## Portland State University <br> PDXScholar

6-1989

# Recoil Distributions in Particle Transfer 

J.H. McGuire<br>Kansas State University<br>Jack C. Straton<br>Kansas State University<br>W.J. Axmann<br>Kansas State University<br>T. Ishihara<br>University of Tsukuba<br>E. Horsdal<br>University of Aarhus,

# Let us know how access to this document benefits you. 

Follow this and additional works at: http://pdxscholar.library.pdx.edu/phy_fac
Part of the Atomic, Molecular and Optical Physics Commons

## Citation Details

McGuire, J.H., J.C. Straton, W.J. Axmann, T. Ishihara, and E. Horsdal, "Recoil distributions in particle transfer." Physical review letters 62 (1989): 2933-2936.

# Recoil Distributions in Particle Transfer 

J. H. McGuire, J. C. Straton, and W. J. Axmann<br>Department of Physics, Kansas State University, Manhattan, Kansas 66506<br>T. Ishihara<br>Institute of Applied Physics, University of Tsukuba, Tsukuba, Ibaraki 305, Japan<br>E. Horsdal<br>Institute of Physics, University of Aarhus, Aarhus C, Denmark<br>(Received 27 January 1989)


#### Abstract

Classical Thomas peaks in various fast second-order particle transfer processes are quantum mechanically broadened by energy nonconservation in the intermediate states of collision. This quantum broadening is considered in observable velocity distributions of recoil particles.


PACS numbers: $34.70 .+\mathrm{e}, 03.65 . \mathrm{Nk}, 82.30 . \mathrm{Fi}$

Energy nonconservation in the intermediate states of a system of colliding particles may be large if the collision time is small. Thus energy-nonconserving effects could be significant for reactions in which intermediate states play an important role at high velocities. Fast particle transfer is such a reaction. In fast particle transfer reactions it is now well known ${ }^{1-8}$ that second Born terms, which include a sum over the intermediate states of the system, dominate over other Born terms including the first Born term, which excludes contributions from intermediate states. The theoretical prediction that second Born terms can dominate even at moderately high velocities in a certain range of projectile scattering angles was confirmed by the observation of the Thomas peak in cross sections, differential in the scattering angle of the projectile, for particle capture. ${ }^{9-12}$ In this paper we detail the quantum broadening of classical Thomas peaks in various second-order transfer processes and give a physical interpretation. For three-body transfer, the broadening of the Thomas peak occurs along a locus determined by overall energy and momentum conservation. If a fourth particle is present this locus broadens into a ridge, and, in addition, there arises a secondary ridge whose crest corresponds to energy conservation in intermediate states of the collision. The classical Thomas peak is obtained by enforcing both overall energy and momentum conservation and conservation of energy of the intermediate states. We concentrate on velocity distributions of the recoil particle since in atomic collisions the quantum broadening for the recoil particle tends to be easier to observe than corresponding broadening for the projectile.
Particle transfer in a one-step process is classically forbidden by conservation of energy and momentum at high collision velocities. Consequently, the simplest classical mechanism for particle transfer is the two-step process proposed ${ }^{13}$ by Thomas in 1927. The kinematic diagram corresponding both to the classical model of Thomas and to second Born terms is shown in Fig. 1, where particle 1,

2, or 3 scatters twice. Here, since particles 1 and 2 go off together with the same velocity, the entire collision is coplanar. If all the masses and the incident velocity $\mathbf{v}$ are known, then there are six unknowns, $\mathbf{v}^{\prime}, \mathbf{v}_{f}$, and $\mathbf{v}_{3}$ as defined by Fig. 1. Conservation of momentum gives two equations of constraint for each collision. Conservation of overall energy gives a fifth constraint. And conservation of energy in the intermediate state gives a sixth constraint. With six equations of constraint, all six unknowns may be completely determined. For example, for $p^{+}+\mathrm{H} \rightarrow \mathrm{H}+p^{+}$it is easily verified that $\alpha=\left(m_{e}\right)$ $\left.M_{p}\right) \sin \left(60^{\circ}\right), \beta=60^{\circ}$, and $\gamma=120^{\circ}$, where $M^{\prime}=\tilde{M}_{f}$ $=M_{2}=m_{e}$, an electron mass, and $M_{1}=M_{3}=M_{p}$, a proton mass. The standard Thomas peak at $\alpha=0.027^{\circ}$ has been observed. ${ }^{12}$

