)} \tag{28}$$

where $\mathbf{M}_i$ is the sub-block matrix of $\tilde{\mathbf{H}}_1\tilde{\mathbf{H}}_1^\dagger$ after removing the ith row and the mth column. Then, we have

$$(-1)^m\mathbf{h}_1\mathbf{h}_m^\dagger \times \frac{\det(\tilde{\mathbf{H}}_1\tilde{\mathbf{H}}_m^\dagger)}{\det(\tilde{\mathbf{H}}_1\tilde{\mathbf{H}}_1^\dagger)} = \frac{\sum_{i=2}^{M}(-1)^{i-1}\det(\mathbf{M}_i)\mathbf{h}_i(\mathbf{h}_1^\dagger\mathbf{h}_1)\mathbf{h}_m^\dagger}{\sum_{j=2}^{M}(-1)^{j-1}\det(\mathbf{M}_j)\mathbf{h}_j\mathbf{h}_m^\dagger}. \tag{29}$$

Taking the average on both sides and using the fact that $\mathrm{E}[\mathbf{h}_1^\dagger\mathbf{h}_1] = \mathbf{I}$ (where we have assumed every channel element of $\mathbf{h}_i$ is independent to each other and that $\mathbf{h}_i$ are uncorrelated with themselves), we obtain

$$\mathrm{E}\left[(-1)^m\mathbf{h}_1\mathbf{h}_m^\dagger \times \frac{\det(\tilde{\mathbf{H}}_1\tilde{\mathbf{H}}_m^\dagger)}{\det(\tilde{\mathbf{H}}_1\tilde{\mathbf{H}}_1^\dagger)}\right] = 1. \tag{30}$$

Knowing also that $\mathrm{E}[\mathbf{h}_1\mathbf{h}_1^\dagger] = n_T$, we end up with $\Omega = \mathrm{E}[\beta_1^2] = n_T - M + 1$ and Theorem 3.2 has been proven.

Now, consider β_1^4 using (27). We get

$$\begin{aligned}\beta_1^4 = {} & (\mathbf{h}_1\mathbf{h}_1^\dagger)^2 + 2(\mathbf{h}_1\mathbf{h}_1^\dagger)\sum_{m=2}^{M}(-1)^{m+1}\mathbf{h}_1\mathbf{h}_m^\dagger\frac{\det(\tilde{\mathbf{H}}_1\tilde{\mathbf{H}}_m^\dagger)}{\det(\tilde{\mathbf{H}}_1\tilde{\mathbf{H}}_1^\dagger)} \\ & + \sum_{i=2}^{M}\left[\frac{\mathbf{h}_1\mathbf{h}_i^\dagger\det(\tilde{\mathbf{H}}_1\tilde{\mathbf{H}}_i^\dagger)}{\det(\tilde{\mathbf{H}}_1\tilde{\mathbf{H}}_1^\dagger)}\right]^2 \\ & + \sum_{\substack{i,j=2 \\ i\neq j}}^{M}\frac{(-1)^{i+j}\mathbf{h}_1\mathbf{h}_i^\dagger\det(\tilde{\mathbf{H}}_1\tilde{\mathbf{H}}_i^\dagger)\mathbf{h}_1\mathbf{h}_j^\dagger\det(\tilde{\mathbf{H}}_1\tilde{\mathbf{H}}_j^\dagger)}{\left[\det(\tilde{\mathbf{H}}_1\tilde{\mathbf{H}}_1^\dagger)\right]^2}.\end{aligned} \tag{31}$$

The expectation of β_1^4 can be analyzed by considering the expectation of each term on the RHS. Using the following results:

$$\mathrm{E}[(\mathbf{h}_1\mathbf{h}_1^\dagger)^2] = n_T^2 + n_T, \tag{32}$$

$$\mathrm{E}\left[\frac{(\mathbf{h}_1\mathbf{h}_1^\dagger)(\mathbf{h}_1\mathbf{h}_m^\dagger)\det(\tilde{\mathbf{H}}_1\tilde{\mathbf{H}}_m^\dagger)}{\det(\tilde{\mathbf{H}}_1\tilde{\mathbf{H}}_1^\dagger)}\right] = (-1)^m(n_T + 1), \tag{33}$$

$$\mathrm{E}\left[\left(\frac{\mathbf{h}_1\mathbf{h}_i^\dagger\det(\tilde{\mathbf{H}}_1\tilde{\mathbf{H}}_i^\dagger)}{\det(\tilde{\mathbf{H}}_1\tilde{\mathbf{H}}_1^\dagger)}\right)^2\right] = 2, \tag{34}$$

$$\mathrm{E}\left[\frac{(\mathbf{h}_1\mathbf{h}_i^\dagger)\det(\tilde{\mathbf{H}}_1\tilde{\mathbf{H}}_i^\dagger)(\mathbf{h}_1\mathbf{h}_j^\dagger)\det(\tilde{\mathbf{H}}_1\tilde{\mathbf{H}}_j^\dagger)}{\left[\det(\tilde{\mathbf{H}}_1\tilde{\mathbf{H}}_1^\dagger)\right]^2}\right] = (-1)^{i+j}, \tag{35}$$

