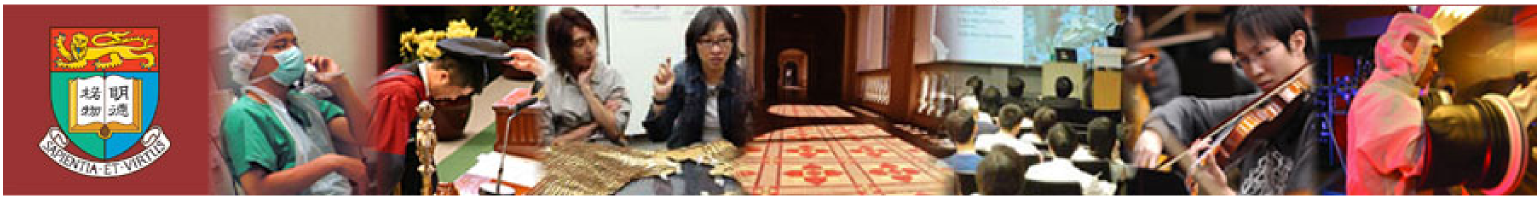

Title	Decentralized H_{∞}-controller design for nonlinear systems
Author(s)	Yang, GH; Wang, J; Soh, CB; Lam, J
Citation	Ieee Transactions On Automatic Control, 1999, v. 44 n. 3, p. 578-583
Issued Date	1999
URL	http://hdl.handle.net/10722/43026
Rights	©1999 IEEE. Personal use of this material is permitted. However, permission to reprint/republish this material for advertising or promotional purposes or for creating new collective works for resale or redistribution to servers or lists, or to reuse any copyrighted component of this work in other works must be obtained from the IEEE.

the PPC policy. In this case, one approach is to identify a stable service period and use it as a supervisor for another, more intuitive (e.g., PCLB) policy. As a supervisor, the purpose of the PPC policy is simply to guarantee stability of the system. In practice, the supervising PPC policy is invoked when some measure of system performance (e.g., the sum of the buffer levels) exceeds some preset threshold (i.e., once the sum of FMS buffer levels grows beyond some preset limit, a predefined, stable periodic service sequence is implemented).

Next, note that in simulation studies for the policies introduced here we have uncovered some interesting points [5]. First, in attempting to formulate a general rule of thumb for determining the suitability of PC control for a given FMS, we make the following observation: The less variance there is among processing rates along individual paths, the better PC policies will perform (with respect to distributed policies). The reason for this is that because PC policies mandate that all buffers on a given path be processed at a single rate (the minimum processing rate of all buffers on the path), any buffers on the path that are able to be processed at a faster rate than the minimum processing rate are constrained to be processed at a lower rate than they would be processed at in a distributed control scheme. In general, then, for systems with very high processing rate "skew" along individual paths, we may be wiser to choose a distributed policy. However, it may be possible to choose paths intelligently so as to minimize the adverse affects of processing rate skew.

Finally, we would like to emphasize that PC policies will not yield stability for all FMS that would be stable under a distributed scheduling approach where the FMS satisfies a capacity constraint. For example, if there is a high amount of "processing rate skew" along the paths PC policies may not be stable and a distributed policy may be.

References

[1] J. R. Perkins and P. Kumar, "Stable, distributed, real-time scheduling of flexible manufacturing/assembly/disassembly systems," *IEEE Trans. Automat. Contr.*, vol. 34, pp. 139–148, Feb. 1989.

[2] P. Kumar and T. J. Seidman, "Dynamic instabilities and stabilization methods in distributed real-time scheduling of manufacturing systems," *IEEE Trans. Automat. Contr.*, vol. 35, pp. 289–298, Mar. 1990.

[3] S. H. Lu and P. Kumar, "Distributed scheduling based on due dates and buffer priorities," *IEEE Trans. Automat. Contr.*, vol. 36, pp. 1406–1416, Dec. 1991.

[4] C. Humes, Jr., "A regular stabilization technique: Kumar–Seidman revisited," *IEEE Trans. Automat. Contr.*, vol. 39, pp. 191–196, Jan. 1994.

[5] K. M. Passino and K. L. Burgess, *Stability Analysis of Discrete Event Systems*. New York: Wiley, 1998.

Decentralized H_∞-Controller Design for Nonlinear Systems

Guang-Hong Yang, Jianliang Wang, C. B. Soh, and James Lam

***Abstract*—This paper considers the decentralized H_∞-controller design problem for nonlinear systems. Sufficient conditions for the solution of the problem are presented in terms of solutions of Hamilton–Jacobi inequalities. The resulting design guarantees local asymptotic stability and ensures a predetermined L_2-gain bound on the closed-loop system.**

***Index Terms*—Decentralized control, Hamilton–Jacobi inequality, nonlinear H_∞ control, nonlinear system.**

I. Introduction

In the area of the decentralized control of large scale systems, numerous important advances have been accomplished in the past two decades [9], [11]. Recently, the decentralized H_∞-control problem for linear systems has been considered in [7], [8], [10], and [13]. In particular, Veillette *et al.* [13] presented a decentralized H_∞-controller design procedure in terms of solutions of the modified algebraic Riccati equations, and the result has also been extended to discrete-time linear systems [7]. In [8], another sufficient condition for the decentralized H_∞-control problem is derived, under which the decentralized solution can be constructed from the central controller solution from the standard H_∞-control theory in [2].

In recent years, the problem of central controller design to solve the H_∞-control problem (or in short, the central H_∞-control problem) for nonlinear systems has been extensively investigated by several authors [1], [3]–[6], [12]. In particular, Van der Schaft [12] has shown that the solution of the H_∞-control problem via state feedback can be determined from the solution of a Hamilton–Jacobi equation (or inequality), which is the nonlinear version of the Riccati equation for the corresponding linear H_∞-control problem. In the case of measurement feedback, a set of sufficient conditions has also been given in [1], [4], and [6] in terms of the solutions of a pair of Hamilton-Jacobi inequalities, and the necessity of these sufficient conditions has been discussed in [1] and [5].

In this paper, we consider the decentralized H_∞-control problem for nonlinear systems by using the Hamilton–Jacobi inequality approach. The results given in this paper are extensions of existing results on the linear decentralized H_∞-control problem [13], [8] and nonlinear central H_∞-control problem [4]. The paper is organized as follows. The system description and problem statement are given in Section II. The main results are given in Section III, followed by a numerical example in Section IV to illustrate the design procedure and the effectiveness of the proposed method. Finally, some concluding remarks are given in Section V.

