A SIMULATION MODEL FOR HF CHANNELS THAT ARE CHARACTERIZED BY NAKAGAMI m-FADING, $m < 1$*

Kun-Wah Yip and *Tung-Sang Ng*
kwyip@eee.hku.hk tsng@eee.hku.hk

Department of Electrical and Electronic Engineering
The University of Hong Kong
Pokfulam Road, Hong Kong
Fax: + 852 + 2559 + 8738
Phone: + 852 + 2857 + 8406

Abstract **— Experimental results have indicated that the Nakagami m-distribution with the fading parameter m in the range $0.5 < m < 1$ can be used to model the fading characteristic of a HF channel when its fading is more severe than Rayleigh fading. In this paper, we propose a mathematical model for simulating a HF channel that is characterized by m-fading, $m < 1$. The procedure for implementation is given. Numerical results demonstrate that statistical properties of the samples generated from the proposed model are close to the required ones.**

Index Terms **— Fading channels, HF channels, Simulation, Stochastic processes.**

I. INTRODUCTION

Although Rayleigh distribution has been shown to be appropriate for modeling the short-term fading statistics of a HF ionospheric channel [1], it is possible that the channel fading becomes more severe than Rayleigh fading in some occasions due to high variability of the HF channel from time to time and from one geographical location to another. The phenomenon of fading more severe than Rayleigh fading was observed by Nakagami [2] in a series of channel measurements for some long-distance HF communication links. His measurement results have indicated that the m-distribution with the fading parameter m in the range $0.5 \le m < 1$ is useful for modeling the fading characteristic of a HF channel when the fading is more severe than Rayleigh fading. Note that the Rayleigh distribution is a special case of the m-distribution when $m = 1$.

In the design of a HF communication link, the system designer may want to ensure that the performance of the communication link is satisfactory not only in a Rayleigh fading environment but also in an environment characterized by fading more severe than Rayleigh fading. The need of a robust communication link arises when reliability is a concern such as in tactical command, control and communication systems. To perform computer or hardware simulation of a HF communication link operating over an m-fading channel for $m < 1$, one is required to generate a complex random process that fits a given Doppler power spectrum and that the amplitude follows an m-distribution with $m < 1$. Although there are a number of known techniques and simulation systems that can be used to generate a complex Gaussian process for simulating a Rayleigh fading channel (e.g., [3]-[7]), the technique for simulating an m-fading channel for $m < 1$ has not appeared in the previous literature to the best of authors' knowledge.

In Section II, we propose a mathematical model that enables one to generate a complex random process for simulating an m-fading channel, $m < 1$. Emphasis is given to applications of the proposed model for HF communications. Since the original m-fading channel model [2] does not provide a phase characterization, we assume that the channel phase shift is uniformly distributed over $[0,2\pi)$. This assumption seems reasonable for an m-fading channel where the channel fading is more severe than Rayleigh fading. We should mention that a first-order hidden Markov model

* This work was supported by the Hong Kong Research Grants Council and by the University Research Committee and the CRCG of The University of Hong Kong, Hong Kong.

0-7803-4984-9/98/$10.00

can be used to generate a Nakagami m-fading process as proposed in [8]. However, the generated fading process is real-valued. While this process simulates the behavior of the amplitude for an m-fading channel, the phase is not generated. Our proposed model is capable of generating both amplitude and phase.

Antenna diversity is a well known technique in HF communications for combating adverse effects due to fading. The technique described in this paper is useful for simulating a communication system with antenna diversity when the diversity branches are independently faded. The corresponding technique for simulating diversity channels of correlated fading is given in detail in a fuller version of this paper [9]. In [9], the simulation technique for a multipath, wideband HF channel characterized by m-fading, $m < 1$, is also given.

The rest of the paper is organized as follows. In Section III, details on the implementation of the proposed model to simulation are given. A numerical example illustrating the use of the proposed model follows in Section IV. Finally, conclusions are drawn in Section V.

