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# Decentralized Power System Voltage Stability Proximity Indicator Based on Local Bus Information

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# ABSTRACT

In this paper, a decentralized power system voltage stability proximity indicator is presented. This indicator can consider the occurrence of not only the saddle node bifurcation but also the Hopf bifurcation, while only local bus information is needed for the decentralized voltage stability monitoring. The results of two power system examples discover the possibility of on-line decentralized voltage instability / collapse assessment.

## 1. INTRODUCTION

Voltage stability is the ability of a power system to maintain acceptable voltage at all buses in the system under normal conditions and after being subjected to a disturbance. In the last two decades several major network collapses such as the massive Tokyo system blackout in July 1987 occurred due to voltage instability problem in the world, therefore voltage stability phenomenon has been a major research area for system planning and operation<sup>[1]</sup>. One of the typical issues of concern for the system operators is how close the system to its voltage stability boundary.

In order to measure the distance between current system operation point and the system instability point or critical point, a lot of voltage stability proximity indicators have been proposed, such as the minimum singular value of the power flow Jacobian matrix<sup>[2][8]</sup>, the minimum eigenvalue of the nodal reduced system matrix<sup>[3]</sup>, voltage power sensitivity<sup>[4][5]</sup>, the angle distance between the current stable equilibrium point and the closest unstable equilibrium point of nonlinear power flow equations<sup>[6]</sup>, some kind of energy functions<sup>[7]</sup>, etc. All the above indicators are based on the assumption that voltage instability is caused only by the saddle node bifurcation.

However it has been shown by some examples that electric power systems can experience Hopf bifurcation that is related to voltage control instability on the upper portion of PV curves before the nose point<sup>[10]</sup>. When the load gradually Yixin Ni Felix F. Wu

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increases, a pair of complex eigenvalue related to the voltage control can go through the imaginary axis before the power flow Jacobian becomes singular. In such cases, therefore, the indicators, which ignore the possibility of Hopf bifurcation occurrence, will draw an optimistic conclusion. It should be very dangerous for system operation and control.

- In this paper, a decentralized power system voltage stability proximity indicator based on local bus information is presented. Based on the modified Jacobian matrix model presented in reference [13], this indicator is calculated by determining how 'close' the reduced matrix on frequency domain is to singular in several different frequency intervals. Its main characteristics include:
- It can be considered that the occurrence of not only the saddle node bifurcation but also the Hopf bifurcation in this indicator. When the frequency is restricted to zero, the proposed indicator can be used to measure the distance to saddle node bifurcation point. On the other hand, when frequency is forced not equal to zero, it is the index to show the distance to Hopf bifurcation point.
- It is based on the local bus information, which makes this indicator possible to be applied for decentralized voltage stability on-line monitoring. Since the reactive power can not be transferred to areas far away, voltage instability / collapse events usually appear due to localized shortage of reactive power or voltage control. This makes it possible for decentralized voltage stability on-line monitoring just with the local bus information, while networks with long distance to the area being monitored can be equalized.

In section 2, firstly the modified Jacobian matrix model is presented in frequency domain. And the nodal reduced matrix can be developed with this model. Based on the matrix, the decentralized power system voltage stability proximity indicator is proposed. Finally two examples are presented to illustrate the efficiency of this voltage stability indicator in section 3.

### 2 MATHEMATICAL MODEL

#### 2.1 Modified Jacobian Matrix Model

Consider a typical system model with nB buses, nG generator buses and nL load buses. In this paper, the generators are modeled with two-axis representation. The excitation systems are assumed as IEEE type-I. And the loads are presented with the load recovery model. Thus the overall system model in time domain can be shown as the following differential-algebraic type<sup>[13]</sup>:

$$\begin{cases} x = f(x, y) \\ 0 = g(x, y) \end{cases}$$
(1)

And then the above equations (1) can be linearized at the system operation point as the following form:

$$\begin{cases} \frac{d\Delta x}{dt} = A\Delta x + B\Delta y\\ 0 = C\Delta x + D\Delta y \end{cases}$$
(2)

where the incremental state variables  $\Delta x$  is defined as  $\Delta x = [\Delta x_g, \Delta x_l]^T$ ,  $\Delta x_g$  is the incremental state variables describing dynamics of the synchronous machine and its excitation system, and  $\Delta x_l$ describes the load dynamics, while  $\Delta y$  is defined as  $\Delta y = [\Delta \theta_1, \Delta V_1, \dots, \Delta \theta_{nB}, \Delta V_{nB}]^T$  i.e. the set of incremental phase angle and magnitude of voltage at each of the network buses.

