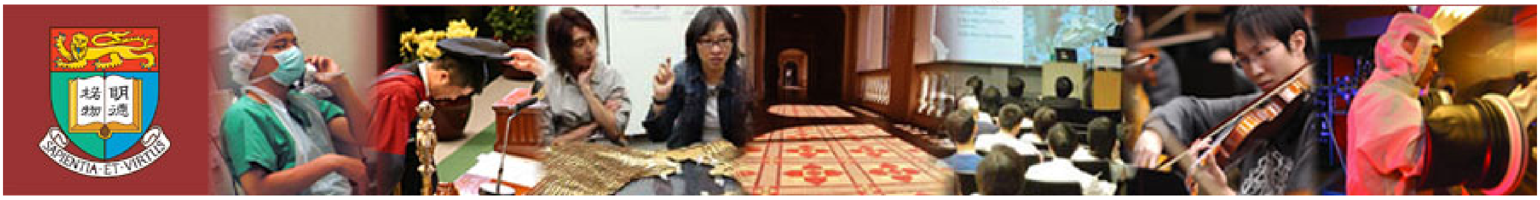

Title	Bound states and Josephson current in mesoscopic s-wave superconductor–normal-metal–d-wave superconductor junctions
Author(s)	Zhu, JX; Wang, ZD; Tang, HX
Citation	Physical Review B (Condensed Matter), 1996, v. 54 n. 10, p. 7354-7359
Issued Date	1996
URL	http://hdl.handle.net/10722/43175
Rights	Physical Review B (Condensed Matter). Copyright © American Physical Society.

PHYSICAL REVIEW B VOLUME 54, NUMBER 10 1 SEPTEMBER 1996-II

Bound states and Josephson current in mesoscopic s-wave superconductor–normal-metal–d-wave superconductor junctions

Jian-Xin Zhu, Z. D. Wang, and H. X. Tang
Department of Physics, University of Hong Kong, Pokfulam Road, Hong Kong

(Received 11 April 1996)

We have investigated the superconducting phase difference dependence of Andreev levels and Josephson current through a mesoscopic normal-metal layer in contact with two superconducting electrodes with s-wave and d-wave pairing symmetry (S_sNS_d junction). It is shown that, regardless of the junction length, due to the sign change of the d-wave order parameter under suitable arrangements, the zero-energy point of Andreev levels for the negative process appears at $\varphi=0$. In particular, at zero temperature, the amplitude of the total Josephson current through the point contact S_sNS_d junction could be enhanced by the sign change of the d-wave order parameter. However, for an S_sNS_d junction of special length, the amplitude of Josephson current may be suppressed by this sign change. Moreover, as a special case, the midgap surface states discovered by Hu [Phys. Rev. Lett. **72**, 1526 (1994)] are recovered naturally. [S0163-1829(96)06933-0]

I. INTRODUCTION

Recently there has been much interest in the pairing symmetry in cuprate high-temperature superconductors. Many theoretical studies[1–12] proposed that the superconducting state of the materials could be characterized by $d_{x^2-y^2}$ symmetry. Such a d-wave pairing state gives rise to an anisotropic energy gap, which not only drops to zero on some nodal points of an essentially cylindrical Fermi surface but also changes sign across the nodes; while an s-wave superconductor has a finite energy gap at all directions of the Fermi surface. Indication for this type of pairing symmetry in high-T_c superconductors comes from experiments on the T^3 dependence of NMR,[13] the linear temperature dependence of the superfluid density observed in high purity crystals of Y-Ba-Cu-O,[14] and the strong anisotropy of the energy gap in angular-resolved photoemission.[15] However, if only the magnitude of a pairing potential is sufficient to determine physical quantities of interest in an experiment, the measured result could be interpreted in terms of either a d-wave gap or a highly anisotropic s-wave gap[16] or an $(s+id)$-wave gap,[17] which could also vanish at the same nodal points. Therefore, to distinguish between a $d_{x^2-y^2}$ wave and strongly anisotropic s-wave gap in high-T_c superconductors, many measurements which look at the relative phase of the pairing potential between different points on the Fermi surface have been designed or theoretically proposed.[18–26] The experiments reported so far which support a d-wave pairing symmetry in high-T_c superconductors include superconducting quantum interference device interferometry,[18–20] tricrystal superconducting ring magnetometry,[21,22] and single junction modulation.[23,24] Theoretically, it was shown in a recent report by Hu[25] that there exist midgap surface states in d-wave superconductors when the pair potential is an odd function of the Fermi wave-vector component normal to the specular surface. This result can be used as a clear signature to distinguish between d-wave and anisotropic s-wave superconductors. As another direct consequence of the sign change of a d-wave superconductor, by investigating the effects of Andreev reflection on the current-voltage characteristic and differential conductance of a normal metal and a d-wave superconductor, Xu, Miller, and Ting[27] found that a zero-bias conductance peak[28] appears when an insulating barrier exists at the interface between the normal metal and the d-wave superconductor. They also predicted bound states within the energy gap and consequent subgap resonances in the differential conductance if the insulating barrier is located in the normal metal several coherence lengths away from the superconductor surface. Along this line, in this work, we investigate the superconducting phase difference dependence of Andreev levels and Josephson current through a mesoscopic normal-metal layer in contact with two superconducting electrodes with s-wave and d-wave pairing symmetry (S_sNS_d junction). It is shown that, regardless of the junction length, due to the sign change of the d-wave order parameter under suitable arrangements, the zero-energy point of Andreev levels for the negative process appears at $\varphi=0$. In particular, at zero temperature, the amplitude of the total Josephson current through the point-contact S_sNS_d junction could be enhanced by the sign change of the d-wave order parameter. However, for an S_sNS_d junction of special length, the amplitude of Josephson current may be suppressed by this sign change. Moreover, the midgap surface states discovered by Hu[25] are naturally recovered.