In general, conservation of overall momentum and energy gives three equations of constraint in the four unknowns $\mathbf{v}_{f}$ and $\mathbf{v}_{3}$, namely,

$$
\begin{align*}
& M_{1} \mathbf{v}=\left(M_{f}+\tilde{M}_{f}\right) \mathbf{v}_{f}+M_{3} \mathbf{v}_{3},  \tag{1}\\
& \frac{1}{2} M_{1} v_{1}^{2}=\frac{1}{2}\left(M_{f}+\tilde{M}_{f}\right) v_{f}^{2}+\frac{1}{2} M_{3} v_{3}^{2}, \tag{2}
\end{align*}
$$

$$
1+(2,3) \rightarrow(1,2)+3
$$



FIG. 1. Particle transfer diagram. The intermediate mass $M^{\prime}$ may equal $M_{1}, M_{2}$, or $M_{3}$. The mass of the upper (lower) particle on the diagram is taken as $M_{f}\left(\tilde{M}_{f}\right)$ in text and $M_{f}$ ( $\tilde{M}_{f}$ ) may equal either $M_{1}\left(M_{2}\right)$ or $M_{2}\left(M_{1}\right)$.
where $M_{f}\left(\tilde{M}_{f}\right)$ is the mass of the upper (lower) particle in the final bound-state system shown in Fig. 1, in which $M^{\prime}$ is the mass of the intermediate particle ( $M_{1}, M_{2}$, or $M_{3}$ ). From Eqs. (1) and (2) it is easily shown that the velocity of the recoil particle is constrained by the condition

$$
\begin{align*}
2 \hat{\mathbf{v}}_{3} \cdot \hat{\mathbf{v}} & =2 \cos \gamma=\frac{M_{1}+M_{2}+M_{3}}{M_{1}} \frac{v_{3}}{v}-\frac{M_{2}}{M_{3}} \frac{v}{v_{3}} \\
& =\frac{M_{1}+M_{2}+M_{3}}{M_{1} M_{3}} M_{2} \frac{M^{\prime}}{M_{2}} K-\frac{M_{2}}{M^{\prime}} \frac{1}{K} \\
& =r \frac{M^{\prime}}{M_{2}} K-\frac{M_{2}}{M^{\prime}} \frac{1}{K} \tag{3}
\end{align*}
$$

where $r=\left(M_{1}+M_{2}+M_{3}\right) M_{2} / M_{1} M_{3}$. Here $K$ is a dimensionless ratio of momenta defined ${ }^{14}$ by

$$
\begin{equation*}
K=\frac{M_{3} v_{3}}{M^{\prime} v} \tag{4}
\end{equation*}
$$

Equation (3) specifies the values of $\mathbf{v}_{3}$ allowed by overall energy and momentum conservation independent of the intermediate states of the system.

In the simplest two-step processes, illustrated in Fig. 1, we impose conservation of energy in the intermediate states, ${ }^{15}$ namely,

$$
\begin{equation*}
E_{\text {total }}=\frac{1}{2} M_{1} v^{2}=\frac{1}{2} M^{\prime} v^{\prime 2}+\frac{1}{2} M_{f} v_{f}^{2} \tag{5}
\end{equation*}
$$

After some algebra, we obtain for $M^{\prime}=M_{1}, M_{2}$, or $M_{3}$ the general condition for intermediate energy conservation, namely,

$$
\begin{equation*}
K=1 \tag{6}
\end{equation*}
$$

This corresponds to $v_{3}=M^{\prime} v / M_{3}$. The classical Thomas peak occurs when both Eqs. (3) and (6) are satisfied.

If the collision time $\Delta t$ is small, then the energy uncertainty $\Delta E$ will be large and the constraint of intermediate energy conservation may be violated within $\pm \Delta E$. For particles of charge $Z_{1},-Z_{2}$, and $Z_{3}$ corresponding to $M_{1}, M_{2}$, and $M_{3}$ in Fig. 1, we estimate $\Delta E \cong \hbar / \Delta t$ $=\hbar v^{\prime} / \Delta r=\Lambda v^{\prime} \hbar / a_{0}$. Here $a_{0} / \Lambda$ is a size characteristic of the system, where $\Lambda=Z_{i} Z_{>}$is an interaction strength in units of the square of the electron charge with $Z_{i}$ as the charge of the intermediate particle and $Z_{>}$as the larger of the projectile or target nuclear charges. Within the constraint of overall energy and momentum conservation, particle transfer is permitted within intermediate energies of order $\pm \Delta E$ away from the classical Thomas peak.

