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# Robustness Analysis of Induction Motor Decoupling Adaptation System

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**Abstract** - By using linear analysis the robustness of the induction motor decoupling adaptation system is studied and quantitative analytical results are given. The simulation results of the system verify the analytical results.

## 1. INTRODUCTION

The high-performance slip-frequency control of an induction motor, which is often called the vector control or decoupling control of induction motor, is considered to be one of the best ac variable speed drives [1,2]. Combining the model decoupling adaptation system technique [3], an induction motor decoupling adaptation system was proposed [4]. However, since the decoupling control of induction motor reaches the decoupling only in steady state [5], the robustness question arises in an induction motor decoupling adaptation system in which an ideal induction motor decoupling state is supposed. This paper analyzes the robustness of the induction motor decoupling adaptation system and gives quantitative analytical results. The example in this paper shows that when the parameters of the adaptation mechanism are located in the robustness region, the induction motor decoupling adaptation system can work very well. The simulation results of the system verify the analytical results.

## 2. DESIGN OF INDUCTION MOTOR DECOUPLING ADAPTATION SYSTEM [6]

A block diagram of an induction motor decoupling adaptation system is given in Fig. 1. It is assumed that under the decoupling control strategy in the  $\alpha$ - $\beta$  axis which is synchronously rotating with the ac power source frequency [1,2]:

$$i_{1\alpha}^{ref} = \frac{\lambda_{2\alpha}^{ref}}{M} = \text{constant} \quad (1)$$

$$i_{1\beta}^{ref} = \frac{L_2}{M\lambda_{2\alpha}^{ref}} \cdot T^{ref} \quad (2)$$

$$\omega_{slip} = \omega_s - \omega_r = \frac{Mr_2^{ref} i_{1\beta}^{ref}}{L_2 \lambda_{2\alpha}^{ref}} \quad (3)$$

$$e_{1\alpha} = r_1 i_{1\alpha}^{ref} - L_\sigma \omega_s i_{1\beta}^{ref} \quad (4)$$

$$e_{1\beta} = r_1 i_{1\beta}^{ref} + L_\sigma \omega_s i_{1\alpha}^{ref} \quad (5)$$

where  $L_\sigma = L_1 - \frac{M^2}{L_2}$

the electric torque can be expressed by

$$T_e = \frac{n_p M^2 i_{1\alpha}^{ref} i_{1\beta}^{ref}}{2L_2} \quad (6)$$

as the rotor mechanical angular equation is

$$\omega_m = \frac{T_e - T_m}{JS - D} \quad (7)$$

where  $T_m$  is load torque. In the design process it is treated as a disturbance. So the simplified induction motor model is

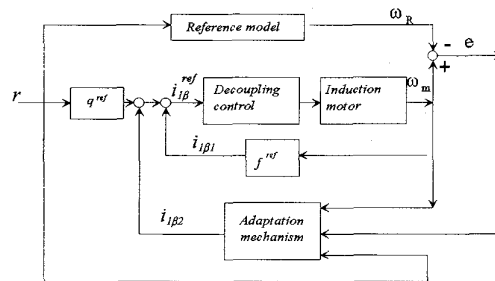


Fig. 1 Induction Motor Decoupling Adaptation System

$$\omega_m = \frac{n_p M^2 i_{1\alpha}^{ref} i_{1\beta}^{ref}}{2L_2 (JS + D)} \quad (8)$$

or,

$$\frac{\omega_m}{i_{1\beta}^{ref}} = \frac{k_g}{JS + D} \quad (9)$$

where

$$k_g = \frac{n_p M^2 i_{1\alpha}^{ref}}{2L_2} \quad (10)$$

Choosing the reference model as

$$G_R(S) = \frac{1}{1+TS} \quad (11)$$

then, according to the model matching condition (when  $e=0$ ), the values of  $f$  and  $g$  in Fig. 1 can be determined:

$$q^{ref} = \frac{J^{ref}}{Tk_g} \quad (12)$$

$$f^{ref} = \frac{J^{ref} - TD}{Tk_g} = q^{ref} - D/k_g \quad (13)$$

when  $e \neq 0$ , the adaptation mechanism will generate an adaptive signal  $i_{1\beta 2}$

$$i_{1\beta 2} = -\Delta f \omega_m + \Delta q r \quad (14)$$

By using Lyapunov stability theory, the adaptive laws of the parameters can be determined

$$\dot{\Delta q} = [-v_{11} e r - v_{21} \frac{d}{dt}(e r)] \frac{J^{ref}}{k_g} \quad (15)$$

$$\dot{\Delta f} = [v_{21} e \omega_m + v_{22} \frac{d}{dt}(e \omega_m)] \frac{J^{ref}}{k_g} \quad (16)$$

