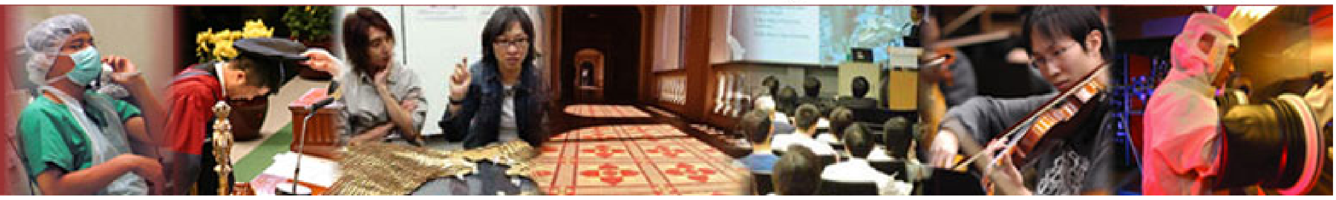

Title	Ginzburg-Landau equations for layered p-wave superconductors
Author(s)	Zhu, JX; Ting, CS; Shen, JL; Wang, ZD
Citation	Physical Review B - Condensed Matter And Materials Physics, 1997, v. 56 n. 21, p. 14093-14101
Issued Date	1997
URL	http://hdl.handle.net/10722/43214
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Ginzburg-Landau equations for layered *p*-wave superconductors

Jian-Xin Zhu and C. S. Ting
Texas Center for Superconductivity, University of Houston, Houston, Texas 77204

J. L. Shen* and Z. D. Wang
Department of Physics, University of Hong Kong, Pokfulam Road, Hong Kong, China

(Received 21 July 1997)

Based on Gor'kov's theory of weakly coupled superconductors, the Ginzburg-Landau equations for layered *p*-wave superconductors are derived, the order parameter of which is assumed to belong to a nontrivial two-dimensional representation. This calculation allows us to microscopically determine the expansion coefficients of the Ginzburg-Landau free-energy functional with respect to the order parameter. The main feature of the vortex solution is briefly discussed. It is found that the extreme condition for the nonaxisymmetric singly quantized vortices is not ensured in the weak-coupling limit. If the discrete crystal symmetry is included, the axisymmetric singly quantized vortex is stable. In addition, the upper critical field is also solely determined within the weak-coupling framework. [S0163-1829(97)01445-8]

I. INTRODUCTION

In an even-parity spin-singlet superconductor, the internal orbital angular momentum and spin of Cooper pairs are, respectively, $L=0,2,\ldots$ and $S=0$; while in an odd-parity spin-triplet superconductor, $L=1,3,\ldots$ and $S=1$. Conventional superconductors refer to those with the pairing symmetry of *s*-wave $(L=0)$ and spin singlet $(S=0)$. Recently, there has been much theoretical and experimental work indicating that the pairing state of high-T_c superconductors is of *d*-wave $(L=2)$ spin-singlet $(S=0)$ symmetry.[1] It is also widely accepted that an anisotropic *p*-wave spin-triplet pairing may be realized in heavy-fermion superconductors. More recently, Sr_2RuO_4 as an example of layered perovskite material was found to exhibit superconductivity with no copper involved.[2] Sr_2RuO_4 has a similar structure to a high-T_c cuprate superconductor. It shares with the cuprate a strong anisotropy in the resistivity ($\rho_c/\rho_{ab}>500$ at low temperatures) and hence provides us with another example of electron-correlated systems of reduced dimensionality. Nevertheless, the superconducting state may have different symmetry from that of cuprate superconductors. Strong correlations lead to the enhancements of mass and susceptibility, the corrections of which agree roughly with those of ^{3}He. Although precise identification of the pairing symmetry in the compound has not yet done, it has been raised by Rice and Sigrist[3] that strong Hund's rule coupling favors triplet over singlet pairing and a strong candidate is the odd-parity pairing state which is the two-dimensional (2D) analog of the Balian-Werthamer[4] (BW) state of ^{3}He. So far, the description of unconventional superconductors with odd-parity triplet pairing symmetry is limited to a phenomenological level,[5] although Scharnberg and Klemm[6] once microscopically studied the upper critical field by considering the solutions with the BW, polar, and Anderson-Brinkman-Morel[7,8] states to the linearized version of the one-component gap equation. In part motivated by the observation of the microscopic derivation of the Ginzberg-Landau (GL) equations for *d*-wave singlet pairing superconductors,[9] the present paper is devoted to the microscopic derivation of the coupled GL equations for the order parameter and supercurrent for a layered superconductor with *p*-wave triplet pairing, which is most suitable to the triplet pairing state of the nontrivial two-dimensional representation Γ_5^- for a square lattice.[3] This calculation allows us to microscopically establish the expansion coefficients of the free energy with respect to the order parameter up to the fourth order. In particular, the values of the nonlinear term coefficients are very important for the determination of the vortex solution. With these microscopically obtained parameters, the main features of the vortex solution to the GL equation could be discussed uniquely. In addition, the final determination of the upper critical field can also be made based on our microscopic theory.

This paper is organized as follows: in Sec. II, a general description of the gap equation for an inhomogeneous superconductor is presented. The explicit derivation of the GL equations for *p*-wave order parameter and supercurrent are given in Sec. III and Sec. IV by assuming that the order parameter belongs to the two-dimensional representation for a two-dimensional square lattice point group. The general vortex solution to the GL equation is discussed in Sec. V. The upper critical field is given in Sec. VI. Finally, a brief conclusion and discussion is given in Sec. VII.

II. GAP FUNCTION

The complete Hamiltonian of the system of electrons in second quantization has the form

$$\mathcal{H}=\int d\mathbf{x}\psi_\sigma^+(\mathbf{x})\left(\frac{(\mathbf{p}+e\mathbf{A}(\mathbf{x}))^2}{2m}-E_F\right)\psi_\sigma(\mathbf{x})+\frac{1}{2}\int\int d\mathbf{x}d\mathbf{x}'\,\psi_\sigma^+(\mathbf{x})\psi_{\sigma'}^+(\mathbf{x}')V(\mathbf{x}-\mathbf{x}')\psi_{\sigma'}(\mathbf{x}')\psi_\sigma(\mathbf{x}), \tag{2.1}$$

where the single-particle energy is measured relative to the the Fermi energy E_F, $\psi_\sigma^+(\mathbf{x})$ and $\psi_\sigma(\mathbf{x})$ are creation and annihilation operators of electrons with spin σ at position $\mathbf{x}$, the repeated indices mean the summation. The quantity $V(\mathbf{x}-\mathbf{x}')$ (<0) is electron-electron interaction which is attractive in a small range near the Fermi surface. Its physical original will not be considered here. Here and after, we choose $\hbar=k_B=c=1$. Throughout the calculation, we are limited to the region $\mu_B H/E_F \ll 1-T/T_c \ll 1$, where μ_B is the Bohr magneton, H is the magnetic field, T and T_c are the temperature and critical temperature. Under this condition, the Pauli paramagnetism effect could be neglected. Within Gor'kov's weak-coupling theory,[10] the equations of motion for the normal and anomalous Green's functions in the frequency space are written as

$$\left(i\omega_n - \frac{(\mathbf{p}+e\mathbf{A})^2}{2m} + E_F \right) \hat{\mathcal{G}}(\mathbf{x},\mathbf{x}';\omega_n) + \int d\mathbf{x}'' \hat{\Delta}(\mathbf{x},\mathbf{x}'') \hat{\mathcal{F}}^{+}(\mathbf{x}'',\mathbf{x}';\omega_n) = \delta(\mathbf{x}-\mathbf{x}'), \tag{2.2}$$

$$\left(-i\omega_n - \frac{(\mathbf{p}-e\mathbf{A})^2}{2m} + E_F \right) \hat{\mathcal{F}}^{+}(\mathbf{x},\mathbf{x}';\omega_n) - \int d\mathbf{x}'' \hat{\Delta}^{*}(\mathbf{x},\mathbf{x}'') \hat{\mathcal{G}}(\mathbf{x}'',\mathbf{x}';\omega_n) = 0, \tag{2.3}$$

where the gap function is defined as

$$\hat{\Delta}^{*}(\mathbf{x},\mathbf{x}') = -V(\mathbf{x},\mathbf{x}') T \sum_n \hat{\mathcal{F}}^{+}(\mathbf{x},\mathbf{x}';\omega_n), \tag{2.4}$$

with the Matsubara frequency $\omega_n=(2n+1)\pi T$.