In two-step processes for transfer, the intermediate particle can in general be particle 1,2 , or 3 . For the case where the intermediate particle is particle 2, i.e., the transferred particle, we show in Fig. 2 the loci for the overall energy and momentum conservation at various values of $r$. We also show the intermediate-energyconserving Thomas peak at $K=1$. Even when the classical Thomas peak is kinematically forbidden, the tail of


FIG. 2. Overall energy- and momentum-conserving loci at $2 \cos \gamma=r K-1 / K$ for various values of $r$, and intermediate-energy-conservation locus at $K=1$ when the intermediate particle is the captured particle, i.e., $M^{\prime}=M_{2}$. These loci are invariant under the interchange of $M_{1}$ and $M_{3}$. In the equivalent figures for $M^{\prime} \neq M_{2}$, the only change is a shift by $M^{\prime} / M_{2}$ of the locus of intermediate energy conservation. The uncertainty principle permits particle transfer within intermediate energies of $\pm \Delta E$ of the Thomas peak along the locus of overall momentum and energy conservation.
the broadening may extend into the physical region ${ }^{16}$ if the energy uncertainty in the intermediate states $\Delta E$ is large enough.

To illustrate the nature of second-order ridges, the distribution of recoil particles, $d \sigma / d \mathbf{v}_{3}$, given by a second Born quantum calculation ${ }^{1}$ for the four-body reaction $p^{+}+\mathrm{He} \rightarrow \mathrm{H}+\mathrm{He}^{++}+e^{-}$is shown in Fig. 3. Here $M_{1}=M_{p}$ and $M^{\prime}=M_{2}=M_{3}=m_{e}$ so that $r \cong 1$. In this case the He nuclear charge $Z_{T}$ localizes the two electrons but plays no direct role in the transfer process. The presence of this fourth particle, however, broadens the constraint of overall three-body energy and momentum conservation. A sharp ridge corresponding to overall energy and momentum conservation, and a broad ridge cresting at $v_{3}=v$ (or $K=1$ ) are both evident for $200-\mathrm{MeV}$ proton impact shown in Fig. 3. The broad ridge corresponds to a pole in the Green's function. ${ }^{2}$ The crest of this ridge occurs when the intermediate energy equals the total energy. The Thomas peak is found at the intersection of these two ridges. The widths of both ridges are consistent with the uncertainty principle. The broad energy-conserving ridge has a width $\Delta E \cong Z_{T} v^{\prime}$


FIG. 3. Differential cross section $d \sigma / d \mathbf{v}_{3}$ in the velocity of the recoil electron for $p^{+}+\mathrm{He} \rightarrow \mathrm{H}+\mathrm{He}^{++}+e^{-}$for $200-$ $\mathrm{MeV} p^{+}$. Here $v^{\prime}{ }_{0}$ is the Bohr velocity. The sharp ridge, which permits energy nonconservation in intermediate states, intersects the broad energy-conserving ridge at $v^{\prime}=v$ and $\gamma=90^{\circ}$. At lower projectile energies both ridges are broader and weaker: The broad peak is indistinct at energies near or below a few MeV where experiments are feasible.
$\times \hbar / a_{0}$, corresponding to the above discussion with $v^{\prime} \cong v \cong v_{3}$. The sharp ridge has a width given by the momentum distribution of the target electron which is proportional to $Z_{T}$. This width goes to zero ${ }^{17}$ as $Z_{T} \rightarrow 0$.

There is also broadening about the Thomas peak in $\alpha$, easily found by repeating the development above. However, for common atomic processes the fractional energy width about the Thomas peak at $K=1$ in $\alpha$, i.e.,

$$
\Delta E / E_{f} \cong\left[\Lambda v^{\prime} \hbar / \frac{1}{2} m^{\prime} v^{2} a_{0}\right]\left[m^{\prime} M_{3} /\left(M_{1} M_{3}-m^{\prime 2}\right)\right]
$$

is usually small, while the fractional energy width in $\gamma$,

$$
\Delta E / E_{3} \cong\left[\Lambda v^{\prime} \hbar / \frac{1}{2} m_{f} v^{2} a_{0}\right]\left(M_{3} / m^{\prime}\right)
$$

is usually large. Thus, a ridge in $\alpha$ is usually less prominent than a ridge in $\gamma$.

Energy-nonconserving second Born amplitudes have been previously examined in the case of the Thomas peak for electron capture from atoms by protons. ${ }^{18-22}$ It was shown ${ }^{18}$ that the energy-nonconserving and energyconserving amplitudes obey a dispersion relation characteristic of resonant processes. ${ }^{19}$ It was also found ${ }^{20}$ that at very large $v$, half of the total cross section for particle transfer is due to energy-nonconserving terms. This implies that half of the total cross section associated with recoil distribution also comes from intermediate energy nonconservation.
Although the effects that we have considered have not
yet been observed experimentally, double-differential measurements for $1-\mathrm{MeV}$ protons on helium are considered to be technically feasible. ${ }^{23}$ As observation of the classical Thomas peak advanced our understanding of second-order effects in particle transfer processes, observation of the effects discussed in this paper could serve to probe energy-nonconserving aspects ${ }^{24}$ of the intermediate states through which a system passes during a collision.