Manuscript received September 11, 1996. Recommended by Associate Editor, J.-B. Pomet.

G.-H. Yang and C. B. Soh are with the School of Electrical and Electronic Engineering, Nanyang Technological University, 639798 Singapore.

J. Wang is with the School of Electrical and Electronic Engineering, Nanyang Technological University, 639798 Singapore (e-mail: ejlwang@ntu.edu.sg).

J. Lam is with the Department of Mechanical Engineering, University of Hong Kong, Hong Kong.

Publisher Item Identifier S 0018-9286(99)01320-3.

II. Problem Statement

Consider a nonlinear system $\mathbf{\Sigma}$ described by equations of the form

$$\mathbf{\Sigma}: \quad \dot{x} = f(x) + g_1(x)w_0 + \sum_{i=1}^{q} g_{2i}(x)u_i \tag{1}$$

$$z = [h_1(x) \quad u_1 \quad \cdots \quad u_q]^T \tag{2}$$

$$y_i = h_{2i}(x) + w_i, \quad i = 1, \cdots q \tag{3}$$

where x is a state vector defined in a neighborhood X of the origin in R^n; the u_i's are the local control inputs with $u_i \in R^{m_i}$; the y_i's are the local measurement with $y_i \in R^{p_i}$; w_0 and the w_i's are square-integrable disturbances; and z is an output to be regulated. The functions $f(x)$, $g_1(x)$, $h_1(x)$, $g_{2i}(x)$, and $h_{2i}(x)$ $(i = 1, \cdots, q)$ are all known smooth mappings of appropriate dimensions, defined in X with $f(0) = 0$, $h_1(0) = 0$, and $h_{2i}(0) = 0$ $(i = 1, \cdots, q)$. For convenience, we denote

$$u = \begin{bmatrix} u_1^T & u_2^T & \cdots & u_q^T \end{bmatrix}^T, \quad y = \begin{bmatrix} y_1^T & y_2^T & \cdots & y_q^T \end{bmatrix}^T$$
$$w_e = \begin{bmatrix} w_0^T & w_1^T & \cdots & w_q^T \end{bmatrix}^T$$
$$g_2(x) = [g_{21}(x) \quad g_{22}(x) \quad \cdots \quad g_{2q}(x)]$$
$$h_2(x) = \begin{bmatrix} h_{21}^T(x) & h_{22}^T(x) & \cdots & h_{2q}^T(x) \end{bmatrix}^T.$$

The decentralized control structure constrains each control input u_i to be generated by an independent controller which uses only the corresponding measurement y_i.

Decentralized H_∞-Controller Design (DHCD) Problem: Given the system $\mathbf{\Sigma}$ described by (1)–(3) and a positive constant γ, find controllers of the following form:

$$\dot{\xi}_i = a_i(\xi_i) + b_i(\xi_i)y_i, \quad \xi_i \in R^{v_i} \tag{4}$$

$$u_i = c_i(\xi_i), \quad i = 1, \cdots, q \tag{5}$$

such that the resulting closed-loop system is locally asymptotically stable and has a local L_2 gain less than or equal to γ.

The following notion of detectability from [3] will be used in the sequel.

Definition 2.1: Suppose $f(0) = 0$ and $h(0) = 0$. The pair $\{f, h\}$ is said to be *locally detectable* if there exists a neighborhood U of the point $x = 0$ such that, if $x(t)$ is any integral curve of $\dot{x} = f(x)$ satisfying $x(0) \in U$, then $h(x(t))$ is defined for all $t \geq 0$ and $h(x(t)) = 0$ for all $t \geq 0$ implies $\lim_{t\to\infty} x(t) = 0$.

For some fundamental notions and results of nonlinear H_∞-control theory, the reader is referred to [4] or [12].

III. Main Results

The main results are presented in the following two subsections. Section III-A gives a solution to the *DHCD* problem, assuming that an observer gain is given. Then, in Section III-B, two methods are given for the observer gain design.

A. A Solution to the DHCD Problem

For the nonlinear system $\mathbf{\Sigma}$ described by (1)–(3) and a smooth positive definite function $V: R^n \to R_+$ [with its Jacobian matrix being $V_x(x)$], denote

$$\alpha_1(x) = \frac{1}{2\gamma^2} g_1^T(x) V_x^T(x) \tag{6}$$

$$\alpha_2(x) = -\frac{1}{2} g_2^T(x) V_x^T(x) = \begin{bmatrix} \alpha_{21}^T(x) & \cdots & \alpha_{2q}^T(x) \end{bmatrix}^T \tag{7}$$

$$\bar{\alpha}_2(\xi) = \begin{bmatrix} \alpha_{21}^T(\xi_1) & \alpha_{22}^T(\xi_2) & \cdots & \alpha_{2q}^T(\xi_q) \end{bmatrix}^T \in R^q \tag{8}$$

$$\bar{g}_1(\xi) = \mathrm{diag}\{g_1(\xi_1), g_1(\xi_2), \cdots, g_1(\xi_q)\} \tag{9}$$

$$\bar{h}_2(\xi) = \mathrm{diag}\{h_{21}(\xi_1), h_{22}(\xi_2), \cdots, h_{2q}(\xi_q)\} \tag{10}$$

$$\bar{f}(\xi) = \begin{bmatrix} f(\xi_1) + g_1(\xi_1)\alpha_1(\xi_1) + g_2(\xi_1)\alpha_2(\xi_1) - g_2(\xi_1)\bar{\alpha}_2(\xi) \\ f(\xi_2) + g_1(\xi_2)\alpha_1(\xi_2) + g_2(\xi_2)\alpha_2(\xi_2) - g_2(\xi_2)\bar{\alpha}_2(\xi) \\ \vdots \\ f(\xi_q) + g_1(\xi_q)\alpha_1(\xi_q) + g_2(\xi_q)\alpha_2(\xi_q) - g_2(\xi_q)\bar{\alpha}_2(\xi) \end{bmatrix} \tag{11}$$

where $\xi = [\xi_1^T \quad \xi_2^T \quad \cdots \quad \xi_q^T]^T$ is an nq dimensional vector with $\xi_i \in R^n$, and define the following matrix which is to be determined later:

$$\bar{L}(\xi) = \mathrm{diag}\{L_1(\xi_1), L_2(\xi_2), \cdots, L_q(\xi_q)\}, \qquad L_i(\xi_i) \in R^{n \times p_i}, \quad \xi_i \in R^n. \tag{12}$$

Then the following theorem presents a sufficient condition for the solution of the *DHCD* problem and gives a controller of order qn.