II. THE SIMULATION MODEL

Let $z(t) = r(t)e^{j\theta(t)}$ be a wide-sense-stationary (WSS) complex random process that is characterized by the autocorrelation function

$$R_z(\Delta t) = E\{z(t)z^*(t+\Delta t)\} \tag{1}$$

and satisfies the following properties:

(i) $r(t)$ is Nakagami m-distributed with the second moment $\Omega = R_z(0)$ and the fading parameter $m = \Omega^2/E\{(r(t)^2-\Omega)^2\}$ limited in the range $m \in [0.5,1)$;

(ii) $\theta(t)$ is uniformly distributed over $[0,2\pi)$; and

(iii) $r(t)$ and $\theta(t)$ are mutually independent.

The probability density function (PDF) of $r(t)$ for an arbitrary time t is given by [2]

$$p(r(t)=r) = \begin{cases} \frac{2}{\Gamma(m)}\left(\frac{m}{\Omega}\right)^m r^{2m-1} e^{-mr^2/\Omega} & r \geq 0 \\ 0 & r < 0 \end{cases} \tag{2}$$

where $\Gamma(\cdot)$ is the gamma function and m is in the range $0.5 \leq m < 1$. Our task is to develop a simulation model that can be used to realize $z(t)$. In the development of this model, the following lemma is required.

Lemma 1: Let r_z be an m-distributed random variable with the second moment $E\{r_z^2\} = \Omega$ and the fading parameter $m \in [0.5,1)$. Let r_w be a Rayleigh distributed random variable with $E\{r_w^2\} = \Omega/m$. If r_z and r_w satisfy the functional relationship

$$r_z = \sqrt{\xi}\, r_w \tag{3}$$

where ξ is a non-negative random variable independent of r_w, then ξ has a standard beta distribution with parameters m and $1 - m$. The PDF of ξ is given by [10]

$$p(\xi) = \begin{cases} \frac{1}{B(m,1-m)}\xi^{m-1}(1-\xi)^{(1-m)-1} & 0 < \xi < 1 \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

where $B(a,b) = \Gamma(a)\Gamma(b)/\Gamma(a+b)$ is a beta function. □

The proof is given in Appendix 1. Let θ be a random variable uniformly distributed over $[0,2\pi)$ and independent of r_z, r_w and ξ. It follows that $w = r_w e^{j\theta}$ is a zero-mean complex Gaussian random variable with uncorrelated real and imaginary parts. Multiplying both sides of (3) by $e^{j\theta}$ gives $z = \xi^{1/2}w$ where $z = r_z e^{j\theta}$. The two expressions of z imply that $z(t)$ can be represented by

$$z(t) = \mu(t)w(t) \tag{5}$$

where (i) $\mu(t)$ is a WSS non-negative random process and $\mu^2(t)$ follows a standard beta distribution with parameters m and $1 - m$, and (ii) $w(t)$, being independent of $\mu(t)$, is a zero-mean WSS complex Gaussian process with uncorrelated real and imaginary parts and with an autocorrelation function

$$R_w(\Delta t) = E\{w(t)w^*(t+\Delta t)\} \tag{6}$$

satisfying $R_w(0) = \Omega/m$. In addition, $\mu(t)$ is required to satisfy the condition

$$\frac{R_z(\Delta t)}{R_z(0)} = \frac{R_\mu(\Delta t)}{R_\mu(0)} \cdot \frac{R_w(\Delta t)}{R_w(0)} \tag{7}$$

where

$$R_\mu(\Delta t) = E\{\mu(t)\mu(t+\Delta t)\}. \tag{8}$$

This condition is obtained by substituting (5) into (1) followed by dividing the resultant expression by $R_z(0)$. The representation of $z(t)$ by (5) is the desired simulation model. One can generate $z(t)$

by generating a complex Gaussian process $w(t)$ and an appropriate square-root-beta random process $\mu(t)$ followed by substituting $w(t)$ and $\mu(t)$ into (5). The complex Gaussian process $w(t)$ can be generated by an established technique, e.g., [3]-[5]. The non-Gaussian process $\mu(t)$ can be generated by a nonlinear transformation of a Gaussian process [11]. Appendix 2 lists the procedure for generating $\mu(t)$.

