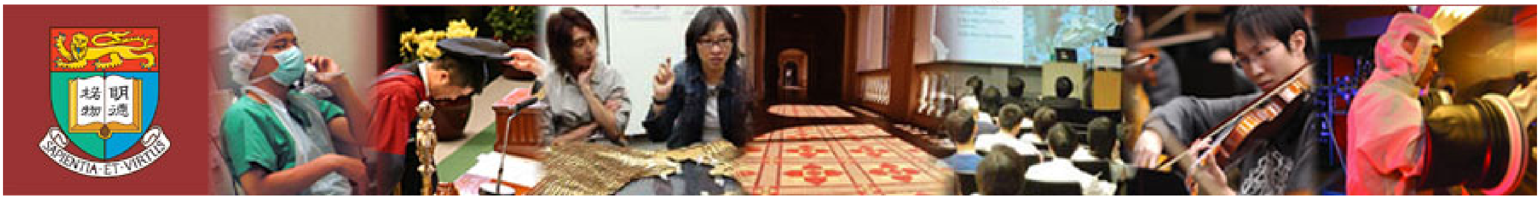

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Robust H^∞ Filtering for Uncertain Markovian Jump Systems With Mode-Dependent Time Delays

Shengyuan Xu, Tongwen Chen, and James Lam

***Abstract*—This note considers the problem of robust H^∞ filtering for uncertain Markovian jump linear systems with time-delays which are time-varying and depend on the system mode. The parameter uncertainties are time-varying norm-bounded. The aim of this problem is to design a Markovian jump linear filter that ensures robust exponential mean-square stability of the filtering error system and a prescribed $\mathcal{L}_2$-induced gain from the noise signals to the estimation error, for all admissible uncertainties. A sufficient condition for the solvability of this problem is obtained. The desired filter can be constructed by solving a set of linear matrix inequalities. An illustrative numerical example is provided to demonstrate the effectiveness of the proposed approach.**

***Index Terms*—H^∞ filtering, linear matrix inequalities, Markovian jump systems, robust filtering, time-delay systems, uncertain systems.**

I. Introduction

State estimator design has been an important research topic and has found many practical applications. It has been recognized that one of the most popular estimation approaches is the celebrated Kalman filtering [1]. A common feature of the standard Kalman filtering algorithm is that an exact internal model of the system is available and the exogenous input signals are assumed to be Gaussian noises with known statistics. In some applications, however, the noise sources may not be exactly known and parameter uncertainties may appear in a system model; these limit the scope of applications of the Kalman filtering technique. To overcome these difficulties, an alternative approach called H_∞ filtering has been introduced. In the H_∞ filtering setting, the noise sources are arbitrary signals with bounded energy or average power, and no exact statistics are required to be known [13]. It has been shown that the H_∞ filtering technique provides both a guaranteed noise attenuation level and robustness against unmodeled dynamics [13].

When parameter uncertainty appears in a system model, the robustness of H_∞ filters has to be taken into account. This has motivated the study of the robust H_∞ filtering problem, which is concerned with the design of estimators ensuring that the filtering error system is asymptotically stable and the $\mathcal{L}_2$-induced gain from the noise signal to the estimation error is below a prescribed level. A great number of results on this topic have been reported in the literature and various approaches have been proposed; see, e.g., [9] and the references therein. Since time delays are frequently encountered in various engineering systems, the robust H_∞ filtering problem for time-delay systems has been considered recently. For example, by a linear matrix inequality (LMI) approach, sufficient conditions for the solvability of this problem was presented in [8] for time-delay systems with structured parameter uncertainty; while in [16], a Riccati-like approach was developed and unstructured parameter uncertainty was considered. Furthermore, when time delays and nonlinearities arise simultaneously in an uncertain system, some sufficient conditions for the existence of robust H_∞ filters were obtained in [17].

On the other hand, a great deal of attention has recently been devoted to the study of Markovian jump linear systems. This class of systems can model stochastic systems with abrupt structural variation resulting from the occurrence of some inner discrete events in the system such as failures and repairs of machine in manufacturing systems, modifications of the operating point of a linearized model of a nonlinear system, and so on. A great number of estimation and control issues concerning these systems have been studied [7], [10]. Some of these results were further extended to Markovian jump systems with time delays in [3], where the time delays are independent of the system modes. Very recently, Markovian jump systems with mode-dependent time delays have been studied. In [3], some robust stability conditions were presented in terms of LMIs. The robust H_∞ control results in the discrete context can be found in [2]. However, the robust H_∞ filtering problem for such systems has not been fully investigated, which is still open and remains challenging.

In this note, we are concerned with the problem of robust H_∞ filtering for Markovian jump linear systems with parameter uncertainties and time delays. The parameter uncertainties are time varying but norm bounded. The time delays are assumed to be time varying, and are dependent on the system mode. The problem we address is the design of Markovian jump linear filters such that for all admissible uncertainties, the resulting error system achieves robust exponential mean-square stability and the $\mathcal{L}_2$-induced gain from the noise signal to the estimation error remains bounded by a prescribed value. It is worth mentioning here that the advantage of a filter with exponential mean-square stability in comparison with a filter with asymptotic mean-square stability lies in that the former provides fast convergence and desirable accuracy in terms of reasonable error covariance of the filtering process [14]. To solve the robust H_∞ filtering problem, an LMI approach is developed and sufficient conditions for the solvability are obtained. The desired filter can be constructed through a convex optimization problem, which can be efficiently handled by using standard numerical algorithms [4].

II. Problem Formulation

Given a probability space $(\Omega, \mathcal{F}, \mathcal{P})$, where Ω is the sample space, $\mathcal{F}$ is the σ-algebra of subsets of the sample space and $\mathcal{P}$ is the probability measure on $\mathcal{F}$. On this probability space $(\Omega, \mathcal{F}, \mathcal{P})$, we consider the following class of uncertain linear stochastic systems with Markovian jump parameters and mode-dependent time delays:

$$(\Sigma): \quad \begin{aligned} \dot{x}(t) &= A(t,r_t)x(t) + A_d(t,r_t)x\left(t-\tau_{r_t}(t)\right) \\ &\quad + B(t,r_t)\omega(t) \\ y(t) &= C(t,r_t)x(t) + C_d(t,r_t)x\left(t-\tau_{r_t}(t)\right) \\ &\quad + D(t,r_t)\omega(t) \qquad (1) \\ z(t) &= L(r_t)x(t) \qquad (2) \\ x(t) &= \varphi(t),\ \forall t \in [-\mu, 0] \qquad (3) \end{aligned}$$

where $x(t) \in \mathbb{R}^n$ is the state; $\omega(t) \in \mathbb{R}^p$ is the noise signal which is assumed to be an arbitrary signal in $\mathcal{L}_2[0,\infty)$, where $\mathcal{L}_2[0,\infty)$ is the space of square-integrable vector functions over $[0,\infty)$; $y(t) \in \mathbb{R}^q$ is the measurement; $z(t) \in \mathbb{R}^m$ is the signal to be estimated; and $\{r_t\}$ is a continuous-time Markovian process with right continuous trajectories and taking values in a finite set $\mathcal{S} = \{1, 2, \ldots, \mathcal{N}\}$ with transition probability matrix $\Pi \triangleq \{\pi_{ij}\}$ given by

$$\Pr\{r_{t+h} = j | r_t = i\} = \begin{cases} \pi_{ij}h + o(h) & i \neq j \\ 1 + \pi_{ii}h + o(h) & i = j \end{cases} \qquad (4)$$