Theorem 3.1: Consider the system $\mathbf{\Sigma}$ described by (1)–(3) and a positive constant γ. Suppose that the following conditions hold.

1) The pair $\{f, h_1\}$ is locally detectable.
2) There exists a C^2 positive definite function $V(x)$, locally defined in a neighborhood of $x = 0$ and vanishing at $x = 0$, which satisfies the Hamilton–Jacobi inequality
$$H_s(x, V_x^T) \triangleq V_x f(x) + h_1^T(x)h_1(x) + \gamma^2 \alpha_1^T(x)\alpha_1(x) - \alpha_2^T(x)\alpha_2(x) \leq 0 \tag{13}$$
where V_x is the Jacobian matrix of $V(x)$.
3) There exist $n \times p_i$ matrix-valued functions $L_i(\xi_i)$ $(i = 1, \cdots, q)$ such that the following Hamilton–Jacobi inequality admits a C^2 positive definite solution $Q(\xi)$ that is locally defined in a neighborhood of $\xi = 0$ and vanishing at $\xi = 0$
$$H_{\mathrm{do}}(\xi, Q_\xi^T) \triangleq Q_\xi[\bar{f}(\xi) - \bar{L}(\xi)\bar{h}_2(\xi)] + \bar{\alpha}_2^T(\xi)\bar{\alpha}_2(\xi) + \frac{1}{4\gamma^2} Q_\xi \bar{g}_1(\xi)\bar{g}_1^T(\xi) Q_\xi^T + \frac{1}{4\gamma^2} Q_\xi \bar{L}(\xi)\bar{L}^T(\xi) Q_\xi^T \leq 0. \tag{14}$$
Furthermore, the Hessian matrix of $H_{\mathrm{do}}(\xi, Q_\xi^T)$ is nonsingular at $\xi = 0$.

Then there exist controllers of the form (4) and (5) that solves the DHCD problem for the system $\mathbf{\Sigma}$. Furthermore, a particular such a controller of order nq is given by

$$\dot{\xi}_i = f(\xi_i) + g_1(\xi_i)\alpha_1(\xi_i) + g_2(\xi_i)\alpha_2(\xi_i) + L_i(\xi_i)(y_i - h_{2i}(\xi_i)) \tag{15}$$

$$u_i = \alpha_{2i}(\xi_i), \quad i = 1, \cdots, q. \tag{16}$$

The following preliminaries are required in the proof of Theorem 3.1.

By applying the controllers given by (15) and (16) to the system $\mathbf{\Sigma}$ of (1)–(3), the resulting closed-loop system $\mathbf{\Sigma}_c$ is described by

$$\mathbf{\Sigma}_c: \quad \dot{x}_e = f_e(x_e) + g_e(x_e)w_e \tag{17}$$

$$z = \begin{bmatrix} h_1^T(x) & \alpha_{21}^T(\xi_1) & \alpha_{22}^T(\xi_2) & \cdots & \alpha_{2q}^T(\xi_q) \end{bmatrix}^T \tag{18}$$

where $x_e = [x^T \quad \xi^T]^T$ is of dimension $n + nq$, and we have the equations shown at the bottom of the next page. Denote (19) and (20), as shown at the bottom of the next page. Then we have the following lemma.

Lemma 3.2: Let $W(x_e) = Q(\bar{x} - \xi)$, where $\bar{x} = [x^T \quad \cdots \quad x^T]^T \in R^{nq}$ with $x \in R^n$. Then under the assumptions of Theorem 3.1, there exists a neighborhood of $x_e = 0$ in which the following inequality holds:

$$W_{x_e} \bar{f}_e(x_e) + h_e^T(x_e)h_e(x_e) + \frac{1}{4\gamma^2} W_{x_e} g_e(x_e) g_e^T(x_e) W_{x_e}^T \leq 0. \tag{21}$$

Proof: Let $e = \bar{x} - \xi$ and $I_c = [I, \cdots, I]^T \in R^{nq\times n}$ where $I \in R^{n\times n}$ is the identity matrix. Then $W_{x_e} = (Q_e(e)I_c, -Q_e(e))$. Note that

$$\begin{aligned} g_2(x)\bar{\alpha}_2(\xi) &= \sum_{i=1}^{q} g_{2i}(x)\alpha_{2i}(\xi_i) \\ &= (g_2(x) - g_2(\xi_j))\bar{\alpha}_2(\xi) \\ &\quad + g_2(\xi_j)\bar{\alpha}_2(\xi), \quad j = 1, \cdots, q. \end{aligned}$$

By (17), (19) and (20), we have

$$\begin{aligned} &H(e,x) \\ &= W_{x_e}\bar{f}_e(x_e) + h_e^T(x_e)h_e(x_e) + \frac{1}{4\gamma^2}W_{x_e}g_e(x_e)g_e^T(x_e)W_{x_e}^T \\ &= Q_e(e)\begin{bmatrix} f(x) + g_1(x)\alpha_1(x) - f(\xi_1) - g_1(\xi_1)\alpha_1(\xi_1) \\ \vdots \\ f(x) + g_1(x)\alpha_1(x) - f(\xi_q) - g_1(\xi_q)\alpha_1(\xi_q) \end{bmatrix} \\ &\quad + Q_e(e)\begin{bmatrix} (g_2(x)-g_2(\xi_1))\bar{\alpha}_2(\xi)+g_2(\xi_1)\bar{\alpha}_2(\xi)-g_2(\xi_1)\alpha_2(\xi_1) \\ \vdots \\ (g_2(x)-g_2(\xi_q))\bar{\alpha}_2(\xi)+g_2(\xi_q)\bar{\alpha}_2(\xi)-g_2(\xi_q)\alpha_2(\xi_q) \end{bmatrix} \\ &\quad - Q_e(e)\bar{L}(\xi)\begin{bmatrix} h_{21}(x) - h_{21}(\xi_1) \\ \vdots \\ h_{2q}(x) - h_{2q}(\xi_q) \end{bmatrix} + [\bar{\alpha}_2(\bar{x}) - \bar{\alpha}_2(\xi)]^T \\ &\quad \times [\bar{\alpha}_2(\bar{x}) - \bar{\alpha}_2(\xi)] + \frac{1}{4\gamma^2}Q_e(e)I_c g_1(x)g_1^T(x)I_c^T Q_e^T(e) \\ &\quad + \frac{1}{4\gamma^2}Q_e(e)\bar{L}(\xi)\bar{L}^T(\xi)Q_e^T(e) \end{aligned} \tag{22}$$

where $\bar{L}(\cdot)$ is as defined in (12). From the above equality, it is easy to see that $H(0,x) = 0$, $[\frac{\partial H(e,x)}{\partial e}]_{e=0} = 0$. Thus, $H(e,x)$ can be expressed as $H(e,x) = e^T R(e,x)e$ for some continuous matrix-valued function $R(e,x)$. By (14) and (22), we have $R(0,0) = [\frac{\partial^2 H_{\rm do}(\xi, Q_\xi^T)}{\partial \xi^2}]_{\xi=0}$. Since the Hessian matrix of $H_{\rm do}(\xi, Q_\xi^T)$ is negative definite, the function $H(e,x)$ is nonpositive in a neighborhood of $(e,x) = (0,0)$, which further implies that the inequality (21) holds. □