III. IMPLEMENTATION FOR SIMULATION

It is desired to realize a complex random process $z(t)$ in order to simulate an m-fading channel, $m < 1$, with a given Doppler power spectrum $S_z(f)$. The power spectrum $S_z(f)$ is related to $R_z(\Delta t)$ by

$$S_z(f) = \int_{-\infty}^{\infty} e^{-j2\pi f(\Delta t)} R_z(\Delta t) d\Delta t. \quad (9)$$

Since Doppler power spectra of HF channels that are of practical interest are bandlimited, we consider only the case of generating a bandlimited process $z(t)$. Let $f_{\max}$ be the frequency such that $S_z(f)$ is zero or negligible for $|f| > f_{\max}$. To realize $z(t)$ one can generate a sequence of equidistant discrete-time samples $z(nT_s)$, $T_s = (\alpha f_{\max})^{-1}$, followed by interpolation to obtain the value of $z(t)$ for an arbitrary time t, where $\alpha \geq 2$ is the oversampling factor. A value of $\alpha \geq 8$ is usually sufficient to generate $z(t)$ without appreciable loss of accuracy [4]. After the value of α is determined, the rest of the procedure except interpolation directly follows from an application of the proposed simulation model. The procedure is given as follows.

1) Compute $R_z(\Delta n \cdot T_s)$, $\Delta n = -N, \cdots, N$, by (9), where N is a sufficiently large integer. This sequence of $R_z(\Delta n \cdot T_s)$ is used in generating $z(nT_s)$'s so that a greater value of N improves the accuracy of the autocorrelation property of the generated $z(nT_s)$'s although more computation is required.
2) Select appropriate values of $R_\mu(\Delta n \cdot T_s)$ and $R_w(\Delta n \cdot T_s)$, $\Delta n = -N, \cdots, N$, that satisfy (7). It is recommended that these autocorrelation values may be selected such that $\mu(nT_s)$'s and $w(nT_s)$'s can be generated with minimal computation effort. Since generation of correlated $\mu(nT_s)$'s usually involves a computationally expensive step of nonlinear transformation of a Gaussian process, it is desirable, whenever possible, to select $R_\mu(\Delta n \cdot T_s)$ values such that $\mu(nT_s)$'s are statistically independent. There are a number of fast algorithms that generate independent beta random variates [10, ch. 25.2] so that independent $\mu(nT_s)$'s can also be efficiently generated. In the special case of independent $\mu(nT_s)$'s,

$$\frac{R_\mu(\Delta n \cdot T_s)}{R_\mu(0)} = K_m, \quad \Delta n \neq 0$$

where

$$K_m = \frac{E\{\mu(t)\} \times E\{\mu(t+\Delta n \cdot T_s)\}}{E\{\mu^2(t)\}}$$
$$= \frac{4}{\pi m}\left[\frac{\Gamma(m+\frac{1}{2})}{\Gamma(m)}\right]^2 \quad (10)$$

Since $|R_z(\Delta n \cdot T_s)/R_z(0)| \leq 1$ and similar inequalities hold for $R_\mu(\cdot)$ and $R_w(\cdot)$, (7) implies that the selected $R_\mu(\Delta n \cdot T_s)$ and $R_w(\Delta n \cdot T_s)$ values must satisfy

$$\left|\frac{R_z(\Delta n \cdot T_s)}{R_z(0)}\right| \leq \left|\frac{R_\mu(\Delta n \cdot T_s)}{R_\mu(0)}\right| \text{ and}$$
$$\left|\frac{R_z(\Delta n \cdot T_s)}{R_z(0)}\right| \leq \left|\frac{R_w(\Delta n \cdot T_s)}{R_w(0)}\right|.$$

The special case of independent $\mu(nT_s)$'s can be used in simulation only if the condition

$$\left|\frac{R_z(\Delta n \cdot T_s)}{R_z(0)}\right| \leq K_m, \quad \Delta n \neq 0 \quad (11)$$

is satisfied.