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where $h > 0$, $\lim_{h\to 0}(o(h)/h) = 0$, and $\pi_{ij} \geq 0$, for $j \neq i$, is the transition rate from mode i at time t to mode j at time $t+h$ and

$$\pi_{ii} = -\sum_{j=1, j\neq i}^{s} \pi_{ij}. \tag{5}$$

In the system (Σ), $\tau_{r_t}(t)$ denotes the time-varying delay when the mode is in τ_t and satisfies

$$0 < \tau_i(t) \leq \mu_i < \infty \quad \dot{\tau}_i(t) \leq h_i < 1 \qquad \forall i \in \mathcal{S} \tag{6}$$

where μ_i and h_i are real constant scalars for any $i \in \mathcal{S}$. In (3), $\mu = \max\{\mu_i,\ i \in \mathcal{S}\}$; $\varphi(t)$ is a vector-valued initial continuous function defined on the interval $[-\mu, 0]$; $A(t, r_t)$, $A_d(t, r_t)$, $B(t, r_t)$, $C(t, r_t)$, $C_d(t, r_t)$, $D(t, r(t))$ and $L(r_t)$ are matrix functions of r_t, and for each $r_t \in \mathcal{S}$

$$\begin{aligned} A(t, r_t) &= A(r_t) + \Delta A(t, r_t) \\ A_d(t, r_t) &= A_d(r_t) + \Delta A_d(t, r_t) \\ B(t, r_t) &= B(r_t) + \Delta B(t, r_t) \\ C(t, r_t) &= C(r_t) + \Delta C(t, r_t) \\ C_d(t, r_t) &= C_d(r_t) + \Delta C_d(t, r_t) \\ D(t, r_t) &= D(r_t) + \Delta D(t, r_t) \end{aligned}$$

where $A(r_t)$, $A_d(r_t)$, $B(r_t)$, $C(r_t)$, $C_d(r_t)$, $D(r(t))$ and $L(r_t)$ are known real constant matrices representing the nominal system for each $r_t \in \mathcal{S}$, and $\Delta A(t, r_t)$, $\Delta A_d(t, r_t)$, $\Delta B(t, r_t)$, $\Delta C(t, r_t)$, $\Delta C_d(t, r_t)$ and $\Delta D(t, r(t))$ are unknown matrices representing time-varying parameter uncertainties, and are assumed to be of the form

$$\begin{aligned} &\begin{bmatrix} \Delta A(t, r_t) & \Delta A_d(t, r_t) & \Delta B(t, r_t) \\ \Delta C(t, r_t) & \Delta C_d(t, r_t) & \Delta D(t, r_t) \end{bmatrix} = \begin{bmatrix} M_1(r_t) \\ M_2(r_t) \end{bmatrix} \\ &\qquad \times F(t, r_t)\left[N_1(r_t) \quad N_2(r_t) \quad N_3(r_t)\right] \qquad \forall r_t \in \mathcal{S} \end{aligned} \tag{7}$$

where $M_1(r_t)$, $M_2(r_t)$, $N_1(r_t)$, $N_2(r_t)$, and $N_3(r_t)$ are known real constant matrices for all $r_t \in \mathcal{S}$, and $F(t, r_t)$, for all $r_t \in \mathcal{S}$, are the uncertain time-varying matrices satisfying

$$F(t, r_t)^T F(t, r_t) \leq I \qquad \forall r_t \in \mathcal{S}. \tag{8}$$

The uncertain matrices $\Delta A(t, r_t)$, $\Delta A_d(t, r_t)$, $\Delta B(t, r_t)$, $\Delta C(t, r_t)$, $\Delta C_d(t, r_t)$ and $\Delta D(t, r(t))$, for each $i \in \mathcal{S}$ are said to be admissible if both (7) and (8) hold.

For the sake of notation simplification, in the sequel, for each possible $r_t = i$, $i \in \mathcal{S}$, a matrix $M(t, r_t)$ will be denoted by $M_i(t)$; for example, $A(t, r_t)$ is denoted by $A_i(t)$, and $B(r_t)$ by B_i, and so on.

Remark 1: It shall be pointed out that the technical assumption on the time-varying delay variables $\tau_i(t)$ satisfying $\dot{\tau}_i(t) \leq h_i < 1$ (for $i \in \mathcal{S}$) is quite standard when dealing with the problems of robust stability analysis and robust stabilization for systems with time-varying delays; see, e.g. [12].

Throughout this note, we adopt the following definition.

Definition 1 [11]: The uncertain Markovian jump system (Σ) is said to achieve robust exponential mean-square stability if, when $\omega(t) = 0$, for any finite $\varphi(t) \in \mathbb{R}^n$ defined on $[-\mu, 0]$, and initial mode $r_0 \in \mathcal{S}$, there exist constant scalars $b > 0$ and $c > 0$ such that

$$\mathcal{E}\left\{|x(t, \varphi, r_0)|^2\right\} \leq b \sup_{-\mu\leq\theta\leq 0} |\varphi(\theta)|^2 e^{-ct} \tag{9}$$

where $\mathcal{E}\{\cdot\}$ denotes the expectation; $x(t, \varphi, r_0)$ denotes the solution of the system (Σ) at time t under the initial conditions $\varphi(t)$ and r_0.

In this note, we are concerned with obtaining an estimate $\hat{z}(t)$, of $z(t)$ via a causal Markovian jump linear filter which guarantees a small estimation error, $z(t) - \hat{z}(t)$, for all nonzero $\omega \in \mathcal{L}_2[0, \infty)$ and all admissible uncertainties. For each $i \in \mathcal{S}$, attention is focused on the design of a linear, exponential mean-square stable, Markovian jump filter of order n with the following form:

$$(\Sigma_f): \quad \dot{\hat{x}}(t) = A_{fi}\hat{x}(t) + B_{fi}y(t) \tag{10}$$

$$\hat{z}(t) = L_i\hat{x}(t) \tag{11}$$

where $\hat{x}(t) \in \mathbb{R}^n$ and $\hat{z}(t) \in \mathbb{R}^q$, for each $i \in \mathcal{S}$, the matrices $\hat{A}_{fi}$ and B_{fi} are to be determined.

Then, the robust filtering problem addressed in this note is formulated as follows: given the uncertain stochastic delay system (Σ) and a prescribed level of noise attenuation $\gamma > 0$, determine an exponentially mean-square stable filter (Σ_f) in the form of (10) and (11) such that the filtering error system achieves robust exponential mean-square stability and

$$\|z - \hat{z}\|_{E_2} < \gamma\|\omega\|_2 \tag{12}$$

under zero-initial conditions for any nonzero $\omega \in \mathcal{L}_2[0, \infty)$, where $\|\cdot\|_2$ represents the usual $\mathcal{L}_2[0, \infty)$ norm, and $\|\cdot\|_{E_2}$ denotes the norm in $\mathcal{L}_2((\Omega, \mathcal{F}, \mathcal{P}), [0, \infty))$, that is

$$\|z - \hat{z}\|_{E_2} = \left(\mathcal{E}\left\{\int_0^\infty |z(t) - \hat{z}(t)|^2\, dt\right\}\right)^{\frac{1}{2}}$$

where the symbol $|\cdot|$ refers to the Euclidean vector norm.

III. Main Results

In this section, a sufficient condition for the existence of robust H_∞ filters is proposed and an LMI approach is developed. Before presenting the main results, we first give the following lemmas which will be used in the proof of our main results.

