Maximum *a posteriori* spatial probability segmentation

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Abstract: An image segmentation algorithm that performs pixel-by-pixel segmentation on an image with consideration of spatial information is described. The spatial information is the joint grey level values of the pixel to be segmented and its neighbouring pixels. The conditional probability that a pixel belongs to a particular class under the condition that the spatial information has been observed is defined to be the *a posteriori* spatial probability. A maximum *a posteriori* spatial probability (MASP) segmentation algorithm is proposed to segment an image such that each pixel is segmented into a particular class when the *a posteriori* spatial probability is maximum. The proposed segmentation algorithm is implemented in an iterative form. During the iteration, a series of intermediate segmented images are produced among which the one that possesses the maximum amount of information in its spatial structure is chosen as the optimum segmented image. Results from segmenting synthetic and practical images demonstrate that the MASP algorithm is capable of achieving better results when compared with other global thresholding methods.

1 Introduction

An image is a collection of spatially ordered picture elements (pixels) representing a scene. The scene is usually assumed to consist of several objects in a background. Each of the background and object parts of the image generates a class of pixels to be characterised by its own statistical properties. An image segmentation process classifies each pixel of an image into a particular class. The major function of segmentation is to transform the raw image data to a form more suitable for the subsequent image processing steps such as image understanding, object identification, or pattern recognition [1].

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The most common approach to image segmentation is thresholding [2–4]. In this approach, a pixel is segmented into one class if its grey-level value is larger than the threshold value; otherwise it is segmented into another class. Thresholding can be categorised as global or local. In the former case, a single global threshold value is employed to threshold the entire image. In the latter case, a local threshold value may be derived from a partial region of the image for thresholding pixels in that region [5]. An important advantage of thresholding is that it can be very fast if the the threshold value is derived from simple statistical properties, such as the histogram, of the image [6]. Utilising the histogram, some thresholding algorithms make no assumption about its form [7, 8] while others may assume a certain functional form for it as an *a priori* information about the image to be segmented [9–11]. There are a number of thresholding methods that employ the entropy concept [12] in determining the optimum threshold value. The entropy of the image histogram [13, 14], the cross entropy of the image [15, 16], and the maximum entropy principle [17, 18] have been utilised in thresholding images with varying degrees of success. To improve thresholding performance beyond that afforded by those histogram-based methods, spatial information of the image may be considered. Some entropy-based thresholding methods adopt this approach and better thresholding results have been reported [19, 24].

A fundamental limitation of thresholding as an approach to segmentation is that pixels having the same grey level value will always be segmented into the same class. As such, the thresholding result is limited by the degree of overlap among the probability density functions (PDF) of the pixels in different classes. When there is no overlap the thresholding results can be perfect. However, when the overlap is substantial the thresholding results will be poor irrespective of the thresholding algorithms employed. Such a performance limitation will be more obvious by examining the ratio of the total number of all possible thresholded images to the total number of all possible segmented images for a particular image. For instance, this ratio is 2^8 to $2^{65,536}$ for an image having 65,536 pixels and 8 bit quantisation for its pixel grey-level values. In other words, the thresholding method only searches over an extremely small portion ($\simeq 10^{-19726}$ in this case) of the total solution space for the best segmentation result. From this point of view it is not surprising that the performance of global thresholding is limited.

To overcome the performance limitation of the global thresholding, spatial information of the image may be utilised and then each pixel classification is to be made individually. Depending on the spatial information, two pixels with the same grey-level value are not necessarily classified into the same class. In deciding which class a pixel is to be segmented into, spatial information related to the joint grey-level values of its neighbouring pixels is made use of in the present paper. Furthermore, the entropy concept is applied determining the spatial information contained in the structure of a segmented image in to identify the optimum segmented image. With a pixel-by-pixel segmentation scheme incorporating these two aspects of spatial information of an image, the limitations inherent in the histogram-based, global thresholding methods will be overcome and better thresholding results can be achieved.

