A New Adaptive Kalman Filter-Based Subspace Tracking Algorithm and Its Application to DOA Estimation

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***Abstract*—This paper presents a new Kalman filter-based subspace tracking algorithm and its application to directions of arrival (DOA) estimation. An autoregressive (AR) process is used to describe the dynamics of the subspace and a new adaptive Kalman filter with variable measurements (KFVM) algorithm is developed to estimate the time-varying subspace recursively from the state-space model and the given observations. For stationary subspace, the proposed algorithm will switch to the conventional PAST to lower the computational complexity. Simulation results show that the adaptive subspace tracking method has a better performance than conventional algorithms in DOA estimation for a wide variety of experimental condition.**

I. INTRODUCTION

Subspace tracking, which refers to the recursive computation of a selected subset of eigenvectors of a Hermitian matrix, plays an important role in a wide variety of signal processing applications. Traditional subspace-based algorithms usually compute either the eigenvalue decomposition (ED) or singular value decomposition (SVD) of the data autocorrelation matrix in order to estimate the signal or noise space [1]. Instead of updating the whole eigen-structure, subspace tracking only works with the signal or noise subspace so as to lower the computational complexity and reduce the storage requirements. These advantages make subspace tracking very attractive and a number of fast subspace tracking algorithms were developed. A very efficient significant subspace tracking algorithms is the project approximation subspace tracking (PAST) approach [2].

The PAST algorithm considers the signal subspace as the solution of an unconstrained minimization problem, which can be simplified to an exponentially weighted least-squares (LS) problem by an appropriate project approximation. The PAST algorithm is implemented using the recursive least-squares (RLS) algorithm. Since conventional RLS doesn't know the system dynamics model, and it only assumes the subspace is slowly time-varying and the estimate is based solely on the observations. As a result, when the subspace changes quickly, RLS will give a poor performance.

In this paper, we propose a new Kalman filter-based subspace tracking algorithm for estimating the signal subspace recursively. Kalman filter is a generalization of the RLS algorithm and it incorporates prior information of the state dynamics into the estimation process. In the context of subspace tracking, the signal subspace is assumed to be the system state, which can be described by some models such as an autoregressive (AR) process. In addition, a new Kalman filter with variable number of measurements (KFVM) is proposed to address the bias-variance tradeoff problem in tracking the time-varying parameters. Instead of using only one measurement in conventional Kalman filter to update the estimate of the signal subspace, the proposed KFVM employs variable number of measurements. A measurement window of appropriate length can help to reduce the variance of estimation due to additive noise, while avoiding excessive bias for non-stationary signals. One can determine the number of measurements using the intersection of confidence intervals (ICI) bandwidth selection [4]. Alternately, the proposed Kalman filter aims to determine the appropriate number of the measurements adaptively according to the approximated derivatives of the system state in order to reduce the complexity in the bandwidth selection algorithm. Basically, when the signal subspace varies rapidly, few measurements will be used to update the state estimate. On the other hand, when the subspace is slow-varying or even static, increasingly more past measurements should be employed. However, a heavy computational complexity will be incurred for large numbers of measurements in the KFVM. Therefore, the KFVM can be switched to the PAST (RLS) algorithm to lessen the arithmetic complexity. This gives a new adaptive subspace tracking algorithm, which composes of KFVM and RLS. We will use the directions of arrival (DOA) estimation to illustrate the tracking performance of the proposed algorithm, as compared with conventional algorithms.

This paper is organized as follows. Section II briefly reviews the basic problem of subspace tracking and the PAST algorithm. The new KFVM algorithm and adaptive measurement number selection are presented in Section III. Section IV introduces the proposed adaptive subspace tracking method using the KFVM and RLS algorithms. Simulation results and comparison are presented in Section V. Conclusions are drawn in Section VI.