Lemma 1: The uncertain Markovian jump system (Σ) achieves robust exponential mean-square stability if there exist matrices $P_1 > 0$, $P_2 > 0, \ldots, P_{\mathcal{N}} > 0$ and $Q > 0$ such that the following LMIs hold for $i = 1, 2, \ldots, \mathcal{N}$

$$\begin{bmatrix} \sum_{j=1}^{\mathcal{N}} \pi_{ij}P_j + A_i(t)^T P_i & \\ +P_i A_i(t) + (1+\eta\mu)Q & P_i A_{di}(t) \\ A_{di}(t)^T P_i & -(1-h_i)Q \end{bmatrix} < 0 \tag{13}$$

where

$$\eta = \max\left\{|\pi_{ii}|, i \in \mathcal{S}\right\}. \tag{14}$$

Proof: To show the robust exponential mean-square stability for the system (Σ) under the conditions of the lemma, we consider the equations in (1) and (3) with $\omega(t) = 0$, that is

$$(\Sigma_1): \quad \dot{x}(t) = A(t, r_t)x(t) + A_d(t, r_t)x\left(t - \tau_{r_t}(t)\right) \tag{15}$$

$$x(t) = \varphi(t),\ \forall t \in [-\mu, 0]. \tag{16}$$

Note that $\{(x(t),\ r_t),\ t \geq 0\}$ is not a Markov process. To cast our model involved into the framework of the Markov processes, we define a new process $\{(x_t,\ r_t),\ t \geq 0\}$ by

$$x_t(s) = x(t+s) \quad t - \tau_{r_t}(t) \leq s \leq t.$$

Then, similar to [3] and [6], we can verify that $\{(x_t,\ r_t),\ t \geq 0\}$ is a Markov process with initial state $(\varphi(\cdot), r_0)$. Now, define a stochastic Lyapunov functional candidate for the system (Σ_1) as

$$V(x_t, r_t) = V_1(x_t, r_t) + V_2(x_t, r_t) + V_3(x_t, r_t) \tag{17}$$

where

$$V_1(x_t, r_t) = x(t)^T P(r_t) x(t)$$
$$V_2(x_t, r_t) = \int_{t-\tau_{r_t}(t)}^{t} x(s)^T Q x(s) ds$$
$$V_3(x_t, r_t) = \eta \int_{-\mu}^{0} \int_{t+\theta}^{t} x(s)^T Q x(s) ds d\theta.$$

Let $\mathcal{A}$ be the weak infinitesimal generator of the random process $\{x_t, r_t\}$. Then, by some calculations, for each $r_t = i, i \in \mathcal{S}$ and any scalar $\beta > 0$, it can be verified that

$$\begin{aligned}\mathcal{A}\left[e^{\beta t} V_1(x_t, i)\right] = & e^{\beta t} x(t)^T \left(\sum_{j=1}^{\mathcal{N}} \pi_{ij} P_j x(t)\right) \\ & + 2e^{\beta t} x(t)^T P_i \left[A_i(t) x(t) \right. \\ & \left. + A_{di}(t) x\left(t - \tau_i(t)\right)\right] \\ & + \beta e^{\beta t} V_1(x_t, i)\end{aligned} \tag{18}$$

$$\begin{aligned}\mathcal{A}\left[e^{\beta t} V_2(x_t, i)\right] = & e^{\beta t} \sum_{j=1}^{\mathcal{N}} \pi_{ij} \int_{t-\tau_j(t)}^{t} x(s)^T Q x(s) ds \\ & + e^{\beta t} x(t)^T Q x(t) \\ & - e^{\beta t} (1 - \dot{\tau}_i(t))\, x\left(t - \tau_i(t)\right)^T Q x\left(t - \tau_i(t)\right) \\ & + \beta e^{\beta t} V_2(x_t, i)\end{aligned} \tag{19}$$

and

$$\begin{aligned}\mathcal{A}\left[e^{\beta t} V_3(x_t, i)\right] = & e^{\beta t} \eta \mu x(t)^T Q x(t) \\ & - e^{\beta t} \int_{t-\mu}^{t} x(s)^T Q x(s) ds \\ & + \beta e^{\beta t} V_3(x_t, i).\end{aligned} \tag{20}$$

Noting $\pi_{ij} \geq 0$, for $j \neq i$ and $\pi_{ii} \leq 0$, we have

$$\begin{aligned}\sum_{j=1}^{\mathcal{N}} \pi_{ij} \int_{t-\tau_j(t)}^{t} x(s)^T Q x(s) ds & \leq \sum_{j \neq 1} \pi_{ij} \int_{t-\mu}^{t} x(s)^T Q x(s) ds \\ & = -\pi_{ii} \int_{t-\mu}^{t} x(s)^T Q x(s) ds \leq \eta \int_{t-\mu}^{t} x(s)^T Q x(s) ds.\end{aligned}$$

From this, together with (6) and (17)–(20), it can be deduced that for each $r_t = i, i \in \mathcal{S}$ and any scalar $\beta > 0$

$$\begin{aligned}\mathcal{A}\left[e^{\beta t} V(x_t, i)\right] \leq & e^{\beta t} x(t)^T \left[\sum_{j=1}^{\mathcal{N}} \pi_{ij} P_j + A_i(t)^T P_i \right. \\ & + P_i A_i(t) + (1 + \eta\mu) Q \\ & \left. + (1 - h_i)^{-1} P_i A_{di}(t) Q^{-1} A_{di}(t)^T P_i \right] x(t) \\ & + \beta e^{\beta t} V(x_t, i)\end{aligned} \tag{21}$$

where we have used

$$\begin{aligned}2x(t)^T P_i A_{di}(t) x\left(t - \tau_i(t)\right) - (1 - h_i) x\left(t - \tau_i(t)\right)^T Q x\left(t - \tau_i(t)\right) \\ \leq (1 - h_i)^{-1} x(t)^T P_i A_{di}(t) Q^{-1} A_{di}(t)^T P_i x(t).\end{aligned}$$

On the other hand, applying the Schur complement formula in (13), we have that there exists a scalar $\alpha > 0$ such that

$$\begin{aligned}\sum_{j=1}^{\mathcal{N}} \pi_{ij} P_j + A_i(t)^T P_i + P_i A_i(t) + (1 + \eta\mu) Q \\ + (1 - h_i)^{-1} P_i A_{di}(t) Q^{-1} A_{di}(t)^T P_i + \alpha I < 0\end{aligned} \tag{22}$$

for each $i \in \mathcal{S}$. Noting (21) and (22), it is easy to show that for each $i \in \mathcal{S}$ and any scalar $\beta > 0$

$$\mathcal{A}\left[e^{\beta t} V(x_t, i)\right] < -\alpha e^{\beta t} |x(t)|^2 + \beta e^{\beta t} V(x_t, i). \tag{23}$$

It follows from this and (17) that

$$\begin{aligned}\mathcal{A}\left[e^{\beta t} V(x_t, i)\right] < & -\alpha e^{\beta t} |x(t)|^2 + \beta e^{\beta t} \left[\alpha_1 |x(t)|^2 \right. \\ & + \lambda_{\max}(Q) \int_{t-\mu}^{t} |x(s)|^2 ds \\ & \left. + \eta \lambda_{\max}(Q) \int_{-\mu}^{0} \int_{t+\theta}^{t} |x(s)|^2 ds d\theta \right].\end{aligned} \tag{24}$$

where

$$\alpha_1 = \max\left\{\lambda_{\max}(P_i),\ i \in \mathcal{S}\right\}.$$

Observe

$$\begin{aligned}\int_{-\mu}^{0} \int_{t+\theta}^{t} |x(s)|^2 ds d\theta & = \int_{t-\mu}^{t} (s - t + \mu) |x(s)|^2 ds \\ & \leq \mu \int_{t-\mu}^{t} |x(s)|^2 ds.\end{aligned}$$

