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# Guidance and Control Elements for Improved Access to Space 

from Planetary Landers to Reusable Launchers

by<br>Pedro V. M. Simplício



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A dissertation submitted to the University of Bristol in accordance with the requirements for award of the degree of Doctor of Philosophy in the Faculty of Engineering.

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#### Abstract

This thesis aims to demonstrate the benefits that specific developments in the field of guidance and control can bring to space exploration. These benefits are mostly related to the improvement of planetary descent \& landing techniques and reduction of launcher development and operation costs. The demonstration employs two application cases that are well-representative of the aforementioned benefits.

The first case study investigates the feasibility of applying robust control tools to design and optimise descent \& landing trajectories. It is based on a sample return mission to the Martian moon Phobos, which is especially challenging because the irregular and poorly-known shape of this body renders its gravitational environment extremely uncertain and variable.

By offering the ability to explicitly account for the effects of such an uncertain environment in a systematic manner, it is shown how robust control synthesis and analysis techniques can be effective in complementing and improving state-of-practice industrial approaches. Due to the conservative character of space industry, especial attention is given to techniques that allow to take advantage of legacy knowledge, such as structured $\mathcal{H}_{\infty}$ optimisation.

The second application is focused on sophisticated guidance and control architectures for reusable launch vehicles, which are seen as a key paradigm for sustainable access to space. Improving launcher performance is also a very demanding task since mission requirements tend to compete against each other due to fundamental couplings between trajectory, actuators and vehicle structure. A novel benchmark and design framework is first developed, featuring a real-time capable guidance algorithm for retro-propulsive descent and pinpoint landing, which relies on recent advances in the domain of convex optimisation.

This reusable launcher benchmark is finally supplemented with dedicated solutions to analyse and minimise the impact of aerodynamic loads, which represent a critical driver of launcher safety and operational availability. More specifically, this part of the thesis showcases the earliest application of robust wind disturbance observation for improved load relief in both ascent and descent flight.


Ao meu avô António Lourenço Menino

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## Author's Declaration

I declare that the work in this dissertation was carried out in accordance with the requirements of the University's Regulations and Code of Practice for Research Degree Programmes and that it has not been submitted for any other academic award. Except where indicated by specific reference in the text, the work is the candidate's own work. Work done in collaboration with, or with the assistance of, others, is indicated as such. Any views expressed in the dissertation are those of the author.

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| AoA | Angle of Attack |
| :--- | :--- |
| APNG | Augmented Proportional Navigation Guidance |
| BCBF | Body-Centred Body-Fixed |
| BPNG | Biased Proportional Navigation Guidance |
| CFD | Computational Fluid Dynamics |
| CG | Centre of Gravity |
| COESA | Committee On Extension to the Standard Atmosphere |
| CP | Centre of Pressure |
| CTVG | Constrained Terminal Velocity Guidance |
| DCM | Direction Cosine Matrix |
| DESCENDO | Descending over Extended envelopes using Successive ConvExificatioN-baseD |
| Optimisation | Descent \& Landing |
| D\&L | German Aerospace Centre |
| DLR | Down-Range Landing |
| DRL | Environment for Autonomous G\&C Landing Experiments |
| $\boldsymbol{E A G L E}$ | Earth-Centred Earth-Fixed |
| ECEF | Earth-Centred Inertial |
| ECI | Earth Gravitational Model |
| EGM | Earth-Moon Lagrangian point |
| EML | Engineering and Physical Sciences Research Council |
| EPSRC | European Space Agency |
| ESA | FeedBack |
| FB |  |


| FEA | Finite Element Analysis |
| :---: | :---: |
| FTVG | Free Terminal Velocity Guidance |
| G\&C | Guidance \& Control |
| GEO | Geosynchronous Earth Orbit |
| G-FOLD | Guidance for Fuel-Optimal Large Diverts |
| GH | Gravity Harmonics |
| GM | Gain Margin |
| GTO | Geostationary Transfer Orbit |
| HF | High-Frequency |
| IACG | Intercept-Angle-Control Guidance |
| IQC | Integral Quadratic Constraints |
| JAXA | Japanese Aerospace Exploration Agency |
| KI | Kinematic Impulsive |
| LB | Lower Bound |
| LEO | Low Earth Orbit |
| LF | Low-Frequency |
| LFT | Linear Fractional Transformation |
| LiDAR | Light Detection And Ranging |
| LMI | Linear Matrix Inequality |
| LOS | Line-Of-Sight |
| LP | Launch Pad |
| LPF | Low-Pass Filter |
| LPO | Libration Point Orbit |
| LPV | Linear Parameter-Varying |
| LR | Load Relief |
| L\&R | Launch \& Recovery |
| LTI | Linear Time-Invariant |
| MB | MultiBody |
| MC | Monte-Carlo |


| MCI | Mass, CG \& Inertia |
| :---: | :---: |
| MDO | Multi-Disciplinary Optimisation |
| MIMO | Multiple-Input Multiple-Output |
| MoI | Moment of Inertia |
| MPC | Model Predictive Control |
| MSV | Maximum Singular Value |
| NASA | National Aeronautics and Space Administration |
| NLP | NonLinear Programming |
| NP | Nominal Performance |
| NPI | Network Partnering Initiative |
| NSTP | National Space Technology Programme |
| OFIGL | Optimal Fixed-Interval Guidance Law |
| OGL | Optimal Guidance Law |
| OSG | Optimal Sliding Guidance |
| PD | Proportional-Derivative |
| PI | Predictive Impulsive |
| PL | PayLoad |
| PM | Phase Margin |
| PNG | Proportional Navigation Guidance |
| PSD | Power Spectral Density |
| PVP | TVC Pivot Point |
| PWA | Piece-Wise Affine |
| QFT | Quantitative Feedback Theory |
| RB | Rate-Bounded |
| RLV | Reusable Launch Vehicle |
| RP | Recovery Pad |
| RP | Robust Performance |
| RS | Robust Stability |
| RT | Reference Trajectory |


| RTLS | Return To Launch Site |
| :---: | :---: |
| rWDO | Robust Wind Disturbance Observer |
| SBNL | Sector-Bounded NonLinearity |
| S/C | SpaceCraft |
| SDK | Spacecraft Dynamics \& Kinematics |
| SISO | Single-Input Single-Output |
| SMC | Sliding Mode Control |
| SOCP | Second-Order Cone Programming |
| SR | Sample Return |
| SSV | Structured Singular Value |
| STM | State Transition Matrix |
| TASC | Technology for AeroSpace Control |
| TRC | Trust Region Constraint |
| TRL | Technology Readiness Level |
| TV | Time-Varying |
| TVC | Thrust Vector Control |
| UB | Upper Bound |
| VRF | Velocity Reference Frame |
| VTVL | Vertical Take-off and Vertical Landing |
| V\&V | Verification \& Validation |
| WC | Worst-Case |
| WDO | Wind Disturbance Observer |
| ZEM | Zero-Effort-Miss |
| ZEV | Zero-Effort-Velocity |

## Nomenclature

Due to the diverse technical areas covered by the thesis, the repetition of a few specific symbols cannot be avoided. In order to clarify the adopted notation, the nomenclature is organised in: generic descent \& landing and robust control notation, additional variables of application I and II (that add to the generic notation but are specific to each application), and subscripts \& superscripts (which are common to the applications and extend the definition of all the variables).

## Generic descent \& Landing notation

| $0_{n \times m}$ | Zero matrix of dimension $n$ by $m$ (implicit if subscripts are omitted) |
| :--- | :--- |
| $\mathbf{a}, \mathbf{w}$ | Control and total acceleration vectors |
| $\mathbf{g}, g, G$ | Gravity acceleration vector, norm and Jacobian matrix |
| $H, J, V$ | Hamiltonian, cost and candidate Lyapunov functions |
| $\mathbf{h}$ | Nonlinear guidance function |
| $\mathbb{I}_{n \times n}$ | Identity matrix of dimension $n$ by $n$ (implicit if subscripts are omitted) |
| $I_{\mathrm{sp}}$ | Engine specific impulse |
| $k$ | Optimisation-based guidance discretisation point index, $k \in[1, \cdots, N]$ |
| $k_{r}, k_{v}$ | Linear guidance gains |
| $m$ | Total mass |
| $n$ | Effective navigation ratio |
| $\mathbf{p}$ | External perturbation vector |
| $\mathbf{p}_{r}, \mathbf{p}_{v}$ | Position and velocity co-state vectors |
| $\mathbb{R}^{n}$ | Space of real numbers with dimension $n$ (implicit if superscript is omitted) |
| $\mathbf{r}, r, R_{c}$ | Relative position vector, norm and target distance |
| $\mathbf{s}$ | Sliding mode state vector |
| $\mathbf{T}, T$ | Thrust force vector and norm |
| $t, t_{\mathrm{go}}$ | Time and time-to-go |
| $T_{\mathrm{S}}$ | Optimisation-based guidance time step |
| $\mathcal{T}_{p}, \Delta t_{p}$ | Set and duration of pre-scheduled manoeuvres |
| $\mathbf{v}, v, V_{c}$ | Relative velocity vector, norm and closing speed |
| $\mathbf{Z E M}, \mathbf{Z E V}$ | Zero-effort-miss and zero-effort-velocity vectors |
| $z, \sigma$ | Mass and thrust convexification variables |
| $\Delta m, \Delta V$ | Change of mass and velocity required for manoeuvre |
| $\delta_{g}, w_{g}$ | Normalised gravity acceleration uncertainty and relative weight |
| $\theta$ | Angle between the vehicle's longitudinal axis and the vertical direction |


| $\boldsymbol{\Lambda}, \lambda$ | Line-of-sight vector and angle |
| :--- | :--- |
| $\tilde{\lambda}$ | Sliding mode weight coefficient |
| $\tau, P_{n}$ | Pseudospectral transcription variable and Legendre polynomial of order $n$ |
| $\phi$ | Nonlinear guidance weight coefficient |

## GENERIC ROBUST CONTROL NOTATION

| $0_{n \times m}$ | Zero matrix of dimension $n$ by $m$ (implicit if subscripts are omitted) |
| :---: | :---: |
| $D, Q$ | Scaling matrices for $\mu$ analysis, contained in the sets $\mathcal{D}$ and $\mathcal{Q}$ |
| $\mathcal{F}_{\mathrm{u}}, \mathcal{F}_{1}$ | Upper and lower LFT operations |
| GM, PM | Linear gain and phase stability margins |
| $\mathbb{I}_{n \times n}$ | Identity matrix of dimension $n$ by $n$ (implicit if subscripts are omitted) |
| $j$ | Imaginary unit |
| $\mathcal{L}_{2}$ | Space of signals with finite 2-norm |
| $M, P, G, H, K$ | Generalised closed-loop and open-loop plant, subsystem, filter and controller. They are commonly described as a space-state with matrices $\{A, B, C, D\}$ or as a transfer function matrix partitioned as $\left[M_{11} M_{12} ; M_{21} M_{22}\right]$ |
| $\\|M\\|_{\infty}$ | $\mathcal{H}_{\infty}$-norm of LTI system $M$, i.e. induced gain of an $\mathcal{L}_{2}$ signal |
| $\mathbb{R}^{n}, \mathbb{C}^{n}$ | Space of real or complex numbers with dimension $n$ (implicit if superscript is omitted) |
| $S, T$ | Sensitivity and complementary sensitivity transfer functions |
| $s, \omega$ | Laplace variable (often dropped for clarity) and angular frequency |
| $t, T$ | Time and time-horizon, $t \in[0, T]$ |
| $W_{i}, W_{o}$ | Input and output frequency-dependent requirement weights |
| $\mathbf{w}, \mathbf{w}_{\Delta}, \hat{\mathbf{w}}_{\Delta}$ | Exogenous input signal, uncertainty channel and its Fourier transform vector (scalar if not boldface) |
| $\mathbf{x}, \mathbf{y}, \mathbf{u}, \mathbf{n}$ | State, output, control and noise input vectors of a generic system |
| $\mathbf{z}, \mathbf{z}_{\Delta}, \hat{\mathbf{z}}_{\Delta}$ | Exogenous output signal, uncertainty channel and its Fourier transform vector (scalar if not boldface) |
| $\\|\mathbf{z}\\|_{2}$ | 2-norm of signal $\mathbf{z}$ (2-norm also assumed if subscript is omitted) |
| $\gamma, K^{*}$ | Generic induced $\mathcal{L}_{2}$ gain (optimisation cost) and associated controller |
| $\Delta, \Delta^{\mathrm{WC}}$ | Structured uncertainty block, $\Delta=\operatorname{diag}\left[\delta_{\mathrm{x}_{1}}, \delta_{\mathrm{x}_{2}}, \ldots, \delta_{\mathrm{x}_{\mathrm{n}}}\right]$, and worst-case uncertainty configuration |
| $\delta_{\mathrm{x}}, w_{\mathrm{x}}$ | Normalised uncertainty of parameter $x$ and relative weight |
| $\mu(M)$ | Structured singular value of $M$ |
| $\Pi$ | IQC multiplier, partitioned as $\left[\Pi_{11} \Pi_{12} ; \Pi_{21} \Pi_{22}\right]$ and contained in the set $\mathcal{D}_{\Pi}$ |
| $\rho_{\text {TV }}$ | Time-varying parameter vector, contained in the set $\mathcal{P}$ |
| $\sigma(M), \rho(M)$ | Singular value and spectral radius of $M$ |
| $\Psi, H$ | Filters applied for the factorisation of multiplier $\Pi$ |

```
Additional variables of Application I
C
f, J
i
\overline{n}}\quad\mathrm{ Gravity harmonics model order
```



```
r,0,\varphi Spherical gravity harmonics coordinates
ti}\quad\mathrm{ Descent time index, t}\mp@subsup{t}{i}{}\in[\mp@subsup{t}{0}{},\mp@subsup{t}{f}{}
U,\nablaU|# Gravity potential of main body and gradient of body * affecting body #
WS},\mp@subsup{W}{A}{}\quad\mathrm{ Weights related to the sensitivity function and actuators block
\mp@subsup{z}{r}{}},\mp@subsup{z}{a}{}\quad\mathrm{ Output signals related to accuracy }\mp@subsup{\gamma}{\mathrm{ acc }}{}\mathrm{ and efficiency }\mp@subsup{\gamma}{\mathrm{ eff }}{}\mathrm{ objectives
\deltax Small perturbation of variable }x\mathrm{ (not to be confused with the uncertainty }\mp@subsup{\delta}{\textrm{x}}{}\mathrm{ )
\nu True anomaly of Phobos around Mars
\rho
*\sigma Gaussian uncertainty range considered (95.5% for 2\sigma, 99.7% for 3\sigma)
\sigma,\alpha,\beta Maximum control authority, nonlinear sector lower and upper bounds
\tau}\quad\mathrm{ Time delay
\phi,\Gamma Nonlinear saturation and dead-zone operators
\omega
```


## Additional variables of Application II

| $\mathrm{a}_{i}^{*}, d_{i}^{*}$ | Thrust acceleration and aerodynamic drag templates at successive convexification iteration $i$ (the drag template is computed as a function of additional variables $\mathbf{v}_{i}^{*}, z_{i}^{*}, \rho_{i}^{*}$ and $C_{D i}^{*}$ ) |
| :---: | :---: |
| $C_{D}, C_{L}, C_{N}, C_{\text {f }}$ | Drag, lift and normal RLV force coefficients, fin force coefficient |
| $C_{N_{\alpha}}, C_{\mathrm{f}_{\alpha}}$ | Normal RLV and fin force gradient coefficients |
| F, M | Generic force and moment vectors |
| $f_{\text {sim }}, f_{\text {gui }}$ | Simulation and guidance update frequencies |
| $h, \rho, a$ | Altitude and altitude-dependent atmospheric density and speed of sound |
| $h_{\mathrm{P}}, N_{\mathrm{P}}$ | Terminal altitude and number of successive convexification iterations |
| $h_{s}, N$ | Recovery start altitude and number of DESCENDO discretisation points |
| $\mathbf{i}_{\#}, \mathbf{j}_{\#}, \mathbf{k}_{\#}$ | Basis vectors of reference frame \# |
| $i$ | Bending mode index, $i \in[1, \cdots, k]$ |
| $j$ | RLV node/station index, $j \in[1, \cdots, n]$ |
| $J, J_{A}, J_{N}$ | Total, axial and normal moment of inertia |
| $k_{\mathrm{p}}, k_{\mathrm{d}}, k_{\phi}$ | Proportional, derivative and roll-rate gains |
| $l, r, J_{0}$ | Beam element length, radius and moment of inertia per unit of mass |


| $L_{\text {w }}$ | Wind disturbance observer |
| :---: | :---: |
| $l_{\alpha}, l_{\mathrm{c}}, l_{\mathrm{f}}$ | Aerodynamic, TVC, and fin control arms |
| $m_{\#}, h_{\#}$ | Mass and height of structural element \# |
| $m_{\text {ctr }}, \Gamma_{\text {ctr }}$ | Pitch control moment and effectiveness matrix |
| $N_{\alpha}, N_{\mathrm{f}}, N_{\alpha^{\prime}}$ | Normal RLV, fin and combined force gradients |
| $Q, Q_{\alpha}, Q_{H}$ | Dynamic pressure, aerodynamic and thermal load indicators |
| $\mathbf{q}_{\#}^{*}, C_{\mathbf{q}_{\#}^{*}}$ | Rotation quaternion from frame * to \# and associated DCM |
| S, R | Reference aerodynamic area and radius |
| s, $\xi$ | Nodal translation and rotation state vectors |
| $t_{\text {sep }}, t_{1}, t_{2}$ | Separation time, relative duration of re-entry and landing burns |
| $\mathbf{v a i r}^{\text {air }}, V, M$ | Air-relative velocity vector, norm and Mach number |
| $v_{\mathrm{w}}, n_{\mathrm{w}}, \mathbf{w}_{\mathrm{E}}$ | Wind disturbance speed and noise signal, total wind disturbance vector |
| $W_{\mathrm{u}}, W_{\mathrm{n}}, W_{\mathrm{e}}$ | Weights related to input perturbations, wind noise and attitude error |
| $\mathbf{x}, x$ | Location in the body-fixed frame, longitudinal coordinate |
| $z$ | Drift motion coordinate |
| $\alpha, \beta, \alpha_{\text {eff }}$ | Angle of attack, sideslip and effective angle of attack. $\alpha$ \# indicates local angle of attack at the RLV location defined by \# (CG assumed if omitted) |
| $\beta_{\mathrm{TVC}}, \beta_{\mathrm{fin}}, \beta_{\mathrm{thr}}$ | TVC, planar fin and cold gas thruster control inputs |
| $\eta, \eta_{i}$ | Generalised modal vector and coordinate of mode $i$ |
| $\eta_{\mathbf{a}}, w_{\eta_{\mathrm{a}}}$ | Trust region constraint cost and associated weight |
| $\theta, \psi, \phi$ | Pitch, yaw and roll angles |
| $\mu_{\alpha}, \mu_{\mathrm{c}}, \mu_{\mathrm{f}}, \mu_{\alpha^{\prime}}$ | Aerodynamic, TVC, fin and combined load proneness coefficients |
| $\nu$ | Ratio between current and initial propellant mass |
| $\varphi, \lambda, \chi$ | Latitude, longitude and launch azimuth |
| $\Phi, \phi_{i j}$ | Modal shape matrix and normalised displacement of mode $i$ at node $j$ |
| $\tau^{j}, a_{\mathrm{r}}^{j}, b_{\mathrm{r}}^{j}$ | Applied torque, stiffness and damping coefficients of connection joint $j$ |
| $\boldsymbol{\Omega}, \omega_{\mathrm{E}}$ | Earth's angular velocity vector and norm |
| $\omega$ | Angular velocity vector |
| $\omega_{\mathrm{f}}, \zeta_{\mathrm{f}}$ | Modal frequency and damping matrices |
| $\omega_{\mathrm{L}}, \zeta_{\mathrm{L}}$ | Natural frequency and damping ratio of low-pass filter |
| $\omega_{\mathrm{N}}, \epsilon_{\mathrm{N}}, \kappa_{\mathrm{N}}$ | Central frequency, selectivity and gain of notch filter |

## Subscripts \& SUPERSCRIPTS

$0 \quad$ Initial, baseline or trim value (depending on the context)
$1,2,3,4$ Planar fin index ( 1 and 2 pitch plane, 3 and 4 yaw plane)
A, $* \sigma \quad$ Relative to actuators block, with SDK uncertainty range specified by $*$ (nominal case assumed if omitted)
aero, EGM Relative to aerodynamics and Earth gravitational models

| burnt, rec | Propellant consumption/required for recovery |
| :---: | :---: |
| CG, CP | Relative to centre of gravity or pressure |
| cmd, cmp | Commanded or compensated value |
| CTV, CVX | Relative to CTVG or convex optimisation-based guidance |
| deriv | Relative to derivative filter (with time constant $\sigma_{\mathrm{d}}$ ) |
| dry | Relative to dry first stage |
| e | Error signal |
| $f$ | Final value |
| FB, LR | Relative to feedback-only or load relief controller |
| fin | Relative to planar fins |
| fuel, oxid | Relative to fuel/oxidizer mass |
| I,E,L,R,B,V | Relative to ECI, ECEF, LP, RP, body-fixed and velocity reference frames |
| $i$ | Relative to plant $i$ |
| $j$ | Relative to RLV node/station $j$ |
| LPF, notch | Relative to low-pass or notch filter |
| min, max | Minimum or maximum limits |
| NOM | Nominal value |
| pert | Perturbation signal |
| PL | Relative to payload |
| prop | Relative to propellant (fuel plus oxidizer) mass |
| ref | Reference value/trajectory |
| RLV | Relative to RLV model |
| rNAV, vNAV | Relative position or velocity estimates |
| S, T | Relative to spacecraft or target |
| SDK,* $\sigma$ | Relative to SDK block, with SDK uncertainty range specified by * (nominal case assumed if omitted) |
| thr | Relative to cold gas thrusters |
| tk | Relative to propellant tanks |
| TVC, PVP | Relative to TVC and TVC pivot point |
| wind | Relative to wind model |
| $\mathrm{x}, \mathrm{y}, \mathrm{z}$ | Component in the $x, y, z$ axes |
| $\perp$ | Normal vector component |
| +, T, * | Pseudo-inverse, transpose and conjugate matrices |
| $\uparrow, \downarrow$ | Relative to ascent or descent controller |
| , | First and second order derivatives |
| , ${ }^{\circ}$ | Estimated or predicted value |
|  | Perturbed value |
|  | Maximum value |



## INTRODUCTION

It is hard to imagine today's life without all the fruits space exploration has brought. Satellite navigation has improved the way we travel whether by land, sea and air. Communications satellites ensure reliable worldwide connectivity even in areas where terrestrial infrastructure is poor, damaged or non-existent. A fleet of Earth observation satellites is constantly monitoring our planet, providing the means to better manage its natural resources and study its climate. Data gathered by spacecraft flying around the Earth and beyond have greatly contributed to our current understanding of the Universe and of our humble place in it. Unfortunately, sending satellites to space - and doing so in a safe and affordable manner - is not an easy task.

### 1.1 Short overview of launching and landing

To provide all the energy needed to put a satellite in orbit, state-of-the-art launch vehicles have a propellant-to-structure mass ratio between 80 and $90 \%$, far larger than the $40 \%$ of a cargo aircraft and even $52 \%$ of a molotov cocktail [Pet12]. Furthermore, launchers are inherently unstable during atmospheric flight, which makes them fundamentally more difficult to control, especially as lighter structures tend to lead to stronger control-structure interactions [WYH08]. These two factors make of rockets highly dangerous and complex machines, becoming even more critical when human spaceflight is envisaged. Because of this, they are also extremely costly to develop and operate.

Hence, flight test opportunities for launchers are significantly less than for aircraft and the subsequent lack of experimental data subjects their development to a higher level of modelling uncertainty. This in turn makes the whole design process even more critical. To tackle it, launcher manufacturers rely on extensive mission preparation stages for each specific trajectory and can operate only within a narrow set of wind and weather conditions. If those conditions are not
met in the day-of-flight, the launch is postponed to a later date, which increases cost even further. Then, after successfully delivering a spacecraft into space, the launcher comes crashing back down to Earth and a new one will have to be built for the next trip. Once again, this is not the case with aircraft as they are able to operate under a much wider range of conditions, independently of the trajectory, and they are not thrown away after each flight.

On the other hand, exploration missions where a probe or lander is to be delivered, e.g. Sample Return (SR) missions, tend to offer a much higher scientific return at the expense of an extra degree of complexity. For instance, in 2012, chemical analyses carried out by the University of Chicago on the lunar material collected by Apollo 14 fifty years earlier brought new elements to the disputed question of the origin of the Moon, casting a new doubt on the most widely-accepted "Giant Impact" hypothesis [ZDDF12].

But Descent \& Landing (D\&L) is extremely challenging, even on the most Earth-like planet of the solar system, Mars. In fact, two out of three missions to the red planet have ended in failure [LJ14]. This is mostly because the Martian atmosphere is about 100 times thinner than Earth's, which compromises the effectiveness of aerodynamic control and deceleration methods, but it is also highly uncertain and time-varying. To make it more challenging, landing accuracy specifications have become more stringent over the years, as illustrated by the required confidence ellipses in Fig. 1.1.


Figure 1.1: Landing ellipses of successful Mars landings on elevation map of Gale Crater (credits: Ryan Anderson, US Geological Survey Astrogeology Science Center). Highlighted in red is the landing target of NASA's Mars Science Laboratory Curiosity.

In order to steer the spacecraft through the harsh Martian environment while coping with communication delays with Earth (ranging between 4 and 24 minutes) as well as with the chance of subsystem failures, Guidance \& Control (G\&C) algorithms have to be highly robust and fully autonomous.

Yet, current endeavours aiming to make space more accessible are also making all the challenges mentioned above ever more present.

### 1.2 Making space more accessible

The most immediate way of making space more accessible to everyone is through space tourism. This concept was pioneered by private company Virgin Galactic and aims to offer anyone who can afford it, an orbital or sub-orbital flight or even a trip around the Moon [Vir19].

Another rapidly-growing interest in space is that of asteroid exploitation, pursued by companies such as Planetary Resources and Deep Space Industries [Gle18]. There are two main reasons why reaching for asteroids is economically appealing. The first one is to transfer heavy-industry activities to space so as to alleviate their harmful impact on Earth. The other one is to harvest asteroids' natural resources which, depending on their composition, range from water (essential for the manufacture of propellant in space) to precious metals like platinum (that are rare on our planet). Additional advantages of asteroid mining are related to their low-gravity environment, where moving the same amount of material requires considerably less energy than on Earth, and to the fact that the ability to harvest and use in situ resources will ultimately become mandatory if we continue to expand into space.

But the most noble motivation for better capabilities to access space probably lies on the discovery and colonisation of other planets. Scientists agree that there are now multiple foreseeable threats with the potential to wipe out humanity and even all life on Earth. Amongst others, the least unlikely include ecological collapse, global pandemic, nuclear catastrophe, asteroid collision and misuse of artificial intelligence [Bos02]. Hence, many believe that to safeguard our survival in case of any of these events, we must colonise other worlds and become a multi-planetary species as soon as possible. The main candidates for the creation of a selfsustainable civilisation are Mars and a few moons of Jupiter and Saturn. But the former, having an adequate atmosphere as well as a land mass and day length comparable to Earth's seems to be the next logical leap of mankind.

Nevertheless, in order to make all these ventures feasible, as well as to ensure the sustainability of today's space activities, two key questions need to be addressed:

I D\&L technologies must be mastered, Mars' current failure ratio of $2 / 3$ is simply not acceptable if we plan to carry people or mine asteroids on a regular basis;

II Launch costs must be slashed down, which is equivalent to say that the efficiency of the development and of the operations process have to be improved.

A more elaborate discussion on the reduction of costs will be provided in Sec. 1.3.2, but the most impactful paradigm shift in this domain is that of launcher reusability, which is also tightly related to the need to master $\mathrm{D} \& \mathrm{~L}$ techniques.

The idea of reusability is to recover launcher stages through a controlled re-entry, descent and pinpoint landing back on Earth and then reuse them for other missions. This idea is not new - the historical illustration of the first Reusable Launch Vehicle (RLV) is NASA's winged Space Shuttle. However, reusing this vehicle proved to be much more difficult than anticipated, and the cost and risk of maintaining it caused this programme to be cancelled in 2011.

But not long after, two private companies, Blue Origin and SpaceX, were founded with plans to make reusable Vertical Take-off and Vertical Landing (VTVL) stages [Bla16], which have now been demonstrated technically feasible. Blue Origin has recovered a sub-orbital vehicle, New Shepard, in November 2015 for the first time and reused it a few months later. SpaceX has successfully recovered the first stage of their Falcon 9 launcher on land in December 2015, on an ocean platform in April 2016 and re-flew an used stage in March 2017. For a broad overview of other past and current experimental recovery concepts, the interested reader is referred to [HHA18].

Thanks to the cost efficiency achieved with reusability [Jon18], these companies are already scaling-up their vehicles towards a new generation of launchers that will open even more directions in space exploration. For example, Blue Origin is developing the New Glenn launcher (Fig. 1.2a) which will be able to bring heavy payloads to the Moon (this capability was lost with the cancellation of the Apollo program) and beyond. A distinctive feature of its design is the inclusion of planar fins for improved control during Launch \& Recovery (L\&R).


Figure 1.2: New generation of reusable heavy-lift launch vehicles

SpaceX is working on their interplanetary transport system (Fig. 1.2b) composed of a reusable Super Heavy Rocket and a Starship that can be refilled in orbit (to further cut down costs). This system is seen as the first enabler for the colonisation of Mars [Mus17]. But it also holds the potential to revolutionise everyday transportation on Earth - flying outside of the atmosphere, SpaceX foresees trips between Paris and New York to last only 30 minutes and access anywhere on the planet in under an hour.

The multi-disciplinary nature of reusability and of space exploration in general makes G\&C applications particularly demanding. Nonetheless, with compatible modelling, synthesis and
analysis tools provided by Systems and Control theory, improvements can be made that greatly contribute to more effective integrated solutions. As described in the following section, the aim of this thesis is precisely to showcase some of these contributions.

### 1.3 Motivation for guidance \& control developments

Because of all the involved risks, launcher and lander manufacturers are historically extremely conservative and the use of simple classical control techniques is by far the most common state-of-practice as they are vastly established and understood.

However, this industrial state-of-practice offers limited capabilities to manage issues such as the impact of uncertainties (which, as seen in Sec. 1.1, play an important role) and specification of Multiple-Input Multiple-Output (MIMO) requirements in a systematic manner. Instead, requirements are tackled iteratively for a single channel at a time, although control adjustments in one channel are typically coupled with the others. Consequently, not only does this approach rely on very time and cost consuming tuning and ad hoc Verification \& Validation (V\&V) cycles, there may also be missing opportunities for performance improvement.

Mathematical tools to address these limitations arose with the advent of the robust control paradigm in the 1980's [DGKF89, SD91, DPZ91, PDB93], which provides an explicit consideration of uncertainties and a multi-channel, multi-requirement design framework with formal guarantees of stability/performance by design. From a safety-oriented point of view, these formal guarantees make robust control even more attractive than newer alternatives based on the learning or identification of unknown system dynamics [NMBR19], such as adaptive or intelligent control. Moreover, a robust control technique called structured $\mathcal{H}_{\infty}$ has been more recently developed [AN06, GA11] and allows controller dimension and structure to be specified. As it will be seen, this is extremely convenient since it facilitates the transfer of this type of approach (offering better performance/robustness properties) to industry while keeping the legacy knowledge from previous control designs.

Despite the aforementioned advantages, there is still a big gap between the academic development and industrial application of robust control techniques. For this reason, one of the thesis' motivations is to work towards narrowing that gap.

In addition to safety concerns, computational capabilities have also limited the adoption of more advanced guidance algorithms, in particular for propelled $\mathrm{D} \& \mathrm{~L}$, which involves solving a fuel-optimal trajectory generation problem with state and control constraints. This problem is highly nonlinear and, until the past decade, it could only be solved in an offline setting, where trajectories are designed on the ground with powerful computers.

However, the increase of computational power available onboard together with a few mathematical developments, mostly in the field of convex optimisation [AP07, BAS10, LL14, MSA16], have enabled representative solutions to be determined online and applied in a closed-loop
fashion. This paradigm shift is now known as Computational Guidance and Control [Spe17] and it represents one of the enablers of Blue Origin and SpaceX successes [Bla16]. It also enables the exploration of less accessible, but more scientifically interesting, landing sites on other planets [SEA17].

Despite the feasibility demonstration of launcher recovery by those private companies, it is noted that the academic understanding of this topic remains largely uncharted and open to improvement, which makes it an important research topic of this thesis.

The thesis aims to demonstrate the benefits that specific G\&C elements can bring to space exploration. These elements arise from the aforementioned motivation to narrow the gap between academic/industrial development of robust control techniques and to increase the understanding of G\&C approaches for D\&L. Throughout the thesis, the potential benefits will be demonstrated by means of two case studies that are well-representative of those benefits, while targeting the key questions of Sec. 1.2: the first one is devoted to the improvement of $\mathrm{D} \& \mathrm{~L}$ on small planetary bodies and the other is aimed at solutions to reduce the cost of launchers.

The investigated G\&C approaches provide a design framework that is transversal to different aerospace systems but, since the two application cases are very specific and challenging, the transversal techniques have to be evolved and tailored. The case studies are linked to two different activities that co-funded the research work; the associated challenges and G\&C elements are introduced in Sec. 1.3.1 and 1.3.2.

### 1.3.1 Application I

## Fuel-optimal descent \& landing on small planetary bodies

This activity was sponsored by the UK Space Agency through a National Space Technology Programme (NSTP) fast-track grant and carried out in cooperation with Airbus Defence \& Space in Stevenage (UK). Its primary objective was to investigate the application of robust control techniques for the design and optimisation of $\mathrm{D} \& \mathrm{~L}$ approaches on small planetary bodies.

As mentioned in Sec. 1.1, SR missions tend to offer a revolutionising degree of scientific return. However, to facilitate the science, they also require a high level of technological development, with $D \& L$ being one of the most critical phases. In the past few years, recent advances have led to several SR missions and studies targeting small planetary bodies such as asteroids, comets and moons. An overview of those missions and guidance strategies is provided in Sec. 2.1.

The growing interest on small planetary bodies is mostly motivated by the fact that their weaker gravity field makes them more accessible than larger planets. To illustrate this, Fig. 1.3 shows the propellant cost of travelling between Earth and Mars systems. This cost is measured through the cumulative change of energy or velocity $(\Delta V)$ that is required to move from one orbit to another.

From this figure, it can be seen that a $\Delta V$ between 9.3 and $10 \mathrm{~km} / \mathrm{s}$ is required just to place a satellite on Low Earth Orbit (LEO). This interval accounts for the energy loss due to


Figure 1.3: $\Delta V$ requirements (in $\mathrm{km} / \mathrm{s}$ ) between Earth and Mars systems (adapted from The case for a mission to Mars' moon Phobos, https://phys.org/). Note: $\Delta V$ values are cumulative; not all possible routes are included.
atmospheric drag of different vehicles. To go to Mars, a further $\Delta V$ of $10.2 \mathrm{~km} / \mathrm{s}$ is needed (if no aerobraking is performed), adding up to a total of about $20 \mathrm{~km} / \mathrm{s}$. Instead, if the destination is one of its moons, Phobos or Deimos, the required $\Delta V$ is about $4.5 \mathrm{~km} / \mathrm{s}$ inferior, which results in a propellant saving of up to $23 \%$.

To reach the surface of these bodies, conventional strategies involve an extended period of forced motion. Yet, propellant requirements can be further alleviated if the D\&L trajectory exploits the dynamics of the gravity field in the vicinity of the target body. However, in opposition to planets, small bodies are often characterised by highly irregular and poorly known shapes, which render their physical environment extremely uncertain and variable. Therefore, any G\&C solution must be robust in the presence of these challenging circumstances.

Moreover, due to the interplanetary distances involved, highly robust algorithms are required to cope with communication delays and with the chance of long-term subsystem degradation. An excellent example of the latter was provided by ESA's Rosetta mission [FPB ${ }^{+} 15$ ] when, in 2014, robust control techniques had to be employed to re-tune Rosetta's control gains after thruster authority degradation occurred since its launch ten years before. The new gains were uploaded to the spacecraft just before its braking and final insertion with the target comet Churyumov-Gerasimenko.

This need for robust G\&C approaches represents the main motivation for this part of the thesis. Due to the long-term scope of D\&L missions and the conservative character of space industry in general, special attention is given to techniques well-oriented towards the state-of-practice, where the integration with legacy knowledge is fundamental.