It is useful to introduce the normal-state Green's function $\hat{\tilde{\mathcal{G}}}_0(\mathbf{x},\mathbf{x}';\omega_n)$ for electrons in the magnetic field $\mathbf{A}$. The equations of motion for $\hat{\tilde{\mathcal{G}}}_0(\mathbf{x},\mathbf{x}';\omega_n)$ can be written in two ways

$$\left(i\omega_n - \frac{(\mathbf{p}+e\mathbf{A})^2}{2m} + E_F \right) \hat{\tilde{\mathcal{G}}}_0(\mathbf{x},\mathbf{x}';\omega_n) = \delta(\mathbf{x}-\mathbf{x}'), \tag{2.5a}$$

$$\left(i\omega_n - \frac{(\mathbf{p}-e\mathbf{A})^2}{2m} + E_F \right) \hat{\tilde{\mathcal{G}}}_0(\mathbf{x}',\mathbf{x};\omega_n) = \delta(\mathbf{x}-\mathbf{x}'). \tag{2.5b}$$

With the aid of $\hat{\tilde{\mathcal{G}}}_0$, we can reduce the system of equations for $\hat{\mathcal{G}}$ and $\hat{\mathcal{F}}^{+}$ to the integral form

$$\hat{\mathcal{G}}(\mathbf{x},\mathbf{x}';\omega_n) = \hat{\tilde{\mathcal{G}}}_0(\mathbf{x},\mathbf{x}';\omega_n) - \int d\mathbf{x}_1 d\mathbf{x}_2 \hat{\tilde{\mathcal{G}}}_0(\mathbf{x},\mathbf{x}_1;\omega_n) \times \hat{\Delta}(\mathbf{x}_1,\mathbf{x}_2) \hat{\mathcal{F}}^{+}(\mathbf{x}_2,\mathbf{x}';\omega_n), \tag{2.6}$$

$$\hat{\mathcal{F}}^{+}(\mathbf{x},\mathbf{x}';\omega_n) = \int d\mathbf{x}_1 d\mathbf{x}_2 \hat{\tilde{\mathcal{G}}}_0(\mathbf{x}_1,\mathbf{x};-\omega_n) \hat{\Delta}^{*}(\mathbf{x}_1,\mathbf{x}_2) \times \hat{\mathcal{G}}(\mathbf{x}_2,\mathbf{x}';\omega_n). \tag{2.7}$$

In the absence of a magnetic field, $\hat{\mathcal{G}}_0(\mathbf{x},\mathbf{x}';\omega_n)$ is a function of the coordinate difference $\mathbf{x}-\mathbf{x}'$ and equals

$$\begin{aligned}\hat{\mathcal{G}}_0(\mathbf{x},\mathbf{x}';\omega_n) &= \left(i\omega_n + \frac{\nabla^2}{2m} + E_F \right)^{-1} \delta(\mathbf{x}-\mathbf{x}')\hat{1} \\ &= \left(i\omega_n + \frac{\nabla^2}{2m} + E_F \right)^{-1} \frac{1}{(2\pi)^d} \times \int d\mathbf{k} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}\hat{1} \\ &= \frac{1}{(2\pi)^d} \int d\mathbf{k} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')} \frac{1}{i\omega_n - \xi_{\mathbf{k}}} \hat{1},\end{aligned} \tag{2.8}$$

where d is the dimension of the system under consideration and $\xi_{\mathbf{k}}=\mathbf{k}^2/2m-E_F$. It is obvious that $\hat{\mathcal{G}}_0(\mathbf{x},\mathbf{x}';\omega_n)$ oscillates at the linear dimension k_F^{-1}, which is much smaller than the penetration depth. On the other hand, the vector potential $\mathbf{A}$ varies slowly at several wavelengths. Therefore, in the semiclassical phase integral approximation,[11] the normal Green's function in the magnetic field could be approximated as

$$\hat{\tilde{\mathcal{G}}}_0(\mathbf{x},\mathbf{x}';\omega_n) = \hat{\mathcal{G}}_0(\mathbf{x},\mathbf{x}';\omega_n) e^{-ie\int_{\mathbf{x}'}^{\mathbf{x}} d\mathbf{s}\cdot\mathbf{A}(\mathbf{s})}, \tag{2.9}$$

where the path integration between $\mathbf{x}'$ and $\mathbf{x}$ is a straight line. Near T_c, the absolute value of the gap is fairly small and we can perform the expansion of the Green's function with respect to the gap function. By expanding $\hat{\mathcal{F}}^{+}$ up to the third power in $|\Delta|$ while $\hat{\mathcal{G}}$ up to the second power, we find

$$\hat{\mathcal{G}}(\mathbf{x},\mathbf{x}';\omega_n) = \hat{\tilde{\mathcal{G}}}_0(\mathbf{x},\mathbf{x}';\omega_n) - \int d\mathbf{x}_1 d\mathbf{x}_2 \hat{\tilde{\mathcal{G}}}_0(\mathbf{x},\mathbf{x}_1;\omega_n) \hat{\Delta}(\mathbf{x}_1,\mathbf{x}_2) \int d\mathbf{x}_3 d\mathbf{x}_4 \hat{\tilde{\mathcal{G}}}_0(\mathbf{x}_3,\mathbf{x}_2;-\omega_n) \hat{\Delta}^{*}(\mathbf{x}_3,\mathbf{x}_4) \hat{\tilde{\mathcal{G}}}_0(\mathbf{x}_4,\mathbf{x}';\omega_n), \tag{2.10}$$

$$\begin{aligned}\hat{\mathcal{F}}^{+}(\mathbf{x},\mathbf{x}';\omega_n) = \int d\mathbf{x}_1 d\mathbf{x}_2 \hat{\tilde{\mathcal{G}}}_0(\mathbf{x}_1,\mathbf{x};-\omega_n) \hat{\Delta}^{*}(\mathbf{x}_1,\mathbf{x}_2) \Big[\hat{\tilde{\mathcal{G}}}_0(\mathbf{x}_2,\mathbf{x}';\omega_n) - \int d\mathbf{x}_3 d\mathbf{x}_4 d\mathbf{x}_5 d\mathbf{x}_6 \hat{\tilde{\mathcal{G}}}_0(\mathbf{x}_2,\mathbf{x}_3;\omega_n) \hat{\Delta}(\mathbf{x}_3,\mathbf{x}_4) \\ \times \hat{\tilde{\mathcal{G}}}_0(\mathbf{x}_5,\mathbf{x}_4;-\omega_n) \hat{\Delta}^{*}(\mathbf{x}_5,\mathbf{x}_6) \hat{\tilde{\mathcal{G}}}_0(\mathbf{x}_6,\mathbf{x}';\omega_n) \Big].\end{aligned} \tag{2.11}$$

Notice that the spin indices have been dropped out, regardless of the pairing symmetry being s, d, or p wave, as long as the $\hat{\Delta}$ matrix is unitary, i.e., the product $\hat{\Delta}\hat{\Delta}^{\dagger}$ is proportional to the unit matrix $\hat{1}$.

