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Biachi type I cosmology in $N=2$, $D=5$ supergravity

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The dynamics and evolution of Bianchi type I space-times are considered in the framework of the four-dimensional truncation of a reduced theory obtained from $N=2$, $D=5$ supergravity. The general solution of the gravitational field equations can be represented in an exact parametric form. All solutions have a singular behavior at the initial or final moment, except when the space-time geometry reduces to the isotropic flat case. Generically the obtained cosmological models describe an anisotropic, expanding or collapsing, singular universe with a noninflationary evolution for all times.

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I. INTRODUCTION

The similarity between the $D=5$ simple supergravity (SUGRA) and $D=11$ SUGRA has been recognized for a long time [1,2]. $D=11$ SUGRA is supposed to play a fundamental role as the low-energy limit of M theory [3] — an expected unified speculation for the well-known five consistent superstring theories. The field content of the $D=11$ SUGRA theory consists of the metric, a single Majorana spin-$\frac{3}{2}$ fermion, along with a (singlet) three-form gauge potential, with neither "$N>1$" extensions nor matter coupling permitted [4]. Simple $D=5$ SUGRA, in addition to the metric $\hat{g}_{AB}$, contains a spin-$\frac{3}{2}$ field $\hat{\Psi}^a_A$ ($a=1,2$ is an internal index) and U(1) gauge field (one-form $\hat{B}_A$) which replaces the three-form gauge field in the $D=11$ SUGRA. The "primeval" likeness comes directly from the fact that the Lagrangians of both SUGRAs are exactly of the same form, except for the numbering of the gauge field indices. In addition, their dimensional reduction to $D=4$ can be carried out in a similar way [5]. Furthermore, the $D=5$ simple SUGRA can be realized as a Calabi-Yau compactification of the $D=11$ SUGRA together with the truncation of the scalar multiplets, which is always necessary since there arises at least one scalar multiplet for any Calabi-Yau compactification [6,7]. Further resemblances between the two SUGRAs are related to the duality groups upon dimensional reduction and the world sheet structure of the solitonic string of the $D=5$ SUGRA [8].

Thus the four-dimensional reduced effective action of the $N=2$, $D=5$ SUGRA contains an additional Maxwell-like $U(1)$ field and a scalar field regarded as external fields in five dimensions which are contributed by $\hat{B}$, in addition to the ones coming from the metric $\hat{g}_{AB}$ as in the traditional scheme for the Kaluza-Klein theory [9–12]. Cosmological solutions to this model have been previously considered by Balbinot, Fabris, and Kerner [11,12]. For the case of spatial homogeneity and isotropy the general solution is nonsingular in the scale factor, but unstable due to the collapse to zero of the size of the fifth dimension [11,12]. Biaxial (with two equal scale factors) anisotropic solutions with a cylindrical homogeneous five-dimensional metric lead to singular solutions with positive gravitational coupling [12]. Recently, an explicit example of a manifestly U-duality covariant M-theory cosmology in five dimensions resulting from compactification on a Calabi-Yau threefold has been obtained in Ref. [13]. Exact static solutions in $N=2$, $D=5$ SUGRA have been found by Pimentel [14], in a metric with cylindrical symmetry, with a particular case corresponding to the exterior of a cosmic string.

The purpose of the present paper is to construct the general solution to the gravitational field equations of the $N=2$, $D=5$ SUGRA as formulated in Refs. [11,12] for an anisotropic triaxial (all directions have unequal scale factors) Bianchi type I space-time. In this case the general solution of the field equations can be expressed in an exact parametric form. For all cosmological solutions, the singularity at the starting and ending time of the evolution cannot be avoided except in the isotropic limit considered in Refs. [11,12]. Nevertheless, in the models analyzed in this paper, the anisotropic Universe has non-inflationary evolution for all times and for all values of parameters.

The present paper is organized as follows. The field equations of our model are written down in Sec. II. In Sec. III the general solution of the field equations is obtained. We discuss our results and conclusions in Sec. IV.