Lemma 3.3: Under the assumption 3) of Theorem 3.1, the equilibrium $\xi = 0$ of the system

$$\dot{\xi} = \bar{f}(\xi) - \bar{L}(\xi)\bar{h}_2(\xi) \tag{23}$$

is locally asymptotically stable.

The proof is straightforward and omitted.

Proof of Theorem 3.1: A straightforward calculation using (13), (16), (6), and (8) yields the following inequality:

$$\begin{aligned} &V_x[f(x) + g_1(x)w_0 + g_2(x)u] + \|h_1(x)\|^2 + \|u\|^2 \\ &\quad - \gamma^2\|w_0\|^2 - \gamma^2\|w\|^2 \\ &\quad \le \sum_{i=1}^{q}\|\alpha_{2i}(\xi_i) - \alpha_{2i}(x)\|^2 - \gamma^2\|w_0 - \alpha_1(x)\|^2 - \gamma^2\|w\|^2. \end{aligned} \tag{24}$$

Let

$$\begin{aligned} U(x_e) &= V(x) + W(x_e), \\ \bar{\alpha}_1(x) &= [[\alpha_1(x)]^T \quad 0 \quad \cdots \quad 0]^T. \end{aligned}$$

Then, by using (17)–(19), (24), and Lemma 3.2, it follows that

$$\begin{aligned} &U_{x_e}(f_e(x_e) + g_e(x_e)w_e) + \|z\|^2 - \gamma^2\|w_e\|^2 \\ &= V_x[f(x) + g_1(x)w_0 + g_2(x)\bar{\alpha}_2(\xi)] + W_{x_e}\big[f_e(x_e) \\ &\quad + g_e(x_e)w_{x_e}\big] + \|z\|^2 - \gamma^2\|w_e\|^2 \\ &\le \sum_{i=1}^{q}\|\alpha_{2i}(\xi_i) - \alpha_{2i}(x)\|^2 - \gamma^2\|w_0 - \alpha_1(x)\|^2 \\ &\quad - \gamma^2\|w\|^2 + W_{x_e}\big[f_e(x_e) + g_e(x_e)w_{x_e}\big] \\ &= -\gamma^2\|w_0 - \alpha_1(x)\|^2 - \gamma^2\|w\|^2 + h_e^T(x_e)h_e(x_e) \\ &\quad + W_{x_e}[\bar{f}_e(x_e) + g_e(x_e)(w_e - \bar{\alpha}_1(x))] \\ &\le -\gamma^2\|w_0 - \alpha_1(x)\|^2 - \gamma^2\|w\|^2 + W_{x_e}\bar{f}_e(x_e) \\ &\quad + h_e^T(x_e)h_e(x_e) + \frac{1}{4\gamma^2}W_{x_e}g_e(x_e)g_e^T(x_e)W_{x_e}^T \\ &\quad + \gamma^2\|w_e - \bar{\alpha}_1(x)\|^2 \\ &= W_{x_e}\bar{f}_e(x_e) + h_e^T(x_e)h_e(x_e) \\ &\quad + \frac{1}{4\gamma^2}W_{x_e}g_e(x_e)g_e^T(x_e)W_{x_e}^T \le 0. \end{aligned} \tag{25}$$

This shows that $U(x_e)$ is a storage function for the closed-loop system $\mathbf{\Sigma}_c$, with respect to the supply rate $s = \gamma^2\|w_e\|^2 - \|z\|^2$. This further implies that the system $\mathbf{\Sigma}_c$ has an L_2 gain less than or equal to γ. Let $w_e = 0$. Then from inequality (25), we have

$$\frac{dU(x_e(t))}{dt} = U_{x_e}f_e(x_e) \le -\|h_1(x(t))\|^2 - \|\bar{\alpha}_2(\xi(t))\|^2. \tag{26}$$

Since $U(x_e)$ is positive definite, it follows that the equilibrium $(x,\xi) = (0,0)$ of the closed-loop system $\mathbf{\Sigma}_c$ is stable. To prove asymptotic stability, note that any trajectory satisfying $\frac{dU(x_e)}{dt} = 0$ is necessarily a trajectory of $\dot{x} = f(x) + g_2(x)\bar{\alpha}_2(\xi)$ such that $x(t)$ is

---

$$\begin{aligned} f_e(x_e) &= \begin{bmatrix} f(x) + \sum_{i=1}^{q} g_{2i}(x)\alpha_{2i}(\xi_i) \\ f(\xi_1) + g_1(\xi_1)\alpha_1(\xi_1) + g_2(\xi_1)\alpha_2(\xi_1) - L_1(\xi_1)h_{21}(\xi_1) + L_1(\xi_1)h_{21}(x) \\ \vdots \\ f(\xi_q) + g_1(\xi_q)\alpha_1(\xi_q) + g_2(\xi_q)\alpha_2(\xi_q) - L_q(\xi_q)h_{2q}(\xi_q) + L_q(\xi_q)h_{2q}(x) \end{bmatrix} \\ g_e(x_e) &= {\rm diag}\{g_1(x), L_1(\xi_1), \cdots, L_q(\xi_q)\} \end{aligned}$$

$$\begin{aligned} \bar{f}_e(x_e) &= f_e(x_e) + [[g_1(x)\alpha_1(x)]^T \quad 0 \quad \cdots \quad 0]^T \\ &= \begin{bmatrix} f(x) + g_1(x)\alpha_1(x) + \sum_{i=1}^{q} g_{2i}(x)\alpha_{2i}(\xi_i) \\ f(\xi_1) + g_1(\xi_1)\alpha_1(\xi_1) + g_2(\xi_1)\alpha_2(\xi_1) - L_1(\xi_1)h_{21}(\xi_1) + L_1(\xi_1)h_{21}(x) \\ \vdots \\ f(\xi_q) + g_1(\xi_q)\alpha_1(\xi_q) + g_2(\xi_q)\alpha_2(\xi_q) - L_q(\xi_q)h_{2q}(\xi_q) + L_q(\xi_q)h_{2q}(x) \end{bmatrix} \end{aligned} \tag{19}$$

