Approximate BER Performance of Generalized Selection Combining in Nakagami-m Fading

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***Abstract*—The error performance of generalized selection combining (GSC), which optimally combines the K highest signal-to-noise ratio (SNR) signals out of L total diversity signals, in Nakagami-m fading was recently evaluated using moment generating function (MGF) of the GSC output SNR. However, no single closed-form expression for the MGF exists for arbitrary K and L. In fact, closed-form expression for the MGF is possible only for individual combination of K and L. In this letter, a single closed-form expression for approximating the MGF is, therefore, derived and employed in evaluating the approximate error performance. Although the approximation is only applicable for GSC with K being a factor of L, it nonetheless achieves a high degree of accuracy.**

***Index Terms*—Bit error rate, generalized selection combining, Nakagami fading.**

I. Introduction

DIVERSITY combining has long been recognized as one of the effective compensation techniques for combating detrimental effects of channel fading. The optimal combining technique is maximal ratio combining (MRC), which maximizes the signal-to-noise ratio (SNR) of the combined signals at the expense of implementation complexity. The selection combining (SC) is, however, considered as the least complicated, but it achieves much lower diversity gain than MRC. In attempting to compromise between error performance and implementation complexity, Kong *et al.* [1], [2] proposed a new diversity combining technique known as the generalized diversity combining (GSC), which optimally combines (as per the rules of MRC) only the K largest signals from L total diversity branches, as shown in Fig. 1(a).

Nakagami-m distribution is considered as one of the most versatile fading distributions. However, most studies have only investigated the error performance of GSC over Rayleigh fading channels (see, e.g., [2]). Recently, Alouini *et al.* [3] presented a bit error rate (BER) analysis of coherent binary signals with GSC over Nakagami-m fading channels for $K = 2$ and $L = 3$ and 4. The error rate analysis was then extended for arbitrary K and L, and M-ary modulations in [4] based on a moment generating function (MGF) approach. In particular, MGF of the GSC output SNR was derived in Nakagami-m fading and employed for evaluating the average BER performance. However, closed-form expression for the MGF was possible only for individual combination of K and L. Moreover, deriving the closed-form expression for the MGF for every different combinations of K and L was a rather tedious process. In this letter, for K being a factor of L, we are able to derive a single closed-form expression for approximating the MGF, and then make a comparison between the exact and the approximate average BER's.

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II. Exact Average BER of GSC

With the assumption of independent and identically distributed (i.i.d.) diversity branches in GSC, the exact MGF of the GSC output SNR in Nakagami-m fading is given by [4]

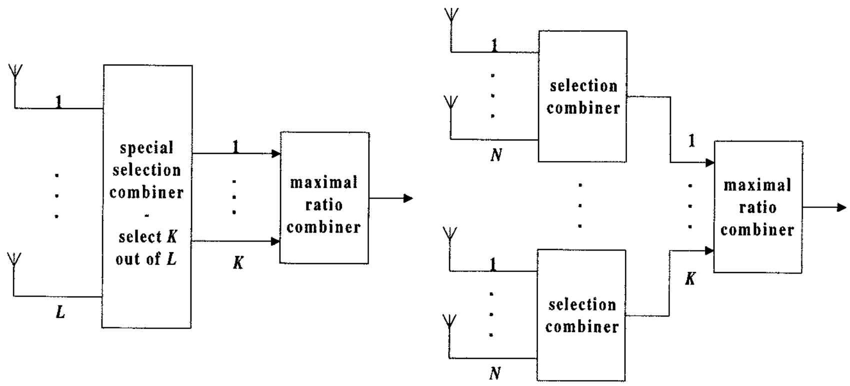
$$M_{\text{exact}}(s) = \frac{L!\Gamma(Lm)}{(\Gamma(m))^L} \int_0^1 \cdots \int_0^1 \left(\prod_{l=1}^{L-1} x_l^{lm-1}\right) \times (H(s; x_1, x_2, \ldots, x_{L-1}))^{-Lm} \, dx_{L-1} \cdots dx_1 \tag{1}$$

where

$$\begin{aligned} &H(s; x_1, x_2, \ldots, x_{L-1}) \\ &\quad = (1-(\Omega/m)s)\{1 + x_{L-1} + x_{L-1}x_{L-2} \\ &\qquad + \cdots + x_{L-1}x_{L-2}\cdots x_{L-K+2}x_{L-K+1}\} + x_{L-1}x_{L-2} \\ &\qquad \cdots x_{L-K+1}x_{L-K} + \cdots + x_{L-1}x_{L-2}\cdots x_2 \\ &\qquad + x_{L-1}x_{L-2}\cdots x_1 \end{aligned} \tag{2}$$