II. SUBSPACE TRACKING AND PAST ALGORITHM

Consider r narrow-band incoherent signal impinging a uniform linear antenna array with N elements, it allows the N-dimensional signal vector $\underline{z}(t)$. Let $\underline{z}(t) \in \mathbb{C}^N$ be the observed data vector sampled at time instant $t = 1,2,\cdots,T$ .Then, $\underline{z}(t)$ can be represented as the following model:

$$\underline{z}(t) = \sum_{i=1}^{r} \underline{a}(\omega_i) s_i(t) + \underline{n}(t) = \mathbf{A}\underline{s}(t) + \underline{n}(t)\,, \qquad (1)$$

where $\underline{a}(\omega_i) = [1, e^{j\omega_i}, \cdots, e^{j(N-1)\omega_i}]$ is the steering or frequency vector, and $\mathbf{A} = [\underline{a}(\omega_1), \underline{a}(\omega_2), \cdots, \underline{a}(\omega_r)]$. $\underline{s}(t) = [s_1(t), s_2(t), \cdots, s_r(t)]^T$ is the random source vector and $\underline{n}(t)$ is an independent and identically distributed (i.i.d.) additive white Gaussian noise (AWGN) vector with correlation matrix $\sigma^2 \mathbf{I}_N$. The auto correlation matrix of $\underline{z}(t)$ is then given by:

$$\mathbf{C}_{zz} = E[\underline{z}(t)\underline{z}^H(t)] = \mathbf{A}\mathbf{C}_{ss}\mathbf{A} + \sigma^2 \mathbf{I}_N\,, \qquad (2)$$

where $\mathbf{C}_{ss} = E[\underline{s}(t)\underline{s}^H(t)]$ is the autocorrelation matrix of $\underline{s}(t)$. Since $\mathbf{C}_{ss}$ is Hermitian, it admits an eigenvalue decomposition (ED) as follows:

$$\mathbf{C}_{zz} = \mathbf{U}\Sigma\mathbf{U}^H\,, \qquad (3)$$

where $\Sigma = diag(\lambda_1, \lambda_2, \cdots, \lambda_N)$ is composed of the eigenvalue λ_i of $\mathbf{C}_{zz}$, and $\mathbf{U} = [\underline{u}_1, \underline{u}_2, \cdots, \underline{u}_N]$ is made up of the eigenvector $\underline{u}_i$

of $\mathbf{C}_{zz}$. If $r \le N$, the eigenvalues can be ordered non-increasingly as $\lambda_1 \ge \cdots \ge \lambda_r \ge \lambda_{r+1} = \cdots \lambda_N = \sigma^2$. The dominant eigenvalues λ_i for $i = 1,2,\cdots,r$ are called the signal eigenvalues, and the corresponding $\underline{u}_i$'s for $i = 1,2,\cdots,r$ are the corresponding signal eigenvectors. On the other hand, λ_i and $\underline{u}_i$ for $i = r+1, r+2,\cdots,N$ are called the noise eigenvalues and noise eigenvectors, respectively. The signal subspace is defined as the column span of the signal eigenvectors: $\mathbf{U}_s = [\underline{u}_1, \underline{u}_2, \cdots, \underline{u}_r]$, and the noise subspace is $\mathbf{U}_n = [\underline{u}_r, \underline{u}_{r+1}, \cdots, \underline{u}_N]$. The goal of subspace tracking is to track the signal or noise subspace instead of the whole eigen-structure to reduce the computational complexity.

It was shown in [2] that subspace tracking can be considered as an unconstrained optimization problem with the cost function:

$$J(\mathbf{W}) = E[\|\underline{z} - \mathbf{W}\mathbf{W}^H \underline{z}\|_2^2] = tr(\mathbf{C}_{zz}) - 2tr(\mathbf{W}^H \mathbf{C}_{zz} \mathbf{W}) + tr(\mathbf{W}^H \mathbf{C}_{zz} \mathbf{W}\mathbf{W}^H \mathbf{W}), \quad (4)$$

where $\mathbf{W} \in \mathbb{C}^{N \times r}$ is a matrix variable with rank r. The minimization of $J(\mathbf{W})$ leads us not only to a unique global minimum, but also to an orthonormal basis of signal subspace. By using the projection approximation, the above problem can be simplified to an unconstrained minimization problem, for which many well-developed algorithms can be applied. Detailed theorem and proof are omitted to save space, and they can be found in [1] – [3].

