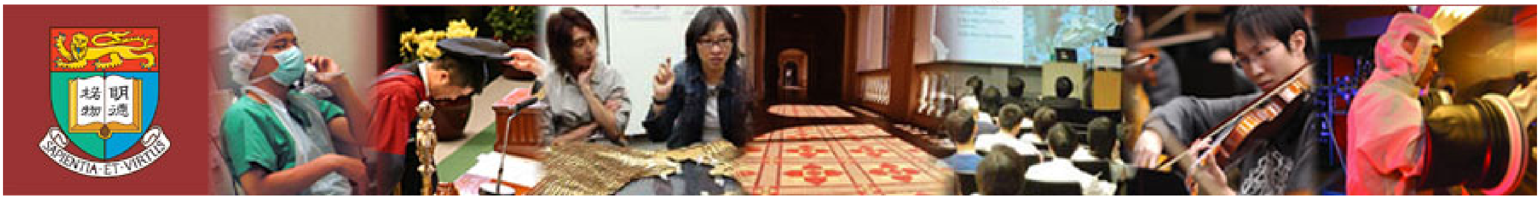

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Radiation transport equations in non-Riemannian space-times

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The transport equations for polarized radiation transfer in non-Riemannian, Weyl-Cartan type space-times are derived, with the effects of both torsion and nonmetricity included. To obtain the basic propagation equations we use the tangent bundle approach. The equations describing the time evolution of the Stokes parameters, of the photon distribution function and of the total polarization degree can be formulated as a system of coupled first-order partial differential equations. As an application of our results we consider the propagation of the cosmological gamma-ray bursts in spatially homogeneous and isotropic spaces with torsion and nonmetricity. For this case the exact general solution of the equation for the polarization degree is obtained, with the effects of the torsion and nonmetricity included. The presence of a non-Riemannian geometrical background in which the electromagnetic fields couple to torsion and/or nonmetricity affect the polarization of photon beams. Consequently, we suggest that the observed polarization of prompt cosmological gamma-ray bursts and of their optical afterglows may have a propagation effect component, due to a torsion/nonmetricity induced birefringence of the vacuum. A cosmological redshift and frequency dependence of the polarization degree of gamma-ray bursts also follows from the model, thus providing a clear observational signature of the torsional/nonmetric effects. On the other hand, observations of the polarization of the gamma-ray bursts can impose strong constraints on the torsion and nonmetricity and discriminate between different theoretical models.

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I. INTRODUCTION

The study of the propagation of electromagnetic radiation in gravitational fields plays an essential role in relativistic astrophysics and cosmology. In order to obtain a correct and consistent description of the radiative transfer processes for photons propagating on cosmological distances or on the cosmic microwave background one must also take into account the effects of the background geometry on the radiation emitted near the surface of neutrons stars or black holes [1]. The radiation follows curved paths according to the underlying geometry and is redshifted or blueshifted. Spatial curvature in some geometries, like, for example, the Kerr geometry, rotates the polarization of photons propagating through them.

Generally, one cannot solve Maxwell's equations exactly for waves propagating in curved space-times or in a relativistic medium. Instead one either uses a geometrical optics approximation or a kinetic description of the multiphoton system, by treating photons as massless classical particles, characterized by a four-momentum p at an event x [2]. In the geometrical optics approximation, which is reasonable in most of the cases, the degree of polarization is not affected by the underlying geometry. The field vector propagate parallel along the light paths. For nonflat curved paths, a rotation of the polarization angle results.

Astrophysicists measure intensities and spectral distributions. These can conveniently be described by a distribution function, which is defined on phase space and is directly related to the spectral intensity. In this picture the propagation of radiation is described by an equation of radiative transfer, which is a differential equation for the photon distribution function. In addition to the above-mentioned quantities, degrees and plane of polarization of the observed radiation provide important means of information about the source and the intervening medium and geometry.

A general relativistic form of the Boltzmann transport equation for particles or radiation interacting with an external medium has been developed by Lindquist [3]. The comoving frame radiation transfer equation in an Eulerian coordinate system was obtained by Riffert [4]. A transport equation for photons in curved Riemannian space-times was derived by Bildhauer [5], by making use of the covariant generalization of the Wigner transformation. Two linear equations on the tangent bundle are obtained whose correction terms to the Liouville equation and the classical mass-shell condition are of the order of the wavelength over the variation length of the background geometry. The comoving frame transfer equation for the Stokes parameters in an arbitrary background metric in an Eulerian coordinate system has been obtained in [6] and applied to the study of the Berry's phase for polarized light in [7].

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A long time ago Brans [8] has suggested that a rotation of the polarization plane of the radiation arises whenever there is shear, by an effect analogous to the Thomas precession. Therefore if there is a large scale anisotropy in the expansion of the Universe, the microwave background radiation is expected to be linearly polarized. In Friedmann-Robertson-Walker models with small expansion anisotropy the observed rotation of the polarization plane would be appreciable and constant over the celestial sphere in closed (Bianchi type IX) models or it should vanish in flat (type I) and open (type V) models [9]. Hence the study of the polarization of the cosmic microwave background radiation can imposes strong limits on the anisotropy and other fundamental physical parameters of the Universe. A rigorous first-order solution with respect to the anisotropy for the equation of polarized radiation transfer in a homogeneous anisotropic Universe was obtained in [10]. The degree of polarization of the background radiation is very sensitive to the recombination dynamics and to the reheating epoch. For a Bianchi type-II space-time with a frozen-in magnetic field the polarization and anisotropy properties of the cosmic microwave background radiation have been studied in [11]. Faraday-rotation effects, such as the rotation of the linear polarization plane, are found to be independent of the spatial curvature effect and are thus the same as in type-I models.

Almost immediately after the birth of general relativity (GR), more general geometries, with nonmetricity and torsion, have been proposed by Weyl and Cartan, in order to incorporate in a geometric framework the effects of the electromagnetism and of the angular momentum. Later on, these extensions of the classical GR have been incorporated in the different gauge theoretical formulations of gravity [12]. There are different gauge theories of gravity in dependence of the choice of the gravitational gauge group and of the gravitational Lagrangian. One of the most studied gauge theories of gravity is the Poincaré gauge theory, which requires a generalization of the Riemannian geometry and the introduction of the torsion of the space-time [13]. Nonsymmetric gravitation theories, in which the gravitational field is described by a nonsymmetric metric tensor, whose antisymmetric part couples directly to the electromagnetic field, have also been proposed [14].

The study of the propagation of light in a gravitational field has provided the classic observational tests of general relativity. The modifications of the geometry due to the non-Riemannian effects, like torsion and nonmetricity, further affect the motion of photons, by inducing a birefringence of the vacuum which can modify the polarization of the photons as they propagate through the gravitational field. Thus the study of the polarization of photons from astrophysical or cosmological sources can provide a valuable tool for discriminating between the different modifications and extensions of general relativity. Ni [15] has shown that nonmetric gravitational fields can single out linear polarization states of light that propagate with different speeds. He also proposed to use pulsar polarization data to impose constraints on nonmetric theories. Limits on the gravity-induced polarization of the Zeeman components of the solar spectral lines have been obtained in [16]. The polarization of light from magnetic white dwarfs can be used to impose constraints on the gravity-induced birefringence of space in nonmetric gravitational theories [17]. By using the magnetic white dwarf Grw $+70°$ 8247 polarization data one can obtain the constraint $l_*^2 < (4.9\ \mathrm{km})^2$, where l_* is the nonsymmetric gravitational theory charge.