3) Based on the selected $R_\mu(\Delta n \cdot T_s)$ values, generate $\mu(nT_s)$'s by the method given in Appendix 2. If $\mu(nT_s)$'s are independent, other efficient algorithms can be used in this special case.
4) Generate complex-Gaussian distributed samples $w(nT_s)$ by a known technique.
5) Compute $z(nT_s)$ by (5).

Finally, $z(t)$ for an arbitrary time t can be computed by interpolation. If $\alpha \geq 8$, it has been suggested in [4] that piecewise-constant

Table 1. Autocorrelation values used in the numerical example. [$R_z(0)=1$, $R_\mu(0)=0.5$, $R_w(0)=2$]

Δn	$\frac{R_Z(\Delta n\cdot T_S)}{R_Z(0)}$	$\frac{R_\mu(\Delta n\cdot T_S)}{R_\mu(0)}$	$\frac{R_W(\Delta n\cdot T_S)}{R_W(0)}$	$\rho(\Delta n\cdot T_s)$
0	1.0000	1.0000	1.0000	1.0
1	0.9459	0.9570	0.9884	0.8
2	0.7915	0.9178	0.8624	0.6
3	0.5595	0.8808	0.6352	0.4
4	0.2831	0.8452	0.3349	0.2
5	0.0000	0.8106	0.0000	0.0
6	−0.2537	0.8106	−0.3129	0.0
7	−0.4493	0.8106	−0.5543	0.0

interpolation, i.e., $z(t) = z(nT_s)$ for $nT_s \le t < (n+1)T_s$, suffices without causing appreciable loss of accuracy.

IV. NUMERICAL EXAMPLE

A realization of $z(t)$ was generated based on the procedure given in Section III. For illustration purpose, we assumed that $m = 0.5$ and $\Omega = 1$. A double-Gaussian spectrum was used:

$$S_z(f)=\frac{1}{2\sqrt{2\pi}f_\Delta}[\exp(-\frac{f-\gamma f_{\max}}{\sqrt{2}f_\Delta}) + \exp(-\frac{f+\gamma f_{\max}}{\sqrt{2}f_\Delta})] \quad (12)$$

$$\leftrightarrow R_z(\Delta t)=\cos(2\pi\gamma f_{\max}\Delta t)\times\exp[-2(\pi f_\Delta \Delta t)^2]$$

where $f_\Delta = (1-\gamma)f_{\max}/3$ and we arbitrarily selected $\gamma = 0.5$. The double-Gaussian spectrum was selected because it is appropriate for modeling Doppler power spectra of HF channels [1]. Although a symmetric spectrum is used here for simplicity, realistic spectra for HF channels are usually asymmetric [1]. An oversampling ratio of $\alpha = 10$ was employed.