The aim of subspace tracking is to recursively estimate the signal subspace at time t from the subspace estimate $\mathbf{W}(t-1)$ at $t-1$ and the current data vector $\underline{z}(t)$. Let $\underline{h}(t) = \mathbf{W}^H(t-1)\underline{z}(t)$, the cost function (4) becomes a typical LS function:

$$J(\mathbf{W}) = E[\|\underline{z}(t) - \mathbf{W}(t)\underline{h}(t)\|_2^2]. \quad (5)$$

Obviously, a set of adaptive filtering algorithms, such as LMS and RLS, can be used to solve the LS cost function (5) for the subspace vectors. PAST and PASTd algorithms proposed in [2] are just derived from RLS algorithm. In this paper, we will introduce a novel Kalman filter frame instead of RLS to achieve a better tracking performance and a better flexibility for different scenarios.

III. Kalman Filter with Variable Measurements

Consider the linear state-space model as follows:

$$x(t) = \mathbf{F}(t)x(t-1) + w(t), \quad (6)$$

$$y(t) = \mathbf{H}(t)x(t) + \varepsilon(t), \quad (7)$$

where $x(t)$ and $y(t)$ are respectively the state vector and the observation vector at time instant t. $\mathbf{F}(t)$ and $\mathbf{H}(t)$ are respectively the state transition matrix and the observation matrix. The state noise vector $w(t)$ and the observation noise vector $\varepsilon(t)$ are zero mean Gaussian noise with covariance matrices $\mathbf{Q}(t)$ and $\mathbf{R}(t)$ respectively. An optimal state estimator in the least mean squares criterion for the state-space model can be computed by the standard Kalman filter recursions:

$$\hat{x}(t+1/t) = \mathbf{F}(t)\hat{x}(t/t), \quad (8)$$

$$\mathbf{P}(t+1/t) = \mathbf{F}(t)\mathbf{P}(t/t)\mathbf{F}(t)^T + \mathbf{Q}(t), \quad (9)$$

$$e(t) = y(t) - \mathbf{H}(t)\hat{x}(t/t-1), \quad (10)$$

$$\mathbf{K}(t) = \mathbf{P}(t/t-1)\mathbf{H}(t)^T \cdot [\mathbf{H}(t)\mathbf{P}(t/t-1)\mathbf{H}(t)^T + \mathbf{R}(t)]^{-1}, \quad (11)$$

$$\hat{x}(t/t) = \hat{x}(t/t-1) + \mathbf{K}(t)e(t), \quad (12)$$

$$\mathbf{P}(t/t) = [\mathbf{I} - \mathbf{K}(t)\mathbf{H}(t)]\mathbf{P}(t/t-1), \quad (13)$$

where $e(t)$ is the prediction error of the observation vector. $\hat{x}(t/k)$ ($k = t-1, t$) represents the estimator of $x(t)$ given the measurements up to time instant k $\{y(j), j \le k\}$, and $\mathbf{P}(t/k)$ is the corresponding covariance matrix of $\hat{x}(t/k)$.

To derive the new KFVM, we apply the equivalence formulation of the Kalman filter algorithm as a particular LS regression problem of Durović and Kovačević [5]. More precisely, (6) and (7) are combined together to yield the following equivalent linear model:

$$\begin{bmatrix} \mathbf{I} \\ \mathbf{H}(t) \end{bmatrix} x(t) = \begin{bmatrix} \mathbf{F}\hat{x}(t-1/t-1) \\ y(t) \end{bmatrix} + \mathbf{E}(t), \quad (14)$$

where $\mathbf{E}(t) = \begin{bmatrix} \mathbf{F}[x(t-1) - \hat{x}(t-1/t-1)] + w(t-1) \\ -\varepsilon(t) \end{bmatrix}$ and $E[\mathbf{E}(t)\mathbf{E}^T(t)] = \begin{bmatrix} \mathbf{P}(t/t-1) & 0 \\ 0 & \mathbf{R}(t) \end{bmatrix} = \mathbf{S}(t)\mathbf{S}^T(t)$. $\mathbf{S}(t)$ can be computed from the Cholesky decomposition of $E[\mathbf{E}(t)\mathbf{E}^T(t)]$. By multiplying both sides of (14) by $\mathbf{S}^{-1}(t)$, we get the following linear regression:

$$\mathbf{Y}(t) = \mathbf{X}(t)\beta(t) + \xi(t), \quad (15)$$

where $\mathbf{X}(t) = \mathbf{S}^{-1}(t)\begin{bmatrix} \mathbf{I} \\ \mathbf{H}(t) \end{bmatrix}$, $\mathbf{Y}(t) = \mathbf{S}^{-1}(t)\begin{bmatrix} \mathbf{F}\hat{x}(t-1) \\ y(t) \end{bmatrix}$, $\beta(t) = x(t)$, and $\xi(t) = -\mathbf{S}^{-1}(t)\mathbf{E}(t)$. Note that $\mathbf{E}(t)$ is whitened by $\mathbf{S}^{-1}(t)$ and the residual $\xi(t)$ satisfies $E[\xi(t)\xi^T(t)] = \mathbf{I}$. It can be seen that (15) is a standard LS regression problem with solution:

$$\hat{\beta}(t) = \hat{x}(t/t) = (\mathbf{X}^T(t)\mathbf{X}(t))^{-1}\mathbf{X}^T(t)\mathbf{Y}(t), \quad (16)$$

and the covariance matrix of estimating $\beta(t)$ is

$$E[(\beta(t) - \hat{\beta}(t))(\beta(t) - \hat{\beta}(t))^T] = \mathbf{P}(t/t) = (\mathbf{X}^T(t)\mathbf{X}(t))^{-1}. \quad (17)$$

Hence, the Kalman filter can also be thought of as the solution to a LS problem with $\hat{\beta}(t) = \hat{x}(t/t)$ and $\mathbf{P}(t/t) = \mathrm{cov}(\hat{\beta}(t))$. Equations (15)–(17) form an equivalent Kalman filter recursion based on LS estimation.

To derive the proposed Kalman filter with variable measurement algorithm, let's rewrite (15) as

$$\left\{\mathbf{S}^{-1}(t)\begin{bmatrix} \mathbf{F}\hat{x}(t-1) \\ y(t) \end{bmatrix}\right\} = \left\{\mathbf{S}^{-1}(t)\begin{bmatrix} \mathbf{I} \\ \mathbf{H}(t) \end{bmatrix}\right\} x(t) + \xi(t). \quad (18)$$

The lower part of the equation is equivalent to a conventional LS estimation of $x(t)$ from the available measurement. The upper part is a regularization term that imposes a smoothness constraint from the state dynamic into the LS problem. If $\mathbf{F}$ is an identity matrix, (18) is equivalent to the LMS algorithm with a certain kind of diagonal loading. Another observation is that only one measurement is used to update the state vector. Hence, the bias error will be low especially when the system is fast time-varying. On the other hand, if the system is time-invariant or slowly time-varying, including more past measurements can help to reduce the estimation variance. These observations motivate us to develop a new Kalman filter algorithm with variable number of measurements (KFVM) to achieve the best bias-variance tradeoff for time-varying environment.

Suppose the measurements used for tracking the state estimate are: $[y(t-L+1),\cdots,y(t-1),y(t)]$, where L is the number of measurements used to update the state estimate. Including all these measurements in (15) gives:

$\mathbf{Y}(t) = \mathbf{S}^{-1}(t)\left[\{\mathbf{F}\hat{x}(t-1)\}^T, y(t-L+1), \cdots, y(t-1), y(t)\right]^T$ and $\mathbf{X}(t) = \mathbf{S}^{-1}(t)\left[\mathbf{I}, \mathbf{H}^T(t-L+1), \cdots, \mathbf{H}^T(t-1), \mathbf{H}^T(t)\right]^T$. Note that $\mathbf{S}^{-1}(t)$ is now obtained from $\begin{bmatrix} \mathbf{P}(t/t-1) & \mathbf{0} \\ \mathbf{0} & diag\{\mathbf{R}(t-L+1), \cdots, \mathbf{R}(t-1), \mathbf{R}(t)\} \end{bmatrix}$. The linear LS problem (15) in block-update form can be solved using (16). However, the QR decomposition (QRD) should be used to reduce the arithmetic complexity of the algorithm and improve the numerical stability in finite precision arithmetic.