The paper is arranged as follows. In Sec. II, we present the solution to the Bogoliubov–de Gennes equations for an s-wave superconductor–normal-metal–d-wave superconductor junction. In Sec. III, we calculate the bound states between an insulating barrier and the superconducting electrodes, and the Andreev levels for a clean junction. The Josephson current through the junction is computed in Sec. IV. Finally, conclusions are given in Sec. V.

II. BOGOLIUBOV–DE GENNES EQUATIONS FOR AN S_sNS_d JUNCTION

To relate the physics to high-T_c superconductors, we assume that the system under consideration is two-dimensional. A non-s-wave superconducting order parameter

0163-1829/96/54(10)/7354(6)/$10.00

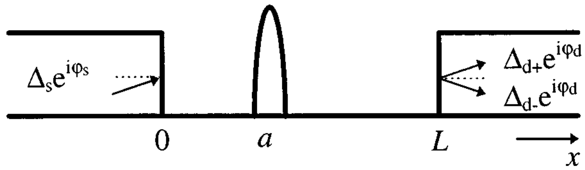

FIG. 1. Scattering potential of an S_sNS_d junction.

(or pair potential) not only depends on the center-of-mass coordinates $\mathbf{R}$ but also on the relative coordinates $\mathbf{s}$, or after a Fourier transform, on the relative wave vector $\mathbf{k}$, which in the weak-coupling theory is fixed on the Fermi surface such that only its direction $\hat{\mathbf{k}}_F = \mathbf{k}_F/|\mathbf{k}_F|$ is a variable. In particular, the pair potential of a two-dimensional $d_{x^2-y^2}$-wave superconductor could be expressed as

$$\Delta_{d_0}|\cos(2\phi)|e^{i(\varphi_d+\varphi_J)}, \tag{1}$$

where $\Delta_{d_0}>0$, ϕ is the azimuthal angle, φ_d is the ordinary phase, and the gauge-invariant phase is

$$\varphi_J = \begin{cases} 0, & \text{for } \cos(2\phi)>0, \\ \pi, & \text{for } \cos(2\phi)<0. \end{cases} \tag{2}$$

If quasiparticles travel in a bulk superconductor along a straight line, they will feel a constant pair potential. However, for a normal-metal–superconductor junction, these quasiparticles, e.g., electronlike excitations, will be partially reflected as electronlike and holelike excitations and partially transmitted through the interface between the normal-metal and superconductor. If the order parameter in the superconductor has a d-wave symmetry, the effective pair potentials experienced by the reflected electronlike excitations and holelike excitations are different from each other due to the change of the wave vector after the reflection, and they even have opposite signs under appropriate arrangements, which does not happen if the order parameter of the superconductor has an either isotropic or strongly anisotropic s-wave symmetry. As shown in Fig. 1, we consider a mesoscopic normal-metal layer spanning between two superconducting electrodes with s-wave and d-wave pairing symmetry (S_sNS_d junction), and assume that the x axis is normal to the interfaces of normal-metal and superconductors, and the y axis is parallel to the interfaces. The S_sNS_d junction is modeled by a step change in the pair potential

$$\Delta(x) = \begin{cases} \Delta_s e^{i\varphi_s}, & x<0, \\ 0, & 0<x<L, \\ \Delta_{d_\pm} e^{i\varphi_d}, & x>L. \end{cases} \tag{3}$$

The order parameters of two superconducting banks have isotropic s-wave symmetry and $d_{x^2-y^2}$-wave symmetry, respectively. We denote by $\Delta_{d\pm}$ the effective pair potentials experienced differently by the electronlike and holelike excitations in the d-wave superconducting region. If the crystalline a axis of the $d_{x^2-y^2}$-wave superconductor, along which the magnitude of the pair potential is arranged to reach a maximum, is misoriented with an angle α with respect to the normal direction of the interface, and for definiteness, a beam of electronlike excitations are incident from the left superconducting electrode with an angle θ with respect to the normal direction of the interface, we can write

$$\Delta_{d_\pm} = \Delta_{d_0}|\cos(2\theta \mp 2\alpha)|e^{i\varphi_{J_\pm}}. \tag{4}$$

For our purpose, we also introduce a δ-function impurity which is located in the normal conducting region

$$V(x) = V_s\delta(x-a), \quad 0 \leqslant a \leqslant L. \tag{5}$$

The motion of elementary quasiparticles in the S_sNS_d junction is described by the Bogoliubov–de Gennes equations[29,30]

$$\mathscr{H}_e(\mathbf{r})u(\mathbf{r}) + \int d\mathbf{r}'\,\Delta(\mathbf{s},\mathbf{R})v(\mathbf{r}') = Eu(\mathbf{r}), \tag{6a}$$

$$-\mathscr{H}_e^*(\mathbf{r})v(\mathbf{r}) + \int d\mathbf{r}'\,\Delta^*(\mathbf{s},\mathbf{R})u(\mathbf{r}') = Ev(\mathbf{r}), \tag{6b}$$

where $\mathbf{s}\equiv\mathbf{r}-\mathbf{r}'$ and $\mathbf{R}\equiv(\mathbf{r}+\mathbf{r}')/2$, and the single electron Hamiltonian $\mathscr{H}_e(\mathbf{r})$ in the absence of vector potential is

$$\mathscr{H}_e(\mathbf{r}) = -\frac{\hbar^2}{2m_e}\nabla_{\mathbf{r}}^2 + V(\mathbf{r}) - E_F. \tag{7}$$

Here the excitation energy E is measured relative to the Fermi energy E_F. As a special case, $\Delta(\mathbf{s},\mathbf{R}) = \Delta_s\delta(\mathbf{r}-\mathbf{r}')$ when the order parameter has an isotropic s-wave symmetry.

