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PN CODE TRACKING FOR EXPONENTIALLY WEIGHTED CHIP DESPREADING SEQUENCE

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ABSTRACT

This paper presents the code tracking performance on the use of weighted chip despreading PN sequence in a coherent delay locked tracking scheme for direct-sequence spread-spectrum (DS/SS) systems. In the delay locked loop (DLL), the conventional rectangular chip waveform for early and late despreading PN sequences is replaced by weighted exponential chip waveform. The chip weight (γ) is tuned to optimise the DLL performance. The early-late spacing is considered as one chip time. This scheme is analysed for both ideal and band limited received signals in the presence of additive white Gaussian noise.

I. INTRODUCTION

DS/SS systems have become a popular choice for commercial applications such as mobile communications and positioning systems. In such a DS/SS system, synchronising the locally generated PN code sequence to the PN code sequence in the received signal is crucial. The delay lock technique is widely used for tracking purposes. During normal operations, low bit error rate transmission is ensured only if the tracking error is small in the DLL [1]. Therefore it is necessary to keep the tracking error of the DLL as small as possible. Conventional DLL uses rectangular chip waveform to generate the early and late PN sequences. The use of weighted exponential PN pulses is studied in this paper. The technique of using despreading sequence with adjustable chip waveforms was introduced to reduce the multiple access interference (MAI) for DS-CDMA systems [2-4]. The current work demonstrate that this technique can also improve the tracking performance of the delay locked loop.

II. SYSTEM MODEL

Since we are considering the coherent delay locked loop, it is possible to analyse the signals in the base band. For ease of analysis, we assume that the received and the locally generated carrier phases are aligned with negligible error. The block diagram of the DLL in base band is given in Fig. 1. The base band received signal $r(t)$ in a DS/SS systems corrupted by additive white noise can be expressed as

$$r(t) = \sqrt{P}C(t) + n(t) \tag{1}$$

where P is the received signal power, $C(t)$ is the spreading code, $n(t)$ is the white noise with two sided power spectral density of $N_o/2$. The data signal is dropped in this analysis for simplicity. The modulation type is assumed to be BPSK with the conventional rectangular chip spreading waveform. The early-late spacing is considered as one chip time. The early and late despreading sequences are obtained by the shifted versions of locally generated PN sequence $\hat{C}(t)$ given by

$$\hat{C}(t) = \sum_{k=-\infty}^{\infty} a_k p(t-kT_c) \tag{2}$$

where $p(t)=1$ for $0 \le t < T_c$ and $p(t)=0$ otherwise, a_k is a binary value (± 1) and T_c is the chip time. Now consider the exponential weighted despreading chip waveform $\hat{p}(t)$ instead of $p(t)$ in (2).

$$\hat{p}(t) = \begin{cases} e^{-\gamma\left|\frac{t}{T_c}-\frac{1}{2}\right|} & 0 \le t < T_c \\ 0 & otherwise \end{cases} \tag{3}$$

The chip weight γ $(\gamma \ge 0)$ will be adjusted to optimise the DLL performance. When $\gamma = 0$, the shape of $\hat{p}(t)$ is rectangular and the system is therefore conventional. The waveform $\hat{p}(t)$ is assumed to be time limited to the chip duration and cause no inter-chip interference. Fig. 2 shows the chip arrangements of a early and late PN pulses for $\gamma = 4$ with an incoming PN pulse.

III. ANALYSIS

The cross correlation function $R(\tau)$ of $C(t)$ and $\hat{C}(t)$ is given as

$$R(\tau) \triangleq \frac{1}{NT_c}\int_0^{NT_c} C(t)\hat{C}(t+\tau T_c)dt \tag{4}$$

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we assume that the PN code length N is very long so that $R(\tau)$ vanishes when $|\tau| \geq 1$. Combining (3) and (4), $R(\tau)$ becomes

$$R(\tau)=\begin{cases} \frac{1}{\gamma}\left\{2-e^{-\gamma\left(\frac{1}{2}-|\tau|\right)}-e^{-\frac{\gamma}{2}}\right\} & |\tau|<\frac{1}{2} \\ \frac{1}{\gamma}\left\{e^{-\gamma\left(|\tau|-\frac{1}{2}\right)}-e^{-\frac{\gamma}{2}}\right\} & \frac{1}{2}\leq|\tau|<1 \\ 0 & otherwise \end{cases} \quad (5)$$

