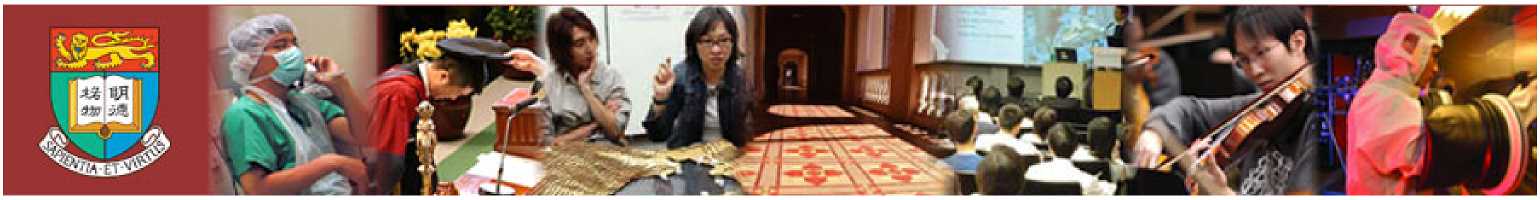

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Matching Patterns of Line Segments by Eigenvector Decomposition

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Abstract

This paper presents an algorithm for matching line segments in two images which are related by an affine transformation. Images are represented as patterns of line segments. Areas defined between line segments are used to describe a line pattern in the form of a proximity matrix. Matches are determined by comparing the feature vectors obtained from eigenvalue decomposition of the proximity matrices. Reliable matches of line segments are obtained in both synthetic and real images.

1. Introduction

Feature matching has always been a very difficult problem in computer vision and many different solutions have been proposed. Scott and Longuet-Higgins [1] have proposed an algorithm for matching two 2D point patterns. In their algorithm, point patterns are described by their inter-image distances in the form of a proximity matrix. However, it does not work well when large rotation is introduced between images. In order to overcome this problem, Shapiro and Brady [2] used intra-image distance instead of inter-image distance. Excellent results are obtained for translation and rotation motions. In order to cope with affine transformations, Xue and Teoh [3] used area ratios as entries of proximity matrices. As area ratio is affine invariant, the method works well for affine transformations. However, area ratios defined depend on the convex hull of the point set, therefore the result deteriorates if some outer most points are missing.

All the above methods are point-based. Points are very sensitive to noise and more difficult to extract accurately than lines. Park, Lee and Lee [4] proposed a method for line matching. However the method involves complicated preliminary tests, and the proximity matrix is not affine invariant.

In this paper, an eigenvector approach is proposed for line matching between two line patterns. The method is new in that areas defined between line segments are used to represent line patterns. It works well for patterns under affine transformation, even in cases with missing features.

2. The New Eigenvector Approach

The ratio of two areas is a well-known affine invariant. It can be shown that corresponding areas under an affine transformation are related by a scale factor. Consider three points (x_1, y_1), (x_2, y_2) and (x_3, y_3), the area enclosed by them is

$$S^a = \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{vmatrix} \tag{1}$$

Suppose (w_1, v_1), (w_2, v_2) and (w_3, v_3) are the corresponding points of (x_1, y_1), (x_2, y_2) and (x_3, y_3) after applying an affine transformation A:

$$A = \begin{bmatrix} r_{11} & r_{12} & t_1 \\ r_{21} & r_{22} & t_2 \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

Then,

$$\begin{bmatrix} w_i \\ v_i \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix}\begin{bmatrix} x_i \\ y_i \end{bmatrix} + \begin{bmatrix} t_1 \\ t_2 \end{bmatrix}, \ (i = 1,2,3), \tag{3}$$

The area enclosed by (w_1, v_1), (w_2, v_2) and (w_3, v_3) is

$$S^b = \frac{1}{2}\begin{vmatrix} w_1 & w_2 & w_3 \\ v_1 & v_2 & v_3 \\ 1 & 1 & 1 \end{vmatrix}$$

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$$= \frac{1}{2}\begin{vmatrix} r_{11} & r_{12} & t_1 \\ r_{21} & r_{22} & t_2 \\ 0 & 0 & 1 \end{vmatrix}\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \left(r_{11}r_{22} - r_{21}r_{12}\right)S^a = \alpha S^a \qquad (4)$$

From equation (4), S^b / S^a is a constant α determined by A.

In our algorithm, images are represented as line segments. First, edges are extracted from images. The thinned edges are represented as chains and straight lines segments are then fitted to the chains under prescribed error tolerence.