where  $v_{11} > 0$ ,  $v_{12} > 0$ ,  $v_{21} > 0$ ,  $v_{22} > 0$ , and the system of Fig. 1 is global stability under the assuming (13). When  $J$  varies from  $J^{ref}$ , there should be

$$\Delta f = \Delta q \quad (17)$$

so it is convenient to choose

$$v_{11} = v_{12} = v_a \frac{J^{ref}}{k_g} = v_1 \quad (18)$$

$$v_{21} = v_{22} = v_b \frac{J^{ref}}{k_g} = v_2 \quad (19)$$

in (15) (16), so that the adaptive laws of the parameters are

$$\Delta q = -v_1 \int_0^t e r d\tau - v_2 e r \quad (20)$$

$$\Delta f = v_1 \int_0^t e \omega_m d\tau + v_2 e \omega_m \quad (21)$$

where  $v_1 > 0$ ,  $v_2 > 0$  and  $\Delta q(0) = \Delta f(0) = e(0) = 0$ .

### 3. ROBUSTNESS ANALYSIS OF AN INDUCTION MOTOR DECOUPLING ADAPTATION SYSTEM

It is noted that (6) is only held in steady state. So it should be determined whether the design of the induction motor decoupling adaptation system is valid when a simplified system model is used in the designing stage. This is the robustness issue of the induction motor decoupling adaptation system. As the system of Fig. 1 is a nonlinear system, in order to use linear analytical techniques which can gives quantitative results, the system needs to be linearized at one operating point.

Let  $r = \text{constant}$  and there exist

$$f = f^{ref} \quad (22)$$

$$q = q^{ref} \quad (23)$$

so that

$$e = e^{ref} = 0 \quad (24)$$

under this condition

$$\omega_R = \omega_R^{ref} = r \quad (25)$$

$$i_{1\beta}^{ref} = i_{1\beta}^{ref 0} = q^{ref} r - f^{ref} \omega_m^{ref} \quad (26)$$

$$\omega_m = \omega_m^{ref} = G_p^{ref}(S) i_{1\beta}^{ref} - \frac{T_m}{JS + D} = r \quad (27)$$

where  $G_p^{ref}(S)$  is the induction motor transfer function with the decoupling control strategy (1)-(4) at operating point [6]. Combining (26) and (27),

$$\omega_m^{ref} = \left[ G_p^{ref}(S) q^{ref} r - \frac{T_m}{JS + D} \right] + [1 + f^{ref} G_p^{ref}(S)] = r \quad (28)$$

when  $e \neq 0$ , there should be

$$\omega_m = \omega_m^{ref} + \Delta \omega_m \quad (29)$$

then,

$$e = \omega_m - \omega_R = \omega_m^{ref} + \Delta \omega_m - \omega_R^{ref} = \Delta \omega_m \quad (30)$$

so

$$\Delta e = e - e^{ref} = e = \Delta \omega_m \quad (31)$$

Combining (20), (21), (29), (31) and omitting the square term of  $\Delta$ :

$$\Delta q = -\left(v_1 \int_0^t \Delta e d\tau + v_2 \Delta e\right) r \quad (32)$$

$$\Delta f = \left(v_1 \int_0^t \Delta e d\tau + v_2 \Delta e\right) \omega_m^{ref} \quad (33)$$

From Fig. 1 and (14),

$$i_{1\beta}^{ref} = (q^{ref} + \Delta q) r - (f^{ref} + \Delta f) (\omega_m^{ref} + \Delta \omega_m) \quad (34)$$

Omitting the square term of  $\Delta$ :

$$\begin{aligned} i_{1\beta}^{ref} &= q^{ref} r - f^{ref} \omega_m^{ref} + \Delta q r - \Delta f \omega_m^{ref} - f^{ref} \Delta \omega_m \\ &= i_{1\beta}^{ref0} + \Delta q \cdot r - \Delta f \cdot \omega_m^{ref} - f^{ref} \Delta \omega_m \end{aligned} \quad (35)$$

so,

$$\Delta i_{1\beta}^{ref} = i_{1\beta}^{ref} - i_{1\beta}^{ref0} = \Delta q \cdot r - \Delta f \cdot \omega_m^{ref} - f^{ref} \Delta \omega_m \quad (36)$$

From (27),

$$\begin{aligned} \Delta \omega_m &= G_p^{ref}(S) \Delta i_{1\beta}^{ref} - \frac{\Delta T_m}{JS + D} \\ &= G_p^{ref}(S) [\Delta q \cdot r - \Delta f \cdot \omega_m^{ref} - f^{ref} \Delta \omega_m] - \frac{\Delta T_m}{JS + D} \end{aligned} \quad (37)$$

so,

$$\Delta \omega_m = \frac{G_p^{ref}(S) [r \Delta q - \omega_m^{ref} \Delta f] - \frac{\Delta T_m}{JS + D}}{1 + f^{ref} G_p^{ref}(S)} \quad (38)$$

