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Geometric Quantum Computation and Multiqubit Entanglement with Superconducting Qubits inside a Cavity

Shi-Liang Zhu,[1,2] Z. D. Wang,[3,4] and Paolo Zanardi[2]

[1]School of Physics and Telecommunication Engineering, South China Normal University, Guangzhou, China
[2]Institute for Scientific Interchange Foundation, Viale Settimio Severo 65, I-10133 Torino, Italy
[3]Department of Physics, University of Hong Kong, Pokfulam Road, Hong Kong, China
[4]National Laboratory of Solid State Microstructures, Nanjing University, Nanjing, China

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We analyze a new scheme for quantum information processing, with superconducting charge qubits coupled through a cavity mode, in which quantum manipulations are insensitive to the state of the cavity. We illustrate how to physically implement universal quantum computation as well as multiqubit entanglement based on unconventional geometric phase shifts in this scalable solid-state system. Some quantum error-correcting codes can also be easily constructed using the same technique. In view of the gate dependence on just global geometric features and the insensitivity to the state of cavity modes, the proposed quantum operations may result in high-fidelity quantum information processing.

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Superconducting qubits have recently attracted significant interest because of their potential suitability for integrated devices in quantum information processing [1–4]. So far, experimental research on quantum information processing using this kind of system has mostly focused on the behavior of a single isolated qubit as the decoherence time of this solid-state system is quite short [1]. Only very recently significant achievements on superconducting two-charge-qubit systems were reported, i.e., realization of an entangled state for two qubits, and implementation of a conditional gate [2]. Also note that multiparticle entanglement was experimentally reported only in photons and trapped ions. Implementation of a universal set of high-fidelity quantum gates and generation of multiqubit entanglement will be the next significant and very challenging steps towards quantum information processing based on this scalable solid-state approach.

In this Letter, by designing a device consisting of superconducting charge qubits coupled through a cavity, we propose a feasible scheme to implement a universal set of quantum gates and produce the Greenberger-Horne-Zeilinger (GHZ) state [5] based on unconventional geometric phases. In addition, we find that this scheme is also applicable to construct quantum error-correcting codes (QECC) [6]. In particular, we show how to realize the geometric evolution operators $U_{x,y}(\gamma) = \exp(i\gamma J_{x,y}^2)$ with $J_{x,y}$ collective operators and γ an unconventional geometric phase [7], which may have some inherent fault-tolerant features due to the fact that unconventional geometric phases depend only on some global geometric property [7–13]. It is also shown that in terms of $U_{x,y}(\gamma)$ of the present system one is able to achieve a universal set of quantum gates, to generate even-N-qubit entanglement simultaneously by one operation [14], and to construct the QECC [15]. All of these are essential ingredients in quantum information processing. Implementation of these tasks based on the same set of geometric quantum operators may simplify the experimental operations. Apart from the high-fidelity advantage, the geometric scheme proposed here has other distinctive merits: the operator is insensitive to the state of the cavity modes [14,16], is tolerant to device parameter nonuniformity in quantum computation, and the process can be fast [17,18].

Superconducting nanocircuits coupling through cavities have been shown to be a promising solid-state system for implementation of quantum computation and quantum communication [19–22]. One of advantages for qubits placed in a cavity is that the cavity can also protect the qubits from the environment, which is important for a useful operation of qubits especially in the scaling up of the solid-state devices. Moreover, an architecture using one-dimensional transmission line resonators to reach the strong coupling limit between cavity and superconducting nanocircuits was theoretically proposed in Ref. [21] and then experimentally achieved in Ref. [22]. In the present work, we address a system with N specially designed superconducting charge qubits coupled through a high-quality single-mode cavity, as shown in Fig. 1. Clearly, as shown in Fig. 1(a), the newly designed single qubit is significantly different from those studied before [1,2,10,19]. The jth qubit consists of a small superconducting box with n_j excess Cooper-pair charges, formed by two SQUIDs with Josephson coupling energies $E_j^{\epsilon l}$ ($\epsilon = a, b; l = 1, 2$) and a π-phase junction, and each SQUID is penetrated by a magnetic flux ϕ_j^{ϵ}. A control gate voltage V_j^g is connected to the system via a capacitor C_j^g. The Hamiltonian of the jth qubit reads

$$H = E_{\mathrm{ch}}(n_j - \bar{n}_j)^2 - \sum_{\epsilon l} E_j^{\epsilon l} \cos\varphi_j^{\epsilon l}, \qquad (1)$$