This, together with (24), implies that for each $i \in \mathcal{S}$ and any scalar $\beta > 0$

$$\begin{aligned}\mathcal{A}\left[e^{\beta t} V(x_t, i)\right] < & (-\alpha + \alpha_1 \beta) e^{\beta t} |x(t)|^2 \\ & + \beta e^{\beta t} (\mu\eta + 1) \lambda_{\max}(Q) \int_{t-\mu}^{t} |x(s)|^2 ds.\end{aligned}$$

Therefore, by using Dynkin's formula [3], [5], we have that for any $T > 0$, $\beta > 0$, and each $r(t) = i, i \in \mathcal{S}$

$$\begin{aligned}\mathcal{E}\left\{e^{\beta T} V(x_T, i)\right\} - V(x_0, r_0) = & \mathcal{E}\left\{\int_0^T \mathcal{A}\left[e^{\beta s} V(x_s, i)\right] ds\right\} \\ < & (-\alpha + \alpha_1 \beta) \int_0^T e^{\beta t} |x(t)|^2 dt \\ & + \beta (\mu\eta + 1) \lambda_{\max}(Q) \\ & \times \int_0^T e^{\beta t} \int_{t-\mu}^{t} |x(s)|^2 ds dt.\end{aligned} \tag{25}$$

By setting

$$\alpha_2 = \mu(\mu\eta + 1) \lambda_{\max}(Q)$$

and noting

$$\begin{aligned}\int_0^T e^{\beta t}\int_{t-\mu}^{t} |x(s)|^2\, ds dt &\le \int_{-\mu}^{0} \mu e^{\beta(s+\mu)}\, |x(s)|^2\, ds\\ &+ \int_0^{T-\mu} \mu e^{\beta(s+\mu)}\, |x(s)|^2\, ds\\ &+ \int_{T-\mu}^{T} \mu e^{\beta(s+\mu)}\, |x(s)|^2\, ds\\ &= \mu \int_{-\mu}^{T} e^{\beta(s+\mu)}\, |x(t)|^2\, dt\end{aligned}$$

we have that for each $i \in \mathcal{S}$ and any scalar $\beta > 0$

$$\begin{aligned}\mathcal{E}\left\{e^{\beta T}V(x_T, i)\right\} &< V(x_0, r_0) + (-\alpha + \alpha_1\beta)\\ &\times \int_0^T e^{\beta t}|x(t)|^2 dt + \alpha_2\beta\\ &\times \int_{-\mu}^{T} e^{\beta(t+\mu)}|x(t)|^2 dt\\ &\le V(x_0, r_0) + \left(-\alpha + \alpha_1\beta + \alpha_2\beta e^{\beta\mu}\right)\\ &\times \int_0^T e^{\beta t}|x(t)|^2 dt + \alpha_2\beta e^{\beta\mu}\\ &\times \int_{-\mu}^{0} |x(t)|^2 dt. \end{aligned}\tag{26}$$

Now, choose $\beta > 0$ to be the unique solution to the following equation:

$$-\alpha + \alpha_1\beta + \alpha_2\beta e^{\beta\mu} = 0.$$

Then, by this and (26), we have

$$\mathcal{E}\left\{e^{\beta T}V(x_T, i)\right\} < V(x_0, r_0) + \alpha_2\beta e^{\beta\mu}\int_{-\mu}^{0} |x(t)|^2\, dt$$

and, hence, the uncertain stochastic system (Σ) achieves robust exponential mean-square stability. This completes the proof. □

Lemma 2 [15]: Let $\mathcal{D}$ and $\mathcal{H}$ be F real matrices of appropriate dimensions with F satisfying $F^TF \le I$. Then, for any scalar $\epsilon > 0$ and vectors $x,\, y \in \mathbb{R}^n$

$$2x^T\mathcal{D}F\mathcal{H}y \le \epsilon^{-1}x^T\mathcal{D}\mathcal{D}^Tx + \epsilon y^T\mathcal{H}^T\mathcal{H}y.$$

Now, we are in a position to give the robust H_∞ filtering results.

Theorem 1: Consider the uncertain Markovian jump system (Σ) and let $\gamma > 0$ be a given scalar. Then, there exists a Markovian jump linear filter (Σ_f) in the form of (10) and (11) such that, for all admissible uncertainties, the filtering error system is robustly exponentially mean-square stable and (12) holds for any nonzero $\omega \in \mathcal{L}_2[0, \infty)$, if for each $i \in \mathcal{S}$ there exist matrices $Q > 0$, $X_i > 0$, $Y_i > 0$, W_i, Z_i and scalars $\epsilon_i > 0$ such that the LMIs shown in (27)–(30) at the bottom of the page hold, and the scalar η is given in (14). In this case, a suitable robust H_∞ filter (Σ_f) in the form of (10) and (11) has parameters as follows:

$$A_{fi} = Y_i^{-1}W_i \quad B_{fi} = Y_i^{-1}Z_i, \qquad i \in \mathcal{S}. \tag{31}$$

Proof: Let $\tilde{x}(t) = x(t) - \hat{x}(t)$ then from the system (Σ) and filter (Σ_f), it can be verified that for each $i \in \mathcal{S}$

$$\begin{aligned}\dot{\tilde{x}}(t) &= A_{fi}\tilde{x}(t) + [A_i(t) - A_{fi} - B_{fi}C_i(t)]\, x(t)\\ &+ [A_{di}(t) - B_{fi}C_{di}(t)]\, x(t-\tau_i(t))\\ &+ [B_i(t) - B_{fi}D_i(t)]\, \omega(t).\end{aligned}$$

Define

$$e(t) = \left[x(t)^T,\ \tilde{x}(t)^T\right]^T \qquad \tilde{z}(t) = z(t) - \hat{z}(t).$$

Then, for each $r_t = i \in \mathcal{S}$, the filtering error dynamics from the systems (Σ) and (Σ_f) is described by

$$(\tilde{\Sigma}): \quad \begin{aligned}\dot{e}(t) &= \tilde{A}_i(t)e(t) + \tilde{A}_{di}(t)He\,(t-\tau_i(t))\\ &+ \tilde{B}_i(t)\omega(t)\end{aligned} \tag{32}$$

$$\tilde{z}(t) = \tilde{L}_ie(t) \tag{33}$$

where

$$\tilde{A}_i(t) = \tilde{A}_i + \Delta\tilde{A}_i(t) \quad \tilde{A}_{di}(t) = \tilde{A}_{di} + \Delta\tilde{A}_{di}(t)$$
$$\tilde{B}_i(t) = \tilde{B}_i + \Delta\tilde{B}_i(t)$$

$$\begin{bmatrix} \Omega_i & A_i^TY_i - W_i^T - C_i^TZ_i^T & X_iA_{di} + \epsilon_iN_{1i}^TN_{2i} & X_iB_i + \epsilon_iN_{1i}^TN_{3i} & X_iM_{1i}\\ Y_iA_i - W_i - Z_iC_i & \Phi_i & Y_iA_{di} - Z_iC_{di} & Y_iB_i - Z_iD_i & Y_iM_{1i} - Z_iM_{2i}\\ A_{di}^TX_i + \epsilon_iN_{2i}^TN_{1i} & A_{di}^TY_i - C_{di}^TZ_i^T & V_i & \epsilon_iN_{2i}^TN_{3i} & 0\\ B_i^TX_i + \epsilon_iN_{3i}^TN_{1i} & B_i^TY_i - D_i^TZ_i^T & \epsilon_iN_{3i}^TN_{2i} & \epsilon_iN_{3i}^TN_{3i} - \gamma^2I & 0\\ M_{1i}^TX_i & M_{1i}^TY_i - M_{2i}^TZ_i^T & 0 & 0 & -\epsilon_iI \end{bmatrix} < 0 \tag{27}$$