where Ω is the average SNR per bit per branch, m is the fading severity parameter and $\Gamma(\dot{})$ denotes the gamma function. No single closed-form expression exists for (1) because both the number of integrals in (1) and the number of terms in (2) depend on K and L of particular GSC. Therefore, closed-form expression for the exact MGF has to be derived for every combination of K and L. For instance, in the cases of: 1) $K = 2$ and $L = 4$ and 2) $K = 3$ and $L = 6$, the exact MGF's, with integer m, can be, respectively, manipulated into closed-form as shown in (3a) and (3b) at the bottom of the next page, where $H = 1 - (\Omega/m)s$. For large values of K and L, the above derivation of the exact MGF is thus tremendously tedious. The exact average BER $\overline{P}_{\text{exact}}$ of coherent binary phase-shift keying (BPSK) with GSC in Nakagami-m fading can then be obtained using [4, eq. (3)]

$$\overline{P}_{\text{exact}} = \frac{1}{\pi} \int_0^{\pi/2} M_{\text{exact}}\left(-\frac{1}{\sin^2\phi}\right) d\phi. \tag{4}$$



(a) (b)

Fig. 1. Block diagrams for: (a) a generalized selection combining receiver and (b) a cascade combining receiver.

III. Approximate Average BER of GSC

The inconvenient form of the exact MGF in (1) is resulted because GSC receiver employs a special selection combiner before a maximal ratio combiner. If the special selection combiner can be mimicked by several conventional selection combiners, then the derivation of the exact MGF should be greatly simplified. The closest mimicry possible is by the use of K conventional selection combiners having $N = L/K$ antennas each as shown in Fig. 1(b). In fact, the diversity combining, as shown in Fig. 1(b) is called cascade combining (CC) [5]. Although both GSC and CC are not exactly the same, similar error performance is however expected because of the close similarity in combining operations. Most importantly, the MGF of the CC output SNR in Nakagami-m fading can be expressed in a simple closed-form expression without any integral. Hence the MGF of the CC output SNR (now being called as approximate MGF) is used to approximate the exact MGF of the GSC output SNR. This approximation can be applied to those GSC with K being a factor of L. With i.i.d. diversity branches in all N-branch selection combiners in a CC receiver, the probability density function (pdf) of the instantaneous SNR per bit γ_{SC} at the selection combiner output is given by [6]

$$p_{\gamma_{SC}}(\gamma) = \left(\frac{m}{\Omega}\right)^m \frac{N\gamma^{m-1}}{(\Gamma(m))^N} \left\{\gamma\left(m, \frac{m\gamma}{\Omega}\right)\right\}^{N-1} \times \exp\left(-\frac{m\gamma}{\Omega}\right) \tag{5}$$

where $\gamma(\cdot,\cdot)$ is the incomplete gamma function, given by [3]

$$\gamma\left(m, \frac{m\gamma}{\Omega}\right) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!(m+n)} \left(\frac{m\gamma}{\Omega}\right)^{m+n}. \tag{6}$$

$$M_{\text{exact}}(s) = \frac{4!\Gamma(4m)}{(\Gamma(m))^4} \left\{ \sum_{k=0}^{m-1} \sum_{i=0}^{k} \sum_{r=0}^{i} \sum_{q=0}^{p} \sum_{p=0}^{m+k-i-1} \sum_{t=0}^{m+i+p-r-q-1} \binom{m-1}{k} \binom{k}{i} \binom{i}{r} \binom{p}{q} \binom{m+k-i-1}{p} \times \binom{m+i+p-r-q-1}{t} \right\} \frac{(-1)^{k+p+t}}{(3m+k)} \times \frac{2^{-(m+k-i)} H^{-2m} (2H+1)^{-(m+i+p-r-q)}}{(2m+i+p)(r+q+m+t)(2H+2)^{r+q+m+t}}, \quad \text{for } K=2,\ L=4 \tag{3a}$$