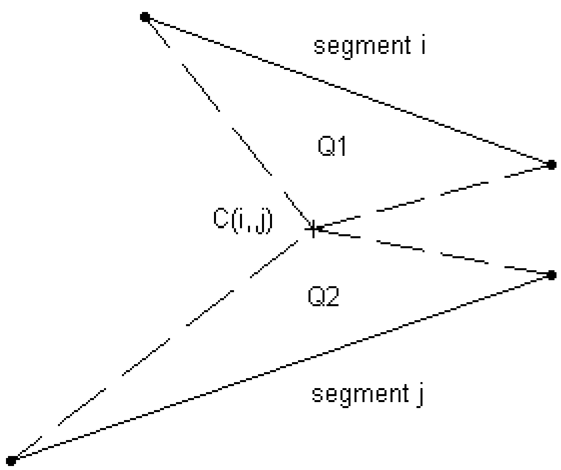

Figure 1. Segment i and segment j

Assuming a and b are two line patterns to be matched. Suppose there are M and N segments in a and b respectively. Consider segment i and segment j, both existing in the same line pattern, as shown in Figure 1, where the two segments are represented by solid lines. The centroid of the two line segments is denoted C(i,j). Let Q_1 be the area enclosed by segment i and the centroid, and Q_2 be the area enclosed by segment j and the centroid. The entry G_{ij}^x ($x = a, b$) of an area matrix G^x is defined to be the average area $\left(Q_1 + Q_2\right)/2$. Hence G^x is square and symmetric. As mentioned above, corresponding areas under affine transformation are related by a factor α. Elements in G^a and G^b are actually corresponding areas arranged in different orders. Scaling G^a by α will turn the corresponding entries in the two area matrices into affine invariants. Since the elements in the two area matrices differ by a scaling factor α, their eigenvalues also differ by α after being arranged in decreasing order. To calculate α, only the first k (k<=min(M,N)) most significant eigenvalues are used

$$\alpha = \sum_{i=1}^{k} \left\| \lambda_i^b \right\|_2 \Big/ \sum_{m=1}^{k} \left\| \lambda_m^a \right\|_2 \qquad (5)$$

Proximity matrices H^x are formed by applying Gaussian weightings to the elements of the scaled area matrices:

$$H_{mn}^a = \exp\left(-\alpha G_{mn}^a / 2\sigma^2\right),$$
$$H_{ij}^b = \exp\left(-G_{ij}^b / 2\sigma^2\right),$$
$$\text{for } 1 \le m, n \le M, 1 \le i, j \le N \qquad (6)$$

The purpose of Gaussian weightings is to localize the interactions between segments. Since larger G_{ij}^x means greater distance between segments i and j, the Gaussian weightings make entries in G^x with greater values less dominating. σ controls the size of neighbourhood of interactions. Since G^x is square, positive definite and symmetric, it can be decomposed by an eigenvalue decomposition into

$$G = VDV^T$$

with

$$V = \left[E_1 \mid E_2 \mid E_3 \mid \ldots \mid E_m\right]$$

where the columns V are the eigenvectors of G and D is a diagonal matrix containing the eigenvalues in decreasing order. The rows of V are extracted as feature vectors F as follow:

$$V = \left[F_1^T, F_2^T, \ldots F_m^T\right]^T$$

After truncation and sign correction, these feature vectors are used to calculate the distance Z_{ij} [3], defined as

$$Z_{ij} = \left\| F_i^a - F_j^b \right\|^2 \text{ for } 1 \le i \le M, 1 \le j \le N \qquad (7)$$

If distance Z_{ij} is zero, it indicates a perfect match of segment i in pattern a with segment j in pattern b. In practice any Z_{ij} below a prescribed threshold and is the minimum for both row i and column j of Z is accepted as a match.

3. Experimental Results

Here we demonstrate our algorithm by using two sets of images: the first set is cropped from an image 'trees' available in Matlab image processing toolbox, the other set contains a box.

The edges of the image 'trees' (shown in Figure 2) is extracted, cropped and divided into 133 line segments (shown in Figure 3). Dots represent the end points of line segments; circles represent the midpoints of line segments. A second pattern is formed by applying an affine transformation to the pattern of line segments in Figure 3 (shown in Figure 4). The two patterns of line segments are therefore related by an affine

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transformation. All 133 segments are matched correctly. The result is shown in Figure 5, where 'o' and '+' are used to mark matched segment midpoints of the two patterns and solid lines represent correspondences of segments. Line segments are not shown for better visualization.