The plots of $R(\tau)$ for $\gamma = 0$, 2 and 4 are given in Fig. 3. Since we limit the maximum value of $\hat{p}(t)$ to 1, the total power of the local PN sequence is reduced with γ. The characteristic function $S(\varepsilon)$ of the DLL is given by the difference of the early and late versions of $R(\tau)$.

$$S(\varepsilon) = R\left(\varepsilon-\frac{1}{2}\right)-R\left(\varepsilon+\frac{1}{2}\right) \quad (6)$$

where ε is the phase error between the received and the locally generated PN sequences normalised to T_c. The characteristic curves are presented in Fig. 4. We note that the rate of change of $R(\tau)$ at $\tau = \pm 1/2$ in Fig. 3 is unchanged by increasing γ and so is $S(\varepsilon)$ at $\varepsilon = 0$. We also observe that the linear region of $S(\varepsilon)$ at $\varepsilon = 0$ is reduced by increasing γ. Therefore non-linear analysis is more appropriate for higher values of γ. Referring to the block diagram of the DLL given in Fig. 1, the error signal $e(t)$ can be obtained as follows.

$$e(t)=\sqrt{P}S(\varepsilon)+\left\{\hat{C}\left(t+\varepsilon T_c-\frac{1}{2}T_c\right)-\hat{C}\left(t+\varepsilon T_c+\frac{1}{2}T_c\right)\right\}n(t) \quad (7)$$

The *rms* tracking error is determined by using standard linear loop analysis. Detailed derivation can be found for conventional system in [5, chapter 4]. The results obtained by linear analysis is generally considered as a good approximation of the actual performance in the region of SNR which we are interested in. The high signal to noise ratio assumption will results in smaller tracking error of the DLL and this enables us to express $S(\varepsilon)$ to be a linear curve at $\varepsilon = 0$. For small tracking errors, $e(t)$ in (7) is reduced to

$$e(t) = \sqrt{P}S'(0)\{\varepsilon+n'(t)\} \quad (8)$$

where

$$S'(0) = \left.\frac{\partial S(\varepsilon)}{\partial \varepsilon}\right|_{\varepsilon=0} \quad (9)$$

and the noise process $n'(t)$ is defined by

$$n'(t)=\frac{1}{\sqrt{PS'(0)}}\left\{\hat{C}\left(t+\varepsilon T_c-\frac{1}{2}T_c\right)-\hat{C}\left(t+\varepsilon T_c+\frac{1}{2}T_c\right)\right\}n(t) \quad (10)$$

For ideal received signal, $S'(0)=2$. Since the noise process $n(t)$ is white, the autocorrelation function of the noise process $n'(t)$ becomes

$$\begin{aligned} R_{n'}(\tau) &= E\{n'(t)n'(t+\tau T_c)\} \\ &= \Omega(\gamma)N_0\delta(\tau)/P \end{aligned} \quad (11)$$

where delta function $\delta(\tau)=1$ for $\tau=0$ and $\delta(\tau)=0$ otherwise and

$$\Omega(\gamma) = R_{\hat{C}}(0)/S'(0)^2 \quad (12)$$

The auto correlation function of $\hat{C}(t)$ is denoted by $R_{\hat{C}}(\tau)$. The cross correlation value of early and late PN sequences is negligible since they are separated by one chip time. We note that the parameter $\Omega(\gamma)$ is a function of γ. The expressions for $R_{\hat{C}}(0)$ in (12) is obtained as

