

Title	On-line adaptation of the SCHMM parameters based on the segmental quasi-bayes learning for speech recognition		
Author(s)	Huo, Q; Chan, C; Lee, CH		
Citation	IEEE Transactions on Speech and Audio Processing, 1996, v. 4 n. 2, p. 141-144		
Issued Date	1996		
URL	http://hdl.handle.net/10722/43628		
Rights	Creative Commons: Attribution 3.0 Hong Kong License		

On-Line Adaptation of the SCHMM Parameters Based on the Segmental Quasi-Bayes Learning for Speech Recognition

Qiang Huo, Chorkin Chan, and Chin-Hui Lee

Abstract— In this correspondence, on-line quasi-Bayes adaptation of the mixture coefficients and mean vectors in semicontinuous hidden Markov model (SCHMM) is studied. The viability of the proposed algorithm is confirmed and the related practical issues are addressed in a specific application of on-line speaker adaptation using a 26-word English alphabet vocabulary.

I. INTRODUCTION

In many speech recognition systems, there usually exists a performance gap between the recognition accuracies on training and on testing data. One major reason lies in the possible mismatch between the underlying acoustic characteristics associated with the training and testing conditions. This mismatch may arise from inter- and intraspeaker variabilities, transducer, channel and other environmental variabilities, and many other phonetic and linguistic effects due to a task mismatch problem. To bridge this performance gap, one possible solution is to design a speech recognition system that are robust to the above types of acoustic mismatch, and this has been a long standing objective of many researchers over the past 20 years. Another way to reduce the possible acoustic mismatch between the training and testing conditions is to adopt the so called *adaptive learning* approach. The scenario is like this: starting from a pretrained (e.g., speakerindependent) speech recognition system, for a new user (or a group of users) to use the system for a specific task, a small number of adaptation data is collected from the user, and these data are used to construct a speaker adaptive system for the speaker in the particular environment for that specific application. By doing so, the mismatch between training and testing can generally be reduced. The most fascinating adaptation scheme with great practical value is the so called on-line (or incremental, sequential) adaptation, and this scheme makes the recognition system continuously adapted to the new adaptation data (possibly derived from actual test utterances) without the requirement of the storage of previous training data. It is this kind of approach that this correspondence focuses on.

Recently, Bayesian adaptive learning of Hidden Markov Model (HMM) parameters has been proposed and adopted in a number of speech recognition applications. A theoretical framework of Bayesian learning was first proposed by Lee *et al.* [8] for estimating the mean and covariance matrix parameters of a continuous density HMM (CDHMM) with a multivariate Gaussian state observation density. It was then extended to handle all the parameters of a CDHMM with mixture Gaussian state observation densities [1], [7], [2] as well as the parameters of discrete HMM's (DHMM's) and semicontinuous HMM's (SCHMM's, also called tied-mixture HMM's) [3]–[6]. It

Manuscript received June 25, 1994; revised October 27, 1995. The associate editor coordinating the review of this paper and approving it for publication was Prof. Kuldip K. Paliwal.

Q. Huo was with the Department of Computer Science, The University of Hong Kong, Hong Kong, He is now with the ATR Interpreting Telecommunications Research Laboratories, Kyoto 619-02, Japan.

C. Chan is with the Department of Computer Science, The University of Hong Kong, Hong Kong.

C. H. Lee is with the Speech Research Department, AT&T Bell Laboratories, Murray Hill, NJ 07974 USA.

Publisher Item Identifier S 1063-6676(96)02443-1.

was shown that, for HMM-based speech recognition applications, the MAP framework provides an effective way for combining adaptation data and the prior knowledge, and then creating a set of adaptive HMM's to cope with the new acoustic conditions in the test data. The prior knowledge, which is embodied in a set of seed HMM's as well as in the assumed distributions of the model parameters being adapted, is made use of to mitigate the effect of adaptation data shortage to improve the system robustness. This approach works in a block (or batch) adaptation mode using a history of all the adaptation data. It finds applications such as fast adaptation to new speaker(s) and/or new speaking environments where only a small amount of adaptation data is needed. In order to consider the long-term and short-term adaptations simultaneously, an ideal adaptation approach should work in an incremental adaptation mode. A related work is conducted by Matsuoka and Lee in [9]. They used the segmental MAP algorithm to conduct the so called on-line adaptation. Due to its missing mechanism of updating the hyperparameters of the prior distribution incrementally, all the previously seen adaptation data need to be stored. A full-scale on-line adaptation approach should be able to update both the hyperparameters of the prior distribution and the HMM parameters themselves simultaneously upon the presentation of the latest adaptation data. An on-line adaptation approach of this nature for SCHMM parameters is presented in this correspondence. It is based on the segmental quasi-Bayes estimation algorithm for the mixture coefficients of SCHMM recently developed in [5] and [6].