From Eqs. (2.4) and (2.11), the gap equation is obtained

$$\Delta^*(\mathbf{x},\mathbf{x}')=\Delta_I^*(\mathbf{x},\mathbf{x}')+\Delta_{II}^*(\mathbf{x},\mathbf{x}'), \tag{2.12}$$

where

$$\Delta_I^*(\mathbf{x},\mathbf{x}')=-V(\mathbf{x},\mathbf{x}')T\sum_n\int d\mathbf{x}_1 d\mathbf{x}_2\widetilde{\mathcal{G}}_0(\mathbf{x}_1,\mathbf{x};-\omega_n)\Delta^*(\mathbf{x}_1,\mathbf{x}_2)\widetilde{\mathcal{G}}_0(\mathbf{x}_2,\mathbf{x}';\omega_n), \tag{2.13}$$

$$\Delta_{II}^*(\mathbf{x},\mathbf{x}')=V(\mathbf{x},\mathbf{x}')T\sum_n\int d\mathbf{x}_1 d\mathbf{x}_2 d\mathbf{x}_3 d\mathbf{x}_4 d\mathbf{x}_5 d\mathbf{x}_6\widetilde{\mathcal{G}}_0(\mathbf{x}_1,\mathbf{x};-\omega_n)\Delta^*(\mathbf{x}_1,\mathbf{x}_2)\widetilde{\mathcal{G}}_0(\mathbf{x}_2,\mathbf{x}_3;\omega_n)\Delta(\mathbf{x}_3,\mathbf{x}_4)$$

$$\times\widetilde{\mathcal{G}}_0(\mathbf{x}_5,\mathbf{x}_4;-\omega_n)\Delta^*(\mathbf{x}_5,\mathbf{x}_6)\widetilde{\mathcal{G}}_0(\mathbf{x}_6,\mathbf{x}';\omega_n). \tag{2.14}$$

Because of the strongly two-dimensional electronic structure of Sr_2RuO_4, we consider the triplet superconductivity in two dimensions and use the simplification of a cylindrical Fermi surface, although quantum oscillations[12] and band-structure calculations[13] show that the Fermi surface consists of three approximately cylindrical pieces. In the center-of-mass coordinate system, Eq. (2.13) becomes

$$\Delta_I^*(\mathbf{R},\mathbf{r})=-V(\mathbf{r})T\sum_n\int d\mathbf{R}'d\mathbf{r}'\mathcal{G}_0(\mathbf{R}'+\mathbf{r}'/2-\mathbf{R}-\mathbf{r}/2;-\omega_n)\mathcal{G}_0(\mathbf{R}'-\mathbf{r}'/2-\mathbf{R}+\mathbf{r}/2;\omega_n)$$

$$\times e^{-i(\mathbf{R}-\mathbf{R}')\cdot\mathbf{\Pi}-i(\mathbf{r}-\mathbf{r}')\cdot(-i\nabla_{\mathbf{r}})}\Delta^*(\mathbf{R},\mathbf{r}), \tag{2.15}$$

where we have used the lemma[11] extended to the bilocal function

$$e^{-ie\left(\int_{\mathbf{x}}^{\mathbf{x}_1}+\int_{\mathbf{x}'}^{\mathbf{x}_2}\right)d\mathbf{s}\cdot\mathbf{A}(\mathbf{s})}\Delta^*(\mathbf{x}_1,\mathbf{x}_2)=e^{-i(\mathbf{x}_1-\mathbf{x})\cdot[i\nabla_{\mathbf{x}}+e\mathbf{A}(\mathbf{x})]-i(\mathbf{x}_2-\mathbf{x}')\cdot[i\nabla_{\mathbf{x}'}+e\mathbf{A}(\mathbf{x}')]}\Delta^*(\mathbf{x},\mathbf{x}'), \tag{2.16}$$

and assumed the slow variation of the magnetic field $\mathbf{A}(\mathbf{R}+\mathbf{r}/2)\approx\mathbf{A}(\mathbf{R}-\mathbf{r}/2)\approx\mathbf{A}(\mathbf{R})$, and $\mathbf{\Pi}=-i\nabla_{\mathbf{R}}-2e\mathbf{A}(\mathbf{R})$. Performing the Fourier transform with respect to the relative coordinate, we obtain

$$\Delta_I^*(\mathbf{R},\mathbf{k})=\int d\mathbf{r}e^{-i\mathbf{k}\cdot\mathbf{r}}\Delta^*(\mathbf{R},\mathbf{r})=-T\sum_n\int d\mathbf{r}e^{-i\mathbf{k}\cdot\mathbf{r}}V(\mathbf{r})\int d\mathbf{R}'d\mathbf{r}'\int\frac{d\mathbf{p}d\mathbf{q}d\mathbf{k}'}{(2\pi)^6}$$

$$\times e^{i\mathbf{p}\cdot(\mathbf{R}'+\mathbf{r}'/2-\mathbf{R}-\mathbf{r}/2)+i\mathbf{q}\cdot(\mathbf{R}'-\mathbf{r}'/2-\mathbf{R}+\mathbf{r}/2)}\frac{1}{-i\omega_n-\xi_{\mathbf{p}}}\frac{1}{i\omega_n-\xi_{\mathbf{q}}}e^{-i(\mathbf{R}-\mathbf{R}')\cdot\mathbf{\Pi}+i\mathbf{k}'\cdot\mathbf{r}'}\Delta^*(\mathbf{R},\mathbf{k}'). \tag{2.17}$$

Expanding in powers of $\mathbf{\Pi}$ to the second order, we can write the above equations in terms of a constant term Δ_{Ic}^* and a gradient term Δ_{Ig}^*

$$\Delta_I^*(\mathbf{R},\mathbf{k})=\Delta_{Ic}^*(\mathbf{R},\mathbf{k})+\Delta_{Ig}^*(\mathbf{R},\mathbf{k}), \tag{2.18}$$

where

$$\Delta_{Ic}^*(\mathbf{R},\mathbf{k})=-T\sum_n\int\frac{d\mathbf{k}'}{(2\pi)^2}V(\mathbf{k}-\mathbf{k}')\frac{1}{\omega_n^2+\xi_{\mathbf{k}'}^2}\Delta^*(\mathbf{R},\mathbf{k}'), \tag{2.19}$$

and

$$\Delta_{Ig}^*(\mathbf{R},\mathbf{k})=-T\sum_n\int\frac{d\mathbf{k}'}{2(2\pi)^2}V(\mathbf{k}-\mathbf{k}')\left\{\frac{1}{(2m)^2}\frac{2\xi_{\mathbf{k}'}^2-6\omega_n^2}{(\omega_n^2+\xi_{\mathbf{k}'}^2)^3}(k_x'\Pi_x+k_y'\Pi_y)^2-\frac{1}{2m}\frac{\xi_{\mathbf{k}'}\mathbf{\Pi}^2}{(\omega_n^2+\xi_{\mathbf{k}'}^2)^2}\right\}\Delta^*(\mathbf{R},\mathbf{k}'). \tag{2.20}$$

$\Delta^*_{II}(\mathbf{x},\mathbf{x}')$ given by Eq. (2.14) can be calculated by introducing the center-of-mass coordinates and relative coordinates, and neglecting effects of the magnetic field since $\Delta^*_{II}(\mathbf{x},\mathbf{x}')$ itself has already been of small values,

$$\Delta^*_{II}(\mathbf{R},\mathbf{r})=T\sum_n \int \frac{d\mathbf{k}'}{(2\pi)^2}e^{i\mathbf{k}'\cdot\mathbf{r}}V(\mathbf{r})|\Delta^*(\mathbf{R},\mathbf{k}')|^2 \times\Delta^*(\mathbf{R},\mathbf{k}')\frac{1}{(\omega_n^2+\xi_{\mathbf{k}'}^2)^2}. \tag{2.21}$$

Performing the Fourier transform with respect to the relative coordinate, we obtain

$$\Delta^*_{II}(\mathbf{R},\mathbf{k})=\int d\mathbf{r}e^{-i\mathbf{k}\cdot\mathbf{r}}\Delta^*_{II}(\mathbf{R},\mathbf{r}) =T\sum_n \int \frac{d\mathbf{k}'}{(2\pi)^2}V(\mathbf{k}-\mathbf{k}')|\Delta^*(\mathbf{R},\mathbf{k}')|^2 \times\Delta^*(\mathbf{R},\mathbf{k}')\frac{1}{(\omega_n^2+\xi_{\mathbf{k}'}^2)^2}. \tag{2.22}$$