where

$$\Omega_i = \sum_{j=1}^{s}\pi_{ij}X_j + A_i^TX_i + X_iA_i + (1+\eta\mu)Q + \epsilon_iN_{1i}^TN_{1i} \tag{28}$$

$$\Phi_i = W_i + W_i^T + \sum_{j=1}^{s}\pi_{ij}Y_j + L_i^TL_i \tag{29}$$

$$V_i = \epsilon_iN_{2i}^TN_{2i} - (1-h_i)Q \tag{30}$$

$$\tilde{A}_i = \begin{bmatrix} A_i & 0 \\ A_i - A_{fi} - B_{fi}C_i & A_{fi} \end{bmatrix}$$
$$\Delta\tilde{A}_i(t) = \begin{bmatrix} \Delta A_i(t) & 0 \\ \Delta A_i(t) - B_{fi}\Delta C_i(t) & 0 \end{bmatrix}$$
$$\tilde{A}_{di} = \begin{bmatrix} A_{di} \\ A_{di} - B_{fi}C_{di} \end{bmatrix}$$
$$\Delta\tilde{A}_{di}(t) = \begin{bmatrix} \Delta A_{di}(t) \\ \Delta A_{di}(t) - B_{fi}\Delta C_{di}(t) \end{bmatrix} \quad H = [\, I \quad 0 \,]$$
$$\tilde{B}_i = \begin{bmatrix} B_i \\ B_i - B_{fi}D_i \end{bmatrix}$$
$$\Delta\tilde{B}_i(t) = \begin{bmatrix} \Delta B_i(t) \\ \Delta B_i(t) - B_{fi}\Delta D_i(t) \end{bmatrix} \quad \tilde{L}_i = [\, 0 \quad L_i \,].$$

Now, we establish the robust exponential mean-square stability of the filtering error system $(\tilde{\Sigma})$ in (32) and (33) under the conditions of the theorem. To this end, we consider (32) with $\omega(t) = 0$,that is

$$\dot{e}(t) = \tilde{A}_i(t)e(t) + \tilde{A}_{di}(t)He\,(t - \tau_i(t)). \tag{34}$$

For each $r_t = i, i \in \mathcal{S}$, we define a matrix $\tilde{P}_i > 0$ by

$$\tilde{P}_i = \begin{bmatrix} X_i & 0 \\ 0 & Y_i \end{bmatrix}.$$

Then, with the parameters in (31), it can be verified that, for each $i \in \mathcal{S}$, the LMIs in (27) can be rewritten as shown in (35)–(38) at the bottom of the page.

It is easy to see that (35) implies that for each $i \in \mathcal{S}$

$$\begin{bmatrix} \tilde{\Omega}_i + \epsilon_i^{-1}\tilde{P}_i\tilde{M}_{1i}\tilde{M}_{1i}^T\tilde{P}_i + \epsilon_i\tilde{N}_{1i}^T\tilde{N}_{1i} & \tilde{P}_i\tilde{A}_{di} + \epsilon_i\tilde{N}_{1i}^T\tilde{N}_{2i} \\ \tilde{A}_{di}^T\tilde{P}_i + \epsilon_i\tilde{N}_{2i}^T\tilde{N}_{1i} & \epsilon_i\tilde{N}_{2i}^T\tilde{N}_{2i} - (1-h_i)Q \end{bmatrix} < 0. \tag{39}$$

On the other hand, noting

$$[\, \Delta\tilde{A}_i(t)^T \quad \Delta\tilde{A}_{di}(t) \quad \Delta\tilde{B}_i(t) \,] = \tilde{M}_{1i}F_i(t)[\, \tilde{N}_{1i} \quad \tilde{N}_{2i} \quad \tilde{N}_{3i} \,]$$

and using Lemma 2, we have

$$\begin{bmatrix} \Delta\tilde{A}_i(t)^T\tilde{P}_i + \tilde{P}_i\Delta\tilde{A}_i(t) & \tilde{P}_i\Delta\tilde{A}_{di}(t) \\ \Delta\tilde{A}_{di}(t)^T\tilde{P}_i & 0 \end{bmatrix} \le \epsilon_i^{-1}\begin{bmatrix} \tilde{P}_i\tilde{M}_{1i} \\ 0 \end{bmatrix}[\, \tilde{M}_{1i}^T\tilde{P}_{1i} \quad 0 \,] + \epsilon_i\begin{bmatrix} \tilde{N}_{1i}^T \\ \tilde{N}_{2i}^T \end{bmatrix}[\, \tilde{N}_{1i} \quad \tilde{N}_{2i} \,].$$

It then follows from this and the LMIs in (39) that for each $i \in \mathcal{S}$, (40), shown at the bottom of the page, holds. Define a stochastic Lyapunov functional candidate for (34)

$$V(e_t, r_t) = e(t)^T\tilde{P}(r_t)e(t) + \int_{t-\tau_{r_t}}^{t} e(s)^T H^T Q H e(s)ds + \eta\int_{-\mu}^{0}\int_{t+\theta}^{t} e(s)^T H^T Q H e(s)ds d\theta. \tag{41}$$

Then, by noting (40) and following a similar line as in the proof of Lemma 1, we can deduce that the filtering error system $(\tilde{\Sigma})$ achieves robust exponential mean-square stability.

Next, we will show that (12) is satisfied for any nonzero $\omega \in \mathcal{L}_2[0,\infty)$. To this end, we set

$$J(T) = \mathcal{E}\left\{\int_0^T \left[\tilde{z}(t)^T\tilde{z}(t) - \gamma^2\omega(t)^T\omega(t)\right]dt\right\} \tag{42}$$

$$\begin{bmatrix} \tilde{\Omega}_i + \epsilon_i\tilde{N}_{1i}^T\tilde{N}_{1i} + \tilde{L}_i^T\tilde{L}_i & \tilde{P}_i\tilde{A}_{di} + \epsilon_i\tilde{N}_{1i}^T\tilde{N}_{2i} & \tilde{P}_i\tilde{B}_i + \epsilon_i\tilde{N}_{1i}^T\tilde{N}_{3i} & \tilde{P}_i\tilde{M}_{1i} \\ \tilde{A}_{di}^T\tilde{P}_i + \epsilon_i\tilde{N}_{2i}^T\tilde{N}_{1i} & \epsilon_i\tilde{N}_{2i}^T\tilde{N}_{2i} - (1-h_i)Q & \epsilon_i\tilde{N}_{2i}^T\tilde{N}_{3i} & 0 \\ \tilde{B}_i^T\tilde{P}_i + \epsilon_i\tilde{N}_{3i}^T\tilde{N}_{1i} & \epsilon_i\tilde{N}_{3i}^T\tilde{N}_{2i} & \epsilon_i\tilde{N}_{3i}^T\tilde{N}_{3i} - \gamma^2 I & 0 \\ \tilde{M}_{1i}^T\tilde{P}_i & 0 & 0 & -\epsilon_i I \end{bmatrix} < 0 \tag{35}$$

where

$$\tilde{\Omega}_i = \sum_{j=1}^{\mathcal{N}}\pi_{ij}\tilde{P}_j + \tilde{A}_i^T\tilde{P}_i + \tilde{P}_i\tilde{A}_i + (1+\eta\mu)H^TQH \tag{36}$$
$$\tilde{M}_{1i} = \begin{bmatrix} M_{1i} \\ M_{1i} - B_{fi}M_{2i} \end{bmatrix} \quad \tilde{M}_{2i} = M_{3i} - D_{fi}M_{2i} \tag{37}$$
$$\tilde{N}_{1i} = [\, N_{1i} \quad 0 \,] \quad \tilde{N}_{2i} = N_{2i} \quad \tilde{N}_{3i} = N_{3i}. \tag{38}$$