The rest of the correspondence is organized as follows. In Section II, the theoretical formulation of segmental quasi-Bayes learning of mixture coefficients in SCHMM is briefly presented and discussed. Two on-line adaptation procedures are presented in Section III. In Section IV, practical issues of on-line Bayesian adaptation of the SCHMM parameters are investigated in the context of a speaker adaptation application, and the related experimental results are presented and discussed. Finally, concluding remarks are given in Section V.

II. SEGMENTAL QUASI-BAYES ESTIMATE

Consider an N-state SCHMM with parameter vector $\lambda = (\pi, A, \theta)$, where π is the initial state distribution, A is the state transition matrix, and θ is the parameter vector composed of mixture parameters $\theta_i = \{\omega_{ik}, m_k, r_k\}_{k=1,2,\dots,K}$ for each state *i* with the state observation probability density function (PDF) being a mixture of a *common* set of K Gaussian PDF's shared by all the HMM states. For state *i*, its observation PDF has the form of

$$p_i(x_t|\theta_i) = \sum_{k=1}^K \omega_{ik} f_k(x_t) = \sum_{k=1}^K \omega_{ik} \mathcal{N}(x_t|m_k, r_k)$$
(1)

where $\mathcal{N}(x|m_k, r_k)$ is the *k*th normal mix and, with m_k being the *D*-dimensional mean vector and r_k being the $D \times D$ precision (inverse covariance) matrix. Each state observation density differs from another by its corresponding mixture coefficients, ω_{ik} , which satisfy the constraint $\sum_{k=1}^{K} \omega_{ik} = 1$.

For an observation sequence $\mathbf{x} = (x_1, x_2, \dots, x_T)$, let $\mathbf{s} = (s_1, s_2, \dots, s_T)$ be the unobserved associated state sequence. By maximizing the joint posterior density of the parameters λ and state sequence \mathbf{s} , $p(\lambda, \mathbf{s} | \mathbf{x})$, one has

$$\lambda = \arg \max \max p(\lambda, \mathbf{s} | \mathbf{x}) = \arg \max \max p(\mathbf{x}, \mathbf{s} | \lambda) g(\lambda) \quad (2)$$

where $g(\lambda)$ is the prior density for parameter λ and $\tilde{\lambda}$ is called the segmental MAP estimate of λ [8]. It can be shown that by starting

1063--6676/96\$05.00 © 1996 IEEE

with any estimate $\lambda^{(l)}$, alternate maximization over s and λ gives a sequence of estimates with nondecreasing values of $p(\lambda, \mathbf{s}|\mathbf{x})$, i.e. $p(\lambda^{(l+1)}, \mathbf{s}^{(l+1)}|\mathbf{x}) > p(\lambda^{(l)}, \mathbf{s}^{(l)}|\mathbf{x})$ with

$$\mathbf{s}^{(l)} = \arg\max p(\mathbf{x}, \mathbf{s} | \lambda^{(l)}) \tag{3}$$

$$\lambda^{(l+1)} = \arg\max_{\lambda} p(\mathbf{x}, \mathbf{s}^{(l)} | \lambda) g(\lambda).$$
(4)

The most likely state sequence $s^{(l)}$ is decoded by means of the Viterbi algorithm. The maximization over λ in (4) is usually accomplished with an EM algorithm which itself is an iterative algorithm and very time consuming [8,2,6]. We have proposed previously [5,6] and summarize here an approximate but efficient solution which is called the quasi-Bayes method which estimates the mixture coefficients alone.