II. FIELD EQUATIONS, GEOMETRY AND CONSEQUENCES

The bosonic sector of $N=2$, $D=5$ SUGRA contains the five-dimensional metric $\hat{g}_{AB}$ and U(1) gauge field $\hat{B}_A$ described by a Lagrangian which possesses a nonvanishing Chern-Simons term [1]

$$\hat{\mathcal{L}}=\sqrt{-\hat{g}}\left\{\hat{R}-\frac{1}{4}\hat{F}_{AB}\hat{F}^{AB}\right\}-\frac{1}{12\sqrt{3}}\epsilon^{ABCDE}\hat{F}_{AB}\hat{F}_{CD}\hat{B}_E, \tag{1}$$

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where $\hat{F}_{AB}=2\partial_{[A}\hat{B}_{B]}$. In this paper we use the following conventions and notations. The variables with hats are five-dimensional objects all other variables are four dimensional. Upper case indices $A,B,\ldots,$ are used for five-dimensional space-time, greek indices $\mu,\nu,\ldots,$ and low case indices $i,j,\ldots,$ are for four-dimensional space-time and three-dimensional space, respectively. The signature is $(-,+,+,+,+)$.

Assuming that the five dimensional space-time has locally the structure of $M^4\times S^1$ with a four-dimensional space-time M^4 whose spatial sections are homogeneous and asymptotic flat, then the five-dimensional metric can be decomposed along the standard Kaluza-Klein pattern

$$d\hat{s}^2=\phi^2(dx_4+A_\mu dx^\mu)^2+g_{\mu\nu}dx^\mu dx^\nu, \tag{2}$$

where the scale factor ϕ and Kaluza-Klein vector A_μ are functions depending on x^μ only.

Looking for a ''ground state'' configuration we set, following Refs. [11,12], the Kaluza-Klein vector A_μ equal to zero and take the one-form potential $\hat{B}_A$ to be $\hat{B}_\mu=0$ and $\hat{B}_4=\sqrt{3}\psi(x^\mu)$. Under this ansatz, the five-dimensional gravitational field equations for Eq. (1) reduce to a set of four-dimensional equations

$$R_{\mu\nu}-\phi^{-1}D_\mu D_\nu\phi-\frac{1}{2}\phi^{-2}[3\partial_\mu\psi\partial_\nu\psi-g_{\mu\nu}(\partial\psi)^2]=0, \tag{3}$$

$$D^2\phi+\phi^{-1}(\partial\psi)^2=0, \tag{4}$$

$$D^2\psi-\phi^{-1}\partial_\mu\phi\partial^\mu\psi=0, \tag{5}$$

where D denotes the four-dimensional covariant derivative with respect to the metric $g_{\mu\nu}$. Equivalently, the field equations can be rederived, in the string frame, from the four-dimensional Lagrangian [11,12]

$$\mathcal{L}=\sqrt{-g}\,\phi\left\{R-\frac{3}{2}\phi^{-2}(\partial\psi)^2\right\}, \tag{6}$$

via variation with respect to the fields $g_{\mu\nu}$, ϕ and ψ. In the Lagrangian (6), the scale factor ϕ is an analogue of the Brans-Dicke field whereas the origin of ψ is purely supersymmetric.

The line element of an anisotropic homogeneous flat Bianchi type I space-time is given by

$$ds^2=-dt^2+a_1^2(t)dx^2+a_2^2(t)dy^2+a_3^2(t)dz^2. \tag{7}$$

Defining the ''volume scale factor'' $V:=\Pi_i a_i$, ''directional Hubble factors'' $H_i:=\dot{a}_i/a_i$, and ''average Hubble factor'' $H:=\frac{1}{3}\Sigma_i H_i$, one can promptly find the relation $3H=\dot{V}/V$, where dot means the derivative with respect to time t. In terms of those variables, the field equations (3) and the equations of motion for ϕ and ψ (4),(5) coupling with the anisotropic Bianchi type I geometry take the concise forms

$$3\dot{H}+\sum_i H_i^2+\phi^{-1}\ddot{\phi}+\phi^{-2}\dot{\psi}^2=0, \tag{8}$$

$$V^{-1}\frac{d}{dt}(VH_i)+H_i\phi^{-1}\dot{\phi}-\frac{1}{2}\phi^{-2}\dot{\psi}^2=0,\quad i=1,2,3, \tag{9}$$

$$V^{-1}\frac{d}{dt}(V\dot{\phi})+\phi^{-1}\dot{\psi}^2=0, \tag{10}$$

$$V^{-1}\frac{d}{dt}(V\dot{\psi})-\phi^{-1}\dot{\phi}\dot{\psi}=0. \tag{11}$$

The physical quantities of interest in cosmology are the expansion scalar θ, the mean anisotropy parameter A, the shear scalar σ^2, and the deceleration parameter q defined as [15]

$$\theta:=3H,\quad A:=\frac{1}{3}\sum_i\left(\frac{H-H_i}{H}\right)^2,$$

$$\sigma^2:=\frac{1}{2}\left(\sum_i H_i^2-3H^2\right),\quad q:=\frac{d}{dt}H^{-1}-1. \tag{12}$$