2 Maximum *a posteriori* spatial probability segmentation

Consider an image that is a representation of a scene having K–1 objects in a background. The image may be regarded as a collection of some spatially ordered pixels each of which is denoted as X. Associated with each pixel X is a class parameter $x \in \{1, 2, ..., K\}$ denoting which class the pixel X belongs to. Without loss of generality, we assume that the grey-level values of the pixels have been discretised to values 1, 2, ..., L and that the grey-level value of the pixel X is denoted as $g \in \{1, 2, ..., L\}$. From the view point of a general segmentation process, which classifies X according to observation information, g is observable while x is unknown. Based on the observed g values, the segmentation process segments each pixel X into one of the K classes.

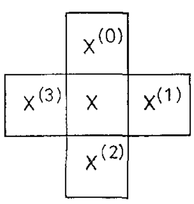

Fig. 1 *Four pixel nearest-neighbour lattice scheme*

Neglecting the pixels on the image boundary, associated with each pixel X are M neighbouring pixels $X^{(1)}$, $X^{(2)}$, ..., $X^{(M)}$. The positions of the M neighbouring pixels usually bear a fixed relationship with that of X. For example, the positions of four neighbouring pixels relative to X defined in the 'nearest-neighbour' lattice scheme [25] is depicted in Fig. 1. We denote the joint class parameters of the M neighbouring pixels as the neighbourhood configuration $\mathbf{N} = (x^{(1)}, x^{(2)}, ..., x^{(M)})$; and the joint observed grey-level values of the M neighbouring pixels as $\mathbf{G} = (g^{(1)}, g^{(2)}, ..., g^{(M)})$ where $x^{(m)}$ and $g^{(m)}$ are, respectively, the class parameter and the grey-level value of the neighbouring pixel $X^{(m)}$, $m \in \{1, 2, ..., M\}$. The observation data $\mathbf{D}$ that will be utilised by the segmentation algorithm for segmenting the pixel X are g and $\mathbf{G}$, i.e. $\mathbf{D} = (g, \mathbf{G})$. Finally, we denote the probability density function (PDF) of the grey-level distribution of the kth-class pixels as $f_k(g)$, $k \in \{1, 2, ..., K\}$, i.e.

$$P(X = g|x = k) = f_k(g) \tag{1}$$

Consider the conditional probability P_k that the pixel X belongs to class k given the observation data $\mathbf{D}$. This probability can be written as

$$P_k = P(x = k|\mathbf{D}) = P(x = k|g, \mathbf{G}), \quad k \in \{1, 2, \ldots, K\} \tag{2}$$

Eqn. 2 describes a set of probabilities that are conditioned on the spatial information about pixel X and they may be referred to as the *a posteriori* spatial probabilities. If the values of $P(x = k \mid g, \mathbf{G})$ are known for all k and $(g, \mathbf{G})$, the pixel may be segmented into class k^* where $P(x = k^* \mid g, \mathbf{G})$ is the maximum. When an image is segmented in this way, the probability of segmentation error will be minimum and the segmentation result should be optimum when measured in terms of segmentation error rate. We denote this segmentation scheme as maximum *a posteriori* spatial probability (MASP) segmentation.

In the most general sense, the probability of '$x = k$' may depend on the entire (global) image rather than on the joint local observation $(g, \mathbf{G})$ only. To justify the MASP approach, we model the image as a Markov random field, where the global dependence property is reduced to a local dependence [26]. Furthermore, by increasing or decreasing the number of pixels contained in $\mathbf{G}$, the extent to which a pixel's classification depends on its neighbourhood pixel grey-level values may be adjusted from nondependence ($\mathbf{D} = g$, the grey-level value of the pixel to be segmented) to global dependence ($\mathbf{D}$ = entire image). Hence the MASP approach has great flexibility in utilising spatial information of an image.