we have

$$\mathrm{E}[\beta_1^4] = n_T^2 + M^2 + 3n_T - 3M - 2Mn_T + 2. \tag{36}$$

As a consequence, the diversity order of the channel, Ψ, can be expressed as

$$\Psi = \frac{\Omega^2}{\mathrm{E}[(\beta_1^2 - \Omega)^2]} = \frac{\Omega^2}{\mathrm{E}[\beta_1^4] - \Omega^2} \tag{37}$$

$$= n_T - M + 1. \tag{38}$$

From (37) to (38), we have used (22) and (36). This completes the proof of Theorem 3.3.

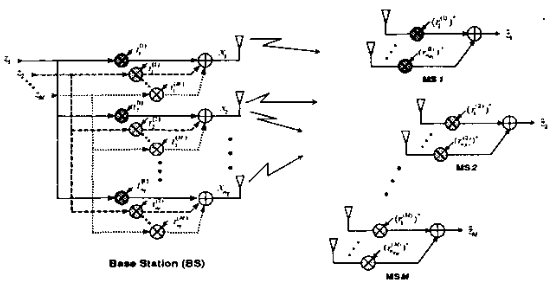

Fig. 1. System configuration of a multi-user MIMO antenna system.

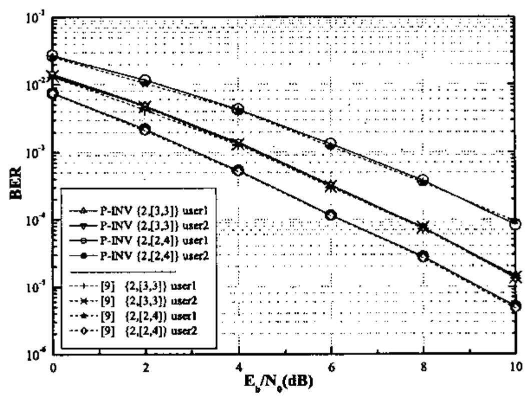

Fig. 2. Performance comparison of the proposed scheme and the method in [9] in flat Rayleigh fading channels.

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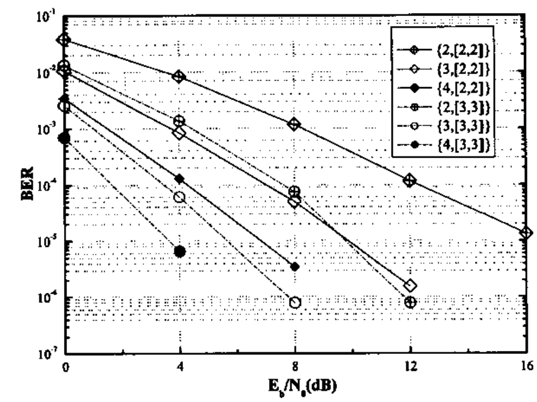

Fig. 3. Performance comparison of the proposed scheme with additional number of transmit and receive antennas in flat Rayleigh fading channels.

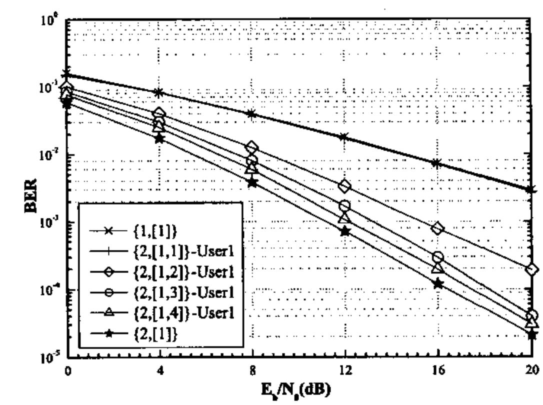

Fig. 4. Average BER performance of user 1 with increasing number of antennas of user 2.

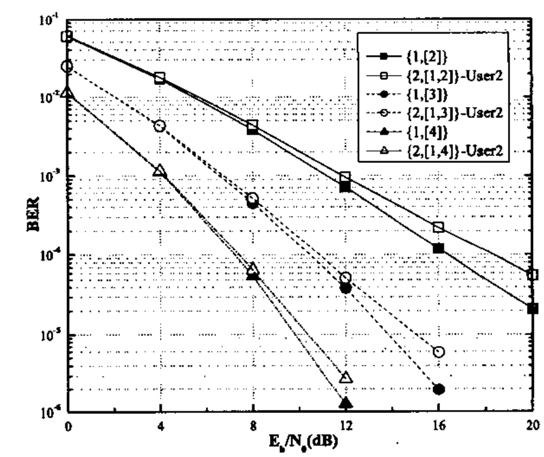

Fig. 5. Average BER performance of user 2 with increasing number of antennas of user 2.