In conventional Maxwell-Lorentz electrodynamics, the propagation of light is influenced by the metric only, and not by the torsion T of the space-time. However, there is a possibility of interaction between light and torsion if the latter is nonminimally coupled to the electromagnetic field F by means of a Lagrangian of the form $\sim l^2F^2T^2$, where l is a coupling constant. Several such couplings have been proposed and analyzed recently. A supplementary Lagrangian of the form $L = \lambda_0 l^2(T_\alpha \wedge F)T^\alpha \wedge F$ was considered in [18], and it was shown that it can yield birefringence in the vacuum. Nonminimally coupled homogeneous and isotropic torsion field in a Friedmann-Robertson-Walker geometry affects the speed of light, with the photons propagating with a torsion-dependent speed. In fact, torsion generates three major effects affecting the propagation of light: it produces an axion field that induces an optical activity into space-time, modifies the light cone structure that yields birefringence of the vacuum and modifies the speed of light in a torsion-dependent way [19]. An example of a metric-affine gauge theory of gravity in which torsion couples nonminimally to the electromagnetic field was considered in [20]. The coupling causes a phase difference to accumulate between different polarization states of light as they propagate through the metric-affine gravitational field. The model has been constrained by using solar spectropolarimetric observations, which allow one to set an upper bound on the relevant coupling constant k, $k^2 < (2.5\ \mathrm{km})^2$.

The confirmation that at least some gamma-ray bursts (GRBs) are indeed at cosmological distances raises the possibility that observations of these could provide interesting constraints on the fundamental laws of physics (for reviews on GRBs, see [21–24]). The fine-scale time structure and hard spectra of GRB emissions are very sensitive to the possible dispersion of electromagnetic waves *in vacuo*. Hence the study of short-duration photon bursts propagating over cosmological distances is the most promising way to probe the quantum gravitational and/or the effects related to the existence of extra dimensions [25]. The modification of the group velocity of the photons by the quantum effects would affect the simultaneity of the arrival times of photons with different energies. Thus, given a distant, transient source of photons, one could measure the differences in the arrival times of sharp transitions in the signals in different energy bands and use this information to constrain quantum gravity and/or multidimensional effects [26].

The announcement of the results of the polarization measurements of the prompt gamma rays and of the optical afterglows of gamma-ray bursts has attracted great interest recently. Coburn and Boggs [27] claimed the measurement of a very large linear polarization, $80 \pm 20\%$, of the prompt gamma rays from an extremely bright burst, GRB 021206. However, this polarization measurement has been criticized by Rutledge and Fox [28], who obtained an upper limit of only $<4.1\%$ at 90% confidence from the same data, while in Wigger *et al.* [29] a value of 41% (+ 57% − 44%) is found. One the other hand, the polarization measurements of the optical afterglows are much more convincing. The first positive detection of the polarization was for GRB 990510 with the degree of $1.7 \pm 0.2\%$, and since then polarized emission has been measured in several other afterglows (see [30] for a review).

It is the purpose of the present paper to generalize the transport equations for polarized radiation [3–6] for the case of the polarized radiation propagating in a space-time manifold with non-Riemannian geometry. More exactly, we shall consider the case of the so-called Weyl-Cartan space-times, whose geometrical structure is described by three tensors: the metric, the torsion and the nonmetricity [13]. In this case the propagation equations for the radiation distribution function, for the Stokes parameters, for the linear polarization and for the degree of the total polarization can be formulated as a system of a first-order partial differential equations on the photon phase space, with the non-Riemannian effects included via the contorsion tensor.

As an astrophysical application of the obtained equations we consider the propagation of gamma-ray bursts in a non-Riemannian geometrical background. The general solution of the propagation equation for the total polarization degree can be obtained in an exact form. The presence of a non-Riemannian geometrical background, in which the electromagnetic fields couple to torsion and/or nonmetricity affect the polarization of photon beams. The polarization of the beam is generally a function of time, with non-Riemannian effects induced via the contorsion tensor. Since most of the gamma-ray bursts have a cosmological origin, due to the long propagation times, the influence of the torsion and nonmetricity could significantly affect their polarization. Therefore we suggest that the observed polarization of the prompt gamma-ray bursts and of their optical afterglows could also contain a propagation effect component, due to a torsion/nonmetricity induced birefringence of the vacuum. If such a component could be unambiguously detected, this would provide a significant test for the existence of the non-Riemannian geometrical effects in our Universe. However, there are many conventional astrophysical mechanisms that could explain the polarization of the gamma-ray bursts, and affect the propagation of the electromagnetic radiation. The uncertainties in the knowledge of the astrophysical environment in which photons propagate, and the difficulty in separating the different physical effects contributing to the polarization of the gamma-ray bursts make the practical detection of the torsion or nonmetricity an extremely difficult and very challenging observational task.

The present paper is organized as follows. The transport equations for polarized light in Weyl-Cartan space-times are derived in Sec. II. The general solution for the total polarization degree for a homogeneous and isotropic background geometry is obtained in Sec. III. Constraints on torsion and nonmetricity obtained from the observed polarization of gamma-ray bursts are obtained in Sec. IV. In Sec. V we discuss and conclude our results.

II. TRANSPORT EQUATIONS FOR POLARIZED LIGHT IN WEYL-CARTAN SPACE-TIMES

Relativistic transport theory finds its most elegant and natural expression in terms of geometric structures defined in the tangent bundle over the space-time manifold.

Consider a time-oriented Lorentzian four-dimensional space-time manifold M, with metric g of signature $(+,-,-,-)$. The tangent bundle $T(M)$ is a real vector bundle whose fibers at a point $x \in M$ is given by the tangent space $T_x(M)$. If X is a tangent vector and s is a section of $T(M)$, then a connection ∇ is a rule $\nabla_X(s)$ for taking the directional derivative of s in the direction X satisfying the properties of linearity in s and X, behaving like a first-order differential operator and being tensorial in X. The curvature operator is defined as $R(X,Y) = \nabla_x\nabla_Y(s) - \nabla_Y\nabla_X(s) - \nabla_{[X,Y]}(s)$, where $R(\partial/\partial x^\mu, \partial/\partial x^\nu)(\mathbf{e}_i) = \mathbf{e}_j R^j_{i\mu\nu}$, where $\{\mathbf{e}_1, \mathbf{e}_2, \dots, \mathbf{e}_k\}$ is a local frame defined on each neighborhood $U \subset M$. The curvature operator satisfies the properties of multilinearity, antisymmetry and tensoriality. The torsion operator on $T(M)$ is defined as $T(X,Y) = \nabla_X Y - \nabla_Y X - [X,Y]$. This is a vector field with components $T(\partial/\partial x^\mu, \partial/\partial x^\nu) = (\Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\nu\mu})(\partial/\partial x^\lambda) = T^\lambda_{\mu\nu}(\partial/\partial x^\lambda)$, where $\Gamma^\lambda_{\mu\nu}$ are the Christoffel symbols on $T(M)$ and $T^\lambda_{\mu\nu}$ is the torsion tensor [31]. Moreover, we assume that the space-time manifold M is equipped with a conformal structure, i.e. with a class $[g]$ of conformally equivalent Lorentz metrics (and not a definite metric as in general relativity). This corresponds to the requirement that it should only be possible to compare lengths at one and the same world point. We also suppose that the connection ∇ respects the conformal structure. Differentially this means that for any $g \in [g]$ the covariant derivative ∇g should be proportional to g: $\nabla g = -2A \otimes g$ ($\nabla_\lambda g_{\mu\nu} = -2A_\lambda g_{\mu\nu} = -Q_{\lambda\mu\nu}$), where $A = A_\mu dx^\mu$ is a differential 1-form and Q is called the nonmetricity [32]. A change of calibration of the metric induces a gauge transformation for A: $g \to \exp(2\lambda)g$, $A \to A - d\lambda$. Therefore in terms of the torsion and nonmetricity the connection $\Gamma^{\lambda}_{\mu\nu}$ on M can be expressed as

$$\Gamma^{\lambda}_{\mu\nu} = \gamma^{\lambda}_{\mu\nu} + T^{\lambda}_{\mu\nu} + T_{\mu\nu}{}^{\lambda} + T_{\nu\mu}{}^{\lambda} + \frac{1}{2}(Q_{\mu\nu}{}^{\lambda} - Q_{\mu}{}^{\lambda}{}_{\nu} - Q_{\nu}{}^{\lambda}{}_{\mu}), \quad (1)$$

where $\gamma^{\lambda}_{\mu\nu}$ is the Christoffel symbol computed from the metric g by using the general relativistic prescription, $\gamma^{\mu}_{\alpha\beta} = g^{\mu\nu}(\partial_\alpha g_{\nu\beta} + \partial_\beta g_{\nu\alpha} - \partial_\nu g_{\alpha\beta})$. Generally, we may represent the connection as $\Gamma^{\lambda}_{\mu\nu} = \gamma^{\lambda}_{\mu\nu} - K_{\nu\mu}{}^{\lambda}$, where $K_{\nu\mu}{}^{\lambda}$, the contorsion tensor, is a function of the torsion T and of the nonmetricity Q [13].