We gratefully acknowledge discussions with M Lieber, R. Schuch, and J. Giese. This work was supported by the Division of Chemical Sciences, Office of Energy Research, U.S. Department of Energy.
${ }^{1}$ T. Ishihara and J. H. McGuire, Phys. Rev. A 38, 3310 (1988).
${ }^{2}$ J. S. Briggs and K. Taulbjerg, J. Phys. B 12, 2565 (1979).
${ }^{3}$ R. Shakeshaft and L. Spruch, J. Phys. B 11, L457 (1978).
${ }^{4}$ J. Macek and S. Alston, Phys. Rev. A 26, 250 (1982).
${ }^{5}$ J. E. Miraglia, R. D. Piacentinni, R. D. Rivarola, and A. Salin, J. Phys. B 14, 1191 (1981).
${ }^{6}$ J. Burgdorfer and L. J. Dube, Phys. Rev. A 31, 634 (1985).
${ }^{7}$ D. S. F. Crothers and J. F. McCann, J. Phys. B 17, L177 (1984).
${ }^{8}$ D. H. Jakubassa-Amundsen, Z. Phys. A 316, 161 (1984).
${ }^{9}$ C. J. Cook, N. R. A. Smyth, and O. Heinz, J. Chem. Phys. 63, 1218 (1975).
${ }^{10}$ M. Brenig and I. A. Sellin, Bull. Am. Phys. Soc. 76, 601 (1981).
${ }^{11}$ E. Horsdal-Pedersen, C. L. Cocke, and M. Stockli, Phys. Rev. Lett. 50, 1910 (1983).
${ }^{12}$ H. Vogt, R. Schuch, E. Justiniano, M. Schulz, and W. Schwab, Phys. Rev. Lett. 57, 2256 (1986).
${ }^{13}$ L. H. Thomas, Proc. Roy. Soc. London A 144, 5161 (1927).
${ }^{14}$ Alternatively, one could use $K^{\prime}=M_{3 v^{3}} / M_{2 v}$, independent of the intermediate mass $M^{\prime}$. Then Eq. (3) is more simply $2 \cos \gamma$ $=K^{\prime}-1 / K^{\prime}$. However, we choose $K=M_{3} v^{\prime} / M^{\prime} v$ since $K=1$ corresponds to intermediate energy conservation for all $\boldsymbol{M}^{\prime}$.
${ }^{15}$ Of course momentum is conserved in each intermediate step for central potentials, e.g., $\left|\mathbf{R}-R^{\prime}\right|^{-1}$, which are translationally invariant.
${ }^{16}$ Certain symmetries may, however, impose restrictions: Some cancellation can occur, for example, when particles 1 and 2 have equal masses much smaller than $M_{3}$ and opposite charges. Compare R. Shakeshaft and J. M. Wadehra, Phys. Rev. A 22, 968 (1980); N. C. Deb, J. H. McGuire, and N. C. Sil, Phys. Rev. A 36, 1082 (1987).
${ }^{17}$ For a three-body system any width to the sharp peak is due to preparation of the projectile and target wave packets and due to detectors.
${ }^{18}$ J. H. McGuire and O. L. Weaver, J. Phys. B 17, L583 (1984).
${ }^{19}$ O. L. Weaver and J. H. McGuire, Phys. Rev. A 32, 1435 (1985).
${ }^{20}$ J. H. McGuire, P. R. Simony, O. L. Weaver, and J. Macek, Phys. Rev. A 26, 1109 (1982).
${ }^{21}$ S. Alston, Phys. Rev. A 38, 6092 (1988).
${ }^{22}$ J. Macek, Phys. Rev. A 37, 2365 (1988).
${ }^{23}$ R. Schuch (private communication); J. P. Giese (private communication).
${ }^{24} \mathrm{We}$ note that the energy fluctuation may be computed classically, namely,

$$
\Delta \mathscr{E}=E^{\prime}=E_{\text {total }}=\left(K^{2}-1\right) E_{\text {total }}\left(\frac{M_{f} M^{\prime}}{M_{1} M_{3}}\right)\left(\frac{\tilde{M}_{f}+M_{3}}{M_{1}+M_{2}}\right) .
$$

If a cross section were to remain large (e.g., due to higher Born terms) for some $\Delta \mathscr{E}$ greater than the quantum uncertainty $\Delta E$, then it would be possible to study cross sections at specific nonconserving intermediate energies to a relative accuracy of $\Delta E / \Delta \mathscr{E}$.