$$R_{\hat{C}}(0) = \frac{1}{\gamma}\left\{1-e^{-\gamma}\right\} \quad (13)$$

Since $S'(0)$ is independent of γ for ideal received signal, the closed loop transfer function $H(j2\pi f)$ and the closed loop bandwidth $W_L = \int_0^\infty |H(j2\pi f)|^2 df$ are also independent of γ and both are the same as that of conventional system $(\gamma = 0)$. Taking Fourier transform on $R_{n'}(\tau)$ gives the power spectral density of $n'(t)$ which is flat with density of $\Omega(\gamma)N_0/P$ as indicated by equation (11). The *rms* tracking error (σ) of the DLL then becomes

$$\sigma = \sqrt{\frac{2\Omega(\gamma)N_o W_L}{P}} \quad (14)$$

III. BAND LIMITED RECEIVED SIGNAL

In practice the received signal $r(t)$ in (1) is band limited. The transmission channel consists of a transmitter filter, a receiver filter and a propagation medium. The low pass transfer functions of these are denoted by $H_T(f)$, $H_R(f)$ and $H_M(f)$ respectively.

The transmission channel low pass transfer function denoted by $H_C(f)$ becomes [6]

$$H_C(f) = H_T(f)H_M(f)H_R(f) \tag{15}$$

The function $H_M(f)$ depends on the channel environment. The impulse response of the transmission channel $h_l(t)$ is obtained by the inverse fourier transform of $H_C(f)$. The incoming band limited chip wave form $\overline{p}(t)$, given by $p(t)\otimes h_l(t)$ where $\otimes$ is the convolution operator, will not be limited in the range of $0 \le t < T_c$ due to the influence of $h_l(t)$. Therefore $R(\tau)$ in (4) is not limited in the range $|\tau| < 1$.

The band limited received signals are modelled by assuming ideal low pass transfer function for $H_C(f)$. Even though this is a rough approximation, it serves the purpose of showing the effects of band limitation on the received signal. For illustration purpose, assume

$$H_C(f) = \begin{cases} 1 & |f| \le \dfrac{2}{T_C} \\ 0 & otherwise \end{cases} \tag{16}$$

i.e., the bandwidth of the channel is assumed to be twice that of the main lobe spectrum of the spreading PN sequence. Using the expression of $h_l(t)$ for (16), $\overline{p}(t)$ and hence $R(\tau)$ in (4) are obtained. The parameter $S'(0)$ is then computed using numerical integration. The plot of $S'(0)$ against γ is given in Fig. 5. It is clear that band limitation affects $S'(0)$ and thus it depends on γ. We note that this change in $S'(0)$ will also cause variation in $H(j2\pi f)$ and hence in W_L. However, these parameters can be adjusted by the sensitivity gain of the voltage controlled clock in the DLL. Since the band width of $H_C(f)$ is much greater than W_L, band limitation in the additive noise process $n(t)$ can be neglected because of the flat power spectral density at $f = 0$. Thus tracking error in (14) can be computed for band limited signals.

IV. NUMERICAL EXAMPLE

The effect of γ on the *rms* tracking error in (14) can be studied by investigating $\Omega(\gamma)$. The plot of $\Omega(\gamma)$ against the chip weight γ for ideal received signal is given by the dotted line in Fig. 6. Note that for conventional DLL, $\Omega(0) = 1/4$. This curve shows that $\Omega(\gamma)$ is reduced as γ increases, although not linearly. It follows from equation (14) that the *rms* tracking error will be reduced when the other parameters remain unchange. For the band limited case, using numerical value of $S'(0)$ given in Fig. 5, $\Omega(\gamma)$ is obtained and also plotted by the solid line in Fig. 6. This graph shows that $\Omega(\gamma)$ has a minimum value and so is the *rms* tracking error (σ). Therefore an optimum value for γ exists. The tracking error (σ) of the DLL in (14) against loop SNR is plotted in Fig. 7. The loop SNR is defined by P/N_oW_L. From the results in Fig. 7, it is clear that for a specific tracking error, band limitation requires an increase of SNR by approximately 1.2 dB when rectangular chip despreading waveform is used. The results also indicate that the required SNR has been reduced by about 6.4 dB by employing the exponentially weighted chip despreading waveform adjusted to the optimum value of chip weight γ.