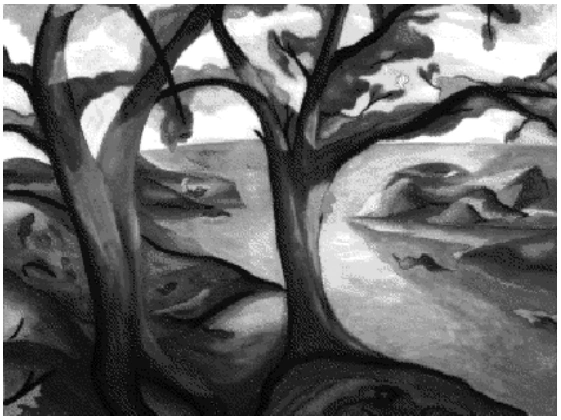

Figure 2. 'Trees'

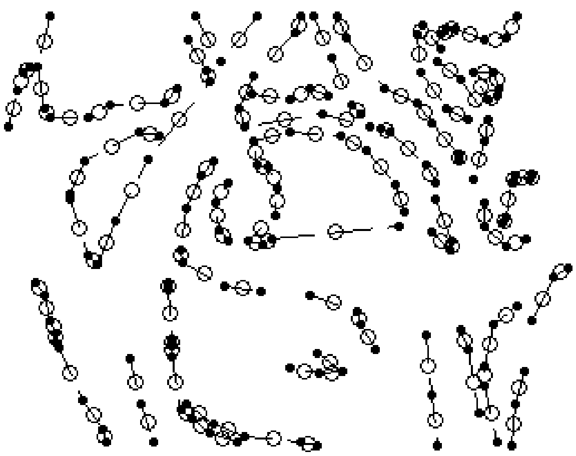

Figure 3. Pattern of line segments extracted from 'trees'

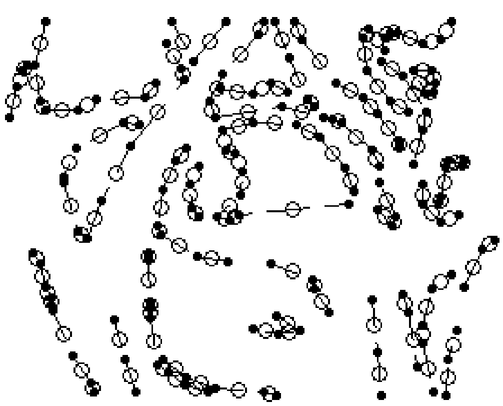

Figure 4. Pattern of line segments obtained by applying an affine transformation to Figure 3

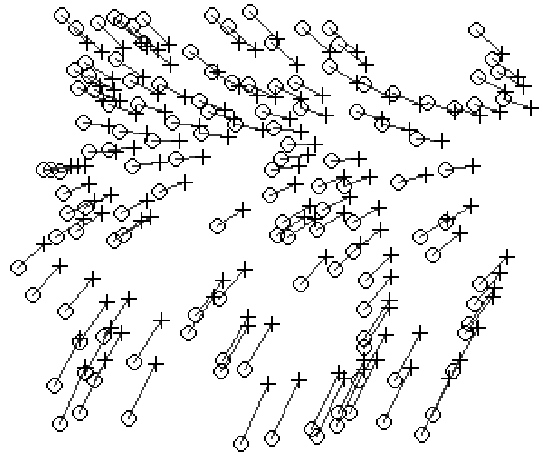

Figure 5. Matched line segments of 'trees'

In the second example, image of a box with triangles on its faces are taken by a digital camera. These images are taken at different angles and are not affinely related. We will refer to the first image as 'box 1' and the second image as 'box 2', which are shown in Figures 6 and 7 respectively. Patterns of line segments are extracted from these images (shown in Figures 8 and 9), where dots represent the end points of line segments, and circles represent the midpoints of line segments. As seen in the figures, there are 18 segments extracted form both 'box 1' and 'box 2'. The matched result is shown in Figure. 10. Symbols are used with the same meanings as in Figure. 5 except that the two patterns of segments are drawn in dotted and dashed lines respectively. All 18 segments are matched correctly. Therefore it can be seen that the method works well even in cases with real images which are not exactly related by affine transformations.

4. Conclusion

We have proposed a new line matching algorithm making use of areas defined between line segments. These areas are then scaled to be affine invariant. Proximity matrices are calculated from these areas. Matches are obtained by comparing feature vectors obtained from eigenvalue decomposition of the proximity matrices. Reliable matches of line segments are obtained in both synthetic and real images.