In the space-time M the instantaneous state of a photon is given by a four-momentum $p \in T_x(M)$ at an event $x \in M$. The one-particle phase space P_γ is a subset of the tangent bundle given by [5]

$$P_\gamma := \{(x, p) | x \in M, p \in T_x(M), p^2 = 0\}. \quad (2)$$

A state of a multiphoton system is described by a continuous, non-negative distribution function $f(x, p)$, defined on P_γ, and which gives the number dN of the particles of the system which cross a certain spacelike volume dV at x, and whose 4-momenta p lie within a corresponding three-surface element $d\vec{p}$ in the momentum space. The mean value of f gives the average number of occupied photon states (x, p). Macroscopic, observable quantities can be defined as moments of f.

Let $\{x^\alpha\}$, $\alpha = 0, 1, 2, 3$ be a local coordinate system in M, defined in some open set $U \subset M$. Then $\{\partial/\partial x^\alpha\}$ is the corresponding natural basis for tangent vectors. We express each tangent vector p in U in terms of this basis as $p = p^\alpha(\partial/\partial x^\alpha)$ and define a system of local coordinates $\{z^A\}$, $A = 0, \ldots, 7$ in $T_U(M)$ as $z^\alpha = x^\alpha$, $z^{\alpha+4} = p^\alpha$. This defines a natural basis in the tangent space given by $\{\partial/\partial z^A\} = \{\partial/\partial x^\alpha, \partial/\partial p^\alpha\}$. A vertical vector field over TM is given by $\pi = p^\alpha \partial/\partial p^\alpha$. The geodesic flow field σ, which can be constructed over the tangent bundle, is defined as $\sigma = p^\alpha \partial/\partial x^\alpha - p^\alpha p^\gamma \Gamma^{\beta}_{\alpha\gamma} \partial/\partial p^\beta = p^\alpha D_\alpha$, where $D_\alpha = \partial/\partial x^\alpha - p^\gamma \Gamma^{\beta}_{\alpha\gamma} \partial/\partial p^\beta$. Physically, σ describes the phase flow for a stream of particles whose motion through space-time is geodesic.

Therefore the transport equation for the propagation of a photon beam in a curved arbitrary non-Riemannian space-time is given by

$$\left(p^\alpha \frac{\partial}{\partial x^\alpha} - p^\alpha p^\beta \Gamma^{i}_{\alpha\beta} \frac{\partial}{\partial p^i}\right) f = 0. \quad (3)$$

By taking explicitly into account the decomposition of the connection in an arbitrary non-Riemannian space-time with torsion and nonmetricity, Eq. (3) for the photon distribution function takes the form

$$\left(p^\alpha \frac{\partial}{\partial x^\alpha} - p^\alpha p^\beta \gamma^{i}_{\alpha\beta} \frac{\partial}{\partial p^i}\right) f(x^\mu, p^\nu) + p^\alpha p^\beta K_{\alpha\beta}{}^{i} \frac{\partial f(x^\mu, p^\nu)}{\partial p^i} = 0. \quad (4)$$

To describe polarized or partially polarized radiation, the distribution function has to be generalized to a distribution tensor $f_{\alpha\beta}(x, p) = \sum_{a,b=1}^{2} f_{ab} e^{(a)}_{\alpha} e^{(b)}_{\beta}$, where $e^{(a)}_{\alpha}$ ($a = 1, 2$) two spacelike unit vectors orthogonal to the direction of propagation. f_{ab} is a Hermitian and positive matrix, satisfying the conditions $\mathrm{Tr}(f_{ab}(x, p)) = f(x, p)$, $\det f_{ab}(x, p) \leq f(x, p)$, and $\bar{f}_{ab} = f_{ba}$. The matrix f_{ab} can be parametrized by the Stokes parameters ξ_i, ξ_i: $P_\gamma \to [-1, 1]$, $i = 1, 2, 3$, three measurable quantities, which describe polarization as [33]

$$f_{ab} = \frac{1}{2} f(x, p) \begin{pmatrix} 1 + \xi_3 & \xi_1 + i\xi_2 \\ \xi_1 + i\xi_2 & 1 + \xi_3 \end{pmatrix} = \frac{1}{2} f(x, p)(1 + \xi \cdot \sigma), \quad (5)$$

where $\sigma = (\sigma_1 \sigma_2 \sigma_3)$, with σ_i the Pauli matrices, and $\xi = (\xi_1 \xi_2 \xi_3)$.

In a spacelike hypersurface element Ω of P_γ at (x, p) the number of photons linearly polarized along the $e^{(1)}$ and $e^{(2)}$ axes are $\frac{1}{2}(1 + \xi_3) f \Omega$ and $\frac{1}{2}(1 - \xi_3) f \Omega$. $\frac{1}{2}(1 + \xi_1) f \Omega$ is the result of the measurement of the linear polarization in a direction at $\pi/4$ to the $e^{(1)}$ axis. $P_L = (\xi_1^2 + \xi_2^2)^{1/2}$ is the degree of linear polarization. The parameter ξ_2, the antisymmetric part of f_{ab}, represents the degree of circular polarization. A measure for the right-hand and left-hand circular polarization is $(1 + \xi_2)/2$ and $(1 - \xi_2)/2$, respectively. The polarization degree is given by $P = (\sum_{i=1}^{3} \xi_i^2)^{1/2}$. The polarization angle is defined as $\Psi = \tan(\xi_1/\xi_3)/2$ [5].

The transport equation satisfied by the distribution tensor $f_{\mu\nu}(x, p)$, describing polarized radiation, is [3,5]

$$\left(p^\alpha \nabla_\alpha - p^\alpha p^\beta \Gamma^{i}_{\alpha\beta} \frac{\partial}{\partial p^i}\right) f_{\mu\nu}(x, p) = 0. \quad (6)$$

In a general non-Riemannian space-time with nonvanishing torsion and nonmetricity the equation satisfied by the distribution tensor takes the form

$$\left(p^\alpha \frac{\partial}{\partial x^\alpha} - p^\alpha p^\beta \gamma^{i}_{\alpha\beta} \frac{\partial}{\partial p^i}\right) f_{\mu\nu} + p^\alpha p^\beta K_{\alpha\beta}{}^{i} \frac{\partial f_{\mu\nu}}{\partial p^i} - p^\alpha \gamma^{\lambda}_{\mu\alpha} f_{\lambda\nu} - p^\alpha \gamma^{\lambda}_{\nu\alpha} f_{\mu\lambda} + p^\alpha K_{\mu\alpha}{}^{\lambda} f_{\lambda\nu} + p^\alpha K_{\nu\alpha}{}^{\lambda} f_{\mu\lambda} = 0. \quad (7)$$

As in the Riemannian space-time we may introduce a pseudo-orthonormal basis of vectors $e_\alpha = e^{a}_{\alpha} \partial_a$ (a tetrad), and the dual basis $\theta^\alpha = e^{\alpha}_{a} dx^a$ of the one-forms. The components e^{a}_{α} and their reciprocals e^{α}_{a} satisfy the relations

$e^a_\alpha e^\alpha_b = \delta^a_b$ and $e^i_\alpha e^\beta_i = \delta^\beta_\alpha$, respectively. The metric tensor can be expressed as $g_{ab} = e^\alpha_a e^\beta_b \eta_{\alpha\beta}$, where $\eta_{\alpha\beta}$ is the Minkowski metric tensor [31]. The object of anholonomy $\Omega_{ab}{}^c = e^\alpha_a e^\beta_b \partial_{[i} e_{f]}{}^c$, $\Omega_{abc} = \Omega_{ab}{}^d g_{dc}$ measures the noncommutativity of the tetrad basis. The connection expressed in these anholonomic coordinates is [13,31]

$$\Gamma_{abc} = \Gamma^d_{ab} g_{dc} = -\Omega_{abc} + \Omega_{bca} - \Omega_{cab} - K_{abc}. \quad (8)$$

Any tangent vector p at x can be expressed as $p = p^\alpha e_\alpha$. If we set $e_a = e^\alpha_a e_\alpha$, then $p^a = e^a_\alpha p^\alpha$ and $p^\alpha = e^\alpha_a p^a$. In terms of the tetrad components the transport equations can be expressed in the forms

$$p^a D_a f = 0, \qquad p^a D_a f_{bc} - p^a \Gamma^d_{ba} f_{dc} - p^a \Gamma^d_{ca} f_{bd} = 0, \quad (9)$$

where $D_a = \partial_a - \Gamma^b_{ac} p^c \partial / \partial p^b$.