Suppose that matrix D isn't singular then

 $\Delta y = -D^{-1}C\Delta x.$  (3) Replacing variable  $\Delta y$  in equation (2) with (3), yields:

$$\frac{d\Delta x}{dt} = (A - BD^{-1}C)\Delta x \tag{4}$$

Generally, after defining the system matrix  $\tilde{A}$  as  $\tilde{A} = A - BD^{-1}C$  the steady-state stability of the operating point can be determined by the eigenvalues of the system matrix.

Using the Laplace transformation, equation (4) in time domain can be changed to the following form in frequency domain.

$$\begin{cases} sI\Delta x = A\Delta x + B\Delta y \\ 0 = C\Delta x + D\Delta y \end{cases}$$
(5)

where I is the unit matrix with n dimensions, and  $s=j\omega$ .

Define that 
$$D(s)=D+C(sI-A)^{-1}B$$
 (6)  
as the modified Jacobian matrix. And the trajectory  
of  $det[D(s)]$  in complex plane, when  $\omega$  drift along  
the imaginary axis within the interval  $[0 + \infty]$ , is  
called characteristic trajectory of equation (5). The  
following conclusions can be drawn that:

(a) Matrix D(s) can be obtained by only modifying the 2x2 diagonal blocks of load flow Jacobian matrix. And each of these 2x2 diagonal blocks is determined by the dynamics of generators or loads at the corresponding buses. Thus we call the equation (5) in frequency domain modified Jacobian matrix model.

(b) It can be proved that no matter whether the operating point is saddle node or Hopf bifurcation point, the characteristic trajectory of equation (5) passes through the origin point in complex plane.

#### 2.2 Nodal Reduced Matrix

Generally unlike real power in the electric power systems, the reactive power can't be transferred through a long distance. Voltage instability / collapse events usually appear as the localized shortage of reactive power or voltage control. Therefore it is reasonable to ask whether it is possible to decentralized monitor and assess system voltage stability decentralized, where only the nodal bus information is used. Based on the above idea, a nodal reduced matrix is developed.

Suppose for any bus *i*, matrix D(s) in equation (6) can be reformulated as the following blocked matrix form.

$$\begin{bmatrix} D_{AA}(s) & D_{AB}(s) \\ D_{BA}(s) & D_{BB}(s) \end{bmatrix}$$
(7)

where  $D_{BB}(s) \in \mathbb{C}^{2x_2}$  is the 2x2 block matrix, whose related incremental variables is  $\Delta \theta_i$  and  $\Delta V_i$ .

Now the nodal reduced matrix of bus *i* is defined as  

$$D'_{RR}(s) = D_{RR}(s) - D_{RA}(s)D_{A}^{-1}(s)D_{AR}(s)$$
 (8)

Since det 
$$D(s) = \det D_{AA}(s) \det D'_{BB}(s)$$
 (9)

it can be proved that when the system operation point is saddle node or Hopf bifurcation point, there much exist a  $\omega_0$  ( $0 \le \omega_0 < +\infty$ ) and the nodal reduced matrix  $D'_{BB}(j \omega_0)$  at  $\omega_0$  is singular.

Therefore a new decentralized power system voltage stability proximity indicator is proposed as following:

$$L_B = \min_{0 \le \omega < +\infty} \left| \det D'_{BB}(j\omega) \right|$$
(10)

With this indicator, no matter whether the system operation point is saddle node or Hopf bifurcation point,  $L_B=0$ . After applying the above indicator to two example systems, it can be found that  $L_B$  approaches to zero with almost quadratic function to the load factor, which makes it possible for online assessment of the system voltage stability.

### 3. EXAMPLES

In this section, the indicator of equation (10) is evaluated for both the three machines system and the New England system.