Applying Bayes' rule to eqn. 2, P_k can be written as:

$$P_k = P(x = k|g, \mathbf{G}) = \frac{P(g, \mathbf{G}|x = k) \times P(x = k)}{P(g, \mathbf{G})} \tag{3}$$

When comparing the magnitudes among all the P_k values to determine the maximum, there is no need to consider the denominator in eqn. 3 since it is independent of k. Only the values of $P(g, \mathbf{G} \mid x = k)$ and $P(x = k)$ need to be computed. The terms $P(x = k)$ may be estimated from a particular segmented version of the image, as will be described in the following Section. On account of the extremely large dimension of the joint grey-level probability distribution, the estimation of $P(g, \mathbf{G} \mid x = k)$ presents a problem since there will not be a sufficient number of samples. For instance, with a four-pixel neighbourhood and 8 bits of grey-level quantisation, the dimension of $P(g, \mathbf{G} \mid x = k)$ will amount to 2^{40}. With an image having only 65,536 pixels, the number of samples is insufficient for confident estimates of the probabilities $P(g, \mathbf{G} \mid x = k)$ to be made. Such a dimensionality problem and the associated computational challenge are commonly experienced by segmentation algorithms incorporating spatial information of an image. A possible approach is to express the interdependence of image grey-level values in terms of a model like the autoregressive moving average (ARMA) model [27] or the autobinomial model [28], and the model parameters are estimated by standard estimation methods [29]. Possible solutions include simulated annealing, segmentation by iterated conditional modes (ICM) or maximiser of posterior marginals (MPM) [26]. To render the problem computationally tractable without modelling the image explicitly, we propose to retain only the class interdependence among pixels while neglecting their grey-level interdependence. Consequently, the following two assumptions about the image are made:

(i) given an image pixel X, the probability that its neighbourhood assumes a particular neighbourhood configuration $\mathbf{N}$ depends on which class the pixel X belongs to;

(ii) given an image pixel X, the probability that it has a certain grey-level value depends only on which class it belongs to, but does not depend on the grey-level values of its neighbourhood pixels.

Assumption (i) is concerned with geometric features of the scene, and hence will be valid for a single image, or for multiple images of a scene provided the latter remains stationary during the time over which the images are acquired. The second assumption may not be valid for those images exhibiting pixel grey-level values interdependence. Employing assumption (ii) will mean the loss of a certain amount of image information but such a disadvantage may be more than offset by the advantage of having a computationally tractable solution when assumption (ii) is applied.

If it is known that a pixel X belongs to class k, then all the possible neighbourhood configurations $\mathbf{N}$ are mutually exclusive events and applying the total probability theorem [30], $P(g, \mathbf{G} \mid x = k)$ in eqn. 3 is written as

$$P(g, \mathbf{G}|x = k) = \sum_{\text{all } \mathbf{N}} P(\mathbf{N}|x = k) \times P(g, \mathbf{G}|x = k, \mathbf{N}) \tag{4}$$

Making use of assumptions (i) and (ii) discussed, eqn. 4 can be written as

$$P(g, \mathbf{G}|x = k) = \sum_{\text{all } \mathbf{N}} P(\mathbf{N}|x = k) \times f_k(g) \prod_{m=1}^{M} f_{x^{(m)}}(g^{(m)}) \tag{5}$$

A similar expansion for the term $P(x = k| \mathbf{D})$ has been proposed by Kittler and Hancock [31] and is known as an 'evidence combining' formula. Since we are going to employ a maximum *a posteriori* probability approach to determine the optimum segmentation results, we shall concentrate on the expansion of the term $P(g, \mathbf{G} \mid x = k)$ as shown in eqn. 5.

The dimension of $P(\mathbf{N} \mid x = k)$ is K^M, which is usually a relatively small number for most practical images. For instance, if binary segmentation is to be performed with four-pixel neighbourhood, the dimension of $P(\mathbf{N} \mid x = k)$, which is K^M, is only $2^4 = 16$. The values of $P(\mathbf{N} \mid x = k)$ can now be confidently estimated from a segmented version of the image, which is known as a provisional estimate of the true scene [28]. After the PDFs $f_k(g)$ of the classes of pixels have been estimated in a similar way, the product $P(g, \mathbf{G} \mid x = k) \times P(x = k)$ in eqn. 3 can be computed for each k and then the MASP segmentation process can be performed by segmenting X into class k^* where the product $P(g, \mathbf{G} \mid x = k^*) \times P(x = k^*)$ is the maximum. With all image pixels having been classified, the provisional estimate can be replaced by the new segmentation result and then the MASP segmentation cycle can be repeated. Finally, an iterative segmentation algorithm results. This 'estimate and then segment' approach is similar to the iterated conditional modes (ICM) method proposed by Besag [28].