As an application of the tetrad formalism briefly described above we consider the transformation from the coordinate basis $e_\alpha = \partial_\alpha$ to the basis b_α of an arbitrary coordinate system, obtained from a Lorentz boost of the coordinate basis, so that $b_\alpha = \hat{L}^\beta_\alpha e_\beta$, $e_\alpha = L^\beta_\alpha b_\beta$, with the transformation matrices $\hat{L}^\beta_\alpha$ and L^β_α satisfying the condition $L^\alpha_\mu \hat{L}^\beta_\alpha = \delta^\beta_\mu$. The explicit form of the matrix components is given by $\hat{L}^b_0 = u^b$, $\hat{L}^0_\alpha = \lambda a_\alpha u_\alpha$ and $\hat{L}^\beta_\alpha = a_\alpha(\delta^\beta_\alpha + k u^\beta u_\alpha)$, where $a_\alpha = \sqrt{g^{\alpha\alpha}}$, $\lambda = \sqrt{-g^{00}}$, and $k = (1 + u^0\sqrt{-g_{00}})^{-1}$. The components of the four-momentum with respect to b_α are $\hat{p}^\alpha = L^\alpha_\beta p^\beta$. The photon four-momentum is described by its frequency, defined as $\nu = -p \cdot u = -\hat{p}^a b_a \cdot u = \hat{p}^0$, and two angle for the direction of propagation. Hence, the photon four-momentum can be described in the basis $\{b_\alpha\}$ as $\hat{p}^\alpha = \nu(1, \hat{n})$. To parametrize the unit three-vector $\hat{n}$ we introduce spherical coordinates in the momentum space so that

$$\hat{n}^1 = \mu, \qquad \hat{n}^2 = \sqrt{1-\mu^2}\cos\phi, \qquad \hat{n}^3 = \sqrt{1-\mu^2}\sin\phi. \quad (10)$$

By assuming that the observer places a two-dimensional screen normal to the propagation direction of the photon beam in his rest frame, we can perform a further transformation of the basis so that in the new basis $d_\alpha = \hat{D}^\beta_\alpha b_\beta$ the spatial propagation direction of the photon is now the z axis. The transformation matrix $\hat{D}^\beta_\alpha$ is given by [6]

$$\hat{D}^\beta_\alpha = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{1-\mu^2} & 0 & \mu \\ 0 & -\mu\cos\phi & \sin\phi & \sqrt{1-\mu^2}\cos\phi \\ 0 & -\mu\sin\phi & -\cos\phi & \sqrt{1-\mu^2}\sin\phi \end{pmatrix}. \quad (11)$$

The components of the four-momentum in the new basis $\{d_\alpha\}$are $\bar{p}^\alpha = \nu(1,0,0,1)$. With respect to the initial basis $e_\alpha = \partial_\alpha$we have $d_\alpha = \hat{A}^\beta_\alpha e_\beta$, where $\hat{A}^\beta_\alpha = \hat{L}^\beta_\mu \hat{D}^\mu_\alpha$. The inverse transformation is given by $e_\alpha = A^\beta_\alpha d_\beta$. The components of the four-momentum in the two basis are related by the transformations $\bar{p}^\alpha = A^\alpha_\beta p^\beta = D^\alpha_\beta \hat{p}^\beta$.

With respect to the basis $\{d_\alpha\}$ the transport equation can be written as

$$\left(\hat{A}^\alpha_\beta \bar{p}^\beta \frac{\partial}{\partial x^\alpha} - \bar{p}^\alpha \bar{p}^\beta \bar{\gamma}^i_{\alpha\beta} \frac{\partial}{\partial \bar{p}^i}\right)\bar{f}_{ab} + \bar{p}^\alpha \bar{p}^\beta \bar{K}_{\alpha\beta}{}^i \frac{\partial \bar{f}_{ab}}{\partial \bar{p}^i} - \bar{p}^\alpha \bar{\gamma}^c_{a\alpha} \bar{f}_{cb} - \bar{p}^\alpha \bar{\gamma}^c_{b\alpha} \bar{f}_{ac} + \bar{p}^\alpha \bar{K}^c_{a\alpha} \bar{f}_{cb} + \bar{p}^\alpha \bar{K}^c_{b\alpha} \bar{f}_{ac} = 0, \quad (12)$$

where $\bar{K}_{\alpha\beta}{}^i$ are the components of the contorsion tensor with respect to $\{d_\alpha\}$. The new connection coefficients $\bar{\gamma}^\lambda_{\alpha\beta}$ and $\bar{K}_{\alpha\beta}{}^\lambda$ can be calculated by means of the transformations

$$\bar{\gamma}^{\ \lambda}_{\alpha\beta} = A^\lambda_\mu \hat{A}^\eta_\beta (\hat{A}^\nu_\alpha \gamma^\mu_{\eta\nu} + \partial_\eta \hat{A}^\mu_\alpha), \quad (13)$$

and

$$\bar{K}_{\ \alpha\beta}{}^\lambda = A^\lambda_\mu \hat{A}^\eta_\beta (\hat{A}^\nu_\alpha \bar{K}_{\eta\nu}{}^\mu + \partial_\eta \hat{A}^\mu_\alpha), \quad (14)$$

respectively.

By denoting

$$D = \frac{1}{\nu}\left(\hat{A}^\alpha_\beta \bar{p}^\beta \frac{\partial}{\partial x^\alpha} - \bar{p}^\alpha \bar{p}^\beta \bar{\gamma}^i_{\alpha\beta} \frac{\partial}{\partial \bar{p}^i} + \bar{p}^\alpha \bar{p}^\beta \bar{K}_{\alpha\beta}{}^i \frac{\partial \bar{f}_{ab}}{\partial \bar{p}^i}\right), \quad (15)$$

it follows that in a non-Riemannian space-time the radiation distribution function and the Stokes parameters satisfy the equations

$$Df = 0, \qquad D\xi_1 - 2(\bar{\gamma}^1_{20} + \bar{\gamma}^2_{23})\xi_3 + 2(\bar{K}^1_{20} + \bar{K}^2_{23})\xi_3 = 0, \quad (16)$$

$$D\xi_2 = 0, \qquad D\xi_3 + 2(\bar{\gamma}^1_{20} + \bar{\gamma}^2_{23})\xi_1 - 2(\bar{K}^1_{20} + \bar{K}^2_{23})\xi_1 = 0. \quad (17)$$

The evolution equations describing the degrees of the linear polarization P_L and the total polarization P are

$$DP_L = 0, \qquad DP = 0. \quad (18)$$

In the limit of the zero contorsion $K_{\alpha\beta}{}^i \to 0$, corresponding to the transition to the Riemannian geometry, from Eqs. (16)–(18) we recover the transport equations given in [5,6].