In the WKBJ approximation, Eq. (6) reduces to the Andreev equations[30–32]

$$E\tilde{u}(\mathbf{r}) = -i(\hbar^2/m_e)\mathbf{k_F}\cdot\nabla\tilde{u}(\mathbf{r}) + \Delta(\hat{\mathbf{k}}_F,\mathbf{r})\tilde{v}(\mathbf{r}), \tag{8a}$$

$$E\tilde{v}(\mathbf{r}) = i(\hbar^2/m_e)\mathbf{k_F}\cdot\nabla\tilde{v}(\mathbf{r}) + \Delta^*(\hat{\mathbf{k}}_F,\mathbf{r})\tilde{u}(\mathbf{r}), \tag{8b}$$

where

$$\begin{pmatrix}\tilde{u}(\mathbf{r})\\ \tilde{v}(\mathbf{r})\end{pmatrix} = e^{-i\mathbf{k}_F\cdot\mathbf{r}}\begin{pmatrix}u(\mathbf{r})\\ v(\mathbf{r})\end{pmatrix}. \tag{9}$$

In the presence of an incident electronlike excitation and considering the translational invariance of the system along the y axis, we find the wave function $\Psi\equiv(u(\mathbf{r}),v(\mathbf{r}))^{\text{Trans}}$, by solving the Andreev equations (8),

$$\Psi = e^{ik_Fy\sin\theta}\left[\begin{pmatrix}\hat{u}_se^{i\varphi_s}\\ \hat{v}_s\end{pmatrix}e^{ik_{s_e}x} + A\begin{pmatrix}\hat{u}_se^{i\varphi_s}\\ \hat{v}_s\end{pmatrix}e^{-ik_{s_e}x} + \left(B-\frac{\hat{v}_s}{\hat{u}_s}\right)\right.$$
$$\left.\times\begin{pmatrix}\hat{v}_se^{i\varphi_s}\\ \hat{u}_s\end{pmatrix}e^{ik_{s_h}x}\right], \tag{10a}$$

for $x<0$;

$$\Psi = e^{ik_Fy\sin\theta}\left[\left(B-\frac{\hat{v}_s}{\hat{u}_s}+\frac{\hat{u}_s}{\hat{v}_s}\right)\begin{pmatrix}\hat{v}_se^{i\varphi_s}\\ 0\end{pmatrix}e^{ik_{N_e}x}\right.$$
$$\left.+A\begin{pmatrix}\hat{u}_se^{i\varphi_s}\\ 0\end{pmatrix}e^{-ik_{N_e}x} + A\begin{pmatrix}0\\ \hat{v}_s\end{pmatrix}e^{-ik_{N_h}x} + B\begin{pmatrix}0\\ \hat{u}_s\end{pmatrix}e^{ik_{N_h}x}\right], \tag{10b}$$

for $0<x<a$;

$$\Psi=e^{ik_Fy\sin\theta}\left[C\begin{pmatrix}\hat{u}_{d_+}e^{i(\varphi_d+\varphi_{J_+})}\\ 0\end{pmatrix}e^{ik_{N_e}(x-L)}+D\begin{pmatrix}\hat{v}_{d_-}e^{i(\varphi_d+\varphi_{J_-})}\\ 0\end{pmatrix}e^{-ik_{N_e}(x-L)}+C\begin{pmatrix}0\\ \hat{v}_{d_+}\end{pmatrix}e^{ik_{N_h}(x-L)}+D\begin{pmatrix}0\\ \hat{u}_{d_-}\end{pmatrix}e^{-ik_{N_h}x}\right], \tag{10c}$$

for $a<x<L$;

$$\Psi=e^{ik_Fy\sin\theta}\left[C\begin{pmatrix}\hat{u}_{d_+}e^{i(\varphi_d+\varphi_{J_+})}\\ \hat{v}_{d_+}\end{pmatrix}e^{ik_{d_e}(x-L)}+D\begin{pmatrix}\hat{v}_{d_-}e^{i(\varphi_d+\varphi_{J_-})}\\ \hat{u}_{d_-}\end{pmatrix}e^{-ik_{d_h}(x-L)}\right], \tag{10d}$$

for $L<x$. Here the ''coherence factors'' in the superconducting regions are, respectively,

$$\hat{u}^2_{s,d_\pm}=\frac{1}{2}\left(1+\frac{\sqrt{E^2-\Delta^2_{s,d_\pm}}}{E}\right),\hat{v}^2_{s,d_\pm}=\frac{1}{2}\left(1-\frac{\sqrt{E^2-\Delta^2_{s,d_\pm}}}{E}\right), \tag{11}$$

and the wave vectors are determined from the dispersion law,

$$k_{N_{e,h}}=k_F\cos\theta\pm\frac{m_eE}{\hbar^2k_F\cos\theta}, \tag{12}$$

$$k_{s_{e,h}}=k_F\cos\theta\pm\frac{m_e\sqrt{E^2-\Delta_s^2}}{\hbar^2k_F\cos\theta}, \tag{13}$$

and

$$k_{d_{e,h}}=k_F\cos\theta\pm\frac{m_e\sqrt{E^2-\Delta^2_{d_\pm}}}{\hbar^2k_F\cos\theta}. \tag{14}$$

To obtain the wave function, we have assumed that the Fermi energy is much greater than the pair potentials so that the difference between the wave vectors can be neglected except those appearing in exponents.