As a special case, consider an image where there is no spatial regularity for the true scene, and there is no interdependence among the grey-level values of the image pixels. All the $P(\mathbf{N} \mid x = k)$ values in eqn. 5 are identical and the MASP segmentation process will segment a pixel into class k^* if $f_{k^*}(g) \times P(x = k^*)$ is maximum. This is just the result of a histogram-based maximum-likelihood global thresholding method [18] where no spatial information about the image has been taken into account.

To segment one pixel by the MASP method, $K \times K^M \times M$ multiplications, $K \times (K^M - 1)$ additions and $K - 1$ comparisons are required. For a single run of binary segmentation ($K = 2$) with a four-pixel neighbourhood scheme ($M = 4$), approximately 8.4 million multiplications are required for an image having 65,536 pixels, where the relatively fast addition and comparison operations are neglected. Hence it may be noted that segmentation by the MASP method still requires quite substantial computation.

3 Segmented scene spatial entropy

In Section 2 it is seen that the MASP segmentation method is suitable for implementation in an iterative form. Such an iterative method will produce a series of intermediate segmented images before it terminates. For any practical segmentation algorithm implementing the MASP method there must be some way to select the best segmented image from these intermediate segmented images. For this purpose, the segmented-scene spatial entropy (SSE) [32] is employed as a selection criterion.

Given a segmented image, denote the segmented pixel as Y. Associated with Y is a segmented class parameter y that denotes which class Y belongs to. Similar to notations employed in Section 2, there will be a neighbourhood configuration $\mathbf{N}_i$, $i \in \{1, 2, ..., K^M\}$, associated with each segmented pixel. If the segmented pixels are treated as random variables without regard to their spatial relationship, the uncertainty about which segmented class the segmented pixels belong to is given by

$$H_0 = -\sum_{k=1}^{K} P(y = k) \log P(y = k) \tag{6}$$

If the neighbourhood configuration of the pixel Y is observed to be $\mathbf{N}_i$, the probability that the segmented pixel Y belongs to segmented class k is given by the conditional probability $P(y = k \mid \mathbf{N}_i)$. The uncertainty about which segmented class the segmented pixel Y belongs to is given by

$$H(\mathbf{Y}|\mathbf{N}_i) = -\sum_{k=1}^{K} P(y = k|\mathbf{N}_i) \log P(y = k|\mathbf{N}_i) \tag{7}$$

The uncertainty averaged over all possible neighbourhood configurations is

$$H(\mathbf{Y}|\mathbf{N}) = \sum_{i=1}^{K^M} P(\mathbf{N}_i) H(\mathbf{Y}|\mathbf{N}_i) \tag{8}$$

In general, $H(\mathbf{Y} \mid \mathbf{N})$ will not be greater than H_0 since additional information has been provided for in determining which segmented class a segmented pixel belongs to. The difference

$$H_1 = H_0 - H(\mathbf{Y}|\mathbf{N}) \tag{9}$$

measures the information contained in the spatial structure of the segmented scene and is referred to as the segmented scene spatial entropy (SSE). A large value of SSE implies that the segmented scene exhibits strong spatial structure, and hence is compatible with the feature that the original scene is composed of K–1 objects in a background. The SSE value H_1 will be taken as a criterion for selecting the optimum segmented image. A practical iterative segmentation algorithm is proposed in Section 4.

4 Iterative MASP segmentation algorithm

We propose an iterative MASP segmentation algorithm in the following:

Step 1 threshold the image with a suitably chosen global threshold value, resulting in an initial segmented image (the provisional estimate of the true scene)

Step 2 based on the segmented image obtained in the previous step, make the following estimations:

μ_k: sample mean of the grey-level values of all pixels segmented into class k

σ_k: sample standard deviation of the grey-level values of all pixels segmented into class k

$f_k(g)$: $1/(\sigma_k\sqrt{2\pi}) \exp -(g - \mu_k)^2/2\sigma_k^2$

$P(x = k)$: the ratio of the number of pixels segmented into class k to the total number of pixels in the image

$P(\mathbf{N} \mid x = k)$: the ratio of the number of pixels which have been segmented into class k and have neighbourhood configuration $\mathbf{N}$ to the total number of pixels segmented into class k.