III. RADIATION TRANSFER IN ISOTROPIC AND HOMOGENEOUS WEYL-CARTAN SPACE-TIMES

In order to analyze the influence of the non-Riemannian background on the propagation properties of the electromagnetic radiation we adopt the simplifying assumption of an isotropic, homogeneous and flat Friedmann-Robertson-Walker type geometry, that is, we assume that the metric of the Universe is given by

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2). \tag{19}$$

Generally in the comoving reference frame the torsion tensor is determined by two functions of time $T(t)$ and $\bar{T}(t)$, so that $T_{0\alpha\beta} = (T(t)/l)\delta_{\alpha\beta}$ and $T_{\alpha\beta\gamma} = (\bar{T}(t)/l)\varepsilon_{\alpha\beta\gamma}$, where l is a coupling constant and $\varepsilon_{\alpha\beta\gamma}$ is the totally antisymmetric tensor in three dimensions [19]. By supposing that the theory is invariant under space inversions we have $\bar{T}(t) \equiv 0$. Therefore the only nonvanishing components of the torsion are $T(t) = T^1_{10} = T^2_{20} = T^3_{30}$. The nonmetricity torsion is defined by three functions of time $Q_i(t)$, $i = 1, 2, 3$, so that, in an anholonomic basis we have $\bar{Q}_{110} = \bar{Q}_{220} = \bar{Q}_{330} = Q_1(t)$, $\bar{Q}_{000} = Q_2(t)$, and $\bar{Q}_{011} = \bar{Q}_{022} = \bar{Q}_{033} = Q_3(t)$ [34].

By denoting $K(t) = -(T(t) + Q_3(t))$, the transport equation of the total polarization degree P of the gamma rays in a non-Riemannian space-time is given by

$$\left[\frac{\partial}{\partial t} + \frac{\mu}{a}\frac{\partial}{\partial x} + \frac{\sqrt{1-\mu^2}\cos\phi}{a}\frac{\partial}{\partial y} + \frac{\sqrt{1-\mu^2}\sin\phi}{a}\frac{\partial}{\partial z} - \nu\left(\frac{\dot{a}}{a} - K(t)\right)\frac{\partial}{\partial \nu}\right]P = 0. \tag{20}$$

We assume that the photon is emitted at a point $Q = (x_0, y_0, z_0)$ with frequency ν_0 in the direction (μ_0, ϕ_0) and is observed at the point $O = (x, y, z)$ with the corresponding four-momentum quantities (ν, μ, ϕ).

Equation (20) represents a first-order partial differential equations with the characteristics given by

$$dt = \frac{adx}{\mu} = \frac{ady}{\sqrt{1-\mu^2}\cos\phi} = \frac{adz}{\sqrt{1-\mu^2}\sin\phi} = -\frac{d\nu}{\nu[\frac{\dot{a}}{a} + K(t)]}, \tag{21}$$

which can be immediately integrated to give

$$\sqrt{1-\mu^2}\cos\phi x - \mu y = \sqrt{1-\mu_0^2}\cos\phi_0 x_0 - \mu_0 y_0 = \text{constant}, \tag{22}$$

$$\sqrt{1-\mu^2}\sin\phi x - \mu z = \sqrt{1-\mu_0^2}\sin\phi_0 x_0 - \mu_0 z_0 = \text{constant}, \tag{23}$$

$$\sin\phi y - \cos\phi z = \sin\phi_0 y_0 - \cos\phi_0 z_0 = \text{constant}, \tag{24}$$

$$\ln(a\nu) + \int K(t)dt = \text{constant}, \tag{25}$$

$$\begin{aligned} x - \mu\int\frac{dt}{a} &= \text{constant}, \\ y - \sqrt{1-\mu^2}\cos\phi\int\frac{dt}{a} &= \text{constant}, \\ z - \sqrt{1-\mu^2}\sin\phi\int\frac{dt}{a} &= \text{constant}. \end{aligned} \tag{26}$$

Hence the general solution of Eq. (20) is given by

$$\begin{aligned} P = P\Big(&\sqrt{1-\mu^2}\cos\phi x - \mu y, \sqrt{1-\mu^2}\sin\phi x - \mu z, \sin\phi y - \cos\phi z, x - \mu\int\frac{dt}{a}, y \\ &- \sqrt{1-\mu^2}\cos\phi\int\frac{dt}{a}, z - \sqrt{1-\mu^2}\sin\phi\int\frac{dt}{a}, \ln(a\nu) + \int K(t)dt\Big). \end{aligned} \tag{27}$$

For $K(t) \equiv 0$, Eq. (25) gives the usual redshift relation $a\nu = a_0\nu_0$ and Eq. (27) describes the propagation of radiation in the Riemannian space-time of general relativity.

In the following we denote by P_0 the value of the polarization corresponding to the propagation of the electromagnetic radiation in a curved Riemannian space-time, $P_0 = P|_{K(t)\equiv 0}$. We also make the simplifying assumption that generally the dependence of the polarization on the term $\ln(a\nu) + \int K(t)dt$ is linear, so that

$$P = P_0\left[1 + \frac{1}{\ln(a\nu)}\int K(t)dt\right]. \tag{28}$$

In order to obtain a quantitative characterization of the non-Riemannian effects on the radiation propagation we introduce a parameter δ, describing the variation of the polarization due to the propagation effects, and defined as

$$\delta = \frac{P - P_0}{P_0}. \tag{29}$$

In terms of the contorsion tensor, by taking into account that at the receiver $a = a_{\rm rec}$ and by denoting the observed frequency of the radiation by $\nu_{\rm rec}$, we obtain

$$\delta = \frac{1}{\ln(a_{\rm rec}\nu_{\rm rec})}\int K(t)dt. \tag{30}$$

The integration over t can be converted to integration over the redshift z, by using the equality $dt = (dt/da)\times(da/dz)dz = -d_H(z)dz/(1+z)$, where $d_H(z) = (\dot{a}/a)^{-1} = H^{-1}(z)$, where H is the Hubble function. We assume that generally the Hubble function can be written as

$$H(z) = H_0\sqrt{\Omega_{m,0}(1+z)^3 + \Omega_\Lambda + \Omega_{K,0}f(z)}, \tag{31}$$

where $H_0 = 3.24 \times 10^{-18} h\ \mathrm{s}^{-1}$, with $0.5 < h < 1$, $\Omega_{m,0} \approx 0.3$ is the present day matter density parameter, $\Omega_\Lambda \approx 0.7$ is the dark energy density parameter and $\Omega_{K,0}$ is the density parameter formally associated with the non-Riemannian effects. $f(z)$ describes the time variation of the contorsion. $\Omega_{K,0}$ and $f(z)$ are strongly model-dependent quantities and generally they depend on the assumed functional form for the contorsion $K(t)$. Therefore in terms of the redshift, the parameter δ can be expressed as

$$\delta(z) = -\frac{t_H}{\ln(a_{\mathrm{rec}}\nu_{\mathrm{rec}})} \times \int_z^0 \frac{K(z)dz}{(1+z)\sqrt{\Omega_{m,0}(1+z)^3 + \Omega_\Lambda + \Omega_{K,0} f(z)}}, \tag{32}$$

where $t_H = 1/H_0 = 3.09 \times 10^{17} h^{-1}$ s is the Hubble time.

IV. CONSTRAINING THE TORSION AND THE NON-METRICITY WITH GAMMA-RAY BURSTS

As an application of the formalism developed in the previous section we consider the possible effects of a non-Riemannian structure of the space-time on the propagation of the gamma-ray bursts. Gamma-ray bursts (GRBs) are cosmic gamma-ray emissions with typical fluxes of the order of 10^{-5} to 5×10^{-4} erg cm^{-2} with the rise time as low as 10^{-4} s and the duration of bursts from 10^{-2} to 10^3 s. The distribution of the bursts is isotropic and they are believed to have a cosmological origin, recent observations suggesting that GRBs might originate at extragalactic distances [21]. The large inferred distances imply isotropic energy losses as large as 3×10^{53} erg for GRB 971214 and 3.4×10^{54} erg for GRB 990123 [22].

The widely accepted interpretation of the phenomenology of γ-ray bursts is that the observable effects are due to the dissipation of the kinetic energy of a relativistically expanding fireball, whose primal cause is not yet known [23].