III. GINZBURG-LANDAU EQUATIONS FOR ORDER PARAMETER

We use the weak-coupling approach and take the odd-parity attractive interaction[4]

$$V(\mathbf{k}-\mathbf{k}')=-V_p\hat{\mathbf{k}}\cdot\hat{\mathbf{k}}', \tag{3.1}$$

where we assume that the pair formation occurs near the Fermi surface and $\hat{k}_{x,y}=k_{x,y}/k_F$. For such an odd-parity interaction, the gap function can be expressed as[5]

$$\hat{\Delta}(\mathbf{k})=i[\mathbf{d}(\mathbf{k})\cdot\hat{\sigma}]\hat{\sigma}_y, \tag{3.2}$$

where $\hat{\sigma}$ denotes the Pauli matrices and $\mathbf{d}(\mathbf{k})$ is the vectorial function odd in $\mathbf{k}$. There are four 1D (Γ^-_{1-4}) and one 2D odd-parity representations of a 2D square lattice point group C_{4v}.[3] The pairing states belonging to the 1D representation (Γ^-_{1-4}) are so-called equal spin pairing states and these states do not break the time-reversal symmetry, as indicated by the orbital part $\mathbf{d}(\mathbf{k})$ of the order parameter. The pairing state belonging to the 2D representation (Γ^-_5) lies in the basal plane and is the analog to the Anderson-Brinkman-Morel state, however, this order parameter breaks the time-reversal symmetry and is twofold degenerate. In the absence of the spin-orbit coupling, the superconducting state could belong either to the 1D or 2D representations. Here we do not attempt to determine the symmetry of the pairing state in the above-mentioned layered superconductor. We assume instead that the order parameter belongs to the 2D representation. Due to the twofold degeneracy of the Γ^-_5 odd-parity state, we should expand the order parameter in terms of these two degenerate pure states. On the other hand, one can expect no induced s-wave component in the p-wave order parameter because in the absence of spin-orbit coupling, the singlet and triplet spinor wave function are orthogonal. This situation is different from the d-wave superconductor, in which an s-wave spin singlet component could be induced.[9,14] Consequently, the order parameter for the Γ^-_5 odd-parity state can be written in the form

$$\Delta^*(\mathbf{R},\mathbf{k})=\Delta^*_1(\mathbf{R})\hat{k}_+ +\Delta^*_2(\mathbf{R})\hat{k}_-, \tag{3.3}$$

where $\hat{k}_\pm=\hat{k}_x\pm i\hat{k}_y$.

Substituting Eqs. (3.1) and (3.3) into Eqs. (2.19), (2.20), and (2.22), we obtain

$$\Delta^*_{Ic}(\mathbf{R},\mathbf{k})=\lambda_p\ln\frac{2e^\gamma\omega_D}{\pi T}[\Delta^*_1\hat{k}_+ +\Delta^*_2\hat{k}_-], \tag{3.4}$$

where $\lambda=N(0)V_p/2$ with $N(0)$ is the 2D density of states at the Fermi surface for each spin direction, $\gamma=0.5772$ is the Euler constant, ω_D is the Debye frequency.

$$\Delta^*_{Ig}(\mathbf{R},\mathbf{k})=-\lambda_p\alpha v_F^2\{\hat{k}_+\mathbf{\Pi}^2/4+\hat{k}_-\Pi_+^2/8\}\Delta^*_1 -\lambda_p\alpha v_F^2\{\hat{k}_-\mathbf{\Pi}^2/4+\hat{k}_+\Pi_-^2/8\}\Delta^*_2, \tag{3.5}$$

where $\alpha=7\zeta(3)/8(\pi T)^2$, $v_F=k_F/m$, and $\Pi_\pm=\Pi_x\pm i\Pi_y$. Similarly, we have

$$\Delta^*_{II}=-\lambda_p\alpha\{(|\Delta_1|^2+2|\Delta_2|^2)\Delta^*_1\hat{k}_+ +(2|\Delta_1|^2 +|\Delta_2|^2)\Delta^*_2\hat{k}_-\}. \tag{3.6}$$

Comparing both sides of the gap equation for terms proportional to $\hat{k}_+$ and $\hat{k}_-$, we obtain the GL equation for the gap function

$$-\lambda_p\ln(T_c/T)\Delta^*_1+\lambda_p\alpha[v_F^2\mathbf{\Pi}^2\Delta^*_1/4+v_F^2\Pi_-^2\Delta^*_2/8+|\Delta_1|^2\Delta^*_1 +2|\Delta_2|^2\Delta^*_1]=0, \tag{3.7a}$$

$$-\lambda_p\ln(T_c/T)\Delta^*_2+\lambda_p\alpha[v_F^2\mathbf{\Pi}^2\Delta^*_2/4+v_F^2\Pi_+^2\Delta^*_1/8+|\Delta_2|^2\Delta^*_2 +2|\Delta_1|^2\Delta^*_2]=0. \tag{3.7b}$$

The transition temperature T_c is determined by

$$\lambda_d\ln\frac{2e^\gamma\omega_D}{\pi T_c}=1. \tag{3.8}$$

Note that for the interaction given in Eq. (3.1), these two degenerate pairing states have the identical transition temperature T_c. For the d-wave superconductor case, due to the repulsive s-channel interaction, a Padé approximation should be used to eliminate the unphysical results.[9] Obviously, there is only one attractive interaction in our case and thus it is unnecessary to do the Padé approximation.

IV. GINZBURG-LANDAU EQUATION FOR SUPERCURRENT

The current density can be written directly in terms of Green's function of the system

$$\mathbf{j}(\mathbf{x})=-\frac{eT}{mi}\sum_n(\nabla_\mathbf{x}-\nabla_{\mathbf{x}'})\mathcal{G}(\mathbf{x},\mathbf{x}';\omega_n)|_{\mathbf{x}'\to\mathbf{x}} -\frac{2e^2T}{m}\mathbf{A}(\mathbf{x})\sum_n\mathcal{G}(\mathbf{x},\mathbf{x};\omega_n). \tag{4.1}$$

Substituting Eq. (2.10) into Eq. (4.1), we obtain

$$\mathbf{j}(\mathbf{x}) = -\frac{eT}{mi}\sum_n (\nabla_{\mathbf{x}} - \nabla_{\mathbf{x}'})\tilde{\mathcal{G}}_0(\mathbf{x},\mathbf{x}';\omega_n)\Big|_{\mathbf{x}'\to\mathbf{x}} - \frac{2e^2T}{m}\mathbf{A}(\mathbf{x})\sum_n \tilde{\mathcal{G}}_0(\mathbf{x},\mathbf{x};\omega_n) + \frac{eT}{mi}\sum_n \int d\mathbf{x}_1 d\mathbf{x}_2 d\mathbf{x}_3 d\mathbf{x}_4 (\nabla_{\mathbf{x}} - \nabla_{\mathbf{x}'})$$

$$\times \tilde{\mathcal{G}}_0(\mathbf{x},\mathbf{x}_1;\omega_n)\Delta(\mathbf{x}_1,\mathbf{x}_2)\tilde{\mathcal{G}}_0(\mathbf{x}_3,\mathbf{x}_2;-\omega_n)\Delta^*(\mathbf{x}_3,\mathbf{x}_4)\tilde{\mathcal{G}}_0(\mathbf{x}_4,\mathbf{x}';\omega_n)\Big|_{\mathbf{x}'\to\mathbf{x}}$$

$$+\frac{2e^2T}{m}\mathbf{A}(\mathbf{x})\sum_n \int d\mathbf{x}_1 d\mathbf{x}_2 d\mathbf{x}_3 d\mathbf{x}_4 \tilde{\mathcal{G}}_0(\mathbf{x},\mathbf{x}_1;\omega_n)\Delta(\mathbf{x}_1,\mathbf{x}_2)\tilde{\mathcal{G}}_0(\mathbf{x}_3,\mathbf{x}_2;-\omega_n)\Delta^*(\mathbf{x}_3,\mathbf{x}_4)\tilde{\mathcal{G}}_0(\mathbf{x}_4,\mathbf{x};\omega_n). \quad (4.2)$$