The sign of the deceleration parameter indicates whether the cosmological model inflates. A positive sign corresponds to standard decelerating models whereas a negative sign indicates inflationary behavior.

III. GENERAL SOLUTION OF THE FIELD EQUATIONS

Equation (11) can immediately be integrated to give

$$V\dot{\psi}=\omega\phi, \tag{13}$$

with ω — a constant of integration. From Eqs. (10) and (11) one can find that the expressions of the fields $\phi(t)$ and $\psi(t)$ have the following form:

$$\phi(t)=\phi_0\cos\left(\omega\int\frac{dt}{V}+\omega_0\right), \tag{14}$$

$$\psi(t)=\psi_0+\phi_0\sin\left(\omega\int\frac{dt}{V}+\omega_0\right), \tag{15}$$

where ϕ_0, ψ_0 and ω_0 are constants of integration.

By summing equations (9) one gets

$$V^{-1}\frac{d}{dt}(VH)+H\phi^{-1}\dot{\phi}-\frac{1}{2}\phi^{-2}\dot{\psi}^2=0, \tag{16}$$

which can be transformed, by using Eqs. (13) and (14), into the following differential-integral equation describing the dynamics and evolution of a triaxial Bianchi type I space-time in $N=2$, $D=5$ SUGRA:

$$\ddot{V}=\omega\frac{\dot{V}}{V}\tan\left(\omega\int\frac{dt}{V}+\omega_0\right)+\frac{3}{2}\frac{\omega^2}{V}. \tag{17}$$

Furthermore, by subtracting Eq. (16) from Eqs. (9), one can solve for the H_i as

$$H_i=H+\frac{K_i}{\phi V},\quad i=1,2,3, \tag{18}$$

where K_i are constants of integration satisfying the following consistency condition:

$$\sum_i K_i=0. \tag{19}$$

Therefore the physical quantities of interest (12) reduce to

$$A=\frac{K^2}{3\phi^2V^2H^2},\quad \sigma^2=\frac{3}{2}AH^2, \tag{20}$$

where $K^2=\Sigma_i K_i^2$.

By introducing a new variable η related to the physical time t by means of the transformation $d\eta := dt/V$ and by denoting $u:=\dot{V}=dV/(Vd\eta)$, Eq. (17) reduces to a first order linear differential equation for the unknown function u:

$$\frac{du}{d\eta}=\omega\tan(\omega\eta+\omega_0)u+\frac{3}{2}\omega^2, \tag{21}$$

whose general solution is given by

$$u=C\cos^{-1}(\omega\eta+\omega_0)+\frac{3}{2}\omega\tan(\omega\eta+\omega_0), \tag{22}$$

where C is an arbitrary constant of integration.

Defining a new parameter $\zeta(\eta):=\omega\eta+\omega_0$, we can represent the general solution of the field equations for a Bianchi type I space-time in the $N=2,\ D=5$ SUGRA in the following exact parametric form:

$$t=t_0+\frac{V_0}{\omega}\int\frac{(1+\sin\zeta)^{\beta}}{(1-\sin\zeta)^{\gamma}}d\zeta, \tag{23}$$

$$V=V_0\frac{(1+\sin\zeta)^{\beta}}{(1-\sin\zeta)^{\gamma}}, \tag{24}$$

$$H=\frac{\omega}{2V_0}(\alpha+\sin\zeta)\frac{(1-\sin\zeta)^{\gamma-1/2}}{(1+\sin\zeta)^{\beta+1/2}}, \tag{25}$$

$$a_i=a_{i0}\frac{(1+\sin\zeta)^{\beta/3+K_i/2\omega\phi_0}}{(1-\sin\zeta)^{\gamma/3+K_i/2\omega\phi_0}},\quad i=1,2,3, \tag{26}$$