V. CONCLUSION

The use of weighted exponential chip waveform for early-late despreading PN sequence in a coherent DLL tracking scheme for DS/SS systems has been analysed for both ideal and band limited received signals. It has been shown that the proposed technique reduces the tracking error significantly when the chip weight is adjusted to its optimum value.

ACKNOWLEDGEMENT

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REFERENCES

[1] Marvin K. Simon, Jim K. Omura, Robert A. Scholtz and barry K. Levitt, "Spread Spectrum Communications," Vol. III, Computer Science Press, Rockville, 1985.

[2] Y. Huang and T.S. Ng, "Performance of Coherent Receiver with Weighted Despreading Sequence for DS-CDMA," Electron. Lett., Vol. 33, pp. 23-25, Jan. 1997.

[3] Yuejin Huang and T.S. Ng, "A DS-CDMA with power control error using weighted despreading sequences over a multi-path Rayleigh fading channel," accepted for publication, IEEE Transactions on Vehicular Technology.

[4] Yuejin Huang and T.S. Ng, "A DS-CDMA system using despreading sequences weighted

by adjustable chip waveforms," revised manuscript being considered by, IEEE Transactions on Communications.

[5] Roger L. Peterson, Rodger E. Ziemer and David E. Borth, "Introduction to Spread Spectrum Communications," Prentice-Hall, 1995.

[6] C. Caini, L. Calandrino and A. Vanelli Coralli, "Effects of Channel Distortion on Code Tracking Error in DS Spread Spectrum Systems," Proceedings of the IEEE International Symposium on spread spectrum techniques and applications, pp 1012-1016, September 1996.

Figure Captions

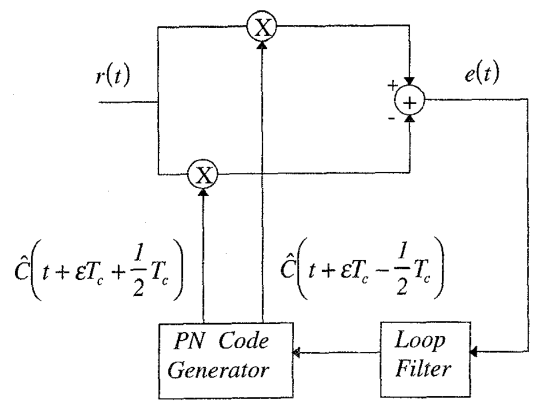

Fig. 1: The block diagram of the coherent DLL

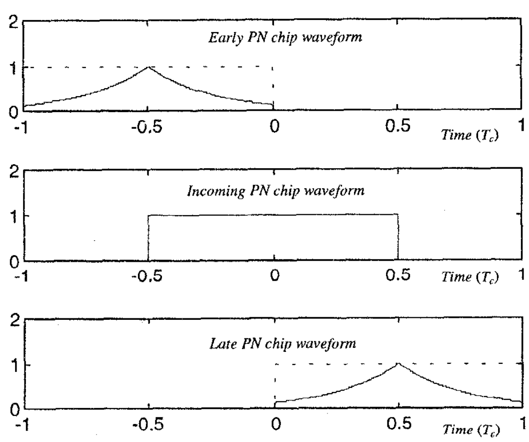

Fig. 2: The early, late and incoming PN pulses for $\gamma = 4$.

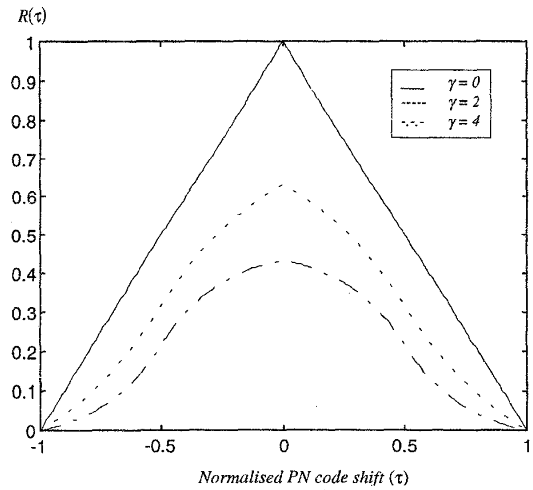

Fig. 3: The plots of cross correlation $R(\tau)$ against phase shift τ.