If the number of measurements L is a constant for all time instants, we refer the resultant Kalman filter algorithm as the Kalman filter with multi-measurements (KFMM). When $L = 1$, KFMM will reduce to the conventional Kalman filter. The problem then is: how to choose an adaptive time varying L for the KFVM to achieve the best bias-variance tradeoff. As mentioned above, L is determined according to the variations of the subspace.

Inspired by the variable forgetting factor (VFF) method for RLS algorithms, we will develop a new control scheme to select L adaptively. The proposed control scheme is based on the approximated derivatives of the system state, which was first proposed in the GP-APA algorithm for LMS-type algorithms [6]. The proposed scheme is given as follows:

$$\hat{c}(t) = \hat{x}(t-1) - \tilde{x}(t-1), \quad (19)$$

$$\tilde{x}(t) = \eta\tilde{x}(t-1) + (1-\eta)\hat{x}(t-1), \quad (20)$$

where $\hat{x}(t)$ is the state estimate and $\hat{c}(t)$ is its approximated time derivative. η is the forgetting factor ($0 < \eta \le 1$) for calculating the smoothed tap weight $\tilde{x}(t)$. The l_1 norm of $\hat{c}(t)$, $\|\hat{c}(t)\|_1$, will decrease and converge gradually from its initial value to a very small value when the algorithm is about to converge to the signal subspace of a static environment. Therefore, it serves as a measure of the variation in the signal subspace, or states. To determine the number of measurements $L(t)$, we compute the absolute value of the approximate derivative of $\|\hat{c}(t)\|_1$ as

$$G_c(t) = \left|\|\hat{c}(t)\|_1 - \|\hat{c}(t-1)\|_1\right|, \quad (21)$$

and then $\overline{G}_c(t)$, a smoothed version of $G_c(t)$, by averaging it over a time window of length T_s. The initial value of $\overline{G}_c(t)$, denoted by $\overline{G}_{c0}$, is obtained by averaging the first T_s data. By normalizing $\overline{G}_c(t)$ with $\overline{G}_{c0}$, we get $\overline{G}_N(t)$, which is a more stable measure of the subspace variation. Denote the lower and upper bounds of $L(t)$ as L_L and L_U, $L(t)$ is updated at each iteration as:

$$L(t) = L_L + [1 - \overline{G}_N(t)](L_U - L_L). \quad (22)$$

Hence, the new KFVM algorithm can be obtained by using $L(t)$ number of measurements to estimate the system state. Alternately, $L(t)$ can also be determined using the intersection of confidence intervals (ICI) rule [4], at higher arithmetic complexity.

IV. Kalman Filter-based Subspace Tracking

As mentioned earlier, the conventional PAST method, which is based on the RLS algorithm, is able to track the stationary or slowly time-varying subspaces very well. However, when the signal subspace varies acutely, the tracking performance will deteriorate. To address the problem, a dynamical function based on an AR model for $\mathbf{W}$ will be introduced in the state-space model. For notation convenience, the state dynamics will be presented in terms of $\mathbf{W}^T$, instead of $\mathbf{W}$:

$$\mathbf{W}^T(t) = \mathbf{F}(t)\mathbf{W}^T(t-1) + \mathbf{\Delta}(t), \quad (23)$$

where $\mathbf{F}(t)$ is the state transition matrix. The innovation matrix is given by $\mathbf{\Delta}(t)$, and it is modeled as a ($r \times N$) AWGN process.

