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Abstract

Technology Forecasting using Data Envelopment Analysis (TFDEA) provides an effective means to forecast technological capability over time without the burden of fixed *a priori* weighting schemes. However, there are situations where result reproduction can be a challenge as first pointed out in a previous *Technological Forecasting and Social Change* article [11]. When using a commonly used extension of TFDEA, there are circumstances where multiple optimal solutions can complicate analysis. This paper addresses this issue through extending the TFDEA model in a manner consistent with common Data Envelopment Analysis (DEA) techniques. The extension is then demonstrated using datasets from previous publications on fighter jet and commercial airplane technology where the issue of multiple optima has been observed. The results indicate that traditional TFDEA may generate varying forecasts depending on the software used, which can be dealt with by introducing a secondary objective function. Therefore, researchers should explicitly state which secondary objective function they are using for the TFDEA applications.

Keywords

Technology Planning and Forecasting

Time Series Forecasting

Data Envelopment Analysis

Introduction

As frontier analysis methods become popular in modern benchmarking studies, their emphasis on taking advantage of extreme data points has been widely used in the technological forecasting field as well. Technology Forecasting using Data Envelopment Analysis (TFDEA), which falls under this category, has been applied to various areas covering public sectors (U.S. fighter jet development [11] and worldwide wireless protocol adoption [13,16]) as well as cutting edge industries including microprocessor [3], commercial airplane [15], and flat panel [17] since the first introduction in *PICMET '01* [4].

TFDEA application has spread and it is actively being examined by researchers worldwide. Kim *et al.* [14] proposed a resampling technique using Constrained Canonical Correlation Analysis (CCCA) that could make more reliable forecasts for battle tank development. Durmusoglu and Dereli [10] introduced a modified TFDEA model that can employ a dynamic Rate of Change (RoC) by fitting a cubic regression into the RoC calculation. Tudorie [20] applied TFDEA to capture the technological progress and future performance of the Electric Vehicle (EV) technologies. It was found that Battery Electric Vehicles (BEV) showed relatively more accurate forecast than the Hybrid Electric Vehicle (HEV). Shin [19] proposed a hybrid TFDEA model with a growth curve in an attempt to take the maturity level of each technology attribute into account. Cole [6], in his dissertation, compared TFDEA with a hyper-plane model and multi-dimensional growth model (MDGM) to develop an integrated projection model for battery technology. He found that the TFDEA results to be less biased and yield more normally distributed residuals than the other two forecasting methods.

However, the issue of multiple optima can occur in DEA-related models primarily due to the issue of degeneracy in linear programming. The envelopment form of the traditional DEA model can result in non-unique or weakly efficient targets of performance in the envelopment model, which is typically resolved by introducing a secondary phase of slack maximization [1]. Similarly, the two-step procedure is used to identify weights associated with a full dimensional efficient facet (FDEF) in the multiplier model [7]. An extension of

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DEA termed cross-efficiency or cross-evaluation is similarly affected by multiple optima, and a formal examination and resolution of the issue was developed by Doyle and Green [9].

Likewise, it was determined by Inman, Anderson, and Harmon (subsequently referred to as IAH) in *Technological Forecasting and Social Change, vol.73, no.9* [11] that a variation of TFDEA can at times result in non-unique solutions and cause difficulty with reproducing calculations in certain cases. That is, different forecast results may be obtained from traditional TFDEA depending, for instance, on the software used. IAH illustrated the issue and recommended that this be addressed in future work. This paper aims to address this issue and extend the methodology as inspired by a common DEA technique to resolve the issue of non-unique solutions.

Explanation of Dynamic Frontier Year

The original implementation of TFDEA [3] used a static frontier year based on the year at which the analysis was conducted. Later, Inman [12] introduced the concept, referred to here as a dynamic frontier year, that uses a combination of dates associated with the products defining the frontier. To illustrate this point, the following example of four products released in three different years is illustrated in Figure 1. The fourth product, d, is a set of specifications for a product with an expected release date of 2013.