By applying the Viterbi algorithm to the training data, sets of observations (e.g., x_1, x_2, \dots, x_T) associated with each HMM state can be identified. Conditional on $\omega_i = (\omega_{i1}, \omega_{i2}, \dots, \omega_{iK})$ and density functions f_1, f_2, \dots, f_K , each x_n is assumed independently observed with the PDF as shown in (1). Assuming that the prior density for ω_i has the form of a Dirichlet density

$$g(\omega_i) = D(\omega_i | \nu_{i1}^{(0)}, \cdots, \nu_{iK}^{(0)}) \propto \prod_{k=1}^K \omega_{ik}^{\nu_{ik}^{(0)}-1}$$
(5)

where $\nu_{ik}^{(0)} > 0$, for $k = 1, \dots, K$, and " \propto " denotes proportionality. Given the sequence of observations, the maximization with respect to $\{\omega_{ik}\}$ in (4) can be solved by using the following quasi-Bayes recursive updating formula for one pass of the adaptation data:

 $\tilde{\omega}_{ik}^{(n)}$

$$\hat{\omega}_{ik}^{(n)} = \frac{\nu_{ik}^{(n)}}{\sum_{m=1}^{K} \nu_{im}^{(n)}} \tag{6}$$

or

$$V = rac{
u_{ik}^{(n)} - 1}{\sum_{m=1}^{K} (
u_{im}^{(n)} - 1)}$$

where

$$\nu_{ik}^{(n)} = \nu_{ik}^{(n-1)} + p_{ik}(x_n)$$
(8)

and

$$p_{ik}(x_n) = \frac{f_k(x_n)\nu_{ik}^{(n-1)}}{\sum_{m=1}^{K} f_m(x_n)\nu_{im}^{(n-1)}}.$$
(9)

Both (6) and (7) can serve as the updating formula for the mixture coefficients in the segmental quasi-Bayes learning for SCHMM's. Equation (8) is used as the updating formula of the hyperparameters. In the sense that the approximate posterior distribution has a mean identical to that of the true distribution, the convergence properties of the quasi-Bayes method can be established [10].

Note that apart from its computational efficiency, another advantage of the segmental quasi-Bayes method over the segmental MAP one is due to its *sequential* nature in updating both the hyperparameters of the prior distribution and the SCHMM parameters. This makes the so-called *on-line adaptation* of the mixture coefficients very natural under the framework of the quasi-Bayes method.

III. On-LINEADAPTATION PROCEDURES

As is well known, mixture coefficients are very important parameters in modeling speech units in SCHMM. By using the above quasi-Bayes method, the on-line adaptation procedure for the mixture coefficients only can be readily obtained as follows:

- 1) Given seed models, the initial hyperparameters of the mixture coefficients are computed.
- 2) Obtain new adaptation token(s).
- Conduct state labeling to identify sets of observations associated with each HMM state.
- 4) Conduct quasi-Bayes estimation of the mixture coefficients.
- 5) Repeat 3) and 4) several times, and then update the hyperparameters of the prior distributions of the mixture coefficients.
- 6) Go to Step 2).

Apart from the mixture coefficients, the adaptation of the mean vectors of the Gaussian mixture components is also very important [4], [6]. However, the previously proposed algorithm can only be theoretically justified in the case of fixed mixture components. On the other hand, it has been shown in [4] and [6] that the mean vectors of the common Gaussian densities in SCHMM can be rapidly and effectively estimated even with a limited amount of training data by the conventional ML (maximum likelihood) training. Thus a pragmatic on-line adaptation procedure which combines the quasi-Bayes adaptation of the mixture coefficients and the adaptation of the mean vectors can be as follows:

- 1) Given seed models, the initial hyperparameters of the mixture coefficients are computed.
- 2) Obtain new adaptation token(s) and push it (them) into the "history data buffer."Conduct state labeling to identify sets of observations associated with each HMM state.
- 3) Conduct guasi-Bayes estimation of the mixture coefficients.
- 4) Fix the other parameters and ML-train the mean vectors of the mixture components with the adaptation data in the "history data buffer."
- 5) Repeat 3)-5) several times, and then update the hyperparameters of the prior distributions of the mixture coefficients.
- 6) Go to 2).

(7)

IV. SPEAKER ADAPTATION EXPERIMENTS

A. Experimental Setup

We will compare the so-called *batch adaptation* scheme and the *on-line adaptation* scheme in this section using a series of speaker adaptation experiments to substantiate the viability of the proposed techniques. The 26 letters of the English alphabet are chosen to form the vocabulary for all experiments. Two severely mismatched databases are used for evaluating the adaptation algorithms. For speaker independent (SI) training and prior density estimation, the OGI ISOLET database produced by 150 speakers (75 females and 75 males) is used. Each speaker utters each of the letters twice. For speaker dependent (SD) or adaptive (SA) training and testing, the TI46 isolated word corpus produced by 12 speakers (eight females and four males) is used. Each person utters each of the letters 26 times, 10 of them used for SD/SA training and the remaining 16 tokens for testing. Readers are referred to [3]–[6] for further details.