The proposed models for the energy source involve merger of binary neutron stars, capture of neutron stars by black holes, differentially rotating neutron stars or neutron star-quark star conversion, etc. ([23] and references therein). However, the most popular model involves the violent formation of an approximately one solar mass black hole, surrounded by a similarly massive debris torus. The formation of the black hole and debris torus may take place through the coalescence of a compact binary or the collapse of a quickly rotating massive stellar core [22]. There are still many open problems concerning GRBs, from which the most important is the problem of the source of the large energy emission during the bursts.

The recent observations of the polarization of the GRB's are considered to be of fundamental importance for the understanding of the nature and properties of these phenomena. The polarization of the afterglows is well established. Typically, the polarization of the afterglows is observed to be at the 1%–3% level, with constant or smoothly variable level and position angle when associated with a relatively smooth light curve. The usual explanation for the polarized radiation of afterglows is the synchrotron emission from the fireball when some kind of asymmetry is present. There are mainly two kinds of asymmetry considered. One class includes the models assuming that ordered magnetic fields play a crucial role. The magnetic fields can be either locally ordered, which corresponds to the magnetic domain model [35], or even entirely aligned within the ejecta, which is magnetized by the central engine [36]. In addition, small regions in which the magnetic field has some degree of order could be amplified by scintillation [37], or by gravitational microlensing [38]. A different class of models postulates that the fireball is collimated. In this case the observer likely sees the fireball off-axis, as the probability of being exactly on-axis is vanishingly small. When the line of sight makes an angle with the collimation axis, the asymmetry and hence a net polarization may arise. The magnetic fields, compressed in the plane normal to the motion, are postulated to be distributed in a random but anisotropic manner [37,39].

A strong gamma-ray polarization may indicate a strongly magnetized central engine, either in pure Poynting-flux-dominated form [40] or in conventional hydrodynamical form, but with a globally organized magnetic field configuration [41]. Models involving inverse Compton scattering with offset beaming angles can also give rise to large degrees of polarization in gamma rays [42]. There are also indications that the polarization degree and position angle may evolve significantly with time for optical transients [43]. Two-component jet models have also been proposed to explain the observed polarization of the afterglows [44].

Hence still there are no definite physical models to consistently predict or explain the observed polarization of the gamma-ray bursts.

Since a non-Riemannian geometrical structure of the space-time manifold cannot be excluded *a priori*, the possible effect of a nonzero contorsion must also be taken into account in the description of the propagation of the gamma-ray bursts. In the following we investigate the effect of different choices of the torsion and nonmetricity, corresponding to different physical and cosmological models, on the parameter $\delta = (P - P_0)/P_0$, describing the deviations in the polarization of the gamma-ray bursts due to the propagation effects in a Weyl-Cartan geometry.

A. Models with constant contorsion

The simplest case that could arise in the analysis of the modifications of the polarization of the gamma-ray bursts due to propagation effects is the one corresponding to a constant contorsion, with $K(z) = K_A =$ constant. Moreover, we ignore the contributions of the torsion and nonmetricity to the density parameter, by taking $\Omega_{K,0} = 0$. Therefore we obtain for δ the simple expression

$$\delta(z) = -\frac{t_H K_A}{\ln(\nu_{\rm rec})} \int_z^0 \frac{dz}{(1+z)\sqrt{\Omega_{m,0}(1+z)^3 + \Omega_\Lambda}}. \quad (33)$$

For this model the variation of δ as a function of z is represented, for different numerical values of the constant contorsion K_0, in Fig. 1.

On the other hand the observational knowledge of δ would allow one to impose some constraints on the value of the contorsion tensor, with

$$K_A \leq \frac{H_0 \ln(a_{\rm rec}\nu_{\rm rec})}{F(z)} \delta, \quad (34)$$

where we denoted $F(z) = \int_0^z dz/(1+z) \times \sqrt{\Omega_{m,0}(1+z)^3 + \Omega_\Lambda}$.

B. The Weyssenhoff spin fluid case

In the framework of the Einstein-Cartan theory, the intrinsic angular momentum (the spin) of a particle is introduced via a generalization of the structure of the space-time, by assuming that the affine connection is non-symmetric. The torsion contributes to the energy momentum of a spin fluid which has the form

$$T_{\mu\nu}^{\rm eff} = (\rho + p - 2s^2)u_\mu u_\nu - (p - s^2)g_{\mu\nu}, \quad (35)$$

where ρ and p are the energy density and the pressure, respectively, and $s^2 = s_{\mu\nu}s^{\mu\nu}$ is the spin. This energy-momentum tensor corresponds to a perfect fluid with spin, called the Weyssenhoff fluid model [13]. The macroscopic spin tensor may be expressed in terms of the spin density tensor $s_{\mu\nu}$ and the four-velocity of the fluid as $\tau_{\mu\nu}^\lambda = u^\lambda s_{\mu\nu}$. For a pressureless fluid ($p = 0$) we have $\rho = mn$ and $s = \hbar n/2$, respectively, where n is the number of particles with mass m per unit volume and s is the spin density. The effect of the spin is dynamically equivalent to introducing into the model some additional non-interacting spin fluid with the energy density $\rho_s = \rho_{0s}/a^6$ [45]. Although the contribution of the spin to the dense matter appears to be negligible small, on a large scale it can produces a "centrifugal force" which is able to prevent the occurrence of singularities in cosmology [13].

By taking into account that for a Weyssenhoff fluid dominated universe the contorsion is proportional to the spin, the parameter δ becomes

$$\delta(z) = -\frac{t_H K_B}{\ln(\nu_{rec})} \times \int_z^0 \frac{(1+z)^2 dz}{\sqrt{\Omega_{m,0}(1+z)^3 + \Omega_{s,0}(1+z)^6 + \Omega_\Lambda}}, \quad (36)$$

where $\Omega_{s,0} = \rho_{s,0}/3H_0^2$ is the density parameter of the spin fluid, with $\rho_{s,0} = \hbar^2 n(0)/16$. S_0 can be related to the mass m of the particles forming the spinning fluid by means of the relation $K_B = \rho_{s,0}/\sqrt{2}m$.

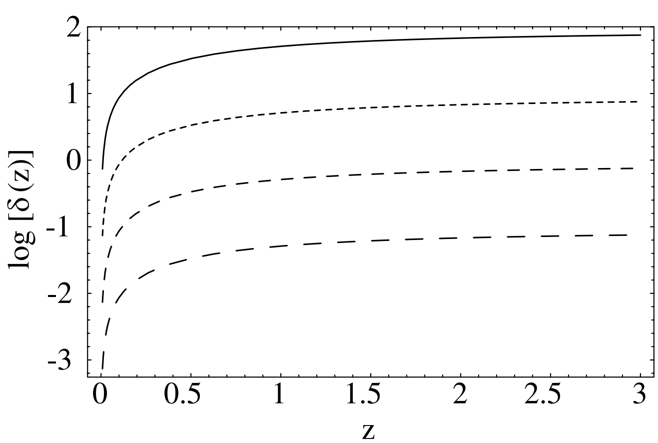

FIG. 1. Variation, as a function of the redshift z, of the parameter δ (in a logarithmic scale) for gamma-ray bursts propagating in a Weyl-Cartan geometry with constant contorsion tensor K_A, for different values of K_A: $K_A = 10^{-14}$ s^{-1} (solid curve), $K_A = 10^{-15}$ s^{-1} (dotted curve), $K_A = 10^{-16}$ s^{-1} (dashed curve) and $K_A = 10^{-17}$ s^{-1} (long dashed curve). For the cosmological parameters we have adopted the values $a_{\rm rec} = 1$, $\Omega_{m,0} = 0.3$ and $\Omega_\Lambda = 0.7$. The frequency at the detector of the gamma-ray bursts is assumed to be $\nu_{\rm rec} = 3 \times 10^{14}$ s^{-1}, corresponding to the optical afterglow emission.