Conceptually, TFDEA estimates RoC and then uses this momentum to forecast future products. The dynamic frontier year can be illustrated by examining product d. It is a 2013 product that is compared to products a, b, and c from 2011 and earlier. Given technological progress, d should outperform these earlier products. Product d's performance is projected to the state of the art surface from current products, namely, a, b, and c. This projected point is labeled $d^{(Target)}$. In an output-oriented model, with a single input (perhaps manufacturing cost) and a single output (such as a performance proxy), it is easy to visualize the projection and measure the values. This is simply the ratio of the heights of the vertical lines. One would characterize

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the output-oriented efficiency of product d to be examined at the current time of the forecast,

$$\phi_d^C = \frac{Y_{d(\text{Target})}}{Y_{d^{2013}}} = \frac{6}{10} = 0.6.$$ This indicates that the best convex combination of comparison for d using earlier

products performs only 60% as well as d. Conversely, d is $\frac{1}{.6} = \frac{5}{3} = 1.\overline{6}$ times better than the target based

on earlier technology.
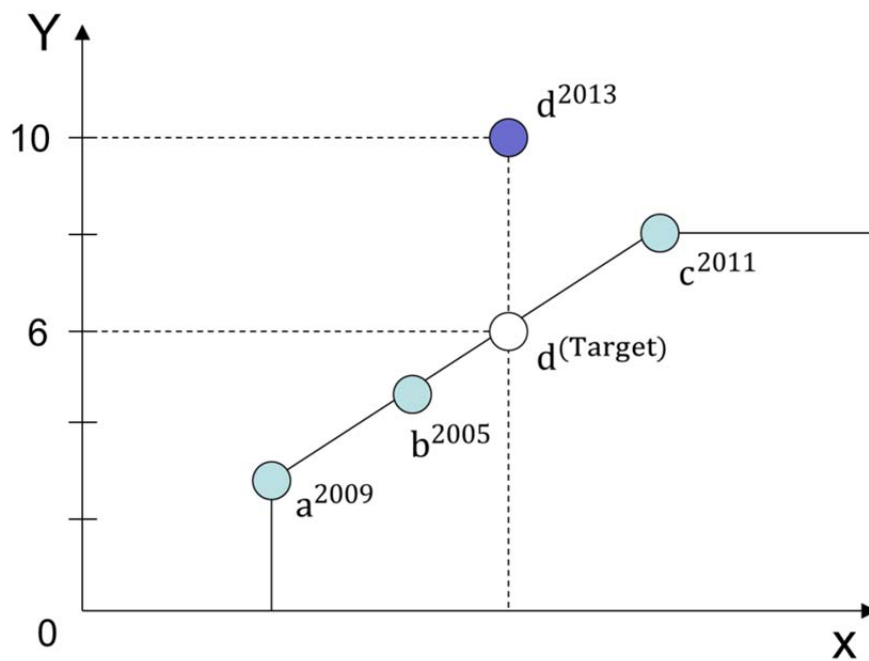
Figure 1. Illustrative Example of TFDEA with Multiple Optima

Next, let us extend the example by assuming that products had been improving in the past by 15% a

year. Therefore, the average RoC is 1.15 ($\bar{\gamma}$ =1.15).

Now product d's specifications may be used to forecast the expected release date based upon the

average RoC. Given the RoC, $\bar{\gamma}$ =1.15, and the amount by which it surpasses current technologies of $1.\overline{6}$, it

is a straightforward calculation to find the number of years by which it exceeds the current state of the art:

$1.\overline{6} = 1.15^n$. Taking the natural logarithm of both sides results in $\ln(1.\overline{6}) = \ln(1.15^n) = n\ln(1.15)$ and then $n = \ln(1.\overline{6})/\ln(1.15) = 3.655$ years. This indicates that, based on past trends, one should expect 3.655 years of advancement to achieve the performance level of d.

When using the original static frontier year concept, the 3.655 years would be relative to the date of the forecast being made, 2011, resulting in an expected release of product d to be in 2014.655, or about a year and a half after the product's expected release date of 2013. This indicates that it is an aggressive set of specifications, and if it reaches the market on time and with current specifications, is likely to be better than competitors. On the other hand, there may be significant technical risk since it requires exceeding the past technological advancement.

Potential for Multiple Optima

The dynamic frontier concept recognizes that target points may be composed of products from different years, and it may not be appropriate to assume that the current time period is the basis for the best way of estimating the year of the target. In the example provided, is 2011 really the best estimate of the age of the technologies that d is being compared against? Product d is being compared against some combination of the three products a, b, and c, all from different years. If it is being compared against just a and c, then the weighted average of the years is approximately 2010.4, which gives a forecasted date of about 2014.