where we have denoted $\alpha=2C/3\omega$, $\beta=\frac{3}{4}(\alpha-1)$, $\gamma=\frac{3}{4}(\alpha+1)$ and the a_{i0} are arbitrary constants of integration while $V_0=\Pi_i a_{i0}$. The observationally important physical quantities are given by

$$A=\frac{4K^2}{3\phi_0^2\omega^2}(\alpha+\sin\zeta)^{-2}, \tag{27}$$

$$\sigma^2=\frac{K^2}{2\phi_0^2V_0^2}\frac{(1-\sin\zeta)^{2\gamma-1}}{(1+\sin\zeta)^{2\beta+1}}, \tag{28}$$

$$q=2\left\{1-\frac{1+\alpha\sin\zeta}{(\alpha+\sin\zeta)^2}\right\}. \tag{29}$$

Finally, the field equation (8) gives a consistency condition relating the constants K^2, ω, α, and ϕ_0:

$$K^2=\frac{3}{2}\phi_0^2\omega^2(\alpha^2-1), \tag{30}$$

leading to

$$\alpha\geqslant 1\ \text{or}\ \alpha\leqslant -1. \tag{31}$$

It is worth noting that these two classes of solutions corresponding to positive or negative values of α and ω are not independent. Indeed, they can be related via a ''duality'' transformation by changing the signs of ω, α, and ζ so that $t(\omega,\alpha,\zeta)=t(-\omega,-\alpha,-\zeta)$, $a_i(\omega,\alpha,\zeta)=a_i(-\omega,-\alpha,-\zeta)$, $i=1,2,3$, $V(\alpha,\zeta)=V(-\alpha,-\zeta)$, etc. This duality relation can be obtained by a simple inspection of Eqs. (23)–(26) and, therefore, all physical quantities are invariant with respect to this transformation. Moreover, the physical properties of the cosmological models presented here are strongly dependent on the signs of the parameters α and ω. Nevertheless, due to the duality transformation, hereafter we will consider, without loss of generality, the cases with positive α only.

For some particular values of α, the general solutions can be expressed in an exact nonparametric form, for instance, an exact class solutions can be obtained for $\alpha=\pm\frac{5}{3}$. By introducing a new time variable $\tau:=3\sqrt{2}\omega/V_0t$, and choosing $t_0=\mp V_0/3\sqrt{2}\omega$, the exact solution in $N=2,D=5$ SUGRA for the Bianchi type I space-time is given by

$$\tau=\pm\left[\cos^{-3}\left(\frac{\zeta}{2}\pm\frac{\pi}{4}\right)-1\right], \tag{32}$$

$$V=\frac{V_0}{2\sqrt{2}}(1\pm\tau)[(1\pm\tau)^{2/3}-1]^{1/2}, \tag{33}$$

$$H=\frac{\sqrt{2}\omega}{V_0}\frac{\frac{4}{3}(1\pm\tau)^{2/3}-1}{(1\pm\tau)[(1\pm\tau)^{2/3}-1]}, \tag{34}$$

$$a_i=\frac{a_{i0}}{\sqrt{2}}(1\pm\tau)^{1/3}[(1\pm\tau)^{2/3}-1]^{1/6\pm K_i/2\omega\phi_0}, \tag{35}$$

$$A=\frac{8}{9}(1\pm\tau)^{4/3}\left[\frac{4}{3}(1\pm\tau)^{2/3}-1\right]^{-2}, \tag{36}$$

$$\sigma^2=\frac{8\omega^2}{3V_0^2}(1\pm\tau)^{-2/3}[(1\pm\tau)^{2/3}-1]^{-2}, \tag{37}$$

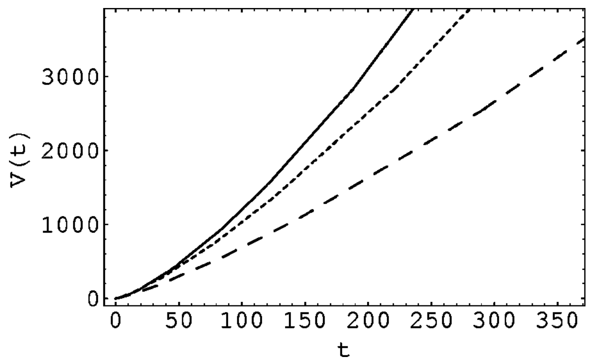

FIG. 1. Behavior of the volume scale factor of the Bianchi type I space-time for different values of $\alpha>1$ ($V_0=1$ and $\omega=1$): $\alpha=3/2$ (solid curve), $\alpha=2$ (dotted curve), and $\alpha=5/2$ (dashed curve).