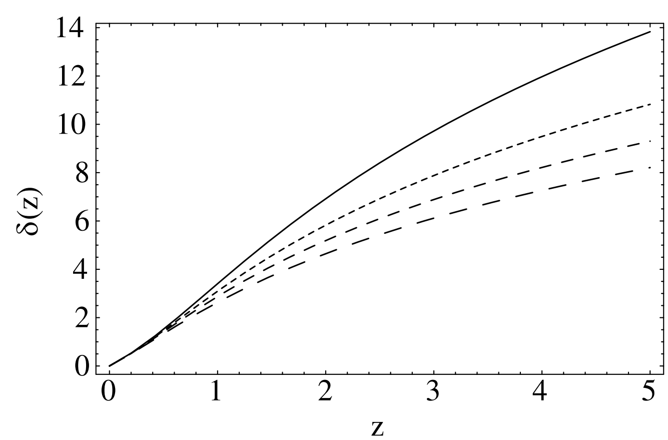

FIG. 2. The parameter δ as a function of the redshift z for gamma-ray bursts propagating in a Weyssenhoff spinning fluid filled universe, with the torsion tensor proportional to the spin density, for different values of the cosmological parameters: $\Omega_{m,0} = 0.25$, $\Omega_\Lambda = 0.7$ and $\Omega_{s,0} = 0.05$ (solid curve), $\Omega_{m,0} = 0.20$, $\Omega_\Lambda = 0.7$ and $\Omega_{s,0} = 0.1$ (dotted curve), $\Omega_{m,0} = 0.15$, $\Omega_\Lambda = 0.7$ and $\Omega_{s,0} = 0.15$ (short dashed curve) and $\Omega_{m,0} = 0.20$, $\Omega_\Lambda = 0.6$ and $\Omega_{s,0} = 0.20$ (long dashed curve). The frequency at the detector of the gamma-ray bursts is assumed to be $\nu_{\rm rec} = 3 \times 10^{14}$ s^{-1}, corresponding to the optical afterglow emission. For the constant K_B we have adopted the value $K_B = 2.5 \times 10^{-16}$ s^{-1}

The variation of the parameter δ for a Weyssenhoff fluid filled universe is represented, for different values of the cosmological parameters, in Fig. 2.

The study of the polarization of the gamma-ray bursts can impose some constraints on s. Once the functional forms of the polarization of the gamma-ray bursts at the source and observer are known, from Eq. (36) one can evaluate the basic physical parameters characterizing the Weyssenhoff fluid and obtain some restrictions on the spin density of the particles.

C. Models with modified double duality ansatz

An extensive study of the spatially homogeneous and $SO(3)$-isotropic cosmologies in the framework of the 10 parameter Lagrangian of the Poincaré gauge theory was performed in [46]. By evaluating the action in the tangent spaces induced by the isotropy group it follows that the torsion tensor has two nonvanishing components $h(t)$ and $f(t)$. By excluding the parity violating terms in the Lagrangian the field equations are invariant under the discrete transformation $f \to -f$ and $s \to -s$, where s is the spin scalar. This symmetry rules out classical spin distributions like the Weyssenhoff fluid we have previously discussed. It is difficult to find a physical interpretation of a spin tensor within this model and therefore the case of a vanishing spin was considered in detail.

A simple class of solutions of the Einstein-Cartan field equations can be obtained by assuming that the axial torsion is nonvanishing and the modified double duality ansatz $\Phi = \Phi_0 =$ constant and $\Psi = 0$ is satisfied, where $\Phi = M + N = R/6$ and $\Psi = fH + F = R_{\alpha\beta\gamma\delta}\varepsilon^{\alpha\beta\gamma\delta}$, with $H = h + \dot{a}/a$, $M = \dot{H} + (\dot{a}/a)H$, $N = H^2 + k/a^2 - f^2/4$ and $F = [\dot{f} + (\dot{a}/a)f]$. By further imposing the constraints $H = 0$ and $k = 0$, it follows that the only nonvanishing component of the torsion is $h = -\dot{a}/a$, and the field equations reduce to $(\dot{a}/a)^2 = (4\pi G/9c_4)\rho - c_0/6c_4$, $c_0, c_4 =$ constants, and the conservation law for the energy and pressure. In this model the function $K(t)$, describing the effect of the torsion on the propagation of the gamma-ray bursts is given by $K(t) = -K_C \ln a(t)$, where K_C is an integration constant. In terms of the redshift we have $K(z) = K_C \ln(1 + z)$. The constants $-c_0/6c_4$ can be interpreted as an effective cosmological constant $\Lambda_{\rm eff}$, generated by the presence of the torsion. Therefore in this model the parameter $\delta(z)$ is given by

$$\delta(z) = -\frac{t_H K_C}{\ln(\nu_{\rm rec})}\int_0^z \frac{\ln(1+z)dz}{(1+z)\sqrt{\Omega_{m,0}(1+z)^3 + \Omega_\Lambda}}, \quad (37)$$

where $\Omega_\Lambda = \Lambda_{\rm eff}/\rho_c$ is the mass density parameter associated to the torsion-generated cosmological constant.

The variation of δ as a function of the redshift is presented in Fig. 3.

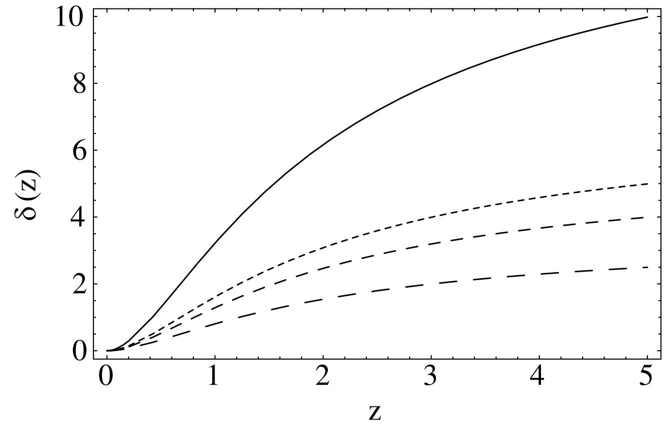

FIG. 3. The parameter δ as a function of the redshift z for gamma-ray bursts propagating in a Riemann-Cartan space-time, with vanishing spin density and the modified double duality ansatz satisfied, for different values of the constant K_C: $K_C = 2 \times 10^{-15}$ s^{-1} (solid curve), $K_C = 10^{-15}$ s^{-1} (dotted curve), $K_C = 8 \times 10^{-16}$ s^{-1} (dashed curve) and $K_C = 5 \times 10^{-16}$ s^{-1} (long dashed curve). For the cosmological parameters we have adopted the values $a_{\rm rec} = 1$, $\Omega_{m,0} = 0.3$ and $\Omega_\Lambda = 0.7$. The frequency at the detector of the gamma-ray bursts is assumed to be $\nu_{\rm rec} = 3 \times 10^{14}$ s^{-1}, corresponding to the optical afterglow emission.

V. DISCUSSIONS AND FINAL REMARKS

In the present paper we have generalized the radiation propagation equations for polarized photons in curved space-times from Riemannian geometries to Weyl-Cartan type geometries, with torsion and nonmetricity. The non-Riemannian background enters in the equations via the contorsion tensor and, for particles propagating on cosmological distances, can induce a significant and observationally detectable change in the photon polarization state. Of course, to fully evaluate this effect one should know the behavior of the contorsion in different classes of generalized gravitation theories with torsion. For the Poincaré gauge theory and for vanishing nonmetricity spatially homogeneous and $SO(3)$-isotropic exact solutions for a 10-parameter Lagrangian have been studied in [46], while the effect of the nonmetricity on the cosmological evolution has been considered in [34]. All these solutions are strongly model dependent and generally have a complicated mathematical form.

On the other hand the study of the polarization of the prompt cosmological gamma-ray bursts and of their afterglows offers, at least in principle, the possibility of observationally testing Poincaré type gauge theories.

Assuming, as an extreme and perhaps unrealistic case, that the initial gamma-ray emission is totally unpolarized, as predicted by some models of the gamma-ray burst emission (see [21–24] and references therein), the detected polarization amount could be mainly due to the propagation effects of the photons traveling on cosmological distances in a non-Riemannian geometrical background. The presence of the contorsion in the radiation transport equations in curved space-times could generate a supplementary polarization of the photon beam. If the polarization of the gamma-ray bursts is indeed due to the coupling between photons and geometry, then the observed degree of the polarization must be redshift dependent, since photons traveling on longer distances will be more affected. The smallness of the observed effect even for high redshift sources ($P = 1.7 \pm 0.2\%$) also seems to suggest mainly a propagation effect.