Again using the projection approximation, as in (5), $\underline{z}(t)$ is represented as $\mathbf{W}(t)\underline{h}(t)$, corrupted by a residual error vector. Hence, we have:

$$\underline{z}^T(t) = \underline{h}^T(t)\mathbf{W}^T(t) + \Psi(t), \quad (24)$$

where $\underline{h}(t) = \hat{\mathbf{W}}^H(t-1)\underline{z}(t)$ and $\Psi(t)$ is a $(1 \times N)$ AWGN vector representing the residual error. From (24), the state transition matrix is seen to be $\mathbf{H}(t) = \underline{h}^T(t) = \underline{z}^T(t)\hat{\mathbf{W}}^*(t-1)$, where $*$ means the complex conjugate operator. Note that the idea of observation matrix estimate $\mathbf{H}(t) = \underline{z}^T(t)\hat{\mathbf{W}}^*(t-1)$ originates from the conventional PAST algorithm. However, in our Kalman filter-based algorithm, a better estimate for the observation matrix is given by $\mathbf{H}(t) = \underline{z}^T(t)\hat{\mathbf{W}}^*(t/t-1)$.

Equation (23) and (24) constitute the linear state-space model for our Kalman filter-based subspace tracking algorithm. Comparing (23) and (24) with the state-space equations in (6) and (7), we can see that: $\mathbf{W}^T(t)$ is the state matrix, $\underline{z}^T(t)$ is the observation vector, $\mathbf{\Delta}(t)$ is the state noise matrix and $\Psi(t)$ is the observation noise vector.

Using our KFVM algorithm, a variable block of past measurements, $\underline{z}(t-L(t)+1), \cdots, \underline{z}(t)$, can be used to update the subspace matrix $\mathbf{W}(t)$ at time instant t. The number of measurements is time varying and it can be adjusted using the control scheme in the previous section. However, when too many measurements are used in the KFVM updating process, the complexity will increase considerably. Since a large $L(t)$ means that the signal subspace changes very slowly or even remains stationary, therefore, a large number of measurements are used to obtain a small estimation variance. As the RLS algorithm (PAST) can perform well in tracking a steady subspace while having a much lower complexity than KFVM, it is advantageous to be switched to a RLS-based algorithm to avoid the excessive complexity of using a large number of measurements in KFVM. In the proposed control scheme, a threshold $\overline{L}$ can be set for $L(t)$, so that when $L(t) \ge \overline{L}$, the RLS will be employed for the state estimation at time t, instead of the KFVM with a large value of $L(t)$.

V. Simulation Results

A classical application of subspace tracking is in the Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) [7] for the estimation of the directions of arrival (DOA) of plane waves impinging on a uniform linear antenna array. Here, we will evaluate the performance of the proposed KFVM plus RLS algorithm using this DOA tracking application.

The observation signal vector $\underline{z}(t)$, $t = 1, 2, \cdots, 600s$, is composed of $r = 3$ narrow-band sources impinging on a linear uniform antenna array with $N = 8$ sensors. The background observation noise is assumed to be an AWGN with a SNR of 20dB. The parameters for the adaptive selection of the number of measurements are: $T_s = 100$, $L_L = 1$, $L_U = 32$, and $\overline{L} = 0.9L_U$. In the state-space model in (23) and (24), the state transition matrix and the observation matrix are chosen as $\mathbf{F}(t)=\mathbf{I}_r$ and $\mathbf{H}(t)=\underline{z}^T(t)\hat{\mathbf{W}}^*(t/t-1)$, respectively. The covariance matrices of the state noise $\mathbf{\Delta}(t)$ and the observation noise $\Psi(t)$ can be estimated recursively using the algorithm proposed in [5].

Several different algorithms are tested to compare their tracking performances: PAST, KFMM with $L=1$ and 32, and KFVM plus RLS. The ESPRIT algorithm is employed to compute the DOA from the signal subspace estimate. The mean square difference (MSD), defined as the sum of the square of the DOA estimate errors, is used to evaluate the convergence behavior of the algorithms. All the curves in Fig. 2 are averaged over 100 independent runs.

The DOAs in our simulation are generated as:

$$\theta(t)=\begin{cases}\theta(t-1)+10^{-4}\cdot[2,-1,-2]^T+v_1(t) & t\le 300\\ \theta(t-1)+v_2(t) & t>300\end{cases}, \tag{27}$$

where $v_1(t)$ and $v_2(t)$ are two zero mean Gaussian noise vectors with covariance matrices $2.5\times10^{-7}\mathbf{I}_r$ and $0.9\times10^{-7}\mathbf{I}_r$, respectively. The initial DOA is given as $\theta_0=[-0.18,0.1,0.2]^T$. We can see from Fig. 1 that the DOAs comprise two segments: the fast-varying DOAs in the first half of time ($t\le 300$), and the slow-varying DOAs in the second half ($t>300$).

