# Reply to "Comment on 'Quantum convolutional error-correcting codes' "

H. F. Chau*

*Department of Physics, University of Hong Kong, Pokfulam Road, Hong Kong and Center of Theoretical and Computational Physics, University of Hong Kong, Pokfulam Road, Hong Kong*

(Received 29 March 2005; published 5 August 2005)

In their Comment, de Almeida and Palazzo [Phys. Rev. A **72**, 026301 (2005)] discovered an error in my earlier paper concerning the construction of quantum convolutional codes [Phys. Rev. A **58**, 905 (1998)]. This error can be repaired by modifying the method of code construction.

DOI: 10.1103/PhysRevA.72.026302 PACS number(s): 03.67.Pp, 89.70.+c

de Almeida and Palazzo [1] found a counterexample showing the invalidity of Theorems 2 and 3 in my earlier paper [2]. Their counterexample is correct; and the source of error lies with the proof of Lemma 2 in Ref. [2]. In fact, Lemma 2 is not correct and Theorem 3 should be modified as follows. (It is straightforward to extend the modified theorem to cover the case of qudits.)

**Theorem 3.** Let $C_1$ be a classical (block or convolutional) code of rate $r_1$ and distance $d_1$ and let $C_2$ be the $[n_2 = d_2, 1, d_2]$ majority vote classical code of rate $r_2$, namely, the one that maps $|t\rangle$ to $\otimes_{j=1}^{n_2}|t\rangle$. We construct a quantum code $C$ by first encoding a quantum state by $C_1$, then by applying a Hadamard transform to every resultant qubit, and finally by encoding each of the Hadamard transformed qubits by $C_2$. The rate and minimum distance of code $C$ equal $r_1 r_2$ and $\min(d_1^{\perp}, d_2)$, respectively, where $d_1^{\perp}$ is the minimum distance of the (classical) dual code of $C_1$.

*Proof.* Clearly, the rate of code $C$ equals $r_1 r_2$. So, we only need to show that its minimum distance is $\min(d_1^{\perp}, d_2)$. Let us examine the classical code $C_1$ and the quantum code $C$ in the stabilizer formalism. We denote the operation of applying $\sigma_x$ ($\sigma_z$) to the $i$th qubit by $X_i$ ($Z_i$). The encoded operation that flips the spin of the $i$th unencoded qubit for the classical code $C_1$ can be expressed in the form $X_1^{f_i(1)} \circ X_2^{f_i(2)} \circ \cdots \equiv \Pi_{j \geq 1} X_j^{f_i(j)}$, where $f_i$ is a binary valued function. The dual code of $C_1$ is a linear space spanned by vectors in the form $\Pi_{j \geq 1} X_j^{g_s(j)}$, where $g_s$'s are some binary valued functions. Since $C_2$ is the majority vote code, from our construction of $C$, the encoded operation that flips the spin (shifts the phase) of the $i$th unencoded qubit for the quantum code $C$ is given by $\Pi_{j \geq 1} \Pi_{k=1}^{n_2} Z_{n_2(j-1)+k}^{f_i(j)}$ ($\Pi_{j \geq 1} \Pi_{k=1}^{n_2} X_{n_2(j-1)+k}^{f_i(j)}$). Furthermore, the stabilizer of $C$ equals the span of $\{Z_{n_2(m-1)+1} \circ Z_{n_2(m-1)+\ell}, \Pi_{j \geq 1} \Pi_{k=1}^{n_2} X_{n_2(j-1)+k}^{g_s(j)} : \ell = 2, 3, \ldots, n_2$ and $m, s \geq 1\}$. So just like CCS codes, the spin-flip and phase-shift errors in the quantum code $C$ can be corrected separately. After explicitly writing down the encoded operations and the generators of the stabilizer for the (degenerate) quantum code $C$, it is straightforward to check that $C$ detects all spin errors happening to less than $d_2$ qubits. Moreover, all phase-shift errors involving less than $\min(d_1^{\perp}, d_2)$ qubits are in the stabilizer. Thus, the minimum distance of code $C$ is $\min(d_1^{\perp}, d_2)$. □

This work is supported by the RGC Grant HKU 7010/04P of the HKSAR Government.

*Electronic address: hfchau@hkusua.hku.hk

[1] A. C. A. de Almeida and R. Palazzo, Jr., Phys. Rev. A **72**, 026301 (2005).

[2] H. F. Chau, Phys. Rev. A **58**, 905 (1998).

 