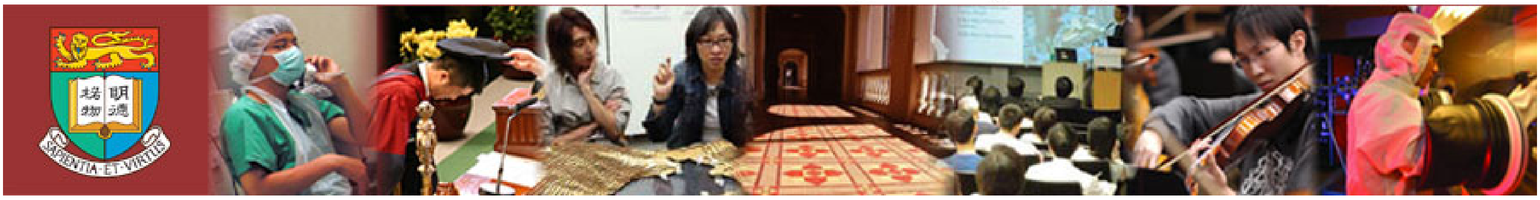

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Performance Analysis of Bandwidth Efficient Coherent Modulation Schemes with *L*-fold MRC and SC in Nakagami-*m* Fading Channels

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ABSTRACT

This paper presents the average symbol error rates (SERs) of the 16-ary phase shift keying (16PSK), 16-ary amplitude phase shift keying (16APSK) and 16-ary quadrature amplitude modulation (16QAM) with L-fold maximal ratio combining (MRC) and selection combining (SC) space diversity receptions over Nakagami-m fading channels. Numerical results manifested error performance improvement when L-fold MRC and SC diversity receptions were employed. Error performance improvement attributed to an L=2 MRC diversity reception is comparable with that attributed to an L=4 SC diversity reception for the above three 16-ary modulation systems in a Nakagami-m fading channel with m of 5.

I. INTRODUCTION

Over the years, compensation techniques for multipath channel fading in wireless communications have attracted much attention (see, e.g., [1-2] and references therein). Diversity combining, which skillfully combines multiple replicas of the received signals, has long been recognized as one of the effective compensation techniques for combating detrimental effects of channel fading. Two of the well known methods to combine these multipath components are MRC and SC [1]. While MRC is known as the optimal combining technique by maximizing the signal-to-noise ratio (SNR) of the combined signal, SC is considered as the easiest method of combining multipath components by selecting the diversity branch with the largest SNR.

In consideration of the mobile communications channel, Nakagami-m distribution [3] is regarded as one of the most versatile fading channel models. In these mobile communications channels, bandwidth efficient modulation schemes are usually employed as a result of rising number of mobile users and increasing popularity of multimedia applications. Coherent modulation schemes are not commonly used in mobile fading channels because of channel amplitude and phase variations. However, reliable fading estimation may still be available by the use of pilot tone. Therefore, it is still of interest to evaluate the error performance of coherent modulation schemes for its superiority over the noncoherent or differentially-coherent modulations. The aim of this paper is thus to present some closed form expressions for the average SER of the three bandwidth efficient coherent modulation schemes (16PSK, 16APSK and 16QAM) with L-fold MRC and SC space diversity receptions over Nakagami-m fading channels.

This paper is organized as follows. Section II will describe some backgrounds of the systems. The error performance analyses of 16PSK, 16APSK and 16QAM with L-fold MRC and SC over Nakagami-m fading channels are presented in Section III – V, respectively. Section VI presents the numerical results, and finally, some concluding remarks are given in Section VII.

II. BACKGROUND

In consideration of a communication system where the mobile communications fading channel is modeled as Nakagami-m distribution, the probability density function (pdf) of the instantaneous received SNR per symbol ρ_i on the ith branch of a receiver with diversity combining is given by [3]

$$p(\rho_i) = \left(\frac{m}{\Omega}\right)^m \frac{\rho_i^{\,m-1}}{\Gamma(m)} \exp\left(-\frac{m}{\Omega}\rho_i\right) \tag{1}$$

where m is the fading severity parameter with values from 0.5 to ∞, Ω is the average SNR per symbol per branch and $\Gamma(.)$ denotes the gamma function. Note that all branches are assumed to have the same values of m and Ω. With L-fold diversity combining implemented in the receiver and equal noise power in all the L branches, the instantaneous SNR per symbol γ at the output of the receiver is given by $\gamma = \sum_{i=1}^{L} \rho_i$ for MRC and $\gamma = \max(\rho_1, \rho_2, \cdots, \rho_L)$ for SC. The pdf of γ for the receiver with L-fold MRC diversity reception can be expressed as [4]