$$h_e(x_e) = [\alpha_{21}(x) - \alpha_{21}(\xi_1) \quad \cdots \quad \alpha_{2q}(x) - \alpha_{2q}(\xi_q)]^T \tag{20}$$

bounded, and $h_1(x(t)) = 0$ and $\bar{\alpha}_2(\xi) = 0$ for all $t \geq 0$. From the detectability of the pair $\{f, h_1\}$, it follows that $\lim_{t\to\infty} x(t) = 0$. Thus, the ω-limit set of such a trajectory is a subset of

$$M = \{(x, \xi) : x = 0, \bar{\alpha}_2(\xi) = 0\}.$$

Any initial condition on this ω-limit set yields a trajectory in which $x(t) = 0$ for all $t \geq 0$, while from (17) and $\bar{\alpha}_2(\xi) = 0$, $\xi(t)$ is a trajectory of

$$\dot{\xi} = \bar{f}_0(\xi) - \bar{L}(\xi)\bar{h}_2(\xi) = \bar{f}(\xi) - \bar{L}(\xi)\bar{h}_2(\xi)$$

where

$$\bar{f}_0(\xi) = \begin{bmatrix} f(\xi_1) + g_1(\xi_1)\alpha_1(\xi_1) + g_2(\xi_1)\alpha_2(\xi_1) \\ f(\xi_2) + g_1(\xi_2)\alpha_1(\xi_2) + g_2(\xi_2)\alpha_2(\xi_2) \\ \vdots \\ f(\xi_q) + g_1(\xi_q)\alpha_1(\xi_q) + g_2(\xi_q)\alpha_2(\xi_q) \end{bmatrix}.$$

From Lemma 3.3, it follows that $\lim_{t\to\infty} \xi(t) = 0$ and by the invariance principle, the closed-loop system $\mathbf{\Sigma}_c$ is locally asymptotically stable. □

Remark 3.4: Theorem 3.1 presents an approach to solve the *DHCD* problem for nonlinear systems. If $q = 1$, then Theorem 3.1 becomes [4, Lemma 3.2], which is a solution to the central H_∞ control problem for the system $\mathbf{\Sigma}$. It should be noted that the inequality (14) in Theorem 3.1 still contains undetermined observer gains $L_i(\xi_i)$'s. The problem of how to design these observer gains will be treated in the next subsection.

B. Observer Gain Design

Now we look at how to design these observer gains $L_i(\xi_i)$, $i = 1, 2, \cdots, q$. Two methods will be presented. The first method makes use of the centralized observer design result in [4], and the idea is similar to that of Paz [8] for linear local observers. But our result here is for nonlinear systems. The second method gives a decentralized observer design in terms of solutions to a matrix inequality that implies the Hamilton–Jacobi inequality (14) in condition 3) of Theorem 3.1.

B.1 The First Method: First, we review the result for nonlinear centralized observer design of [4]. Suppose that the assumptions 1) and 2) of Theorem 3.1 hold. Suppose that there exists a C^2 positive definite function $S(x)$, locally defined in a neighborhood of $x = 0$ and vanishing at $x = 0$, which satisfies the Hamilton-Jacobi inequality

$$\begin{aligned} H_o\big(x, S_x^T\big) &\triangleq S_x[f(x) + g_1(x)\alpha_1(x)] + \alpha_2^T(x)\alpha_2(x) \\ &\quad - \gamma^2 h_2^T(x)h_2(x) + \frac{1}{4\gamma^2} S_x g_1(x) g_1^T(x) S_x^T \leq 0 \end{aligned}$$

and the Hessian matrix of $H_o(x, S_x^T)$ is nonsingular at $x = 0$. Suppose also that there exists a C^2 matrix-valued function $L(x) \in R^{n\times p}$ with $p = \sum_{i=1}^q p_i$ such that

$$S_x L(x) = 2\gamma^2 h_2^T(x). \tag{27}$$

Then, from in [4, Th. 3.1], the centralized controller

$$\begin{aligned} \dot{\eta} &= f(\eta) + g_1(\eta)\alpha_1(\eta) + g_2(\eta)\alpha_2(\eta) + L(\eta)(y - h_2(\eta)) \\ u &= \alpha_2(\eta) \end{aligned}$$

locally stabilizes the system $\mathbf{\Sigma}$ and guarantees that the resulting closed-loop system has an L_2 gain less than or equal to γ.

For decentralized nonlinear control, we have the following result. Note that, in the decentralized case, $L(x) = [L_1(x)\ \ L_2(x)\ \ \cdots\ \ L_q(x)]$ with $L_i(x) \in R^{n\times p_i}$, $i = 1, \cdots, q$, and condition (27) becomes

$$S_x L_i(x) = 2\gamma^2 h_{2i}^T(x), \qquad i = 1, \cdots, q. \tag{28}$$

Then by using Theorem 3.1, we have the following corollary which is an extension of Theorem 2.1 in [8] for the linear *DHCD* problem.

Corollary 3.5: Suppose that assumptions 1) and 2) of Theorem 3.1 hold, and that the local observer gains $L_i(\xi_i), i = 1, 2, \cdots, q$ given in (28) are such that the Hamilton–Jacobi inequality (14) has a C^2 local positive definite solution $Q(\xi)$ in a neighborhood of $\xi = 0$, vanishing at $\xi = 0$, and the Hessian matrix of $H_{\rm do}(\xi, Q_\xi^T)$ is nonsingular at $\xi = 0$. Then the controller given by (15) and (16) with the above observer gains solves the *DHCD* problem for the nonlinear system $\mathbf{\Sigma}$.