Step 3 based on the estimations made in step 2, perform MASP segmentation for every pixel as described in Section 2; a new segmented image is obtained; calculate and record the SSE H_1 for this new segmented image

Step 4 if the new segmented image is different from the segmented image produced in the previous step, goto step 2; otherwise goto step 5

Step 5 Among the several segmented images produced, the segmented image with the maximum H_1 is taken to be the final segmentation result

5 Results and discussion

To assess the performance of the MASP algorithm proposed in the previous Section, a simulation study is carried out in which synthetic images are segmented. The advantage of employing synthetic images for investigation of segmentation algorithms is that the image characteristics may be adjusted easily. Furthermore, since the true scene is known, an objective and quantitative assessment of segmentation results by measuring the segmentation error rate, which is defined as ratio of the number of erroneously-segmented pixels to the total number of image pixels, is possible.

The neighbourhood scheme adopted is the nearest four-neighbourhood scheme as depicted in Fig. 1. For the synthetic image, the true scene is a circular object in a background. Image size is 256 × 256 and the grey-level value ranges from 0 to 255 inclusive. The grey-level values of the object pixels and background pixels are generated randomly following a normal distribution PDF [33], where the mean value and the standard deviation of the grey-level values of the object and background pixels are denoted as μ_0, σ_0, μ_1 and σ_1, respectively. The object size relative to the entire image size is denoted as α, $1 > \alpha > 0$. The MASP algorithm is initialised by thresholding the image with a suitably chosen threshold value. Related research results show that the final MASP segmentation result is not very sensitive to the choice of the initial threshold value; in the present work we employ a maximum segmented-image entropy thresholding algorithm [Note 1] to threshold the image.

For the purpose of performance comparison, two other segmentation methods are implemented. The first method is the ideal minimum error (IMINE) global thresholding of the image. This method requires that the object and background PDFs as well as the value of α are known, and a threshold value that results in the minimum segmentation error rate is searched for over the entire grey-level values range. The second method filters the image by a median filter [27] and then thresholds the filtered image by OTSU's method [8]. This method will be abbreviated as MFOT method. The median filter mask is the four-neighbourhood scheme as shown in Fig. 1, plus the pixel to be segmented. When compared to the MASP method, the MFOT method utilises information of the same neighbourhood pixels but in a different way, and hence the performance of the two methods should be different.

Consider the segmentation results for a synthesised image with image parameters $\alpha = 0.2$, $\mu_0 = 100$, $\sigma_0 = 20$, $\mu_1 = 140$, $\sigma_1 = 20$. The grey-scale image, its histogram, the IMINE-, MFOT- and the MASP-segmented images are shown in Figs. 2*a*–2*e*, respectively. For the MASP algorithm, it takes 11 iterations to terminate and the segmented image generated in the 7th iteration has the maximum SSE, so this is identified as the optimum segmented image. The segmentation error rates achieved by the IMINE, the MFOT and the MASP methods are, respectively, 11.47, 11.36 and 1.31%. From these results, it is seen that the MFOT is slightly better than the IMINE thresholding method. This is reasonable since a certain amount of spatial information of the image has been utilised. However, the MASP algorithm gives far better results in that an almost 9:1 improvement in segmentation error rate over the IMINE method has been achieved.

The second synthetic image to be segmented has the same true scene as the previous one, but the grey-level values of the object and background pixels are generated randomly according to an exponential distribution [33]. The relative size of the object is 0.25. The sample mean and standard deviation of the grey-level values of the object pixels are, respectively, 100 and 20. The sample mean and standard deviation of the grey-level values of the background pixels are, respectively, 120 and 20. For the MASP algorithm, it takes 25 iterations to terminate and the optimum segmented image is generated in the 4th iteration where the SSE is maximum. The grey-scale image, its histogram, the IMINE-, MFOT- and the MASP-segmented images are shown in Figs. 3*a*–3*e*, respectively. The segmentation error rates of the IMINE-, MFOT- and MASP-segmented images are, respectively, 21.35, 45.74 and 13.08%. For this case, the MFOT result is not as good as the IMINE result, but the MASP result is still better than the two results.