Combining (38), (32), (33),

$$\Delta \omega_m = \frac{-G_p^{ref}(S) \left[ v_1 \int_0^t \Delta e d\tau + v_2 \Delta e \right] \left[ r^2 + (\omega_m^{ref})^2 \right] - \frac{\Delta T_m}{JS + D}}{1 + f^{ref} G_p^{ref}(S)} \quad (39)$$

Changing (39) into  $S$  operator form and using (31), noticing  $\Delta e(0) = \Delta \omega_m(0) = 0$ ,

$$\Delta e(S) = \frac{-G_p^{ref}(S) \left( \frac{v_1}{S} + v_2 \right) \left[ r^2 + (\omega_m^{ref})^2 \right] \Delta e(S) - \frac{\Delta T_m(S)}{JS + D}}{1 + f^{ref} G_p^{ref}(S)} \quad (40)$$

or

$$\Delta e(S) = \frac{-\Delta T_m(S)}{\left[ 1 + G_p^{ref}(S) \left[ f^{ref} + \left( \frac{v_1}{S} + v_2 \right) \left( r^2 + (\omega_m^{ref})^2 \right) \right] \right] (JS + D)} \quad (41)$$

the characteristic equation of Fig. 1 is

$$\left\{ 1 + G_p^{ref}(S) \left[ f^{ref} + \left( \frac{v_1}{S} + v_2 \right) \left( r^2 + (\omega_m^{ref})^2 \right) \right] \right\} (JS + D) = 0 \quad (42)$$

From the root-locus of (42), the stability and the dynamics characteristic of Fig. 1 can be determined. The robustness parameters  $v_1$  and  $v_2$  can be found from the root-locus of (42) if they exist.

#### 4. EXAMPLE

1KW, 100V, 8.4A, 60 Hz, 4 poles, 1710rpm		
Primary resistance	$r_1$	0.49 $\Omega$
Secondary resistance	$r_2$	0.45 $\Omega$
Primary self-inductance	$L_1$	38.8 mH
Secondary self-inductance	$L_2$	35.4 mH
Mutual inductance	$M$	35.4 mH
Total inertia	$J^{ref}$	0.024 Nmsec <sup>2</sup> /rad
Viscous-friction coefficient	$D$	0.0011 Nmsec/rad
Rated $\alpha$ axis primary current	$i_{1\alpha}^{ref}$	6.83 A
Rated $\beta$ axis primary current	$i_{1\beta}^{ref}$	11.54 A

Table 1  
Parameters of the test motor

The parameters of the test motor are listed in Table 1 and  $L_1$ ,  $L_2$ , and  $M$  should multiply a factor (2/3) in  $\alpha$ - $\beta$  axis. Choosing  $T=0.25$  s in the reference model, the  $f^{ref}$  and  $q^{ref}$  can be calculated from (12), (13):

$$f^{ref} = 0.130834, \quad q^{ref} = 0.132350$$

(42) has 11 roots but only 10 roots need to be calculated because one stable root is  $S=-D/J$  and the root-locus plot is calculated by a computer program. If the desired working speed of the motor is 2000 rpm, then

$$r = \omega_m^{ref} = 2000(2\pi/60) = 209 \text{ (rad/s)}$$

Case one :

when  $v_1=10^{-4}$ ,  $v_2=10^{-4}$ ,  $J=J^{ref}$ ,  $r_2=r_2^{ref}$ , there are two unstable roots in (42) and the root-locus is shown in Fig.2a, so the system of Fig.1 will be unstable. The simulation results of Fig. 2b, 2c and 2d also prove the instability of the system. Note that this case is in a "decoupling state", and according to the design strategy, the system should be globally stable. But as noted in [5,6], the decoupling control strategy (1)-(5) only realized "static" decoupling or "nearly dynamics decoupling" when  $L_\sigma \rightarrow 0$ . When  $L_\sigma \neq 0$ , it is necessary to choose the adaptation parameter  $v_1, v_2$  very carefully because the global stability is not guaranteed. On the other hand, this shows that the linear analysis presented in this paper works very well.

Case two :

when  $v_1=10^{-3}$ ,  $v_2=10^{-5}$ ,  $J=J^{ref}$ ,  $r_2=(1/2)r_2^{ref}$ , the root-locus of (42) in Fig. 3a shows that the system still has good dynamics characteristics and loaded ability when the load changing and the decoupling condition is destroyed. This shows that the system has a strong robust character in the region of some adaptation parameters and the robustness analysis works.