Also, a conceptual separation between guidance and control is usually adopted by industry, in which the control element is either over-simplified or non-existent. As it will be seen, this simplification is only practical if the target body is well known or if guidance algorithms do not rely on its natural dynamics, which leaves significant room for performance improvement.

Although a generic framework for $D \& L$ on small bodies is pursued in this thesis, the scenario of a candidate Phobos SR mission led by Airbus (Fig. 1.4) is employed as an illustrative case of the proposed strategies on system design and operation. This Martian moon has been receiving significant attention from the international community, not only because of the wide scientific interest to solve the unknowns surrounding its formation, but also as a technological precursor for future manned and unmanned exploration missions targeting the Martian System $\left[\mathrm{BRB}^{+} 14\right]$.


Figure 1.4: Artist's concept of a lander on Phobos (credits: Airbus Defence \& Space)
In addition to all the support related to the Phobos SR mission, the collaboration with Airbus Defence \& Space (Stevenage) on this activity was especially crucial in ensuring that the proposed G\&C strategies are deemed feasible to adopt in short-term future missions.

### 1.3.2 Application II <br> Reusable launcher guidance \& control with active load relief

This application was sponsored by the European Space Agency (ESA) through a Network Partnering Initiative (NPI) doctoral grant and carried out in cooperation with the German Aerospace Centre (DLR) in Bremen. As mentioned before, the end goal is to explore G\&C functionalities to improve the cost (and performance) of launch vehicles.

As introduced in Sec. 1.2, reusability is currently seen as a key driver for more affordable launcher operations. The first objective of this part of the thesis is therefore to explore the
field of Computational Guidance and Control and mature the understanding and application of onboard $\mathrm{D} \& \mathrm{~L}$ guidance algorithms. Once again, these algorithms must be fuel-optimal to further minimise costs, but also suitable for the challenging environment of retro-propulsive entry, descent and pinpoint landing.

The second objective is to supplement these algorithms with robust attitude controllers, exploiting the benefits of robust control. By offering the ability to explicitly account for uncertainties as well as improving the management of MIMO specifications in a systematic manner, then the mission design and V\&V efforts can be greatly reduced, while simultaneously increasing wind resilience. This in turn allows to decrease the turnaround time of the vehicle and increase its operational availability and safety.

In addition, loads arising from the aerodynamic forces exerted on the vehicle must be maintained below certain structural integrity limits for a set of admissible wind conditions. When Load Relief (LR) is not properly addressed at control design stage, launcher manufacturers account for the safety limits by over-dimensioning the structure or constraining the admissible day-of-flight wind conditions. The design of active LR controllers is a well-established practice in aeronautics but, despite the critical effect of wind, not in launchers.

In this thesis, an improved LR capability is achieved by augmenting a fixed control structure with a robust Wind Disturbance Observer (WDO) for onboard wind estimation. This approach is an excellent example of the advantage of exploiting control structure, mentioned in the beginning of Sec. 1.3: while the state-of-practice robust control procedure to improve LR performance would involve increasing the order (and complexity) of a feedback controller, the wind observer augmentation provides a more gradual, industry-oriented approach, in which heritage launcher control knowledge can be complemented using an element with clear physical meaning. This simpler element can then be efficiently designed using (full-order) robust control techniques, as proposed in Sec. 10.3.

The inclusion of the WDO element is expected to enable further improvements in wind resilience, as well as help decrease the structural mass. The latter, in turn, allows for an increase of payload mass and subsequent reduction of the launch cost per kilogram of payload. However, the holistic optimisation of RLV performance is particularly demanding. One reason for this is that different requirements tend to compete against each other and a successful design is a result of an acceptable trade-off between them - as it will be seen, design choices that minimise propellant consumption are likely to subject the vehicle to higher aerodynamic loads and touchdown errors, and vice-versa. Another reason is that trajectory, control actuation and launcher structure are intrinsically coupled, as exemplified by Fig. 1.5.

In short, the vehicle is steered via Thrust Vector Control (TVC), aided by two pairs of fins under low thrust and two pairs of cold gas thrusters under low dynamic pressure conditions. Not only do these mechanisms influence L\&R performance, they also induce excitations that are propagated along the vehicle. Predicting these excitations is critical to be able to minimise the


Figure 1.5: Inherent launcher interactions. These interactions make guidance and control particularly challenging.
oscillations transmitted to the payload, but also very challenging because the vehicle is subject to uncertainty, time-varying effects (largely related to Mass, CG \& Inertia (MCI) variations caused by propellant burn), as well as distributed aerodynamic loads and structural elasticity (jointly known as aeroelasticity).

These effects become even more adverse for the next-generation of launchers, as lighter (and more flexible) structures tend to lead to stronger control-induced interactions [SYA11, MG16]. Aeroelastic couplings are expected to be even stronger during D\&L since the launcher will be flying with quasi-emptied propellant tanks, which reduces the separation between flexible and rigid-mode frequencies. All these effects are further worsened by wind gusts, also extremely uncertain and time-varying, and must be compensated for by the G\&C algorithms.

The ability to accurately capture these interactions is fundamental for the complete understanding of reusable flight mechanics and successful G\&C design. Several programmes and studies have addressed the problem of RLV performance optimisation [BH09, BGF14, TBLG15, SSBD17, DSE ${ }^{+}$17], but they are mostly focused on the application of Multi-Disciplinary Optimisation (MDO) methods to determine combined L\&R reference trajectories, staging conditions and preliminary vehicle configurations that allow delivering the highest payload while keeping aerodynamic and thermal loads at reasonable levels.

In this thesis, an alternative design framework based on the coupled assessment of flight mechanics, guidance and control is proposed. To do so, before any G\&C design, an RLV benchmark will be developed. It is noted that, while this is not an industrial-level simulator
(with detailed aerodynamic, thermal and structural models), it is a very sophisticated one capturing all the essential behaviours with sufficient representativeness to allow verifying the results to a confidence level acceptable by industry.

The benchmark simulates the L\&R trajectory of a VTVL booster used as first stage of a lightweight, non-winged launcher injecting a $1,100 \mathrm{~kg}$ satellite in a quasi-polar orbit at 800 km . The same recovery scenarios employed by SpaceX are implemented: Down-Range Landing (DRL), in which the reusable stage lands close to its unpropelled impact site, and Return To Launch Site (RTLS), where the booster uses an additional firing to return to its launch site.

The cooperation with DLR (Bremen) on this activity is particularly interesting because it opens the door to the possibility of flight testing algorithms developed throughout the thesis using their own VTVL demonstrator called Environment for Autonomous G\&C Landing Experiments (EAGLE) [SDT19] and depicted in Fig. 1.6.


Figure 1.6: DLR's EAGLE demonstrator

### 1.4 Organisation of the thesis

As introduced in the previous section, this thesis is composed of two application cases arising from two different activities, but sharing the same G\&C design framework. The work breakdown of these activities and the interplay between them is clarified in Fig. 1.7.

This figure shows the structure of the UK Space Agency activity (Application I) on the lefthand side, ESA's activity (Application II) on the right-hand side and the transversal framework in the middle. In addition, the temporal allocation of tasks is highlighted using different colours and the overall workflow is clarified using arrows.

The first year (blue blocks) started with the review of $\mathrm{D} \& \mathrm{~L}$ guidance as well as robust control modelling, analysis and synthesis techniques. The former techniques were then applied for the development of a systematic closed-loop guidance tuning methodology, which was verified


Figure 1.7: Organisation of the activities (arrows indicate the overall workflow)
using a high-fidelity industrial simulator of the dynamics in the vicinity of Phobos developed by Airbus Defence \& Space (Stevenage).

The robust control framework was then employed for the synthesis and analysis of control compensators in the first part of the second year (green blocks), marking the end of the first activity. Control compensation proved to be essential for open-loop guidance architectures (an elaborate discussion on G\&C architectures is provided in Sec. 2.1) and an added-value for closed-loop laws.

In addition, to initiate and support ESA's activity, the second part of the second year was dedicated to the development of a multi-disciplinary benchmark for the study of reusable flight mechanics. Since there are inherent interactions between trajectory and flight mechanics, the knowledge acquired during the review of $\mathrm{D} \& \mathrm{~L}$ techniques was fundamental for the implementation of a suitable launcher recovery law and for the development of a more sophisticated algorithm coined Descending over Extended envelopes using Successive ConvExificatioN-baseD Optimisation (DESCENDO).

The third year (yellow blocks) started with the design of active LR controllers based on wind disturbance observation. To achieve this, the set of robust control techniques reviewed in the first year was fundamental, as well as the knowledge from previous activities on launcher TVC control in ascent flight. It is highlighted that the study of the combined use of TVC and
fins in descent flight is particularly novel in launcher control literature. Descent flight control of a vertical take-off and landing vehicle has been addressed by Boelitz [Boe99], but only TVC was considered.

The last task corresponded to the inclusion of aeroelasticity effects (couplings between structural flexibility and distributed aerodynamics) in the model of the launcher by relying on a MultiBody (MB) dynamics approach. This refined model provided a higher-fidelity analysis of the LR controllers under potential load cases extracted from the benchmark.

Finally, ESA's activity accounts for the possibility/potential to verify algorithms developed throughout the thesis (red block) using the EAGLE demonstrator of Fig. 1.6. The current plan is to flight test the robust wind disturbance observers mentioned above through concise campaigns after submitting the thesis.

The thesis report is also organised around the two application cases, see Fig. 1.8, which provides the layout of the thesis. Following the present introduction, Chapter 2 provides a broad state-of-the-art review of G\&C strategies and techniques in support of both cases. Developments on each application take place independently but through a common sequence: development of benchmark/models, followed by development of guidance and control functionalities, finishing with dedicated analysis and conclusions for each case.


Figure 1.8: Organisation of the report (arrows indicate the overall workflow)

With this structure in mind, Chapter 3 presents the models relative to Application I, Chapter 4 proposes a performance-oriented guidance tuning methodology for space $\mathrm{D} \& \mathrm{~L}$, Chapter 5 is dedicated to the development of control compensators for further performance improvements. Chapter 6 provides thorough robustness analyses of those compensators and Chapter 7 draws the main conclusions.

Regarding Application II, Chapter 8 reports the development of the RLV benchmark and G\&C framework, Chapter 9 and Chapter 10 are respectively focused on improving the guidance and control algorithms implemented with the benchmark. Chapter 11 then refines the performance analysis by accounting for effects related to aeroelastic loads and Chapter 12 concludes this application.

To conclude the thesis, the conclusions drawn in Chapter 7 and 12 are reviewed in Chapter 13 and recommendations for future work are provided based on the encountered room for improvement.

### 1.5 Main contributions

In accordance with the motivations stated in Sec. 1.3 for the two activities that constitute this thesis, its high-level contribution to the state-of-the-art is a Technology Readiness Level (TRL) increase of G\&C approaches for planetary landers and reusable launchers.

This outcome is broken down into the following achievements (in order of appearance):

1. Reconciliation of $\mathbf{G} \& \mathbf{C}$ architectures for $\mathbf{D} \& \mathbf{L}$, enabling the integration of legacy industry knowledge with sophisticated design techniques;
2. Development of a methodology for performance quantification and optimisation of guidance laws based on the generation of trade-off maps;
3. Earliest application of robust control synthesis techniques to spacecraft orbital (translational) control, as they were well-established for attitude control only;
4. Demonstration of how robust control analysis techniques can be effective to validate modelling choices and complement state-of-practice V\&V methods;
5. Development of a novel benchmark/framework for RLV G\&C design that allows to exploit fundamental couplings between flight mechanics and closed-loop algorithms;
6. Design and validation of an onboard convex optimisation-based recovery guidance algorithm specifically tailored to the extended flight envelope encountered by RLVs;
7. Earliest application of robust wind disturbance observation to improve the control of launcher aerodynamic loads in ascent and descent flight;
8. Extension of the RLV benchmark with a MB model that enables more accurate load predictions and faster design iterations between trajectory, structure and control.

These accomplishments have been disseminated through several journals, conferences and invited presentations. A list of these publications is provided in Appendix A.

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## Guidance \& Control Background

This chapter provides a state-of-the-art survey of methods and tools that are the foundation to technological developments performed in the remainder of the thesis. Section 2.1 is devoted to the Descent \& Landing (D\&L) problem and associated guidance strategies, while Sec. 2.2 introduces the robust control modelling, analysis and synthesis techniques to be subsequently employed.

### 2.1 Descent \& Landing (D\&L) strategies

The D\&L strategies presented in this section cover applications to both small asteroids and larger planetary bodies, as it is noted that, while techniques for each category have been developed rather independently, much insight can be gained by examining them jointly. Hence, in contrast to surveys available in the literature, e.g. [HGW12, GHW13], the present one is much ampler and techniques are compared in an implementation-oriented manner.

For each category, relevant space missions are presented and guidance laws are described. In addition, emphasis is placed on a parametric description of the algorithms (Sec. 2.1.4.4), which is convenient for their optimisation, as well as on the impact of uncertainties and inaccuracies (Sec. 2.1.5) that can lead to performance or mission loss if not addressed properly. This survey is also available in $\left[\mathrm{SMJ}^{+} 17 \mathrm{~b}, \mathrm{SMJ}^{+} 18 \mathrm{a}\right]$.

### 2.1.1 $\mathrm{D} \& \mathrm{~L}$ problem formulation

The D\&L problem geometry is depicted in Fig. 2.1 for planar motion but without loss of generality. It describes a spacecraft approaching a moving body target subject to the influence of a larger one. Here, the $X$ and $Y$ axes arbitrarily define an inertial reference frame with origin at
the centre of the larger body. For this problem, it is assumed that the spacecraft has a dedicated attitude control system that maintains a nadir pointing during the descent (this is actually a requirement of visual-navigation systems). Couplings are then considered at actuator level by reserving a fraction of the available thruster authority (10 to $20 \%$ ) for attitude control.


Figure 2.1: D\&L problem geometry showing a spacecraft approaching a moving body subject to the influence of a larger one

The position and velocity of the target in Fig. 2.1, $\mathbf{r}_{T}(t) \in \mathbb{R}^{3}$ and $\mathbf{v}_{T}(t) \in \mathbb{R}^{3}$, are described as follows:

$$
\begin{align*}
\dot{\mathbf{r}}_{T}(t) & =\mathbf{v}_{T}(t) \\
\dot{\mathbf{v}}_{T}(t) & =\mathbf{g}_{T}\left(\mathbf{r}_{T}\right) \tag{2.1}
\end{align*}
$$

where $\mathbf{g}_{T}\left(\mathbf{r}_{T}\right) \in \mathbb{R}^{3}$ is the gravitational acceleration acting on the target, which is generically expressed as a partial derivative of the gravity potential of the main body. Also, $r_{T}(t)=\left\|\mathbf{r}_{T}(t)\right\|$ and $v_{T}(t)=\left\|\mathbf{v}_{T}(t)\right\|$.

In a similar way, the position and velocity of the impactor or lander spacecraft in the same frame, $\mathbf{r}_{S}(t) \in \mathbb{R}^{3}$ with $r_{S}(t)=\left\|\mathbf{r}_{S}(t)\right\|$ and $\mathbf{v}_{S}(t) \in \mathbb{R}^{3}$ with $v_{S}(t)=\left\|\mathbf{v}_{S}(t)\right\|$, are modelled as [HGW11]:

$$
\begin{align*}
\dot{\mathbf{r}}_{S}(t) & =\mathbf{v}_{S}(t)  \tag{2.2}\\
\dot{\mathbf{v}}_{S}(t) & =\mathbf{g}_{S}\left(\mathbf{r}_{T}, \mathbf{r}_{S}\right)+\mathbf{a}(t)+\mathbf{p}(t)
\end{align*}
$$

where $\mathbf{g}_{S}\left(\mathbf{r}_{T}, \mathbf{r}_{S}\right) \in \mathbb{R}^{3}$ is the gravitational acceleration felt by the spacecraft due to the main and target bodies, $\mathbf{a}(t) \in \mathbb{R}^{3}$ is the control acceleration provided by the spacecraft thrusters and $\mathbf{p}(t) \in \mathbb{R}^{3}$ represents any external perturbations (e.g. third-body perturbations) and unknowns.

Defining relative position as $\mathbf{r}(t)=\mathbf{r}_{S}(t)-\mathbf{r}_{T}(t)$ and relative velocity as $\mathbf{v}(t)=\mathbf{v}_{S}(t)-\mathbf{v}_{T}(t)$, the relative motion between spacecraft and target is expressed as:

$$
\begin{align*}
\dot{\mathbf{r}}(t) & =\mathbf{v}(t)  \tag{2.3}\\
\dot{\mathbf{v}}(t) & =\mathbf{g}\left(\mathbf{r}_{T}, \mathbf{r}_{S}\right)+\mathbf{a}(t)+\mathbf{p}(t)
\end{align*}
$$

where:

$$
\begin{equation*}
\mathbf{g}\left(\mathbf{r}_{T}, \mathbf{r}_{S}\right)=\mathbf{g}_{S}\left(\mathbf{r}_{T}, \mathbf{r}_{S}\right)-\mathbf{g}_{T}\left(\mathbf{r}_{T}\right) \tag{2.4}
\end{equation*}
$$

is the apparent gravitational acceleration. The closing velocity of the spacecraft is defined as $V_{c}(t)=-\|\mathbf{v}(t)\|$. Moreover, $\mathbf{g}\left(\mathbf{r}_{T}, \mathbf{r}_{S}\right)$ can be computed with different levels of accuracy, e.g. assuming Keplerian forces only or including detailed representations of the inhomogeneity of bodies via Gravity Harmonics (GH), with all the inaccuracies contained in $\mathbf{p}(t)$. Also, note that it is important that non-inertial effects are accounted for in $\mathbf{g}\left(\mathbf{r}_{T}, \mathbf{r}_{S}\right)$ when the frame of Fig. 2.1 is rotating with the system.

Based on this figure, the $\mathbf{D} \& \mathbf{L}$ problem is defined as the computation of the acceleration input $\mathbf{a}(t)$ between initial and final times, i.e. $t=t_{0}$ and $t=t_{f}$, that is able to:

- Bring the relative position and velocity from the initial boundary conditions $\mathbf{r}\left(t_{0}\right)=\mathbf{r}_{0}$ and $\mathbf{v}\left(t_{0}\right)=\mathbf{v}_{0}$ to the final conditions $\mathbf{r}\left(t_{f}\right)=\mathbf{r}_{f}$ and $\mathbf{v}\left(t_{f}\right)=\mathbf{v}_{f}$;
- Cope with the effects of uncertainties and perturbations. In the case of small bodies, these effects are mostly caused by an inexact gravitational model, thruster realisation errors and inaccurate sensor measurements.

Two additional important concepts to describe the D\&L problem that must be introduced now are the time duration from a given instance $t$ until the end of the manoeuvre, known as time-to-go, $t_{\mathrm{go}}(t)=t_{f}-t$, and the Line-Of-Sight (LOS) vector $\boldsymbol{\Lambda}(t) \in \mathbb{R}^{3}$, which is the direction from target to spacecraft [HPW10] and is given by:

$$
\begin{equation*}
\boldsymbol{\Lambda}(t)=\frac{\mathbf{r}(t)}{r(t)} \tag{2.5}
\end{equation*}
$$

For the planar illustration of Fig. 2.1, the LOS is represented by a single angle and angular rate:

$$
\begin{equation*}
\lambda(t)=\arctan \frac{r_{y}(t)}{r_{x}(t)}, \quad \dot{\lambda}(t)=\frac{r_{x}(t) \dot{r}_{y}(t)+r_{y}(t) \dot{r}_{x}(t)}{r^{2}(t)} \tag{2.6}
\end{equation*}
$$

and, in this case:

$$
\boldsymbol{\Lambda}(t)=\left[\begin{array}{c}
\cos \lambda(t)  \tag{2.7}\\
\sin \lambda(t)
\end{array}\right], \quad \dot{\boldsymbol{\Lambda}}(t)=\dot{\lambda}(t)\left[\begin{array}{c}
-\sin \lambda(t) \\
\cos \lambda(t)
\end{array}\right]
$$

In addition, and specially important to reconcile the diverse guidance laws, the concept of zero-effort errors that was first defined in [EBR08] must be introduced:

- Zero-Effort-Miss (ZEM) is the position error at the end-of-mission if no corrective manoeuvres are made after time $t$ :

$$
\begin{equation*}
\mathbf{Z E M}(t)=\mathbf{r}_{f}-\mathbf{r}\left(t_{f}\right) \mid \mathbf{a}(\tau)=0 \forall \tau \in\left[t, t_{f}\right] \tag{2.8}
\end{equation*}
$$

- Zero-Effort-Velocity (ZEV) is the velocity error at the end-of-mission if no corrective manoeuvres are made after time $t$ :

$$
\begin{equation*}
\mathbf{Z E V}(t)=\mathbf{v}_{f}-\mathbf{v}\left(t_{f}\right) \mid \mathbf{a}(\tau)=0 \forall \tau \in\left[t, t_{f}\right] \tag{2.9}
\end{equation*}
$$

Position and velocity can be propagated using Eq. (2.3) in the absence of corrective manoeuvres, and then the ZEM and ZEV equations become:

$$
\begin{align*}
& \mathbf{Z E M}(t)=\mathbf{r}_{f}-\left[\mathbf{r}(t)+\left(t_{f}-t\right) \mathbf{v}(t)+\int_{t}^{t_{f}}\left(t_{f}-\tau\right) \mathbf{g}(\tau) \mathrm{d} \tau\right] \\
& \mathbf{Z E V}(t)=\mathbf{v}_{f}-\left[\mathbf{v}(t)+\int_{t}^{t_{f}} \mathbf{g}(\tau) \mathrm{d} \tau\right] \tag{2.10}
\end{align*}
$$

To obtain these analytical expressions for ZEM and ZEV, the apparent acceleration is typically assumed to be known as an explicit function $\mathbf{g}(t)$ of time. However, as the acceleration is more generally given as a function of position, Eq. (2.4), the computation of ZEM and ZEV has to be performed numerically or alternatively must be approximated [HGW12]. For example, the value of ZEM and ZEV can be estimated from a linearised time-varying State Transition Matrix (STM) [Bat87]. While if the gravitational force is not significant, then it can be neglected for ZEM and ZEV computations. However, if the gravitational force cannot be neglected but changes slowly during the manoeuvre, it can be assumed constant and equal to $\mathbf{g}$, yielding:

$$
\begin{align*}
& \mathbf{Z E M}(t)=\mathbf{r}_{f}-\left[\mathbf{r}(t)+t_{\mathrm{go}}(t) \mathbf{v}(t)+\frac{1}{2} t_{\mathrm{go}}^{2}(t) \mathbf{g}\right]  \tag{2.11}\\
& \mathbf{Z E V}(t)=\mathbf{v}_{f}-\left[\mathbf{v}(t)+t_{\mathrm{go}}(t) \mathbf{g}\right]
\end{align*}
$$

For the development of guidance laws, spacecraft mass $m(t)$ is often also assumed constant while the control acceleration is assumed unconstrained. These approximations can be compensated for by an inner control loop (see Sec. 2.1.2). However, for the purpose of Verification \& Validation (V\&V), the actual spacecraft acceleration results from:

$$
\begin{equation*}
\mathbf{a}(t)=\frac{\mathbf{T}(t)}{m(t)} \tag{2.12}
\end{equation*}
$$

in which the available thrust force $\mathbf{T}(t) \in \mathbb{R}^{3}$ is limited:

$$
\begin{equation*}
0 \leq T_{\min } \leq\|\mathbf{T}(t)\| \leq T_{\max } \tag{2.13}
\end{equation*}
$$

and the mass variation is given by the rocket equation [Gre70]:

$$
\begin{equation*}
\dot{m}(t)=-\frac{1}{I_{\mathrm{sp}} g_{0}}\|\mathbf{T}(t)\| \Rightarrow m(t)=m\left(t_{0}\right) \exp \left(-\frac{1}{I_{\mathrm{sp}} g_{0}} \int_{t_{0}}^{t_{f}}\|\mathbf{a}(\tau)\| \mathrm{d} \tau\right) \tag{2.14}
\end{equation*}
$$

where $I_{\mathrm{sp}}$ is the specific impulse of the thrusters, $g_{0}$ is the gravitational acceleration at the surface of the Earth and $\Delta m=m\left(t_{0}\right)-m\left(t_{f}\right)$ is the propellant mass consumed for the manoeuvre.

### 2.1.2 G\&C architectures

Arising from different domains in control theory and from the understanding of Space Mission Analysis, different specialist terms are found in the literature on D\&L techniques. Thus, before
proceeding with the survey, this section clarifies key Guidance \& Control (G\&C) concepts. This clarification is supported by the two block diagrams of Fig. 2.2.

Referring to Fig. 2.2, the Spacecraft Dynamics \& Kinematics (SDK) block lies essentially on the simulation of the relative translational motion of Eq. (2.3). The complexity of this simulation depends on that of Eq. (2.4), since $\mathbf{g}\left(\mathbf{r}_{T}, \mathbf{r}_{S}\right)$ can be highly nonlinear for very accurate gravitational models.


Figure 2.2: G\&C architectures for D\&L. Dashed lines indicate information flow for closed-loop guidance.

The SDK block is fed by the spacecraft actuators, which are commanded by the guidance and/or control systems. The actuators account for realisation errors of the spacecraft thrusters due to mounting errors and gas-dynamics properties, as well as for maximum thruster capability, e.g. Eq. (2.13), and limited authority (if also employed for position-dependent attitude control). Furthermore, the outputs of the SDK block are measured and filtered by the sensors \& navigation subsystem before being used for G\&C. This process introduces noise and quantisation errors in the relative position and velocity estimates, $\hat{\mathbf{r}}(t)$ and $\hat{\mathbf{v}}(t)$.

Focusing now on Fig. 2.2a, two different paradigms for planetary descent can be defined:

- Open-loop guidance, also known as implicit guidance, which refers to the case when a reference trajectory $\left\{\mathbf{r}_{\mathrm{ref}}(t), \mathbf{v}_{\mathrm{ref}}(t)\right\}$ and thruster profile $\mathbf{a}_{\mathrm{ref}}(t)$ are generated before, and remain unchanged during, the descent. In the space domain, these references are designed based on Mission Analysis considerations;
- Closed-loop guidance, also known as explicit guidance, when the thruster profile $\mathbf{a}_{\mathrm{ref}}(t)$ is computed in real-time to correct the trajectory based on onboard measurements, $\hat{\mathbf{r}}(t)$ and
$\hat{\mathbf{v}}(t)$. This feedback of information is represented with dashed lines in the figure. In this case, the guidance subsystem may also be responsible for the computation of auxiliary variables such as the LOS or zero-effort errors, i.e. Eq. (2.5) and (2.10).

Regardless of the type of guidance that precedes it, the architecture may also be augmented with a control compensator, as illustrated in Fig. 2.2b, to further alleviate trajectory errors. As it will be detailed later on, this alleviation is achieved by introducing an additional acceleration vector command $\mathbf{a}_{\mathrm{cmp}}(t)$ that compensates for deviations between reference trajectory and measured states.

### 2.1.3 Space missions and categories

In recent years, a renewed interest in small planetary bodies has led to several studies and missions (refer to Fig. 2.3 for a broad overview of applications). There are mainly two different purposes behind these studies and missions. On the one hand, there is the exploitation of hypervelocity impact with a spacecraft as a mitigation strategy against objects on a course for potential collision with Earth. Notable examples of this type of missions include NASA's Deep Impact Spacecraft [Kub03], which successfully hit comet Tempel 1 on July 2005 at $10 \mathrm{~km} / \mathrm{s}$, and ESA's Asteroid Impact Mission [FLS ${ }^{+}$15], undergoing preliminary design phase but planned to rendezvous with the Didymos binary system and observe closely the collision with an impactor.

On the other hand, there is also the interest of touch-and-go or landing on planetary bodies instead of impacting, as the scientific return in general is much higher. Successful missions in this category include NASA's Stardust [BMRS04], launched in 1999 and the first Sample Return (SR) mission to collect comet and cosmic dust samples, JAXA's Hayabusa [YKH ${ }^{+} 09$ ], a mission that landed on Itokawa asteroid on November 2005 returning to Earth five years after, and ESA's Rosetta [GFUW14], which performed a rendezvous with comet Churyumov-Gerasimenko and delivered a lander for on-site analysis on November 2014. In addition, NASA has launched OSIRIS-REx [WAC ${ }^{+}$18] in September 2016, an SR spacecraft that will reach the near-Earth asteroid Bennu, while ESA is also studying the feasibility of an SR mission to Phobos [ $\mathrm{BRB}^{+}$14]. And more recently, JAXA's Hayabusa-2 [KH15] successfully delivered two small rovers and a lander on asteroid Ryugu in late 2018, an impactor followed by touchdown for sample collection in 2019, and is scheduled to return to Earth in 2020.

For both interception and landing on small bodies, autonomous systems are mandatory to guide the spacecraft through very uncertain operational environments while coping with long communication delays with Earth. The earliest known method is inspired by the missile interception problem. It is known as Proportional Navigation Guidance (PNG) and introduced in [Zar94], where a method of augmenting it when the target acceleration is known or can be assumed is also provided. In addition, guidance using predictive manoeuvres based on linear orbital perturbation theory [Bat87] is proven possible and complemented with PNG in [GFPC08].


Figure 2.3: Examples of D\&L applications

However, most of the work on traditional closed-loop guidance for small bodies recasts the problem as optimal feedback control with terminal constraints only (i.e. without path constraints), which is solved with the Pontryagin maximum principle in [Bat87] or through calculus of variations in [D’S97]. This type of laws, known as Optimal Guidance Laws (OGLs), has been continuously developed for different terminal boundary conditions (e.g. constrained velocity, free velocity, constrained intercept-angle, etc.) and also related to the classical PNG laws (see, for example, [HGW11]).

Additionally, and due to the highly uncertain character of the operational environments, OGLs have been recently augmented with nonlinear terms based on Sliding Mode Control (SMC) theory in order to increase their robustness in the presence of inaccurate measurements and unmodelled dynamics [EBR08]. Because of the achievable robustness improvement, this topic has been evolving and applied to different scenarios in the past few years [FSCC11, FGWS13].

In parallel, different D\&L techniques have been developed and applied for the exploration of larger bodies, such as the Moon and Mars. These approaches are not as demanding as for
the asteroid intercept problem since the curvature of the planet can often be neglected and its gravity field is relatively uniform and well known. Hence, the first-generation of Mars probes that successfully reached its surface, from NASA's Viking 1 in 1976 to Phoenix in 2008, relied on an unguided descent phase. As a consequence, these systems generated a landing uncertainty ellipse in the order of 500 km by 100 km [LJ14].

When the mission has to satisfy more stringent requirements such as very high landing accuracy or crew safety (in the case of manned missions), then the strategies used are generally based by solving a trajectory-generation problem with both terminal and path constraints. This type of approach begun with the US Apollo and Russian Luna programs and continues with the next-generation Mars landers (for which the capability of pinpoint landing the spacecraft in hazardous sites with high scientific value is mandatory, thus requiring an uncertainty ellipse down to $100 \mathrm{~m}[\mathrm{LJ} 14]$ ), and is also used on Earth by Vertical Take-off and Vertical Landing (VTVL) vehicles as introduced in Sec. 1.2.

The aforementioned problem is typically nonlinear and challenging to solve and, until the past decade, its application was only feasible in an offline setting. To simplify the problem, during the Apollo program, an acceleration profile that is a quadratic function of time [Klu74] was chosen. This profile was not optimal in the sense that no cost function was optimised, but the quadratic coefficients could be computed analytically from the terminal boundary conditions for a pre-specified descent duration. This approach was modified for the NASA's Mars Science Laboratory Curiosity in 2011, by adding a line-search over the powered descent duration so as to minimise propellant consumption [PAW06, WSM06]. In addition to these simplifications and extensions, augmenting the polynomial order of an open-loop guidance law renders the computation of the coefficients under-determined and thus, this allows choosing them so as to optimise a desired cost function.

A plethora of optimisation algorithms can be applied to solve this augmented-coefficient optimisation problem offline while enforcing different path constraints (such as minimum altitude or maximum actuation). However, the increase of computational power available onboard in recent years has enabled representative solutions to be determined and applied in a closed-loop fashion, leading to a paradigm shift known as Computational Guidance and Control [Spe17]. This is supplemented by specific semi-analytical algorithms (e.g. [LLA13, LLA14]) or by further mathematical developments in the domain of convex optimisation and pseudospectral methods.

Nonetheless, methods for extraterrestrial autonomous pinpoint landing have very little heritage and have only recently been demonstrated with NASA's VTVL platform Xombie, which uses a vision system to determine its location and the Guidance for Fuel-Optimal Large Diverts (G-FOLD) algorithm to optimally fly to the landing site $\left[\mathrm{AAC}^{+} 13, \mathrm{SRV}^{+} 14\right]$.

Although developed in parallel, these solutions have the potential to complement more traditional techniques targeting small bodies such as Phobos which, due to their irregular shapes and mass distributions, are characterised by very variable and uncertain gravitational fields, with
complex orbits stable only in certain regions [LS03]. This complementarity between techniques will be exploited throughout the thesis.

### 2.1.4 Review of guidance techniques

This section is dedicated to the description of the techniques introduced in Sec. 2.1.3. They are divided in open-loop (Sec. 2.1.4.1), traditional closed-loop (Sec. 2.1.4.2) and computational closed-loop guidance (Sec. 2.1.4.3). An overall summary is also provided in Sec. 2.1.4.4.

### 2.1.4.1 Open-loop guidance

Open-loop guidance relies on the design of a reference trajectory and thruster profile before initiating the descent. Depending on the type of profile, open-loop guidance laws can be further classified as quadratic or as optimal with path constraints, both described next.

## Quadratic

As previously mentioned, for landing on larger planetary bodies where the gravity field is well known and can be assumed constant, open-loop guidance techniques often suffice. For the Apollo program, emphasis was placed on developing computationally feasible guidance laws rather than looking for complex energy-optimal solutions. Hence, the Apollo guidance law is simply defined as a quadratic function of time [Klu74, WSM06]:

$$
\begin{equation*}
\mathbf{a}(t)=\mathbf{C}_{0}+\mathbf{C}_{1} t+\mathbf{C}_{2} t^{2} \tag{2.15}
\end{equation*}
$$

where $\mathbf{C}_{i} \in \mathbb{R}^{3}$ are coefficients to be determined. Velocity and position are obtained assuming a constant gravity field and integrating the above acceleration:

$$
\begin{align*}
\mathbf{v}(t) & =\mathbf{v}_{0}+\left(\mathbf{C}_{0}+\mathbf{g}\right) t+\frac{1}{2} \mathbf{C}_{1} t^{2}+\frac{1}{3} \mathbf{C}_{2} t^{3}  \tag{2.16}\\
\mathbf{r}(t) & =\mathbf{r}_{0}+\mathbf{v}_{0} t+\frac{1}{2}\left(\mathbf{C}_{0}+\mathbf{g}\right) t^{2}+\frac{1}{6} \mathbf{C}_{1} t^{3}+\frac{1}{12} \mathbf{C}_{2} t^{4} \tag{2.17}
\end{align*}
$$

Applying the boundary conditions at the end-of-mission $\left(t=t_{f}\right)$ as defined in Sec. 2.1.1, the unknown coefficients are obtained by solving the linear system:

$$
\left[\begin{array}{ccc}
\mathbb{I} & t_{f} \mathbb{I} & t_{f}^{2} \mathbb{I}  \tag{2.18}\\
t_{f} \mathbb{I} & \frac{t_{f}^{2}}{2} & \frac{t_{f}^{3}}{3} \\
t_{f}^{2} & \mathbb{I} & \frac{t_{f}^{3}}{6}
\end{array} \frac{t_{f}^{4}}{12} \mathbb{I}\right]\left[\begin{array}{c}
\mathbf{C}_{0}+\mathbf{g} \\
\mathbf{C}_{1} \\
\mathbf{C}_{2}
\end{array}\right]=\left[\begin{array}{c}
\dot{\mathbf{v}}\left(t_{f}\right) \\
\mathbf{v}_{f}-\mathbf{v}_{0} \\
\mathbf{r}_{f}-\mathbf{r}_{0}-\mathbf{v}_{0} t_{f}
\end{array}\right]
$$

where $\mathbb{I}$ is the identity matrix of suitable dimension. The solutions are then given by:

$$
\begin{align*}
& \mathbf{C}_{0}=\dot{\mathbf{v}}\left(t_{f}\right)-\mathbf{g}-\frac{6}{t_{f}}\left(\mathbf{v}_{f}-\mathbf{v}_{0}\right)+\frac{12}{t_{f}^{2}}\left(\mathbf{r}_{f}-\mathbf{r}_{0}-\mathbf{v}_{0} t_{f}\right) \\
& \mathbf{C}_{1}=-\frac{6}{t_{f}} \dot{\mathbf{v}}\left(t_{f}\right)+\frac{30}{t_{f}^{2}}\left(\mathbf{v}_{f}-\mathbf{v}_{0}\right)-\frac{48}{t_{f}^{3}}\left(\mathbf{r}_{f}-\mathbf{r}_{0}-\mathbf{v}_{0} t_{f}\right)  \tag{2.19}\\
& \mathbf{C}_{2}=\frac{6}{t_{f}^{2}} \dot{\mathbf{v}}\left(t_{f}\right)-\frac{24}{t_{f}^{3}}\left(\mathbf{v}_{f}-\mathbf{v}_{0}\right)+\frac{36}{t_{f}^{4}}\left(\mathbf{r}_{f}-\mathbf{r}_{0}-\mathbf{v}_{0} t_{f}\right)
\end{align*}
$$

In this type of guidance, the end-of-mission time (or the time-to-go, $t_{\mathrm{go}}(t)=t_{f}-t$ ) is a free parameter that must be specified.