On the other hand, product d's target could be equally well formed from a combination of products b and c. In this case, the weighted average of the years of release would be closer to b's release date, let us say about 2008.5, resulting in a forecasted release of about 2012.

Not only are there two potential dynamic frontier year targets of 2008.5 and 2010.4 resulting in different forecasts, combinations of products a and b could be used simultaneously with varying contributions

from c to result in an infinite number of combinations between 2008.5 and 2010.4. Each of the alternate solutions would result in either a different RoC for past product or a different forecasted year of release for future product, which causes difficulty in replicating results. This phenomenon was found for five of the fighter jets and nine of the commercial airplanes.

Conceptually, it is simple to resolve the issue of multiple optima by providing a secondary objective function. The first objective function is to calculate the performance relative to the state of the art surface for each product. The secondary objective function is to select either the minimum or the maximum target year. An unambiguous resolution to this issue of multiple optima requires greater mathematical formalism, which will be provided in the following section.

TFDEA Formulation

To simplify management of results and to be consistent with current implementations, the TFDEA formulation can be written as a single, larger, linear program in the following manner as shown by (1)-(7). It rewrites the series of smaller linear programs as one large linear program. First, DEA problems with n decision making units, DMUs, typically require n separate linear programs but these can be combined into a single, larger linear program. This is done by adding a subscript, k, to each of the variables, summing all the objective functions, and incorporating all of the constraints.

In TFDEA, we need an efficiency score at the time of release and against what is assumed to be the current time (or the period at which the frontier is considered frozen). These are denoted by an R or C respectively.

First, there are three important data components. The release date for product k is defined as t_k. The ith input or "structural characteristics" of product k is $X_{i,k}$. Similarly, $Y_{r,k}$ is the rth output or functional characteristic of product k.

$$\max \quad \sum_{k=1}^{n} \left(\phi_k^R + \phi_k^C \right) \qquad (1)$$

$$s.t. \quad \sum_{j=1}^{n} \lambda_{j,k}^h = 1, \quad h \in \{R, C\}, k \in \{1, 2, \ldots, n\} \qquad (2)$$

$$\sum_{j=1}^{n} X_{i,j} \lambda_{j,k}^h \le X_{i,k}, \quad h \in \{R, C\}, k \in \{1, 2, \ldots, n\}, i \in \{Inputs\} \qquad (3)$$

$$\sum_{j=1}^{n} Y_{r,j} \lambda_{j,k}^h \ge \phi_k^h Y_{r,k}, \quad h \in \{R, C\}, k \in \{1, 2, \ldots, n\}, r \in \{Outputs\} \qquad (4)$$

$$\lambda_{j,k}^R = 0 \quad \forall (j,k) \,|\, t_j > t_k \qquad (5)$$

$$\lambda_{j,k}^C = 0 \quad \forall (j,k) \,|\, t_j > T \qquad (6)$$

$$\lambda_{j,k}^h \ge 0, \quad \forall (j,k), h \in \{R, C\} \qquad (7)$$

The variable, ϕ_k^R, measures the amount by which product k is surpassed by the technology available at the time of release for product k. This time is denoted as t_k. A value, $\phi_k^R = 1.0$, indicates that product k is state of the art at the time product k is released. A value of $\phi_k^R > 1.0$ then indicates the amount by which all of the outputs of product k must be increased in order to be state of the art at the time of its release.

Similarly, ϕ_k^C measures the performance of product k relative to the state of the art at the fixed time T. For example, a value of $\phi_k^C = 1.5$ indicates that the product is being outperformed by products available at time T by 50%. Conversely, a value of $\phi_k^C = 0.8$ indicates that the state of the art available at time T only performs 80% as well as product k.

The objective function (1) is a sum of the maximum of each of the *2n* linear programs. Each of these linear programs is essentially a standard output-oriented, variable returns to scale (VRS), envelopment DEA

model. Given that all the variables are continuous and the modest number of variables and constraints, a single large problem can be solved very quickly with modern optimization software and computer hardware.