A second important effect, which follows from the consideration of the non-Riemannian structure of the space-time is related to the prediction of the frequency dependence of the polarization of gamma-ray bursts. Since the dependence of the frequency is logarithmic, the total polarization degree of the gamma-ray component should be smaller by a factor of around 2 than the polarization degree of the optical afterglow emission. Together with the cosmological redshift dependence, the frequency dependence of the polarization degree of gamma-ray bursts provides a clear observational signature of the possible presence of non-Riemannian, Weyl-Cartan type geometrical features in our Universe.

On the other hand, if the initial polarization degree of the gamma-ray bursts could be exactly predicted by some photon emission models, then the comparison of the polarization of the gamma-ray bursts at the observer and emitter could impose some strong constraints on the deviations of the geometry of the Universe with respect to the Riemannian background. The torsional effects generated by the spin fluid modify the background geometry and affect the propagation of the photons. The study of the polarization of the gamma-ray bursts could also impose some limits on the spin density and overall rotation of the Universe.

However, the practical implementation of an observational program aiming at detecting torsion and nonmetricity from the polarization of the gamma-ray bursts could be extremely difficult. From the present observational point of view, the polarization measurements of the gamma-ray bursts and the interpretation of the data are still controversial. There is no general agreement on the detected value of the polarization degree [27–29], and, since at the moment the data quality is insufficient to constrain the polarization degree in most of the cases [29], a major improvement in the observational techniques is required. On the other hand, there are many astrophysical mechanisms which can induce an initial polarization of the photons from the cosmological gamma-ray bursts, which makes the task of distinguishing between the different involved physical processes extremely difficult. The propagation of the photons takes place on cosmological distances in a cosmic environment whose properties also present many uncertainties. However, when a larger amount of high precision spectropolarimetric data from gamma-ray bursts will be available, the possibility of testing the foundations of general relativity and other more general gravitational theories could become a reality.

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[1] H. Riffert and P. Meszaros, Astrophys. J. **325**, 207 (1988).
[2] R. A. Breuer and J. Ehlers, Proc. R. Soc. A **370**, 389 (1980); R. A. Breuer and J. Ehlers, Proc. R. Soc. A **374**, 65 (1981).
[3] R. W. Lindquist, Ann. Phys. (N.Y.) **37**, 487 (1966).
[4] H. Riffert, Astrophys. J. **310**, 729 (1986).
[5] S. Bildhauer, Classical Quantum Gravity **6**, 1171 (1989).
[6] S. Bildhauer, Astron. Astrophys. **219**, 25 (1989).
[7] S. Bildhauer, Classical Quantum Gravity **7**, 2367 (1990).
[8] C. H. Brans, Astrophys. J. **197**, 1 (1975).
[9] R. A. Matzner and B. W. Tolman, Phys. Rev. D **26**, 2951 (1982).
[10] M. M. Basko and A. G. Polnarev, Mon. Not. R. Astron. Soc. **191**, 207 (1980).
[11] R. Fabbri and M. Tamburrano, Astron. Astrophys. **179**, 11 (1987).
[12] F. W. Hehl, G. D. McCrea, E. W. Mielke, and Y. W. Ne'eman, Phys. Rep. **258**, 1 (1995).
[13] F. W. Hehl, P. von der Heyde, G. D. Kerlick, and J. Nester, Rev. Mod. Phys. **48**, 393 (1976).
[14] J. W. Moffat and E. Woolgar, Phys. Rev. D **37**, 918 (1988).
[15] W.-T. Ni, in Precision Measurements and Fundamental Constants II, NBS Spec. Pub. No. 617, edited by B. N. Taylor and W. D. Philips (U.S. GPO, Washington, DC, 1984).
[16] M. D. Gabriel, M. P. Haugan, R. B. Mann, and J. H. Palmer, Phys. Rev. Lett. **67**, 2123 (1991).
[17] S. K. Solanski, M. P. Haugan, and R. B. Mann, Phys. Rev. D **59**, 047101 (1999).
[18] G. F. Rubilar, Y. N. Obukhov, and F. W. Hehl, Classical Quantum Gravity **20**, L185 (2003).
[19] Y. Itin and F. W. Hehl, Phys. Rev. D **68**, 127701 (2003).
[20] S. K. Solanki, O. Preuss, M. P. Haugan, A. Gandorfer, H. P. Povel, P. Steiner , K. Stucki, P. N. Bernasconi, and D. Soltau, Phys. Rev. D **69**, 062001 (2004).
[21] K. S. Cheng and T. Lu, Chin. J. Astron. Astrophys. **1**, 1 (2001).
[22] P. Meszaros, Annu. Rev. Astron. Astrophys. **40**, 137 (2002).
[23] B. Zhang and P. Meszaros, Int. J. Mod. Phys. A **19**, 2385

(2004).
[24] T. Piran, Rev. Mod. Phys. **76**, 1143 (2005).
[25] T. Harko and K. S. Cheng, Astrophys. J. **611**, 633 (2004).
[26] K. S. Cheng and T. Harko, Astropart. Phys. **22**, 297 (2004).
[27] W. Coburn and S. E. Boggs, Nature (London) **423**, 415 (2003).
[28] R. E. Rutledge and D. B. Fox, Mon. Not. R. Astron. Soc. **350**, 1288 (2004).
[29] C. Wigger, W. Hajdas, K. Arzner, M. Güdel, and A. Zehnder, Astrophys. J. **613**, 1088 (2004).
[30] S. Covino, D. Lazzati, G. Ghisellini, and D. Malesani, in Proceedings of the Third Rome Workshop on Gamma-Ray Bursts in the Afterglow Era, 2002, ASP Conference Series Vol. 312, edited by M. Feroci, N. Masettit, and L. Piro (Astronomical Society of the Pacific, San Francisco, to be published).
[31] T. Eguchi, P. B. Gilkey, and A. J. Hanson, Phys. Rep. **66**, 213 (1980).
[32] J. Audretsch, F. Gähler, and N. Straumann, Commun. Math. Phys. **95**, 41 (1984).
[33] C. Acquista and J. L. Anderson, Astrophys. J. **191**, 567 (1974).
[34] A. V. Minkevich and A. S. Garkun, gr-qc/9805007.
[35] A. Gruzinov and E. Waxman, Astrophys. J. **511**, 852 (1999).
[36] J. Granot and A. Königl, Astrophys. J. **594**, L83 (2003).
[37] M. V. Medvedev and A. Loeb, Astrophys. J. **526**, 697 (1999).
[38] A. Loeb and R. Perna, Astrophys. J. **495**, 597 (1998).
[39] R. A. M. J. Wijers *et al.*, Astrophys. J. **523**, L33 (1999).
[40] M. Lyutikov, V. I. Pariev, and R. Blandford, Astrophys. J. **597**, 998 (2003).
[41] E. Waxman, Nature (London) **423**, 388 (2003); J. Granot, Astrophys. J. **596**, L17 (2003).
[42] N. J. Shaviv and A. Dar, Astrophys. J. **447**, 863 (1995); D. Eichler and A. Levinson, Astrophys. J. **596**, L147 (2003); D. Lazzati, E. Rossi, G. Ghisellini, and M. J. Rees, Mon. Not. R. Astron. Soc. **347**, L1 (2004).
[43] A. J. Barth *et al.*, Astrophys. J. **584**, L47 (2003); J. Greiner *et al.*, Nature (London) **426**, 157 (2003); E. Rol *et al.*, Astron. Astrophys. **405**, L23 (2003).
[44] X. F. Wu, Z. G. Dai, Y. F. Huang, and T. Lu, Mon. Not. R. Astron. Soc. **357**, 1197 (2005).
[45] M. Szydlowski and A. Krawiec, Phys. Rev. D **70**, 043510 (2004).
[46] H. Goenner and F. Müller-Hoissen, Classical Quantum Gravity **1**, 651 (1984).