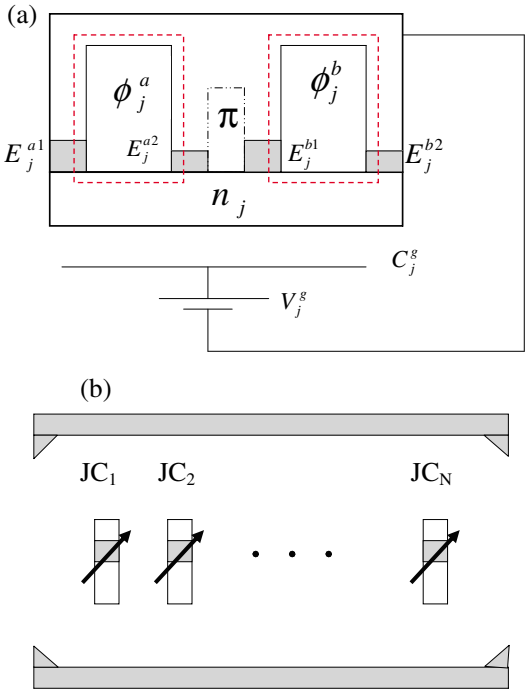

FIG. 1 (color online). Josephson charge qubits. (a) A single Josephson charge qubit. (b) A series of Josephson charge qubits coupled through a cavity.

where $E_{\rm ch} = 2e^2/C_j$ is the charging energy with C_j being the total capacitance of the box; $\bar{n}_j = C_j^g V_j^g/2$ is the induced charge and can be controlled by changing V_j^g. The gauge-invariant phase difference $\varphi_j^{\epsilon l} = \Theta_j - (2\pi/\phi_0) \times \int \mathbf{A}_j^{\epsilon l} \cdot d\mathbf{l}$, with Θ_j being the phase difference of the superconducting wave function across the junctions in a particular gauge and $\mathbf{A}_j^{\epsilon l}$ being the vector potential in the same gauge. Assuming that Josephson junctions are placed inside a single-mode resonant cavity, we have $(2\pi/\phi_0) \times \int \mathbf{A}_j^{\epsilon l} \cdot d\mathbf{l} = (2\pi/\phi_0) \int \mathbf{A}_j^{\prime\epsilon l} \cdot d\mathbf{l} + g_j(a + a^\dagger)$, where a and $a^\dagger$ are the creation and annihilation operators for the single mode, $\phi_0 = \pi\hbar/e$ is the flux quantum, g_j is the coupling constant between the junctions and the cavity, and $\mathbf{A}_j^{\prime\epsilon l}$ is the vector potential determined by the magnetic flux ϕ_j^ϵ penetrating the ϵth SQUID hole in the jth qubit.

At temperatures much lower than the charging energy and the gate voltage being tuned close to a degeneracy ($\bar{n}_j \sim 1/2$), the relevant physics is captured by considering only the two charge eigenstates $n_j = 0, 1$, which constitute the basis $\{|0\rangle, |1\rangle\}$ of the computational space of the qubit. If we have N such qubits located inside a single-mode cavity [Fig. 1(b)] with frequency $\omega_c/2\pi$, to a good approximation, the whole system can be considered as N two-state systems coupled to a quantum harmonic oscillator. On the other hand, $\varphi_j^{\epsilon l}$ are determined from the flux quantization for two independent loops and an effective π-phase junction between $a2$ and $b1$ for each qubit [see Fig. 1(a)]; that is, $\varphi_j^{a1} - \varphi_j^{a2} = \Phi_j^a$, $\varphi_j^{b1} - \varphi_j^{b2} = \Phi_j^b$, and $\varphi_j^{a2} - \varphi_j^{b1} = \pi$, where the flux $\Phi_j^\epsilon = \tilde{\phi}_j^{\ \epsilon} + g_j(a + a^\dagger)$ with $\tilde{\phi}_j^{\ \epsilon} = 2\pi\phi_j^\epsilon/\phi_0$. Moreover, the average phase drop $\sum_{\epsilon l} \varphi_j^{\epsilon l}/4$ is equal to Θ_j, which conjugates to the Cooper-pair number n_j. Here we assume that $E_j^{al} = E_j^{bl} = E_j^l$ and $g_j^\epsilon = g_j$. In this case, the whole system can be described by the Hamiltonian $H = H_0 + H_{\rm int}^l$, where

$$H_0 = \hbar\omega_c\left(a^\dagger a + \frac{1}{2}\right) + \sum_j^N E_{\bar{n}_j}\sigma_j^z/2, \tag{2}$$

$$H_{\rm int}^l = -\frac{1}{2}\sum_{lj}[i(-1)^l E_j^l(1 - e^{i\Gamma_j^l})e^{-i\phi_j^-/2}\sigma^+ + \text{H.c.}]. \tag{3}$$