At the impurity, the amplitudes of the outgoing waves are related to the incoming waves by the scattering matrix for both electrons and holes,[33,34] i.e.,

$$\begin{pmatrix}A\hat{u}_se^{i\varphi_s}e^{-ik_{N_e}a}\\ C\hat{u}_{d_+}e^{i(\varphi_d+\varphi_{J_+})}e^{ik_{N_e}(a-L)}\end{pmatrix}=\begin{pmatrix}r & t\\ t & r\end{pmatrix}\begin{pmatrix}\left(B-\frac{\hat{v}_s}{\hat{u}_s}+\frac{\hat{u}_s}{\hat{v}_s}\right)\hat{v}_se^{i\varphi_s}e^{ik_{N_e}a}\\ D\hat{v}_{d_-}e^{i(\varphi_d+\varphi_{J_-})}e^{-ik_{N_e}(a-L)}\end{pmatrix}, \tag{15}$$

and

$$\begin{pmatrix}B\hat{u}_se^{ik_{N_h}a}\\ D\hat{u}_{d_-}e^{-ik_{N_h}(a-L)}\end{pmatrix}=\begin{pmatrix}r^* & t^*\\ t^* & r^*\end{pmatrix}\begin{pmatrix}A\hat{v}_se^{-ik_{N_h}a}\\ C\hat{v}_{d_+}e^{ik_{N_h}(a-L)}\end{pmatrix}, \tag{16}$$

where the transmission and reflection amplitudes are

$$t=\frac{1}{1+i(m_eV_s/\hbar^2k_F\cos\theta)}, \tag{17}$$

and

$$r=-\frac{m_eV_s/\hbar^2k_F\cos\theta}{1+i(m_eV_s/\hbar^2k_F\cos\theta)}. \tag{18}$$

III. MIDGAP STATES AND ANDREEV LEVELS

In the limit that the amplitude of the scattering potential is infinitely large, which is equivalent to an insulator inserted into the normal-metal layer, the transmission and reflection amplitudes then become $t=0$ and $r=-1$, respectively. In this case, the s- and d-wave superconducting regions are decoupled into two independent systems which share the same insulator layer. By computing the poles of B and C determined from Eqs. (15) and (16) and with the help of Eq. (11), we find

$$E=\Delta_s\cos\left(\frac{2m_eaE}{\hbar^2k_F\cos\theta}\right) \tag{19}$$

for the bound-state energies ($|E|<\Delta_s$) in the s-wave superconductor–normal-metal–insulator system, and

$$-\cos^{-1}\left(\frac{E}{|\Delta_{d_+}|}\right)-\cos^{-1}\left(\frac{E}{|\Delta_{d_-}|}\right)+\frac{4m_e(L-a)E}{\hbar^2k_F\cos\theta}=\varphi_{J_+}-\varphi_{J_-} \tag{20}$$

for the bound-state energies $[|E|<\min(|\Delta_{d_+}|,|\Delta_{d_-}|)]$ in the insulator–normal-metal–d-wave superconductor system. Both of these two groups of bound states stem from the normal reflection at the infinite barrier potential and the Andreev reflection at the interface between the normal metal and the superconducting electrode. It follows from Eq. (19) that no zero-energy bound state is trapped between the infinite barrier potential and the s-wave superconducting bank. However, since $\varphi_{J_+}-\varphi_{J_-}=0,\pm\pi$, Eq. (20) could imply a zero-energy bound state trapped between the barrier potential and the d-wave superconducting bank, which lies on the Fermi surface. To illustrate this point, we take $\alpha=\theta=\pi/4$ which yields $\varphi_{J_+}=0$ and $\varphi_{J_-}=\pi$, and $|\Delta_{d_+}|=|\Delta_{d_-}|=\Delta_{d_0}$. We therefore find

$$E=-\Delta_{d_0}\sin[2\sqrt{2}m_e(L-a)E/\hbar^2k_F], \tag{21}$$

which evidently has a zero-energy solution. Such a midgap state was originally discovered by Hu[25] on a $\{110\}$ surface of a $d_{x_a^2-x_b^2}$-wave superconductor, which is separated from a vacuum or an insulator by a clean normal-metal overlayer.