To compare the performance of the three methods systematically, a series of images are synthesised with these parameters: $\alpha = 0.25$; mean of the object pixel grey-level values = 100; standard deviation of the

Note 1: Leung, C.K. and Lam, F.K.: 'Maximum segmented-image entropy thresholding', submitted to *Comput. Vis. Graph. Image Process.: Graph. Models Image Process.*

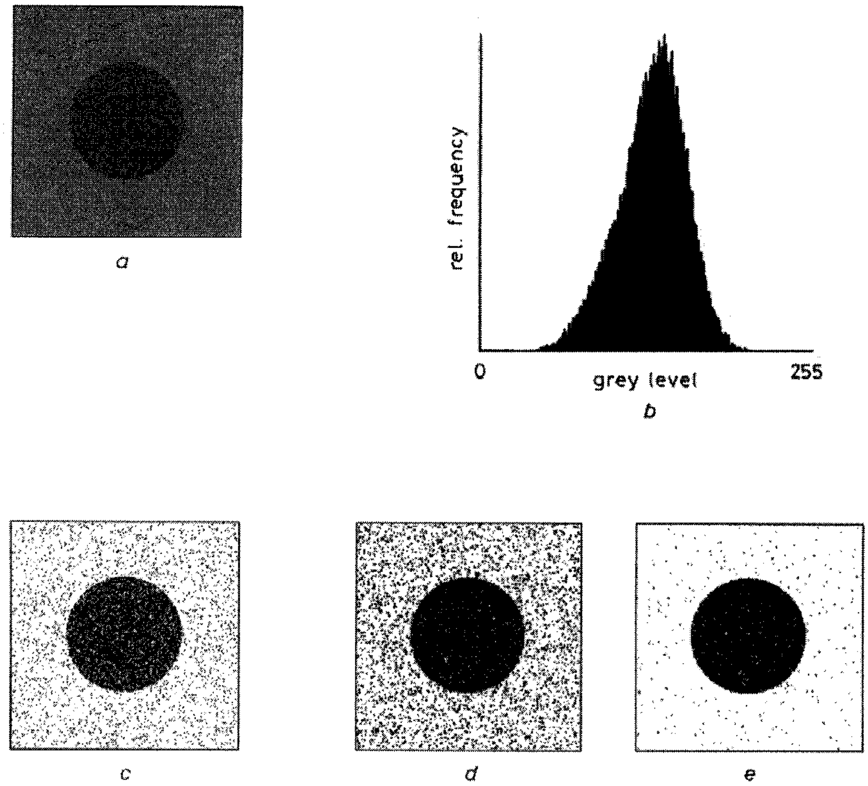

Fig. 2 *Segmentation results of synthesised image*
1 byte per pixel, 256 × 256 pixels
Image parameters: $\alpha = 0.2$, $\mu_0 = 100$, $\sigma_0 = 20$, $\mu_1 = 140$, $\sigma_1 = 20$, gaussian PDFs
a Grey-scale image
b Histogram of *a*
c IMINE-thresholded image, segmentation error = 11.47%
d MFOT-segmented image, segmentation error = 11.36%
e MASP-segmented image, segmentation error = 1.31 %