$$\begin{bmatrix} \sum_{j=1}^{\mathcal{N}}\pi_{ij}\tilde{P}_j + \tilde{A}_i(t)^T\tilde{P}_i + \tilde{P}_i\tilde{A}_i(t) + (1+\eta\mu)H^TQH & \tilde{P}_i\tilde{A}_{di}(t) \\ \tilde{A}_{di}(t)^T\tilde{P}_i & -(1-h_i)Q \end{bmatrix} \le \begin{bmatrix} \tilde{\Omega}_i + \epsilon_i^{-1}\tilde{P}_i\tilde{M}_{1i}\tilde{M}_{1i}^T\tilde{P}_i + \epsilon_i\tilde{N}_{1i}^T\tilde{N}_{1i} & \tilde{P}_i\tilde{A}_{di} + \epsilon_i\tilde{N}_{1i}^T\tilde{N}_{2i} \\ \tilde{A}_{di}^T\tilde{P}_i + \epsilon_i\tilde{N}_{2i}^T\tilde{N}_{1i} & \epsilon_i\tilde{N}_{2i}^T\tilde{N}_{2i} - (1-h_i)Q \end{bmatrix} < 0. \tag{40}$$

for $T > 0$. Observe for any $i \in \mathcal{S}$, $\omega(t) \neq 0$

$$\begin{aligned}\mathcal{A}V(e_t, i) \leq\ & e(t)^T \left[\sum_{j=1}^{\mathcal{N}} \pi_{ij}\tilde{P}_j + (1+\eta\mu)H^TQH\right] e(t) \\ & - (1-h_i)x(t-\tau_i(t))^T Qx(t-\tau_i(t)) \\ & + 2e(t)^T\tilde{P}_i \left[\left(\tilde{A}_i + \Delta\tilde{A}_i(t)\right)e(t)\right. \\ & + \left(\tilde{A}_{di} + \Delta\tilde{A}_{di}(t)\right)x(t-\tau_i(t)) \\ & \left. + \left(\tilde{B}_i + \Delta\tilde{B}_i(t)\right)\omega(t)\right]. \end{aligned} \tag{43}$$

Using Lemma 2 again, we obtain

$$\begin{aligned}2e(t)^T\tilde{P}_i&\left[\Delta\tilde{A}_i(t)e(t) + \Delta\tilde{A}_{di}(t)x(t-\tau(t)) + \Delta\tilde{B}_i(t)\omega(t)\right] \\ &\leq \epsilon_i^{-1}e(t)^T\tilde{P}_i\tilde{M}_{1i}\tilde{M}_{1i}^T\tilde{P}_ie(t) + \epsilon_i\Xi(t)^T\bar{N}^T\bar{N}\Xi(t)\end{aligned} \tag{44}$$

where

$$\Xi(t) = \left[\, e(t)^T \quad x(t-\tau_i(t))^T \quad \omega(t)^T \,\right]^T$$
$$\bar{N} = \left[\, \tilde{N}_{1i} \quad \tilde{N}_{2i} \quad \tilde{N}_{3i} \,\right].$$

Thus, under zero initial conditions, it follows from (42)–(44) that for any $T > 0$, $i \in \mathcal{S}$, $\omega(t) \neq 0$

$$\begin{aligned}J(T) &= \mathcal{E}\left\{\int_0^T \left[\tilde{z}(t)^T\tilde{z}(t) - \gamma^2\omega(t)^T\omega(t) + \mathcal{A}V(e_t, i)\right]dt\right\} \\ &\quad - \mathcal{E}\left\{\int_0^T \mathcal{A}V(e_t, i)dt\right\} \\ &\leq \mathcal{E}\left\{\int_0^T \Xi(t)^T\Theta\Xi(t)dt\right\}\end{aligned} \tag{45}$$

where

$$\Theta = \begin{bmatrix}\tilde{\Omega} + \epsilon_i^{-1}\tilde{P}_i\tilde{M}_{1i}\tilde{M}_{1i}^T\tilde{P}_i + \tilde{L}_i^T\tilde{L}_i & P_i\tilde{A}_{di} & P_i\tilde{B}_i \\ \tilde{A}_{di}^TP_i & -(1-h_i)Q & 0 \\ \tilde{B}_i^TP_i & 0 & -\gamma^2I\end{bmatrix} + \epsilon_i\bar{N}^T\bar{N}.$$

Now, applying the Schur complement formula to (35), we have $\Theta < 0$. This together with (45) implies that $J(T) < 0$ and, hence, (12) holds for any nonzero $\omega \in \mathcal{L}_2[0, \infty)$. This completes the proof. □

Remark 2: Theorem 1 provides sufficient conditions for the solvability of the robust H_∞ filtering problem for uncertain Markovian jump systems with mode-dependent time delays, and the desired filter can be constructed by solving a set of LMIs. We remark that these LMIs can be solved by means of numerically efficient convex programming algorithms [4].

Remark 3: As pointed out earlier that a filter with exponential mean-square stability can provide faster convergence and desirable accuracy in terms of reasonable error covariance of the filtering process compared with a filter with asymptotic mean-square stability [14]. However, it is worth pointing out that the introduction of the exponential mean-square stability instead of the usual asymptotic mean-square stability might lead to conservative solutions.

In the case when $\mathcal{S} = \{1\}$, that is, there is only one mode in operation, we have $\pi_{ii} = 0$, $i \in \mathcal{S} = \{1\}$, and the system (Σ) reduces to the following uncertain time-delay system with no jumping parameters:

$$(\Sigma_2): \quad \begin{aligned}\dot{x}(t) &= [A + \Delta A(t)]\,x(t) \\ &\quad + [A_d + \Delta A_d(t)]\,x(t-\tau(t)) \\ &\quad + [B + \Delta B(t)]\,\omega(t) \\ y(t) &= [C + \Delta C(t)]\,x(t) \\ &\quad + [C_d + \Delta C_d(t)]\,x(t-\tau(t)) \\ &\quad + [D + \Delta D(t)]\,\omega(t) \\ z(t) &= Lx(t) \\ x(t) &= \varphi(t), \forall t \in [-\mu,\ 0]\end{aligned}$$

where $\Delta A(t)$, $\Delta A_d(t)$, $\Delta B(t)$, $\Delta C(t)$, $\Delta C_d(t)$ and $\Delta D(t)$ are unknown matrices, and are assumed to be of the form

$$\begin{bmatrix}\Delta A(t) & \Delta A_d(t) & \Delta B(t) \\ \Delta C(t) & \Delta C_d(t) & \Delta D(t)\end{bmatrix} = \begin{bmatrix}M_1 \\ M_2\end{bmatrix} F(t)[N_1\ N_2\ N_3]$$

where M_1, M_2, M_3, N_1, and N_2 are known constant matrices, and $F(t)$ is the uncertain time-varying matrices satisfying $F(t)^TF(t) \leq I$. The time-varying delay $\tau(t)$ satisfies $0 < \tau(t) \leq \mu < \infty$ and $\dot{\tau}(t) \leq h < 1$. In this case, Theorem 1 reduces to the following result.