## Optimal with path constraints

Planetary descent guidance with a direct consideration of physical state and control constraints can also be implemented in an open-loop fashion. The basis for these sophisticated implicit guidance methods is actually the quadratic acceleration law of Eq. (2.15), in which all the coefficients are completely determined, but in this case augmenting its order to $N>2$ :

$$
\begin{equation*}
\mathbf{a}(t)=\mathbf{C}_{0}+\mathbf{C}_{1} t+\ldots+\mathbf{C}_{N} t^{N} \tag{2.20}
\end{equation*}
$$

This higher order polynomial structure for the acceleration allows for significant improvements [PAW06, SSW07], since the terminal linear system of Eq. (2.18) is now under-determined:

$$
\left[\begin{array}{cccc}
\mathbb{I} & t_{f} \mathbb{I} & \ldots & t_{f}^{N} \mathbb{I}  \tag{2.21}\\
t_{f} \mathbb{I} & \frac{t_{f}^{2}}{2} \mathbb{I} & \ldots & \frac{t_{f}^{N+1}}{(N+1)} \mathbb{I} \\
\frac{t_{f}^{2}}{2} \mathbb{I} & \frac{t_{f}^{3}}{6} \mathbb{I} & \ldots & \frac{t_{f}^{N+2}}{(N+1)(N+2)} \mathbb{I}
\end{array}\right]\left[\begin{array}{c}
\mathbf{C}_{0}+\mathbf{g} \\
\mathbf{C}_{1} \\
\vdots \\
\mathbf{C}_{N}
\end{array}\right]=\left[\begin{array}{c}
\dot{\mathbf{v}}\left(t_{f}\right) \\
\mathbf{v}_{f}-\mathbf{v}_{0} \\
\mathbf{r}_{f}-\mathbf{r}_{0}-\mathbf{v}_{0} t_{f}
\end{array}\right]
$$

Therefore, the solution of the acceleration coefficients $\mathbf{C}_{i} \in \mathbb{R}^{3}$ and of $t_{f}$ can be determined so as to optimise a specified cost function (e.g. propellant consumption) subject to the linear system of Eq. (2.21), as well as to ensure satisfaction of path state and control constraints. These may include bounded thrust force, Eq. (2.13), admissible mass variation, subsurface flight avoidance $\left(r(t) \geq h_{\min }\right)$ or any additional position and velocity constraints (e.g. obstacle avoidance, glide-slope angle or maximum velocity) all given as generalised linear inequalities.

Depending on the formulation of the constrained optimisation problem, a myriad solvers are available and, depending on the solver, parametric descriptions of the acceleration profile different to Eq. (2.20) may also be considered.

### 2.1.4.2 Traditional closed-loop guidance

In traditional closed-loop guidance, the thruster profile is computed as a closed-form solution to actively correct the descent trajectory. Depending on the type of solution, guidance laws can be further classified as [i] proportional, [ii] predictive and hybrid, [iii] optimal without path constraints and [iv] nonlinear robust.

## Proportional

The earliest known guidance strategy for the interception of small bodies is inspired by the missile interception problem and is known as PNG. PNG laws and their most basic variations are introduced in [Zar94] and thoroughly described in [HGW12, GHW13]. The principle of PNG is to drive the LOS rate to zero by applying a proportional acceleration perpendicularly to the LOS direction:

$$
\begin{equation*}
\mathbf{a}(t)=n V_{c}(t) \dot{\boldsymbol{\Lambda}}(t) \tag{2.22}
\end{equation*}
$$

where $n$ is the effective navigation ratio, a tunable parameter typically chosen between 3 and 5 . Smaller values result in reduced propellant consumption whereas larger values are adopted for improved robustness at the expense of higher acceleration commands. For the planar case, i.e. Eq. (2.7), the acceleration becomes:

$$
\mathbf{a}(t)=n V_{c}(t) \dot{\lambda}(t)\left[\begin{array}{c}
-\sin \lambda(t)  \tag{2.23}\\
\cos \lambda(t)
\end{array}\right]
$$

This guidance law does not require the target or spacecraft accelerations to be zero, but its performance is improved if the contribution of the gravitational environment is deducted. This results in the Augmented Proportional Navigation Guidance (APNG) law:

$$
\begin{equation*}
\mathbf{a}(t)=n V_{c}(t) \dot{\boldsymbol{\Lambda}}(t)-\frac{n}{2} \mathbf{g}_{\perp}(t) \tag{2.24}
\end{equation*}
$$

where again $\mathbf{g}_{\perp}(t)$ are the components of apparent gravity perpendicular to the LOS. For the planar case, the APNG law simplifies into:

$$
\mathbf{a}(t)=n\left(V_{c}(t) \dot{\lambda}(t)-\frac{1}{2} g_{\perp}(t)\right)\left[\begin{array}{c}
-\sin \lambda(t)  \tag{2.25}\\
\cos \lambda(t)
\end{array}\right]
$$

with $g_{\perp}(t)=[-\sin \lambda(t) \cos \lambda(t)] \mathbf{g}(t)$. Moreover, employing the definitions of time-to-go and zero-effort-miss, Eq. (2.10), it can be shown that the APNG law becomes:

$$
\begin{equation*}
\mathbf{a}(t)=\frac{n}{t_{\mathrm{go}}^{2}(t)} \mathbf{Z E M}(t) \tag{2.26}
\end{equation*}
$$

where the time-to-go cannot be controlled, but is computed as $\operatorname{tgo}_{\mathrm{go}}(t)=r(t) / V_{c}(t)$.
In [KLH98], an adaptation of the APNG law known as Biased Proportional Navigation Guidance (BPNG) is also explored. This biased law allows to constrain the impact (final) LOS angle to $\lambda_{f}$ resulting in:

$$
\mathbf{a}(t)=\left(4 V_{c}(t) \dot{\lambda}(t)-g_{\perp}(t)+2 V_{c}(t) \frac{\lambda(t)-\lambda_{f}}{t_{\mathrm{go}}(t)}\right)\left[\begin{array}{c}
-\sin \lambda(t)  \tag{2.27}\\
\cos \lambda(t)
\end{array}\right]
$$

Finally, for thrusters with no continuous throttling ability (only on-off), PNG laws have to be applied using a discrete scheme, through a pulse-modulation method [Wie08]. In this case, the guidance is known as Pulsed PNG.

## Predictive and hybrid

A more propellant-efficient method to perform pulsed navigation is to compute impulsive velocity corrections, which eliminate the predicted end-of-mission error by using linearised orbital perturbation theory [Bat87] and apply them only at pre-scheduled firing times $\mathcal{T}_{p}$. Acceleration commands are provided as:

$$
\mathbf{a}(t)= \begin{cases}\frac{\Delta \mathbf{v}_{S}(t)}{\Delta t_{p}}, & \text { for } t \in \mathcal{T}_{p}  \tag{2.28}\\ 0, & \text { otherwise }\end{cases}
$$

where $\Delta t_{p}$ is the duration of the correction and $\mathcal{T}_{p}$ is the set of pre-scheduled times. This method is designated predictive navigation and it is assessed in [GFPC08] and detailed in [HPW10].

If full information on the relative position and velocity vectors is available, then the guidance is known as Predictive Impulsive (PI) and, recalling Eq. (2.5), the velocity correction to be applied is:

$$
\begin{equation*}
\Delta \mathbf{v}_{S}(t)=V_{c}(t) \breve{\boldsymbol{\Lambda}}\left(t_{f}\right)-\mathbf{v}(t)=V_{c}(t) \frac{\breve{\mathbf{r}}\left(t_{f}\right)}{\breve{r}\left(t_{f}\right)}-\mathbf{v}(t) \tag{2.29}
\end{equation*}
$$

where the superscript ${ }$ indicates a predicted term. The term $\breve{\mathbf{r}}\left(t_{f}\right)$ is the predicted relative end-of-mission position, propagated from time $t$ through a linearised time-varying STM yielding:

$$
\begin{equation*}
\breve{\mathbf{r}}\left(t_{f}\right) \approx\left(\mathbb{I}+\frac{1}{2} G(t) t_{\mathrm{go}}^{2}(t)\right) \mathbf{r}(t) \tag{2.30}
\end{equation*}
$$

where $G(t)$ is the gravity Jacobian along the reference orbit [Bat87]:

$$
\begin{equation*}
G(t)=\left.\frac{\partial \mathbf{g}(\mathbf{r}, t)}{\partial \mathbf{r}(t)}\right|_{\mathbf{r}(t)=\mathbf{r}_{\mathrm{ref}}(t)} \tag{2.31}
\end{equation*}
$$

In the case that measurements or estimates of the relative states are not available, then the kinematics of the system must be estimated using for example optical LOS information:

$$
\begin{align*}
\mathbf{r}(t) & =V_{c}(t) t_{\mathrm{go}}(t) \boldsymbol{\Lambda}(t) \\
\mathbf{v}(t) & =V_{c}(t) t_{\mathrm{go}}(t) \dot{\boldsymbol{\Lambda}}(t)+V_{c}(t) \boldsymbol{\Lambda}(t) \tag{2.32}
\end{align*}
$$

In this case, the guidance law is known as Kinematic Impulsive (KI) and it is obtained by substituting Eq. (2.32) into the velocity correction of Eq. (2.29):

$$
\begin{equation*}
\Delta \mathbf{v}_{S}(t)=V_{c}(t) \frac{\breve{\mathbf{r}}\left(t_{f}\right)}{\breve{r}\left(t_{f}\right)}-V_{c}(t)\left(t_{\mathrm{go}}(t) \dot{\boldsymbol{\Lambda}}(t)+\boldsymbol{\Lambda}(t)\right) \tag{2.33}
\end{equation*}
$$

as well as into the predicted end-of-mission position, Eq. (2.30):

$$
\begin{equation*}
\breve{\mathbf{r}}\left(t_{f}\right)=V_{c}(t) t_{\mathrm{go}}(t)\left(\mathbb{I}+\frac{1}{2} G(t) t_{\mathrm{go}}^{2}(t)\right) \boldsymbol{\Lambda}(t) \tag{2.34}
\end{equation*}
$$

With respect to the firing times $\mathcal{T}_{p}$ of the thrusters, earlier firings will lead to less propellant consumption because of the propagated corrective effect - even if the sensed information
is less accurate when the spacecraft is away from the target. On the other hand, in these cases, performance tends to degrade near the end-of-mission as further gravity linearisations cause increasing approximation errors. For this reason, a hybrid guidance scheme is typically adopted, which implements mid-course predictive corrections and then switches to terminal PNG [GFPC08].

## Optimal without path constraints

Most of the work on guidance laws for small bodies recasts the problem as optimal feedback control, resulting in a class known as Optimal Guidance Laws. This problem can be solved using either the Pontryagin maximum principle [Bat87] or calculus of variations [D'S97], although these derivations assume $\mathbf{r}_{f}=\mathbf{v}_{f}=0$. OGLs have been generalised in [HGW11] as described in this subsection.

The objective of the optimal control problem is to find the acceleration profile $\mathbf{a}(t)$ that minimises the actuation effort, formulated as the cost function:

$$
\begin{equation*}
J(\mathbf{a}(t))=\int_{t}^{t_{f}} L(\mathbf{x}(\tau), \mathbf{a}(\tau)) \mathrm{d} \tau=\int_{t}^{t_{f}} \frac{1}{2} \mathbf{a}^{\mathrm{T}}(\tau) \mathbf{a}(\tau) \mathrm{d} \tau \tag{2.35}
\end{equation*}
$$

subject to the dynamics of the system in Eq. (2.3):

$$
\dot{\mathbf{x}}(t)=\mathbf{f}(\mathbf{x}(t), \mathbf{a}(t)) \quad \Leftrightarrow\left[\begin{array}{c}
\dot{\mathbf{r}}(t)  \tag{2.36}\\
\dot{\mathbf{v}}(t)
\end{array}\right]=\left[\begin{array}{c}
\mathbf{v}(t) \\
\mathbf{g}\left(\mathbf{r}_{T}, \mathbf{r}_{S}\right)+\mathbf{a}(t)
\end{array}\right]
$$

and the terminal boundary conditions introduced in Sec. 2.1.1.
In order to solve this problem, it is convenient to define position and velocity co-states, $\mathbf{p}_{r}(t) \in \mathbb{R}^{3}$ and $\mathbf{p}_{v}(t) \in \mathbb{R}^{3}$, which are described by:

$$
\begin{align*}
{\left[\begin{array}{cc}
\dot{\mathbf{p}}_{r}(t) & \dot{\mathbf{p}}_{v}(t)
\end{array}\right] } & =-\left[\begin{array}{ll}
\mathbf{p}_{r}(t) & \mathbf{p}_{v}(t)
\end{array}\right] \frac{\partial \mathbf{f}(\mathbf{x}(t), \mathbf{a}(t))}{\partial \mathbf{x}(t)}+\frac{\partial L(\mathbf{x}(t), \mathbf{a}(t))}{\partial \mathbf{x}(t)}=  \tag{2.37}\\
& =-\left[\begin{array}{ll}
0 & \mathbf{p}_{r}(t)
\end{array}\right]
\end{align*}
$$

These are then solved via integration until the terminal co-states are obtained:

$$
\begin{align*}
& \mathbf{p}_{r}(t)=\mathbf{p}_{r}\left(t_{f}\right)  \tag{2.38}\\
& \mathbf{p}_{v}(t)=\mathbf{p}_{v}\left(t_{f}\right)+\mathbf{p}_{r}\left(t_{f}\right) t_{\mathrm{go}}(t)
\end{align*}
$$

In addition, defining the Hamiltonian function $H(\mathbf{x}(t), \mathbf{a}(t))$ as:

$$
\begin{align*}
H(\mathbf{x}(t), \mathbf{a}(t)) & =\left[\begin{array}{ll}
\mathbf{p}_{r}(t) & \mathbf{p}_{v}(t)
\end{array}\right] \mathbf{f}(\mathbf{x}(t), \mathbf{a}(t))-L(\mathbf{x}(t), \mathbf{a}(t))= \\
& =\left[\begin{array}{ll}
\mathbf{p}_{r}\left(t_{f}\right) & \mathbf{p}_{v}\left(t_{f}\right)+\mathbf{p}_{r}\left(t_{f}\right) t_{\mathrm{go}}(t)
\end{array}\right]\left[\begin{array}{c}
\mathbf{v}(t) \\
\mathbf{g}\left(\mathbf{r}_{T}, \mathbf{r}_{S}\right)+\mathbf{a}(t)
\end{array}\right]-\frac{1}{2} \mathbf{a}^{\mathrm{T}}(t) \mathbf{a}(t) \tag{2.39}
\end{align*}
$$

the Pontryagin maximum principle states that the optimal acceleration profile, which minimises $J(\mathbf{a}(t))$ in Eq. (2.35), is the one that cancels out the Hamiltonian derivative:

$$
\begin{equation*}
\left.\frac{\partial H(\mathbf{x}(t), \mathbf{a}(t))}{\partial \mathbf{a}(t)}\right|_{\text {optimal }}=0 \Rightarrow \mathbf{a}(t)=\mathbf{p}_{v}\left(t_{f}\right)+\mathbf{p}_{r}\left(t_{f}\right) t_{\mathrm{go}}(t) \tag{2.40}
\end{equation*}
$$

From Eq. (2.40), it is observed that the optimal acceleration law is a linear function of the time-to-go (and thus of $t$ ) that depends only on the terminal co-states. These are determined from the terminal states by integrating the acceleration twice, exactly as was done in Sec. 2.1.4.1.

To obtain a closed-form solution, a constant gravity field is assumed, yielding the expression for the so-called Constrained Terminal Velocity Guidance (CTVG) law:

$$
\begin{equation*}
\mathbf{a}(t)=6 \frac{\mathbf{r}_{f}-\left[\mathbf{r}(t)+\mathbf{v}_{f} t_{\mathrm{go}}(t)\right]}{t_{\mathrm{go}}^{2}(t)}+4 \frac{\mathbf{v}_{f}-\mathbf{v}(t)}{t_{\mathrm{go}}(t)}-\mathbf{g} \tag{2.41}
\end{equation*}
$$

For the case when the terminal velocity is unconstrained (e.g. no soft landing is required), the corresponding co-state is zero, $\mathbf{p}_{v}\left(t_{f}\right)=0$, and the so-called Free Terminal Velocity Guidance (FTVG) solution can be found:

$$
\begin{equation*}
\mathbf{a}(t)=3 \frac{\mathbf{r}_{f}-\mathbf{r}(t)}{t_{\mathrm{go}}^{2}(t)}-3 \frac{\mathbf{v}(t)}{t_{\mathrm{go}}(t)}-\frac{3}{2} \mathbf{g} \tag{2.42}
\end{equation*}
$$

Furthermore, if the zero-effort equation (2.10) is analytically integrated for a constant gravity field, the CTVG and FTVG laws can be re-written as:

$$
\begin{align*}
\mathbf{a}(t) & =\frac{6}{t_{\mathrm{go}}^{2}(t)} \mathbf{Z E M}(t)-\frac{2}{t_{\mathrm{go}}(t)} \mathbf{Z E V}(t)  \tag{2.43}\\
\mathbf{a}(t) & =\frac{3}{t_{\mathrm{go}}^{2}(t)} \mathbf{Z E M}(t) \tag{2.44}
\end{align*}
$$

Comparing Eq. (2.26) and (2.44), it is concluded that FTVG corresponds to the APNG law for a navigation ratio $n=3$, which means that this represents the optimal value of proportional laws. In addition, writing the CTVG and FTVG laws generically as a function of ZEM and ZEV avoids the constant gravity assumption at the expense of requiring a more accurate integration of the zero-effort errors. If such an integration is still too complex, a candidate alternative is to perform the computation with respect to a set of intermediate waypoints interpolated on a reference trajectory.

Note also that OGL laws allow to define state constraints only at terminal conditions, but not during the manoeuvre itself (path constraints). Also, they do not directly impose any kind of control constraints. This shortcoming is the main motivation for the constrained guidance strategies introduced in Sec. 2.1.4.1. Nevertheless, several developments of the OGLs can be found in the literature to indirectly account for specific types of constraints:

- In [EBR08], an OGL is derived for a fixed-interval guidance manoeuvre, with a continuous firing that burns-out several seconds before touching down. It is then called Optimal Fixed-Interval Guidance Law (OFIGL).
- In [HW11], an optimal law to control the impact (or glide-slope) angle is obtained by leaving the terminal velocity in the desired direction free and constraining the remaining components to zero. This law is called Intercept-Angle-Control Guidance (IACG) and it is shown to yield the BPNG law of Eq. (2.27) for the case of planar motion.
- In [GHW12], a methodology to cope with thrust and power limited engines is proposed combining the OGLs with the generation of intermediate optimal waypoints.
- In [GHW13], the case when the landing site is not specified is tackled by indirectly incorporating the constraint $\mathbf{r}_{f}^{\mathrm{T}} \mathbf{r}_{f}=R_{T}^{2}$ as a weighted index in the cost function $J(\mathbf{a}(t))$, where $R_{T}$ is the radius of the target.
- In [ZX14], a weighted term is added to the cost function $J(\mathbf{a}(t))$ to penalise trajectories that go under a certain altitude with respect to the target and even below its surface (subsurface flight).


## Nonlinear robust

In opposition to the guidance methods presented in Sec. 2.1.4.1, the techniques from Sec. 2.1.4.2 have relied on closed-loop feedback control based on the assumption that the parameters on which they depend are fully known. But, in practice, inaccurate measurements or unmodelled dynamics will affect the results negatively. To address this issue, in [EBR08] it is proposed to augment the energy-optimal laws with advancements in the field of nonlinear control. This augmentation is rooted on nonlinear Sliding Mode Control theory [Lev07, SEFL14] and results in guidance algorithms that are globally stable in uncertain dynamical environments for which an upper bound of the perturbing acceleration $\mathbf{p}(t)$ is known. These algorithms are named Optimal Sliding Guidance (OSG) and are further developed in [FSCC11, FGWS13].

OSG laws are derived by defining a sliding surface as a linear combination of tracking (zero-effort) errors:

$$
\begin{equation*}
\mathbf{s}(t)=\mathbf{Z E V}(t)+\tilde{\lambda}(t) \mathbf{Z E M}(t) \tag{2.45}
\end{equation*}
$$

where $\tilde{\lambda}(t)>0 \forall t \in\left[t_{0}, t_{f}\right]$. The dynamics of this sliding surface is given by:

$$
\begin{equation*}
\dot{\mathbf{s}}(t)=\mathbf{Z} \dot{\mathbf{E}} \mathbf{V}(t)+\tilde{\lambda}(t) \mathbf{Z \dot { \mathbf { E } } \mathbf { M }}(t)=-\mathbf{a}(t)-\tilde{\lambda}(t) \operatorname{tgo}_{\mathrm{go}}(t) \mathbf{a}(t) \tag{2.46}
\end{equation*}
$$

This results in the guidance law re-constructed in such a way that the system is always asymptotically driven to the sliding surface, i.e. $\mathbf{s}(t) \rightarrow 0$. The rate of convergence corresponds to $\mathrm{e}^{-\tilde{\lambda}(t) t}$ and, when the sliding surface is reached, the system is said to be in sliding phase. The new guidance law can be made as simple as a switching between two states (back or forth towards the sliding surface) and therefore very robust. It is constructed using Lyapunov's direct method by selecting $V(\mathbf{s})=\frac{1}{2} \mathbf{s}^{\mathrm{T}}(t) \mathbf{s}(t)$ as the candidate Lyapunov function with $V(0)=0$ and $V(\mathbf{s})>0$ for $\mathbf{s}(t)>0$, as well as imposing:

$$
\begin{equation*}
\dot{V}(\mathbf{s})=\mathbf{s}^{\mathrm{T}}(t) \dot{\mathbf{s}}(t) \leq 0 \tag{2.47}
\end{equation*}
$$

For the CTVG case, substituting the optimal acceleration of Eq. (2.43) into (2.46) gives:

$$
\begin{equation*}
\dot{\mathbf{s}}(t)=-\frac{1}{t_{\mathrm{go}}(t)} K \mathbf{s}(t) \tag{2.48}
\end{equation*}
$$

where $K=4$. The acceleration input can be augmented, for example, simply as:

$$
\begin{equation*}
\mathbf{a}(t)=\frac{6}{t_{\mathrm{go}}^{2}(t)} \mathbf{Z} \mathbf{E M}(t)-\frac{2}{t_{\mathrm{go}}(t)} \mathbf{Z} \mathbf{E V}(t)-\frac{\phi}{t_{\mathrm{go}}(t)} \operatorname{sign} \mathbf{s}(t) \tag{2.49}
\end{equation*}
$$

which gives the following $\dot{\mathbf{s}}(t)$, satisfying now Eq. (2.47):

$$
\begin{equation*}
\dot{\mathbf{s}}(t)=-\frac{1}{t_{\mathrm{go}}(t)}[K \mathbf{s}(t)+\phi \operatorname{sign} \mathbf{s}(t)] \tag{2.50}
\end{equation*}
$$

Finally, using Lyapunov's second method [FSCC11, FGWS13] it can be proven that the algorithm is globally stable when $\phi \geq\|\mathbf{p}(t)\| \forall t \in\left[t_{0}, t_{f}\right]$. Moreover, the nonlinear term provides an additional acceleration command, but only in off-nominal cases. In these cases, the motion is brought inside the sliding surface at the expense of a larger control effort.

The OSG concept can also be extended to the FTVG cases by defining:

$$
\begin{align*}
\mathbf{s}(t) & =\mathbf{Z E M}(t)  \tag{2.51}\\
\dot{\mathbf{s}}(t) & =-t_{\mathrm{go}}(t) \mathbf{a}(t)=-\frac{1}{t_{\mathrm{go}}(t)} K \mathbf{s}(t) \tag{2.52}
\end{align*}
$$

with $K=3$ and modifying the acceleration law of Eq. (2.44) to:

$$
\begin{equation*}
\mathbf{a}(t)=\frac{3}{t_{\mathrm{go}_{0}}^{2}(t)} \mathbf{Z E M}(t)-\frac{\phi}{t_{\mathrm{go}}(t)} \operatorname{sign} \mathbf{s}(t) \tag{2.53}
\end{equation*}
$$

The main shortcoming of Eq. (2.49) and (2.53) is that the augmentation with the discontinuous term $\operatorname{sign} \mathbf{s}(t)$ can degenerate in the system chattering around the sliding surface, which massively reduces its performance. To overcome this phenomenon, continuous chattering-free augmentations known as higher-order sliding controllers have been presented over the last years [FCW13, FWGS15]. Additional research has also been carried out in the SMC field with its application to increase the robustness of open-loop guidance laws [OS08, OS09].

### 2.1.4.3 Computational closed-loop guidance

With computational guidance, solutions that minimise Eq. (2.35) while meeting desired state and control constraints are computed in real-time. Two classes of algorithms are available to achieve this in an efficient manner: convex optimisation and pseudospectral methods.

## Convex optimisation

Convex optimisation-based guidance entails solving a reformulation of the constrained fueloptimal trajectory generation problem in the convex programming framework. The main difficulty lies in the process of converting non-convex state and control constraints into the convex form, which is known as lossless convexification.

Lossless convexification (see [AP05, AP07, BAS10] and references therein) is a procedure that can be used to relax non-convex constraints into a convex form, and then prove the
equivalence of the resulting optimal control problem. This convex formulation also facilitates the characterisation of controllable and reachable conditions [EDA15, DRA16] (e.g. how far can the vehicle perform a soft landing given the fuel available).

The convex problem, once discretised, becomes a Second-Order Cone Programming (SOCP), for which powerful interior-point solvers exist [NN94, Stu02] with guaranteed convergence to the optimal solution of the original problem within a finite number of iterations. This approach, where a fuel-optimal control input is computed based on the predicted trajectory, enables tackling the D\&L problem in a Model Predictive Control (MPC) setting [PBB15, EPK ${ }^{+} 17$, WCW19].

There are two sources of non-convexity that are inherent to the trajectory-generation problem: [i] the propellant-depletion dynamics, which has an exponential dependence on time as evidenced by Eq. (2.14), and [ii] thrust magnitude and pointing constraints, due to the norm operator in Eq. (2.13). As introduced in [AP07, BAS10], the first one is directly convexified using the following change of coordinates:

$$
\begin{equation*}
z(t)=\ln m(t), \quad \dot{z}(t)=\frac{\dot{m}(t)}{m(t)} \tag{2.54}
\end{equation*}
$$

while the latter requires the introduction of an additional optimisation constraint defined using two new variables:

$$
\begin{equation*}
\mathbf{a}(t)=\frac{\mathbf{T}_{\mathrm{CVX}}(t)}{m(t)}, \quad \sigma(t)=\frac{\left\|\mathbf{T}_{\mathrm{CVX}}(t)\right\|}{m(t)} \tag{2.55}
\end{equation*}
$$

To solve the $\mathrm{D} \& \mathrm{~L}$ problem, the optimisation variables are discretised into $N$ points. The objective is then to find a discrete thrust acceleration profile $\mathbf{a}[k](k \in[1, \cdots, N])$ that minimises the vehicle's fuel consumption, which is equivalent to maximising its final mass or $z[N]$. This specific problem is well established based on [SABJH16, JMBS17] and formally defined as:

$$
\begin{equation*}
\min _{\mathbf{a}, \sigma} \sum_{k=1}^{N} \sigma[k]=\max _{\mathbf{a}, \sigma} z[N] \tag{2.56}
\end{equation*}
$$

Its main constraints are described in the following paragraphs.
Similar to CTVG, the optimisation problem is subject to initial and final boundary states at $k=1$ and $k=N$, respectively. The former specifies the current mass, position, velocity and thrust acceleration, i.e. $z[1]=\ln m(t), \mathbf{r}[1]=\mathbf{r}(t), \mathbf{v}[1]=\mathbf{v}(t)$ and $\mathbf{a}[1]=\mathbf{a}(t)$, while the latter specifies the conditions at touchdown. In this case, the final position and velocity must coincide with the desired values, $\mathbf{r}[N]=\mathbf{r}_{f}$ and $\mathbf{v}[N]=\mathbf{v}_{f}$, and the final thrust acceleration vector is required to have a positive vertical component only, $\mathbf{a}_{x, y}[N]=0_{2 \times 1}$ and $\mathbf{a}_{z}[N] \geq 0$, so that the vehicle lands with an upright orientation.

The optimisation problem includes also the dynamics equations that dictate the timeevolution of the aforementioned states. These equations are discretised using time-interval $T_{\mathrm{S}}$ and the knowledge that acceleration is linearly interpolated between two consecutive points.

More precisely, the discrete motion of Eq. (2.3) is given by:

$$
\begin{align*}
\mathbf{r}[k+1] & =\mathbf{r}[k]+T_{\mathrm{S}} \mathbf{v}[k]+\frac{T_{\mathrm{S}}^{2}}{3}\left(\mathbf{w}[k]+\frac{\mathbf{w}[k+1]}{2}\right)  \tag{2.57}\\
\mathbf{v}[k+1] & =\mathbf{v}[k]+\frac{T_{\mathrm{S}}}{2}(\mathbf{w}[k]+\mathbf{w}[k+1])
\end{align*}
$$

and the mass-depletion dynamics of Eq. (2.14), following the change of variables of Eq. (2.54) and (2.55), becomes:

$$
\begin{equation*}
z[k+1]=z[k]-\frac{1}{I_{\mathrm{sp}} g_{0}} \frac{T_{\mathrm{S}}}{2}(\sigma[k]+\sigma[k+1]) \tag{2.58}
\end{equation*}
$$

A set of auxiliary variables $\mathbf{w}[k]$ and $\sigma[k]$ are then defined. The former variable gathers the vehicle's acceleration contributions:

$$
\begin{equation*}
\mathbf{w}[k]=\mathbf{a}[k]+\mathbf{g}(t) \tag{2.59}
\end{equation*}
$$

where $\mathbf{a}[k]$ is given by Eq. (2.55) and the gravity acceleration $\mathbf{g}(t)$ is assumed constant from time $t$ onwards. Also, the inequality:

$$
\begin{equation*}
\|\mathbf{a}[k]\| \leq \sigma[k] \tag{2.60}
\end{equation*}
$$

is introduced as part of lossless convexification procedure [AP07, BAS10], with $\sigma[k] \rightarrow\|\mathbf{a}[k]\|$ when $z[N]$ is maximised.

In addition, control constraints are employed to bound the direction and magnitude of the thrust force. The direction constraint indirectly limits the angle between the vehicle's longitudinal axis and the vertical direction to $\theta_{\max }$ via:

$$
\begin{equation*}
\mathbf{a}_{z}[k] \geq \frac{\left\|\mathbf{a}_{x, y}[k]\right\|}{\tan \theta_{\max }} \tag{2.61}
\end{equation*}
$$

Lower and upper thrust magnitude limits are given by $\left\{T_{\min }, T_{\max }\right\}$. To preserve convexity, constant mass $m(t)$ is assumed from $t$ onwards, which is not restrictive since the actual limit values can be re-adjusted, and the constraint becomes:

$$
\begin{equation*}
\frac{T_{\mathrm{min}}}{m(t)} \leq \sigma[k] \leq \frac{T_{\mathrm{max}}}{m(t)} \tag{2.62}
\end{equation*}
$$

The optimisation problem subject to the constraints introduced above is then formulated in SOCP 2.1 (see next page). The inclusion of additional constraints in this problem is straightforward if they are convex (e.g. subsurface flight avoidance and glideslope limits) or if they can be converted via lossless convexification. If not, a technique known as successive convexification can be applied to approximate any remaining nonlinearities.

Successive convexification (see [LL14, MSA16] and references therein) constitutes an iterative process in which non-convex terms are sequentially linearised using information from the previous SOCP solution. To facilitate convergence, a Trust Region Constraint (TRC) is typically imposed between consecutive iterations.

## SOCP 2.1

$\max _{\mathbf{a}, \sigma} z[N], \quad$ subject to:
Boundary conditions

$$
\begin{array}{ll}
z[1]=\ln m(t), & \mathbf{r}[1]=\mathbf{r}(t), \\
\mathbf{v}[1]=\mathbf{v}(t), & \mathbf{a}[1]=\mathbf{a}(t) \\
\mathbf{r}[N]=\mathbf{r}_{f}, & \mathbf{v}[N]=\mathbf{v}_{f}, \\
\mathbf{a}_{x, y}[N]=0_{2 \times 1}, & \mathbf{a}_{z}[N] \geq 0
\end{array}
$$

$$
\text { Dynamics equations, } \quad \forall k \in[1, \cdots, N-1]
$$

$$
\mathbf{r}[k+1]=\mathbf{r}[k]+T_{\mathrm{S}} \mathbf{v}[k]+\frac{T_{\mathrm{S}}^{2}}{3}\left(\mathbf{w}[k]+\frac{\mathbf{w}[k+1]}{2}\right)
$$

$$
\mathbf{v}[k+1]=\mathbf{v}[k]+\frac{T_{\mathrm{S}}}{2}(\mathbf{w}[k]+\mathbf{w}[k+1])
$$

$$
z[k+1]=z[k]-\frac{1}{I_{\mathrm{sp}} g_{0}} \frac{T_{\mathrm{S}}}{2}(\sigma[k]+\sigma[k+1])
$$

$$
\text { Surrogate variables, } \quad \forall k \in[1, \cdots, N]
$$

$$
\mathbf{w}[k]=\mathbf{a}[k]+\mathbf{g}(t)
$$

$$
\|\mathbf{a}[k]\| \leq \sigma[k]
$$

Control constraints, $\quad \forall k \in[1, \cdots, N-1]$
$\mathbf{a}_{z}[k] \geq \frac{\left\|\mathbf{a}_{x, y}[k]\right\|}{\tan \theta_{\max }}$

$$
\frac{T_{\min }}{m(t)} \leq \sigma[k] \leq \frac{T_{\max }}{m(t)}
$$

This technique allows to handle certain features that, depending on the target body, may play a critical role for D\&L, such as high-order gravitational harmonics, nonlinear aerodynamic forces, non-convex keep-out zones and also free final-time. This will be further exploited in the second part of the thesis.

Moreover, this approach effectively enables extending the 3 degrees-of-freedom (3-DoF) problem to 6-DoF by incorporating attitude kinematics and decoupling the thrust vector from the attitude of the spacecraft [SEA17, SA18]. An alternative approach was proposed in [LM15, LM17] based on Piece-Wise Affine (PWA) approximations of the nonlinear 6-DoF dynamics and an MPC formulation with dual quaternions. Its main shortcoming is that, since it relies on PWA approximations, the resolution of the discretised dynamics has a drastic impact on the quality of the obtained solutions.

## Pseudospectral methods

An alternative approach to convex optimisation arose with the development of pseudospectral optimal control in [FR08]. Pseudospectral methods allow to transform the infinite-dimensional problem of minimising Eq. (2.35) with state and control constraints into a discrete, finitedimensional NonLinear Programming (NLP) problem, which can be solved using several off-
the-shelf solvers. These methods are particularly suitable for aerospace applications due to the guaranteed spectral (i.e. quasi-exponential) convergence of their solution for smooth problems.