$$p_{MRC}(\gamma) = \left(\frac{m}{\Omega}\right)^{Lm} \frac{\gamma^{Lm-1}}{\Gamma(Lm)} \exp\left(-\frac{m}{\Omega}\gamma\right) \tag{2}$$

and the pdf of γ for the case of L-fold SC diversity reception is given by [5]

$$p_{SC}(\gamma)=\left(\frac{m}{\Omega}\right)^m \frac{L\gamma^{m-1}}{[\Gamma(m)]^L}\left\{\gamma\left(m,\frac{m\gamma}{\Omega}\right)\right\}^{L-1}\exp\left(-\frac{m}{\Omega}\gamma\right) \tag{3}$$

where $\gamma(\alpha, \beta)$ is the incomplete gamma function [6].

The use of (3) in error performance derivation will not result in a closed form solution because of its complicated form. However, (3) can be rewritten as in (4) for integer m [7]

$$p_{SC}(\gamma)=\frac{L}{\Gamma(m)}\sum_{l=0}^{L-1}(-1)^l\binom{L-1}{l}\exp\left(-(l+1)\frac{m}{\Omega}\gamma\right) \times\sum_{k=0}^{l(m-1)} b_k^l\left(\frac{m}{\Omega}\right)^{m+k}\gamma^{m+k-1} \tag{4}$$

where

$$b_0^l=1,\quad b_1^l=l,\quad b_{l(m-1)}^l=\frac{1}{(\Gamma(m))^l}$$
$$b_k^l=\frac{1}{k}\sum_{j=1}^{J_0}\frac{j(l+1)-k}{j!}b_{k-j}^l \tag{5}$$

and $J_0=\min(k,m-1),\ 2\le k\le l(m-1)-1$. With the channel statistics in (2) and (4), the average SER at the output of the receiver can be calculated by averaging the conditional probability of error over the pdf of γ, i.e.,

$$\overline{P}=\int_0^\infty P_e(\gamma)p(\gamma)\,d\gamma \tag{6}$$

where $P_e(\gamma)$ is the conditional probability of symbol error for a particular communications system on the assumption that γ is known. Before going into the derivation of the average SER, three general integrals are evaluated here and they can be applied to the derivation of average SER for the three 16-ary modulations. Let the first integral, $I_{MRC}(A)$, be

$$I_{MRC}(A)=\int_0^\infty erfc\left(\sqrt{A\gamma}\right)p_{MRC}(\gamma)\,d\gamma \tag{7}$$

where *erfc*(.) is the complementary error function. After substituting (2) into (7) and using a relation in [8], $I_{MRC}(A)$ becomes

$$I_{MRC}(A)=2\left(\frac{1-\mu_c}{2}\right)^{Lm}\sum_{k=0}^{Lm-1}\binom{Lm-1+k}{k}\left(\frac{1+\mu_c}{2}\right)^k \tag{8}$$

where

$$\mu_c=\sqrt{\frac{\Omega/m}{1/A+\Omega/m}}. \tag{9}$$

Now, the second integral $I_{SC}(A)$ is given by

$$I_{SC}(A)=\int_0^\infty erfc\left(\sqrt{A\gamma}\right)p_{SC}(\gamma)\,d\gamma. \tag{10}$$

After much simplifications, (10) can be written as

$$I_{SC}(A)=2L\sum_{l=0}^{L-1}\sum_{k=0}^{l(m-1)}\sum_{n=0}^{m+k-1}(-1)^l\binom{L-1}{l}\frac{b_k^l}{(l+1)^{m+k}} \times\frac{\Gamma(m+n+k)}{\Gamma(m)\Gamma(n+1)}\left(\frac{1-\mu_d}{2}\right)^{m+k}\left(\frac{1+\mu_d}{2}\right)^n \tag{11}$$

where

$$\mu_d=\sqrt{\frac{\Omega/m(l+1)}{1/A+\Omega/m(l+1)}}. \tag{12}$$

Finally, the following integral is considered,

$$I_1(p,q,r)=\int_0^\infty erfc^2\left(\sqrt{p\gamma}\right)exp(-q\gamma)\gamma^{r-1}\,d\gamma \tag{13}$$

and it can be expressed as [9]

$$I_1(p,q,r)=\left(\frac{1}{p}\right)^r\left\{\frac{\Gamma(r)}{(q/p)^r}-\frac{4}{\pi(q/p)^r}\sum_{k=0}^{r-1}\frac{\Gamma(r)}{(2k+1)} \times(q/p)^k\,{}_2F_1\left(\frac{1}{2}+k,1+k,\frac{3}{2}+k;-(q/p)-1\right)\right\} \tag{14}$$

where ${}_2F_1(a,b,c;x)$ is the Gauss hypergeometric function [6].