We first computed $R_z(\Delta n\cdot T_s)$, $\Delta n = -N,\cdots,N$, where N was arbitrarily chosen as $N = 1000$. Note that we only needed to compute $R_z(\Delta n\cdot T_s)$ with Δn a non-negative integer because the particular $R_z(\Delta t)$ chosen in this example is an even function. Values of $R_\mu(\Delta n\cdot T_s)$ and $R_w(\Delta n\cdot T_s)$ satisfying (7) were then selected. Table 1 lists the values of $R_z(\Delta n\cdot T_s)/R_z(0)$, $R_\mu(\Delta n\cdot T_s)/R_\mu(0)$ and $R_w(\Delta n\cdot T_s)/R_w(0)$ for $\Delta n = 0,\cdots,7$, where $R_z(0) = 1$, $R_\mu(0) = 0.5$ and $R_w(0) = 2$. Note that (11) is not satisfied ($K_m = 0.8106$) so that it was not possible to use the special case of independent $\mu(nT_s)$'s in this example. Table 1 also lists the values of $\rho(\Delta n\cdot T_s)$ where $\rho(\Delta t)$ is the autocorrelation function of $y(t)$ and $y(t)$ is the Gaussian process used in the nonlinear transformation for generating $\mu(t)$ (see Appendix 2). The particular pattern of $\rho(\Delta n\cdot T_s)$ was chosen so that $y(t)$ could be conveniently generated by a moving average process. A total of 10^4 discrete-time samples $z(nT_s)$ were generated by the following procedure. The time series $\mu(nT_s)$ was generated by the nonlinear transformation technique given in Appendix 2. The complex Gaussian time series $w(nT_s)$ was generated by taking Fourier transform on the sequence of $R_w(\Delta n\cdot T_s)$ values to obtain an approximation of the power spectrum followed by an application of the harmonic decomposition technique [4]. The value of $z(nT_s)$ was computed from $\mu(nT_s)$ and $w(nT_s)$ by (5). After $z(nT_s)$'s were generated, $z((n+\frac{1}{2})T_s)$, $n = 1,\cdots,10^4$, were computed by piecewise-constant interpolation. A total of 2×10^4 samples, i.e., $z(nT_{\rm sim})$, $n = 2,\cdots,(2\times10^4+1)$, where $T_{\rm sim} = T_s/2$, were finally obtained.

Figs. 1a and 1b plot the cumulative distribution functions (CDFs) of the amplitude and phase, respectively, computed from the 2×10^4 generated $z(nT_{\rm sim})$ samples. The CDF of the m-distribution with $m = 0.5$ and $\Omega = 1$, and that of the uniform distribution for the range $[0,2\pi)$ are also plotted for comparison purpose. Fig. 1c plots the autocorrelation function computed from the generated samples and the one for the double-Gaussian spectrum. From the three figures, it is apparent that the distributions and autocorrelation function of the generated samples fit closely to the desired ones.

Fig. 1. Statistics computed from the generated samples of $z(t)$ in the numerical example: (a) CDF of amplitude, (b) CDF of phase, (c) autocorrelation function. The desired CDFs and autocorrelation function are also plotted.

Fig. 1a

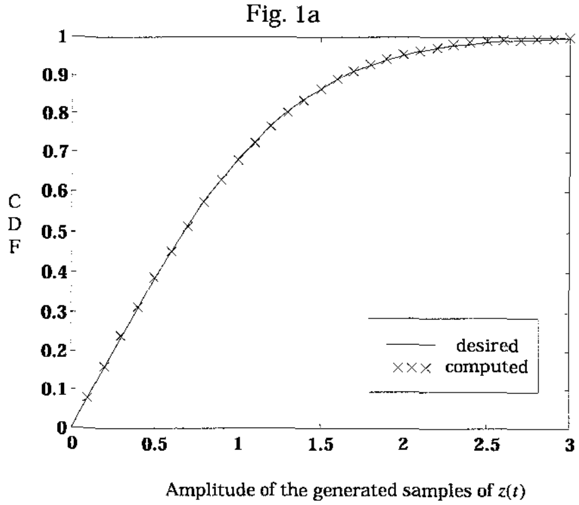
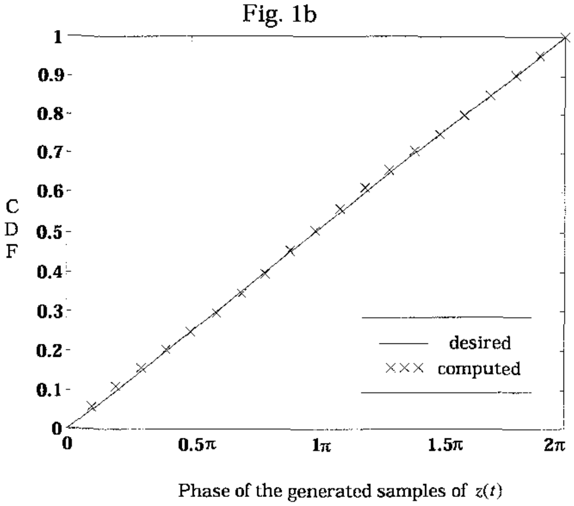