With this approach, the cost function, differential equations and constraints are approximated by being defined at a set of discretisation nodes (known as collocation points) and treated as a set of algebraic constraints, in a process called transcription. To do so, the physical domain of a variable $t \in\left[t_{0}, t_{f}\right]$ is converted into a normalised independent variable $\tau \in[-1,1]$ through the following affine transformation:

$$
\begin{equation*}
\tau=\frac{2}{t_{f}-t_{0}} t-\frac{t_{f}+t_{0}}{t_{f}-t_{0}} \tag{2.63}
\end{equation*}
$$

Different methods can then be employed to compute the location of the collocation points, with one of the simplest corresponding to the roots of a linear combination of Legendre polynomials of order $n$ and $n-1$ as follows:

$$
\begin{equation*}
R_{n}(\tau)=P_{n}(\tau)-P_{n-1}(\tau), \quad \tau \in[-1,1] \tag{2.64}
\end{equation*}
$$

with:

$$
\begin{equation*}
P_{n}(\tau)=\frac{1}{2^{n} n!} \frac{\mathrm{d}^{n}}{\mathrm{~d} \tau^{n}}\left(\tau^{2}-1\right)^{n} \tag{2.65}
\end{equation*}
$$

generating non-uniform grids, where a smaller number of nodes is required to compute a valid solution. Once the domain is discretised, pseudospectral operators for differentiation and integration are also defined. Thanks to the classes of polynomials involved, these operators are more accurate than the standard finite differences for differentiation and trapezoidal rule for integration. Application examples using more sophisticated algorithms, such as the flipped Radau pseudospectral method, can be found in $\left[\mathrm{AOS}^{+} 14\right]$ and [STDB17] for lunar landing and Earth re-entry guidance, respectively.

It is important to note that the quality of the obtained solution is strongly dependent on the Jacobian matrix generated from the transcription process. In [ST13], it is shown how the inherent sparseness of that matrix (due to non-dependencies between states) can be exploited for faster and improved results. Nonetheless, even with a proper transcription, poor scaling of the Jacobian can still lead to numerical difficulties.

Once again, different strategies exist, ranging from ad hoc manual scaling, available with widespread pseudospectral tools like DIDO [Eli15], to more sophisticated self-scaling methods such as Jacobian rows normalisation [Rao09] or projected Jacobian rows normalisation [Sag14].

The real-time implementation of pseudospectral optimisation remains a challenging issue because of the NLP problem involved. To tackle this issue, the hybridization of pseudospectral methods and convex optimisation was recently proposed in [Sag18]. The idea is to combine the more accurate distribution of nodes and pseudospectral operators with the more efficient formulation and computation of SOCP problems. In that reference, this method was shown to remain real-time capable, while providing improved results compared to the standard convex approach.

### 2.1.4.4 Parametric guidance

The open-loop and traditional closed-loop laws addressed in this chapter are summarised in Table 2.1. Although being directed at the same planetary $D \& L$ problem, they present quite different properties as highlighted throughout Sec. 2.1.4.1 and 2.1.4.2.

Table 2.1: Open-loop and traditional closed-loop D\&L guidance laws

| Proportional |  |  |
| :---: | :---: | :---: |
| PNG | $\mathbf{a}(t)=n V_{c}(t) \dot{\boldsymbol{\Lambda}}(t)$ | Eq. (2.22) |
| APNG | $\mathbf{a}(t)=\frac{n}{t_{\mathrm{go}}^{2}(t)} \mathbf{Z E M}(t)$ | Eq. (2.26) |
| Representing the laws as a function of ZEM and ZEV allows to concentrate the knowledge of gravitational forces in these terms |  |  |
| Predictive and hybrid |  |  |
|  | $\Delta \mathbf{v}_{S}(t)=V_{c}(t) \frac{\breve{\mathbf{r}}\left(t_{f}\right)}{\breve{r}\left(t_{f}\right)}-\mathbf{v}(t)$ | Eq. (2.29) |
| KI | $\Delta \mathbf{v}_{S}(t)=V_{c}(t) \frac{\breve{\mathbf{r}}\left(t_{f}\right)}{\breve{r}\left(t_{f}\right)}-V_{c}(t)\left(t_{\mathrm{go}}(t) \dot{\boldsymbol{\Lambda}}(t)+\boldsymbol{\Lambda}(t)\right)$ | Eq. (2.33) |

Pre-scheduled firings are commanded to generate the required $\Delta \mathbf{v}_{S}(t)$ and correct the predicted end-of-mission position $\breve{\mathbf{r}}\left(t_{f}\right)$

## Quadratic

Apollo

$$
\begin{equation*}
\mathbf{a}(t)=\mathbf{C}_{0}+\mathbf{C}_{1} t+\mathbf{C}_{2} t^{2} \tag{2.15}
\end{equation*}
$$

Open-loop trajectory with closed-form solution as a function of terminal conditions

## Optimal without path constraints

CTVG $\quad \mathbf{a}(t)=\frac{6}{t_{\mathrm{go}}^{2}(t)} \mathbf{Z E M}(t)-\frac{2}{t_{\mathrm{go}}(t)} \mathbf{Z E V}(t)$
FTVG $\quad \mathbf{a}(t)=\frac{3}{t_{\mathrm{go}}^{2}(t)} \mathbf{Z E M}(t)$
Specific constraints may be taken into account but not explicitly enforced, e.g. OFIGL [EBR08] and IACG [HW11]

## Optimal with path constraints

Polynomial $\quad \mathbf{a}(t)=\mathbf{C}_{0}+\mathbf{C}_{1} t+\ldots+\mathbf{C}_{N} t^{N}, \quad N>2$
Open-loop trajectory with optimisation-based solution and possibility to explicitly enforce path constraints
Nonlinear robust
Sliding CTVG $\quad \mathbf{a}(t)=\frac{6}{t_{\mathrm{go}_{0}^{2}}^{2}(t)} \mathbf{Z E M}(t)-\frac{2}{t_{\mathrm{go}}(t)} \mathbf{Z E V}(t)-\frac{\phi}{t_{\mathrm{go}}(t)} \operatorname{sign} \mathbf{s}(t)$
Eq. (2.49)
Sliding FTVG $\quad \mathbf{a}(t)=\frac{3}{t_{\mathrm{go}}^{2}(t)} \mathbf{Z E M}(t)-\frac{\phi}{t_{\mathrm{go}}(t)} \operatorname{sign} \mathbf{s}(t)$
Eq. (2.53)
Augmentation with nonlinear function, ensuring stability for $\|\mathbf{p}(t)\| \leq \phi$

Despite their differences, it is observed that traditional closed-loop guidance laws share structural commonalities and can be formalised in terms of LOS kinematics as follows:

$$
\mathbf{a}(t)=\left[\begin{array}{ll}
k_{r} & k_{v}
\end{array}\right] V_{c}(t)\left[\begin{array}{c}
\frac{\boldsymbol{\Lambda}(t)}{t_{\mathrm{go}}(t)}  \tag{2.66}\\
\dot{\boldsymbol{\Lambda}}(t)
\end{array}\right]-\phi \mathbf{h}\left(\boldsymbol{\Lambda}(t), \dot{\boldsymbol{\Lambda}}(t), t_{\mathrm{go}}(t)\right)
$$

Alternatively, they can also be parameterised as a function of zero-effort errors:

$$
\mathbf{a}(t)=\left[\begin{array}{ll}
k_{r} & k_{v}
\end{array}\right]\left[\begin{array}{c}
\frac{\mathbf{Z E M}(t)}{t_{\mathrm{go}}^{2}(t)}  \tag{2.67}\\
\frac{\mathbf{Z E V}(t)}{t_{\mathrm{go}}(t)}
\end{array}\right]-\phi \mathbf{h}\left(\mathbf{Z E M}(t), \mathbf{Z E V}(t), t_{\mathrm{go}}(t)\right)
$$

The two equations above, i.e. Eq. (2.66) and (2.67), clearly show a fixed structure formed by a linear component (parameterised through the gains $k_{r}$ and $k_{v}$ ) and (optionally) by a nonlinear function $\mathbf{h}($.$) (weighted by the constant \phi$ ). As mentioned before, this nonlinear term is introduced to improve robustness properties following the concepts of sliding motion control and it can range from a very simple to a high-order function.

Throughout the first part of the thesis, it will be demonstrated that the two parametrisations presented are particularly convenient for the application of systematic tuning methods. It shall also be remarked that this generalisation does not encapsulate open-loop laws, although these are directly parameterised as time polynomials as in Eq. (2.20).

### 2.1.5 Impact of uncertainties

In Sec. 2.1.4.2, it was anticipated that pure optimal guidance laws are more sensitive to dynamical perturbations and operational uncertainties than their robust counterparts, such as those augmented with SMC. In order to showcase the importance of developing robust guidance laws, this section provides a simple illustration of the effect of uncertainties in planetary D\&L.

The scenario selected to exemplify these effects is taken from reference [HPW10]. It simulates a hyper-velocity interception mission of asteroid Apophis (which will make close approaches to planet Earth in 2029 and 2036) with the interception phase starting at perihelion roughly 1 day before impact. The available thrust force for control is assumed unlimited, but the actuation is switched-off 200 seconds before touch-down. In addition, gravity is assumed constant for ZEM and ZEV computations and all the sensor measurements are assumed ideal. The target asteroid is modelled as a uniform sphere with gravitational coefficient $\mu_{T}=15.35 \mathrm{~m}^{3} / \mathrm{s}^{2}$ and radius $R_{T}=500 \mathrm{~m}$. Note that without any guidance actuation, this scenario results in a miss distance of around $40,000 \mathrm{~km}$.

The main source of uncertainty in this type of scenario is known to lie on the gravitational force experienced by the spacecraft due to the irregular and inaccurately known mass distribution
of the asteroid and to the proneness of variable third-body perturbations. Thus, an uncertainty is introduced around the norm (not direction) of the spacecraft's nominal gravitational acceleration $\mathbf{g}_{S_{\text {NOM }}}\left(\mathbf{r}_{T}, \mathbf{r}_{S}\right)$ assuming a relative uncertainty level $w_{\mathrm{g}}=100 \%$ :

$$
\begin{equation*}
\mathbf{g}_{S}\left(\mathbf{r}_{T}, \mathbf{r}_{S}\right)=\mathbf{g}_{S_{\mathrm{NOM}}}\left(\mathbf{r}_{T}, \mathbf{r}_{S}\right)\left(1+w_{\mathrm{g}} \delta_{\mathrm{g}}\right) \tag{2.68}
\end{equation*}
$$

where $\delta_{\mathrm{g}} \in[-1,+1]$ represents the normalised uncertainty (with $\pm 1$ its maximum/minimum range and 0 its nominal value).

The above uncertainty representation follows the robust control modelling framework known as a Linear Fractional Transformation (LFT). As it will be thoroughly introduced in Sec. 2.2, these LFT models are the cornerstone of robust control as they allow capturing the known part of a system and/or variable (in this case $\mathbf{g}_{S_{\mathrm{NOM}}}$ ) and the uncertain component (in Eq. (2.68) the range of $\delta_{\mathrm{g}}$ ) in a manner amenable for subsequent control synthesis and analysis.

Simulations are depicted for a fixed set of 20 values of $\delta_{\mathrm{g}}$ for the CTVG law (terminal velocity constrained to $0 \mathrm{~m} / \mathrm{s}$, i.e. soft landing) in Fig. 2.4 and for the FTVG law (free terminal velocity) in Fig. 2.5. Both figures show the outcome of the two (pure) optimal laws (given by gray lines) against their SMC-augmented (robust) counterparts (given by red lines) - for the latter a constant $\phi=200$ is used. The same end-of-mission time is commanded for all the cases. The nominal responses, i.e. $\delta_{\mathrm{g}}=0$, are also shown (black lines). The main difference between the CTVG and the FTVG laws is related to the reduction of velocity towards $0 \mathrm{~m} / \mathrm{s}$ imposed by the former (Fig. 2.4b vs. 2.5b). As expected, this manoeuvre is more demanding in terms of control acceleration (Fig. 2.4d vs. 2.5 d ). Note that the linear dependence on time anticipated for the pure optimal laws from Eq. (2.40) is also seen.

For the pure optimal laws, it is also observed that off-nominal spacecraft's gravitational accelerations lead to significantly dispersed trajectories (see Fig. 2.4a and 2.5 a for the heliocentric frame, and Fig. 2.4c and 2.5c for the relative frame). This effect is therefore reflected into a degradation of the landing accuracy. Furthermore, the FTVG law, by employing only position information, Eq. (2.44), shows larger sensitivity to uncertainties, which even causes the spacecraft to miss the asteroid for $\left|\delta_{\mathrm{g}}\right| \geq 0.8$ (in other words, the robust laws improve against gravity uncertainty of at least $80 \%$ larger in norm-size). After augmenting the guidance schemes with SMC, and although the trajectory is still affected by the presence of uncertainties, its degradation is reduced to approximately half for both laws (Fig. 2.4c and 2.5c). Of course, this comes at the expense of higher actuation effort (Fig. 2.4d and 2.5d) in off-nominal cases.

This decrease in sensitivity and consequent increase in control effort is a critical trade-off often encountered in aerospace systems, in which better robustness properties are attained through a compromise in terms of optimality. As briefly mentioned before, the robust control community has particularly well-suited methods to address the aforementioned trade-offs in a systematic manner, while explicitly accounting for uncertainties and disturbances. Hence, this simple analysis serves also as a motivation for the application of those methods in this thesis.


Figure 2.4: Sensitivity of CTVG laws $\left(\mathbf{v}_{f}=0\right)$ to gravitational uncertainty


Figure 2.5: Sensitivity of FTVG laws to gravitational uncertainty. Dashed lines indicate missed interceptions.

### 2.2 Robust control techniques

The robust control framework provides a set of techniques that allow to manage issues such as the impact of uncertainties and specification of Multiple-Input Multiple-Output (MIMO) requirements in a systematic manner. As depicted in Fig. 2.6, these techniques cover all the aspects involved in control design: modelling, synthesis and analysis.


Figure 2.6: Robust control design workflow and tools

The first step in the robust control framework is to model the system, capturing the possible (or expected) uncertainties in a methodical manner. This modelling phase is followed by control synthesis, for which specialised tools are available. As indicated by the arrows in the figure, an acceptable solution is the result of an iterative cycle of control synthesis and analysis, to ensure that design requirements are always fulfilled. Moreover, analysis results also provide valuable insights that can be taken into account to improve the modeling phase.

The modelling, synthesis and analysis techniques employed throughout the thesis are summarised in Fig. 2.6 and reviewed between Sec. 2.2.1 and 2.2.3.

### 2.2.1 Modelling

Stability and performance characteristics of any real system are affected by many dynamical perturbations (uncertainties), ranging from modelling inaccuracies (both, epistemic and random) to external disturbances. Control systems are designed to work with a single nominal plant model, but a successful controller must function properly for all perturbations and operating variations (within a pre-defined set).

### 2.2.1.1 LFT modelling

To capture the effect of modelling uncertainty, a mathematical representation known as Linear Fractional Transformation (LFT) is typically employed in the robust control context. This type of representation is thoroughly described in [DPZ91, ZDG95] and briefly discussed in the following paragraphs.

As a cursory example, assume that a given parameter $x$ has a relative uncertainty range $w_{\mathrm{x}}$ around its nominal value $x_{\mathrm{NOM}}$, then it can be represented as:

$$
\begin{equation*}
x=x_{\mathrm{NOM}}\left(1+w_{\mathrm{x}} \delta_{\mathrm{x}}\right), \quad \delta_{\mathrm{x}} \in[-1,1] \tag{2.69}
\end{equation*}
$$

Following the LFT notation, the product of a signal $w$ with the uncertain parameter $x$ is conventionally represented as shown in Fig. 2.7 and written as:

$$
z=x w=\mathcal{F}_{\mathrm{u}}\{\underbrace{\left[\begin{array}{ll}
M_{11} & M_{12}  \tag{2.70}\\
M_{21} & M_{22}
\end{array}\right]}_{M}, \delta_{\mathrm{x}}\} w=\left(M_{22}+M_{21} \delta_{\mathrm{x}}\left(\mathbb{I}-M_{11} \delta_{\mathrm{x}}\right)^{-1} M_{12}\right) w
$$

where $\mathcal{F}_{\mathrm{u}}$ denotes the upper LFT operation, the nominal system $\left(x_{\mathrm{NOM}}=M_{22}\right)$ is retrieved for $\delta_{\mathrm{x}}=0$, and:

$$
M=\left[\begin{array}{cc}
0 & 1  \tag{2.71}\\
x_{\mathrm{NOM}} w_{\mathrm{x}} & x_{\mathrm{NOM}}
\end{array}\right]
$$



Figure 2.7: Upper LFT representation of an uncertain element
Conversely, a lower LFT operation is also defined as follows:

$$
\begin{equation*}
\mathcal{F}_{1}\left\{M, \delta_{\mathrm{x}}\right\}=M_{11}+M_{12} \delta_{\mathrm{x}}\left(\mathbb{I}-M_{22} \delta_{\mathrm{x}}\right)^{-1} M_{21} \tag{2.72}
\end{equation*}
$$

LFTs are particularly attractive due to their extreme modularity and because typical algebraic operations (e.g., inverse, cascade, parallel and feedback connections) preserve the LFT structure. Moreover, its elements are not limited to scalar parameters and can represent complex-valued matrices. Therefore, in an interconnected system, it is common to isolate what is known as a Linear Time-Invariant (LTI) system, i.e. $M_{22}$, and gather all the "troublemaking" (uncertain, time-varying or nonlinear) elements into a block $\Delta=\operatorname{diag}\left[\delta_{\mathrm{x}_{1}}, \delta_{\mathrm{x}_{2}}, \ldots, \delta_{\mathrm{x}_{\mathrm{n}}}\right]$ with normalised infinity norm, $\|\Delta\|_{\infty} \leq 1$. This block is known as structured uncertainty.

Nowadays, this modelling procedure can be automated thanks to software packages like MATLAB's Robust Control Toolbox [BCPS06] or SMAC [ $\left.\mathrm{BBD}^{+} 16\right]$ from ONERA, the French Aerospace Lab.

### 2.2.1.2 LPV modelling

In addition, when a system is not time-invariant, i.e. when its characteristics change over a certain trajectory (or time), a formalism known as Linear Parameter-Varying (LPV) can be employed to capture this behaviour. This formalism is introduced in the following paragraph, for further details the reader is referred to [Bal02, MB04, MVS $\left.{ }^{+} 10\right]$.

An LPV model of $M(\mathrm{~s})$ is a linear system whose state-space matrices $\{A, B, C, D\}$ are continuous functions of a measurable Time-Varying (TV) parameter vector $\rho_{\mathrm{TV}}(t)$ :

$$
\left[\begin{array}{l}
\dot{\mathbf{x}}(t)  \tag{2.73}\\
\mathbf{z}(t)
\end{array}\right]=\left[\begin{array}{ll}
A\left(\rho_{\mathrm{TV}}(t)\right) & B\left(\rho_{\mathrm{TV}}(t)\right) \\
C\left(\rho_{\mathrm{TV}}(t)\right) & D\left(\rho_{\mathrm{TV}}(t)\right)
\end{array}\right]\left[\begin{array}{c}
\mathbf{x}(t) \\
\mathbf{w}(t)
\end{array}\right], \quad \rho_{\mathrm{TV}}(t) \in \mathcal{P}
$$

where $\mathbf{x}(t)$ is the state vector, $\mathbf{w}(t)$ and $\mathbf{z}(t)$ are vectors of input and output signals, respectively, and $\mathcal{P} \subset \mathbb{R}^{n_{\rho}}$ is a known compact set.

Intuitively, to every time-frozen point of $\rho_{\mathrm{TV}}(t) \in \mathcal{P}$ corresponds an LTI realisation. Moreover, LPV models allow to establish a bridge between linear and nonlinear systems by embedding nonlinear terms (e.g. change of trajectory) in the TV parameter.

The analysis of LPV systems leads to conditions that involve the TV parameter and, if assumed Rate-Bounded (RB), its rate, i.e. $\left(\rho_{\mathrm{TV}}(t), \dot{\rho}_{\mathrm{TV}}(t)\right)$. In this case, a parametric description $(p, q) \in \mathcal{P} \times \dot{\mathcal{P}}$ is introduced to emphasise that such conditions only depend on the compact sets $\mathcal{P}$ and $\dot{\mathcal{P}}$. The manipulation of LPV systems is also made simpler through the use of dedicated toolboxes, e.g. LPVTools [HSP15] from the University of Minnesota. This toolbox also provides tools for LPV control synthesis.

### 2.2.2 Synthesis

As introduced in Sec. 1.3, the use of classical controllers represents the industrial state-of-practice, but offers limited ability to systematically manage (potentially competing) control requirements in a MIMO fashion. Instead, requirements are tackled iteratively for a single channel at a time, although control adjustments in one channel are typically coupled with the others. Consequently, this approach relies on extensive tuning followed by ad hoc Verification \& Validation (V\&V), which renders the design process very time and cost consuming, with solutions often found via brute-force search.

The ability to address the aforementioned limitations arose with the advent of robust control, as explained next. An overview of the primary features of classical and robust control is provided in Table 2.2.

Table 2.2: Comparison of control synthesis paradigms

| Feature | Classical <br> Control | Standard <br> $\mathcal{H}_{\infty}$ Control | Structured <br> $\mathcal{H}_{\infty}$ Control |
| :--- | :---: | :---: | :---: |
| Explicit consideration of <br> plant uncertainties | $\mathrm{No}^{1}$ | Yes | Yes |
| Stability/performance <br> guarantee by design | No $^{1}$ | Yes | Yes |
| Frequency-wise insight on <br> driving perturbations | No $^{1}$ | Yes | Yes |
| Multi-plant, multi-channel, <br> multi-requirement design | No | Limited | Yes |
| Deterministic control <br> solution | No | Yes | Non-smooth |
| Configurable controller <br> size/architecture | Yes | No | Yes |
| Easy handling of design <br> requirements | Often via <br> brute-force | No | Yes, with <br> systune |

### 2.2.2.1 $\quad \mathcal{H}_{\infty}$ synthesis

The objective of robust control synthesis is to find a linear controller $K(\mathrm{~s})$ such that all the desired requirements of a MIMO plant $P(\mathrm{~s})$ are fulfilled, even in the presence of uncertain LTI dynamics $\Delta(\mathrm{s})$. This controller uses an output vector $\mathbf{y}$ and generates an input vector $\mathbf{u}$ to stabilise and control the plant. Similar to Fig. 2.7, the plant may contain a set of exogenous inputs $\mathbf{w}$ (reference and disturbance signals) and regulated outputs $\mathbf{z}$ (error and performance measurements). For ease of design, the perturbation is bounded by $\|\Delta(\mathrm{s})\|_{\infty} \leq 1$ and enters the system via the uncertainty channel $\mathbf{z}_{\Delta} \rightarrow \mathbf{w}_{\Delta}$.

This setup is conventionally represented by the generalised (upper LFT) interconnection of Fig. 2.8, formed by the input-output transfer function $M(s)$ and uncertainty $\Delta(\mathrm{s})$. In that diagram, an input weight $W_{i}(\mathrm{~s})$ is employed to normalise the frequency content of the exogenous signals and an output weight $W_{o}(\mathrm{~s})$ translates the desired requirements in the frequency domain.

[^0]

Figure 2.8: Generalised interconnection in the $\mathcal{H}_{\infty}$ framework

The dynamics of the plant $P(\mathrm{~s})$ is expressed as:

$$
\left[\begin{array}{c}
\mathbf{z}_{\Delta}(\mathrm{s})  \tag{2.74}\\
W_{o}^{-1}(\mathrm{~s}) \mathbf{z}(\mathrm{s})
\end{array}\right]=\underbrace{\left[\begin{array}{ll}
P_{11}(\mathrm{~s}) & P_{12}(\mathrm{~s}) \\
P_{21}(\mathrm{~s}) & P_{22}(\mathrm{~s})
\end{array}\right]}_{P(\mathrm{~s})}\left[\begin{array}{c}
\mathbf{w}_{\Delta}(\mathrm{s}) \\
W_{i}(\mathrm{~s}) \mathbf{w}(\mathrm{s})
\end{array}\right]
$$

and the control loop is closed using the lower LFT operation of Eq. (2.72).
In this case, the transfer function $M(s)$ is given by:

$$
\left[\begin{array}{c}
\mathbf{z}_{\Delta}  \tag{2.75}\\
\mathbf{z}
\end{array}\right]=\left[\begin{array}{cc}
\mathbb{I} & 0 \\
0 & W_{o}
\end{array}\right] \mathcal{F}_{1}\{P, K\}\left[\begin{array}{cc}
\mathbb{I} & 0 \\
0 & W_{i}
\end{array}\right]\left[\begin{array}{c}
\mathbf{w}_{\Delta} \\
\mathbf{w}
\end{array}\right]=\underbrace{\left[\begin{array}{ll}
M_{11} & M_{12} \\
M_{21} & M_{22}
\end{array}\right]}_{M(\mathrm{~s})}\left[\begin{array}{c}
\mathbf{w}_{\Delta} \\
\mathbf{w}
\end{array}\right]
$$

where the dependence on the Laplace variable $s$ is dropped for clarity.
The robust control problem consists then in finding a stabilising controller $K^{*}(\mathrm{~s})$ that minimises the $\mathcal{H}_{\infty}$-norm of $M(s)$ :

$$
\begin{equation*}
\min \|M(\mathrm{~s})\|_{\infty}=\min \sup _{\omega \in \mathbb{R}} \bar{\sigma}(M(j \omega)) \tag{2.76}
\end{equation*}
$$

which corresponds mathematically to its Maximum Singular Value (MSV) $\bar{\sigma}(M(j \omega))$ and physically to the energy amplification from a signal $\mathbf{w} \in \mathcal{L}_{2}$ to $\mathbf{z} \in \mathcal{L}_{2}$. $\mathcal{L}_{2}$ denotes the space of finite-energy functions satisfying:

$$
\begin{equation*}
\|\mathbf{w}\|_{2}^{2}=\int_{0}^{\infty} \mathbf{w}^{\mathrm{T}}(t) \mathbf{w}(t) \mathrm{d} t<\infty \tag{2.77}
\end{equation*}
$$

and it is said that the system $M(s)$ has an induced $\mathcal{L}_{2}$ gain of $\gamma$ if:

$$
\begin{equation*}
\sup _{\mathbf{w} \neq 0} \frac{\|\mathbf{z}\|_{2}}{\|\mathbf{w}\|_{2}}<\gamma \tag{2.78}
\end{equation*}
$$

The output weight $W_{o}(\mathrm{~s})$ normalises the system such that, if all the control requirements are fulfilled, then:

$$
\begin{equation*}
\|M(\mathrm{~s})\|_{\infty}<1 \tag{2.79}
\end{equation*}
$$

and, under one of the most basic formulations of the problem (known as mixed sensitivity [ZDG95]), $W_{o}(\mathrm{~s})$ is a block-diagonal weight that shapes specific functions in frequency:

- Sensitivity $S(j \omega)$, the transfer function from exogenous inputs to tracking errors;
- Complementary sensitivity $T(j \omega)$, the function from exogenous inputs to system outputs;
- Control sensitivity, the transfer function from exogenous inputs to system inputs (control channel), given by the product between the controller $K(j \omega)$ and $S(j \omega)$.

More specifically, the roll-off properties of the sensitivity function define Low-Frequency (LF) tracking capabilities and its peak indicate the attainable closed-loop stability Gain Margin (GM) and Phase Margin (PM) [RLD10]:

$$
\begin{equation*}
\mathrm{GM} \geq \frac{\|S(j \omega)\|_{\infty}}{\|S(j \omega)\|_{\infty}-1}, \quad \mathrm{PM} \geq 2 \arcsin \left(\frac{1}{2\|S(j \omega)\|_{\infty}}\right) \tag{2.80}
\end{equation*}
$$

where $\|S(j \omega)\|_{\infty}$ is the amplitude of the sensitivity peak. On the other hand, the complementary sensitivity function $T(j \omega)$ dictates the controller bandwidth, which determines the closedloop performance, and the High-Frequency (HF) disturbance rejection. Both functions, $S(j \omega)$ and $T(j \omega)$, are bounded by the fundamental relation $S(j \omega)+T(j \omega)=1$, which imposes a design trade-off between the required disturbance attenuation below the controller bandwidth and system performance [Ste03]. Finally, the control sensitivity allows to manage the control actuation effort as a function of frequency.

Well-known solutions of the $\mathcal{H}_{\infty}$ synthesis problem are based on algebraic Riccati equations or Linear Matrix Inequality (LMI) [DGKF89]. This is nowadays a well-established practice for space applications where performance/robustness trade-offs are key [Pig02, $\mathrm{BDG}^{+} 02$, Cha10, $\mathrm{WYF}^{+} 11$ ], but a few practical limitations must be taken into account:

- The controller $K^{*}(\mathrm{~s})$ has the same order of $M(\mathrm{~s})$, therefore high-dimensional plants result in control systems that may not be suitable for onboard implementation (despite the increasing computational power) unless ad hoc model-reduction methods are used for the controller (which are typically challenging without loss of performance);
- A proper definition of $W_{i}(\mathrm{~s})$ and $W_{o}(\mathrm{~s})$ is still a difficult process, especially for non- $\mathcal{H}_{\infty}$ control experts;
- Minimising the whole $\mathcal{H}_{\infty}$-norm of $M(\mathrm{~s})$, including off-diagonal terms, does not allow to manage cross-coupling effects and can introduce conservativeness in the solution when considering ill-conditioned plants or multiple requirements.

It is also important to note that the standard (full-order) $\mathcal{H}_{\infty}$ synthesis formulation assumes that the uncertainty block $\Delta(\mathrm{s})$ is unstructured (i.e. strictly complex). The full-order extension to structured (combined real or complex) uncertainties was proposed in [SD91, PDB93] using the so-called $D-K$ iteration algorithm (also known as $\mu$ synthesis), which employs a combination of $\mathcal{H}_{\infty}$ synthesis and $\mu$ analysis (Sec. 2.2.3.1).

### 2.2.2.2 Structured $\mathcal{H}_{\infty}$ synthesis

Motivated by the necessity to keep the dimension of controllers low by design, an algorithm to tackle the minimisation problem for fixed-order controllers via specialised non-smooth optimisation techniques was first proposed in [BLO03, BHLO06]. In addition, another application was further proposed in [AN06, GA11]. The latter is called structured $\mathcal{H}_{\infty}$ and, besides allowing low-order design, it also permits to specify the structure of the controller. This is extremely convenient since it enables the specification of its architecture based on physical considerations (e.g. need for integrative or derivative action) or heritage from previous designs.

The underlying principle of structured $\mathcal{H}_{\infty}$ synthesis is to represent the desired architecture of the controller as an LFT of low-level tunable elements $a_{j}(\mathrm{~s})$. Since any interconnection of LFTs preserves its structure, the system of Fig. 2.8 can be rearranged by isolating the tunable elements into a block-diagonal $\tilde{K}(\mathrm{~s})$ and absorbing the rest of the structure into a new plant termed $\tilde{P}(\mathrm{~s})$. The structured $\mathcal{H}_{\infty}$ problem consists then in tuning the elements $a_{j}(\mathrm{~s})$ by minimising the global transfer function of Eq. (2.76).

Moreover, it is possible to rearrange the plant $\tilde{P}(\mathrm{~s})$ and weights $W_{i}(\mathrm{~s})$ and $W_{o}(\mathrm{~s})$ so as to map different combinations of input-output channels and, most notably, individual design requirements. These combinations are then encapsulated into a new block-diagonal matrix:

$$
\begin{equation*}
M(\mathrm{~s})=\operatorname{diag}\left[M_{1}(\mathrm{~s}), \ldots, M_{N}(\mathrm{~s})\right] \tag{2.81}
\end{equation*}
$$

and the synthesis problem becomes:

$$
\begin{equation*}
\min \max \left(\left\|M_{1}(\mathrm{~s})\right\|_{\infty}, \ldots,\left\|M_{N}(\mathrm{~s})\right\|_{\infty}\right) \tag{2.82}
\end{equation*}
$$

Note that this new formulation enables the direct handling of multiple requirements, as well as design plants. In addition, this approach allows the controller to be parameterised as a function of a scheduling variable (known as self-scheduled control [SBB13, LSZ14]), which paves the way for structured LPV synthesis.

These advantages come at the expense of transforming the original synthesis problem into a non-smooth mathematical problem due to the composition of the convex but non-differentiable $\mathcal{H}_{\infty}$ cost function with the differentiable but non-convex mappings $a_{j}(\mathrm{~s}) \rightarrow M_{i}(\mathrm{~s})$ [GA11]. To solve the non-smooth problem, structured $\mathcal{H}_{\infty}$ employs local optimisation methods and, to mitigate the local nature of the optimiser, multiple runs are often performed from random starting points. This represents a key issue in industry due to certification concerns and also a
breakdown in the design learning experience (i.e. the assessment of system behaviour changes due to changes in the posing of the problem). Hence, the optimiser initialisation and the choice of the parameters that are free to be tuned may become critical for a successful control design.

The structured $\mathcal{H}_{\infty}$ algorithm is part of the hinfstruct and systune routines available in MATLAB [GA11], with the latter including an easier quantification of $\mathcal{H}_{\infty}$ requirements and handling of multiple control requirements, channels and models. This algorithm is also able to seamlessly account for parametric uncertainties [ADN15]. Alternatives to cope with mixed (real and complex) uncertainties are currently being developed [AAN18].

Despite the apparent ease of hinfstruct or systune, there are numerous options available to set the tuning requirements and design conditions (sometimes seemingly fighting each other, as characteristic of multiple minima optimisation). This wide availability of choice is a risk in the sense that it provides a confidence level that can complicate the design process by over-constraining the optimisation problem. In this thesis, that risk is mitigated by posing the problem as in the standard $\mathcal{H}_{\infty}$ framework and focusing on frequency-domain weight functions $W_{i}(\mathrm{~s})$ and $W_{o}(\mathrm{~s})$ to impose the desired closed-loop properties.

Despite the challenges mentioned above, the effectiveness of structured robust control has been proven through many simulation design problems, two space-flown missions [FPB ${ }^{+}$15, PP15] and recently in piloted flight tests [MS17]. The main strength of structured $\mathcal{H}_{\infty}$ under the scope of these applications is its capability to synthesise a controller that is valid not only for one, but for a set of plants or operating conditions (multi-plant control). Other works from the author on structured $\mathcal{H}_{\infty}$ optimisation can be found in [SNIM18, NMS $\left.^{+} 19\right]$.

Finally, an additional strength of structured $\mathcal{H}_{\infty}$ for space missions lies on its ability to specify the controller structure, which allows: [i] control gains to be changed ad hoc if any re-tuning is required (recall Rosetta's example from Sec. 1.3.1), and [ii] legacy industry knowledge to be kept. Taking advantage of legacy knowledge is a very important point as it enables the transfer of this approach (offering better performance/robustness properties) to industry without representing a disruptive change for the design teams.

### 2.2.3 Analysis

Robustness analysis entails assessing the impact of uncertainties and unmodelled dynamics on closed-loop stability and performance. The Nyquist diagram of a generic Single-Input SingleOutput (SISO) system is provided in Fig. 2.9 to evidence such impact in a graphical manner.

Here, the nominal transfer function is shown in magenta, the critical stability point -1 is marked with a red cross and the required performance is specified by the red exclusion disk enveloping the 6 dB M-circles for $S(j \omega)$ and $T(j \omega)$. Plant perturbations are accounted for as an unstructured uncertainty around the nominal response, which is represented by the green circle with the corresponding spectral radius. Note that this effect is depicted for a single frequency, but it can occur (with varying severity) throughout the spectrum.


Figure 2.9: Illustration of robust SISO stability and performance (adapted from [WYF $\left.{ }^{+} 11\right]$ )

In this setting, the system is nominally stable if the magenta response does not encircle the point -1 and it is said to have Robust Stability (RS) if this criterion does not get compromised when accounting for the green circle instead of the nominal response. In addition, Nominal Performance (NP) is assessed by verifying if the nominal response does not intercept the red exclusion disk. Finally, the system is said to have Robust Performance (RP) if the green circle does not intercept the red exclusion disk.

The systematisation of this assessment to MIMO plants with structured uncertainties was introduced in [SD91] and is presented in Sec. 2.2.3.1. Before that, a more concrete example of launcher stability analysis from the author is discussed in the next paragraphs.

This example, taken from [ $\mathrm{SBM}^{+}$16], is based on Europe's lightweight VEGA launcher [Bia08] during atmospheric flight. Here, Nichols charts are employed for the assessment of classical stability indicators [dVK93], which include (in ascending order of frequency): [i] rigid-body low-frequency gain margin (LF GM), [ii] rigid-body phase margin (PM), [iii] rigid-body highfrequency gain margin (HF GM), [iv] right phase margin of the first bending mode (PM1), $[\mathrm{v}]$ gain peak of the first bending mode ( Pk 1 ), [vi] left phase margin of the first bending mode (PM2), and [vii] gain peak of the second bending mode (Pk2).