Corollary 1: Consider the uncertain time-delay system (Σ_2) and let $\gamma > 0$ be a given scalar. Then, there exists a causal linear filter such that, for all admissible uncertainties, the filtering error system is exponentially stable and

$$\|z - \hat{z}\|_2 < \gamma\|\omega\|_2$$

holds for any nonzero $\omega \in \mathcal{L}_2[0, \infty)$ under zero initial conditions, if there exist matrices $Q > 0$, $X > 0$, $Y > 0$, W, Z, and a scalar $\epsilon > 0$ such that the LMI shown at the bottom of the page holds. In this case, a suitable robust H_∞ filter for the system (Σ_2) is given by

$$\dot{\hat{x}}(t) = A_f\hat{x}(t) + B_fy(t) \quad \hat{z}(t) = L\hat{x}(t) \tag{46}$$

$$\begin{bmatrix}\Omega & A^TY - W^T - C^TZ^T & XA_d + \epsilon N_1^TN_2 & XB + \epsilon N_1^TN_3 & XM_1 \\ YA - W - ZC & \Phi & YA_d - ZC_d & YB - ZD & YM_1 - ZM_2 \\ A_d^TX + \epsilon N_2^TN_1 & A_d^TY - C_d^TZ^T & V & \epsilon N_2^TN_3 & 0 \\ B^TX + \epsilon N_3^TN_1 & B^TY - D^TZ^T & \epsilon N_3^TN_2 & \epsilon N_3^TN_3 - \gamma^2I & 0 \\ M_1^TX & M_1^TY - M_2^TZ^T & 0 & 0 & -\epsilon I\end{bmatrix} < 0$$

where

$$\Omega = A^TX + XA + Q + \epsilon N_1^TN_1,\ \Phi = W + W^T + L^TL,\ V = \epsilon N_2^TN_2 - (1-h)Q.$$

where

$$A_f = Y^{-1}W \quad B_f = Y^{-1}Z.$$

Remark 4: The robust H_∞ filtering problem for uncertain time-delay systems is investigated in [17]. It is noted that the time-delay considered in [17] is constant, and the filter parameter A_f is constrained to be equivalent to the nominal system matrix A. In view of this, the result presented in Corollary 1 here is more general than that in [17, Cor. 1].

IV. Numerical Example

In this section, we shall give a numerical example to demonstrate the applicability of the proposed approach.

Consider an uncertain linear Markovian jump system with mode-dependent time delays in the form (1)–(3) with two modes. For mode 1, the dynamics of the system are described as

$$\begin{aligned}
A_1 &= \begin{bmatrix} -3 & 1 & 0 \\ 0.3 & -2.5 & 1 \\ -0.1 & 0.3 & -3.8 \end{bmatrix}\\
A_{d1} &= \begin{bmatrix} -0.2 & 0.1 & 0.6 \\ 0.5 & -1 & -0.8 \\ 0 & 1 & -2.5 \end{bmatrix}\\
B_1 &= \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}\\
C_1 &= [0.8 \quad 0.3 \quad 0]\\
C_{d1} &= [0.2 \quad -0.3 \quad -0.6]\\
L_1 &= [0.5 \quad -0.1 \quad 1]\\
D_1 &= 0.2\\
M_{11} &= \begin{bmatrix} 0.1 \\ 0 \\ 0.2 \end{bmatrix}\\
M_{21} &= 0.2 \quad N_{11} = [0.2 \quad 0 \quad 0.1]\\
N_{21} &= [0.1 \quad 0.2 \quad 0] \quad N_{31} = 0.2
\end{aligned}$$

and the time-varying delay $\tau_1(t)$ satisfies (6) with $\mu_1 = 1.2, h_1 = 0.2$. For mode 2, the dynamics of the system are described as

$$\begin{aligned}
A_2 &= \begin{bmatrix} -2.5 & 0.5 & -0.1 \\ 0.1 & -3.5 & 0.3 \\ -0.1 & 1 & -2 \end{bmatrix}\\
A_{d2} &= \begin{bmatrix} 0 & -0.3 & 0.6 \\ 0.1 & 0.5 & 0 \\ -0.6 & 1 & -0.8 \end{bmatrix} \quad B_2 = \begin{bmatrix} -0.6 \\ 0.5 \\ 0 \end{bmatrix}\\
C_2 &= [-0.5 \quad 0.2 \quad 0.3] \quad C_{d2} = [0 \quad -0.6 \quad 0.2]\\
L_2 &= [0 \quad 1 \quad 0.6] \quad D_2 = 0.5\\
M_{12} &= \begin{bmatrix} 0.1 \\ 0.1 \\ 0 \end{bmatrix} \quad M_{22} = 0.1 \quad N_{12} = [0.1 \quad 0.1 \quad 0]\\
N_{22} &= [0 \quad -0.1 \quad 0.2] \quad N_{32} = 0.1
\end{aligned}$$

and the time-varying delay $\tau_2(t)$ satisfies (6) with $\mu_2 = 0.5, h_2 = 0.3$. Suppose the transition probability matrix is given by

$$\Pi = \begin{bmatrix} -0.5 & 0.5 \\ 0.3 & -0.3 \end{bmatrix}.$$

The purpose is the design of a robust H_∞ filter in the form of (10) and (11) such that the filtering error system achieves robust mean-square stability and (12) is satisfied. In this example, we assume the noise attenuation level $\gamma = 1.2$.

By resorting to the Matlab LMI Control Toolbox to solve the LMIs in (27), we obtain the solution as follows:

$$\begin{aligned}
X_1 &= \begin{bmatrix} 1.3866 & -0.3430 & 0.0230 \\ -0.3430 & 0.9297 & -0.3753 \\ 0.0230 & -0.3753 & 1.2080 \end{bmatrix}\\
X_2 &= \begin{bmatrix} 1.3027 & -0.2205 & 0.0515 \\ -0.2205 & 0.5002 & -0.1818 \\ 0.0515 & -0.1818 & 1.6222 \end{bmatrix}\\
Y_1 &= \begin{bmatrix} 3.3288 & 3.3944 & -0.0412 \\ 3.3944 & 4.4025 & -0.1197 \\ -0.0412 & -0.1197 & 0.3685 \end{bmatrix}\\
Y_2 &= \begin{bmatrix} 7.6675 & 7.8160 & -0.1527 \\ 7.8160 & 8.9294 & 0.2110 \\ -0.1527 & 0.2110 & 0.7280 \end{bmatrix}\\
W_1 &= \begin{bmatrix} -13.0965 & -6.7241 & 2.9448 \\ -12.2930 & -10.0564 & 2.0232 \\ 0.2390 & -1.3219 & -2.0676 \end{bmatrix}\\
W_2 &= \begin{bmatrix} -23.1622 & -20.9810 & 0.2088 \\ -20.1789 & -24.8354 & 1.0332 \\ -0.8298 & 0.3666 & -0.8119 \end{bmatrix}\\
Z_1 &= \begin{bmatrix} 4.5713 \\ 7.3286 \\ 1.7094 \end{bmatrix} \quad Z_2 = \begin{bmatrix} -5.4160 \\ -4.7364 \\ -0.8827 \end{bmatrix}\\
Q &= \begin{bmatrix} 1.5722 & -0.9511 & 0.4941 \\ -0.9511 & 2.1202 & -1.5764 \\ 0.4941 & -1.5764 & 3.0137 \end{bmatrix}\\
\epsilon_1 &= 0.3878 \quad \epsilon_2 = 4.0350.
\end{aligned}$$

Therefore, by Theorem 1, the corresponding parameters of a suitable robust Markovian H_∞ filter can be chosen as

$$\begin{aligned}
A_{f1} &= \begin{bmatrix} -5.1167 & 1.7777 & 2.3648 \\ 1.1651 & -3.7804 & -1.5225 \\ 0.4543 & -4.6164 & -5.8408 \end{bmatrix}\\
B_{f1} &= \begin{bmatrix} -1.9102 \\ 3.2868 \\ 5.4927 \end{bmatrix}\\
A_{f2} &= \begin{bmatrix} -8.4713 & 1.7986 & -1.6240 \\ 5.2601 & -4.4067 & 1.5824 \\ -4.4416 & 2.1581 & -1.9146 \end{bmatrix}\\
B_{f2} &= \begin{bmatrix} -2.4376 \\ 1.6553 \\ -2.2037 \end{bmatrix}.
\end{aligned}$$

V. Conclusion

In this note, we have studied the problem of robust H_∞ filtering for Markovian jump linear systems with parameter uncertainties and mode-dependent time-varying delays. An LMI approach has been developed to design a Markovian jump linear filter, which guarantees robust exponential mean-square stability of the resulting error system and a prescribed bound on the $\mathcal{L}_2$-induced gain from the noise signal to the estimation error, irrespective of the parameter uncertainties. The desired filter can be constructed through a convex optimization problem.