$$q=2\left\{1-\frac{(1\pm\tau)^{2/3}[4(1\pm\tau)^{2/3}-5]}{6[\frac{4}{3}(1\pm\tau)^{2/3}-1]^2}\right\}. \tag{38}$$

The isotropic limit can be achieved by taking $\alpha=\pm1$ and, consequently, $K_i=0$, $i=1,2,3$. It is worth noting that our solutions reduce to two different types of homogeneous space-times when $\alpha=\pm1$.

For $\alpha=1$, we obtain (by denoting $a_1=a_2=a_3=a$)

$$t=t_0+\frac{V_0}{2\sqrt{2}\omega}\left[\frac{\sin\theta}{\cos^2\theta}+\ln(\tan\theta+\sec\theta)\right], \tag{39}$$

$$a=\frac{a_0}{2\sqrt{2}}\cos^{-3}\theta, \tag{40}$$

where $\theta:=\zeta/2+\pi/4$. Equations (39) and (40), describing a homogeneous flat isotropic space-time interacting with two scalar fields (Kaluza-Klein and supersymmetric), have been previously obtained by Balbinot, Fabris, and Kerner [11] (for an extra choice of the parameter $\omega_0=\pi/2$), who extensively studied their physical properties. This isotropic solution also provides a positive gravitational coupling at the present time.

In the isotropic limit corresponding to $\alpha=-1$, one can obtain another class of isotropic homogeneous flat space-times represented in the following parametric form by

$$t=t_0-\frac{V_0}{2\sqrt{2}\omega}\left[\frac{\cos\theta}{\sin^2\theta}-\ln(\csc\theta-\cot\theta)\right], \tag{41}$$

$$a=\frac{a_0}{2\sqrt{2}}\sin^{-3}\theta. \tag{42}$$

This type of flat space-time has not been previously considered.

IV. DISCUSSIONS AND FINAL REMARKS

In order to study the physical properties of the Bianchi type I universe described by the Eqs. (24)–(26) we need to fix first the range of variation of the parameter ζ. There are no *a priori* limitations in choosing the admissible range of values, thus both positive and negative values are permitted since the variable $\eta=\int dt/V$ can also be negative. But from a physical point of view it is natural to impose the condition such that the gravitational coupling ϕ is always positive during the evolution of the Bianchi type I space-time in $N=2$, $D=5$ SUGRA. Consequently, we shall consider $\zeta\in(-\pi/2,\pi/2)$. With this choice, the universe for $\alpha<-1$ starts its evolution in the infinite past ($t\rightarrow-\infty$) and ends at a finite moment $t=t_0$. For $\alpha>1$ the universe starts at $t=t_0$ and ends in an infinite future with $t\rightarrow\infty$. (All discussion here and hereafter are with respect to positive ω.)

As can be easily seen from Eq. (22), if $\alpha<-1$ we have $\dot{V}<0$ for all t. For these values of the parameters the Bianchi type I anisotropic universe collapses from an initial state characterized by infinite values of the volume scale factor and of the scale factors a_i, $i=1,2,3$, to a singular state with all factors vanishing. But if $\alpha>1$, the universe expands and $\dot{V}>0$ for all t. The expanding Bianchi type I universe starts its evolution at the initial moment, t_0, from a singular state with zero values of the scale factors, $a_i(t_0)=0$, $i=1,2,3$.