III. AVERAGE SER OF 16PSK

One of the most common bandwidth efficient coherent modulation scheme is 16PSK, and the classical conditional SER for 16PSK involve an integral which has to be evaluated numerically. However, there exists a simple approximation given by [10]

$$P_{e,16PSK}(\gamma)\cong erfc\left[\sqrt{\left(\frac{1-Cos(\pi/8)}{2}\right)\gamma}\right]. \tag{15}$$

After the substitution of (15) and (2) into (6), the average SER of 16PSK with L-fold MRC diversity reception over Nakagami-m fading channels can be calculated as

$$\overline{P}_{MRC,16PSK} = \int_0^\infty erfc\left[\sqrt{\left(\frac{1-Cos(\pi/8)}{2}\right)\gamma}\right] p_{MRC}(\gamma)d\gamma \quad (16)$$

One can immediately recognize that (16) can be represented in terms of $I_{MRC}(A)$ and be written as

$$\overline{P}_{MRC,16PSK} = I_{MRC}\left(\frac{1-Cos(\pi/8)}{2}\right). \quad (17)$$

For the case of L-fold SC diversity reception, the average SER can be found by substituting (15) and (4) into (6) and it can be expressed in terms of $I_{SC}(A)$ as

$$\overline{P}_{SC,16PSK} = I_{SC}\left(\frac{1-Cos(\pi/8)}{2}\right). \quad (18)$$

IV. AVERAGE SER OF 16APSK

16APSK (or 16 star QAM), which utilizes two concentric 8PSK rings, has been designed for robust differential reception over typical fading channel. However, the focus of this paper is to evaluate coherent modulations, thus coherent reception of 16APSK is investigated. The conditional SER of 16APSK with coherent reception is given by [11]

$$P_{e,16APSK}(\gamma) \cong \frac{1}{2}\left\{erfc\left(\sqrt{D\gamma}\right)+erfc\left(\sqrt{E\gamma}\right)+erfc\left(\sqrt{F\gamma}\right)\right\} \quad (19)$$

where

$$D = \frac{8\sin^2(\pi/8)}{1+\beta^2}$$

$$E = \frac{2(\beta-1)^2}{1+\beta^2} \text{ and } F = \frac{8\beta^2\sin^2(\pi/8)}{1+\beta^2} \quad (20)$$

and β is the ring ratio. After the substitution of (19) and (2) into (6), the average SER of 16APSK with L-fold MRC diversity reception over Nakagami-m fading channels can be again expressed in terms of $I_{MRC}(A)$ by

$$\overline{P}_{MRC,16APSK} = \frac{1}{2}\left\{I_{MRC}(D)+I_{MRC}(E)+I_{MRC}(F)\right\}. \quad (21)$$

In addition, the average SER for the case of L-fold SC diversity reception is evaluated by substituting (19) and (4) into (6) and it can be simplified as

$$\overline{P}_{SC,16APSK} = \frac{1}{2}\left\{I_{SC}(D)+I_{SC}(E)+I_{SC}(F)\right\}. \quad (22)$$

V. AVERAGE SER OF 16QAM

M-ary QAM has been recently proposed for wireless communications [12], and thus, the error performance of 16QAM is investigated here. The conditional SER of 16QAM is given by [12]

$$P_{e,16QAM}(\gamma) = \frac{3}{2}erfc\left(\sqrt{\frac{2}{5}\gamma}\right) - \frac{9}{16}erfc^2\left(\sqrt{\frac{2}{5}\gamma}\right). \quad (23)$$

With the substitution of (23) and (2) into (6), the average SER of 16QAM with L-fold MRC diversity reception in Nakagami-m fading channels is to be evaluated by

$$\overline{P}_{MRC,16QAM} = \frac{3}{2}\int_0^\infty erfc\left(\sqrt{\frac{2}{5}\gamma}\right)p_{MRC}(\gamma)d\gamma - \frac{9}{16\,\Gamma(Lm)} \times\left(\frac{m}{\Omega}\right)^{Lm}\int_0^\infty \gamma^{Lm-1}\exp\left(-\frac{m}{\Omega}\gamma\right)erfc^2\left(\sqrt{\frac{2}{5}\gamma}\right)d\gamma \quad (24)$$

and it can be simplified as

$$\overline{P}_{MRC,16QAM} = \frac{3}{2}I_{MRC}\left(\frac{2}{5}\right) - \frac{9}{16\,\Gamma(Lm)}\left(\frac{m}{\Omega}\right)^{Lm} \times I_1\left(\frac{2}{5},\frac{m}{\Omega},Lm\right). \quad (25)$$