The VRS is enforced by (2). It requires each product, k, when it is evaluated, to be compared against a construction of other products that sums to one. In other words, it cannot be compared against a much larger product, j, which is simply rescaled to a much smaller size. For example, if product five was much larger than product k, we might see a result of $\lambda_{j,k}^{h} = 0.1$, for say $j=5$ and $\lambda_{j,k}^{h} = 0$ for all other j. The VRS disallows this simple rescaling. The VRS constraint was incorporated into DEA by Banker, Charnes, and Cooper and is widely used in many DEA applications [5].

The input for each product k's evaluation is considered by constraint (3). This constraint ensures that for each evaluation (at time of release-R and at time of current horizon-C), the input used by the target is less than or equal to the amount actually used by product k as denoted by $X_{i,k}$. It should be noted that in IAH, a constant value of one was used as the only input $X_{1,j} = 1$ for all fighter jet j.

The next constraint, (4), relates outputs achievable by the combination of products indicated by the variables λ to be greater than or equal to the level of outputs achieved by product k, multiplied by ϕ. This constraint makes the direct linkage with the objective function components, ϕ.

This is followed by a constraint (5) that says the at-time of release evaluation of product k can only be done by looking at products that were released at the same time product k was released or earlier. This is enforced by setting all multipliers, $\lambda_{j,k}^{R}$, to be zero for any product j that was released after k was released: $t_j > t_k$.

Constraint (6) performs the same role of limiting which products can be used for evaluation but instead uses the analysis time period of T to limit where the products can be drawn for evaluation. These constraints say that products after time period T cannot be used for evaluating the target. In the case of the

fighter jet application of IAH, *T=1960* corresponded to the position of the analyst only using data of released planes up to and including 1960 in order to forecast subsequent aircraft.

Non-negativity of the variables is enforced in (7).

Having solved the above linear program, the various values of ϕ and λ can then be used to arrive at estimates of rates of change, γ. First, though, the dynamic frontier year model, introduced in Inman [12] and used in IAH, also requires the calculation of the frontier year used for evaluating each of the product k's evaluations. These were defined as $t_{k,eff}$ in IAH.

Calculating Rate of Change

For the purpose of this paper, we will redefine $t_{k,eff}$ as E_k^h, to eliminate ambiguity. The corresponding equation (8) is consistent with IAH.

$$E_k^h = \frac{\sum_{j=1}^n t_j \lambda_{j,k}^h}{\sum_{j=1}^n \lambda_{j,k}^h}, \quad \forall k, h \in \{R, C\} \tag{8}$$

In the case of VRS, the denominator of (8) will always be equal to one due to constraint (2). Therefore, the calculation in (8) simplifies under VRS to be simply the following.

$$E_k^h = \sum_{j=1}^n t_j \lambda_{j,k}^h, \quad \forall k, h \in \{R, C\} \tag{9}$$

The rates of change may then be calculated by taking all products that were efficient at the time of release, $\phi_k^R = 1$, but were superseded by technology at time T, $\phi_k^C > 1$. The periodic RoC needed to supersede each product is then calculated as the following.

$$\gamma_k^C = \left(\phi_k^C\right)^{1/\left(E_k^C - t_k\right)} \tag{10}$$

The issue of multiple optima arises due to different possible values of λ resulting in the same objective function values, ϕ. The different values of λ result in different possible effective times E_k, which can create different estimates of the corresponding γ_k values. Therefore, the problem caused by multiple optima can be resolved by modifying the objective function (1) to determine a unique value of E_k.

$$\max \quad \sum_{k=1}^{n} \left[\left(\phi_k^R + \phi_k^C\right) - \varepsilon \left(E_k^R + E_k^C\right)\right] \tag{11}$$

The parameter ε is a non-Archimedean infinitesimal, which is greater than zero but smaller than any finite positive value. The non-Archimedean infinitesimal is commonly used in DEA models [8] to do a slack maximization. It is imperative that ε not be approximated with a finite value. Some early DEA implementations made the mistake of using finite approximations such as $\varepsilon = 10^{-6}$ which resulted in numerical errors [2]. The actual implementation is to use a two-stage preemptive linear programming approach where in the first phase, the objective is the same as in (1), simply maximize all the values of ϕ. The second phase then holds all the variables ϕ fixed and minimizes the sum of the effective years, E_k^R and E_k^C (or equivalently by maximizing negative E_k^R and E_k^C).