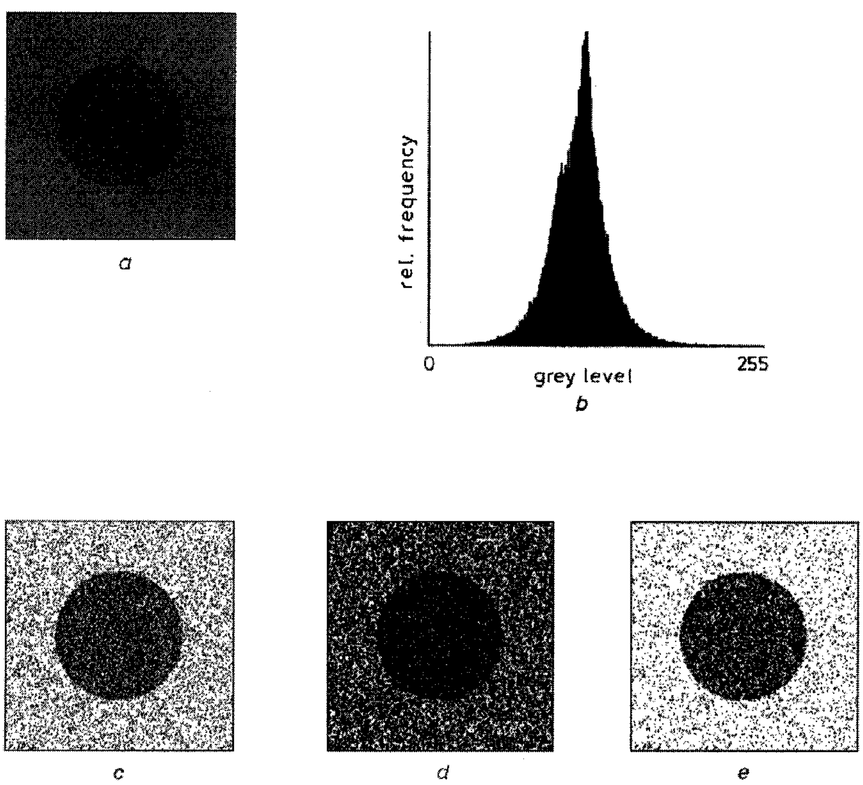

Fig. 3 *Segmentation results of synthesised image*
1 byte per pixel, 256 × 256 pixels
Image parameters: $\alpha = 0.25$, object pixels: mean of grey-level values = 100, standard deviation = 20; background pixels: mean of grey-level values = 120, standard deviation of grey-level values = 20; exponential PDFs
a Grey-scale image
b Histogram of *a*
c IMINE-thresholded image, segmentation error = 21.35%
d MFOT-segmented image, segmentation error = 45.74%
e MASP-segmented image, segmentation error = 13.08%

object pixel grey-level values = standard deviation of the background pixel grey-level values = 20. The mean of the background pixel grey-level values takes values from 120 to 160 in steps of 2. Both gaussian and exponential PDFs are used in synthesising the images. The segmentation error rates achieved by the IMINE, MFOT and MASP methods are plotted against the mean of the background pixel grey-level values in Fig. 4. From these results the performance of the MASP algorithm is seen to be better than IMINE and MFOT.

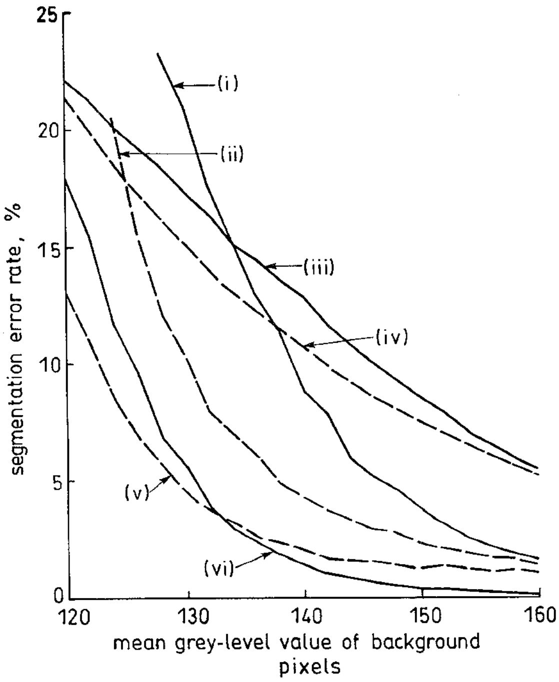

Fig. 4 *Segmentation errors against mean grey-level value of background pixels*
1 byte per pixel, 256 × 256 pixels
Image parameters: $\alpha = 0.25$; object pixels: mean of grey-level values = 100, standard deviation = 20; background pixels: standard deviation of grey-level values = 20
(i) MFOT gaussian PDF
(ii) MFOT exponential PDF
(iii) IMINE gaussian PDF
(iv) IMINE exponential PDF
(v) MASP exponential PDF
(vi) MASP gaussian PDF

Finally, we apply the MASP algorithm to segment a practical image. Since the true scene is unknown and no IMINE thresholding result is obtainable, the image is thresholded by OTSU and the MFOT methods for performance comparison. For the MASP algorithm, it takes nine iterations to terminate and the segmented image generated in the 8th iteration has the maximum SSE. The grey-scale image, its histogram, the OTSU-, MFOT- and MASP-segmented images are shown in Figs. 5*a*–5*e*, respectively. It can be seen that the MASP algorithm makes the 'pills' in the image more conspicuous as a result of reducing the effects of image noise.