Here $E_{\bar{n}_j} = 2E_{\rm ch}(\bar{n}_j - 1/2)$, $\Gamma_j^l = \phi_j^- + i(-1)^{l-1} \times [\phi_j^+ + g(a + a^\dagger)]$, and $\phi_j^\pm = (\tilde{\phi}_j^a \pm \tilde{\phi}_j^b)/2$. A spin notation is used for the qubit j with Pauli matrices $\{\sigma_j^x, \sigma_j^y, \sigma_j^z\}$, and $\sigma_j^\pm = (\sigma_j^x \pm i\sigma_j^y)/2$. It is interesting to note that this Hamiltonian is similar to that used in Ref. [14] when the external magnetic flux $\tilde{\phi}_j^\epsilon$ depends on the time t with a constant rate ω_j^ϵ.

Comparing with the charge qubit made of one SQUID and coupled to the cavity [19], where the initial cavity mode should be in the ground state in performing quantum gates (the task is exceedingly difficult to implement experimentally), one more SQUID being penetrated through another magnetic flux is used here. Two magnetic fluxes for one qubit can be used to manipulate the qubit states. The function of these fluxes is similar to bichromatic lights in quantum computation based on trapped ions [14]: transition paths involving unpopulated cavity states interfere destructively to eliminate the influence of the cavity mode. This phenomenon plays a key role in the implementation of a quantum gate which is insensitive to the initial state of cavity mode and robust against changes in the cavity state occurring during operation. This scenario has been used in the implementation of quantum computation with trapped ions [13,14,16–18,23,24] and in generating experimentally the maximally entangled state for four ions [23]. In the following we illustrate how this strategy can be applied to the present system.

To eliminate the influence of the cavity mode, we now propose to apply two constantly growing external fluxes penetrating the designated SQUIDs in the jth qubit: $\tilde{\phi}_j^a = \omega_j^\phi t - \beta_j^0$ and $\tilde{\phi}_j^b = \omega_j^\phi t - \beta_j^0 - 2k\pi$, where β_j^0 ($\beta_j^0 - 2k\pi$) is the initial value of $\tilde{\phi}_j^a$ ($\tilde{\phi}_j^b$) and k is an integer. Experimentally, one possible way is to place each SQUID just above an approximately inductive circuit with a constant voltage $V_0 = \omega_j^\phi \phi_0/2\pi$. Expanding the Hamiltonian (3) to the first order of g_j in the Lamb-Dicke limit and under the rotating wave approximation as well as in the interaction picture $U_0 = \exp(-iH_0 t)$, with $E_{\bar{n}_j} = 0$ (i.e.,

the qubits are set at the degeneracy point) and $E_j^1 = E_j^2 = E_j$, yields

$$H_{\rm int}^{\eta} = \sum_j^N g_j E_j (a^\dagger e^{i\delta_j t + i\beta_j^0} + \text{H.c.})\sigma_j^{\eta}, \qquad (\eta = x, y), \quad (4)$$

where $\eta = x,\ y$ for $k = 0, 1$, and $\delta_j = \omega_c - \omega_j^{\phi} \ll \omega_j^{\phi}$ [25].

We now show that the Hamiltonian described in Eq. (4) can be used to perform universal quantum computation as well as to produce the GHZ state. Let us first address a simpler case, in which all parameters for different qubits are the same, namely, $E_j^{\epsilon l} = E$, $\delta_j = \delta$, and $\beta_j^0 = \beta^0$. The corresponding Hamiltonian (4) can be rewritten as

$$H_{\rm int}^{\eta} = gE(a^\dagger e^{i\delta t + i\beta^0} + \text{H.c.})J_{\eta}, \qquad (5)$$

where $J_\eta = \sum_j^N \sigma_j^\eta$ is the collective spin operator. By using the magnus formula, the evolution operator $U(t)$ is found as [7]

$$U_\eta(t) = e^{i\gamma(t)J_\eta^2} e^{[\alpha(t,\beta^0)a^\dagger - \alpha^*(t,\beta^0)a]J_\eta}, \qquad (6)$$

where $\alpha(t, \beta^0) = (gE/\hbar\delta)(1 - e^{i\delta t})e^{i\beta^0}$ and $\gamma(t) = (gE/\hbar\delta)^2(\delta t - \sin\delta t)$. Here $\alpha(t, \beta^0)$ is a periodic function of time and vanishes at $t_m = 2m\pi/\delta$ for an integer m. At the time $t = t_m$, the evolution operator is explicitly expressed as

$$U_\eta(\gamma) = \exp[i\gamma J_\eta^2]. \qquad (7)$$

This operator is insensitive to heating by removing the influence of the cavity mode represented by the last exponent in Eq. (6).