$$M_{\text{exact}}(s) = \frac{6!\Gamma(6m)}{(\Gamma(m))^6} \left\{ \sum_{k_5=0}^{m-1} \sum_{i_5=0}^{k_5} \sum_{k_4=0}^{m+k_5-i_5-1} \sum_{i_4=0}^{k_4+i_5} \sum_{k_3=0}^{m+i_5-i_4+k_4-1} \sum_{i_3=0}^{k_3+i_4} \sum_{k_2=0}^{m+i_4-i_3+k_3-1} \sum_{i_2=0}^{k_2+i_3} \sum_{k_1=0}^{m+i_3-i_2+k_2-1} \binom{m-1}{k_5} \binom{k_5}{i_5} \times \binom{k_4+i_5}{i_4} \binom{k_3+i_4}{i_3} \binom{m+k_5-i_5-1}{k_4} \binom{m+i_5-i_4+k_4-1}{k_3} \binom{m+i_4-i_3+k_3-1}{k_2} \times \binom{m+i_3-i_2+k_2-1}{k_1} \times \binom{k_2+i_3}{i_2} \frac{(-1)^{k_5+k_4+k_3+k_2+k_1} 2^{-(m+k_5-i_5)}}{(4m+i_5+k_4)(3m+i_4+k_3)} \times \frac{3^{-(m+i_5-i_4+k_4)} H^{-3m} (3H+1)^{-(m+i_4-i_3+k_3)} (3H+3)^{-(m+i_3+k_1)}}{(2m+i_3+k_2)(m+i_2+k_1)(5m+k_5)(3H+2)^{(m+i_3-i_2+k_2)}} \right\}, \quad \text{for } K=3,\ L=6 \tag{3b}$$

Combining (5) and (6), the MGF of γ_{SC} can then be obtained as

$$\begin{aligned} M_{\gamma_{\mathrm{SC}}}(s) &= \int_0^{\infty} e^{s\gamma} p_{\gamma_{\mathrm{SC}}}(\gamma)\, d\gamma \\ &= \frac{N}{(\Gamma(m))^N} \sum_{n_1=0}^{\infty} \cdots \sum_{n_{L-1}=0}^{\infty} \\ &\quad \times \frac{(-1)^{n_1+\cdots+n_{L-1}} (m/\Omega)^{(N-1)m+n_1+\cdots+n_{L-1}+m}}{\prod_{k=1}^{N-1} n_k!(m+n_k)} \\ &\quad \times \frac{((N-1)m+n_1+\cdots+n_{L-1}+m-1)!}{((m/\Omega)-s)^{Nm+n_1+\cdots+n_{L-1}}}. \end{aligned} \tag{7}$$

Note that the infinite summations in (7) can be shown to be convergent. The approximate MGF $M_{\mathrm{approx}}(s)$ can then be found by raising $M_{\gamma_{\mathrm{SC}}}(s)$ of (7) to the Kth power as []

$$M_{\mathrm{approx}}(s) = \left[M_{\gamma_{\mathrm{SC}}}(s)\right]^K. \tag{8}$$

Unlike the exact MGF in (3a) and (3b), the approximate MGF in (8) is not restricted to integer m. Using (8), the truncation of the infinite summations will, however, inevitably introduce errors, even though they can be made negligible. If infinite summations are not desirable, approximate MGF containing only finite summations can be derived. With integer m (5)becomes [6]

$$\begin{aligned} p_{\gamma_{\mathrm{SC}}}(\gamma) &= \frac{N}{\Gamma(m)} \sum_{l=0}^{N-1} (-1)^l \binom{N-1}{l} \exp\left(-(l+1)\frac{m\gamma}{\Omega}\right) \\ &\quad \times \sum_{k=0}^{l(m-1)} b_k^l \left(\frac{m}{\Omega}\right)^{m+k} \gamma^{m+k-1} \end{aligned} \tag{10}$$

where the notation b follows [6]. Using (9) and following the same derivation steps as in (7) and (8), we obtain the approximate MGF as

$$\begin{aligned} M_{\mathrm{approx}}(s) = \Bigg[&\frac{N}{\Gamma(m)} \sum_{l=0}^{N-1} \sum_{k=0}^{l(m-1)} (-1)^l \binom{N-1}{l} b_k^l \\ &\times \left(\frac{m}{\Omega}\right)^{m+k} \frac{\Gamma(m+k)}{((l+1)(m/\Omega)-s)^{m+k}} \Bigg]^K. \end{aligned} \tag{11}$$

The approximate MGF in (11) has only one closed-form for all K and N, and it is simpler than the exact MGF in (1) or (3a) and (3b). Similarly, the approximate average BER $\overline{P}_{\mathrm{approx}}$ can be evaluated using (4) with $M_{\mathrm{exact}}(s)$ replaced by $M_{\mathrm{approx}}(s)$.