3.1 The Three Machines System

For the three machines system<sup>[11]</sup> shown in Fig. 1, bus 1, 2 and 3's generator excitation systems are all IEEE Type-1 with the same parameters as Table 1. And each of the dynamic loads at load buses 5, 6 and 8 is modeled by the load recovery model with the same parameters as Table 2.



Fig.1 The Three Machines System

	Table 1 T	he excitatio	on system p	parameters	
K <sub>E</sub>	$T_E$	K <sub>F</sub>	$T_F$	KA	$T_A$
1.0	0.314	0.063	0.35	20	0.2
	Table 2	The dynam	nic load pa	rameters	
$T_{p}$	$T_q$	$n_{ps}$	$n_{qs}$	n <sub>pt</sub>	$n_{qt}$
10	10	0.6	0.6	12	1.2

Assume that the real and reactive power injected at each network buses changes in the following pattern: The real power at each load buses increases in steps at the same rate, while the reactive loads remain unchanged. The total amount of increased loads should be distributed by 2 and 3 generators in the same proportion. In this paper, the system loading condition is represented by the loading factor  $\gamma$ . When  $\gamma=0$ , the system is in the normal base loading condition. When each of the real power loads at three load buses increases by one per unit, the loading factor  $\gamma$  will be 1.

When the load factor  $\gamma$  increases step by step, the system voltage stability proximity indicator  $L_B$  is calculated with bus 5, 6 and 8 selected to be the bus *i* in  $L_B$ 's definition. The results ( $L_{B5}$ ,  $L_{B6}$ , and  $L_{B8}$ ) are shown in Fig. 2.

From Fig. 2, it can be found that the indicator  $L_B$  approaches to zero very smoothly and monotonously when load factor  $\gamma$  increases. Also after numerical curve fit manipulation, it can be found that  $L_B$  changes nearly as a quadratic function with  $\gamma$ .



## 3.2 New England System



For the New England System<sup>[12]</sup> shown in Fig. 3, all the load is modeled by the load recovery model with the same parameters as Table 3.

	Table 3 The dynamic load parameters							
$T_p$	$T_{q}$	$n_{ps}$	n <sub>q s</sub>	n <sub>pt</sub>	n <sub>at</sub>			
0.1	0.1	0.6	0.6	1.2	1.2			

After making the assumption that the real power at load buses 3, 4, 7 and 8 increases in steps at the same rate, while the reactive loads remain unchanged, while the total amount of increased loads should be distributed by all generators in the same proportion, the load factor  $\gamma$  can be defined as  $\gamma=0$  corresponding to the normal base loading condition. And  $\gamma = 1$  means that each of the real power loads at buses 3, 4, 7 and 8 increases by one per unit

When the load factor  $\gamma$  increases step by step, the proposed system voltage stability proximity indicator  $L_B$  is calculated related to bus 7, 8, 15 and 25 as the selected bus *i*. The results ( $L_{B7}$ ,  $L_{B8}$ ,  $L_{B15}$ , and  $L_{B25}$ ) are shown in Fig 4.



Fig. 4 Voltage Stability Indicator L<sub>B</sub>

The following two conclusion can also been drawn from Fig. 4.

- 5. The indicator  $L_B$  approaches to zero very smoothly and monotonously as load factor  $\gamma$  increases.
- 6. After the numerical curve fit manipulation, the quadratic function relationship between  $L_B$  and  $\gamma$  can also be discovered in this example.

## 4. CONCLUSION

Based on the modified Jacobian matrix model presented in [13], a new decentralized voltage stability proximity indicator is presented. This indicator can consider the occurrence of not only saddle node bifurcation but also Hopf bifurcation. Meanwhile the nodal reduced matrix is employed to calculate the proposed indicator, in which only local bus information is used.

It should be emphasized that by the means of nodal reduced matrix the information of the whole system can be 'concentrated' to the local bus. Therefore the proposed index  $L_B$  can be used as an indicator of the system voltage stability by just using local bus information.

Both three machines and New England example systems show that the proposed indicator can decrease monotonously and very smoothly to zero with the loading factor  $\gamma$ . And a quadratic function relationship between  $L_B$  and  $\gamma$  is found in above two examples, which makes it possible for on-line monitoring and assessment of system voltage stability in the future.

## 5. ACKNOWLEDGMENTS

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