After the cancellation of some terms in Eq. (4.2) and introducing the center-of-mass and relative coordinates, we find

$$\mathbf{j}(\mathbf{R}) = \frac{eT}{mi}\sum_n \int d\mathbf{x}_1 d\mathbf{x}_2 d\mathbf{x}_3 d\mathbf{x}_4 e^{-ie(\int_{\mathbf{x}_1}^{\mathbf{x}} + \int_{\mathbf{x}_2}^{\mathbf{x}_3} + \int_{\mathbf{x}'}^{\mathbf{x}_4})d\mathbf{s}\cdot\mathbf{A}(\mathbf{s})}\Delta(\mathbf{x}_1,\mathbf{x}_2)\Delta^*(\mathbf{x}_3,\mathbf{x}_4)\mathcal{G}_0(\mathbf{x}_3,\mathbf{x}_2;-\omega_n)(\nabla_{\mathbf{x}} - \nabla_{\mathbf{x}'})$$

$$\times[\mathcal{G}_0(\mathbf{x},\mathbf{x}_1;\omega_n)\mathcal{G}_0(\mathbf{x}_4,\mathbf{x}';\omega_n)]\Big|_{\mathbf{x}'\to\mathbf{x}}$$

$$\approx \frac{eT}{mi}\sum_n \int d\mathbf{R}' d\mathbf{r}' d\mathbf{R}'' d\mathbf{r}''[e^{-i(\mathbf{R}'-\mathbf{R})\cdot\mathbf{\Pi}^* + (\mathbf{r}'-\mathbf{r})\cdot\nabla_{\mathbf{r}}}\Delta(\mathbf{R},\mathbf{r})][e^{i(\mathbf{R}''-\mathbf{R})\cdot\mathbf{\Pi} + (\mathbf{r}''-\mathbf{r})\cdot\nabla_{\mathbf{r}}}\Delta^*(\mathbf{R},\mathbf{r})]$$

$$\times \mathcal{G}_0(\mathbf{R}''+\mathbf{r}''/2-\mathbf{R}'+\mathbf{r}'/2;-\omega_n)\nabla_{\mathbf{r}}[\mathcal{G}_0(\mathbf{R}+\mathbf{r}/2-\mathbf{R}'-\mathbf{r}'/2;\omega_n)\mathcal{G}_0(\mathbf{R}''-\mathbf{r}''/2-\mathbf{R}+\mathbf{r}/2;\omega_n)]\Big|_{\mathbf{r}\to 0}$$

$$= \frac{eT}{2m}\sum_n \int d\mathbf{R}' d\mathbf{r}' d\mathbf{R}'' d\mathbf{r}'' \int \frac{d\mathbf{p} d\mathbf{q} d\mathbf{s} d\mathbf{k} d\mathbf{k}'}{(2\pi)^{10}}[e^{-i(\mathbf{R}'-\mathbf{R})\cdot\mathbf{\Pi}^* + (\mathbf{r}'-\mathbf{r})\cdot\nabla_{\mathbf{r}}}e^{i\mathbf{k}\cdot\mathbf{r}}\Delta(\mathbf{R},\mathbf{k})]$$

$$\times[e^{i(\mathbf{R}''-\mathbf{R})\cdot\mathbf{\Pi} + (\mathbf{r}''-\mathbf{r})\cdot\nabla_{\mathbf{r}}}e^{i\mathbf{k}'\cdot\mathbf{r}}\Delta^*(\mathbf{R},\mathbf{k}')]e^{i\mathbf{q}\cdot(\mathbf{R}''-\mathbf{R}'+\mathbf{r}''/2+\mathbf{r}'/2)}(\mathbf{p}+\mathbf{s})e^{i\mathbf{p}\cdot(\mathbf{R}-\mathbf{R}'-\mathbf{r}'/2)}e^{i\mathbf{s}\cdot(\mathbf{R}''-\mathbf{R}-\mathbf{r}''/2)}$$

$$\times \frac{1}{i\omega_n - \xi_{\mathbf{p}}}\,\frac{1}{-i\omega_n - \xi_{\mathbf{q}}}\,\frac{1}{i\omega_n - \xi_{\mathbf{s}}}. \quad (4.3)$$

Expanding $|\mathbf{\Pi}|$ to the first order gives rise to

$$\mathbf{j}(\mathbf{R}) = \mathbf{j}_1(\mathbf{R}) + \mathbf{j}_2(\mathbf{R}) + \mathbf{j}_2^*(\mathbf{R}), \quad (4.4)$$

where

$$\mathbf{j}_1(\mathbf{R}) = \frac{eT}{2m}\sum_n \int d\mathbf{R}' d\mathbf{r}' d\mathbf{R}'' d\mathbf{r}'' \int \frac{d\mathbf{p} d\mathbf{q} d\mathbf{s} d\mathbf{k} d\mathbf{k}'}{(2\pi)^{10}} e^{i\mathbf{k}\cdot\mathbf{r}'}\Delta(\mathbf{R},\mathbf{k})e^{i\mathbf{k}'\cdot\mathbf{r}''}\Delta^*(\mathbf{R},\mathbf{k}')e^{i(\mathbf{p}-\mathbf{s})\cdot\mathbf{R}}$$

$$\times e^{-i(\mathbf{p}+\mathbf{q})\cdot\mathbf{R}'}e^{i(\mathbf{q}+\mathbf{s})\cdot\mathbf{R}''}e^{i(-\mathbf{p}+\mathbf{q})\cdot\mathbf{r}'/2}e^{i(\mathbf{q}-\mathbf{s})\cdot\mathbf{r}''/2}(\mathbf{p}+\mathbf{s})\frac{1}{i\omega_n - \xi_{\mathbf{p}}}\,\frac{1}{-i\omega_n - \xi_{\mathbf{q}}}\,\frac{1}{i\omega_n - \xi_{\mathbf{s}}}$$

$$= \frac{eT}{m}\sum_n \int \frac{d\mathbf{p}}{(2\pi)^2}\mathbf{p}|\Delta(\mathbf{R},\mathbf{p})|^2 \frac{1}{\omega_n^2 + \xi_{\mathbf{p}}^2}\,\frac{1}{i\omega_n - \xi_{\mathbf{p}}} = 0, \quad (4.5)$$