Another possible way to investigate the singularity behavior at the initial moment is to consider the sign of the quantity $R_{\mu\nu}u^\mu u^\nu$, where u^μ is the vector tangent to the geodesics; $u^\mu=(-1,0,0,0)$ for the present model. From the gravitational field equations we easily obtain

$$R_{\mu\nu}u^\mu u^\nu=3H^2(q-A). \tag{43}$$

By using Eqs. (27), (29), and (30) we can express Eq. (43) as

$$R_{\mu\nu}u^\mu u^\nu=6H^2\frac{\sin\zeta}{\alpha+\sin\zeta}. \tag{44}$$

For $\zeta\rightarrow-\pi/2$ the sign of $R_{\mu\nu}u^\mu u^\nu$ is determined by the sign of $1-\alpha$. Therefore we obtain

$$R_{\mu\nu}u^\mu u^\nu<0 \text{ for } \alpha>1. \tag{45}$$

Hence, the energy condition of Hawking-Penrose singularity theorems [16] is not satisfied for the solutions corresponding to Bianchi type I Universes in the four-dimensional reduced two scalar fields theory of $N=2$, $D=5$ SUGRA with $\alpha>1$. Nevertheless, for those solutions an initial singular state is *unavoidable* at the initial moment t_0.

Since for $\alpha>1$ the Bianchi type I Universe starts its evolution at the initial moment $t=t_0$ ($\zeta\rightarrow-\pi/2$) from a singular state, therefore, the presence of a variable gravitational coupling ϕ and of a supersymmetric field ψ in an anisotropic geometry *cannot* remove the initial singularity that mars the big-bang cosmology. At the initial moment the degree of the anisotropy of the space-time is maximal, with the initial value of the anisotropy parameter $A(t_0)=2(\alpha+1)/(\alpha-1)$. For $t>t_0$ the Universe expands and the anisotropy parameter decreases.

The behavior of the volume scale factor, of the anisotropy parameter and of the deceleration parameter is presented for different values of α in Figs. 1–3. The evolution of the Universe is noninflationary, with $q>0$ for all $t>t_0$. Noninfla-

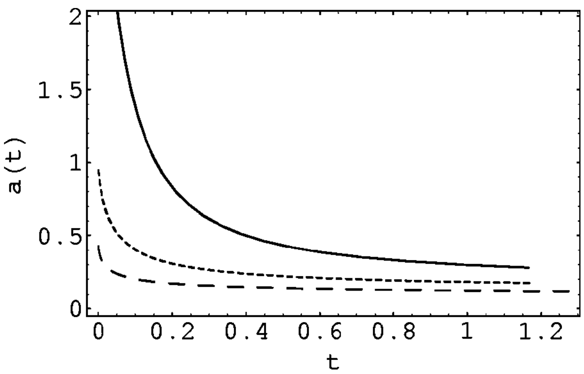

FIG. 2. Time dependence of the parameter $a=(3\phi_0^2\omega^2/4K^2)A$ for different values of α: $\alpha=3/2$ (solid curve), $\alpha=2$ (dotted curve), and $\alpha=5/2$ (dashed curve).

tionary behavior is a generic feature of most of the supersymmetric models. This is due to the general fact that the effective potential for the inflation field σ in SUGRA typically is too curved, growing as $\exp(c\sigma^2/m)$, with c a parameter [typically $O(1)$] and m the stringy Planck mass [17]. The typical values of c make inflation impossible because the inflation mass becomes of the order of the Hubble constant. See also Ref. [17] for a simple realization of hybrid inflation in SUGRA. In the present model the presence of the supersymmetric field ψ prevents the Bianchi type I Universe from inflating.

In the far future, for $\zeta\rightarrow\pi/2$ and $t\rightarrow\infty$ $(\alpha>1)$, we have $V\rightarrow\infty,\ a_i\rightarrow\infty,\ i=1,2,3$. In this limit the anisotropy parameter becomes a non-zero constant and the Universe ends in a still anisotropic phase, but with a decrease in the value of the anisotropy parameter A, as compared with the initial one. Therefore during its evolution the Bianchi type I Universe cannot experience a transition from the anisotropic phase to the isotropic flat geometry. The time evolution of the gravitational coupling ϕ and of the supersymmetric field ψ is represented in Fig. 4. The ϕ field is positive for all values of time.