B.2 The Second Method: Next, we present the second approach to the observer gain design. Since $f(x)$ and $h_{2i}(x)$ $(i = 1, \cdots, q)$ are smooth functions with $f(0) = 0$ and $h_{2i}(0) = 0$, there exist smooth matrix-valued functions $A(x)$ and $C_{2i}(x)$ $(i = 1, \cdots, q)$ such that

$$f(x) = A(x)x, \qquad h_{2i}(x) = C_{2i}(x)x. \tag{29}$$

Theorem 3.6: Under conditions 1) and 2) of Theorem 3.1, we assume that $V_x = 2x^T P(x)$ with $P(x)$ being a C^2 matrix-valued function. Furthermore, we assume that [in place of 3) of Theorem 3.1] there exists a C^2 matrix-valued functions $T(\xi)$, locally defined and nonsingular in a neighborhood of $\xi = 0$, of the form[1]

$$T(\xi) = \begin{bmatrix} T_{11}(\xi_1) & T_{12}(\xi) & \cdots & T_{1q}(\xi) \\ T_{21}(\xi) & T_{22}(\xi_2) & \cdots & T_{2q}(\xi) \\ \vdots & \vdots & \ddots & \vdots \\ T_{q1}(\xi) & T_{q2}(\xi) & \cdots & T_{qq}(\xi_q) \end{bmatrix}, \qquad T_{ii}(\xi_i) \in R^{n\times n} \tag{30}$$

that satisfies the matrix inequality

$$\begin{aligned} &T(\xi)\bar{A}_c^T(\xi) + \bar{A}_c(\xi)T^T(\xi) + T(\xi)K_c^T(\xi)K_c(\xi)T^T(\xi) \\ &\quad - \gamma^2 T(\xi)\bar{C}^T(\xi)\bar{C}(\xi)T^T(\xi) + \gamma^2[T(\xi) \\ &\quad - T_D(\xi)]\bar{C}^T(\xi)\bar{C}(\xi)[T(\xi) - T_D(\xi)]^T + \frac{1}{\gamma^2}\bar{g}_1(\xi)\bar{g}_1^T(\xi) < 0 \end{aligned} \tag{31}$$

where $\bar{g}_1(\xi)$ is as defined in (9), and there exists a positive definite function $Q(\xi)$ with $Q(0) = 0$ such that $Q_\xi = 2\xi^T T^{-1}(\xi)$ where

$$T_D(\xi) = \mathrm{diag}\{T_{11}(\xi_1), T_{22}(\xi_2), \cdots, T_{qq}(\xi_q)\} \tag{32}$$

$$\begin{aligned} \bar{A}_c(\xi) &= \mathrm{diag}\{\bar{A}(\xi_1), \bar{A}(\xi_2), \cdots, \bar{A}(\xi_q)\} \\ &\quad - \begin{bmatrix} g_2(\xi_1) \\ \vdots \\ g_2(\xi_q) \end{bmatrix} \mathrm{diag}\big\{g_{21}^T(\xi_1)P^T(\xi_1), g_{22}^T(\xi_2)P^T(\xi_2), \cdots, \\ &\qquad g_{2q}^T(\xi_q)P^T(\xi_q)\big\} \end{aligned} \tag{33}$$

$$\bar{A}(x) = A(x) + \frac{1}{\gamma^2} g_1(x)g_1^T(x)P^T(x) - g_2(x)g_2^T(x)P^T(x) \tag{34}$$

$$\begin{aligned} K_c(\xi) &= \mathrm{diag}\big\{-g_{21}^T(\xi_1)P^T(\xi_1), -g_{22}^T(\xi_2)P^T(\xi_2), \cdots, \\ &\qquad - g_{2q}^T(\xi_q)P^T(\xi_q)\big\} \end{aligned} \tag{35}$$

$$\bar{C}(\xi) = \mathrm{diag}\{C_{21}(\xi_1), C_{22}(\xi_2), \cdots, C_{2q}(\xi_q)\}. \tag{36}$$

Denote

$$\bar{L}(\xi) = \gamma^2 T_D(\xi)\bar{C}^T(\xi). \tag{37}$$

Then the controller given by (15) and (16) with the observer gain as specified by (37) and (12) solves the *DHCD* problem for the system $\mathbf{\Sigma}$.

[1] Note the special structure in $T(\xi)$ that the diagonal entries $T_{ii}(\xi_i)$ is a function of ξ_i only, not ξ!

Proof: From inequality (32), and definitions (32) and (37), it follows that

$$T(\xi)[\bar{A}_c(\xi) - \bar{L}(\xi)\bar{C}(\xi)]^T + [\bar{A}_c(\xi) - \bar{L}(\xi)\bar{C}(\xi)]T^T(\xi) + T(\xi)K_c^T(\xi)K_c(\xi)T^T(\xi) + \frac{1}{\gamma^2}\bar{g}_1(\xi)\bar{g}_1^T(\xi) + \frac{1}{\gamma^2}\bar{L}(\xi)\bar{L}^T(\xi) < 0$$

which further implies that

$$[\bar{A}_c(\xi) - \bar{L}(\xi)\bar{C}(\xi)]^T[T^{-1}(\xi)]^T + T^{-1}(\xi)[\bar{A}_c(\xi) - \bar{L}(\xi)\bar{C}(\xi)]) + K_c^T(\xi)K_c(\xi) + \frac{1}{\gamma^2}T^{-1}(\xi)\bar{g}_1(\xi)\bar{g}_1^T(\xi)[T^{-1}(\xi)]^T + \frac{1}{\gamma^2}T^{-1}(\xi)\bar{L}(\xi)\bar{L}^T(\xi)[T^{-1}(\xi)]^T < 0.$$

By using definitions (29) and (33)–(36), and by comparing with $H_{\mathrm{do}}(\xi, Q_\xi^T)$ in (14), we can see that the above strict inequality implies that condition 3) of Theorem 3.1 is satisfied. Also, the nonsingularity of the Hessian matrix of $H_{\mathrm{do}}(\xi, Q_\xi^T)$ is implied by the above strict inequality. Thus, the proof is completed from Theorem 3.1. □

It should be noted that, apart from the special structure as mentioned in the footnote to Theorem 3.6, $T(\xi)$ must satisfy the matrix inequality (31) and must also be related to a gradient function [i.e., $Q_\xi = 2\xi^T T^{-1}(\xi)$]. In general, $T(\xi)$ may not be symmetric. Therefore, the task of solving for such a $T(\xi)$ is indeed a nontrivial one. But a constant solution (which renders a linear observer gain) can be obtained easily as follows.