IV. RESULTS

To assess the usefulness of the approximation, the error performance of BPSK with GSC in Nakagami-m fading are evaluated and compared for two cases: 1) the exact MGF with $K=2$ and $L=4$ and the approximate MGF with $K=2$ and $N\,(=L/K)=2$, and 2) the exact MGF with $K=3$ and $L=6$ and the approximate MGF with $K=3$ and $N=2$. Fig. 2 shows the exact and the approximate average BER performance for m of 1 and 2. As can be observed, the difference between the exact and the approximate average BER's is slim, and the accuracy is uniformly good for the entire range of Ω and m.

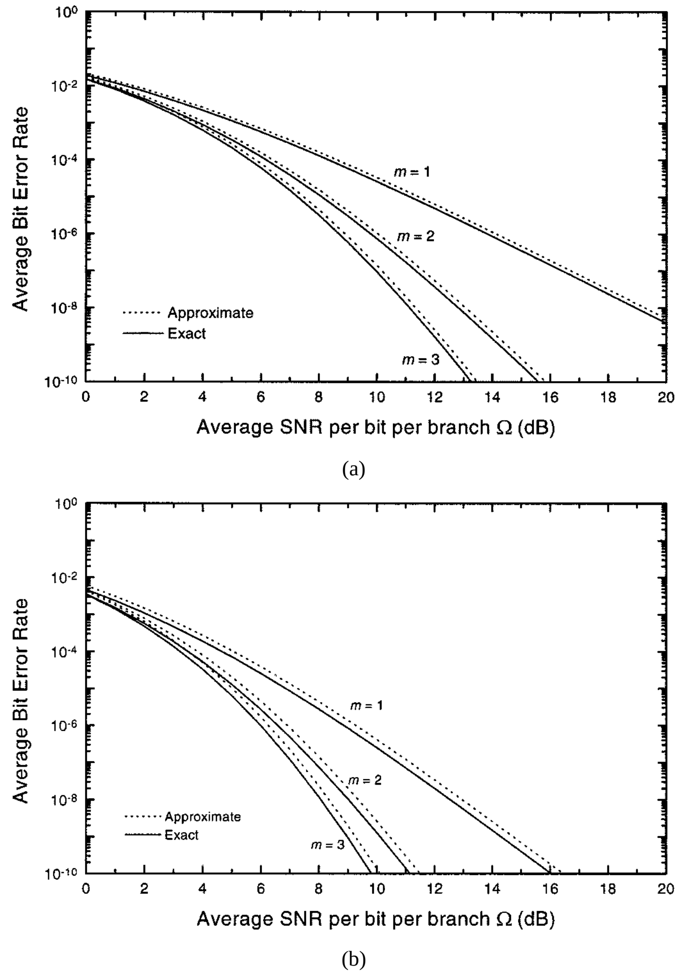


Fig. 2. Exact and approximate average BER's against average SNR per bit per branch Ω for fading parameters m of 1, 2 and 3. (a) GSC with $K=2, L=4$. (b) GSC with $K=3, L=6$.

REFERENCES

[1] N. Kong, T. Eng, and L. B. Milstein, "A selection combining scheme for rake receivers," in *Proc. ICUPC*, Tokyo, Japan, Nov. 1995, pp. 426–429.
[2] T. Eng, N. Kong, and L. B. Milstein, "Comparison of diversity combining techniques for Rayleigh fading channels," *IEEE Trans. Commun.*, vol. 44, pp. 1117–1129, Sept. 1996.
[3] M.-S. Alouini and M. K. Simon, "Performance of coherent receivers with hybrid SC/MRC over Nakagami-m fading channels," *IEEE Trans. Veh. Technol.*, vol. 48, pp. 1155–1164, July 1999.
[4] M.-S. Alouini and M. K. Simon, "Application of the Dirichlet transformation to the performance evaluation of generalized selection combining over Nakagami-m fading channels," *Int. J. Commun. Networks*, vol. 1, pp. 5–13, Mar. 1999.
[5] L. C. Choo and T. T. Tjhung, "BER performance of DQPSK in Nakagami fading with selection diversity and maximal-radio combining," *IEEE Trans. Commun.*, vol. 48, pp. 1618–1621, Oct. 2000.
[6] G. Fedele, "N-branch diversity reception of M-ary DPSK signals in slow and nonselective Nakagami fading," *Eur. Trans. Telecom.*, vol. 7, pp. 119–123, Mar.–Apr. 1996.