Minimizing the effective years: E_k^R and E_k^C has the interpretation of always saying that when there are multiple ways of forming a target on the frontier peer for product k's with the same distance to the frontier (efficiency), use the combination of products that would have the earliest possible effective date (weighted average of product release dates).

Note that, in case of non-VRS model, calculation of (8) would render the objective function to no longer be linear. For computational purposes, the same general secondary goal of minimizing effective years

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can also be pursued by subtracting the denominator of (8) in the objective function as seen in (12). While this substitution is not technically a numerical approximation, it is generally consistent with minimizing effective year and has the advantage of remaining linear.

$$\max \quad \sum_{k=1}^{n}\left[\left(\phi_k^R + \phi_k^C\right) - \varepsilon\left(\left(\sum_{j=1}^{n} t_j \lambda_{j,k}^R - \sum_{j=1}^{n}\lambda_{j,k}^R\right) + \left(\sum_{j=1}^{n} t_j \lambda_{j,k}^C - \sum_{j=1}^{n}\lambda_{j,k}^C\right)\right)\right] \tag{12}$$

Rather than minimizing effective dates, maximizing the effective dates would also result in a unique solution for rates of change. The corresponding objective function is given in (13).

$$\max \quad \sum_{k=1}^{n}\left[\left(\phi_k^R + \phi_k^C\right) + \varepsilon\left(E_k^R + E_k^C\right)\right] \tag{13}$$

Likewise, in the case of non-VRS models, the same transformation can be made as seen in (14).

$$\max \quad \sum_{k=1}^{n}\left[\left(\phi_k^R + \phi_k^C\right) + \varepsilon\left(\left(\sum_{j=1}^{n} t_j \lambda_{j,k}^R - \sum_{j=1}^{n}\lambda_{j,k}^R\right) + \left(\sum_{j=1}^{n} t_j \lambda_{j,k}^C - \sum_{j=1}^{n}\lambda_{j,k}^C\right)\right)\right] \tag{14}$$

In either case of minimizing or maximizing the sum of effective years, the set of equations (8) calculating E_k^h is done using the result of the linear program.

Re-examining U.S. Fighter Jet Development

The above formulations address the problem of non-unique solution in the VRS TFDEA models. The following section provides numerical results demonstrating this on the fighter jet dataset from Martino [18] used by IAH [11] which used four outputs: maximum Mach, mean flying time between failure, payload, and range of BVR missiles with a constant one as an input and a VRS dynamic frontier year.

k	Model	Date of Release	ϕ_k^R	ϕ_k^C	Maximize E			Minimize E		
					Effective Date	Rate of Change	Forecasted Release Date	Effective Date	Rate of Change	Forecasted Release Date
1	F80	1944	1	1.679899	1957.178	1.04015		1957.178	1.04015	
2	F84	1946	1	1.962264	1958.151	1.057045		1958.151	1.057045	
3	F86	1947	1	1.732124	1957.294	1.054818		1957.294	1.054818	
4	F89	1949	1	1.2	1956	1.026388		1956	1.026388	
5	F94	1950	1	1.915759	1955.573	1.123729		1955.573	1.123729	
6	F100	1953	1	1.405405	1956.162	1.113629		1956.162	1.113629	
7*	F101	1954	1	1	1956			1953.5		
8*	F102	1953	1	1	1956			1953		
9*	F104	1954	1	1	1956			1954		
10	F106	1955	1	1	1955			1955		
11	F8	1956	1	1	1956			1956		
12	F5A	1959	1	1	1959			1959		
13	F4E	1967	1	0.585938	1956.07		1964.049	1956.07		1964.049
14*	F14	1971	1	0.2	1956		1980.022	1953		1977.022
15*	F5E	1971	1	0.2	1956		1980.022	1954		1978.022
16	F15	1972	1	0.3	1956		1973.97	1956		1973.97
17	F16	1974	1.0753	0.328518	1955.197		1971.811	1955.197		1971.811
18	F18	1978	1	0.262238	1956.378		1976.356	1956.378		1976.356
19	F20	1982	1	0.241935	1956.581		1977.762	1956.581		1977.762

Table 1: Fighter jet results be secondary objective functions used (* indicates fighter jets with multiple optima solutions)

A couple of interesting points should be noted from the results. Running separate analyses with both secondary objective functions (minimizing and maximizing the effective dates) will determine if there are alternate solutions that can affect the solution. Therefore, it is noted that IAH found both cases of multiple optima: F14 and F5E that affected the result in their forecasts.