For a clean S_sNS_d junction ($V=0$), $t=1$ and $r=0$, we find from Eqs. (15) and (16) that the Andreev level spectrum is determined from

$$-\cos^{-1}\left(\frac{E}{|\Delta_{d_\pm}|}\right)-\cos^{-1}\left(\frac{E}{\Delta_s}\right)\pm(\varphi-\varphi_{J_\pm})+\left(\frac{2L}{\pi\xi_s\cos\theta}\right)\left(\frac{E}{\Delta_s}\right)=2\pi n, \tag{22}$$

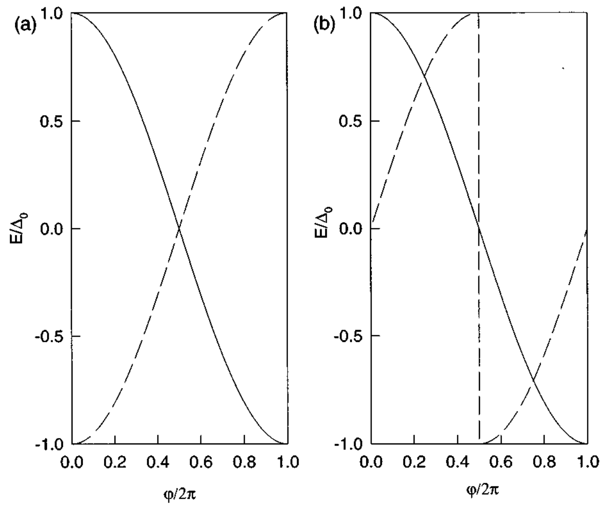

FIG. 2. Andreev levels as a function of the superconducting phase difference for a point-contact junction ($L=0$) under the arrangements $\alpha=\theta=0$ (a) and $\alpha=\theta=\pi/4$ (b).

where $\xi_s=\hbar v_F/\pi\Delta_s$ is the BCS coherence length for the s-wave superconducting electrode, and $\varphi_s-\varphi_d$ is the usual superconducting phase difference, $n=0,\pm1,\pm2,\ldots$. We refer to the upper branch as the positive process and the lower branch as the negative process. It is evident that the Andreev energies are periodic in the superconducting phase difference with period 2π. Equation (22) also shows that, under appropriate arrangements, there may occur a π-phase shift in the dependence of the Andreev energies on the usual superconducting phase difference for an S_sNS_d junction. Such a π-phase shift does not occur in an s-wave superconductor–normal-metal–s-wave superconductor (S_sNS_s) junction, whether the s wave is isotropic or anisotropic. In the following, we assume for simplicity $\Delta_{d_0}=\Delta_s=\Delta_0$, which does not alter the essential physics of interest. In the short or point-contact junction limit ($L\to0$), we obtain the Andreev energies for the positive process when $\alpha=\theta=0$ or $\alpha=\theta=\pi/4$,

$$E^+=\Delta_0\cos(\varphi/2),\quad \varphi\in[0,2\pi]; \tag{23}$$

and for the negative process, when $\alpha=\theta=0$,

$$E^-=-\Delta_0\cos(\varphi/2),\quad \varphi\in[0,2\pi], \tag{24}$$

while when $\alpha=\theta=\pi/4$

$$E^-=\begin{cases}\sin(\varphi/2), & \varphi\in[0,\pi],\\ -\sin(\varphi/2), & \varphi\in[\pi,2\pi].\end{cases} \tag{25}$$

The corresponding result is graphed in Fig. 2(a) for $\alpha=\theta=0$ and in Fig. 2(b) for $\alpha=\theta=\pi/4$. Notice that the Andreev levels for $\alpha=\theta=0$ are equivalent to those for a clean S_sNS_s junction. Consequently, we see that, for a clean S_sNS_s junction, the zero-energy points ($E=0$) of Andreev levels always appear at $\varphi=\pi$ for both the positive and negative processes. Remarkably, for an S_sNS_d junction and under suitable arrangements as shown in Fig. 2(b), the zero-energy point of Andreev levels for the positive process still appears at $\varphi=\pi$, while for the negative process the zero-energy point appears at $\varphi=0$, which is different from those results for an S_sNS_s junction. These zero-energy states exist regardless of the junction length. As a general result, the φ dependence of the Andreev levels for a clean S_sNS_d junction of the junction length $L=2\xi_s$ is plotted in Fig. 3(a) when $\alpha=\theta=0$ and in Fig. 3(b) when $\alpha=\theta=\pi/4$, from which the π-phase shift of the zero-energy point for the negative process could be observed clearly.

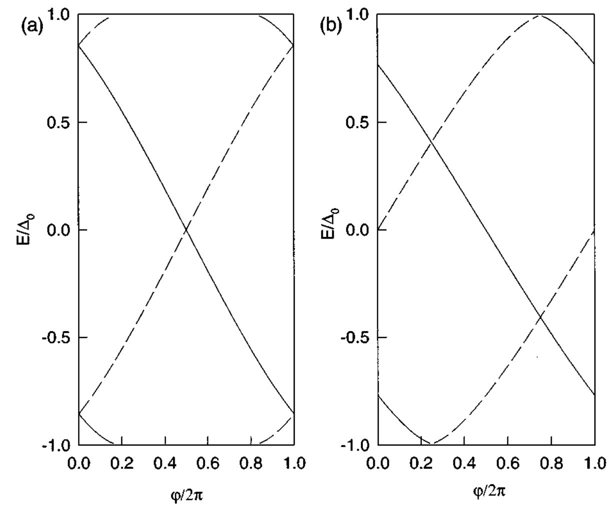

FIG. 3. Same as Fig. 2 except that $L=2\xi_s$.