Nichols charts of VEGA's closed-loop attitude channel are illustrated together with the nominal stability indicators in Fig. 2.10a at distinct instants over the flight (for $t \in[5,110]$ seconds), and the corresponding rigid-body margins are gathered in Fig. 2.10b (note that PM is reported in terms of the equivalent delay margin divided by 10 for readability purposes). This type of representation allows to verify that closed-loop stability under nominal conditions is ensured throughout the flight, although there is a reduction of stability margins when aerodynamic loads become more intense (between 20 and 80 seconds).

(a) Nominal Nichols charts

(b) Rigid-body stability margins

Figure 2.10: Classical launcher stability analysis over the flight

### 2.2.3.1 $\mu$ analysis

The Structured Singular Value (SSV) $\mu$ relies on the LFT representation in the form of Fig. 2.8 to determine if a system $M(\mathrm{~s})$, partitioned as in Eq. (2.70), is stable in the presence of all the normalised LTI uncertainties $\|\Delta(\mathrm{s})\|_{\infty} \leq 1$.

In this case, stability is ensured by the existence of the inverse $\left(\mathbb{I}-M_{11} \Delta\right)^{-1}$ in Eq. (2.70), and verified by computing:

$$
\begin{equation*}
\mu\left(M_{11}\right)=\frac{1}{\min _{\Delta}\left\{\bar{\sigma}(\Delta):\|\Delta\|_{\infty} \leq 1, \operatorname{det}\left(\mathbb{I}-M_{11} \Delta\right)=0\right\}} \tag{2.83}
\end{equation*}
$$

with $\mu\left(M_{11}\right) \in \mathbb{R}^{+}$if a solution exists.
The system is then said to have RS [DPZ91, ZDG95] if and only if its nominal component $M_{22}(\mathrm{~s})$ is stable and:

$$
\begin{equation*}
\mu\left(M_{11}(j \omega)\right)<1, \quad \forall \omega \in \mathbb{R} \tag{2.84}
\end{equation*}
$$

Moreover, the norm of the smallest set of uncertainties that destabilises the system is given by $\left\|\mu\left(M_{11}\right)\right\|_{\infty}^{-1}$. The computation of Eq. (2.83) is known to be polynomial-time hard, hence its estimation (except for a few specific cases) relies on lower and upper bounds, such that:

$$
\begin{equation*}
\max _{Q \in \mathcal{Q}} \rho\left(Q M_{11}\right) \leq \mu\left(M_{11}\right) \leq \inf _{D \in \mathcal{D}} \bar{\sigma}\left(D M_{11} D^{-1}\right) \tag{2.85}
\end{equation*}
$$

where $Q$ and $D$ are scaling matrices from two complex subsets $\mathcal{Q}$ and $\mathcal{D}$ defined to get the bounds as tight as possible [DPZ91], $\rho$ indicates the spectral radius (recall Fig. 2.9) and $\bar{\sigma}$ the MSV (recall Eq. (2.76)). All the algorithms necessary for $\mu$ analysis are also available with MATLAB's Robust Control Toolbox [BCPS06] and ONERA's SMAC [ $\left.\mathrm{BBD}^{+} 16\right]$.

In practice, the two bounds provide an analytical guarantee of existence of at least one perturbation that equals the Lower Bound (LB) degradation and of inexistence of any perturbation that exceeds the Upper Bound (UB) degradation. For more accurate bounds, the size of LFT models shall be kept as small as possible while capturing the most relevant physical phenomena of the real system and their interplay with the uncertainties.

Furthermore, the SSV $\mu$ can be employed for robust performance analysis, i.e. to check if closed-loop requirements defined as in Eq. (2.79) remain fulfilled under the effect of the allowable LTI uncertainties $\|\Delta(\mathrm{s})\|_{\infty} \leq 1$. Without uncertainties, NP is assessed by verifying if Eq. (2.79) holds for $\Delta(\mathrm{s})=0$ over frequency, which is equivalent to:

$$
\begin{equation*}
\bar{\sigma}\left(M_{22}(j \omega)\right)<1, \quad \forall \omega \in \mathbb{R} \tag{2.86}
\end{equation*}
$$

Under the presence of uncertainties, the performance channel $\mathbf{z} \rightarrow \mathbf{w}$ is closed through a fictitious complex perturbation $\Delta_{\mathrm{p}}(\mathrm{s})$ as shown in Fig. 2.11, and the $\mu$ test is now applied to $M(\mathrm{~s})$ instead of $M_{11}(\mathrm{~s})$, considering the augmented uncertainty $\operatorname{diag}\left[\Delta(\mathrm{s}), \Delta_{\mathrm{p}}(\mathrm{s})\right]$.


Figure 2.11: Generalised interconnection for RP analysis

Similar to the stability case, the system with requirements defined as in Eq. (2.79) is said to have RP if and only if it has nominal stability and:

$$
\begin{equation*}
\mu(M(j \omega))<1, \quad \forall \omega \in \mathbb{R} \tag{2.87}
\end{equation*}
$$

with smaller values of $\mu(M)$ indicating better RP properties. It is also important to note that, as indicated by the upper LFT operation of Eq. (2.70), $M(\mathrm{~s})$ is directly influenced by $M_{11}(\mathrm{~s})$, $M_{22}(\mathrm{~s})$ and by their interplay through $M_{12}(\mathrm{~s})$ and $M_{21}(\mathrm{~s})$, thus a system cannot have RP without having RS and NP. In other words:

$$
\begin{equation*}
\underbrace{\mu(M)}_{\mathrm{RP}} \geq \max \{\underbrace{\mu\left(M_{11}\right)}_{\mathrm{RS}}, \underbrace{\bar{\sigma}\left(M_{22}\right)}_{\mathrm{NP}}\} \tag{2.88}
\end{equation*}
$$

Additional valuable insights can also be extracted using $\mu$ analysis, which will be thoroughly exploited in the thesis:

- In addition to the binomial RS/RP check, $\mu$ offers a frequency-wise insight on the stability/performance degradation mechanisms;
- The LB can be used to obtain critical uncertainties $\Delta^{\mathrm{WC}}(\mathrm{s})$, which allow to identify Worst-Case (WC) configurations [MRR $\left.{ }^{+} 15\right]$ and complement traditional sampling-based V\&V methods such as Monte-Carlo (MC) by narrowing the sampling around critical areas;
- The possibility to calculate $\mathrm{RS} / \mathrm{RP}$ sensitivities [MBP05] that quantify the impact of each uncertainty in the solution of $\mu$, allowing to validate the meaningfulness of the results and to support considerations often derived from an engineering perspective.

Coming back to VEGA's example from [ $\left.\mathrm{SBM}^{+} 16\right]$, the WC closed-loop system can be constructed from the results of $\mu$ analysis and its stability properties may be analysed as before. The comparison between nominal and WC response is illustrated in Fig. 2.12a. This Nichols chart shows that the system remains stable, but the WC combination of uncertain parameters amounts to a profound stability degradation in the LF area, with a critical reduction of gain and delay margins.

The same approach allows to determine the WC stability margins for other instants of time, which are plotted against the nominal ones of Fig. 2.10b in Fig. 2.12b. Similar to what is shown in Fig. 2.12a for $t=60$ seconds, the WC conditions found via $\mu$ lead to a considerable reduction of LF GM and also PM throughout the flight and to the consequent (negative) increase of HF GM as the Nichols plot is essentially shifted down.


Figure 2.12: Nominal and WC launcher stability indicators

Figure 2.12a also depicts the response of 10000 MC random LFT samples. These results clarify the effectiveness of the $\mu$ algorithm which, in a single shot, was able to identify conditions for a more intense degradation of stability. WC margins tend to be very realistic as they are derived from the LB peak of $\mu$, but it is understood that this methodology is inherently conservative and that these margins are generally associated to rather unlikely configurations. A more sophisticated approach known as probabilistic $\mu$ has been proposed in [KP98] to estimate the WC likelihood and applied in [MBR15, FAP17].

It is highlighted that the $\mu$ framework is limited to LTI uncertainties. For this particular case, once the WC perturbation $\Delta^{\mathrm{WC}}(\mathrm{s})$ of a system $M(\mathrm{~s})$ is determined, its induced WC $\mathcal{L}_{2}$ gain, defined in Eq. (2.78), corresponds to:

$$
\begin{equation*}
\gamma=\left\|\mathcal{F}_{\mathrm{u}}\left\{M(\mathrm{~s}), \Delta^{\mathrm{WC}}(\mathrm{~s})\right\}\right\|_{\infty} \tag{2.89}
\end{equation*}
$$

### 2.2.3.2 IQC analysis

In order to extend the induced $\mathcal{L}_{2}$ gain computation to systems with nonlinear perturbations (such as time-varying uncertainties, time delays and limited control authority), an analytical methodology has been proposed in [MR97] based on the concept of Integral Quadratic Constraints (IQC).

The underlying principle of the IQC framework is that the finite-energy signals $\mathbf{w}_{\Delta} \in \mathcal{L}_{2}$ and $\mathbf{z}_{\Delta} \in \mathcal{L}_{2}$, also depicted in Fig. 2.8 and with Fourier transforms $\hat{\mathbf{w}}_{\Delta}(j \omega)$ and $\hat{\mathbf{z}}_{\Delta}(j \omega)$, are said to satisfy the IQC defined by multiplier $\Pi(j \omega)$ if:

$$
\int_{-\infty}^{+\infty}\left[\begin{array}{c}
\hat{\mathbf{z}}_{\Delta}(j \omega)  \tag{2.90}\\
\hat{\mathbf{w}}_{\Delta}(j \omega)
\end{array}\right]^{*} \underbrace{\left[\begin{array}{ll}
\Pi_{11}(j \omega) & \Pi_{12}(j \omega) \\
\Pi_{12}^{*}(j \omega) & \Pi_{22}(j \omega)
\end{array}\right]}_{\Pi(j \omega)}\left[\begin{array}{c}
\hat{\mathbf{z}}_{\Delta}(j \omega) \\
\hat{\mathbf{w}}_{\Delta}(j \omega)
\end{array}\right] \mathrm{d} \omega \geq 0
$$

where $\Pi(j \omega): j \mathbb{R} \rightarrow \mathbb{C}$ is a measurable Hermitian-valued function, usually chosen amongst a set of rational functions bounded on the imaginary axis, and the superscript $*$ represents the complex conjugate transpose.

Using this setup, any bounded and causal nonlinear perturbation $\Delta$ can be described by an IQC in the form of Eq. (2.90), also known as soft IQC, provided that it holds for all $\mathbf{z}_{\Delta} \in \mathcal{L}_{2}$ and $\mathbf{w}_{\Delta}=\Delta\left(\mathbf{z}_{\Delta}\right)$ (i.e. $\mathbf{w}_{\Delta}$ denotes the output of $\Delta$ when excited by $\mathbf{z}_{\Delta}$ ). For a given uncertainty description, the set $\mathcal{D}_{\Pi}$ of all suitable multipliers $\Pi(j \omega)$ is given by:

$$
\mathcal{D}_{\Pi}=\left\{\Pi(j \omega): \int_{-\infty}^{+\infty}\left[\begin{array}{c}
\hat{\mathbf{z}}_{\Delta}(j \omega)  \tag{2.91}\\
\epsilon \hat{\mathbf{w}}_{\Delta}(j \omega)
\end{array}\right]^{*} \Pi(j \omega)\left[\begin{array}{c}
\hat{\mathbf{z}}_{\Delta}(j \omega) \\
\epsilon \hat{\mathbf{w}}_{\Delta}(j \omega)
\end{array}\right] \mathrm{d} \omega \geq 0, \forall \epsilon \in[0,1], \mathbf{w}_{\Delta}=\Delta\left(\mathbf{z}_{\Delta}\right)\right\}
$$

where the term $\epsilon$ is introduced because IQC theory requires $\mathcal{D}_{\Pi}$ to be star-shaped with respect to the origin. The structure of $\Pi(j \omega)$ is chosen based on the type of perturbation being investigated. A list of multipliers describing common perturbations can be found in [MR97].

The frequency-domain formulation of Eq. (2.90) is known to have a time-domain equivalent through the application of Parseval's theorem and factorisation of $\Pi(j \omega)$ as:

$$
\begin{equation*}
\Pi(j \omega)=\Psi^{*}(j \omega) H \Psi(j \omega) \tag{2.92}
\end{equation*}
$$

where $\Psi(j \omega)$ is a tall stable filter applied to $\mathbf{w}_{\Delta}$ and $\mathbf{z}_{\Delta}$, and $H=H^{\mathrm{T}}$ is a real matrix variable that defines the structure of the multiplier. The graphical interpretation of this setup is provided in Fig. 2.13.


Figure 2.13: Graphical interpretation of the IQC analysis setup
In this case, the signals $\mathbf{z}_{\Delta} \in \mathcal{L}_{2}$ and $\mathbf{w}_{\Delta}=\Delta\left(\mathbf{z}_{\Delta}\right)$ are said to satisfy the IQC defined by $(\Psi, H)$, also known as hard IQC, for a finite time-horizon $T>0$ if:

$$
\int_{0}^{T}\left(\Psi\left[\begin{array}{c}
\hat{\mathbf{z}}_{\Delta}(t)  \tag{2.93}\\
\hat{\mathbf{w}}_{\Delta}(t)
\end{array}\right]\right)^{\mathrm{T}} H\left(\Psi\left[\begin{array}{c}
\hat{\mathbf{z}}_{\Delta}(t) \\
\hat{\mathbf{w}}_{\Delta}(t)
\end{array}\right]\right) \mathrm{d} t \geq 0
$$

The filter $\Psi(j \omega)$ can be understood as the IQC generalisation of the $Q$ and $D$ scaling matrices employed in $\mu$ analysis.

Within this framework, analysing whether a nominally stable system $M(\mathrm{~s})$ has an induced WC $\mathcal{L}_{2}$ gain of $\gamma$ amounts to verifying whether the following inequality is satisfied:

$$
\begin{align*}
& {\left[\begin{array}{c}
M(j \omega) \\
\mathbb{I}
\end{array}\right]^{*} \tilde{\Pi}(j \omega)\left[\begin{array}{c}
M(j \omega) \\
\mathbb{I}
\end{array}\right] \mathrm{d} \omega<0, \quad \forall \omega \in \mathbb{R}}  \tag{2.94}\\
& \text { with } \tilde{\Pi}(j \omega)=\left[\begin{array}{cccc}
\Pi_{11}(j \omega) & 0 & \Pi_{12}(j \omega) & 0 \\
0 & \mathbb{I} & 0 & 0 \\
\Pi_{12}^{*}(j \omega) & 0 & \Pi_{22}(j \omega) & 0 \\
0 & 0 & 0 & -\gamma^{2} \mathbb{I}
\end{array}\right] \tag{2.95}
\end{align*}
$$

In Eq. (2.94), the behaviour of the $\mathbf{z}_{\Delta}$ and $\Delta\left(\mathbf{z}_{\Delta}\right)$ channels is directly reflected into the $[M(j \omega) \mathbb{I}]^{\mathrm{T}}$ structure. To solve the IQC problem, this inequality is often reformulated as a single LMI using the Kalman-Yakubovich-Popov lemma [IH05].

As a cursory example of this formulation, consider that a system $M(\mathrm{~s})$ is solely perturbed by a (rate-bounded) TV parameter such that it can be parameterised as introduced in Sec. 2.2.1.2. In this particular case, the generalised LTI bounded real lemma [WYPB96] states that $M(\mathrm{~s})$ is exponentially stable and its induced $\mathcal{L}_{2}$ gain is bounded by $\gamma>0$ if there exists a continuously differentiable TV parameter-dependent matrix $P(p)$, such that $\forall(p, q) \in \mathcal{P} \times \dot{\mathcal{P}}$ :

$$
\begin{gather*}
P(p)>0, \\
{\left[\begin{array}{cc}
P(p) A(p)+A^{\mathrm{T}}(p) P(p)+\sum_{i=1}^{n_{\rho}} \frac{\partial P(p)}{\partial p_{i}} q_{i} & P(p) B(p) \\
B^{\mathrm{T}}(p) P(p) & -\mathbb{I}
\end{array}\right]+\frac{1}{\gamma^{2}}\left[\begin{array}{c}
C^{\mathrm{T}}(p) \\
D^{\mathrm{T}}(p)
\end{array}\right]\left[\begin{array}{ll}
C(p) & D(p)
\end{array}\right]<0} \tag{2.96}
\end{gather*}
$$

where $\{A(p), B(p), C(p), D(p)\}$ represent the state-space matrices of $M(\mathrm{~s})$. The ability of IQCs to generalise uncertain and time-varying effects under the same unified framework also paves the way for robust LPV synthesis [PS15].

In opposition to the $\mu$ approach, for which very efficient algorithms exist, IQC analysis is less straightforward mostly due to the choice of the filter $\Psi(j \omega)$ in Eq. (2.92). While static filters may be over-conservative and render Eq. (2.94) non-solvable, adding dynamics leads to ambiguity and computational scalability issues. To facilitate the process, dedicated toolboxes are available, such as IQC $\beta$ [KMJR04] from the authors of [MR97], and [BBD $\left.{ }^{+} 16\right]$ from ONERA.

## Application I

## Fuel-optimal Descent \& Landing on Small Planetary Bodies



## System Modelling for Landing on Phobos

In this chapter, all the models needed for the first application of the thesis are presented. Sections 3.1 and 3.2 present respectively the high-fidelity benchmark and the reference Descent \& Landing (D\&L) trajectories employed for the landing on Phobos case study. Using these inputs, an approach to derive Linear Time-Invariant (LTI) models from the high-fidelity dynamics that can be employed for Guidance \& Control (G\&C) design purposes is provided in Sec. 3.3.

The linear models are then augmented to account for gravitational uncertainties and Time-Varying (TV) effects via Linear Fractional Transformation (LFT) and Linear ParameterVarying (LPV) modelling in Sec. 3.4 and 3.5, respectively. To complete the chapter, actuator and navigation subsystems included with the benchmark are characterised and modelled in Sec. 3.6.

### 3.1 High-fidelity dynamics in the vicinity of Phobos

Mars' largest moon Phobos is a small body, orbiting the red planet at a mean altitude of less than $6,000 \mathrm{~km}$ and a period of about 7 hours and 40 minutes. An overview of physical and orbital parameters of the Mars-Phobos system is provided in Appendix B. One interesting property is that, just like our Moon, Phobos is tidally locked due to the long-term effect of the planet's gravity gradient, meaning that its revolution and rotation periods are the same and that it always shows the same face to Mars.

Phobos is also particularly lightweight (its mass is 8 orders of magnitude smaller than Mars) and close to the red planet, which causes the planet's sphere of influence (Fig. 3.1a) to end just 3.5 km above the moon. Hence, there is no possibility for Keplerian orbits around Phobos and the third-body perturbation of Mars cannot be neglected, making landing on this asteroid extremely challenging.

(a) Mars' sphere of influence around Phobos. Axes normalised by mean distance Mars-Phobos. Continuous black line represents Phobos ellipsoid and dashed line its mean sphere.

Figure 3.1: Highly inhomogeneous gravity field of Phobos (credits: Airbus Defence \& Space)

Furthermore, due to the irregular shape and mass distribution of Phobos, the gravity of the moon cannot be accurately accounted for by a spherical (Keplerian) field, and requires to be described using a Gravity Harmonics (GH) model. The difference between these two models over Phobos' surface is illustrated in Fig. 3.1b.

Using spherical coordinates $(r, \theta, \varphi)$ for distance to barycentre, co-latitude and longitude, as well as $R$ for a reference radius and $\mu_{\mathrm{g}}$ for the gravitational constant, the GH gravity potential is described by the spherical series expansion [ZB15]:

$$
\begin{equation*}
U(r, \theta, \varphi)=-\frac{\mu_{\mathrm{g}}}{R} \sum_{n=0}^{\bar{n}}\left(\frac{R}{r}\right)^{n+1} \sum_{m=0}^{n} C_{n}^{m}(\varphi) P_{n}^{m}(\cos \theta) \tag{3.1}
\end{equation*}
$$

where:

$$
\begin{align*}
P_{n}^{m}(x) & =\left(1-x^{2}\right)^{m / 2} \frac{\mathrm{~d}^{m}}{\mathrm{~d} x^{m}} P_{n}(x) \\
P_{n}(x) & =\frac{1}{2^{n} n!} \frac{\mathrm{d}^{n}}{\mathrm{~d} x^{n}}\left(x^{2}-1\right)^{n} \tag{3.2}
\end{align*}
$$

are the associated Legendre polynomials and:

$$
\begin{equation*}
C_{n}^{m}(\varphi)=C_{n, m} \cos m \varphi+S_{n, m} \sin m \varphi \tag{3.3}
\end{equation*}
$$

is the double expansion of GH coefficients $C_{n, m}$ and $S_{n, m}$.
For Phobos, $\bar{n}=4$ is assumed to suffice, which leads to 28 coefficients plus the Keplerian term $C_{0}^{0}(\varphi)=1$. Estimates of Phobos' GH coefficients are available from ground observations but, due to its complex gravitational environment and distance to Earth, 19 of them are known with a large range of uncertainty. In this work, each coefficient is assumed to follow a Gaussian
distribution with standard deviation equal to its nominal value. This assumption is further discussed in [ZB15], the nominal GH coefficients are also provided in Appendix B.

As demonstrated in Sec. 2.1.5, it is mandatory that every candidate G\&C algorithm is verified not only for nominal GH coefficients, but also for all admissible values within the uncertainty range mentioned above. In practice, this range may be significantly reduced before D\&L if an observation phase of the target body is included in the mission. The possibility of G\&C retuning upon the availability of updated estimates is actually an added-value of the methodologies proposed in Chapter 4 and 5.

Having in mind the small eccentricity of Phobos' orbit around Mars ( $e_{\mathrm{Pho}} \approx 0.0156$ ), the nonlinear dynamics of a spacecraft flying in the vicinity of the Mars-Phobos system can be described as a circular restricted three-body problem. In this case, using a Body-Centred BodyFixed (BCBF) frame with origin at the moon's barycentre, the spacecraft motion is written as [ZB15]:

$$
\left[\begin{array}{c}
\dot{\mathbf{r}}(t)  \tag{3.4}\\
\dot{\mathbf{v}}(t) \\
\dot{\nu}(t)
\end{array}\right]=\mathbf{f}(\mathbf{r}(t), \mathbf{v}(t), \nu(t))+\left[\begin{array}{c}
0_{3 \times 3} \\
\mathbb{I}_{3 \times 3} \\
0_{1 \times 3}
\end{array}\right] \mathbf{a}(t)
$$

where the state vector $[\mathbf{r}(t) \mathbf{v}(t) \nu(t)]^{\mathrm{T}}$ gathers the position and velocity of the spacecraft with respect to Phobos, as well as the true anomaly of the latter around Mars, while the control vector $\mathbf{a}(t)$ represents any propulsive acceleration generated by the spacecraft in the BCBF frame.

The equation above is equivalent to Eq. (2.3), with the complex gravity potential description of Eq. (3.1) for Phobos and Mars embedded in $\mathbf{f}(\mathbf{r}(t), \mathbf{v}(t), \nu(t))$ through the dynamics of $\mathbf{v}(t)$. Dropping the dependence on $t$ for clarity, this is given by:

$$
\begin{equation*}
\dot{\mathbf{v}}=\nabla U_{\mathrm{S} / \mathrm{C}}^{\mathrm{Pho}}(\mathbf{r})+\nabla U_{\mathrm{S} / \mathrm{C}}^{\mathrm{Mars}}(\mathbf{r}, \nu)-\nabla U_{\mathrm{Pho}}^{\mathrm{Mars}}(\nu)-\omega_{\mathrm{Pho}} \times\left(\omega_{\mathrm{Pho}} \times \mathbf{r}\right)-2 \omega_{\mathrm{Pho}} \times \mathbf{v} \tag{3.5}
\end{equation*}
$$

Here, the first three terms $\boldsymbol{\nabla} U_{\#}^{*}$ represent respectively the gravity of Phobos on the spacecraft, Mars on the spacecraft and Mars on Phobos, computed as a function of the position $\mathbf{r}$ of the spacecraft relative to Phobos and of the latter's location around Mars via its true anomaly $\nu$.

The calculation above becomes particularly complex due to all the frame transformations involved. Details on these transformations are outside the scope of this thesis, but can be found in [ZB15]. The last two terms of Eq. (3.5) account for the non-inertial acceleration caused by the fact that the BCBF frame is rotating with Phobos at a rate $\omega_{\text {Pho }}$. Finally, based on this rate, the true anomaly of Phobos on its orbit around Mars is propagated as follows:

$$
\begin{equation*}
\dot{\nu}=\omega_{\mathrm{Pho}} \frac{\left(1+e_{\mathrm{Pho}} \cos \nu\right)^{2}}{\left(1-e_{\mathrm{Pho}}^{2}\right)^{3 / 2}} \tag{3.6}
\end{equation*}
$$

For this thesis, all the Spacecraft Dynamics \& Kinematics (SDK) computations are performed using a high-fidelity simulator developed by Airbus Defence \& Space for this project based on
their space operational experience $\left[\mathrm{JZS}^{+} 17, \mathrm{JZS}^{+} 18\right]$. This simulator is part of their industrial testing, simulation and assessment facility and will be employed for system development and validation in case of a future Phobos Sample Return (SR) mission.

### 3.2 Reference trajectory design

Two guidance paradigms for D\&L have been defined in Sec. 2.1.2: [i] open-loop (or implicit), employed when a Reference Trajectory (RT) $\left\{\mathbf{r}_{\text {ref }}(t), \mathbf{v}_{\text {ref }}(t)\right\}$ and thruster profile $\mathbf{a}_{\text {ref }}(t)$ are generated before, and remain unchanged during, the descent, or [ii] closed-loop (or explicit), which refers to the case when the thruster profile is computed in real-time to correct the trajectory based on onboard measurements (as it was shown in Fig. 2.2 by dashed lines). Regardless of the type of guidance, the architecture may be augmented with a control compensator which introduces an additional acceleration vector command $\mathbf{a}_{\mathrm{cmp}}(t)$ to further alleviate trajectory errors.

For G\&C design, it is therefore convenient to have adequate RTs defining not only the aforementioned profiles $\left\{\mathbf{r}_{\text {ref }}(t), \mathbf{v}_{\text {ref }}(t), \mathbf{a}_{\text {ref }}(t)\right\}$, but also the set of design points where any control compensator must be able to operate. In this study, a set of RTs has also been calculated by Airbus Defence \& Space and provided together with the high-fidelity benchmark introduced in the previous section. These trajectories were designed via the following process $\left[\mathrm{JZS}^{+} 17, \mathrm{JZS}^{+} 18\right]$ :

1. Identification of unstable manifolds that originate at an invariant Libration Point Orbit (LPO) in the three-body system and that intersect Phobos. Examples of such manifolds are depicted in Fig. 3.2a;
2. Selection of the manifolds that reach the moon with higher incidence angle (i.e. the most vertical ones) as initial trajectory guesses. These trajectories are ballistic (i.e. require $\Delta V=0$ ), but their final (touchdown) speed may be too high. Typical sample return missions require a touchdown velocity inferior to $1.5 \mathrm{~m} / \mathrm{s}$ vertically and $1.0 \mathrm{~m} / \mathrm{s}$ horizontally;
3. Parametrisation of a fixed-order polynomial acceleration profile aimed at bringing the touchdown speed to zero. The fixed-order structure will lead to sub-optimal solutions, but allows the use of finite-dimensional NonLinear Programming (NLP) algorithms for a faster optimisation;
4. Computation of the acceleration profile and associated RT (starting from the initial guesses from point 2) that minimise the required $\Delta V$ :

$$
\begin{equation*}
\Delta V=\int_{t_{0}}^{t_{f}}\left[\mathbf{a}^{\mathrm{T}}(\tau) \mathbf{a}(\tau)\right]^{1 / 2} \mathrm{~d} \tau \tag{3.7}
\end{equation*}
$$

while keeping an admissible level of error on the final states. To simplify the optimisation runs, a few approximations are introduced in the dynamics of the Mars-Phobos system,
hence it may not be possible to accurately track the resulting RT in open-loop (as it will be seen in Chapter 5).

The three optimised trajectories that will be considered throughout the thesis for the Phobos application are shown (together with their required $\Delta V$ value) in Fig. 3.2b. All of them will be employed for closed-loop Verification \& Validation (V\&V), but only that termed RT1 will be used for G\&C synthesis purposes.

(a) Candidate manifolds originated at LPOs that intersect Phobos

(b) Optimised trajectories and $\Delta V$ requirements (all the trajectories have a duration of less than 2 hours)

Figure 3.2: Reference landing trajectories

### 3.3 Orbital perturbation model

For G\&C synthesis purposes, it is also convenient to have a linear representation of Eq. (3.4). In this thesis, such a representation is accomplished through the application of linearised orbital perturbation theory [Bat87].

According to the orbital perturbation theory, state and control variables can be defined at different operating points along a given trajectory as the sum of a reference (desired) value and small perturbations (deviations):

$$
\begin{align*}
\mathbf{r}(t) & =\mathbf{r}_{\mathrm{ref}}(t)+\delta \mathbf{r}(t) \\
\mathbf{v}(t) & =\mathbf{v}_{\mathrm{ref}}(t)+\delta \mathbf{v}(t)  \tag{3.8}\\
\nu(t) & =\nu_{\mathrm{ref}}(t)+\delta \nu(t) \\
\mathbf{a}(t) & =\mathbf{a}_{\mathrm{ref}}(t)+\delta \mathbf{a}(t)
\end{align*}
$$

The dynamics of these perturbations are then approximated by the $1^{\text {st }}$ order terms of the Taylor series expansion of $\mathbf{f}(\mathbf{r}(t), \mathbf{v}(t), \nu(t))$ around the reference points:

$$
\left[\begin{array}{c}
\delta \dot{\mathbf{r}}(t)  \tag{3.9}\\
\delta \dot{\mathbf{v}}(t) \\
\delta \dot{\nu}(t)
\end{array}\right]=J_{\mathbf{f}}(t)\left[\begin{array}{c}
\delta \mathbf{r}(t) \\
\delta \mathbf{v}(t) \\
\delta \nu(t)
\end{array}\right]+\left[\begin{array}{c}
0_{3 \times 3} \\
\mathbb{I}_{3 \times 3} \\
0_{1 \times 3}
\end{array}\right] \delta \mathbf{a}(t)
$$

where the Jacobian matrix is given by:

$$
\left.J_{\mathbf{f}}(t)=\left[\begin{array}{lll}
\frac{\partial \mathbf{f}}{\partial \mathbf{r}} & \frac{\partial \mathbf{f}}{\partial \mathbf{v}} & \frac{\partial \mathbf{f}}{\partial \nu}
\end{array}\right] \right\rvert\, \begin{aligned}
& \mathbf{r}=\mathbf{r}_{\mathrm{ref}}(t)  \tag{3.10}\\
& \mathbf{v}=\mathbf{v}_{\mathrm{ref}}(t) \\
& \nu=\nu_{\mathrm{ref}}(t)
\end{aligned}
$$

which is computed via finite differences due to the complexity of $\mathbf{f}(\mathbf{r}(t), \mathbf{v}(t), \nu(t))$. Performing this linearisation at different instants of time $t_{i}, i=\{1, \ldots, N\}$ along an RT allows to generate a set of LTI SDK models termed $G_{\text {SDK }}^{i}(\mathrm{~s})$, with the following general state-space description:

$$
\left[\begin{array}{c}
\dot{\mathrm{x}}_{\mathrm{SDK}}(\mathrm{~s})  \tag{3.11}\\
\hline \delta \mathbf{r}(\mathrm{s}) \\
\delta \mathbf{v}(\mathrm{s})
\end{array}\right]=\left[\begin{array}{c|c}
J_{\mathbf{f}}^{i} & 0_{3 \times 3} \\
& \mathbb{I}_{3 \times 3} \\
0_{1 \times 3} \\
\hline \mathbb{I}_{6 \times 6} & 0_{6 \times 1}
\end{array} 0_{6 \times 3} .\left[\begin{array}{c}
\mathbf{x}_{\mathrm{SDK}}(\mathrm{~s}) \\
\hline \delta \mathbf{a}(\mathrm{s})
\end{array}\right]\right.
$$

where $J_{\mathbf{f}}^{i}=J_{\mathbf{f}}\left(t_{i}\right)$ and $\mathbf{x}_{\mathrm{SDK}}(\mathrm{s})$ is the internal state vector. The pole-zero map of $G_{\mathrm{SDK}}^{i}(\mathrm{~s})$ along RT1 (refer to Fig. 3.2b) and the magnitude response of its first channel $\delta a_{\mathrm{x}} \rightarrow \delta r_{\mathrm{x}}$ over time are depicted in Fig. 3.3.

The linear analysis shown in this figure confirms the complexity of the evolution of $G_{\mathrm{SDK}}^{i}(\mathrm{~s})$, with its poles (Fig. 3.3a) and magnitude peaks (Fig. 3.3b) changing smoothly but nonmonotonically over time and with higher characteristic frequencies towards the final descent time, where the gravity pull of Phobos becomes more intense. It is also confirmed from Fig. 3.3a that the system is open-loop unstable, since any deviation from the reference trajectory will cause the spacecraft to be pulled towards either Mars or Phobos.

### 3.4 Inclusion of gravitational uncertainties

As introduced in Sec. 3.1, 19 out of the 28 GH coefficients of Phobos are highly inaccurately known. This means that the computation of $J_{\mathbf{f}}^{i}$ and thus the description of $G_{\text {SDK }}^{i}$ (s) in Eq. (3.11) is subject to a high level of uncertainty. To capture the effect of this uncertainty, the LFT representation introduced in Sec. 2.2.1.1 will be employed.

To build representative LFT models of $G_{\text {SDK }}^{i}(\mathrm{~s})$, a three-step procedure has been developed. These steps encompass uncertainty selection, plant interpolation and LFT verification and they are detailed in the following paragraphs. The developed procedure must be repeated for every time instant of interest. Here, 10 design points $i=\{1, \ldots, 10\}: t_{i} \in\left[t_{0}, t_{f}\right]$ have been chosen with uniform intervals in terms of closing speed $\left\|\mathbf{v}_{\text {ref }}(t)\right\|$. The reason for this choice is twofold:

- System information becomes more refined closer to the final descent time, where the dynamics of $G_{\mathrm{SDK}}^{i}(\mathrm{~s})$ becomes faster (recalling Fig. 3.3a);
- The time-evolution depicted in Fig. 3.3b seems to be strongly determined by $\left\|\mathbf{v}_{\text {ref }}(t)\right\|$, as Fig. 3.5 will show.


Figure 3.3: Linear analysis of $G_{\text {SDK }}^{i}(\mathrm{~s})$ for RT1 with $t_{i} \in\left[t_{0}, t_{f}\right]$

### 3.4.1 Sensitivity-based uncertainty selection

In order to minimise the size of the resulting LFT model, only the GH coefficients of Eq. (3.3) with higher impact on $J_{\mathbf{f}}^{i}$ are selected and denoted $\rho_{\mathrm{GH}}$. In order to methodically perform this selection, two distinct criteria are applied: [i] impact of each coefficient on the nonlinear open-loop simulation of Eq. (3.4), and [ii] relative weight of each coefficient on dedicated interpolations (see next step). With the two criteria providing consistent results, the following set of 9 coefficients is selected:

$$
\rho_{\mathrm{GH}}=\left[\begin{array}{lllllllll}
C_{3,0} & C_{3,1} & C_{3,2} & C_{4,0} & C_{4,1} & C_{4,2} & C_{4,3} & S_{3,3} & S_{4,3} \tag{3.12}
\end{array}\right]
$$

This choice will be further revisited at the control analysis stage (Sec. 6.1) based on insights from a closed-loop sensitivity analysis.

### 3.4.2 Jacobian sampling and interpolation

Once the set of uncertainties is selected, an appropriate number of dispersed samples is generated and the Jacobian matrix of Eq. (3.10) is evaluated for each sample. Depending on the dispersion ranges considered, models with different levels of conservativeness can be obtained. As mentioned in Sec. 3.1, each GH coefficient is assumed to follow a Gaussian distribution with standard deviation equal to its nominal value.

Here, two cases are considered: $2 \sigma$ and $3 \sigma$ dispersions, capturing respectively 95.5 and $99.7 \%$ of uncertainty. For each case, a matrix with polynomial dependence on the uncertain parameters $J_{\mathbf{f}}^{i}\left(\rho_{\mathrm{GH}}\right)$ is then constructed.