## 5. CONCLUSION

The results of this paper show that it is effective and convenient to study a nonlinear adaptive control system by linear analysis. The example shows that the induction motor decoupling adaptation system has a strong robustness in the region of some adaptation parameters and quantitative analytical results were given. The simulation results of this system verify the analytical results.

## 6. NOMENCLATURE

$I_{1\alpha}$	Exciting current
$I_{1\beta}$	Torque current
$\lambda_2$	Secondary flux
$L_1$	Primary inductance
$L_2$	Secondary inductance
$M$	Magnetizing inductance
$r_1$	Primary resistance
$r_2$	Secondary resistance
$T_e$	Electrical torque
$T_m$	Loaded torque
$\omega_s$	Supply frequency

$\omega_r$	Rotor mechanical angular speed
$\omega_{slip}$	Slip frequency
$e_{1\alpha}$	primary voltage in $\alpha$ - $\beta$ axis
$e_{1\beta}$	secondary voltage in $\alpha$ - $\beta$ axis
$ref$	Reference value or the nominal value
$\Delta$	Small variation
$S$	differential operator or Laplace operator
$n_p$	number of poles
$J$	Total inertia
$D$	Viscosity resistance
$r$	Motor speed command
$\omega_R$	Reference model output
$G_R(S)$	Reference model transfer function

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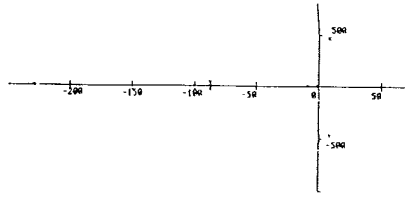


Fig. 2a Root-locus of equ.(41)

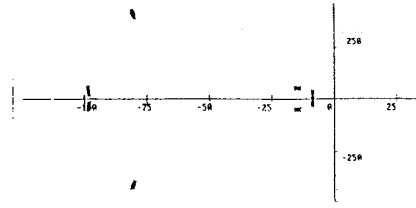


Fig. 3a Root-locus of equ.(41)

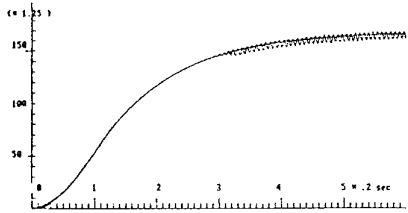


Fig. 2b Transient response of  $\omega_R$  and  $\omega_m$

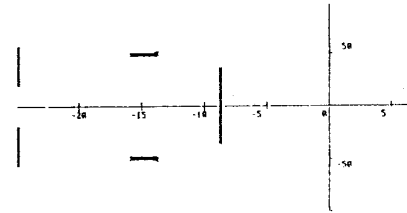


Fig. 3b Main root-locus of equ.(41)

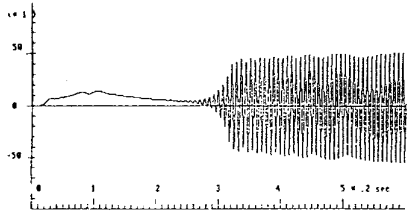


Fig. 2c Transient response of  $T_e$

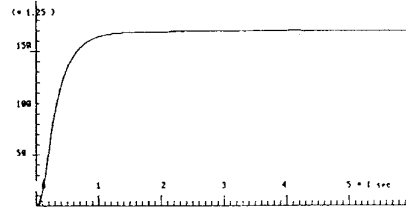


Fig. 3c Transient response of  $\omega_R$  and  $\omega_m$

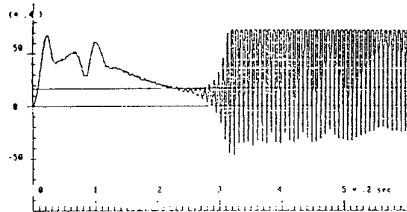


Fig. 2d Transient response of  $i_{\alpha}^{ref}$  and  $i_{\beta}^{ref}$

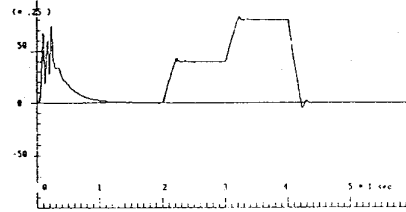


Fig. 3d Transient response of  $T_e$

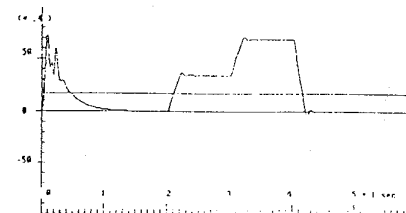


Fig. 3e Transient response of  $i_{\alpha}^{ref}$  and  $i_{\beta}^{ref}$