Corollary 3.7: Under conditions 1) and 2) of Theorem 3.1, we assume that $V_x = 2x^T P(x)$ with $P(x)$ being a C^2 matrix-valued function. Furthermore, we assume that [in place of 3) of Theorem 3.1] there exists a symmetric positive definite matrix $\bar{T}$ that satisfies the matrix inequality

$$\bar{T}\bar{A}_c^T(0) + \bar{A}_c(0)\bar{T} + \bar{T}K_c^T(0)K_c(0)\bar{T} - \gamma^2\bar{T}\bar{C}^T(0)\bar{C}(0)\bar{T} + [\bar{T} - \bar{T}_D]\bar{C}^T(0)\bar{C}(0)[\bar{T} - \bar{T}_D] + \frac{1}{\gamma^2}\bar{g}_1(0)\bar{g}_1^T(0) < 0 \quad (38)$$

where

$$\bar{T} = \begin{bmatrix} \bar{T}_{11} & \bar{T}_{12} & \cdots & \bar{T}_{1q} \\ \bar{T}_{12} & \bar{T}_{22} & \cdots & \bar{T}_{2q} \\ \vdots & \vdots & \ddots & \vdots \\ \bar{T}_{1q} & \bar{T}_{2q} & \cdots & \bar{T}_{qq} \end{bmatrix}, \quad \bar{T}_D = \mathrm{diag}\{\bar{T}_{11}, \bar{T}_{22}, \cdots, \bar{T}_{qq}\}$$

$$T_{ii} = T_{ii}^T \in R^{n\times n}. \quad (39)$$

Denote

$$\bar{L} = \gamma^2\bar{T}_D\bar{C}^T(0). \quad (40)$$

Then the controller given by (15) and (16) with the above observer gain in (40) solves the DHCD problem for the system $\mathbf{\Sigma}$.

Remark 3.8: Theorem 3.6 presents a design method for local observer gains $L_i(\xi_i)$ $(i = 1, \cdots, q)$, which is based on the existence of solutions of the form (30) to the nonlinear matrix inequality (31). From Corollary 3.7, the linear local observer gains can be designed by solving a linear matrix inequality. These results generalize to nonlinear decentralized control systems the results given in [13] for linear decentralized control systems. The results in [13] are given in terms of solutions of modified algebraic Riccati equations.

IV. An Example

Consider the following nonlinear system:

$$\dot{x} = \begin{bmatrix} -2x_1 + x_1x_2^2 \\ x_2^3 \end{bmatrix}x + \begin{bmatrix} 1 \\ x_1 \end{bmatrix}w_0 + \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$z = \begin{bmatrix} x_1 & x_2^4 & u_1 & u_2 \end{bmatrix}^T$$

$$y = \begin{bmatrix} 2x_1 + 2x_2 \\ 2x_1 \end{bmatrix} + \begin{bmatrix} w_{11} \\ w_{12} \end{bmatrix}$$

which is of the form $\mathbf{\Sigma}$ in (1)–(3). Hence, $q = 2$. We will design a decentralized controller for the above system by using Theorem 3.1 and Corollary 3.7.

For this example, it is easy to check that $\{f, h_1\}$ is locally detectable. By solving (13), the following positive definite solutions are obtained with a minimum γ value of $\gamma_d = 0.48$:

$$V(x) = 0.3642x_1^2 - 0.03706x_1x_2 + 0.04331x_2^2 - 0.04442x_1^3 + 0.1324x_1^2x_2 - 0.08808x_1x_2^2 + 0.04536x_2^3 + 0.1612x_1^4 - 0.7842x_1^3x_2 + 2.1051x_1^2x_2^2 - 2.1938x_1x_2^3 + 1.1772x_2^4.$$

The observer gain is taken as linear by solving (38) of Corollary 3.7

$$\bar{L} = \begin{bmatrix} L_1 & 0 \\ 0 & L_2 \end{bmatrix} = \begin{bmatrix} 0.3645 & 0.4029 & 0 & 0 \\ 0 & 0 & 0.6231 & -0.2519 \end{bmatrix}^T.$$

Finally the controller is given by (15) and (16) with $\alpha_1(\xi_i)$ and $\alpha_2(\xi_i)$ as defined in (6) and (7).

As there is no viable method for computing the H_∞-norm of an affine nonlinear system, the following approximate L_2-gain $\gamma_{\max}$ is computed:

$$\gamma_{\max} = \max_{t\geq 0}\sqrt{\frac{\int_0^t \left[z_1^2(\tau) + z_2^2(\tau) + z_3^2(\tau) + z_4^2(\tau)\right]d\tau}{\int_0^t \left[w_0^2(\tau) + w_{11}^2(\tau) + w_{12}^2(\tau)\right]d\tau}}.$$

If the H_∞-norm of the closed-loop system is γ_r, then obviously $0 \le \gamma_{\max} \le \gamma_r \le \gamma_d$.

In computer simulations, the plant disturbance $w_0(t)$ is taken as $w_0(t) = A \cdot \sin(4\pi t)$, the measurement noise $w_{11}(t)$ is taken as a square wave of an amplitude $\pm A$ and a frequency 10 Hz, and $w_{12}(t)$ is taken as a saw-tooth wave of an amplitude A and a frequency of 7 Hz. Here the amplitude A is variable. The initial conditions of the system and the controller are all set to zero. The simulation results are given in Table I. Here, "Nonlinear" refers to the controller that we designed just now, and "Linear" refers to the controller after dropping all nonlinear terms in our nonlinear controller (namely, the controller by using the linear methods of [13]).

TABLE I
HERE "—" INDICATES THAT THE SYSTEM BECOMES UNSTABLE

γ_{max}	$A = 0.1$	$A = 0.17$	$A = 0.18$	$A = 0.23$
Linear	0.1726	0.1732	–	–
Nonlinear	0.1726	0.1728	0.1728	0.1767

It can be seen from Table I that, for small disturbances/noises $(A = 0.1)$, the linear and the nonlinear controllers give the same closed-loop disturbance attenuation $\gamma_{\max}$. Also the L_2-attenuation $\gamma_{\max}$ is less than the designed value of $\gamma_d = 0.48$. When the disturbances/noises are increased to $A = 0.18$, the linear controller fails to stabilize the system, but the nonlinear controller not only stabilizes the system but also provides adequate disturbance attenuation. In fact, the nonlinear controller can meet the design specification for a disturbance level of up to $A = 0.23$.

Of course, the nonlinear method presented in this paper is a local one. This is confirmed by the simulation result that if the disturbance/noise level is increased beyond $A = 0.23$, the nonlinear controller fails to stabilize the system.

V. Conclusion

This paper presents a sufficient condition for the *DHCD* problem for nonlinear systems. The resulting controller guarantees local asymptotic stability and provides a predetermined L_2-gain bound on the closed-loop system. Two design methods of the local observers are given: one is based on the centralized observer gain and another one is related to the solution of the matrix inequalities. The results are extensions of those in [8] and [13] for the case of linear systems.