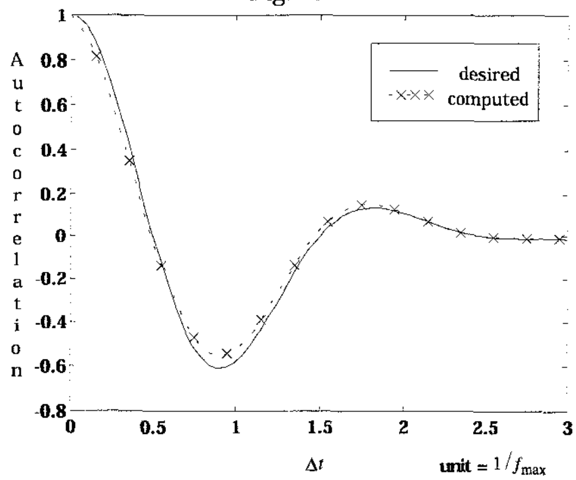
Fig. 1b



V. CONCLUSIONS

We have proposed a simulation model for m-fading channels, $m < 1$. The model reveals that an m-fading channel, $m < 1$, having a given Doppler power spectrum can be simulated by generating a complex Gaussian process and a square-root-beta random process. In the numerical example, it has been shown that

Fig. 1c



statistical properties of the samples generated from the use of the proposed model are close to the required ones.

APPENDIX 1 PROOF OF LEMMA 1

Let $c_z = r_z^2$ and $c_w = r_w^2$. The proof can be accomplished by taking Mellin transform on both sides of $c_z = \xi c_w$ and noting that the Mellin transform of ξc_w is the product of the Mellin transform of ξ and that of c_w. An alternative proof can also be obtained by using the fact that c_z and c_w are gamma random variables and that a beta random variable can be expressed as a ratio of two gamma random variables [10, ch. 25.2].

APPENDIX 2 REALIZATION OF $\mu(t)$

The non-Gaussian process $\mu(t)$ characterized by statistical properties given in Section II can be realized by a nonlinear transformation of a Gaussian random process [11]. Let the functions F and Φ be defined as

$$F(x) = I_{x^2}(m, 1-m), \quad 0 \le x \le 1$$

and

$$\Phi(x) = (2\pi)^{-1/2} \int_{-\infty}^{x} e^{-t^2/2} dt,$$

respectively, where $I_u(a,b) = (B(a,b))^{-1} \int_0^u t^{a-1}(1-t)^{b-1} dt$ is the incomplete beta function and m is the fading parameter. The function F^{-1} is the inverse of F. Let $y(t)$ be a

WSS zero-mean unit-variance Gaussian random process with an autocorrelation function

$$\rho(\Delta t) = E\{y(t)y(t+\Delta t)\}.$$

It is known that $\mu(t)$ can be generated by the nonlinear transformation [11]

$$\mu(t) = g(y(t)) \tag{13}$$

where $g(x) = F^{-1}(\Phi(x))$. In addition, $R_\mu(\Delta t)$ and $\rho(\Delta t)$ are related by [11]

$$R_\mu(\Delta t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(y_1)g(y_2)\frac{1}{2\pi\sqrt{1-\rho(\Delta t)^2}} \cdot\exp\left[-\frac{y_1^2+y_2^2-2\rho(\Delta t)y_1y_2}{2(1-\rho(\Delta t)^2)}\right]dy_1dy_2,$$

$$\rho(\Delta t) \neq \pm 1. \tag{14a}$$

In special cases of $\rho(\Delta t) = \pm 1$, it is easy to show that

$$R_\mu(\Delta t) = \begin{cases} m, & \rho(\Delta t) = +1 \\ \int_0^1 F^{-1}(x)F^{-1}(1-x)dx, & \rho(\Delta t) = -1. \end{cases} \tag{14b}$$

Since a knowledge of $R_\mu(\Delta t)$ is known, the corresponding autocorrelation function $\rho(\Delta t)$ can be obtained. A knowledge of $\rho(\Delta t)$ is required in the generation of the Gaussian process $y(t)$. This Gaussian process can be generated by a known technique. After $y(t)$ is generated, one can realize $\mu(t)$ by substituting the generated $y(t)$ into (13).