Fig. 2 shows that in the first segment, the performance of KFMM improves with decreasing L due to the increased bias error. KFVM plus RLS can achieve a result comparable to KFMM with $L=1$, which gives the best result. However, the conventional PAST algorithm will diverge when DOAs change rapidly. After time instant 300 when the DOAs change slowly, PAST, KFMM with $L=32$, and KFVM plus RLS all have a fast convergence speed and achieve a small MSD. If L is too small, e.g. $L=1$, the corresponding KFMM yields a considerably higher DOA estimation error due to increased estimation variance. Fig. 3 gives a realization of the number of measurements selection in KFVM. We can see that when $t\le 300$, L tends to have a small value for fast-varying DOA tracking. On the contrary, when DOAs come to the stationary segment, a large value of L is chosen to allow more measurements to be included to estimate the subspace. The RLS algorithm will be employed when $L(t)\ge\bar{L}$ to reduce the arithmetic complexity because sufficient observations are now available to determine the signal subspace accurately.

From the above simulation results, we can conclude that the PAST and Kalman filter with a large number of measurements are suitable for tracking time-invariant or slow-varying subspace. When the subspace varies rapidly, Kalman filter with fewer measurements can yield a good tracking result. The proposed KFVM plus RLS adaptive algorithm adaptively combines their advantages and is able to achieve a better result for a wide variety of signal subspace variations.

VI. Conclusion

A novel Kalman filter-based algorithm with variable measurements is introduced in this paper. The proposed KFVM is applied to the subspace tracking problem and it yields a new tacking algorithm with improved tracking performance for a wide variety of signal subspace variations. The number of measurements in the KFVM is determined adaptively according to the approximated state estimate derivatives. When the number of measurements is large, the RLS can be employed in the KFVM to reduce the arithmetic complexity. The advantages of the proposed algorithm are demonstrated using a DOA estimation application.

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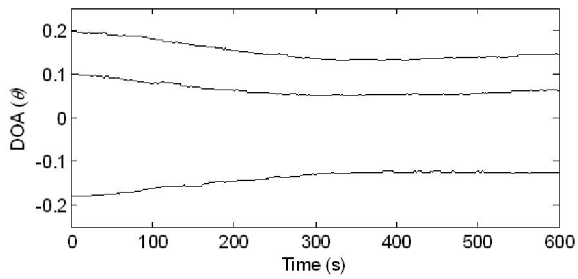

Fig. 1. Direction of Arrials (DOA) model.

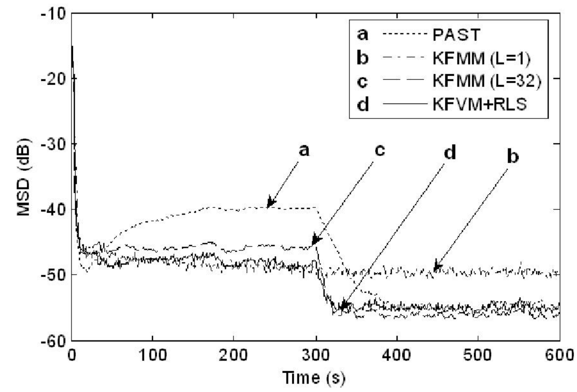

Fig. 2. MSD of DOA estimation using different algorithms.

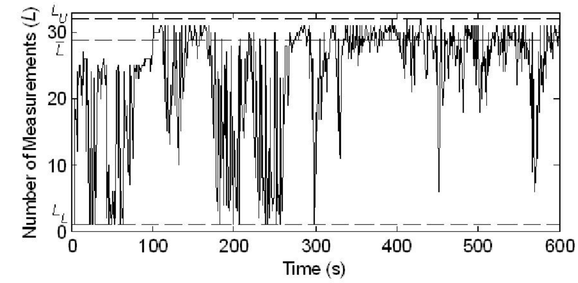

Fig. 3. Number of measurements used in KFVM for subspace estimate.