References

[1] J. Ball, J. W. Helton, and M. L. Walker, "H_∞ control for nonlinear systems via output feedback," *IEEE Trans. Automat. Contr.*, vol. 38, pp. 546–559, 1993.

[2] J. C. Doyle, K. Glover, P. P. Khargonekar, and B. A. Francis, "State space solutions to standard H_2 and H_∞ control problems," *IEEE Trans. Automat. Contr.*, vol. 34, pp. 831–847, 1989.

[3] A. Isidori and A. Astolfi, "Disturbance attenuation and H_∞ control via measurement feedback in nonlinear systems," *IEEE Trans. Automat. Contr.*, vol. 37, pp. 1283–1293, 1992.

[4] A. Isidori, "H_∞ control via measurement feedback for affine nonlinear systems," *Int. J. Robust and Nonlinear Contr.*, vol. 4, pp. 553–574, 1994.

[5] ——, "A necessary condition for nonlinear H_∞ control via measurement feedback," *Syst. Contr. Lett.*, vol. 23, pp. 169–177, 1994.

[6] W. M. Lu and J. C. Doyle, "H_∞ control of nonlinear systems via output feedback: Controller parameterization," *IEEE Trans. Automat. Contr.*, vol. 39, pp. 2517–2521, 1994.

[7] R. A. Paz and J. V. Medanić, "Decentralized H_∞ norm bounding control for discrete-time systems," in *Proc. Amer. Contr. Conf.*, Chicago, IL, 1992, pp. 3274–3278.

[8] R. A. Paz, "Decentralized H_∞ control," in *Proc. Amer. Control Conf.*, San Francisco, CA, 1993, pp. 2381–2385.

[9] N. R. Sandell, P. Varaiya, M. Athans, and M. G. Safonov, "Survey of decentralized control methods for large scale systems," *IEEE Trans. Automat. Contr.*, vol. 23, pp. 108–128, 1978.

[10] M. H. Shor, W. R. Perkins, and J. V. Medanić, "Design of reliable decentralized controllers: A unified continuous/discrete formulation," *Int. J. Contr.*, vol. 56, pp. 943–965, 1992.

[11] D. D. Siljak, *Decentralized Control of Complex Systems*. San Diego, CA: Academic, 1991.

[12] A. J. Van der Schaft, "L_2-gain analysis on nonlinear systems and nonlinear state feedback H_∞ control," *IEEE Trans. Automat. Contr.*, vol. 37, pp. 770–784, 1992.

[13] R. J. Veillette, J. V. Medanić, and W. R. Perkins, "Design of reliable control systems," *IEEE Trans. Automat. Contr.*, vol. 37, pp. 290–304, 1992.

A Methodology for the Design of Optimal Traffic Shapers in Communication Networks

Venkat Anantharam and Takis Konstantopoulos

***Abstract*—The authors consider the problem of optimally regulating the source traffic in a communication network to simultaneously satisfy a finite number of affine burstiness constraints. They prove that an optimal solution is a series connection of correspondingly dimensioned "leaky buckets." They propose a simple "fork-join" implementation of the optimal solution and study extensions to the problem of optimally shaping the traffic flow to meet a burstiness constraint specified by a concave increasing function. A consequence of their optimality results is that permutations of leaky buckets in a series connection are input–output equivalent.**

***Index Terms*—Communication networks, flow control, Skorokhod reflection mapping.**

I. Preliminaries

In this paper we consider the problem of designing flow control schemes in a communication network. Flow control is necessary for the regulation and shaping of a source traffic stream, which must interact and share network resources with other traffic streams after it is admitted. Therefore, one normally requires the admitted flow to satisfy certain "burstiness" or "shaping" constraints. It is also desirable that the controller be optimal, in that the offered traffic is transmitted as quickly as possible.

A general model for a traffic process is a nonnegative sigma-finite Borel measure A on the time axis $\mathbb{R}_+$. This is represented by an increasing right-continuous process $\{A_t,\, t \ge 0\}$; the interpretation is that for $0 \le s \le t$, $A_t - A_s$ gives the volume of traffic (in cells) on the time interval $(s, t]$. Write $\mathcal{M}$ for the collection of such processes. We write A^S for the restriction of A on $S \subseteq \mathbb{R}_+$, defined by $A_t^S := \int_{S\cap[0,\,t]} dA_s$. We also define a partial ordering on $\mathcal{M}$ by $A \le B \iff A_t \le B_t$, for all $t \ge 0$. We say that $A \in \mathcal{M}$ is $(\sigma_{i0},\, \sigma_i,\, \rho_i)_{i=1,\cdots,n}$ *constrained* iff, for all $0 \le s \le t$

$$A_t \le \min_{1\le i\le n}\{\sigma_{i0} + \rho_i t\},\; A_t - A_s \le \min_{1\le i\le n}\{\sigma_i + \rho_i(t-s)\}. \quad (1)$$

Here, $\sigma_i \ge \sigma_{i0} \ge 0$, $\rho_i \ge 0$, for all i. For $n = 1$ we simply say that A is $(\sigma_0,\, \sigma,\, \rho)$ constrained. The above definitions are discussed in Anantharam [1] and Cruz [4], [5], and they also closely match the standard shaping descriptors that have been adopted in practice for high-speed networks. More generally, for $f_0,\, f$ arbitrary concave increasing functions from $\mathbb{R}_+$ into $\mathbb{R}_+$, we say that A is $(f_0,\, f)$ *constrained* iff

$$A_t \le f_0(t), \quad A_t - A_s \le f(t-s). \quad (2)$$

Of course, (1) is a convenient special case of (2). A *traffic regulator*, or *flow controller* is simply a map $\varphi: \mathcal{M} \to \mathcal{M}$. Some properties that such a map may possess are as follows.

Manuscript received January 6, 1998. Recommended by Associate Editor, W.-B. Gong. This work was supported in part by NSF under Grant NCR 88-57731, NSF Research Initiation Award NCR-9211343, NSF Faculty Career Development Award NCR-9502582, and the Texas Higher Education Coordinating Board under Grant ARP-224.

V. Anantharam is with the Department of EECS, University of California, Berkeley, CA 94720 USA (e-mail: ananth@eecs.berkeley.edu).

T. Konstantopoulos is with the ECE Department, University of Texas, Austin, TX 78712 USA (e-mail: takis@alea.ece.utexas.edu).

Publisher Item Identifier S 0018-9286(99)01280-5.