Interestingly, the evolution operators (7) can provide a set of universal quantum gates. It is well-known that, for achieving universal quantum computation, we need to realize two noncommuting single-qubit gates and one nontrivial two-qubit gate. When only two qubits are considered, it is straightforward to check that U_x (or U_y) is a nontrivial two-qubit gate. U_x can be used to produce an entangled state from an untangled state. For example, the maximally entangled state $(|00\rangle - i|11\rangle)/\sqrt{2}$ is derived when $U(\pi/2)$ is directly applied to the untangled state $|00\rangle$. Moreover, a controlled-NOT gate is explicitly generated by U_x plus single-qubit rotations in Ref. [14]. We now work out how to use the operators $U_{x,y}$ to achieve a set of noncommuting single-qubit rotations if one of the qubits is an auxiliary qubit. Denoting $|\psi_{\pm}^{x,y}\rangle_a$ as the eigenstate of the operator $\sigma_{x,y}$ for the auxiliary qubit with eigenvalue ± 1, and choosing the initial state as one of the eigenstates, we then have

$$U_{x,y}|\psi\rangle|\psi_{\pm}^{x,y}\rangle_a = e^{2i\gamma}[e^{\pm 2i\gamma\sigma_{x,y}}|\psi\rangle]|\psi_{\pm}^{x,y}\rangle_a. \qquad (8)$$

It is clear from Eq. (8) that an effective single-qubit gate $U_{x,y}^{(1)} = \exp(\pm 2i\gamma\sigma_{x,y})$ is obtained (up to an irrelevant overall phase) by application of $U_{x,y}$ to two qubits, but one of them is an auxiliary qubit with a fixed initial state and then is disregarded after the gate operation. Since $|\psi_{\pm}^{x}\rangle_a$ and $|\psi_{\pm}^{y}\rangle_a$ are the ground states of the Hamiltonian at $(\bar{n}_a = 1/2, \phi^- = 0)$ and $(\bar{n}_a = 1/2, \phi^- = \pi)$, respectively, it is rather easy to experimentally realize the required initial state in the present system. Certainly, $U_x^{(1)}$ and $U_y^{(1)}$ are noncommuting and consist of the well-known single-qubit rotations. Although one more auxiliary qubit is required, all important operations required (all logical gates and construction of QECC are addressed below) can be achieved with similar manipulations, which may simplify experimental operations. In addition, comparing with qubits coupled through capacitance, a nearest neighbor interaction, the distinctive merit is that the nontrivial two-qubit gate proposed here may act on any pair of qubits.

It is worth pointing out that a nontrivial two-qubit gate can also be implemented when the parameters g_j and E_j are dependent on j; thus the method is also insensitive to fabrication errors often appearing in realistic solid-state experiments. We still consider N charge qubits in a cavity, but only two of them are resonant with the cavity and thus are controlled by the Hamiltonian (4). So only the two resonant qubits are relevant and the evolution operator is given by

$$U(t) = e^{i\sum_{j,l=1}^{2}\gamma_{jl}\sigma_j^\eta\sigma_l^\eta} e^{[\sum_{j=1}^{2}\alpha_j(t,\beta_j^0)\sigma_j^\eta a^\dagger - \text{H.c.}]}, \qquad (9)$$