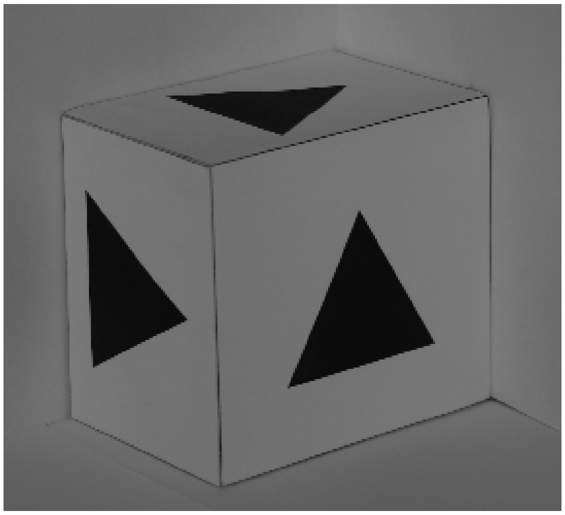

Figure 6. 'Box 1'

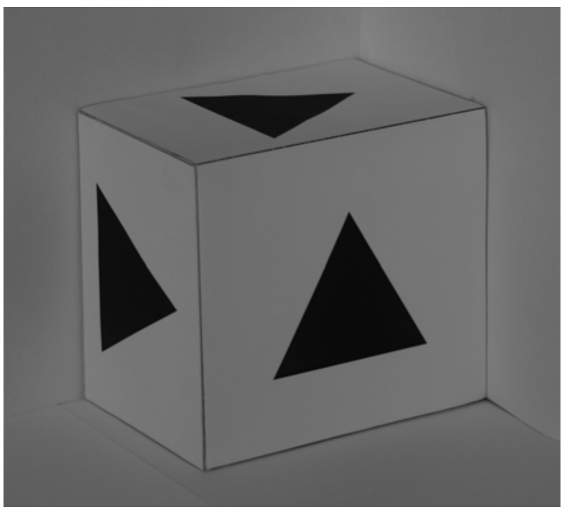

Figure 7. 'Box 2'

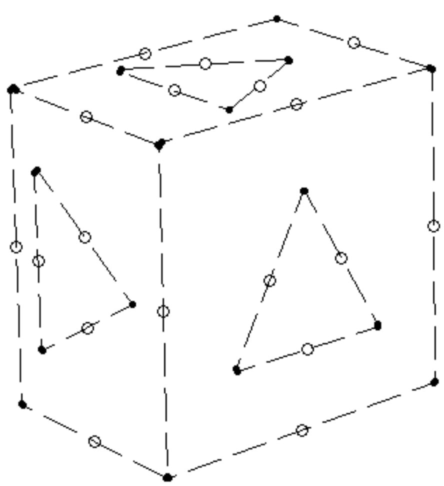

Figure 8. Pattern of line segments extracted from 'Box 1'

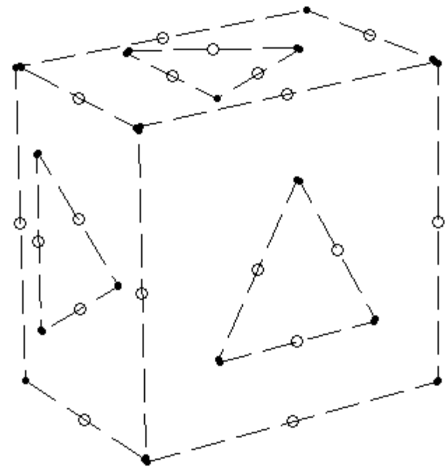

Figure 9. Pattern of line segments extracted from 'Box 2'

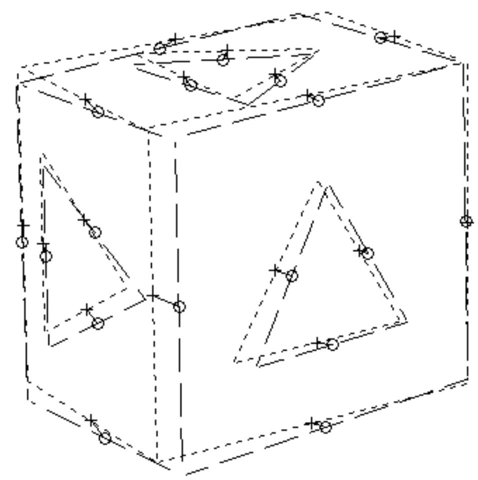

Figure 10. Matched line segments of box

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