The average SER for the case with L-fold SC diversity reception is calculated by substituting (23) and (4) into (6) as

$$\overline{P}_{SC,16QAM} = \frac{3}{2}\int_0^\infty erfc\left(\sqrt{\frac{2}{5}\gamma}\right)p_{SC}(\gamma)d\gamma - \frac{9L}{16\,\Gamma(m)}\sum_{l=0}^{L-1}\sum_{k=0}^{l(m-1)}(-1)^l b_k^l\binom{L-1}{l}\left(\frac{m}{\Omega}\right)^{m+k} \times\int_0^\infty \gamma^{m+k-1}\exp\left(-(l+1)\frac{m}{\Omega}\gamma\right)erfc^2\left(\sqrt{\frac{2}{5}\gamma}\right)d\gamma \quad (26)$$

and finally (26) can be written as

$$\overline{P}_{SC,16QAM} = \frac{3}{2}I_{SC}\left(\frac{2}{5}\right) - \frac{9L}{16\,\Gamma(m)}\sum_{l=0}^{L-1}\sum_{k=0}^{l(m-1)}(-1)^l \times b_k^l\binom{L-1}{l}\left(\frac{m}{\Omega}\right)^{m+k} I_1\left(\frac{2}{5},(l+1)\frac{m}{\Omega},m+k\right). \quad (27)$$

VI. NUMERICAL RESULTS

By using the closed form expressions in (17), (18), (21), (22), (25) and (27), the numerical values of the average SERs for the three 16-ary coherent modulation schemes with L=1,2,3,4 diversity branches are plotted in figures 1 to 6. Figures 1 and 2 show the average SER of 16PSK with both MRC and SC diversity receptions in Nakagami-m fading channels for m=5 and m=10, respectively. Figures 3 and 4 show the corresponding numerical values for 16APSK with β=2, and the cases for 16QAM are shown in figures 5 and 6.

As for the case of 16PSK in figure 1, the required average SNRs per symbol for an average SER of 10^{-5} are 23.5dB, 21dB and 19.4dB for L=2,3,4 MRC diversity receptions. The corresponding values for SC diversity receptions are 26dB, 24.7dB and 24dB. Note that the required SNR per symbol for the case of no diversity (i.e., L=1) is 29.7dB. As expected, the performance improvement attributed to MRC is more significant than that to SC for the same number of diversity branches. Observed from these numerical values, an increase of L from 2 to 3 rendered more performance improvement than from 3 to 4. It indicates that the performance improvement is somewhat retained when additional diversity branch is employed to an existing high number of diversity branches system. Also, an additional diversity branch in MRC calls for more improvement than if it is added to SC when both have the same number of branches. Thus, the performance improvement by employing MRC is even more significant than SC when L is large. One final observation is that the error performance of using L=2 MRC diversity reception is always better than that of using L=4 SC diversity reception for m=10. However, the error performance of using L=2 MRC and L=4 SC diversity receptions are very close for m=5 and there is a cross-over point at average SER of around 10^{-7}; the error performance of using L=2 MRC is better than that of using L=4 SC when the average SER is above 10^{-7}, the opposite is true when the average SER is below 10^{-7}. Thus, an L=4 SC diversity system can mimic an L=2 MRC diversity system when the channel experiences severe fading. Similar conclusions can also be drawn from the other two modulations.

VII. CONCLUSIONS

This paper derived some closed form expressions for the average SER of 16PSK, 16APSK and 16QAM with L-fold MRC and SC (with integer m) diversity receptions over Nakagami-m fading channels. Results show that MRC perform significantly better than SC for the same number of branches and this is more pronounced when the number of diversity branches is large. Results also show that the error performance of L=2 MRC and L=4 SC diversity systems are comparable when the fading severity parameter m is small.

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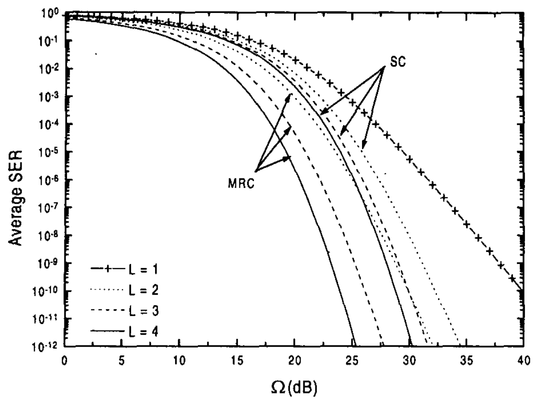

Figure 1. Average SER (versus Ω) of *L*-fold MRC and SC for 16PSK in Nakagami-*m* fading channel, *m*=5.