B. Experimental Results

To examine the viability and effect of the above procedure for on-line adapting the mixture coefficients of SCHMM only, a series of comparative experiments are conducted. The first experiment is to recognize the English alphabet subset of TI46 with the SI system trained with speech tokens from OGI ISOLET. The average

IEEE TRANSACTIONS ON SPEECH AND AUDIO PROCESSING, VOL. 4, NO. 2, MARCH 1996

OF SCHMMS ONLY (SI RECOGNITION RATE: 47.8%)							
Tokens	SEG-ML	SEG-MAP	QB-BL	QB-OL			
1	56.3	61.5	62.0	62.0			
2	62.5	65.1	65.0	65.4			
3	65.9	67.2	66.8	67.4			
4	67.0	68.0	67.8	68.2			
5	68.4	69.1	69.0	69.1			
6	68.3	68.9	69.0	69.3			
7	68.6	69.4	69.3	69.5			
8	69.4	69.7	70.0	69.9			
9	70.3	70.5	70.1	70.1			
10	70.7	70.8	70.4	70.6			

TABLE I PERFORMANCE COMPARISON (% CORRECT) OF SEVERAL SEGMENTAL ADAPTATION SCHEMES FOR THE MIXTURE COEFFICIENTS OF SCHEMES ONLY (CL PERCORPUTED) PURE 47.8%)

TABLE II
ERFORMANCE COMPARISON (% CORRECT) OF SEVERAL ADAPTATION
SCHEMES FOR THE MIXTURE COEFFICIENTS AND THE MEAN
VECTORS OF SCHMMS (SI RECOGNITION RATE: 47.8%)

PE

Tokens	SEG-ML	0L-1	OL-2	OL-3
1	63.6	66.8	66.8	66.8
2	70.4	71.5	72.4	72.4
3	74.2	72.3	73.2	73.9
4	76.2	73.7	73.9	75.0
5	77.3	73.7	74.2	75.3
6	76.1	73.6	74.9	75.6
7	77.1	75.9	76.1	76.2
8	76.9	76.0	76.6	76.0
9	77.7	76.4	76.8	77.0
10	78.5	77.1	77.2	77.4

recognition rate is 47.8%. For simplicity, in SA/SD training, Gaussian mixture component PDF's and the transition probabilities are fixed to that of the SI system.¹ In SA training, the hyperparameters of the prior distribution of the mixture coefficients are estimated with the *ad hoc* method discussed in [3]. The remaining experimental setups are as follows: "SEG-ML" stands for SD segmental ML (*k*-means) training of the mixture coefficients and "SEG-MAP" corresponds to its MAP counterpart. "QB-BL" stands for SA segmental quasi-Bayes block adaptation of the mixture coefficients, and "QB-OL" refers to its on-line adaptation counterpart. The average word recognition rates for the 12 speakers are summarized in Table I. The rows in Table I correspond to the numbers of training tokens used for each SD and SA cases.

The first observation from Table I is that the SD recognition rate of only one training token is better than that of the SI system and this fact is a good indication of the serious mismatch between the two corpora. A second observation is that when using the same amount of training data, SA training outperforms SD training in most of the cases tested. This implies that SA training utilizes the adaptation data more effectively than SD training, especially in cases of insufficient training data. A third observation is that the recognizer performance with the segmental quasi-Bayes method is not much different from that with the segmental MAP method, and this fact also shows the viability of the quasi-Bayes approximation in maximizing the righthand side of (4). As a fourth observation, by comparing the results of "QB-OL" and that of "QB-BL," it is noticed that the on-line adaptation results are similar to the one based on the batch adaptation scheme. This confirms the effectiveness of the on-line adaptation scheme of the mixture coefficients.

To examine the effects of the above pragmatic on-line adaptation procedure for mixture coefficients and mean vectors, a series of comparative experiments are also conducted. Once again, for simplicity, the transition probabilities and the covariance matrices of the Gaussian mixture components are fixed to that of the SI system. In an on-line ML training of mean vectors, different block sizes of the "history data buffer" is examined. In the particular experimental setup here, the cases with buffer size of 1, 2, and 3 token(s) per letter have been tried. The related experimental results (the average word

¹Since the SD system uses some of the SI-trained parameters, actually it can also be treated as a SA system. In the following, we use the term "SD" loosely just for simplicity, since it is not likely to cause confusion.

recognition rates for 12 speakers) are summarized in Table II. The rows in Table II correspond to the numbers of training tokens used for each SD and SA cases. "SEG-ML" stands for SD segmental ML (k-means) training of the mixture coefficients and the mean vectors. "OL-1" corresponds to on-line adaptation of the mixture coefficients and the mean vectors with the history data buffer size being one. Similarly, "OL-2" and "OL-3" refer to, respectively, the cases with buffer sizes of two and three.