All these investigations have been carried out in an IBM personal computer with a Pentium CPU running at 75MHz clock speed. The programming language used is Microsoft's Quick-Basic compiler version 4.2. With this set-up, it is observed that one pass of the MASP algorithm takes two minutes. The total execution time for a particular image depends on the number of passes required to reach termination.

6 Conclusions

An image segmentation algorithm known as maximum *a posteriori* spatial probability (MASP) algorithm has been described. The MASP algorithm performs pixel-by-pixel segmentation by maximising both the *a poste-*

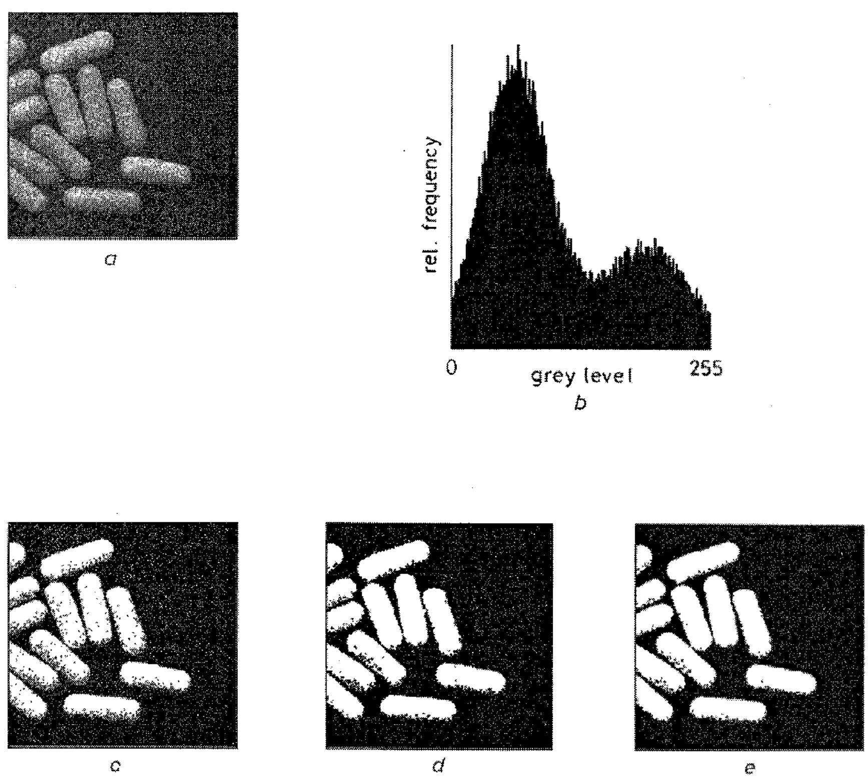

Fig. 5 *Segmentation results of a practical image*
1 byte per pixel, 256 × 256 pixels
a Grey-scale image
b Histogram of (*a*)
c OTSU-thresholded image
d MFOT-segmented image
e MASP-segmented image

riori spatial probability and the segmented scene spatial entropy. Spatial information in the form of joint occurrence of neighbourhood pixels' grey levels is made use of in the proposed method. The grey-level interdependence between pixels is neglected while the class interdependence is retained to make the algorithm computationally feasible. As far as the segmentation results for synthetic images are concerned, it is shown that the MASP algorithm is able to perform better than an ideal minimum error global thresholding method and a simple segmentation method utilising spatial information of an image. As illustrated in Fig. 4, the improvement in performance over the ideal minimum error global thresholding method is found to be very substantial when the PDFs of the different classes of pixels have a separation of more than 1.5 standard deviation of the grey-level values. Where computation time is not a major concern, this algorithm is worthy of consideration for segmenting images to achieve better segmentation result.

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