In general, analytical evaluation of $\rho(\Delta t)$ is seldom possible. To numerically invert (14a), it is required to compute the double integral (14a) many times. Numerical integration of (14a) is made easy by using the transformation $y_1 = x_1$ and $y_2 = \rho(\Delta t)x_1 + (1-\rho(\Delta t)^2)^{1/2}x_2$ followed by another transformation $x_1 = (2s)^{1/2}\cos\theta$ and $x_2 = (2s)^{1/2}\sin\theta$. It follows that (14a) can be simplified to

$$R_\mu(\Delta t) = \frac{1}{2\pi}\int_0^{2\pi}\int_0^{\infty} g(\sqrt{2s}\cos\theta) \cdot g(\rho(\Delta t)\sqrt{2s}\cos\theta + \sqrt{1-\rho(\Delta t)^2}\sqrt{2s}\sin\theta) \cdot e^{-s}dsd\theta,$$

$$\rho(\Delta t) \neq \pm 1$$

The inner and outer integrals can be readily computed by the Gauss-Laguerre rule and the Simpson's rule, respectively.

REFERENCES

[1] C. C. Watterson, J. R. Juroshek and W. D. Bensema, "Experimental confirmation of an HF channel model," *IEEE Trans. on Commun. Technol.*, vol. COM-18, pp. 792-803, Dec. 1970.

[2] M. Nakagami, "The *m*-distribution — a general formula of intensity distribution of rapid fading" in *Statistical Methods of Radio Wave Propagation*, C. C. Hoffman, Ed., Pergamon, Oxford, England, 1960.

[3] P. Hoeher, "A statistical discrete-time model for the WSSUS multipath channel," *IEEE Trans. Veh. Technol.*, vol. VT-41, pp. 461-468, Nov. 1992.

[4] P. M. Crespo and J. Jiménez, "Computer simulation of radio channels using a harmonic decomposition technique," *IEEE Trans. Veh. Technol.*, vol. VT-44, pp. 414-419, Aug. 1995.

[5] K. W. Yip and T. S. Ng, "Efficient simulation of digital transmission over WSSUS channels," *IEEE Trans. Commun.*, vol. COM-43, pp. 2907-2913, Dec. 1995.

[6] K. W. Yip and T. S. Ng, "Karhunen-Loève expansion of the WSSUS channel output and its application to efficient simulation," *IEEE J. Select. Areas in Commun.*, vol. SAC-15, pp. 640-646, May 1997.

[7] J. F. Mastrangelo, J. J. Lemmon, L. E. Vogler, J. A. Hoffmeyer, L. E. Pratt and C. J. Behm, "A new wideband high frequency channel simulation system," *IEEE Trans. Commun.*, vol. COM-45, pp. 26-34, Jan. 1997.

[8] H. Kong and E. Shwedyk, "A hidden Markov model (HMM)-based MAP receiver for Nakagami fading channels," in *Proceedings of 1995 IEEE International Symposium on Information Theory*, p. 210, Whistler, BC, Canada, 17-22 Sep. 1995.

[9] K. W. Yip and T. S. Ng, "A simulation model for Nakagami *m*-fading channels, $m < 1$," submitted to *IEEE Trans. Commun.*

[10] N. L. Johnson, S. Kotz and N. Balakrishnan, *Continuous univariate distributions*, vol. 2, 2nd Ed., Wiley, New York, 1995.

[11] M. Grigoriu, *Applied non-Gaussian processes*, Prentice-Hall, New Jersey, 1995.