IV. SUPERCURRENT THROUGH THE S_sNS_d JUNCTION

The Josephson current induced by the superconducting phase difference consists of the current $I_d(\varphi)$ carried by quasiparticles occupying each discrete Andreev level and $I_c(\varphi)$ carried by quasiparticles flowing in the continuum levels, that is,

$$I(\varphi)=I_d(\varphi)+I_c(\varphi). \tag{26}$$

The current contributed from the Andreev levels is given by[35]

$$I_d(\varphi)=\sum_n\{I_n^+(\varphi)f(E_n^+(\varphi))+I_n^-(\varphi)f(E_n^-(\varphi))\}, \tag{27}$$

where the Fermi distribution function $f(E)=1/[1+\exp(E/k_BT)]$, and

$$I_n^\pm(\varphi)=\frac{2e}{\hbar}\frac{dE_n^\pm}{d\varphi}. \tag{28}$$

The continuum contribution can be calculated by using the transmission formalism of van Wees, Lenssen, and Harmans,[36] which is analogous to the Landauer-Büttiker formalism for currents through mesoscopic normal conductors.[37] For two special cases of $\alpha=\theta=0$ and $\alpha=\theta=\pi/4$, where $|\Delta_{d_+}|=|\Delta_{d_-}|=\Delta_0$, we find this continuum contribution to the current to be

$$I_c(\varphi)=\frac{2e}{h}\left(\int_{-\infty}^{-\Delta_0}+\int_{\Delta_0}^{\infty}\right)|\hat{u}_0^2-\hat{v}_0^2|\left(\frac{1}{\Pi(E,\varphi-\varphi_{J_+})}-\frac{1}{\Pi(E,-\varphi+\varphi_{J_-})}\right)f(E)dE, \tag{29}$$

where

$$\Pi(E,\varphi)=\hat{u}_0^4+\hat{v}_0^4-2\hat{u}_0^2\hat{v}_0^2\cos\left[\varphi+\left(\frac{2L}{\pi\xi_s\cos\theta}\right)\left(\frac{E}{\Delta_0}\right)\right], \tag{30}$$

with $\hat{u}_0$ and $\hat{v}_0$ given in the form of Eq. (11). In the short or point-contact junction limit ($L\rightarrow 0$), there is no continuum contribution when $\alpha=\theta=0$ because Π is an even function of the usual phase difference, and the total current is only determined from the discrete spectrum, which, at zero temperature ($k_BT=0$), can be obtained analytically

$$I(\varphi)=I_d(\varphi)=\begin{cases}\frac{e\Delta_0}{\hbar}\sin(\varphi/2), & \varphi\in[0,\pi],\\ -\frac{e\Delta_0}{\hbar}\sin(\varphi/2), & \varphi\in[\pi,2\pi].\end{cases} \tag{31}$$

In the case of $\alpha=\theta=\pi/4$, the current ($k_BT=0$) contributed from the Andreev levels is given by

$$I_d(\varphi)=\begin{cases}0, & \varphi\in[0,\pi],\\ -\frac{e\Delta_0}{\hbar}\sqrt{2}\sin(\varphi+\pi/4), & \varphi\in[\pi,2\pi].\end{cases} \tag{32}$$

Equation (32) shows that the discrete spectrum contribution is limited to one half of the period, due to the sign change of the d-wave order parameter. In addition, due to this sign change, the continuum spectrum makes also a contribution to the Josephson current when $\alpha=\theta=\pi/4$. These results are significantly different from the case of $\alpha=\theta=0$. At zero temperature, the Josephson current corresponding to Fig. 2 is plotted in Fig. 4. Figure 4(a) actually shows the result for a clean point-contact S_sNS_s junction. Fig. 4(b) shows the result for a clean point-contact junction under the arrangement that $\alpha=\theta=\pi/4$. Comparing Fig. 4(b) with 4(a), we see that the amplitude of the Josephson current for a point-contact S_sNS_d junction at zero temperature could be enhanced by the sign change of the d-wave order parameter. Corresponding to Fig. 3, the Josephson current through a clean S_sNS_d junction of the length $L=2\xi_0$ is plotted in Fig. 4. Certainly, upon the sign change of the d-wave order parameter, the pattern of the current-phase characteristic for an S_sNS_d junction is quite different from that for an S_sNS_s junction. In this special case, the amplitude of the Josephson current is however suppressed by this sign change.

V. CONCLUSIONS AND DISCUSSIONS

We have studied the superconducting phase difference dependence of Andreev levels and Josephson current in a clean S_sNS_d junction. By inserting an insulating barrier in the normal conducting region, we obtain the Andreev levels and the midgap surface states in a unified fashion. It is shown that, the Andreev energies are periodic in the superconducting phase difference with period 2π. In particular, regardless of the junction length, due to the sign change of the d-wave order parameter under suitable arrangements, the zero-energy point of Andreev levels for the negative process appears at $\varphi=0$, which differs from those results for an S_sNS_s junction where the zero-energy points of Andreev levels always appear at $\varphi=\pi$. Moreover, for a point-contact S_sNS_d junction at zero temperature, the π phase shift in the superconducting phase difference dependence of Andreev levels leads to the discrete spectrum contribution to the Josephson current only in one half of the period, and nonvanishing continuum spectrum contribution to the Josephson current. Consequently, relative to that for a point-contact S_sNS_s junction the amplitude of the total Josephson current through the point-contact S_sNS_d junction could be enhanced by the sign change of the d-wave order parameter. For an S_sNS_d junction of special length, the amplitude of Josephson