This task was accomplished using the orthogonal least-squares interpolation routine provided with the Approximation of Polynomial and Rational-type for Indeterminate Coefficients via Optimisation Tools (APRICOT) library [RHB14], which is now openly available [ $\left.\mathrm{BBD}^{+} 16\right]$. After several tests, the maximum degree of the interpolating polynomials was fixed to 1, enabling the quantification of their dependence on each uncertainty coefficient.

### 3.4.3 LFT transformation and verification

The final step consists in converting the LTI system of Eq. (3.11) with $J_{\mathbf{f}}^{i}\left(\rho_{\mathrm{GH}}\right)$ into an LFT. This is also an embedded capability of the APRICOT library, which adjusts the necessary repetitions of each coefficient in $\rho_{\mathrm{GH}}$ to meet a pre-specified approximation error.

In the end, two sets of models, $G_{\mathrm{SDK}, 2 \sigma}^{i}(\mathrm{~s})$ and $G_{\mathrm{SDK}, 3 \sigma}^{i}(\mathrm{~s})$, are obtained for the points $i=\{1, \ldots, 10\}$, together with corresponding $\Delta_{\mathrm{SDK}, 2 \sigma}^{i}(\mathrm{~s})$ and $\Delta_{\mathrm{SDK}, 3 \sigma}^{i}(\mathrm{~s})$ blocks. Each block contains the uncertainties $\delta c_{n, m}(\mathrm{~s})$ and $\delta s_{n, m}(\mathrm{~s})$ (with $n$ and $m$ indicating the selected coefficients shown in Eq. (3.12)), normalised to $[-1,1]$ and repeated as necessary. The sum of these repetitions determines the size of the block and, the larger it is, the more elaborate the impact of the uncertainty in the dynamics of the system (and the more complicated to analyse).

It is therefore reasonable to expect that the uncertainty size changes with the level of conservativeness, but also along the trajectory as the dynamics become more complex closer to Phobos. This behaviour is depicted in Fig. 3.4a, where it is seen that the size of $\Delta_{\mathrm{SDK}, 2 \sigma}^{i}(\mathrm{~s})$ is never larger than $\Delta_{\mathrm{SDK}, 3 \sigma}^{i}(\mathrm{~s})$ and that they both tend to increase with time (i.e. towards Phobos), ranging from a size of 3 (when only some of the 9 uncertainties have a significant role) to 27 (when all the uncertainties appear repeated 3 times).

Finally, the accuracy of the LFTs is verified by comparing frequency response samples obtained from the LFT and from equally dispersed LTI realisations. An example of such a comparison is shown in Fig. 3.4b for the singular values of $G_{\mathrm{SDK}, 3 \sigma}^{10}(\mathrm{~s})$, i.e. the largest model. In this case, LFT accuracy (uppermost subplot of Fig. 3.4b) is confirmed by the good matching with the LTI responses (bottom subplot).

### 3.5 Inclusion of time-varying effects

The models developed in Sec. 3.3 and 3.4 are time-invariant, i.e. their characteristics are assumed to be frozen and locally valid at the instant of time $t_{i}$ for which they are derived. In practice, however, this is not the case, and the behaviour actually changes globally over the trajectory (and time). As introduced in Sec. 2.2.1.2, this TV effect can be captured using the LPV modelling paradigm.


Figure 3.4: Results of LFT verification step

The generation of an LPV model $G_{\text {SDK }}\left(\mathrm{s}, \rho_{\mathrm{TV}}(t)\right)$ follows similar lines to those described in Sec. 3.4. Here, the TV parameter can be directly defined as $\rho_{\mathrm{TV}}(t)=t$, the 10 Jacobian matrices can be written as a function of this parameter, i.e. $J_{\mathbf{f}}^{i}\left(\rho_{\mathrm{TV}}(t)\right), i=\{1, \ldots, 10\}$, and then interpolated. The 10 design points along RT1 and the corresponding closing velocity are illustrated in Fig. 3.5. The interpolation was performed using the LPVTools library [HSP15].


Figure 3.5: Closing speed of RT1 and design points chosen

Alternatively, the inclusion of GH uncertainties and TV effects can be merged through the combined interpolation of $J_{\mathbf{f}}^{i}\left(\left[\rho_{\mathrm{GH}}, \rho_{\mathrm{TV}}(t)\right]\right)$. This formulation was however not considered since, as it will be seen in Sec. 6.2, the impact of TV effects is much less significant than that of uncertainties.

### 3.6 Actuator and navigation models

In addition to simulating the physical environment in the vicinity of Phobos, Airbus' high-fidelity benchmark also included a set of actuator and navigation subsystems. The effects of these two subsystems have been summarised in Sec. 2.1.2 and are further detailed below.

Actuators introduce thruster realisation errors, which are time (and trajectory) dependent, but assumed to be bounded. In this case, actuation errors can be captured following the LFT approach, i.e. defining a simple model $G_{\mathrm{A}}(\mathrm{s})$ :

$$
\mathbf{a}(\mathrm{s})=\left(\mathbb{I}_{3 \times 3}+\left[\begin{array}{ccc}
w_{\mathrm{ax}} \delta_{\mathrm{ax}}(\mathrm{~s}) & w_{\mathrm{aod}} \delta_{\mathrm{aod}}(\mathrm{~s}) & w_{\mathrm{aod}} \delta_{\mathrm{aod}}(\mathrm{~s})  \tag{3.13}\\
w_{\mathrm{aod}} \delta_{\mathrm{aod}}(\mathrm{~s}) & w_{\mathrm{ay}} \delta_{\mathrm{ay}}(\mathrm{~s}) & w_{\mathrm{aod}} \delta_{\mathrm{aod}}(\mathrm{~s}) \\
w_{\mathrm{aod}} \delta_{\mathrm{aod}}(\mathrm{~s}) & w_{\mathrm{aod}} \delta_{\mathrm{aod}}(\mathrm{~s}) & w_{\mathrm{az}} \delta_{\mathrm{az}}(\mathrm{~s})
\end{array}\right]\right) \mathbf{a}_{\mathrm{cmd}}(\mathrm{~s})
$$

and uncertainty block:

$$
\begin{equation*}
\Delta_{\mathrm{A}}(\mathrm{~s})=\operatorname{diag}\left[\delta_{\mathrm{ax}}(\mathrm{~s}), \delta_{\mathrm{ay}}(\mathrm{~s}), \delta_{\mathrm{az}}(\mathrm{~s}), \mathbb{I}_{3 \times 3} \delta_{\mathrm{aod}}(\mathrm{~s})\right] \tag{3.14}
\end{equation*}
$$

Here, $\delta_{\text {ax }}, \delta_{\text {ay }}, \delta_{\text {az }}$ are uncertainties affecting the three axes and $\delta_{\text {aod }}$ represents off-diagonal effects. The relative ranges of $\Delta_{\mathrm{A}}(\mathrm{s})$ were established from Monte-Carlo (MC) time simulations of the high-fidelity benchmark to:

$$
\begin{equation*}
w_{\mathrm{ax}}=0.06, \quad w_{\mathrm{ay}}=0.11, \quad w_{\mathrm{az}}=0.12, \quad w_{\mathrm{aod}}=0.14 \tag{3.15}
\end{equation*}
$$

and then normalised to $[-1,1]$ with $M(\mathrm{~s})$ absorbing these scaling factors (recall Fig. 2.7).
Since uncertainty ranges are relative, $G_{\mathrm{A}}(\mathrm{s})$ is equally applicable to full acceleration commands $\mathbf{a}_{\mathrm{cmd}}(t)$ as well as to acceleration command perturbations $\delta \mathbf{a}_{\mathrm{cmd}}(t)=\mathbf{a}_{\mathrm{cmd}}(t)-\mathbf{a}_{\mathrm{ref}}(t)$, as per Eq. (3.8).

The sensors \& navigation subsystem, on the other hand, introduces two different perturbations: a quantisation and a noise error. The former effect is due to the fact that position and velocity estimates, $\hat{\mathbf{r}}(t)$ and $\hat{\mathbf{v}}(t)$ (recall Fig. 2.2), are updated every 60 seconds, injecting a non-smooth signal into the system. To attenuate it, these estimates are filtered by a first-order Low-Pass Filter (LPF) $G_{\text {LPF }}(\mathrm{s})$ with 0.05 Hz bandwidth.

The noise error is accounted for by colouring white noise signals $\mathbf{n}_{\mathrm{r}}(t)$ and $\mathbf{n}_{\mathrm{V}}(t)$ through appropriate filters $G_{\mathrm{rNAV}}(\mathrm{s})$ and $G_{\mathrm{vNAV}}(\mathrm{s})$. Their transfer functions were chosen such that the Power Spectral Density (PSD) produced by the actual, nonlinear sensors \& navigation block is recovered. The resulting transfer functions are given by:

$$
\begin{align*}
& G_{\mathrm{rNAV}}(\mathrm{~s})=40 \frac{0.001(\mathrm{~s}+1)}{\mathrm{s}+0.001} \mathbb{I}_{3 \times 3}  \tag{3.16}\\
& G_{\mathrm{vNAV}}(\mathrm{~s})=\frac{1}{30} \frac{0.03(\mathrm{~s}+1)}{\mathrm{s}+0.03} \mathbb{I}_{3 \times 3}
\end{align*}
$$

This is exemplified in Fig. 3.6 for $G_{\mathrm{rNAV}}(\mathrm{s})$.
Finally, and similar to the actuator model, the noise shaping filters are equally applicable to full and perturbed estimates since reference values $\mathbf{r}_{\text {ref }}(t)$ and $\mathbf{v}_{\text {ref }}(t)$ are known without error, therefore $\hat{\mathbf{r}}(t)-\mathbf{r}(t)=\delta \hat{\mathbf{r}}(t)-\delta \mathbf{r}(t)$ and $\hat{\mathbf{v}}(t)-\mathbf{v}(t)=\delta \hat{\mathbf{v}}(t)-\delta \mathbf{v}(t)$.


Figure 3.6: Actual (blue) vs. synthetic (red) position noise PSD


## Systematic Closed-Loop Guidance Tuning

Throughout the survey of Sec. 2.1.4, it was seen that most of the work on closed-loop guidance for small bodies recasts the Descent \& Landing (D\&L) problem as optimal feedback control with terminal constraints, for which optimal conditions can be analytically derived using the Pontryagin maximum principle or through calculus of variations. Nevertheless, these optimal conditions are only practical under the assumption of simplified and well-known gravity fields, which is not the case with Phobos.

From the surveyed techniques, a reconciliation (in exactitude, an underlying parametric generalisation) of $\mathrm{D} \& \mathrm{~L}$ guidance laws was identified in Sec. 2.1.4.4. Although a simple step, this parametrisation enables the possibility of applying systematic tuning methodologies, which may prove to be a paradigm change in the current state-of-practice for $\mathrm{D} \& \mathrm{~L}$ on small bodies.

In this chapter, a tuning methodology that relies on the identified parametric generalisation is presented and applied to Phobos D\&L guidance. This approach employs a systematic evaluation of the high-fidelity model of Sec. 3.1 over the design parameter space to generate trade-off maps that enable a clear performance quantification of candidate solutions. In addition, this approach provides a valuable understanding of the system dynamics that supports the application of robust control tools, including the structured $\mathcal{H}_{\infty}$ optimisation framework preceded in Sec. 2.2.2.2.

The chapter begins with the introduction of the baseline guidance technique in Sec. 4.1. The proposed methodology is then thorougly described in Sec. 4.2, applied to distinct D\&L trajectories in Sec. 4.3 and verified in Sec. 4.4. Following its verification, the reconciliation of the tuning methodology with structured $\mathcal{H}_{\infty}$ optimisation is presented in Sec. 4.5. Comparative results between the two approaches are then provided in Sec. 4.6.

The results presented in this chapter have been published in $\left[\mathrm{SMJ}^{+} 17 \mathrm{a}, \mathrm{SMJ}^{+} 19\right]$.

### 4.1 Parametric CTVG implementation

As mentioned above, this chapter is based on the parametric generalisation of D\&L guidance laws identified in Eq. (2.67). Using zero-effort error coordinates, the parametrisation is given by:

$$
\mathbf{a}(t)=\left[\begin{array}{ll}
k_{r} & k_{v}
\end{array}\right]\left[\begin{array}{c}
\frac{\operatorname{ZEM}(t)}{t_{\mathrm{go}}^{2}(t)}  \tag{4.1}\\
\frac{\mathbf{Z E V}(t)}{t_{\mathrm{go}}(t)}
\end{array}\right]
$$

and, depending on the choice of $\left\{k_{r}, k_{v}\right\}$, the corresponding guidance law yields different properties. As seen before, a nonlinear term $\mathbf{h}($.$) can also be considered to improve robustness$ properties, but this option was only exploited by Airbus' team and it is not included in the thesis for simplicity.

For the case of Constrained Terminal Velocity Guidance (CTVG), required for the specification of soft landing, standard values of $\{6,-2\}$ have been analytically derived, as detailed in Sec. 2.1.4.2, based on the cost function of Eq. (2.35) assuming simplified and well-known gravity fields. Note that the latter cost function does not represent a direct minimisation of $\Delta V$ (Eq. (3.7)), but it is generally easier to solve and provides a representative solution [JZS $\left.{ }^{+} 17, \mathrm{JZS}^{+} 18\right]$.

The estimation of Zero-Effort-Miss (ZEM) and Zero-Effort-Velocity (ZEV), Eq. (2.10), can become challenging without some approximations which, depending on their level of conservativeness, can affect the ability of the guidance law to accurately enforce the terminal conditions (i.e. to minimise touchdown errors). In order to mitigate this effect, zero-effort errors can also be defined with respect to a set of intermediate waypoints interpolated along a given Reference Trajectory (RT).

The waypoint-based approach is illustrated in Fig. 4.1. It has been implemented with a fixed guidance time-horizon $t_{\mathrm{h}}=500$ seconds using the optimised D\&L trajectories of Sec. 3.2. As anticipated in that section, those trajectories may not be feasible in open-loop due to modelling inaccuracies, but they can be effectively followed in a closed-loop setup. In Sec. 4.4, it will be shown that this results in total $\Delta V$ values slightly different from those in Fig. 3.2b.


Figure 4.1: Illustration of waypoint-based CTVG

Care must also be taken in Eq. (4.1) to avoid a singularity as $t_{\mathrm{go}}(t) \rightarrow 0$. The most effective way to do this is by switching-off the guidance commands immediately before the end-of-mission. The exact instant of time represents a trade-off between allowable touchdown error and maximum thrust authority.

### 4.2 Proposed tuning approach

A successful guidance tuning is then translated in an appropriate trade-off between acceptable touchdown (position and velocity) accuracy and the total $\Delta V$ needed for the $\mathrm{D} \& \mathrm{~L}$ manoeuvre. As mentioned before, using results from optimal control theory, standard closed-loop guidance gains can be analytically derived. Nevertheless, these gains are only practical under the assumption of simplified and well-known gravity fields.

In order to provide not just a systematic tuning methodology, but also a clear understanding of the aforementioned trade-off, a simulation-based approach is proposed in this section for the case of highly complex and perturbed gravitational environments. This approach relies on the availability of a nonlinear, high-fidelity simulation model (in the present case the Airbus' benchmark introduced in Sec. 3.1) to evaluate three key performance indicators:

- $R_{c}$ - Target distance (position error) at touchdown, i.e. $\left\|\mathbf{r}_{f}-\mathbf{r}\left(t_{f}\right)\right\|$;
- $V_{c}-$ Closing speed (velocity error) at touchdown, i.e. $\left\|\mathbf{v}_{f}-\mathbf{v}\left(t_{f}\right)\right\|$;
- $\Delta V$ - Total $\Delta V$, given by Eq. (3.7).

The proposed tuning approach is illustrated using the parametric guidance generalisation of Sec. 4.1 with intermediate waypoints along RT1 (recall Fig. 3.2b). It is described in the following paragraphs.

### 4.2.1 Step 1 - Nominal indicators

The first step of this approach is to evaluate the three performance indicators in nominal conditions over a parameter grid of $\left\{k_{r}, k_{v}\right\}$. This is represented for RT1 by the three plots of Fig. 4.2, where $k_{r} \in[3.5,6.5]$ and $k_{v} \in[-3.0,-1.5]$. These plots provide a clear visualisation of the tuning trade-off mentioned before. In exactitude, a choice of gains that minimises the touchdown error (either in terms of position and velocity) will maximise the required $\Delta V$ and vice-versa.

### 4.2.2 Step 2 - Dispersed indicators

The same principle is then employed to quantify the dispersed performance obtained with each pair $\left\{k_{r}, k_{v}\right\}$ by analysing the standard deviation of the key indicators for 100 random samples of the 19 uncertain Gravity Harmonics (GH) coefficients in Eq. (3.3) with Gaussian distributions.

The outcome of this analysis is provided in Fig. 4.3, where it is possible to observe that certain guidance combinations are associated with intense indicator peaks, which must be avoided.


Figure 4.2: Nominal performance indicators at touchdown for RT1


Figure 4.3: Dispersed performance indicators at touchdown for RT1

### 4.2.3 Step 3 - Trade-off maps

The final step is to generate a tuning trade-off map featuring both nominal and dispersed information by overlapping the contour plots of the previous two figures. Such trade-off map is depicted in Fig. 4.4 for RT1.

Focusing on Fig. 4.4a, blue and red solid lines represent the contour plots of respectively closing speed (from Fig. 4.2b) and total $\Delta V$ (from Fig. 4.2c). Note that target distance could have been used instead of closing speed since their nominal and dispersed trends are similar. As mentioned before, a minimisation of $V_{c}$ requires an increment of $\Delta V$ and vice-versa, but solutions exist such that minor degradations in $V_{c}$ allow for high $\Delta V$ improvements.

Furthermore, the map of $\Delta V$ has a global minimum (under $8.8 \mathrm{~m} / \mathrm{s}$ ), but it coincides with the area where $V_{c}$ becomes significantly higher. For this reason, the transition area corresponds to a peak of $V_{c}$ dispersion (see Fig. 4.3b), which is depicted in the trade-off map of Fig. 4.4a using dashed black lines. In the same map, the contours of $\Delta V$ dispersion (from Fig. 4.3c) are
represented using dashed magenta lines. This plot allows to identify a global maximum (close to the standard gains $\{6,-2\}$ and marked with a red $\times$ ) and a local minimum next to the nominal $\Delta V$ minimum.


Figure 4.4: Guidance trade-off map for RT1 $(\times / \diamond$ indicate standard/revised gains)

For an easier visualisation of the observations above, Fig. 4.4b highlights the undesirable tuning regions of Fig. 4.4a. This clearly leads to the conclusion that the standard choice of gains is not the most suitable (at least for the studied Phobos mission) since it is associated with relatively high values of both nominal and dispersed $\Delta V$.

Hence, the proposed tuning trade-off process and maps can be systematically employed for a more favourable selection of guidance gains. For example, choosing $\left\{k_{r}, k_{v}\right\}=\{5,-2.35\}$, marked with a green $\diamond$ in Fig. 4.4a and 4.4 b , allows $\Delta V$ to be reduced from 15.9 to less than $9 \mathrm{~m} / \mathrm{s}$ while only increasing $V_{c}$ from slightly more than 0.088 to approximately $0.092 \mathrm{~m} / \mathrm{s}$. At the same time, although it does not correspond to the minimum of $\Delta V$ dispersion, this choice reduces its value from around 0.9 to less than $0.88 \mathrm{~m} / \mathrm{s}$ and, most importantly, keeps a safety margin with respect to the peaks of nominal and dispersed $V_{c}$.

The exact nominal values can be read directly from the map, but the dispersed indicators shall go through a more extensive Monte-Carlo (MC) validation since, for the sake of computational efficiency, the trade-off map is based on a very limited number (100) of simulations per guidance solution $\left\{k_{r}, k_{v}\right\}$. Before performing such a validation, the applicability of the proposed approach to other D\&L trajectories is assessed in the next section.

### 4.3 Applicability to other trajectories

Following the same procedure of Sec. 4.2, trade-off maps for RT2 and RT3 (Fig. 4.5a and 4.5b) can be generated. As before, the same standard and revised choices of gains are marked with $\times$ and $\diamond$, respectively.


Figure 4.5: Guidance trade-off maps for other trajectories ( $\times / \diamond$ indicate standard/revised gains)

For both trajectories, it can be seen that, similar to RT1, switching from standard to revised choice of gains results in a large decrease of $\Delta V$ (about $3.5 \mathrm{~m} / \mathrm{s}$ for RT2 and $2.5 \mathrm{~m} / \mathrm{s}$ for RT3) with a minor increase of $V_{c}$ (approximately $1 \mathrm{~mm} / \mathrm{s}$ for both). The main difference between these two trajectories and RT1 is the absence of $V_{c}$ dispersion peaks, denoted by the dashed black lines. In the present cases, this indicator is inversely proportional to $k_{r}$, which seems to indicate the existence of a terrain hazard in RT1 where the spacecraft collides for certain guidance solutions.

Also, it is possible to verify that, contrary to RT1, the revised choice of gain is not the best for RT2 and RT3 in terms of $\Delta V$ dispersion (dashed magenta lines), since it is associated with higher values than the standard gains. However, the impact of this behaviour is much less significant than the nominal $\Delta V$ reduction and, for simplicity, the same revised gains are kept for the rest of the chapter.

For a real-world application, the practical implementation of the proposed tuning methodology is envisaged as follows:

1. High-fidelity simulation and guidance trade-off maps are generated on the ground, before the mission, for a set of candidate reference trajectories;
2. Once the spacecraft approaches the target and local analyses are carried out, the most suitable reference trajectory can be selected;
3. The most performing tuning selection is then determined using the appropriate trade-off map and implemented before initiating the $\mathrm{D} \& \mathrm{~L}$ manoeuvre.

The main benefit of this approach is that guidance tuning is able to account for the actual $\Delta V$ available and acceptable landing accuracy (e.g. based on visual observations) at the time, without involving any major real-time computation.

### 4.4 Verification and discussion

To validate the results discussed in the previous two sections, the revised choice of guidance gains $\left\{k_{r}, k_{v}\right\}=\{5,-2.35\}$ is now tested and compared with the standard guidance selection $\left\{k_{r}, k_{v}\right\}=\{6,-2\}$ using the benchmark introduced in Sec. 3.1 and the parametric guidance of Sec. 4.1. Each guidance tuning is simulated against the same 1000 MC samples of the 19 GH coefficients with Gaussian distributions for the three RTs. As mentioned in Sec. 4.3, the revised gains have been optimised for RT1 but are kept the same for RT2 and RT3 for the sake of simplicity. Moreover, since the focus of this analysis is on the impact of the GH coefficients, no additional perturbations from actuators or navigation are included in the simulations.

The outcome of this validation campaign is depicted in Fig. 4.6, with a different column for each RT and using darker and lighter lines for results using respectively the standard or the revised gains. For each case, the target distance $\left\|\mathbf{r}_{f}-\mathbf{r}(t)\right\|$, closing speed $\left\|\mathbf{v}_{f}-\mathbf{v}(t)\right\|$ and required $\Delta V$ are given. Trajectories in 3D are not shown since differences amongst them are so small that they cannot be distinguished from those in Fig. 3.2b.

From Fig. 4.6 it is clear that, for the three trajectories, the revised gain selection results in significant $\Delta V$ savings with a minimal impact on position and velocity errors. Since the revised gains have been optimised for RT 1 , it is natural that $\Delta V$ savings are higher for this trajectory than for RT2 and RT3. For a detailed comparison, the average and standard deviation of the final values found in the MC simulations are recorded in Table 4.1. Note that these indicators are directly related to the ones employed for the generation of the guidance trade-off maps in Sec. 4.2.

As anticipated for RT1, the revised gains enable a significant reduction in both average and dispersed $\Delta V$ at the expense of a minor increase in average and dispersed velocity error. In fact, in terms of average indicators, a $\Delta V$ reduction of $44.5 \%$ is achieved with a velocity error increase of only $3.4 \%$. As mentioned before, these indicators can be directly read from the trade-off map of Fig. 4.4. On the contrary, the dispersed indicators do not correspond exactly
to the values of Fig. 4.4 because the trade-off map was generated from a smaller number of simulations (100), but the mismatch is relatively small $(\approx 9 \%)$.


Figure 4.6: High-fidelity simulation of 1000 MC runs per trajectory with different gains

Table 4.1: Performance comparison of 1000 MC runs per trajectory with different gains

|  | RT1 |  | RT2 |  | RT3 |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Standard gains | Avg. | Std. | Avg. | Std. | Avg. | Std. |
| $R_{c}(\mathrm{~m})$ | 1.118 | $8.15 \times 10^{-5}$ | 11.949 | $1.82 \times 10^{-4}$ | 39.053 | $1.72 \times 10^{-4}$ |
| $V_{c}(\mathrm{~m} / \mathrm{s})$ | 0.089 | $4.91 \times 10^{-9}$ | 0.200 | $1.45 \times 10^{-8}$ | 0.404 | $2.19 \times 10^{-8}$ |
| $\Delta V(\mathrm{~m} / \mathrm{s})$ | 15.905 | $9.96 \times 10^{-1}$ | 12.574 | $6.25 \times 10^{-1}$ | 10.586 | $7.02 \times 10^{-1}$ |
| Revised gains | Avg. | Std. | Avg. | Std. | Avg. | Std. |
| $R_{c}(\mathrm{~m})$ | 1.117 | $9.80 \times 10^{-5}$ | 11.946 | $2.19 \times 10^{-4}$ | 39.050 | $2.07 \times 10^{-4}$ |
| $V_{c}(\mathrm{~m} / \mathrm{s})$ | 0.092 | $6.32 \times 10^{-9}$ | 0.202 | $1.76 \times 10^{-8}$ | 0.405 | $2.64 \times 10^{-8}$ |
| $\Delta V(\mathrm{~m} / \mathrm{s})$ | 8.819 | $9.71 \times 10^{-1}$ | 9.009 | $8.00 \times 10^{-1}$ | 8.073 | $8.24 \times 10^{-1}$ |

Similar conclusions then hold for RT2 and RT3, i.e. the revised gains enable a significant reduction in terms of average $\Delta V$ at the expense of a minor increase in average and dispersed velocity errors as well as dispersed $\Delta V$, as anticipated from Sec. 4.3. The increase of the latter
is however very small ( $0.18 \mathrm{~m} / \mathrm{s}$ for RT2 and $0.12 \mathrm{~m} / \mathrm{s}$ for RT3) and is clearly outweighed by the improvements of the average $\Delta V$ ( 28.5 and $23.7 \%$, respectively).

With the aforementioned observations in mind, not only does this section validate the proposed guidance tuning methodology, it also confirms that the state-of-practice tuning selection, based on the assumption of simplified and well-known gravity fields, is not fuel-optimal for the case of Phobos.

### 4.5 Reconciliation with structured $\mathcal{H}_{\infty}$ optimisation

The closed-loop guidance laws considered in this chapter, Eq. (4.1), have a fixed structure parameterised by tunable gains $\left\{k_{r}, k_{v}\right\}$, which makes them perfect candidates for the application of the structured $\mathcal{H}_{\infty}$ optimisation paradigm.

The main strengths and drawbacks of structured $\mathcal{H}_{\infty}$ are summarised in Table 2.2. In addition to the ability to specify the structure of the guidance law, its application to the D\&L problem offers two advantages:

- As it is founded on robust control framework, it allows to explicitly account for uncertain gravitational elements;
- It is able to handle directly and simultaneously multiple tuning requirements and design models, providing solutions that are guaranteed for a set of operating points or RTs.

The main difficulty in the application of structured $\mathcal{H}_{\infty}$ optimisation to this problem lies on its formulation in the $\mathcal{H}_{\infty}$ framework. As introduced in Sec. 2.1.4.2, state-of-practice guidance solutions have been found by recasting it as an optimal control problem with constrained terminal position and velocity. This is substantially different from the structured $\mathcal{H}_{\infty}$ problem, which aims at minimising the induced $\mathcal{H}_{\infty}$-norm of pre-specified input-output control channels. In this case, the problem of Eq. (2.82) is written as:

$$
\begin{equation*}
\min _{k_{r}, k_{v}} \max \left(\left\|M_{1}(\mathrm{~s})\right\|_{\infty}, \ldots,\left\|M_{N}(\mathrm{~s})\right\|_{\infty}\right) \tag{4.2}
\end{equation*}
$$

where $M_{i}(\mathrm{~s})$, with $i=\{1, \ldots, N\}$, are Linear Time-Invariant (LTI) representations of the system dynamics at different operating points along one or more RTs.

All the necessary system interconnections to address this guidance tuning problem have been implemented in Simulink, as depicted in Fig. 4.7. Using this setup, all the models $M_{i}$ (s) can be retrieved using Simulink's slTuner interface, which allows to automatically create linear models of blocks featuring tunable parameters (in this case $\left\{k_{r}, k_{v}\right\}$ ).

The linear representations $M_{i}(\mathrm{~s})$ include not only the Spacecraft Dynamics \& Kinematics (SDK) block (recall Sec. 3.3), but also the structure of the guidance algorithm. Furthermore, the SDK block can be conveniently replaced by the orbital perturbation model of Eq. (3.11) to consider a multi-plant design, adding in this case the required transformation of Eq. (3.8)
between total and perturbed variables. In addition to the parametric structure, i.e. $\left\{k_{r}, k_{v}\right\}$, the guidance algorithm block includes estimators for the apparent gravity (Eq. (3.5)) and zero-effort errors (Eq. (2.10)).


Figure 4.7: Simulink model for structured guidance tuning

Furthermore, the Simulink model highlights (with green and orange blocks) input-output control channels, which provide the basis for the definition of tuning objectives. The specification of these objectives is a key factor in $\mathcal{H}_{\infty}$-based approaches, often resulting in an iterative process. The adopted process is described below and encompasses the following steps:

1. Input-output channel specification as functions of $\left\{k_{r}, k_{v}\right\}$;
2. Problem/trade-off quantification using $\mathcal{H}_{\infty}$-norm objectives;
3. Selection of applicable set of nominal design points or trajectories;
4. Uncertainty specification using Linear Fractional Transformation (LFT) models.

The first step is the selection of input and output channels with respect to which the tuning objectives are to be defined. These channels must contain the gains $\left\{k_{r}, k_{v}\right\}$ on their path and be physically related to the intended tuning objectives. Following the same reasoning of Sec. 4.2, two competing requirements are addressed: accuracy (i.e. minimisation of touchdown error) and efficiency (i.e. minimisation of propellant consumption). To achieve this, the touchdown position vector $\mathbf{r}_{f}$ is selected as control input and two dedicated output signals are defined:

- $\mathbf{z}_{r}(t)=\mathbf{r}_{f}-\mathbf{r}(t)$, measuring the deviation with respect to touchdown site. Remark: a velocity deviation could have also been included, but it was found to be redundant and hence not considered for the sake of simplicity;
- $z_{a}(t)=\mathbf{a}^{\mathrm{T}}(t) \mathbf{a}(t)\left(\frac{t_{\mathrm{go}}(t)}{10^{-3}}\right)^{2}$, quantifying the instantaneous actuation effort commanded to compensate for the former deviation. Remark: the acceleration signal is scaled by $t_{\mathrm{go}}(t)$ to normalise this indicator throughout the descent (recall that actuation effort tends to increase as $\operatorname{tgo}_{\mathrm{go}}(t) \rightarrow 0$ ) and by a constant factor of $10^{3}$ to operate with more suitable units ( $\mathrm{mm} / \mathrm{s}^{2}$ instead of $\mathrm{m} / \mathrm{s}^{2}$ ).

Although the selection of the channels is driven by engineering objectives (e.g. minimise a specific signal), the "best" configuration is not always clear, and different channels' choices might be needed depending on the optimisation outcome. Here, objectives are defined by constraining the $\mathcal{H}_{\infty}$-norm of the control channels as follows:

$$
\begin{align*}
& \left\|M_{\mathbf{r}_{f} \rightarrow \mathbf{z}_{r}}(\mathrm{~s})\right\|_{\infty}<\frac{\gamma_{\mathrm{acc}}}{\alpha} \\
& \left\|M_{\mathbf{r}_{f} \rightarrow z_{a}}(\mathrm{~s})\right\|_{\infty}<\frac{\gamma_{\mathrm{eff}}}{\alpha-1} \tag{4.3}
\end{align*}
$$

In these equations, $M_{\mathbf{r}_{f} \rightarrow \mathbf{z}_{r}}(\mathrm{~s})$ and $M_{\mathbf{r}_{f} \rightarrow z_{a}}(\mathrm{~s})$ are the control channel transfer functions (obtained with slTuner) which depend on $\left\{k_{r}, k_{v}\right\}$, and the parameters $\gamma_{\mathrm{acc}}>0$ and $\gamma_{\mathrm{eff}}>0$ represent the constraints associated with accuracy and efficiency requirements.

The constant $\alpha \in] 0,1[$ is a parameter that allows to exploit the underlying trade-off between the two requirements: when $\alpha \rightarrow 1$ the optimiser focuses on accuracy and when $\alpha \rightarrow 0$ on efficiency. Note that the interval is open in order to prevent the singularity in Eq. (4.3). $\gamma_{\text {acc }}$ and $\gamma_{\text {eff }}$ can be frequency-dependent but, for simplicity, constant values of 0.16 and 25 , respectively, have been adopted. Once again, these values were chosen iteratively and may need revisiting based on the models employed for the optimisation.

Selecting the linear models to be considered is another key step in the process. Different design plants $M_{i}(\mathrm{~s})$ can be specified through the findop routine, which runs the Simulink model up to the desired operating point and provides the LTI representation at that point. Following the multi-plant approach of structured $\mathcal{H}_{\infty}$, those plants are then aggregated in a block-diagonal structure. Here, all the design models have been considered except the last one (at $t=t_{f}$ ) since it leads to $t_{g o}=0$. This choice is also affected by how tight the performance and robustness specifications are defined.

Finally, robustness against gravitational uncertainties can be explicitly accounted for by replacing the linear SDK models in Fig. 4.7 by the LFTs developed in Sec. 3.4. The LFT models can be injected using the BlockSubs field of slTuner. At each step of the iterative process, the systune routine is then called to find optimal gains $\left\{k_{r}, k_{v}\right\}$ that meet the requirements of Eq. (4.3) at every chosen operating point under nominal or dispersed gravitational conditions.

### 4.6 Comparison of results

The guidance tuning results obtained for RT1 using the setup described in the previous section are depicted in Fig. 4.8, which shows the optimal gains $\left\{k_{r}, k_{v}\right\}$ for different values of $\left.\alpha \in\right] 0,1[$
(and therefore different objective combinations of efficiency and accuracy) given in percentage terms. In this figure, continuous lines represent the nominal (NOM) solution and dashed lines that derived using the LFT models.

To generate this figure, the two gains are allowed to vary within the same intervals considered in Sec. 4.2 (indicated by thin horizontal, black lines) and initialised with their state-of-practice values $\{6,-2\}$. The multiple random initialisation capability of systune was also investigated, but the solutions found by the optimiser remained the same, indicating that it is unlikely that these solutions correspond to local minima.


Figure 4.8: Guidance tuning trade-off results for RT1, ranging from $100 \%$ efficiency $(\alpha \rightarrow 0)$ to $100 \%$ accuracy $(\alpha \rightarrow 1)$

From Fig. 4.8, it is clear that extreme tuning objectives ( $100 \%$ efficiency or $100 \%$ accuracy) lead to a completely opposite trend in optimal gains. This trade-off was already anticipated based on the understanding provided by the trade-off maps. Note also that, for $100 \%$ accuracy, the optimal value of $k_{v}$ would reach its lower limit if $k_{r}$ was allowed to have higher values.

Between the two extremes, there is a smooth transition of optimal gains as a function of the objective combination. This transition is different for the nominal and LFT solutions. For a clearer interpretation of results, the structured $\mathcal{H}_{\infty}$ tuning solutions of Fig. 4.8 are plotted in cyan on top of the trade-off map of RT1 (from Fig. 4.4) in Fig. 4.9, and referred to as "stune". Intermediate trade-off points are also highlighted.

From Fig. 4.9, it is confirmed that the difference between optimal nominal and LFT results is consistent with the understanding provided by the trade-off map, that is, LFT solutions are shifted towards the top right corner of the plot, away from the dispersion peaks of both touchdown error and total $\Delta V$.

The main observation, however, is that, using the structured $\mathcal{H}_{\infty}$ guidance tuning methodology, the standard gain selection of $\{6,-2\}$ (given by the red $\times$ ) could be successfully recovered. This takes place for nominal conditions, the same under which the gains were analytically derived, and for a combination of $83 \%$ accuracy / $17 \%$ efficiency.