In addition, three cases: F101, F102, and F104 were identified to have alternate effective dates though they are not affecting the RoC. This is because they didn't show technological advancement between their introductions and frontier year of 1960. As previously discussed, effective dates determine the time span to calculate the RoC from past technologies as well as the reference point of forecast for future technologies. Hence, the issue of multiple optima can affect RoCs and/or forecast results. Fighter jet case is an example in which forecast results are affected by differing reference points of forecast without being influenced by alternate RoCs.

While not related to the specific issue of multiple optima, the F16 was interesting. It was found to be efficient at the time of the forecast being made, T=1960, but not at the time of release, 1974. This indicates that while it surpassed the performance of pre-1960 fighter jets ($\phi_{F16}^{C} = 0.328518 < 1$), by the time it was released in 1974, the four post-1960 fighter jets advanced the state of the art surface such that the F16 was not considered state of the art at time of release ($\phi_{F16}^{R} = 1.0753 > 1$).

Table 2 provides numerical results calculated using three different linear programming engines, Xpress-MP, GLPK, and lpSolveAPI consisting of three cases: a base case without a secondary objective function and then cases of minimizing and maximizing the sum of effective dates. It shows that a variety of solutions are obtained with different mean absolute deviations depending upon the software used and the presence of a secondary objective function. In the absence of a way to differentiate them, each is equally correct. It is important to note that the use of one of the secondary objective functions eliminates the issue of

different software providing different solutions. This is critical for researchers to be able to replicate the results.

		Secondary Objective Used		
		None	Maximize Sum of E	Minimize Sum of E
	Xpress-MP	4.005272	4.433844	3.719558
LP Engines	**GLPK**	3.862410	4.433844	3.719558
	lpSolveAPI	3.862415	4.433844	3.719558

Table 2: Mean absolute deviation comparison of forecasts for post-1960 fighter jets by secondary objective function used.

In this application, the post-1960 fighter jets are best forecasted by minimizing the sum of effective dates. This secondary objective function results in the lowest mean absolute deviation of 3.719558 regardless of linear programming engine used. This indicates that one can expect 3.7 years of forecasting error using this model in fighter jet development planning.

Re-examining Commercial Airplane Development

This section provides another numerical result from secondary objective on the commercial airplane dataset from Lamb, Anderson and Daim [15] (subsequently referred to as LAD) which used five outputs: travel range, passenger capacity, passenger fuel efficiency, maximum speed, and cruising speed with a constant one as input and a VRS dynamic frontier year.

k	Model	Date of Release	ϕ_k^R	ϕ_k^C	Maximize E			Minimize E		
					Effective Date	Rate of Change	Forecasted Release Date	Effective Date	Rate of Change	Forecasted Release Date
1*	DC8-55	1965	1	1.036782	1994.447	1.001227		1987.184	1.001630	
2	DC8-62	1966	1	1.012435	1989	1.000537		1989.000	1.000537	
3*	747-100	1969	1	1.010078	2001.643	1.000307		1984.786	1.000635	
4*	747-200	1971	1	1.010078	2001.643	1.000327		1985.446	1.000694	
5*	DC10-30	1972	1.018338	1.036782	1993.863			1987.379		
6*	DC10-40	1973	1.018338	1.036782	1993.863			1987.379		
7*	L1011-TriStar 500	1979	1	1.011211	1995.353	1.000682		1986.882	1.001415	
8*	747-300	1983	1	1	1991.348			1983.000		
9	767-200ER	1984	1.016489	1.046803	1998.970			1998.970		
10	767-300ER	1988	1	1.033580	2003.539	1.002128		2003.539	1.002128	
11	747-400	1989	1	1	1989			1989.000		
12	MD-11	1990	1.017806	1.032138	1995.059			1995.059		
13	A330-300	1993	1	1	1993			1993.000		
14	A340-200	1993	1	1.029601	1999.282	1.004655		1999.282	1.004655	
15	A340-300	1993	1	1.024546	2003.678	1.002274		2003.678	1.002274	
16	MD-11ER	1996	1.016926	1.030304	1995.889			1995.889		
17	777-200ER	1997	1	1.009587	1995.828			1995.828		
18*	777-300	1998	1.011211	1.011211	2001.041			1984.986		
19*	A330-200	1998	1.036782	1.036782	1993.733			1986.199		
20	A340-600	2002	1	1.006869	2004.330	1.002942		2004.330	1.002942	
21	A340-500	2003	1	1.014706	2006.196	1.004579		2006.196	1.004579	
22	777-300ER	2004	1	1	2004			2004.000		
23	777-200LR	2006	1	1	2006			2006.000		
24	A380-800	2007	1	1	2007			2007.000		
25	787-8 Dreamliner	2010	1	0.989973	2003.293		2008.425	2003.293		2007.988
26	747-8	2011	1	0.975083	2001.877		2014.725	2001.877		2013.631
27	787-9 Dreamliner	2013	1	0.956494	2001.638		2024.287	2001.638		2022.359
28	A350-900	2013	1	0.959307	2000.536		2021.690	2000.536		2019.889