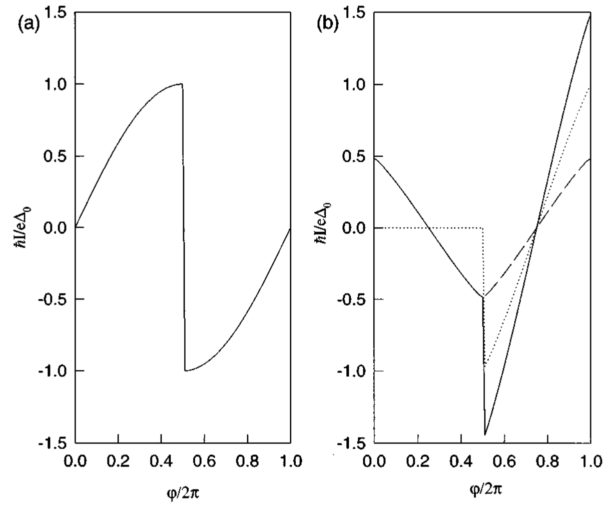

FIG. 4. Josephson current as a function of the superconducting phase difference for a point-contact junction ($L=0$) under the arrangements $\alpha=\theta=0$ (a) and $\alpha=\theta=\pi/4$ (b). The total Josephson current (solid) consists of the contributions from discrete (dotted) and continuum (dashed) spectrum.

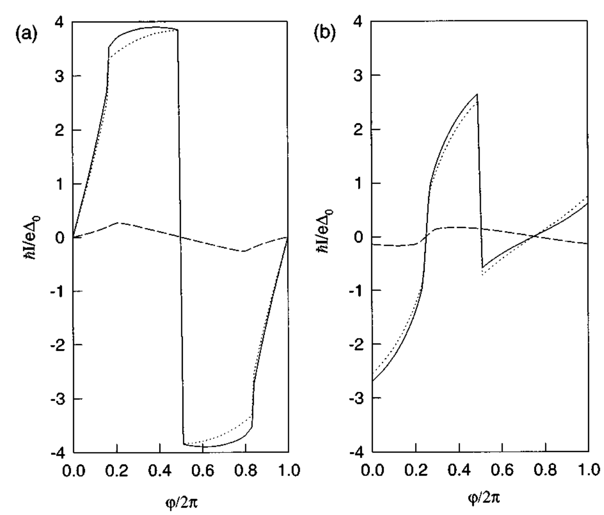

FIG. 5. Same as Fig. 4 except that $L=2\xi_s$.

current may be suppressed by this sign change. In addition, the midgap surface states is also discussed as a special case.

Finally, we would like to make several remarks on our calculations: (i) The WKBJ approximation which we have made is valid under the condition that the superconducting coherence length ξ is much greater than the inverse Fermi wave vector k_F^{-1} because the wave function oscillates on the scale of k_F^{-1} while the anisotropic energy gap varies on the scale of ξ. (ii) The Andreev approximation, which is used in most works on the Josephson effect in superconductor–normal-metal–superconductor junctions, requires that the Fermi energy is much greater than the energy gap. In high-T_c superconductors, the ratio of the Fermi energy to the energy gap is not so large as that in conventional superconductors. (iii) The mismatch of the Fermi wave vectors in normal conducting and superconducting regions has been neglected. (iv) We are only concerned with two very typical cases, i.e., $\alpha=\theta=0$ and $\alpha=\theta=\pi/4$. The computation will be complicated if the values of the energy gaps in two superconducting electrodes are not identical to each other. (v) The effects of finite temperature and disorder on the bound states and the Josephson current in an S_sNS_d junction has not been touched yet. However, the main results of this work do not deviate too much if these effects are taken into account.

ACKNOWLEDGMENTS

One of the authors (J.-X.Z.) is grateful to the Robert Black College of the University of Hong Kong for an offer of the Wang Gungwu Scholarship, and also for the Hung Hing Ying Scholarship of the University of Hong Kong. This work was supported by the RGC grant of Hong Kong under Grant No. HKU262/95P and the CRCG grant at the University of Hong Kong.