REFERENCES

[1] B. D. O. Anderson and J. B. Moore, *Optimal Filtering*. Upper Saddle River, NJ: Prentice-Hall, 1979.

[2] E. K. Boukas and Z. K. Liu, "Robust H_∞ control of discrete-time Markovian jump linear systems with mode-dependent time-delays," *IEEE Trans. Automat. Contr.*, vol. 46, pp. 1918–1924, Dec. 2001.

[3] ——, "Robust stability and stabilizability of Markov jump linear uncertain systems with mode-dependent time delays," *J. Optim. Theory Appl.*, vol. 109, pp. 587–600, 2001.

[4] S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in Systems and Control Theory*, ser. SIAM Studies in Applied Mathematics. Philadelphia, PA: SIAM, 1994.

[5] Y.-Y. Cao and J. Lam, "Robust H_∞ control of uncertain Markovian jump systems with time-delay," *IEEE Trans. Automat. Contr.*, vol. 45, pp. 77–83, Jan. 2001.

[6] M. H. A. Davis, *Markov Models and Optimization*. London, U.K.: Chapman and Hall, 1993.

[7] C. E. De Souza and M. D. Fragoso, "H_∞ control for linear systems with Markovian jumping parameters," *Control Theory Adv. Technol.*, vol. 9, pp. 457–466, 1993.

[8] C. E. De Souza, R. M. Palhares, and P. L. D. Peres, "Robust H_∞ filter design for uncertain linear systems with multiple time-varying state delays," *IEEE Trans. Signal Processing*, vol. 49, pp. 569–576, Mar. 2001.

[9] C. E. De Souza, L. Xie, and Y. Wang, "H_∞ filtering for a class of uncertain nonlinear systems," *Syst. Control Lett.*, vol. 20, pp. 419–426, 1993.

[10] V. Dragan and T. Morozan, "Stability and robust stabilization to linear stochastic systems described by differential equations with Markov jumping and multiplicative white noise," *Stoch. Anal. Appl.*, vol. 20, pp. 33–92, 2002.

[11] X. Feng, K. A. Loparo, Y. Ji, and H. J. Chizeck, "Stochastic stability properties of jump linear systems," *IEEE Trans. Automat. Control*, vol. 37, pp. 38–53, Jan. 1992.

[12] H. K. Kim and H. B. Park, "H_∞ state feedback control for generalized continuous/discrete time-delay system," *Automatica*, vol. 35, pp. 1443–1451, 1999.

[13] K. M. Nagpal and P. P. Khargonekar, "Filtering and smoothing in an H_∞ setting," *IEEE Trans. Automat. Contr.*, vol. 36, pp. 152–166, Jan. 1991.

[14] K. Reif and R. Unbehauen, "The extended Kalman filter as an exponential observer for nonlinear systems," *IEEE Trans. Signal Processing*, vol. 47, pp. 2324–2328, Aug. 1999.

[15] Y. Wang, L. Xie, and C. E. De Souza, "Robust control of a class of uncertain nonlinear systems," *Syst. Control Lett.*, vol. 19, pp. 139–149, 1992.

[16] Z. Wang, B. Huang, and H. Unbehauen, "Robust H_∞ observer design of linear time-delay systems with parametric uncertainty," *Syst. Control Lett.*, vol. 42, pp. 303–312, 2001.

[17] S. Xu and P. Van Dooren, "Robust H_∞ filtering for a class of nonlinear systems with state delay and parameter uncertainty," *Int. J. Control*, vol. 75, pp. 766–774, 2002.

Comments on "Robust Stabilization of a Class of Time-Delay Nonlinear Systems"

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I. INTRODUCTION

In this paper, we point out an error in [1], which will show that the main result of the paper cannot stand. In [1], the author considered the problem of robust stabilization of a class of triangular nonlinear system, and developed an iterative procedure of constructing a stabilization controller based on the constructive use of appropriate Lyapunov–Krasovskii functions. However, we show in this note that there is an error in [1], which leads to the failure of the iterative procedure, so the conclusion cannot stand.

II. MAIN RESULTS

In [1], the following system was considered:

$$\sum \begin{cases} \dot{x}_i(t) = F_i(w_i(t)) + H_i(w_i(t-\tau)) \\ \qquad + G_i(w_i(t))\, x_{i+1} \\ \dot{x}_n(t) = F_n(w_n(t)) + H_n(w_n(t-\tau)) \\ \qquad + G_n(w_n(t))\, u \end{cases} \tag{1}$$

and then the following nonlinear coordinates transformation was introduced:

$$\begin{cases} x_1(t) = z_1(t) \\ x_i(t) = z_i(t) + \phi_{i-1}(r_{i-1}(t)), \qquad \text{for } (i = 2, \ldots, n) \end{cases} \tag{2}$$

then (1), under the transformation, became

$$\begin{aligned} \dot{z}_1(t) &= f_1(z_1(t)) + h_1(z_1(t-\tau)) \\ &\quad + g_1(z_1(t))\,[z_2 + \phi_1(z_1(t))] \end{aligned} \tag{3}$$

$$\begin{aligned} \dot{z}_i(t) &= f_i(r_i(t)) + h_i(r_i(t-\tau)) \\ &\quad + g_i(r_i(t))\,[z_{i+1} + \phi_i(r_i(t))] + M_i(r_i(t))\, r_i^t(t) \\ &\quad + D_{i-1}(r_{i-1}(t))\, \tilde{H}_{i-1}^t(r_{i-1}(t-\tau)), \\ &\quad \text{for } (i = 2, \ldots, n-1) \end{aligned} \tag{4}$$

$$\begin{aligned} \dot{z}_n(t) &= f_n(r_n(t)) + h_n(r_n(t-\tau)) + g_n(r_n(t))\, u \\ &\quad + M_n(r_n(t))\, r_n^t(t) \\ &\quad + D_{n-1}(r_{n-1}(t))\, \tilde{H}_{n-1}^t(r_{n-1}(t-\tau)) \end{aligned} \tag{5}$$

where $f_i(r_i(t))$, $h_i(r_i(t-\tau))$ and $g_i(r_i(t))$ were functions $F_i(w_i(t))$, $H_i(w_i(t-\tau))$ and $G_i(w_i(t))$ in new coordinates, respectively. $M_i(r_i(t))\, r_i^t(t)$, $\tilde{H}_{i-1}^t(r_{i-1}(t-\tau))$ and $D_{i-1}(r_{i-1}(t))$ were defined in [1], and the time delay part of equations was required to satisfy the following assumption of [1, Assumption C_2]:

$$|h_i(r_i(t-\tau))| <= \sum_{j=1}^{i} |z_j(t-\tau)|\, \tilde{\rho}_{ij}(r_j(t-\tau)) \tag{6}$$

where $\tilde{\rho}_{ij}(r_j(t-\tau))$ were known smooth nonlinear functions.

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