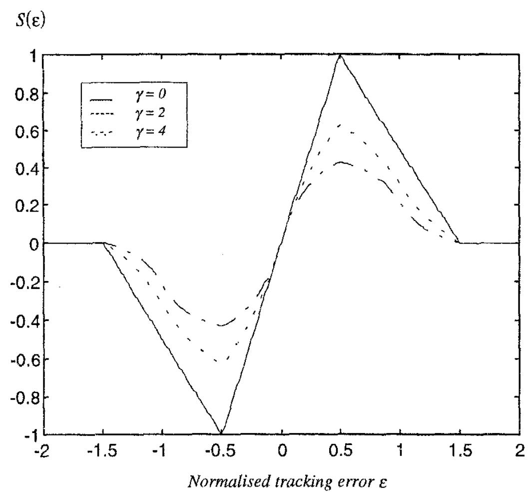

Fig. 4: The characteristic curves of the DLL.

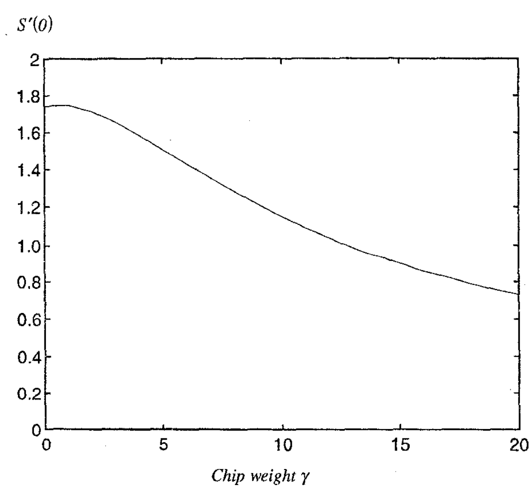

Fig. 5: The parameter $S'(0)$ against chip weight γ.

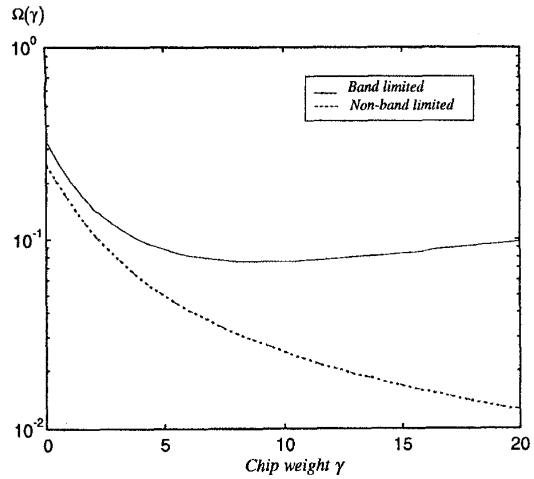

Fig. 6: The parameter $\Omega(\gamma)$ against chip weight γ.

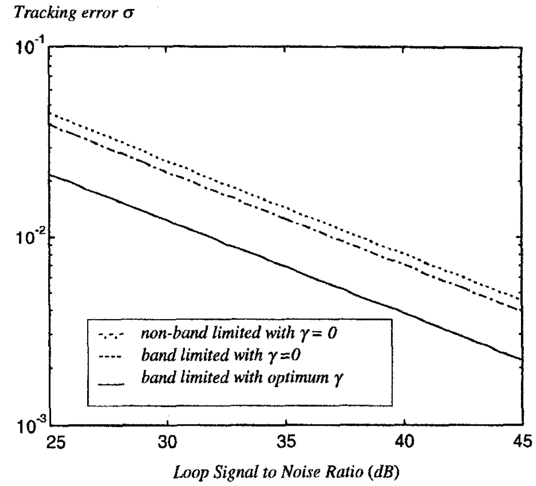

Fig. 7: The tracking error (σ) against loop signal to noise ratio.