[1] N. E. Bickers, D. J. Scalapino, and R. T. Scalettar, Int. J. Mod. Phys. B **1**, 687 (1987); N. E. Bickers, D. J. Scalapino, and S. R. White, Phys. Rev. Lett. **62**, 961 (1989).
[2] C. Gros, R. Joynt, and T. M. Rice, Z. Phys. B **68**, 425 (1987); C. Gros, Phys. Rev. B **38**, 931 (1988).
[3] Z. Y. Weng, T. K. Lee, and C. S. Ting, Phys. Rev. B **38**, 6561 (1988).
[4] P. Monthoux, A. V. Balatsky, and D. Pines, Phys. Rev. Lett. **67**, 3448 (1991); Phys. Rev. B **46**, 14 803 (1992); P. Monthoux and D. Pines, *ibid.* **47**, 6069 (1993).
[5] M. Sigrist and T. M. Rice, J. Phys. Soc. Jpn. **61**, 4283 (1992).
[6] N. Bulut and D. J. Scalapino, Phys. Rev. Lett. **68**, 706 (1992); Phys. Rev. B **45**, 2371 (1992).
[7] T. Moriya, Y. Takahashi, and K. Ueda, Physica C (Amsterdam) **185-189**, 114 (1991); K. Ueda, T. Moriya, and Y. Takahashi, in *Electronic Properties and Mechanisms of High-T_c Superconductors*, edited by T. Oguchi (North-Holland, Amsterdam, 1992), p. 145.
[8] D. Thelen, D. Pines, and J. P. Lu, Phys. Rev. B **47**, 9151 (1993).
[9] D. S. Rokhar, Phys. Rev. Lett. **70**, 493 (1993).
[10] E. Dagotto and J. Riera, Phys. Rev. Lett. **70**, 682 (1993).
[11] K. Maki and H. Won, Phys. Rev. Lett. **72**, 1758 (1994).
[12] T. P. Devereaux, D. Einzel, B. Stadlober, R. Hackl, D. H. Leach, and J. J. Neumeirer, Phys. Rev. Lett. **72**, 396 (1994); T. P. Devereaux and D. Einzel, Phys. Rev. B **51**, 16 336 (1995).
[13] J. A. Martindale, S. E. Barrett, K. E. O'Hara, C. P. Slichter, W. C. Lee, and D. M. Ginsberg, Phys. Rev. B **47**, 9155 (1993); M. Takigawa, J. L. Smith, and W. L. Hults, *ibid.* **44**, 7764 (1991); Physica C **185**, 1105 (1991).
[14] W. N. Hardy, D. A. Bonn, D. C. Morgan, R. Liang, and K. Zhang, Phys. Rev. Lett. **70**, 3999 (1993).
[15] Z.-X. Shen, D. S. Dessau, B. O. Wells, D. M. King, W. E. Spicer, A. J. Arko, D. Marshall, L. W. Lombardo, A. Kapitulnik, P. Dickinson, S. Doniach, J. DiCarlo, A. G. Loeser, and C. H. Park, Phys. Rev. Lett. **70**, 1553 (1993).
[16] S. Chakravarty, A. Sudbø, P. W. Anderson, and S. Strong, Science **261**, 337 (1993).
[17] G. Kotliar, Phys. Rev. B **37**, 3664 (1988); Q. P. Li, B. E. C. Koltenbah, and R. Joynt, *ibid.* **48**, 437 (1993).
[18] D. A. Wollman, D. J. Van Harlingen, W. C. Lee, D. M. Ginsberg, and A. J. Leggett, Phys. Rev. Lett. **71**, 2134 (1993).
[19] A. Mathai, Y. Gim, R. C. Black, A. Amar, and F. C. Wellstood, Phys. Rev. Lett. **74**, 4523 (1995).
[20] D. A. Brawner and H. R. Ott, Phys. Rev. B **50**, 6530 (1994).
[21] C. C. Tsuei, J. R. Kirtley, C. C. Chi, L. S. Yu-Jahnes, A. Gupta, T. Shaw, J. Z. Sun, and M. B. Ketchen, Phys. Rev. Lett. **73**, 593 (1994); J. R. Kirtley, C. C. Tsuei, J. Z. Sun, C. C. Chi, L. S. Yu-Jahnes, A. Gupta, M. Rupp, and M. B. Ketchen, Nature (London) **373**, 225 (1995).
[22] C. C. Tsuei, J. R. Kirtley, M. Rupp, J. Z. Sun, A. Gupta, M. B. Ketchen, C. A. Wang, and Z. F. Ren, J. H. Wang, and M. Bhushan, Science **271**, 329 (1996).
[23] J. H. Miller, Jr., Q. Y. Ying, Z. G. Zou, N. Q. Fan, J. H. Xu, M. F. Davis, and J. C. Wolfe, Phys. Rev. Lett. **74**, 2347 (1995).
[24] D. A. Wollman, D. J. Van Harlingen, J. Giapintzakis, and D. M. Ginsberg, Phys. Rev. Lett. **74**, 797 (1995).
[25] C. R. Hu, Phys. Rev. Lett. **72**, 1526 (1994).
[26] Junren Shi, Jinming Dong, D. Y. Xing, and Z. D. Wang (unpublished).
[27] J. H. Xu, J. H. Miller, and C. S. Ting, Phys. Rev. B **53**, 3604 (1996).
[28] Y. Tanaka and S. Kashiwaya, Phys. Rev. Lett. **74**, 3451 (1995).
[29] P. G. de Gennes, *Superconductivity of Metals and Alloys* (Benjamin, New York, 1966).
[30] Chr. Bruder, Phys. Rev. B **41**, 4017 (1990).
[31] A. F. Andreev, Zh. Éksp. Teor. Fiz. **46**, 1823 (1964) [Sov. Phys. JETP **19**, 1228 (1964)].
[32] J. Kurkijärvi and D. Rainer, in *Modern Problems in Condensed Matter Sciences*, edited by W. P. Halperin and L. P. Pitaerskii (Elsevier, Amsterdam, 1989).
[33] M. Cahay, M. McLennan, and S. Datta, Phys. Rev. B **37**, 10 125 (1988).
[34] P. F. Bagwell, Phys. Rev. B **46**, 12 573 (1992).
[35] C. W. J. Beenakker and H. van Houten, in *Nanostructures and Mesoscopic Systems*, edited by W. P. Kirk and M. A. Reed (Academic, New York, 1992).
[36] B. J. van Wees, K. M. H. Lenssen, and C. J. P. M. Harmans, Phys. Rev. B **44**, 470 (1991).
[37] R. Landauer, Philos. Mag. **21**, 863 (1970); M. Büttiker, Phys. Rev. Lett. **57**, 1761 (1986).