Table 3: Commercial airplane results by secondary objective functions used. (* indicates airplanes with multiple optima solutions)

Unlike the fighter jet case, the multiple optima issue here is affecting the RoC calculation which then causes alternate forecasted release dates. In other words, different effective dates only occur in pre-2008 airplanes that the model results in diverged estimates of RoCs, which eventually leads to different forecasts in post-2008 airplanes. As in the earlier fighter jet case, airplanes that were not the state of the art when they were released: DC10-30, DC10-40, 777-300, and A330-200 are not taken into consideration with regard to the RoC calculation. Likewise, an airplane which has been on the state of the art frontier since its release: 747-300 is not affecting RoC calculation because it doesn't show the technological progress within timeframe. Therefore, multiple optima are captured in those airplanes but do not affect the results.

Although post-2008 airplanes don't suffer from multiple optima, the forecasted release dates vary due to the alternate RoCs identified from multiple optima of pre-2008 airplanes. That is, the commercial airplane case is an example in which forecast results are affected by differing RoCs without being influenced by alternate reference points.

It should also be noted that LAD included the 777-200ER in calculating the RoC since it was one of the state of the arts at the time of release ($\phi^R_{777-200ER} = 1$) and was surpassed by the state of the art in 2007 ($\phi^C_{777-200ER} = 1.009587 > 1$). However, its effective date ($E^C_{777-200ER} = 1995.828$) turned out to be earlier than its release date ($t_{777-200ER} = 1997$), which resulted in RoC less than 1 ($\gamma^C_{777-200ER} = 0.991894$). This attenuated the average RoC and, consequently, caused worse results than the other model presented in the paper. This numerical issue is outside the scope of the current paper and will be the subject of future research.

Table 4, in the same manner, provides numerical results calculated using three different linear programming engines with and without secondary objective function. It can be seen here again that results vary depending on the software used without secondary objective function. The results from LAD correspond to the Maximize the Sum of Effective Dates.

In this application, however, the post-2008 commercial airplanes are best forecasted by minimizing the sum of effective dates. This secondary objective function results in the lowest mean absolute deviation of 5.223051.

		Secondary Objective Used		
		None	Maximize Sum of E	Minimize Sum of E
LP Engines	Xpress-MP	5.615992	6.319297	5.223051
	GLPK	6.319126	6.319297	5.223051
	lpSolveAPI	5.586473	6.319297	5.223051

Table 4: Mean absolute deviation comparison of forecasts for post-2007 commercial airplanes by secondary objective function used.

Conclusion

This paper addresses a key issue that must be considered in any TFDEA application that uses a dynamic frontier year approach by way of a secondary objective function to differentiate between multiple optima. Researchers should explicitly state which secondary objective function they are using. Not using a secondary objective function may result in difficulty reproducing results.

In theory, multiple optima occur either due to weakly efficient technology or due to efficient but not an extreme technology. However, characteristics of those two types of technologies have not received extensive attention. This suggests a direction of future research that could explore the conditions and frequency of multiple optima whereby unique technology clusters may be classified. In addition, current results for both cases indicated that minimizing the effective year provided more accurate results, but this is insufficient to give a general advice recommending minimizing over maximizing. Therefore, it is worthwhile to investigate additional cases to compare characteristics of secondary objectives. Finally, non-VRS

applications that render the objective function non-linear need to be tested to validate the performance of linear approximation program.

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