Figure 4.9: Visualisation of tuning results (of Fig. 4.8) over the trade-off map (of Fig. 4.4)

The fact that the ratio between accuracy and efficiency requirements is so high in this case is again related to the way the optimal D\&L control problem (Sec. 2.1.4.2) was formulated: since terminal position and velocity are constrained variables in that problem, it is reasonable that accuracy requirements have a stronger impact in the $\mathcal{H}_{\infty}$-norm trade-off.

Nevertheless, for this scenario, the structured tuning approach was unable to capture more performing guidance solutions, such as the one marked with the green $\diamond$, obtained using the trade-off maps and systematic methodology proposed before. This is due to the loss of highly nonlinear effects with the linearisations performed by slTuner, as well as of their propagation throughout the D\&L trajectory.

The aforementioned limitation of this method is in fact endemic to all LTI-based design and tuning approaches. For that reason, the applicability of structured $\mathcal{H}_{\infty}$ optimisation to problems where the propagation of highly nonlinear dynamics is key remains an open research question. This applicability can be tackled either by adopting a different type of formulation (e.g. structured LPV) for the guidance problem or by modifying the behaviour of the structured $\mathcal{H}_{\infty}$ algorithm itself.


## Synthesis of Robust Control Compensators

The conceptual separation between guidance and control introduced in Sec. 2.1.2 has been assumed throughout this thesis. Following this logic, Chapter 4 was dedicated to the assessment and optimisation of different Descent \& Landing (D\&L) closed-loop guidance laws. In the present chapter, improved $D \& L$ strategies are obtained through the augmentation with a simple yet robust control compensator, which enables the adoption of simpler (e.g. open-loop) guidance schemes.

In fact, significant room for improvement can be achieved at this level since the state-ofpractice in $\mathrm{D} \& \mathrm{~L}$ control compensation is conventionally over-simplified or even non-existent, which is only practical if the target body is well-known or if guidance algorithms do not rely on its natural dynamics. The application of robust control techniques is nowadays a well-established practice for spacecraft attitude (rotational) control synthesis, but not for orbital (translational) control.

The proposed design approach is further motivated by, and based on, the structured $\mathcal{H}_{\infty}$ optimisation paradigm which, as explained in Sec. 2.2.2, is particularly well-oriented towards the industrial state-of-practice. This point has been central during the whole thesis, since the applicability of legacy knowledge is fundamental within the space industry.

Controller structure, performance requirements, synthesis steps and nominal results are presented from Sec. 5.1 to 5.4. Throughout these sections, the suitability of structured $\mathcal{H}_{\infty}$ to the D\&L problem, as well its main challenges, are further discussed. The robustness of the designed compensators will then be thoroughly assessed in Chapter 6 using the analysis tools of Sec. 2.2.3 together with high-fidelity Monte-Carlo (MC) campaigns.

The results presented in this chapter have been published in $\left[\mathrm{SMJ}^{+} 18 \mathrm{~b}, \mathrm{SMJ}^{+} 18 \mathrm{c}\right]$.

### 5.1 Control structure

The scope of D\&L control compensation was introduced in Sec. 2.1.2. The compensator's objective is to provide an additional acceleration command $\mathbf{a}_{\mathrm{cmp}}(t)$ to compensate for deviations with respect to a given Reference Trajectory (RT). This task must be achieved regardless of the type of guidance (i.e. open or closed-loop) and of the trajectory to be followed.

In Chapter 3, it was seen that RTs are convenient to define a set of design points (i.e. apparent gravity Jacobian matrices), which are then augmented via the inclusion of gravitational uncertainties. In this case, control compensators can be designed using structured $\mathcal{H}_{\infty}$ optimisation to exploit the advantages highlighted in Table 2.2.

To fulfil the control objective mentioned above, the internal architecture chosen for the compensator (as introduced in Fig. 2.2b) is evidenced in Fig. 5.1.


Figure 5.1: Control compensator architecture
At the core of the compensator, there is a Linear Time-Invariant (LTI) controller $K^{i}(\mathrm{~s})$ that must be designed to track commanded deviations $\delta \mathbf{r}_{\mathrm{cmd}}(t)$ using position and velocity deviation measurements $\delta \hat{\mathbf{r}}(t)$ and $\delta \hat{\mathbf{v}}(t)$ relative to the RT $\left\{\mathbf{r}_{\mathrm{ref}}(t), \mathbf{v}_{\mathrm{ref}}(t)\right\}$. These signals are filtered by a first-order low-pass filter $G_{\mathrm{LPF}}(\mathrm{s})$, introduced in Sec. 3.6, to attenuate the quantisation effect injected by the navigation algorithms.

An additional velocity command $\delta \mathbf{v}_{\mathrm{cmd}}(t)$ could have been included, but it was found to be redundant and hence not considered. For this case, the commanded deviation signal is fixed to zero, $\delta \mathbf{r}_{\mathrm{cmd}}(t)=0$, but it can also be employed to command small changes of landing site without the need for redesigning the trajectory.

With such a structure (fed by position and velocity deviations and providing an acceleration command $\mathbf{a}_{\text {cmp }}(t)$ that is summed to a reference acceleration profile $\left.\mathbf{a}_{\mathrm{ref}}(t)\right)$ the controller can be designed based on the orbital perturbation model of Sec. 3.3 by realising that the acceleration command is actually a perturbation, i.e. $\mathbf{a}_{\mathrm{cmp}}(t)=\delta \mathbf{a}(t)$. The closed-loop interconnection is then depicted in Fig. 5.2.

In addition to the blocks already present in the previous figure, Fig. 5.2 features actuator's model $G_{\mathrm{A}}(\mathrm{s})$ and uncertainty $\Delta_{\mathrm{A}}(\mathrm{s})$, and navigation noise-shaping filters $G_{\mathrm{rNAV}}(\mathrm{s})$ and $G_{\mathrm{vNAV}}(\mathrm{s})$, all of them described in Sec. 3.6. It also includes the orbital perturbation models $G_{\mathrm{SDK}, *}^{i}(\mathrm{~s})$ and


Figure 5.2: Closed-loop model for control synthesis
gravitational uncertainty $\Delta_{\text {SDK,* }}^{i}(\mathrm{~s})$ developed in Sec. 3.4. Here, the superscript $i=\{1, \ldots, 10\}$ specifies different designs point along the RT and $*$ represents either $2 \sigma$ or $3 \sigma$, depending on the Linear Fractional Transformation (LFT) conservativeness at each point.

Generically speaking, a different LTI controller $K^{i}(\mathrm{~s})$ can be synthesised and implemented for each design point $i$. Nevertheless, both hinfstruct and systune enable additional alternative strategies (with increasing level of complexity):

- Single-plant controller, when synthesised so as to fulfil the control requirements for a single design point $i=i^{*}$, but then applied throughout the trajectory. It is therefore clear that this controller may perform inappropriately at other trajectory points $i \neq i^{*}$.
- Scheduled controller, if a different controller is designed for each LTI point $i=\{1, \ldots, 10\}$ and then interpolated ad hoc to cover all the trajectory. This interpolation can however become challenging for dynamic compensators and implementation problems may arise.
- Multi-plant controller, when a single controller is synthesised so as to fulfil the control requirements at all the design points $i=\{1, \ldots, 10\}$ simultaneously. The feasibility of this approach naturally depends on how different the design points are from each other.
- Self-scheduled controller [SBB13, LSZ14], similar to the multi-plant controller, but parameterised as a function of a scheduling variable. It performs as a scheduled controller, but with an interpolation law explicitly enforced.

As it will be demonstrated in Chapter 6, the multi-plant controller synthesis approach proved to be very successful for this D\&L problem. Hence, in Fig. 5.1 and 5.2 there will be only one controller $K^{i}(\mathrm{~s})=K(\mathrm{~s})$ for all the design points $i=\{1, \ldots, 10\}$.

### 5.2 Control requirements

In order to ensure a successful control compensation, the controller $K(\mathrm{~s})$ must be designed in such a way that, even under an uncertain gravitational environment and perturbed actuator and sensor responses, two driving requirements are fulfilled:

- Tracking, minimising the error $\delta \mathbf{r}_{\mathrm{e}}(\mathrm{s})=\delta \mathbf{r}_{\mathrm{cmd}}(\mathrm{s})-\delta \mathbf{r}(\mathrm{s})$;
- Actuation, minimising the overall control effort $\delta \mathbf{a}(\mathrm{s})$.

As described in Sec. 2.2.2, control requirements are posed by rearranging the design problem into the generalised framework of Fig. 2.8, reproduced below for ease of readability. More specifically, comparing the latter figure with Fig. 5.2, one can identify as exogenous inputs:

$$
\begin{equation*}
\mathbf{w}(\mathrm{s})=\left[\delta \mathbf{r}_{\mathrm{cmd}}(\mathrm{~s}) \mathbf{n}_{\mathrm{r}}(\mathrm{~s}) \mathbf{n}_{\mathrm{v}}(\mathrm{~s})\right]^{\mathrm{T}} \tag{5.1}
\end{equation*}
$$

and as regulated outputs:

$$
\begin{equation*}
\mathbf{z}(\mathrm{s})=\left[\delta \mathbf{r}_{\mathrm{e}}(\mathrm{~s}) \delta \mathbf{a}(\mathrm{s})\right]^{\mathrm{T}} \tag{5.2}
\end{equation*}
$$

In addition, all the uncertain elements are combined into a single uncertainty block:

$$
\begin{equation*}
\Delta^{i}(\mathrm{~s})=\operatorname{diag}\left[\Delta_{\mathrm{SDK}, *}^{i}(\mathrm{~s}), \Delta_{\mathrm{A}}(\mathrm{~s})\right] \tag{5.3}
\end{equation*}
$$

and everything else is gathered in the LTI system $M^{i}(\mathrm{~s})$, again for $i=\{1, \ldots, 10\}$. Control requirements are then defined through the appropriate choice of frequency-dependent weights $W_{i}(\mathrm{~s})$ and $W_{o}(\mathrm{~s})$, also contained in $M^{i}(\mathrm{~s})$ as shown in Fig. 2.8.


Figure 2.8: Generalised interconnection in the $\mathcal{H}_{\infty}$ framework (repeated from page 45)
Weight selection is a key factor for the synthesis, but also one of the most time-consuming, typically involving several iterations. For standard $\mathcal{H}_{\infty}$ synthesis, weight complexity must be chosen as low as possible to limit the controller order, but this does not apply to structured $\mathcal{H}_{\infty}$. In addition, thanks to the multi-plant approach of the latter, it is possible to use different weights for different design points $i$. Still, the number of weights shall be the lowest possible in order to
reduce the designer's work (i.e. fewer parameters to tune) and the risk of over-constraining the optimisation with conflicting requirements.

For this problem, weights were chosen to be the same for all the design points (in general, this might not be possible). The input weight $W_{i}(\mathrm{~s})$ is employed for a differential scaling within the input signal w(s) of Eq. (5.1). For this specific case, with main focus on the effect of gravitational uncertainties, no noise has been considered for design, therefore $W_{i}(\mathrm{~s})=\operatorname{diag}\left[\mathbb{I}_{3 \times 3}, 0_{3 \times 3}, 0_{3 \times 3}\right]$. Resilience to noise is achieved by adjusting these values accordingly. The output weight $W_{o}(\mathrm{~s})$ is applied to the output signal $\mathbf{z}(\mathrm{s})$ of Eq. (5.2) and partitioned as $W_{o}(\mathrm{~s})=\operatorname{diag}\left[W_{S}(\mathrm{~s}), W_{A}(\mathrm{~s})\right]$, where $W_{S}(\mathrm{~s})$ imposes the tracking requirements via $\delta \mathbf{r}_{\mathrm{e}}(\mathrm{s})$ and $W_{A}(\mathrm{~s})$ the actuation requirements through $\delta \mathbf{a}(\mathrm{s})$. These two requirements represent a generalisation to constrain the sensitivity and control sensitivity transfer functions presented in Sec. 2.2.2.1.

More specifically, as a consequence of Eq. (2.79), the magnitude response of $\delta \mathbf{r}_{\mathrm{e}}(\mathrm{s})$ for a unitary command in $\delta \mathbf{r}_{\mathrm{cmd}}(\mathrm{s})$ is bounded by $W_{S}^{-1}(\mathrm{~s})$. After a few trial-and-error design/analysis iterations, the following characteristics were selected for $W_{S}^{-1}(\mathrm{~s})$ :

- Small Low-Frequency (LF) gain $\left(10^{-3}=-60 \mathrm{~dB}\right)$ for small steady-state error;
- Reasonable High-Frequency (HF) gain $(2 \approx 6 \mathrm{~dB})$ for good stability margins (from Eq. (2.80));
- Roll-over frequency ( $10^{-2} \mathrm{rad} / \mathrm{s}$ ) that is fast enough for the dynamics of the problem;
- Zero off-diagonal terms to minimise cross-coupled interactions.

These characteristics can be imposed by defining the weight as the following generic first-order transfer function [ZDG95]:

$$
\begin{equation*}
W(\mathrm{~s})=K_{\mathrm{LF}} \frac{K_{\mathrm{HF}} \mathrm{~s}+\omega_{\mathrm{B}}}{K_{\mathrm{LF}} \mathrm{~s}+\omega_{\mathrm{B}}} \tag{5.4}
\end{equation*}
$$

where $K_{\mathrm{LF}}$ and $K_{\mathrm{HF}}$ represent the gain of the LF and HF asymptotes, respectively, and $\omega_{\mathrm{B}}$ is the desired bandwidth. Hence, $W_{S}(\mathrm{~s})$ is written as:

$$
\begin{equation*}
W_{S}(\mathrm{~s})=\frac{\frac{1}{2} \mathrm{~s}+10^{-2}}{\mathrm{~s}+10^{-5}} \mathbb{I}_{3 \times 3} \tag{5.5}
\end{equation*}
$$

In a similar fashion, the magnitude response of $\delta \mathbf{a}(\mathrm{s})$ for a unitary command in $\delta \mathbf{r}_{\mathrm{cmd}}(\mathrm{s})$ is bounded by $W_{A}^{-1}(\mathrm{~s})$, and the following characteristics were selected:

- Reasonable LF gain $\left(5.5 \times 10^{-4} \approx-65 \mathrm{~dB}\right)$ which establishes the maximum control effort;
- Small HF gain $\left(10^{-9}=-180 \mathrm{~dB}\right)$ for small reactivity to noisy signals;
- A roll-off frequency $\left(10^{-3} \mathrm{rad} / \mathrm{s}\right)$ able to accommodate the tracking bandwidth;
- Zero off-diagonal terms to minimise cross-coupled interactions.

Hence:

$$
\begin{equation*}
W_{A}(\mathrm{~s})=\frac{\mathrm{s}+\frac{10}{5.5}}{10^{-9} \mathrm{~s}+10^{-3}} \mathbb{I}_{3 \times 3} \tag{5.6}
\end{equation*}
$$

### 5.3 Control synthesis

Following the multi-plant synthesis approach, plants $M^{i}(\mathrm{~s})$ are aggregated in a block-diagonal structure and the hinfstruct routine is called to find a single stabilising controller $K(\mathrm{~s})$ that meets the requirements of Sec. 5.2 at every LTI point $i=\{1, \ldots, 10\}$.

As mentioned in Sec. 2.2.2.2, structured $\mathcal{H}_{\infty}$ is able to seamlessly account for parametric uncertainties $\Delta^{i}(\mathrm{~s})$ in the optimisation problem. In the same section, it was anticipated that the optimiser initialisation and the choice of control parameters that are free to be tuned represent key factors for the success of the optimisation.

Without a suitable initialisation, e.g. using random guesses, the optimiser is often unable to find a solution or ends up converging towards a controller with fast and/or unstable poles, which is not practical to implement. Hence, assuming a third-order controller and partitioning its state-space representation as:

$$
\left[\begin{array}{c}
\dot{\mathbf{x}}_{\mathrm{K}}(\mathrm{~s})  \tag{5.7}\\
\mathbf{a}_{\mathrm{cmp}}(\mathrm{~s})
\end{array}\right]=\left[\begin{array}{c|ccc}
A_{\mathrm{K}} & B 1_{\mathrm{K}} & B 2_{\mathrm{K}} & B 3_{\mathrm{K}} \\
\hline C_{\mathrm{K}} & D 1_{\mathrm{K}} & D 2_{\mathrm{K}} & D 3_{\mathrm{K}}
\end{array}\right]\left[\begin{array}{c}
\mathbf{x}_{\mathrm{K}}(\mathrm{~s}) \\
\delta \mathbf{r}_{\mathrm{cmd}}(\mathrm{~s}) \\
\delta \mathbf{r}_{\mathrm{lpf}}(\mathrm{~s}) \\
\delta \mathbf{v}_{\mathrm{lpf}}(\mathrm{~s})
\end{array}\right]
$$

where $\mathbf{x}_{\mathrm{K}}(\mathrm{s})$ is the internal state vector, initial guesses were chosen as follows:

- $A_{\mathrm{K}}=-10^{-2} \mathbb{I}_{3 \times 3}$ for stable poles with reasonable frequency;
- $B 1_{\mathrm{K}}=-B 2_{\mathrm{K}}=\mathbb{I}_{3 \times 3}$ and $B 3_{\mathrm{K}}=0_{3 \times 3}$ to lead the optimiser towards the consideration of the error $\delta \mathbf{r}_{\mathrm{cmd}}(\mathrm{s})-\delta \mathbf{r}_{\mathrm{lpf}}(\mathrm{s})$;
- $C_{\mathrm{K}}=D 2_{\mathrm{K}}=D 3_{\mathrm{K}}=\mathbb{I}_{3 \times 3}$, respectively responsible for integrative, proportional and derivative control actions;
- $D 1_{\mathrm{K}}=0_{3 \times 3}$, feed-forward term, which should not be needed since control compensation is independent of the reference trajectory.

These matrices can be understood as gains that can be changed ad hoc if any control re-tuning is required.

The choice above results in 72 parameters to be tuned. However, if all of them are left free, the optimiser tends to find physically inappropriate solutions, featuring very different dynamics on each control axis or strong cross-coupled interactions between them. This issue was solved by fixing to zero all the off-diagonal terms of the sub-matrices in Eq. (5.7), which reduces the number of tunable parameters to 24 .

As a consequence, the highly-coupled D\&L dynamics will be tackled by a diagonal (but robust) controller. It is important to note that the fixed controller structure comes with an intrinsic sacrifice of achievable performance compared to standard $\mathcal{H}_{\infty}$ controllers. For the present case, the focus was in studying the viability of structured $\mathcal{H}_{\infty}$ for the Phobos D\&L mission, as it enabled to reconcile legacy control architecture knowledge with sophisticated tuning tools. Additional research has also been conducted in order to find the minimal acceptable structure for Eq. (5.7).

With this strategy, four controllers have been synthesised, each of them accounting for a different set of actuator and/or gravitational uncertainties (recall Fig. 5.2). Naturally, the larger the number of uncertainties considered in the design, the more time consuming the optimisation process becomes. The selection of the four sets of design uncertainties is summarised in Table 5.1. For better results, and referring to the latter table, the controller $K_{\mathrm{A}}(\mathrm{s})$ has been employed as initial condition for the synthesis of $K_{\mathrm{A}, 2 \sigma}(\mathrm{~s})$ and $K_{\mathrm{A}, 3 \sigma}(\mathrm{~s})$.

Table 5.1: Specification of the designed D\&L controllers

| Controller | Uncertainties considered |
| :--- | :--- |
| $K_{0}(\mathrm{~s})$ | None (non-robust controller) |
| $K_{\mathrm{A}}(\mathrm{s})$ | $\Delta_{\mathrm{A}}(\mathrm{s})$ only |
| $K_{\mathrm{A}, 2 \sigma}(\mathrm{~s})$ | $\Delta_{\mathrm{A}}(\mathrm{s})$ and $\Delta_{\mathrm{SDK}, 2 \sigma}^{i}(\mathrm{~s})$ |
| $K_{\mathrm{A}, 3 \sigma}(\mathrm{~s})$ | $\Delta_{\mathrm{A}}(\mathrm{s})$ and $\Delta_{\mathrm{SDK}, 3 \sigma}^{i}(\mathrm{~s})$ |

Before proceeding to an in-depth robustness analysis of the designed controllers in Chapter 6, preliminary tests with models in nominal conditions are provided in the next section.

### 5.4 Nominal analysis

First, an LTI analysis and comparison of closed-loop frequency responses using the four controllers of Table 5.1 is provided in Fig. 5.3. For each case, the figure shows two singular value magnitude plots for: [i] the $\delta \mathbf{r}_{\mathrm{e}}(\mathrm{s})$ (top plot) and $\delta \mathbf{a}(\mathrm{s})$ (bottom plot) transfer functions, [ii] the corresponding $\mathcal{H}_{\infty}$ constraints $W_{S}^{-1}(\mathrm{~s})$ and $W_{A}^{-1}(\mathrm{~s})$, and [iii] the response evolution along RT1 using different colours, similar to Fig. 3.3a. In addition, a distinction is made between diagonal and off-diagonal responses (using continuous and dashed lines), which must be as separated as possible in order to minimise cross-coupled interactions.

From this figure, it is clear that, in nominal conditions, the four controllers meet the design requirements for all the points throughout RT1. It is also visible that the margins with respect to $W_{S}^{-1}(\mathrm{~s})$ and $W_{A}^{-1}(\mathrm{~s})$ become tighter towards the final descent time, especially for off-diagonal channels in the LF region. In fact, for the non-robust controller $K_{0}(\mathrm{~s})$ in Fig. 5.3a, there is not enough separation between diagonal and off-diagonal terms, which anticipates robustness issues.


Figure 5.3: Multi-channel closed-loop singular values along RT1 and $t_{i} \in\left[t_{0}, t_{f}\right]$ in nominal conditions, i.e. $\Delta(\mathrm{s})=0$, against the requirements $W_{S}^{-1}(\mathrm{~s})$ (uppermost subplots) and $W_{A}^{-1}(\mathrm{~s})$ (bottom subplots). The control tracking bandwidth is indicated by vertical, green lines.

This separation is then achieved by the other controllers, which account for cross-couplings introduced by the actuators and/or by the gravitational uncertainties. These controllers (Fig. 5.3b to 5.3 d ) show very similar responses (indeed, major differences are only visible in Chapter 6 using robust performance analysis or highly dispersed simulations), although it is observed that they have incrementally smaller diagonal steady-state errors, at the expense of larger off-diagonal components.

In addition, to demonstrate the effect of control compensation, a high-fidelity simulation of RT1 is now performed with open-loop guidance and nominal conditions. The outcome is depicted in Fig. 5.4, showing reference and actual trajectories, $\mathbf{r}_{\mathrm{ref}}(t)$ and $\mathbf{r}(t)$, as well as arrows for the magnitude and direction, at different points throughout the trajectory, of the acceleration signals: reference $\mathbf{a}_{\mathrm{ref}}(t)$, compensation $\mathbf{a}_{\mathrm{cmp}}(t)$ and total, i.e. $\mathbf{a}_{\mathrm{ref}}(t)+\mathbf{a}_{\mathrm{cmp}}(t)$.


Figure 5.4: Impact of control compensation on trajectory RT1 in nominal conditions
On the left-hand side, Fig. 5.4a, no compensation is employed, i.e. $\mathbf{a}_{\mathrm{cmp}}(t)=0$, so RT1 cannot be followed and the spacecraft crashes on Phobos before reaching the desired landing site. This potential failure, i.e. in the absence of any guidance or control feedback loop, has already been identified and addressed in Sec. 4.1 using an uncompensated closed-loop guidance approach. Otherwise, a successful landing is only accomplished when a control compensator is introduced. This is illustrated in Fig. 5.4 b for $K_{0}(\mathrm{~s})$, but the same result is obtained with any of the other controllers since they have similar nominal performance.

This figure also shows that the acceleration vector becomes larger and normal to the moon's surface towards the final descent time for a lower touchdown speed and higher incidence angle, as desired in Sec. 3.2. But most importantly, it confirms the effectiveness of the control compensation approach: the acceleration command $\mathbf{a}_{\mathrm{cmp}}(t)$, smaller than the reference vector, provides the means to correct the latter and to track the commanded $\mathrm{D} \& \mathrm{~L}$ trajectory.


## Robustness Analysis of Control Compensators

This chapter is dedicated to an in-depth robustness analysis of the control compensators designed in Chapter 5. In addition to the assessment of those controllers, the main objective of this chapter is to demonstrate how the robust control analysis techniques introduced in Sec. 2.2.3 can be employed to complement non-analytical Guidance \& Control (G\&C) Verification \& Validation (V\&V) methods. This demonstration is performed under the presence of Linear Time-Invariant (LTI) uncertainties in Sec. 6.1 and of nonlinear effects in Sec. 6.2.

### 6.1 Analysis against uncertainties

In this section, the impact of gravitational uncertainties on the control compensators designed in the previous chapter is assessed. This assessment is carried out first via the structured singular value $\mu$ in Sec. 6.1.1 and then through high-fidelity Monte-Carlo (MC) simulation in Sec. 6.1.3. Moreover, closed-loop sensitivity information provided by $\mu$ analysis is addressed in Sec. 6.1.2 as a way to validate and complement the uncertainty selection choice made before in Sec. 3.4.

### 6.1.1 Application of $\mu$ analysis to $\mathbf{D} \& \mathbf{L}$

The $\mu$ analysis framework introduced in Sec. 2.2.3.1 is here applied to assess and compare robust stability and performance properties of Descent \& Landing (D\&L) compensation systems using the controllers summarised in Table 5.1. Similar to the analysis carried out in Sec. 5.4, this is achieved by closing the loop with the different multi-plant controllers at every LTI point. Also, the uncertainties considered for analysis do not necessarily need to coincide with the ones accounted for the design of each controller. Indeed, this will be one of the key demonstrations from these analyses.

An overview of all the information provided by $\mu$ analysis is illustrated in Fig. 6.1 at the LTI point $i=8$, which corresponds to a descent time around 6500 seconds, for three pairs formed by examining controller $K_{\mathrm{A}}$ with the two uncertainty sets $\Delta_{\mathrm{A}}$ and diag $\left[\Delta_{\mathrm{A}}, \Delta_{\mathrm{SDK}, 3 \sigma}^{8}\right]$, and controller $K_{\mathrm{A}, 3 \sigma}$ paired with the latter uncertainty set.

In the figure, there are two subplots per assessed pair. The top ones depict the Robust Stability (RS) ( - ), Nominal Performance (NP) ( $\square$ ) and Robust Performance (RP) ( $\triangle$ ) tests of Eq. (2.84), (2.86) and (2.87) over frequency. For RS and RP, there is a dashed and a continuous line representing Lower Bound (LB) and Upper Bound (UB) for the computation of Eq. (2.85) which, for practical effects, show a very good agreement in this scenario. The bottom subplots then show the sensitivity of the RP results against each element of the tested uncertainty, quantified by $\partial \mu\left(M^{i}(j \omega)\right) / \partial \delta_{\mathrm{x}}$ and computed by the $\mu$ algorithm. A higher sensitivity indicates that the corresponding uncertainty has a stronger influence in the level of RP obtained.

Starting with the first plot, Fig. 6.1a, the closed-loop analysis with $K_{\mathrm{A}}$, designed for $\Delta_{\mathrm{A}}$ only, against the effect of $\Delta_{\mathrm{A}}$ is shown. Since all the curves lie below 1 , this implies that NP, RS and RP conditions are met, which is expected since the uncertainties considered for design and analysis are the same. In nominal conditions, NP $(\square)$ is more demanding at low and very high frequencies, which could also be anticipated from the proximity of the closed-loop responses in Fig. 5.3 for the $W_{S}^{-1}$ and $W_{A}^{-1}$ requirements, respectively for those two frequency regions. In addition, stability degradation, i.e. RS $(-\nabla)$, is only affected at Low-Frequency (LF) and there is always a significant margin until instability, i.e. until $\mu\left(M_{11}^{i}\right)=1$.

Finally, RP $(\triangle)$ envelops, and is driven by, the shapes of NP and RS, which is a result of Eq. (2.88). That is, in the presence of uncertainties, closed-loop performance is constrained by NP and by the interplay with the uncertainty channels. In terms of sensitivity (bottom subplot), all the uncertain elements have similar trends, but the off-diagonal uncertainty $\delta_{\text {aod }}$ is the most decisive.

Figure 6.1b shows the same analysis but with $K_{\mathrm{A}}$ now against the combined effect of actuator and Gravity Harmonics (GH) uncertainties, i.e. diag $\left[\Delta_{\mathrm{A}}, \Delta_{\mathrm{SDK}, 3 \sigma}^{8}\right]$. As expected, considering a wider range of uncertainties leads to the same NP, but degraded robustness properties, particularly in terms of RP. As evidenced by the top plot in Fig. 6.1b, the RP condition is no longer met by $K_{\mathrm{A}}$. Also, the elements of $\Delta_{\mathrm{SDK}, 3 \sigma}^{8}$ are now included in the sensitivities (bottom) subplot with continuous lines. In fact, the impact of the latter elements becomes comparable to that of $\Delta_{\mathrm{A}}$, precisely at LF where $\mu\left(M^{8}\right) \approx 1$.

Finally, Fig. 6.1c shows the results against the same set of uncertainties, $\operatorname{diag}\left[\Delta_{\mathrm{A}}, \Delta_{\mathrm{SDK}, 3 \sigma}^{8}\right]$, but now replacing the controller with $K_{\mathrm{A}, 3 \sigma}$, synthesised using the latter set. This leads to all NP, RS and RP conditions being met by a change of closed-loop behaviour at low-frequency. Nevertheless, RP sensitivity frequency responses remain roughly the same, which indicates that this type of insight is not intrinsically dependent on the controller being analysed (when controllers are designed to achieve similar objectives).


Figure 6.1: NP, RS, RP results (uppermost subplots) and $\mu$ sensitivities (bottom subplots) at LTI point $i=8$

In order to conclude this first analytical robustness assessment, RP levels against the complete set of uncertainties are computed along RT1 using the compensators that were not addressed in Fig. 6.1, see Fig. 6.2a for $K_{0}$ and Fig. 6.2b for $K_{\mathrm{A}, 2 \sigma}$.


Figure 6.2: RP results against $\Delta_{\mathrm{A}}$ and $\Delta_{\mathrm{SDK}, 3 \sigma}^{i}$ along RT1 with $t_{i} \in\left[t_{0}, t_{f}\right]$

It is confirmed from Fig. 6.2a that $K_{0}$, which was designed for nominal conditions, exhibits RP peaks well above 1 , hence it is not suitable for this D\&L scenario. This is no longer the case with $K_{\mathrm{A}, 2 \sigma}$, in Fig. 6.2b, whose RP levels lie in-between those of $K_{\mathrm{A}}$ and $K_{\mathrm{A}, 3 \sigma}$. For all the cases, as anticipated from the nominal analysis of Fig. 5.3, performance degradation becomes more intense closer to touchdown.

### 6.1.2 Sensitivity analysis

It was mentioned in Sec. 3.4.1 that an open-loop sensitivity analysis was performed to identify the uncertain GH coefficients with higher impact on the dynamics of the problem. More specifically, this consisted in repeating the interpolation of the Jacobian matrices $J_{\mathbf{f}}^{i}$ for each coefficient separately and counting the number of $J_{\mathbf{f}}^{i}$ terms where that coefficient accounts for a variation above $10 \%$ its nominal value. This allowed to quantify the impact of each parameter and select the 9 most relevant for Linear Fractional Transformation (LFT) generation.

Here, this quantification method is revisited using the closed-loop sensitivity information provided by $\mu$ analysis in Sec. 6.1.1 and the fact that this type of insight is not intrinsically dependent on the controller employed. An overview of the results that can be obtained using the aforementioned considerations is depicted in Fig. 6.3, for the case of controller $K_{\mathrm{A}, 3 \sigma}$ and $3 \sigma$ uncertainty dispersions. The figure shows, per normalised Spacecraft Dynamics \& Kinematics (SDK) uncertainty $\delta_{\mathrm{x}}$ (where x identifies the coefficients selected in Eq. (3.12)) and as a function of the design points $i=\{1, \ldots, 10\}$, the following:

- In yellow, the sensitivity peak values, $\sup _{\omega \in \mathbb{R}} \frac{\partial \mu\left(M^{i}(j \omega)\right)}{\partial \delta_{\mathrm{x}}}$, which are computed via $\mu$ analysis (e.g., for $i=8$, these values correspond to the peaks of the bottom subplot of Fig. 6.1c);
- In dark blue, the number of $J_{\mathbf{f}}^{i}$ terms where $\delta_{\mathrm{x}}$ has a relative LFT interpolation weight larger than $10 \%$ (as explained above), normalised so as to have the same scale as the $\mu$ sensitivities.

Figure 6.3 then provides a visual confirmation that the sensitivity insights obtained using two very different methods (open-loop LFT interpolation and closed-loop $\mu$ analysis) are quite consistent for this scenario. This is particularly true with respect to the detection of cases that have small sensitivities.


Figure 6.3: Sensitivity indicators from $3 \sigma$ LFT interpolation and $\mu$ analysis along RT1

Most importantly, the above figure allows to quantify the relative impact of each GH coefficient on the dynamics of the problem, as well as its evolution through the descent. For example, it is clear that $\delta s_{3,3}$ and $\delta c_{4,0}$ are the most determining in this case, with a growing effect throughout the trajectory, while others, such as $\delta c_{3,2}$ or $\delta c_{4,1}$ are more intense at earlier stages of the descent. LFT models featuring uncertainty sets other than Eq. (3.12) could also be included in the comparison.

Besides being useful for LFT reduction (as in Sec. 3.4), the determination of driving GH coefficients has practical implications with the potential to span from control synthesis to space D\&L mission design. Most notably, it can be employed to support key decisions (such as ad hoc trajectory selection) or refined investigations aimed at reducing the uncertainty range of driving coefficients.

### 6.1.3 Verification and discussion

The robustness of the four control compensators of Table 5.1 is now verified using the nonlinear, high-fidelity benchmark introduced in Sec. 3.1 for the trajectory RT1. Each controller is tested against the same 2000 MC samples of the 19 GH coefficients in Eq. (3.3), randomly sampled with Gaussian distributions. Since the focus of the analysis is on the impact of these coefficients, no additional perturbations from actuators or navigation are included in the simulations.

In addition, special Worst-Cases (WCs) are also tested for each compensator. These WCs correspond to the combinations of uncertainties associated with the RP peaks $\Delta_{\mathrm{SDK}, 3 \sigma}^{i, \mathrm{WC}}$, which are determined via $\mu$ analysis at every LTI point $i=\{1, \ldots, 10\}$. Since only 9 out of the 19 GH coefficients are captured by the LFTs employed for $\mu$ analysis, every combination $\Delta_{\text {SDK,3 }}^{i, \mathrm{WC}}$ is tested with the remaining 10 coefficients simultaneously set to their nominal, $+3 \sigma$ and $-3 \sigma$ values. Discarding all the repeated combinations, this originates between 18 and 24 WCs per compensator.

The outcome of the verification campaigns is depicted in Fig. 6.4, showing nominal simulations with a continuous blue line, MC runs with gray lines and the aforementioned WCs with dashed red lines. For each controller, distance and speed errors with respect to the reference trajectory, $\left|\mathbf{r}(t)-\mathbf{r}_{\text {ref }}(t)\right|$ and $\left|\mathbf{v}(t)-\mathbf{v}_{\text {ref }}(t)\right|$, are plotted, together with the magnitude of the acceleration compensation command $\left|\mathbf{a}_{\mathrm{cmp}}(t)\right|$ and total $\Delta V(t)$, i.e. reference plus compensation. For clarity, failure cases (i.e. spacecraft crashing on or diverging from Phobos) are not included in the plots, but will be accounted for later on.

From Fig. 6.4, it can be confirmed that, while all the compensators behave similarly in nominal conditions, the non-robust controller $K_{0}$ (Fig. 6.4a) performs significantly worse than the others under dispersed conditions, particularly in terms of position and velocity error. Additionally, it is clear that, although determined using a reduced set of uncertainties, the WCs identified via $\mu$ analysis are generally more challenging than the 2000 MC runs. In fact, WCs typically have an extremely low probability of occurrence, but could be foreseen analytically, which demonstrates the added-value of $\mu$ analysis as a complement to MC V\&V tests.

For a detailed comparison, the average and standard deviation of the maximum values encountered during the four MC campaigns are listed in Table 6.1, with the failure ratios of both MC and WC simulations.