Once again, from Table II, it is observed that the recognizer performance with on-line adaptation outperforms that with SD training when the SD training data is insufficient (one and two tokens). The SD performance improves as the number of speaker specific training tokens increases, and the on-line adaptation scheme can follow this increasing trend, although its absolute recognition rate is inferior to the SD one when relatively more SD training tokens (in particular here more than three tokens) are available. As for the effects of the "history data buffer" size, it is observed that the larger the buffer size, the better the on-line adaptation performance. On the other hand, larger buffer size also means more storage is required. From the practical point of view, there will be a compromise in real applications. The on-line adaptation of the mixture coefficients can be used to perform a long-term adaptation or a short-term adaptation. By using the quasi-Bayes learning framework, one can update both the hyperparameters of the prior distribution and the mixture coefficients simultaneously upon the presentation of the current adaptation data. In this way, with each adaptation utterance presented, its effect upon further adaptation is accumulated into the prior distribution. Thus previous adaptation data need not be stored explicitly. All the historical knowledge is represented by the prior distributions and is updated incrementally. The effect of this long-term prior knowledge on the adaptation results can be easily controlled through some *forgetting* mechanism, thus it is equally applicable to short-term adaptation. This mechanism can be implemented by setting up some registers to store the most recent contributions from the adaptation data history. When it becomes time to "forget" about the long-term prior knowledge, the hyperparameters of the prior distributions can be recomputed from the stored recent contributions. The on-line SD ML training of mean vectors can be looked upon as a short-term (or fast) adaptation process to track the latest variations. This kind of on-line adaptation framework will find applications in real world adaptive speech recognition systems.

C. General Discussions

The effects of SA training, in the particular setup here, are not so significant. This is because of the serious mismatch between the two corpora. After more detailed analysis, it is found that the SA effects can be very different among different speakers also. In the Bayesian learning framework, one hopes to use prior distribution of HMM parameters to represent the information of the variability of a certain model among the different speakers, so the SA effects depend heavily on the suitability of the prior distribution to the new speaker. To cope with the mismatch problem between the prior distribution and the new adaptation data, it will be beneficial if some kind of speaker normalization (or signal space equalization) is done first in the acoustic (feature) space before the Bayesian framework is applied to adapt the model parameters and/or if multiple set of prior distributions are adopted in the process of model adaptation provided enough training data are available [4], [6].

V. SUMMARY

In this correspondence, in order to cope with the acoustic mismatch problem between the training and testing conditions, the issues of online adaptation of a SCHMM-based speech recognition system are addressed. A theoretical formulation of the segmental quasi-Bayes learning of the mixture coefficients is presented. A pragmatic online adaptation approach to combine the long-term adaptation of the mixture coefficients and the short-term adaptation of the mean vectors of the Gaussian mixture components is also proposed. The viability of the proposed algorithms is confirmed and the related practical issues are studied in a specific application of on-line supervised speaker adaptation using a 26-word English alphabet vocabulary. The kind of on-line adaptation approach studied in this correspondence is a topic of interest both in theory and in practice. Further research is needed to develop the on-line adaptation method which can update incrementally the hyperparameters of both the mixture coefficients and the mean vectors as well as the covariance matrices in CDHMM and SCHMM cases. Another research topic may be to explore fast adaptation algorithm based on all possible sources of knowledge, e.g., by considering the dependency or correlation between HMM's to help adjust those HMM parameters without adaptation data. Our ultimate goal will be to easily adapt a general model to new task, new speaker and new environment.