$$\mathbf{j}_2(\mathbf{R})=\frac{eT}{2m}\sum_n \int d\mathbf{R}'d\mathbf{r}'d\mathbf{R}''d\mathbf{r}''\int \frac{d\mathbf{p}d\mathbf{q}d\mathbf{s}d\mathbf{k}d\mathbf{k}'}{(2\pi)^{10}}e^{i\mathbf{k}\cdot\mathbf{r}'}[-i(\mathbf{R}'-\mathbf{R})\cdot\mathbf{\Pi}^*\Delta(\mathbf{R},\mathbf{k})]e^{i\mathbf{k}'\cdot\mathbf{r}''}\Delta^*(\mathbf{R},\mathbf{k}')e^{i(\mathbf{p}-\mathbf{s})\cdot\mathbf{R}}e^{-i(\mathbf{p}+\mathbf{q})\cdot\mathbf{R}'}$$
$$\times e^{i(\mathbf{q}+\mathbf{s})\cdot\mathbf{R}''}e^{i(-\mathbf{p}+\mathbf{q})\cdot\mathbf{r}'/2}e^{i(\mathbf{q}-\mathbf{s})\cdot\mathbf{r}''/2}(\mathbf{p}+\mathbf{s})\frac{1}{i\omega_n-\xi_{\mathbf{p}}}\frac{1}{-i\omega_n-\xi_{\mathbf{q}}}\frac{1}{i\omega_n-\xi_{\mathbf{s}}}$$
$$=\frac{2eT}{m}\sum_n \int d\mathbf{R}'\int \frac{d\mathbf{k}d\mathbf{k}'}{(2\pi)^4}(2\mathbf{k})$$
$$\times[-i(\mathbf{R}'-\mathbf{R})\cdot\mathbf{\Pi}^*\Delta(\mathbf{R},\mathbf{k})]\Delta^*(\mathbf{R},\mathbf{k}')e^{-2i(\mathbf{k}-\mathbf{k}')\cdot(\mathbf{R}'-\mathbf{R})}\frac{1}{i\omega_n-\xi_{2\mathbf{k}-\mathbf{k}'}}\frac{1}{-i\omega_n-\xi_{-\mathbf{k}'}}\frac{1}{i\omega_n-\xi_{\mathbf{k}'}}$$
$$=-\frac{2eT}{mi}\sum_n \int d\mathbf{R}'\int \frac{d\mathbf{k}d\mathbf{k}'}{(2\pi)^4}\mathbf{k}[-i\mathbf{\Pi}^*\Delta(\mathbf{R},\mathbf{k})]\cdot[\nabla_{\mathbf{k}}e^{-2i(\mathbf{k}-\mathbf{k}')\cdot(\mathbf{R}'-\mathbf{R})}]\Delta^*(\mathbf{R},\mathbf{k}')\frac{1}{i\omega_n-\xi_{2\mathbf{k}-\mathbf{k}'}}\frac{1}{-i\omega_n-\xi_{-\mathbf{k}'}}\frac{1}{i\omega_n-\xi_{\mathbf{k}'}}$$
$$=\mathbf{e}_x\left\{-\frac{eT}{2m}\sum_n \int \frac{d\mathbf{k}}{(2\pi)^2}\Delta^*(\mathbf{R},\mathbf{k})\frac{1}{\omega_n^2+\xi_{\mathbf{k}}^2}\left[\frac{\Pi_x^*\Delta(\mathbf{R},\mathbf{k})+k_x\nabla_{\mathbf{k}}\cdot\mathbf{\Pi}^*\Delta(\mathbf{R},\mathbf{k})}{i\omega_n-\xi_{\mathbf{k}}}+\frac{2k_x\mathbf{k}\cdot\mathbf{\Pi}^*\Delta(\mathbf{R},\mathbf{k})}{m(i\omega_n-\xi_{\mathbf{k}})^2}\right]\right\}$$
$$+\mathbf{e}_y\left\{-\frac{eT}{2m}\sum_n \int \frac{d\mathbf{k}}{(2\pi)^2}\Delta^*(\mathbf{R},\mathbf{k})\frac{1}{\omega_n^2+\xi_{\mathbf{k}}^2}\left[\frac{\Pi_y^*\Delta(\mathbf{R},\mathbf{k})+k_y\nabla_{\mathbf{k}}\cdot\mathbf{\Pi}^*\Delta(\mathbf{R},\mathbf{k})}{i\omega_n-\xi_{\mathbf{k}}}+\frac{2k_y\mathbf{k}\cdot\mathbf{\Pi}^*\Delta(\mathbf{R},\mathbf{k})}{m(i\omega_n-\xi_{\mathbf{k}})^2}\right]\right\}$$
$$=-\frac{eT}{2m}\sum_n \int \frac{d\mathbf{k}}{(2\pi)^2}\Delta^*(\mathbf{R},\mathbf{k})\frac{1}{\omega_n^2+\xi_{\mathbf{k}}^2}\left[\frac{\mathbf{\Pi}^*\Delta(\mathbf{R},\mathbf{k})+\mathbf{k}\nabla_{\mathbf{k}}\cdot\mathbf{\Pi}^*\Delta(\mathbf{R},\mathbf{k})}{i\omega_n-\xi_{\mathbf{k}}}+\frac{2\mathbf{k}\mathbf{k}\cdot\mathbf{\Pi}^*\Delta(\mathbf{R},\mathbf{k})}{m(i\omega_n-\xi_{\mathbf{k}})^2}\right]. \tag{4.6}$$

Substituting Eq. (3.3) into Eq. (4.6), we obtain

$$\mathbf{j}_2(\mathbf{R})=\mathbf{j}_{2a}(\mathbf{R})+\mathbf{j}_{2b}(\mathbf{R}), \tag{4.7}$$

where

$$\mathbf{j}_{2a}(\mathbf{R})=-\frac{eT}{2m}\sum_n \int \frac{d\mathbf{k}}{(2\pi)^2}\frac{(\Delta_1^*\hat{k}_+ +\Delta_2^*\hat{k}_-)(\mathbf{\Pi}^*+\mathbf{k}\Delta_{\mathbf{k}}\cdot\mathbf{\Pi}^*)(\Delta_1\hat{k}_+ +\Delta_-\hat{k}_-)}{(\omega_n^2+\xi_{\mathbf{k}}^2)(i\omega_n-\xi_{\mathbf{k}})}=0, \tag{4.8}$$

because of

$$\sum_n \int_{-\infty}^{\infty}d\xi\frac{1}{(\omega_n^2+\xi^2)(i\omega_n-\xi)}=0,$$

and

$$\mathbf{j}_{2b}(\mathbf{R})=-\frac{eT}{m^2}\sum_n \int \frac{d\mathbf{k}}{(2\pi)^2}\frac{(\Delta_1^*\hat{k}_+ +\Delta_2^*\hat{k}_-)(\mathbf{\Pi}^*+\mathbf{k}\Delta_{\mathbf{k}}\cdot\mathbf{\Pi}^*)(\Delta_1\hat{k}_+ +\Delta_-\hat{k}_-)}{(\omega_n^2+\xi_{\mathbf{k}}^2)(i\omega_n-\xi_{\mathbf{k}})}=\frac{eE_FN(0)\alpha}{2m}\left\{\Delta_1^*\mathbf{\Pi}^*\Delta_1+\Delta_2^*\mathbf{\Pi}^*\Delta_2\right.$$
$$\left.+\frac{1}{2}(\Delta_1^*\Pi_-^*\Delta_2+\Delta_2^*\Pi_+^*\Delta_1)\mathbf{e}_x+\frac{i}{2}(\Delta_1^*\Pi_-^*\Delta_2-\Delta_2^*\Pi_+^*\Delta_1)\mathbf{e}_y\right\}. \tag{4.9}$$

Finally, we have the supercurrent

$$\mathbf{j}(\mathbf{R})=\frac{eE_FN(0)\alpha}{2m}\left\{\Delta_1^*\Pi^*\Delta_1+\Delta_2^*\mathbf{\Pi}^*\Delta_2+\frac{1}{2}(\Delta_1^*\Pi_-^*\Delta_2+\Delta_2^*\Pi_+^*\Delta_1)\mathbf{e}_x+\frac{i}{2}(\Delta_1^*\Pi_-^*\Delta_2-\Delta_2^*\Pi_+^*\Delta_1)\mathbf{e}_y\right\}+\text{ c.c.} \tag{4.10}$$

Equations (3.7) and (4.10) consititute a complete system of the GL equations.