where $\alpha_j(t, \beta_j^0) = (g_j E_j \hbar\delta_j)(1 - e^{i\delta_j t})e^{i\beta_j^0}$ and $\gamma_{jl}(t) = [g_j g_l E_j E_l/(\hbar^2 d_{jl}\delta_j\delta_l)][\delta_j \sin(d_{jl}t + \beta_{jl}^0) - d_{jl}\sin(\delta_j t + \beta_{jl}^0) - \delta_l \sin\beta_{jl}^0]$ with $d_{jl} = \delta_j - \delta_l$ and $\beta_{jl}^0 = \beta_j^0 - \beta_l^0$ when $d_{jl} \neq 0$. Note the facts that $\sum_j \alpha_j(t, 0) = -\sum_j \alpha_j(t, \pi)$, and $\gamma_{jl}(t)$ is the same for $\beta_j^0 = \beta_l^0$. Therefore, the influence of the cavity mode can also be eliminated by using two operators in succession: two qubits evolve with $\beta_j^0 = \beta_l^0 = 0$ for a first period $\tau/2$ and then with $\beta_j^0 = \beta_l^0 = \pi$ for the second period $\tau/2$. Moreover, the controlled phase gate $\text{diag}(1, 1, 1, -1)$ is obtained when $\gamma_{jl} = -\pi$ after performing one-qubit operations [17].

Remarkably, following the approach addressed in Ref. [7], it is straightforward to find that the phase γ in the operators (7)–(9) satisfies the relation $\gamma = -\gamma_g$ ($\gamma_d = -2\gamma_g$), where γ_g is the geometric phase and γ_d is the dynamic phase accumulated in the evolution. Thus γ is an unconventional geometric phase shift, which consists of both a geometric component and a nonzero dynamic component, but still depends only on global geometric features. Because of this, the high fidelity of the gates in the present system may be experimentally achieved. Note that a recent experiment on trapped ions demonstrated that the operation realized by unconventional geometric phases possesses the high fidelity [13], which really benefits from the geometric features: the phase is determined only by the

path area, not on the exact starting state distributions, path shape, orientation in phase space, or the passage rate to traverse the closed path. Moreover, this operation is robust to small noncyclic perturbations [7,26].

At this stage, we illustrate how to produce the GHZ state in this system. Note that the operator (7) is independent on the number of qubits, by choosing $\gamma = \pi/2$ and an initial state $|\Psi^i\rangle = |00\cdots0\rangle$; the final state $|\Psi^f\rangle = e^{i\pi/2J_x^2}|\Psi^i\rangle$ is found to be a GHZ state given by [14]

$$|\Psi^f\rangle = \frac{1}{\sqrt{2}}[e^{-i\pi/4}|00\cdots0\rangle + e^{i\pi(1/4+N/2)}|11\cdots1\rangle], \quad (10)$$

when N is even. Although the operator (7) does not directly produce the maximally entangled state described in Eq. (10) for odd N, it suffices to get the GHZ state by applying the unitary operator $U = \exp(-i\pi J_x/2)$ in addition to the application of Eq. (7). Starting from Eq. (3), it is not difficult to see that the operation U can be implemented in the system by just choosing ($\bar{n}_j = 1/2$, $\phi_j^- = 0$, $\phi_j^+ \neq 0$). To see the feasibility of the present scheme with current technology, we now use typical values of physical parameters in the system to estimate the operation time τ. We have $\gamma \sim g^2E^2\tau/\hbar^2\delta$. Therefore, $\tau \sim \pi\hbar^2\delta/2g^2E^2$ for $\gamma = \pi/2$, which is about 20 ns for $E = 40$ μeV [2], $\hbar\omega_c = 30$ μeV, $\delta = \omega_c/10$, and $g = 10^{-2}$ [20,21]. Thus τ is significantly less than the decoherence time of qubit ($\sim$0.5 μs) without the protection of the cavity [4] and is much less than the photon lifetime of the cavity mode $\tau_c = Q/\omega_c = 22$ μs, as the quality factor of the cavity $Q = 10^6$ was reported experimentally [27]. In this case the voltage V_0 required to generate the external flux is$9\omega_c\phi_0/20\pi \sim 13$ μV, a typical voltage for manipulating Josephson junctions.

Another useful application of the present geometric approach is to construct QECC in this solid-state system. The operator (7) can also be used for QECC [15]. For example, the Shor coding

$$\alpha\left[\frac{1}{\sqrt{2}}(|000\rangle - |111\rangle)\right]^{\otimes 3} + \beta\left[\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)\right]^{\otimes 3} \quad (11)$$

with $|\alpha|^2 + |\beta|^2 = 1$ is achieved by using the operator (7) plus single-qubit measurement or using the operator (7) and a coupling $\sigma_z\sigma_z$, which arises naturally in charge qubits coupled through capacitors. Therefore, the approach proposed here provides a geometric way to efficiently construct this kind of essential code in quantum information. A significant result here is that implementation of a universal set of quantum gates as well as construction of QECC can be based on the same set of geometric quantum operators with intrinsic fault-tolerant features and then may simplify the experimental operations for quantum computation.

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