REFERENCES

- J.-L. Gauvain and C.-H. Lee, "Bayesian learning for hidden Markov model with Gaussian mixture state observation densities," *Speech Commun.*, vol. 11, Nos. 2-3, pp. 205–213, 1992.
- [2] _____, "Maximum a posteriori estimation for multivariate Gaussian mixture observations of Markov chains," *IEEE Trans. on Speech and Audio Processing*, vol. 2, no. 2, pp. 291–298, Apr. 1994.
 [3] Q. Huo, C. Chan, and C.-H. Lee, "Bayesian learning of the parameters"
- [3] Q. Huo, C. Chan, and C.-H. Lee, "Bayesian learning of the parameters of discrete and tied mixture HMM's for speech recognition," in *Proc. Eurospeech-93*, Berlin, Germany, 1993, pp. III–1567–1570.
- [4] _____, "Bayesian learning of the SCHMM parameters for speech recognition," in *Proc. ICASSP-94* Adelaide, Australia, 1994, pp. I-221-224.
 [5] _____, "Segmental quasi-Bayesian learning of the mixture coefficients
- [5] _____, "Segmental quasi-Bayesian learning of the mixture coefficients in SCHMM for speech recognition," in Proc. 1994 Int. Symp. Speech, Image Processing & Neural Networks, Hong Kong, 1994, pp. 678-681.
- [6] _____, "Bayesian adaptive learning of the parameters of hidden Markov model for speech recognition," *IEEE Trans. Speech and Audio Processing*, vol. 3, no. 5, pp. 334–345, Sept. 1995.

- [7] C.-H. Lee and J.-L. Gauvain, "Speaker adaptation based on MAP estimation of HMM parameters," in *Proc. ICASSP-93* Minneapolis, MO, 1993, pp. II-588-591.
- [8] C.-H. Lee, C.-H. Lin, and B.-H. Juang, "A study on speaker adaptation of the parameters of continuous density hidden Markov models," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 39, no. 4, pp. 806–814, 1991.
- [9] T. Matsuoka and C.-H. Lee, "A study of on-line Bayesian adaptation for HMM-based speech recognition," in *Proc. Eurospeech-93*, 1993, Berlin, Germany, pp. 815–818.
- [10] A. F. M. Smith and U. E. Makov, "A quasi-Bayes sequential procedure for mixtures," *Royal Stat. Soc., J., Series B*, vol. 40, no. 1, pp. 106–112, 1978.

Fast Implementation of MPEG Audio Coder Using Recursive Formula with Fast Discrete Cosine Transforms

Din-Yuen Chan, Jar-Ferr Yang, and Chun-Chin Fang

Abstract—For cosine-modulated multirate subband filtering, we propose two implementation algorithms to reduce the computational complexity. First, we develop a recursive algorithm to further decrease the computation of polyphase subband filtering suggested in Nussbaumer and Vetterli. Since the cosine modulation can be implemented by the discrete cosine transform (DCT), we combine both fast decimation-in-time (DIT) and decimation-in-frequency (DIF) DCT methods to further reduce the computation. Consequently, the recursive formula with the mixed fast-DCT method requires about 13.2% of the multiplications and 41.6% of the additions recommended by ISO Std. ISO-IEC JTCI/SC2/WG11.

I. INTRODUCTION

Cosine-modulated subband filtering, which has been adopted as the MPEG-1 audio standard, has been recognized as one of best coding techniques for achieving high audio quality [4]. The theory of the cosine-modulated subband filtering is described in [2]. Nussbaumer and Vetterli [1] presented the first efficient polyphase filtering implementation of quadrature mirror filters (QMF's) in multirate subband filtering. Beyond this polyphase filtering implementation, we first develop a recursive formula to reduce the computational load in the polyphase filters. Since the cosine modulation of polyphase filtered outputs can be transformed into a DCT-III [5] after some rearrangement, we finally combine fast decimation-in-time (DIT) and decimation-in-frequency (DIF) discrete cosine transform (DCT) algorithms [6], [7] to reduce further the computational complexity.

II. MULTIRATE SUBBAND FILTERING

For multirate cosine-modulated QMF's [1], [2], a lowpass prototype filter with the nominal bandwidth $\pi/2M$ is specified by N coefficients A(k) for $k = 0, 1, 2, \dots, (N-1)$. The coefficients of the *i*th bandpass filter are formed by modulating these coefficients

Manuscript received February 28, 1994; revised October 14, 1995. This work was supported by the National Science Council under Grant NSC80-0404-E006-08, Taiwan, Republic of China. The associate editor coordinating the review of this paper and approving it for publication was Dr. James H. Snyder.

The authors are with the Department of Electrical Engineering, National Cheng Kung University, Tainan, Taiwan, ROC (e-mail: jfyang@eembox.ncku.edu.tw).

Publisher Item Identifier S 1063-6676(96)02447-9.

1063-6676/96\$05.00 © 1996 IEEE