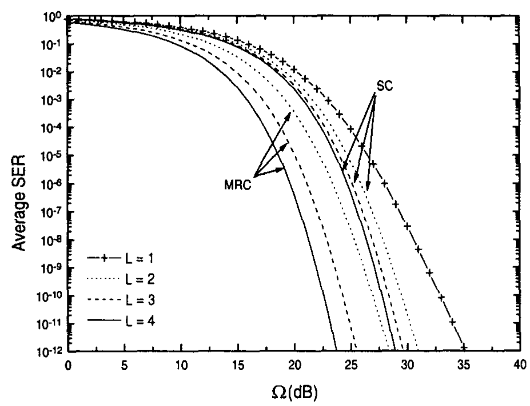

Figure 2. Average SER (versus Ω) of *L*-fold MRC and SC for 16PSK in Nakagami-*m* fading channel, *m*=10.

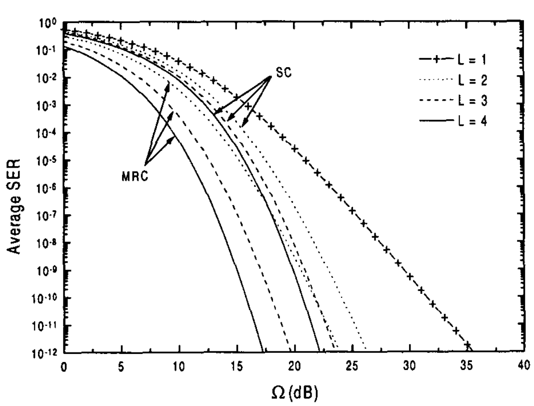

Figure 3. Average SER (versus Ω) of *L*-fold MRC and SC for 16APSK in Nakagami-*m* fading channel, *m*=5.

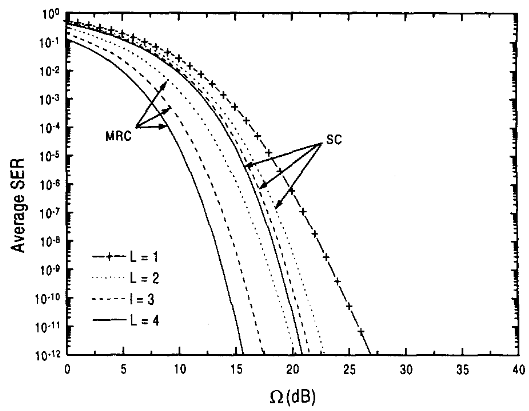

Figure 4. Average SER (versus Ω) of *L*-fold MRC and SC for 16APSK in Nakagami-*m* fading channel,*m*=10.

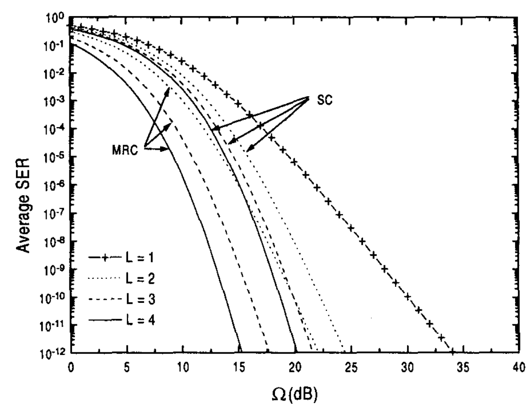

Figure 5. Average SER (versus Ω) of *L*-fold MRC and SC for 16QAM in Nakagami-*m* fading channel, *m*=5.

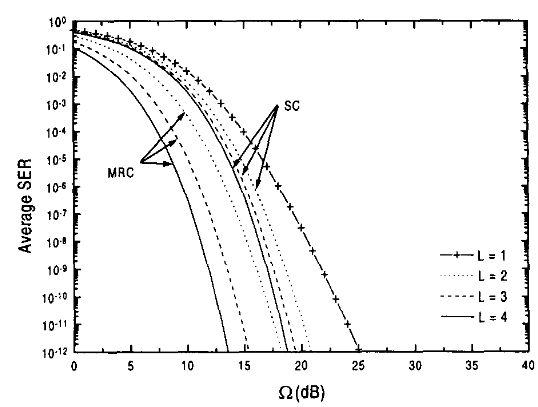

Figure 6. Average SER (versus Ω) of *L*-fold MRC and SC for 16QAM in Nakagami-*m* fading channel,*m*=10.