V. VORTEX SOLUTION TO THE GL EQUATION

From the obtained GL equation, the free-energy density can be constructed

$$f=-\frac{N(0)}{2}\ln(T_c/T)(|\Delta_1|^2+|\Delta_2|^2)+\frac{N(0)}{4}\alpha[(|\Delta_1|^2+|\Delta_2|^2)^2+2|\Delta_1|^2|\Delta_2|^2]+\frac{N(0)}{8}\alpha v_F^2\bigg[|\mathbf{\Pi}\Delta_1^*|^2+|\mathbf{\Pi}\Delta_2^*|^2$$
$$+\frac{1}{2}\Pi_-\Delta_2^*\Pi_+^*\Delta_1+\frac{1}{2}\Pi_+\Delta_1^*\Pi_-^*\Delta_2\bigg]+\frac{\mathbf{h}^2}{8\pi}, \tag{5.1}$$

where $\mathbf{h}$ is the magnetic field. The final coefficients of the free-energy density are found by comparing the supercurrent from its functional derivative with respect to the vector potential with that directly obtained from the Green's function. Therefore, an immediate consequence of the microscopic derivation of the GL equations is that the expansion coefficients of the GL free-energy functional can now be determined microscopically, although the strong-coupling effects may change them. Replacing the order parameter by $\Delta_{1,2}^*=(\eta_1\mp i\eta_2)/\sqrt{2}$, we can rewrite the free-energy functional as

$$f=A(T)(|\eta_1|^2+|\eta_2|^2)+\beta_1(|\eta_1|^2+|\eta_2|^2)^2+\beta_2(\eta_1^*\eta_2-\eta_1\eta_2^*)^2+K_1(|\Pi_x\eta_1|^2+|\Pi_y\eta_2|^2)+K_2(|\Pi_x\eta_2|^2+|\Pi_y\eta_1|^2)$$
$$+K_3(\Pi_x^*\eta_1^*\Pi_y\eta_2+\text{ c.c.})+K_4(\Pi_x^*\eta_2^*\Pi_y\eta_1+\text{ c.c.})+\frac{\mathbf{h}^2}{8\pi}, \tag{5.2}$$

where $A(T)=-\mathcal{N}(0)\ln(T_c/T)/2$, $\beta_1=3\mathcal{N}(0)\alpha/8$, $\beta_2=\mathcal{N}(0)\alpha/8$, $K_1=3\mathcal{N}(0)\alpha v_F^2/16$, $K_2=K_3=K_4=\mathcal{N}(0)\alpha v_F^2/16$. This expression of the GL free-energy density agrees quite well with that constructed from the group-theoretical argument[5] for the Γ_5^- superconducting state in the tetragonal D_{4h} (except $\beta_3=0$) and hexagonal symmetry D_{6h}.

We now study the main feature of the solution to the GL equation. By scaling the order parameter and spatial coordinates in units of $\Delta_0=\sqrt{4/3\alpha}$ and $\xi_0=\sqrt{\alpha}v_F/2$, respectively, the GL equation can be written in a dimensionless form

$$-\ln(T_c/T)\Delta_1^*+\mathbf{\Pi}^2\Delta_1^*+\tfrac{1}{2}\Pi_-^2\Delta_2^*+\tfrac{4}{3}|\Delta_1|^2\Delta_1^*+\tfrac{8}{3}|\Delta_2|^2\Delta_1^*=0, \tag{5.3a}$$

$$-\ln(T_c/T)\Delta_2^*+\mathbf{\Pi}^2\Delta_2^*+\tfrac{1}{2}\Pi_+^2\Delta_1^*+\tfrac{4}{3}|\Delta_2|^2\Delta_2^*+\tfrac{8}{3}|\Delta_1|^2\Delta_2^*=0. \tag{5.3b}$$

In the absence of the magnetic field, the order parameter is found to be $\Delta_1^*=\Delta_2^*=g_0=\sqrt{3\ln(T_c/T)}/2$, which shows clearly that the state Δ_1^* and its time-reversed partner Δ_2^* are degenerate. In the presence of the magnetic field, the time-reversal symmetry is broken and since each component has a different response to the given magnetic field, one component dominates other. Note that, Tokuyasu, Hess, and Sauls[15] have performed a numerical calculation for the two-component time-reversal-breaking superconductor (Γ_5^+). They obtained two classes of vortex solutions depending on the scaled GL parameters $\beta^{\text{THS}}=\beta_2^{\text{THS}}/\beta_1^{\text{THS}}$ and $\tilde{\kappa}=(\kappa_2+\kappa_3)/2\kappa_1$. For large values of β^{THS} and low values of the stiffness ratio $\tilde{\kappa}$, the axial vortices are energetically stable. For small values of β^{THS}, the vortex solutions with nonaxial cores are energetically favorable. The free-energy functional used in Ref. 15 has a different form

$$f=A(T)(|\eta_1|^2+|\eta_2|^2)+\beta_1^{\text{THS}}(|\eta_1|^2+|\eta_2|^2)^2+\beta_2^{\text{THS}}|\eta_1^2-\eta_2^2|^2+\kappa_1(\Pi_i\eta_j)(\Pi_i\eta_j)^*$$
$$+\kappa_2(\Pi_i\eta_i)(\Pi_j\eta_j)^*+\kappa_3(\Pi_i\eta_j)(\Pi_j\eta_i)^*+\frac{\mathbf{h}^2}{8\pi}, \tag{5.4}$$

where the repeated indices ($i,j=1,2$) mean the summation. Comparing Eq. (5.4) with Eq. (5.2), one can find: $\beta_1=\beta_1^{\text{THS}}+\beta_2^{\text{THS}}$, $\beta_2=\beta_2^{\text{THS}}$, $K_1=\kappa_1+\kappa_2+\kappa_3$, $K_2=\kappa_1$, $K_3=\kappa_2$, $K_4=\kappa_3$. Therefore, our microscopic determination of the GL parameters in the weak-coupling limit gives $\beta^{\text{THS}}=1/2$ and $\tilde{\kappa}=1$. By referring to the vortex phase diagram given in Ref. 15, we are able to conclude that in the weak-coupling limit, the vortex solution to the GL equation may belong to the axial symmetry type.

In the cylindrical coordinates, $\mathbf{R}=(r,\theta)$, and by assuming the vector potential to be along the azimuthal direction $\mathbf{A}(r,\theta)=A(r)\hat{\theta}$, the differential operators can be written as

$$\mathbf{\Pi}^2=-\left[\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\left(\frac{1}{r}\frac{\partial}{\partial\theta}-i2e\xi_0A(r\xi_0)\right)^2\right],$$

$$\Pi_\pm^2=-\left[e^{\pm i\theta}\left(\frac{\partial}{\partial r}\pm\frac{i}{r}\frac{\partial}{\partial\theta}\pm 2e\xi_0A(r\xi_0)\right)\right]^2.$$

By a close inspection, we find that the general solution is of the form

$$\Delta_1^*=g_1(r)e^{i(n-1)\theta},\quad \Delta_2^*=g_2(r)e^{i(n+1)\theta}. \tag{5.5}$$

Note that this type of solution is valid only for the existence of the complete rotation symmetry. For a singly quantized vortex, $\Delta_1^*=g_1(r)e^{-i\theta}$ and $\Delta_2^*=g_2(r)e^{i\theta}$; while for a doubly quantized vortex, $\Delta_1^*=g_1(r)e^{-2i\theta}$ and $\Delta_2^*=g_2(r)$ or $\Delta_1^*=g_1(r)$ and $\Delta_2^*=g_2(r)e^{2i\theta}$ depending on whether the external magnetic field $\mathbf{h}_e$ is parallel or antiparallel to the $\hat{\mathbf{z}}$ axis. As for the vector potential $\mathbf{A}$, far away from the vortex, it becomes $A_\infty=\pm\Phi_0/2\pi r\xi_0$ for a singly quantized vortex, while $A_\infty=\pm 2\Phi_0/2\pi r\xi_0$ for a doubly quantized vortex. Far away from the center of the vortex, the boundary condition for the order parameter of both the singly quantized vortex and the doubly quantized vortex, is $g_1(\infty)=g_0$ and $g_2(\infty)=0$ when $\mathbf{h}_e \| -\hat{\mathbf{z}}$ or $g_1(\infty)=0$ and $g_2(\infty)=g_0$ when $\mathbf{h}_e \| \hat{\mathbf{z}}$. Near the center of the singly quantized vortex, the GL equation becomes

$$-\ln(T_c/T)g_1-\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-\frac{1}{r^2}\right)\left(g_1+\frac{1}{2}g_2\right)+\frac{4}{3}g_1^3 +\frac{8}{3}g_2^2 g_1=0, \tag{5.6a}$$

$$-\ln(T_c/T)g_2-\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-\frac{1}{r^2}\right)\left(g_2+\frac{1}{2}g_1\right)+\frac{4}{3}g_2^3 +\frac{8}{3}g_1^2 g_2=0. \tag{5.6b}$$