In the present paper we have investigated the evolution and dynamics of a Bianchi type I space-time in a SUGRA toy model, obtained by dimensional reduction of the $N=2,\ D=5$ SUGRA. The inclusion of the supersymmetric

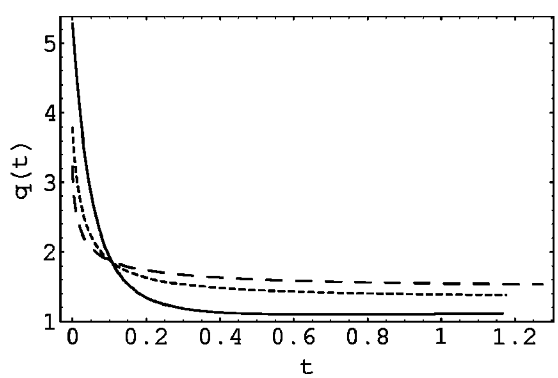

FIG. 3. Evolution of the deceleration parameter q of the Bianchi type I space-time for different values of α: $\alpha=3/2$ (solid curve), $\alpha=2$ (dotted curve), and $\alpha=5/2$ (dashed curve).

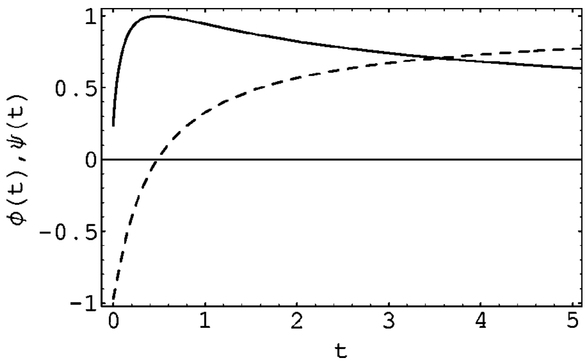

FIG. 4. Time evolution of the gravitational coupling ϕ (solid curve) and of the supersymmetric field ψ (dashed curve) for $\alpha=5/2$ $(\phi_0=1,\ \psi_0=0)$.

term gives some particular features to this cosmological model, by preventing the Universe from inflating and attaining completely isotropy. But globally there is a decrease in the degree of anisotropy of the geometry. Hence this model can be used to describe only a specific, well-determined period of the evolution of our Universe.

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APPENDIX A: SOME EXACT FORMS FOR PHYSICAL TIME

For a large class of values of the parameter α the general solution of the gravitational field equations can be expressed in a closed explicit form: The variation of the physical time t is determined by the integral equation (23). After a trick manipulation, one can rewrite this equation in the following form:

$$t=t_0+\frac{V_0}{\sqrt{2}\omega}\int \sin^{\gamma'-3}\theta\cos^{-\gamma'}\theta d\theta, \tag{A1}$$

where $\theta:=\zeta/2+\pi/4$ and $\gamma':=2\gamma=(3/2)(\alpha+1)$. In general, for arbitrary γ', this integral cannot be closed. Fortunately, for integer values of γ', the physical time t can be expressed in an explicit form as a function of ζ. Some of these exact forms of the time function are listed in the following. [The outcomes for $\alpha\pm 1$ are given in Eqs. (39) and (41).]

For $\alpha<-1$. (i) $\alpha=-\frac{5}{3}$, $(\gamma'=-1)$,

$$t=t_0-\frac{V_0}{3\sqrt{2}\omega}\frac{1}{\sin^3\theta},$$

(ii) $\alpha=-\frac{7}{3}$, $(\gamma'=-2)$,

$$t=t_0-\frac{V_0}{8\sqrt{2}\omega}\left[\frac{\cos\theta(\cos^2\theta+1)}{\sin^4\theta}+\ln(\csc\theta-\cot\theta)\right],$$

(iii) $\alpha = -3$, $(\gamma' = -3)$,

$$t = t_0 - \frac{V_0}{15\sqrt{2}\omega}\left(\frac{5\cos^2 - 2}{\sin^5\theta}\right).$$

For $\alpha > 1$. (i) $\alpha = \frac{5}{3}$, $(\gamma' = 4)$,

$$t = t_0 + \frac{V_0}{3\sqrt{2}\omega}\frac{1}{\cos^3\theta},$$

(ii) $\alpha = \frac{7}{3}$, $(\gamma' = 5)$,

$$t = t_0 + \frac{V_0}{8\sqrt{2}\omega}\left[\frac{\sin\theta(\sin^2\theta + 1)}{\cos^4\theta} - \ln(\tan\theta + \sec\theta)\right],$$

(iii) $\alpha = 3$, $(\gamma' = 6)$,

$$t = t_0 + \frac{V_0}{15\sqrt{2}\omega}\left(\frac{5\sin^2\theta - 2}{\cos^5\theta}\right).$$

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