Therefore, near the center of the vortex, $g_1(r)=c_1 r$ and $g_2(r)=c_2 r$, where the constants c_1 and c_2 are determined by the normalization conditions. This means that the order parameter should be zero at the vortex axis for the singly quantized vortex. For the doubly quantized vortex, the situation is different. If $\mathbf{h}_e \| -\hat{\mathbf{z}}$, the GL equation near the vortex center is

$$-\ln(T_c/T)g_1-\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-\frac{4}{r^2}\right)g_1-\frac{1}{2}\left(\frac{\partial^2}{\partial r^2}-\frac{1}{r}\frac{\partial}{\partial r}\right)g_2 +\frac{4}{3}g_1^3+\frac{8}{3}g_2^2 g_1=0, \tag{5.7a}$$

$$-\ln(T_c/T)g_2-\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}\right)g_2-\frac{1}{2}\left(\frac{\partial^2}{\partial r^2}+\frac{3}{r}\frac{\partial}{\partial r}\right)g_1+\frac{4}{3}g_2^3 +\frac{8}{3}g_1^2 g_2=0. \tag{5.7b}$$

In this case, we find that $g_1=c_1 r^2$ and $g_2=c_2$. Similarly, if $\mathbf{h}_e \| \hat{\mathbf{z}}$, we have $g_1=c_1$ and $g_2=c_2 r^2$. Therefore, the order parameter does not vanish at the doubly quantized vortex axis. This type of vortex is strongly related to the complete rotational symmetry of the system. If the rotational symmetry of the system is reduced by the presence of a crystal field, this type of vortex is no longer stable, and we have only the singular singly quantized vortex present.

Accordingly, the supercurrent circulating around an axisymmetric singly quantized vortex becomes

$$\mathbf{j}=j_0\left[\frac{1}{r}(g_2^2-g_1^2)+\left(g_2\frac{dg_1}{dr}-g_1\frac{dg_2}{dr}\right)-2e\widetilde{A}(g_2-g_1)^2\right]\hat{\theta}, \tag{5.8}$$

where $j_0=2eE_F\mathcal{N}(0)/3m\xi_0$ with $\xi_0=\sqrt{\alpha}v_F/2$, and $\widetilde{A}=A\xi_0$. This result shows that the supercurrent behavior of a p-wave superconductor in the weak-coupling limit is independent of the azimuthal angle and is similar to that of conventional or one-component superconductors.

VI. THE UPPER CRITICAL FIELD

We also wish to discuss the upper critical field H_{c2} for a p-wave superconductor. Consider a magnetic field antiparallel to the $\mathbf{z}$ axis, $\mathbf{H}=(0,0,-H)$, we can choose the vector potential to be $\mathbf{A}=(0,-Hx,0)$. By defining $\widetilde{\Pi}_\pm=\Pi_\pm/2\sqrt{eH}$, we have the commutation relation, $[\widetilde{\Pi}_-,\widetilde{\Pi}_+]=1$. Throwing away the nonlinear terms in Eq. (3.7), we obtain the linearized version of the GL equation for the order parameter

$$\widetilde{K}_1(\widetilde{\Pi}_+\widetilde{\Pi}_-+\widetilde{\Pi}_-\widetilde{\Pi}_+)\psi_1^*+\widetilde{K}_2\Pi_-^2\psi_2^*=\frac{1}{eH}\ln(T_c/T)\psi_1^*, \tag{6.1a}$$

$$\widetilde{K}_1(\widetilde{\Pi}_+\widetilde{\Pi}_-+\widetilde{\Pi}_-\widetilde{\Pi}_+)\psi_2^*+\widetilde{K}_2\Pi_+^2\psi_1^*=\frac{1}{eH}\ln(T_c/T)\psi_2^*, \tag{6.1b}$$

where $\widetilde{K}_1=2\widetilde{K}_2=\alpha v_F^2\Delta_0$, $\psi_{1,2}^*=\Delta_{1,2}^*/\Delta_0$. In view of the commutation relation, we can regard $\widetilde{\Pi}_\pm$ as the creation and annihilation operators in the occupation number space,

$$\widetilde{\Pi}_+|n\rangle=\sqrt{n+1}|n+1\rangle \quad , \qquad \widetilde{\Pi}_-|n\rangle=\sqrt{n}|n-1\rangle. \tag{6.2}$$

Therefore, we expand the wave function in terms of the occupation state, $\psi_{1,2}^*=\Sigma_0^\infty a_n^{(1,2)}|n\rangle$. Substituting them into Eq. (6.1), we obtain a set of linear equations

$$\widetilde{K}_1(n+1)a_n^{(1)}+\widetilde{K}_2\sqrt{(n+2)(n+1)}a_{n+2}^{(2)}=\frac{1}{eH}\ln(T_c/T)a_n^{(1)}, \tag{6.3a}$$

$$\widetilde{K}_1[2(n+2)+1]a_{n+2}^{(2)}+\widetilde{K}_2\sqrt{(n+2)(n+1)}a_n^{(1)} =\frac{1}{eH}\ln(T_c/T)a_{n+2}^{(2)}, \tag{6.3b}$$

whose smallest eigenvalue gives the upper critical field. There are two possible ground states $(\psi_1^*=0,\psi_2^*=a_0^{(2)}|0\rangle)$ and $(\psi_1^*=a_0^{(1)}|0\rangle,\psi_2^*=a_2^{(2)}|2\rangle)$. The corresponding eigenvalues are, respectively,

$$H_{c2}^{\mathrm{I}}=\frac{\ln(T_c/T)}{e\widetilde{K}_1} \quad , \qquad H_{c2}^{\mathrm{II}}=\frac{\sqrt{2}}{3(\sqrt{2}-1)}\,\frac{\ln(T_c/T)}{e\widetilde{K}_1}. \tag{6.4}$$

Clearly, in the weak-coupling limit, the latter state should be the relevant upper critical magnetic field, which also means that when the magnetic field is lower than the upper critical magnetic field the two components will be present simultaneously in the superconductor. This is consistent with the discussion on vortex solutions in the previous section.

VII. CONCLUSION AND DISCUSSION

Based on the weak-coupling theory, we have derived the complete set of two-component Ginzburg-Landau equations for a layered p-wave superconductor. The expansion coefficients of the phenomenological GL free energy with respect to the order parameter are determined up to the fourth order. These coefficients allow us to identify the vortex structure by referring to the vortex phase diagram which was obtained numerically from phenomenological GL equations with freely adjusted coefficients. We find that the microscopically obtained coefficients are within the region where the vortex is of axisymmetry, but are close to the phase boundary. Correspondingly, the supercurrent circulating around a vortex is similar to that in conventional *s*-wave superconductors. Moreover, we have also calculated the upper critical magnetic field and a unique one from two possible values has been figured out.

We have to make several remarks on our derivation. Firstly, our calculations have been performed by making use of simplifications like a cylindrical Fermi surface as well as a simple form of the attractive interaction which is known as the weak-coupling limit. In the strong-coupling limit, the GL expansion coefficients may change appreciably and the nonaxial vortex structures could be expected. Secondly, we are mainly concerned with the derivation of GL equations for the 2D representation of the odd-parity pairing state (Γ_5^-) so that we can directly compare the obtained GL free-energy functional with the phenomenological free-energy functional for the superconducting state composed of two degenerate components. However, it is not difficult to conceive that one can obtain the same form of GL equations of one 1D pairing state (e.g., Γ_1^-) coupled to one of the other 1D pairing states (e.g., Γ_5^-) by still assuming a unitary order-parameter matrix. It can be understood that due to the space inhomogeneity, there arises the possibility for one state to fluctuate into the other one. In principle, a superconducting pairing state should be a linear combination of all basis pairing functions (Γ_1^--Γ_5^-). In this situation, the superconducting pairing state is nonunitary and the derivation for GL equations is greatly complicated and becomes very tedious. Finally, the Pauli paramagnetism, spin-orbit coupling, and related effects on the magnetic properties are under consideration.

ACKNOWLEDGMENTS

This work was supported by a research grant from the Texas Center for Superconductivity at the University of Houston, and by the Robert A. Welch Foundation.

*Permanent address: Institute of Physics, Academia Sinica, Beijing 100080, People's Republic of China.

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