As anticipated from Fig. 6.2a, $K_{0}$ is not suitable for this scenario because of its higher errors, which are further translated into MC and WC failure ratios of 44.15 and $33.33 \%$. The three robust controllers then showed satisfactory and equivalent distribution indicators, but distinct failure ratios. With $K_{\mathrm{A}}$, no failures were obtained from the MC simulations, but $12.5 \%$ of the $\mu$ WCs resulted into failure. The situation improves with $K_{\mathrm{A}, 2 \sigma}$ (Fig. 6.2c), which yields no MC or WC failures.

Finally, results using $K_{\mathrm{A}, 3 \sigma}$ became inferior to the other two robust controllers, with failure ratios of 0.1 and $16.37 \%$. Although this last result may seem counter-intuitive, it is actually a
fairly common pitfall of robust control algorithms, in which controller performance may become restricted (in the nonlinear, real-world case) by an over-conservative robustness specification during the (linear) design phase. It is also important to note that, despite their added-value, RP insights from $\mu$ analysis only have guaranteed validity for the LTI models adopted, hence the execution of MC tests is fundamental for controller validation in the nonlinear case.


Figure 6.4: High-fidelity simulation of 2000 MC runs and WCs from $\mu$ analysis of RT1

Based on Table 6.1, $K_{\mathrm{A}, 2 \sigma}$ is selected as the most suitable controller. To assess its applicability to trajectories other than the one it has been designed for, the same MC uncertainty combinations are executed using $K_{\mathrm{A}, 2 \sigma}$ and trajectories RT2 and RT3 (recall Fig. 3.2b). The same indicators are now gathered in Table 6.2 for these campaigns. Since no detailed $\mu$ analysis was performed along RT2 or RT3, the consideration of WCs is not pursued here.

Table 6.1: Robustness indicators for RT1 using different controllers

|  | $K_{0}$ |  | $K_{\mathrm{A}}$ |  | $K_{\mathrm{A}, 2 \sigma}$ | $K_{\mathrm{A}, 3 \sigma}$ |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Max. MC values | Avg. | Std. | Avg. | Std. | Avg. | Std. | Avg. |
| $\mathbf{r}-\mathbf{r}_{\mathrm{ref}} \mid(\mathrm{m})$ | 0.503 | 0.259 | 0.029 | 0.013 | 0.030 | 0.014 | 0.038 |
| $\left\|\mathbf{v}-\mathbf{v}_{\mathrm{ref}}\right\|(\mathrm{mm} / \mathrm{s})$ | 1.234 | 0.130 | 1.206 | 0.008 | 1.208 | 0.007 | 1.212 |
| $\mathbf{a}_{\mathrm{cmp}} \mid\left(\mathrm{mm} / \mathrm{s}^{2}\right)$ | 0.411 | 0.207 | 0.431 | 0.208 | 0.430 | 0.208 | 0.429 |
| $\Delta V(\mathrm{~m} / \mathrm{s})$ | 8.793 | 0.630 | 8.326 | 0.966 | 8.326 | 0.965 | 8.329 |
| MC failures (\%) | 44.15 | 0.00 | 0.955 |  |  |  |  |
| WC failures (\%) | 33.33 | 12.50 | 0.00 | 0.10 |  |  |  |

Table 6.2: Robustness indicators for different trajectories using $K_{\mathrm{A}, 2 \sigma}$

|  | RT1 |  | RT2 |  | RT3 |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Max. MC values | Avg. | Std. | Avg. | Std. | Avg. | Std. |
| $\left\|\mathbf{r}-\mathbf{r}_{\text {ref }}\right\|(\mathrm{m})$ | 0.030 | 0.014 | 0.026 | 0.010 | 0.026 | 0.011 |
| $\left\|\mathbf{v}-\mathbf{v}_{\text {ref }}\right\|(\mathrm{mm} / \mathrm{s})$ | 1.208 | 0.007 | 0.694 | 0.253 | 0.661 | 0.325 |
| $\left\|\mathbf{a}_{\mathrm{cmp}}\right\|\left(\mathrm{mm} / \mathrm{s}^{2}\right)$ | 0.430 | 0.208 | 0.332 | 0.160 | 0.326 | 0.164 |
| $\Delta V(\mathrm{~m} / \mathrm{s})$ | 8.326 | 0.965 | 8.714 | 0.831 | 7.989 | 0.830 |
| MC failures (\%) | 0.00 |  | 0.00 |  | 0.00 |  |

The results of Table 6.2 show that $K_{\mathrm{A}, 2 \sigma}$ successfully handles the RT2 and RT3 trajectories with similar performance to RT1 and without any failure. The success of this approach lies on the fact that, since the controllers were designed around perturbation variables (recall Sec. 3.3), the effect of trajectory changes is much reduced in comparison with GH uncertainties. Specific differences amongst the indicators are then mostly related to how demanding each trajectory is.

These results also indicate that accounting for the uncertainties encountered along RT1 alone turned out to be enough for the design of a compensator that is equally able to cope with other trajectories. This conclusion is tightly linked to the previous observation that having an all-encompassing uncertainty specification is not necessarily sufficient (nor required) to achieve a successful robust control design.

### 6.2 Analysis against nonlinearities

Following the robustness analysis under the presence of GH uncertainties performed in the previous section, this section focuses on the impact of nonlinear effects on $D \& L$ performance. The analytical assessment of these impacts is generalised by the Integral Quadratic Constraints (IQC) analysis framework introduced in Sec. 2.2.3.2, and is here applied to the case of Time-Varying (TV) effects (Sec. 6.2.1), time delays (Sec. 6.2.2) and limited control authority (Sec. 6.2.3).

### 6.2.1 Impact of time-varying effects

The inclusion of TV effects on the SDK model was addressed in Sec. 3.5, generating the nominal Linear Parameter-Varying (LPV) model $G_{\mathrm{SDK}}\left(\mathrm{s}, \rho_{\mathrm{TV}}(t)\right)$, with $\rho_{\mathrm{TV}}(t)=t$. It was also mentioned that LPV-LFT models with combined consideration of GH uncertainties and TV parameters can be obtained via the interpolation of the Jacobian matrices $J_{\mathbf{f}}^{i}\left(\left[\rho_{\mathrm{GH}}, \rho_{\mathrm{TV}}(t)\right]\right), i=\{1, \ldots, 10\}$.

Nevertheless, the size and complexity of the LFT models for this combined problem render the applicability of the IQC analysis framework (Sec. 2.2.3.2) non-trivial. Alternatively, in order to perform the robust D\&L LPV analysis, the following approach, proposed in [PS15] for verification purposes, is pursued:

1. The worst-case SDK uncertainty $\Delta_{\mathrm{SDK}, 3 \sigma}^{i, \mathrm{WC}}(\mathrm{s})$ is computed using RP $\mu$ analysis (Sec. 6.1.1) at every LTI point;
2. With these uncertainties, the worst-case LTI realisations are defined by:

$$
\begin{equation*}
G_{\mathrm{SDK}}^{i, \mathrm{WC}}(\mathrm{~s})=\mathcal{F}_{\mathrm{u}}\left\{G_{\mathrm{SDK}, 3 \sigma}^{i}(\mathrm{~s}), \Delta_{\mathrm{SDK}, 3 \sigma}^{i, \mathrm{WC}}(\mathrm{~s})\right\}, \quad i=\{1, \ldots, 10\} \tag{6.1}
\end{equation*}
$$

3. A worst-case (not uncertain) LPV model $G_{\mathrm{SDK}}^{\mathrm{WC}}\left(\mathrm{s}, \rho_{\mathrm{TV}}(t)\right)$ is built, as in Sec. 3.5, interpolating the realisations above.

Robustness analysis can now be performed by replacing the LTI SDK models in Fig. 5.2 with the nominal and WC LPV models $G_{\mathrm{SDK}}\left(\mathrm{s}, \rho_{\mathrm{TV}}(t)\right)$ and $G_{\mathrm{SDK}}^{\mathrm{WC}}\left(\mathrm{s}, \rho_{\mathrm{TV}}(t)\right)$. The resulting closed-loop LPV plants are denoted $M_{\mathrm{LPV}}\left(\mathrm{s}, \rho_{\mathrm{TV}}(t)\right)$ and $M_{\mathrm{LPV}}^{\mathrm{WC}}\left(\mathrm{s}, \rho_{\mathrm{TV}}(t)\right)$, respectively. Because the focus of this analysis is on the impact of GH uncertainties and TV effects, the actuator model is fixed to nominal in this case, $\Delta_{\mathrm{A}}(\mathrm{s})=0$.

Since none of the new LPV models contains uncertain elements, the generalised LTI bounded real lemma of Eq. (2.96) provides a sufficient condition to upper bound their induced $\mathcal{L}_{2}$ gain. It is important to note that this approach does not provide the true WC gain of the uncertain LPV system, but it allows to assess the relative impact of GH uncertainties and TV effects in terms of $D \& L$ performance degradation.

The verification of Eq. (2.96) is performed using the same toolbox employed for LPV modelling [HSP15]. This toolbox also provides routines to compute balanced LPV system realisations, which is essential to obtain physically-meaningful results. For LPV analysis, two sub-cases can be further considered:

- The TV parameter rate $\left|\dot{\rho}_{\mathrm{TV}}(t)\right|$ is assumed unbounded. In practice, this assumption may be over-conservative, but it simplifies greatly the computation of Eq. (2.96) since the parameter-dependent matrix $P$ becomes constant, i.e. $\frac{\partial P\left(\rho_{\mathrm{TV}}\right)}{\partial \rho_{\mathrm{TV}}}=0$;
- The TV parameter is assumed Rate-Bounded (RB) and, since $\rho_{\mathrm{TV}}(t)=t$ in this case, it is reasonable to assume $\left|\dot{\rho}_{\mathrm{TV}}(t)\right| \leq 1$. Polynomial TV parameter-dependencies are generally
considered for $P$, and the parametrisation $P=P_{0}+\rho_{\mathrm{TV}} P_{1}+\rho_{\mathrm{TV}}^{2} P_{2}$ is undertaken for this analysis.

The outcomes of this analysis are gathered in Fig. 6.5, with one plot per each of the four control compensators (from Table 5.1), and explained below.


Figure 6.5: Robustness analysis of RT1 with uncertainty and TV effects

In each plot of Fig. 6.5, six indicators are represented along RT1:

- "LTI Nominal", nominal LTI gains $(\square)$, given by Eq. (2.86);
- "LTI WC", WC LTI gains (-■-), given by Eq. (2.89) with $\Delta_{\mathrm{A}}(\mathrm{s})=0$;
- "RB-LPV Nom.", induced $\mathcal{L}_{2}$ gain of $M_{\text {LPV }}$ with RB TV $(\nabla)$;
- "RB-LPV WC", induced $\mathcal{L}_{2}$ gain of $M_{\mathrm{LPV}}^{\mathrm{WC}}$ with RB TV (- - -);
- "LPV Nom.", induced $\mathcal{L}_{2}$ gain of $M_{\text {LPV }}$ with unbounded TV ( $\left.\triangle\right)$;
- "LPV WC", induced $\mathcal{L}_{2}$ gain of $M_{\mathrm{LPV}}^{\mathrm{WC}}$ with unbounded TV (- $\left.\Delta-\right)$.

It is important to note that, while there is a different LTI gain for every plant $i=\{1, \ldots, 10\}$ (or $t_{i} \in\left[t_{0}, t_{f}\right]$ ), each LPV gain corresponds to a single bound over the trajectory. Thus, the last four indicators are always a constant horizontal line in the plots.

Following the considerations made throughout this subsection with respect to system robustness properties, it is then expected that:

- For the same type of system (LTI, RB-LPV or LPV), worst-case $\geq$ nominal induced gains;
- For the same condition (nominal or worst-case), LPV $\geq$ RB-LPV $\geq$ LTI gains.

All these relationships are retrieved from the plots of Fig. 6.5, corroborating the meaningfulness of the analysis:

- For the same marker $(\square, \nabla$ or $\triangle)$, dashed $\geq$ continuous lines;
- For the same type of line (continuous or dashed), $\Delta \geq \nabla \geq \square$.

As a global conclusion, it can be observed that there is a sequential improvement (i.e. decrease of $\gamma$ ) from $K_{0}$ in Fig. 6.5a to $K_{\mathrm{A}, 3 \sigma}$ in Fig. 6.5d. The former is the only controller that does not fulfil the RP requirements since $\gamma>1$. This was already the case with LTI uncertainties (recall Fig. 6.2a), but it gets even worse with TV effects.

Moreover, different types of system ( $\square, \nabla$ or $\Delta$ ) present similar maximum gains for the same conditions (nominal and worst-case), which indicates that, for this scenario, the impact of time variation on D\&L performance is significantly smaller than that of GH uncertainties. In other words, a single robust multi-plant controller suffices to operate in the uncertain TV environment encountered, as confirmed in Sec. 6.1.3.

### 6.2.2 Impact of time delays

The existence of time delays in any real-world control implementation, either introduced by sensing/actuation hardware or by control logics computations, is unavoidable. Thus, in here the impact of a time delay $\tau$ affecting the three components of the compensated acceleration $\mathbf{a}_{\mathrm{cmp}}(\mathrm{s})$ is addressed.

Time delays represent another nonlinear effect, as the time-delayed acceleration command is given by:

$$
\begin{equation*}
\mathbf{a}_{\mathrm{cmp}}^{\prime}(\mathrm{s})=\mathbf{a}_{\mathrm{cmp}}(\mathrm{~s}) \mathrm{e}^{-\mathrm{s} \tau} \tag{6.2}
\end{equation*}
$$

Depending on their actual value and on the characteristics of the system, time delays have the potential to compromise closed-loop stability and their impact must therefore be assessed. This assessment can be performed using IQC analysis as demonstrated in [JS01], but the size of the

LFT models for the present problem makes this approach very demanding from a computational point of view.

Instead, for the combined assessment of GH uncertainties and time delays, the exponential term in Eq. (6.2) will be replaced by a second-order Padé approximation [BCPS06], defined as:

$$
\begin{equation*}
G_{\tau}(\mathrm{s})=\frac{\frac{(\mathrm{s} \tau)^{2}}{12}-\frac{\mathrm{s} \tau}{2}+1}{\frac{(\mathrm{~s} \tau)^{2}}{12}+\frac{\mathrm{s} \tau}{2}+1} \mathbb{I}_{3 \times 3} \tag{6.3}
\end{equation*}
$$

and illustrated in Fig. 6.6.


Figure 6.6: Loop transformation for the analysis of time delays
With this transformation, all the elements enter the system linearly and $\mu$ analysis can be employed for RP assessment. In addition, two modelling alternatives are possible: $[\mathrm{i}] \tau$ is assumed uncertain and considered as an additional uncertainty $\tau \in[0, \bar{\tau}]$, or [ii] only the case corresponding to its maximum value $\tau=\bar{\tau}$ is addressed. While the first one may be more conservative, the latter option is adopted in order to reduce the size of the overall LFT (and keep the bounds of Eq. (2.85) as tight as possible).

The RP results are presented in Fig. 6.7, in a similar fashion to the previous subsection, with one plot per control compensator. Each plot of Fig. 6.7 shows the same (non-delayed) nominal and WC LTI gains of Fig. 6.5 ( $\square$ and - $\square-$, respectively), which are then recalculated for sequentially larger delays up to 4 s (represented with different markers).

All the plots show that different conditions (nominal or worst-case) have very similar gains for the same level of delay, meaning that large time delays are more impactful on $\mathrm{D} \& \mathrm{~L}$ performance than GH uncertainties. The only exception is the non-robust compensator $K_{0}$, Fig. 6.7a, whose performance is mainly degraded by the uncertainties, especially during the second half of the trajectory. Within the other three controllers, performance degradation remains minor $(\gamma<1.2)$ for delays up to 2 s , which are extremely unlikely having in mind the state-of-practice in computational/hardware capabilities.

This formal performance guarantee is particularly valuable as it does not involve any extensive V\&V tests and because analytical results (based on the LTI models) tend to be more conservative than nonlinear, real-world case. In fact, high-fidelity simulations (performed but not depicted here for the sake of conciseness) showed an acceptable performance up to 7 s delay.

### 6.2.3 Impact of limited control authority

The last nonlinearity to be considered in this section is the effect of limited control authority. Under the scope of D\&L compensation, this would be the case when the acceleration command


Figure 6.7: Robustness analysis of RT1 with uncertainty and time delay $\tau$
$\mathbf{a}_{\mathrm{cmp}}(t)$ is constrained so as not to significantly alter the reference acceleration profile (recall Fig. 2.2b), but the same phenomenon often arises in electromechanical systems under the form of actuator saturation or minimum impulse bit.

Here, it is assumed that the three channels $a_{j, \mathrm{cmp}}(t), j \in\{x, y, z\}$ are independently limited by $\sigma$ through:

$$
a_{j, \mathrm{cmp}}^{\prime}(t)=\phi\left(a_{j, \mathrm{cmp}}(t)\right)=\left\{\begin{array}{ll}
a_{j, \mathrm{cmp}}(t), & \text { if }\left|a_{j, \mathrm{cmp}}(t)\right| \leq \sigma  \tag{6.4}\\
\operatorname{sign}\left(a_{j, \mathrm{cmp}}(t)\right) \sigma, & \text { otherwise }
\end{array} \quad j \in\{x, y, z\}\right.
$$

This definition is problematic for robustness analysis since the extraction of the saturation nonlinearity $\phi\left(a_{j, \mathrm{cmp}}(t)\right)$ as a $\Delta$ block (in the interconnection of Fig. 2.13) will inevitably open the loop, making the nominal system $M(\mathrm{~s})$ unstable and therefore Eq. (2.94) inapplicable.

To overcome this issue, a transformation was proposed in $\left[\mathrm{CBB}^{+} 13\right]$, which employs a nonlinear dead-zone perturbation:

$$
w_{\Delta j}(t)=\Gamma\left(z_{\Delta j}(t)\right)=\left\{\begin{array}{ll}
0, & \text { if }\left|z_{\Delta j}(t)\right| \leq \sigma  \tag{6.5}\\
z_{\Delta j}(t)-\operatorname{sign}\left(z_{\Delta j}(t)\right) \sigma, & \text { otherwise }
\end{array} \quad j \in\{x, y, z\}\right.
$$

with $z_{\Delta j}(t)=a_{j, \mathrm{cmp}}(t)$, to define $\phi\left(\mathbf{a}_{\mathrm{cmp}}(t)\right)$ as:

$$
\begin{equation*}
\phi\left(\mathbf{a}_{\mathrm{cmp}}(t)\right)=\mathbf{a}_{\mathrm{cmp}}(t)-\Gamma\left(\mathbf{a}_{\mathrm{cmp}}(t)\right)=\mathbf{a}_{\mathrm{cmp}}^{\prime}(t) \tag{6.6}
\end{equation*}
$$

This transformation is illustrated in Fig. 6.8.


Figure 6.8: Loop transformation for the analysis of limited control authority
It is worth noting a couple of important properties of the new nonlinearity $\Gamma\left(z_{\Delta j}(t)\right)$ :

- Its output is bounded to a unitary-slope sector, i.e. defined in Fig. 6.8 by $[\alpha, \beta]=[0,1]$;
- It is memoryless, i.e. there is only one possible outcome per input.

These two properties enable the characterisation of $\Gamma\left(z_{\Delta j}(t)\right)$ using IQCs (Sec. 2.2.3.2).
More specifically, the bounded sector $[\alpha, \beta]$ is formally defined as:

$$
\begin{equation*}
\alpha z_{\Delta j}^{2}(t) \leq w_{\Delta j}(t) z_{\Delta j}(t) \leq \beta z_{\Delta j}^{2}(t) \tag{6.7}
\end{equation*}
$$

and associated with the Sector-Bounded NonLinearity (SBNL) multiplier [MR97]:

$$
\Pi_{\mathrm{SBNL}}(j \omega)=\left[\begin{array}{cc}
-2 \alpha \beta & \alpha+\beta  \tag{6.8}\\
\alpha+\beta & -2
\end{array}\right]
$$

While this IQC guarantees global RP, the sector of Eq. (6.7) is independent of $\sigma$, which may be over-conservative in practice. To tackle this limitation, an approach exploiting the local nature of the perturbation $\Gamma\left(z_{\Delta j}(t)\right)$ was proposed in [KPS15]. Following this approach, if a norm bound $\gamma_{\Delta}$ on the signal $\mathbf{a}_{\mathrm{cmp}}(t)$ is known, instead of checking over the whole sector $[0,1]$, it is sufficient to consider a smaller sector.

Assuming $\mathbf{w}(t) \in \mathcal{L}_{2}$ (exogenous input signal, not to be confused with the perturbation channel $\left.\mathbf{w}_{\Delta}(t)\right)$, the bound $\gamma_{\Delta}$ corresponds to:

$$
\begin{equation*}
\sup \frac{\left\|\mathbf{a}_{\mathrm{cmp}}(t)\right\|_{2}}{\|\mathbf{w}(t)\|_{2}}<\gamma_{\Delta} \tag{6.9}
\end{equation*}
$$

and can be estimated using IQC analysis. Once $\gamma_{\Delta}$ is known, the sector slope can be reduced from 1 to $1-\epsilon$, i.e. to $[\alpha, \beta]=[0,1-\epsilon]$, with:

$$
\begin{equation*}
0<\epsilon=\frac{\sigma}{\gamma_{\Delta}}<1 \tag{6.10}
\end{equation*}
$$

and the results obtained will be valid for all:

$$
\begin{equation*}
\sigma \geq \gamma_{\Delta} \epsilon>0 \tag{6.11}
\end{equation*}
$$

In addition, this sector reduction offers the mathematical benefit of avoiding $\phi\left(\mathbf{a}_{\mathrm{cmp}}(t)\right) \rightarrow 0$ when $\beta \rightarrow 1$, Eq. (6.6), which would degenerate in the undesirable open-loop condition mentioned above.

The SBNL multiplier holds for both time-invariant and time-varying nonlinear perturbations. To capture the time-invariance of $\Gamma\left(z_{\Delta j}(t)\right)$, assuming $z_{\Delta j}(0)=0$, Eq. (6.5) must satisfy the condition:

$$
\begin{equation*}
\int_{0}^{+\infty} \dot{z}_{\Delta j}(t) w_{\Delta j}(t) \mathrm{d} t=0 \tag{6.12}
\end{equation*}
$$

which can be imposed through the Popov multiplier [MR97]:

$$
\Pi_{\text {Popov }}(j \omega)= \pm\left[\begin{array}{cc}
0 & -j \omega  \tag{6.13}\\
j \omega & 0
\end{array}\right]
$$

A further refinement in the IQC description of limited control authority can be introduced using a sector slope restriction since $\Gamma\left(z_{\Delta j}(t)\right)$ is a monotonic and odd function. This can be achieved with the so-called (dynamic) Zames-Falb multiplier [MR97] but, in order to simplify the analysis, it is not considered. A comparative study of the conservativeness obtained when using different combinations of multipliers is provided in [KPS15]. Additionally, the IQC description can be extended so as to account for GH uncertainties as well but, again for the sake of simplicity, these are not considered in this section.

The IQC description and RP analysis introduced above, namely using the combined SBNL and Popov multipliers, can be conducted with any of the two toolboxes referred in Sec. 2.2.3.2, i.e. [KMJR04] and $\left[\mathrm{BBD}^{+} 16\right]$. Since the latter is more recent and based on a more efficient optimisation algorithm (SeDuMi [Stu99]), this is the one adopted to solve the problems of Eq. (6.9) and (2.94).

The results obtained are provided in Fig. 6.9, which shows the induced $\mathcal{L}_{2}$ gain $\gamma$ as a function of RT1 points (i.e. descent time) and of limited control authority $\sigma$ for the three robust compensators. With $K_{0}$, no feasibility nor unfeasibility of Eq. (2.94) could be found,
which means that no finite $\gamma$ can be guaranteed analytically. This figure shows, across descent time, the degradation of the controllers (i.e. the increase of $\gamma$ ) as the control authority becomes increasingly limited (i.e. the smaller $\sigma$ ).

From Fig. 6.9, a sharp performance degradation can be observed with the three controllers for $\sigma \lesssim 4 \times 10^{-4} \mathrm{~m} / \mathrm{s}^{2}$. For larger values of $\sigma$ (i.e. more control authority available) there are no degradation effects and the nominal performance levels ( $\square$ in Fig. 6.5 or 6.7) are retrieved. Performance degradation takes places slightly later from $K_{\mathrm{A}}$ to $K_{\mathrm{A}, 3 \sigma}$ and it is roughly independent on the mission time.


Figure 6.9: Robustness analysis of RT1 with limited control authority $\sigma$. No finite degradation $\gamma$ could be guaranteed with $K_{0}$.

The three robust controllers are therefore analytically guaranteed to have an appropriate D\&L performance for authority limitations larger than $4 \times 10^{-4} \mathrm{~m} / \mathrm{s}^{2}$. Similar to the delay case of Sec. 6.2.3, this guarantee is relatively conservative as nonlinear simulations showed reasonable performances down to $1 \times 10^{-4} \mathrm{~m} / \mathrm{s}^{2}$. This value is deemed acceptable, but it can be further adjusted through a revision of the design weight in Eq. (5.6).

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## Conclusions of Application I

This part of the thesis was devoted to the reconciliation of G\&C architectures for space D\&L. From the state-of-practice review of guidance techniques, it was verified that technological developments on distinct areas (e.g. systems and control theory vs. space mission analysis) often take place independently but, as demonstrated, having a coherent G\&C design framework is key to facilitate interdisciplinary cross-pollination towards effective planetary descent strategies. For example, it was seen that traditional closed-loop guidance techniques share structural commonalities that are particularly convenient for the application of systematic tuning methods.

It was also shown that special care must be taken to ensure robustness of autonomous D\&L systems on very variable and uncertain environments, which are characteristic of planetary bodies with irregular shapes and mass distributions. This requires the ability to effectively model and account for these uncertain effects when synthesising guidance and control algorithms. Chapter 3 illustrated how the uncertain gravity field of Phobos can be modelled using robust control tools such as LFTs and LPV, paving the way for more reliable planetary landers and less stringent $\mathrm{D} \& \mathrm{~L}$ propellant requirements.

Building up on this knowledge, Chapter 4 developed a tuning methodology for uncompensated closed-loop guidance laws, which employs a systematic evaluation of high-fidelity models to generate trade-off maps that provide clear performance and robustness quantifications of candidate solutions. It was demonstrated that, for a landing on Phobos, propellant consumption savings of around $40 \%$ could be achieved (for similar errors) compared to state-of-practice tuning selections (derived under the assumption of simplified and well-known gravity fields). In addition, trade-off maps can be generated before the mission but only applied once the spacecraft approaches the target body, having in consideration the actual propellant available and without the need for expensive computations, therefore making this approach extremely industry-oriented.

Also with the industrial pertinence in mind, the application of structured $\mathcal{H}_{\infty}$ optimisation was addressed as a way to exploit the fixed structure of $\mathrm{D} \& \mathrm{~L}$ guidance laws. Comparative results were presented regarding the reconciliation and complementarity of the two tuning processes, but research on this topic is still open. This is mostly related to the fact that, although the state-of-practice tuning selections could be recovered by the structured $\mathcal{H}_{\infty}$ approach, superior guidance solutions, identified with trade-off maps, could not be captured due to the propagation of highly nonlinear dynamics inherent to the problem.

Emphasis was then placed on the application of structured $\mathcal{H}_{\infty}$ synthesis to design simple yet effective control compensators in Chapter 5 . This choice was mainly because of its ability to synthesise controllers that are valid for multiple plants, as well as the possibility to specify the controller structure and take advantage of legacy industry knowledge. Design challenges were also highlighted, for example on the sensitivity of structured $\mathcal{H}_{\infty}$ to the choice of initial conditions and control parameters to be tuned.

The main accomplishment of this work is that robust control techniques, which represent non-standard practices for trajectory and orbit G\&C, were successfully tailored to this type of application for the first time. Control compensation proved to be essential for open-loop guidance architectures and an added-value for closed-loop laws (although not shown in the thesis for the sake of conciseness). Because of this, investigations on further D\&L performance improvements have continued within the TASC research group. Moreover, these outcomes led to the industrial transfer of techniques and Airbus' design teams are currently employing structured $\mathcal{H}_{\infty}$ in the preparation of future missions.

To conclude this part, the developed compensators were validated in Chapter 6 via highfidelity MC campaigns, but also through dedicated tests using robust control methods. The latter tests provided an analytical assessment of the impact of gravitational uncertainties and nonlinear issues, including TV effects, time delays and limited control authority, using $\mu$ and IQC analysis, respectively. Amongst them, it was proven that the effect of uncertainties is expressively more impactful than the others in the scenario considered.

Most importantly, it was shown that, more than a robustness assessment tool, $\mu$ analysis can be directly employed to support the complete compensator design process. On the one hand, critical uncertainty combinations can be used to complement and enhance conventional MC verification campaigns and, on the other, closed-loop sensitivities that quantify the impact of each uncertain parameter can be used to support system modelling choices. The practical implications provided by these insights have the potential to span from control synthesis to space D\&L mission design.

## Application II

## Reusable Launcher Guidance \& <br> Control with Active Load Relief



## Coupled Flight Mechanics, Guidance \& Control

 BenchmarkThis part of the thesis starts with the development of a nonlinear benchmark model for the integrated assessment of reusable launcher Guidance \& Control (G\&C) approaches. As introduced in Sec. 1.3.2, the relevance of such a benchmark is related to the fact that state-ofpractice Reusable Launch Vehicle (RLV) performance optimisation studies tend to simplify or even neglect the interactions between flight mechanics and G\&C algorithms and mechanisms. Moreover, in many studies, model data details are not published, which makes it difficult to reproduce results.

The benchmark simulates the Launch \& Recovery (L\&R) trajectory of a liquid-fuel Vertical Take-off and Vertical Landing (VTVL) booster used as first stage of a lightweight, non-winged launcher injecting a $1,100 \mathrm{~kg}$ satellite in a quasi-polar orbit at 800 km . Configuration and mission parameters can be easily modified and assessed thanks to its modular architecture, which makes it extremely versatile to study the aforementioned interactions at user-defined levels of fidelity. This modularity will also be exploited in the following chapters for the development and verification of improved guidance and control techniques.

This chapter begins with the description of all the building blocks of the model in Sec. 8.1. Then, the L\&R mission scenarios under assessment are introduced in Sec. 8.2, as well as a baseline closed-loop guidance law for retro-propulsive entry, descent and pinpoint landing. In addition, Sec. 8.3 showcases how attitude control algorithms can be designed and integrated in the RLV benchmark while taking into account Thrust Vector Control (TVC), planar fins and cold gas thrusters. Finally, Sec. 8.4 presents detailed flight mechanics simulations for both Down-Range Landing (DRL) and Return To Launch Site (RTLS) missions, as well a preliminary controllability analysis for the former method of recovery.

## CHAPTER 8. COUPLED FLIGHT MECHANICS, GUIDANCE \& CONTROL BENCHMARK

The coupled RLV benchmark has been published in [SMB19d] and presented together with a more sophisticated guidance algorithm (developed in Chapter 9) in [SMB19a].

### 8.1 RLV flight mechanics modelling

The RLV flight mechanics model developed in this thesis results from the interconnection of several system building blocks. These elements are depicted in Fig. 8.1 and presented in the following subsections.


Figure 8.1: RLV simulation block interconnections
Reference frames and environment models adopted for gravity, atmosphere and wind are described in Sec. 8.1.1. Then, a detailed description of the equations of motion, aerodynamic calculations and Mass, CG \& Inertia (MCI) evolution is provided in Sec. 8.1.2, 8.1.3 and 8.1.7, respectively. The vehicle is mainly steered via TVC, but two pairs of planar fins are also included to provide attitude control under low thrust, as well as two pairs of cold gas thrusters for pitch and yaw manoeuvring in low dynamic pressure conditions. These actuators are introduced in Sec. 8.1.4, 8.1.5 and 8.1.6, respectively.

Finally, G\&C algorithms are organised in three subsystems. The first one, "launch \& recovery guidance", is responsible for the online generation of thrust and attitude commands. A dedicated discussion on L\&R mission profiles and guidance techniques is provided in Sec. 8.2. Then, the "attitude control" subsystem (responsible for the computation of attitude control moments) and the "control allocation" subsystem (for the allocation among the aforementioned actuators) are presented in Sec. 8.3.

### 8.1.1 Reference frames and environment models

This subsection introduces the reference frames and environment models that are essential to simulate the motion of RLVs during flight. For a thorough description of these frames and their coordinate transformations, the reader is referred to [Gre70] or [RA93].

The first reference frame is the Earth-Centred Inertial (ECI) frame, with basis vectors $\left\{\mathbf{i}_{\mathrm{I}}, \mathbf{j}_{\mathrm{I}}, \mathbf{k}_{\mathrm{I}}\right\}$. Its origin is at the centre of the Earth, $\mathbf{i}_{\mathrm{I}}$ points to the vernal equinox, $\mathbf{k}_{\mathrm{I}}$ to the North pole and $\mathbf{j}_{I}$ completes a right-handed set. Since the Earth's orbital motion around the Sun can be neglected for the study of RLV trajectories, the ECI frame is considered inertial and the equations of motion are referred to it.

With the same origin and equatorial plane, the Earth-Centred Earth-Fixed (ECEF) frame is defined by the set of vectors $\left\{\mathbf{i}_{\mathrm{E}}, \mathbf{j}_{\mathrm{E}}, \mathbf{k}_{\mathrm{E}}\right\}$. This frame rotates with the Earth's angular velocity $\boldsymbol{\Omega}_{\mathrm{I}}=\omega_{\mathrm{E}} \mathbf{k}_{\mathrm{I}}$, keeping $\mathbf{i}_{\mathrm{E}}$ along the Greenwich meridian. It is useful for the computation of position-dependent quantities due to a straightforward conversion between its coordinates and latitude, longitude and altitude $\{\varphi(t), \lambda(t), h(t)\}$.

In this study, simulations are initiated from the European Space Centre situated in French Guiana [IHJH99], with $\{\varphi(0), \lambda(0), h(0)\} \approx\{5.2 \mathrm{deg},-52.8 \mathrm{deg}, 0 \mathrm{~m}\}$. Defining this initial position in the ECI frame as $\mathbf{r}_{\mathrm{I}}(0)$, the initial velocity of the vehicle due to the Earth's rotation is $\mathbf{v}_{\mathrm{I}}(0)=\boldsymbol{\Omega}_{\mathrm{I}} \times \mathbf{r}_{\mathrm{I}}(0)$. Furthermore, the rotation quaternion $\mathbf{q}_{\mathrm{E}}^{\mathrm{I}}(t)$ from ECI to ECEF and the associated Direction Cosine Matrix (DCM) $C_{\mathbf{q}_{\mathrm{E}}^{\mathrm{I}}}(t)$ are related to their initial values as follows:

$$
C_{\mathbf{q}_{\mathrm{E}}^{\mathrm{I}}}(t)=\left[\begin{array}{ccc}
\cos \omega_{\mathrm{E}} t & \sin \omega_{\mathrm{E}} t & 0  \tag{8.1}\\
-\sin \omega_{\mathrm{E}} t & \cos \omega_{\mathrm{E}} t & 0 \\
0 & 0 & 1
\end{array}\right] C_{\mathbf{q}_{\mathrm{E}}^{\mathrm{I}}}(0)
$$

Being an orthonormal transformation, the inverse DCM is defined as $C_{\mathbf{q}_{\mathrm{I}}^{\mathrm{E}}}(t)=C_{\mathbf{q}_{\mathrm{E}}^{\mathrm{I}}}^{-1}(t)=C_{\mathbf{q}_{\mathrm{E}}^{\mathrm{I}}}^{\mathrm{T}}(t)$.
For a more intuitive analysis of launch trajectories, see Fig. 8.2, the Launch Pad (LP) reference frame is fixed at the initial position $\mathbf{r}_{\mathrm{I}}(0)$ and specified by $\left\{\mathbf{i}_{\mathrm{L}}, \mathbf{j}_{\mathrm{L}}, \mathbf{k}_{\mathrm{L}}\right\}$. Here, $\mathbf{k}_{\mathrm{L}}$ is normal to the local horizon, $\mathbf{i}_{\mathrm{L}}$ indicates the direction of launch, with an azimuth $\chi$ relative to the North, and $\mathbf{j}_{\mathrm{L}}$ completes a right-handed set. The reference mission addressed in this study (see Sec. 8.2.1) is based on a satellite injection in a quasi-polar orbit, with $\chi \approx-0.02$ degrees. Equivalently, vectors specifying a Recovery $\operatorname{Pad}(R P)$ reference frame $\left\{\mathbf{i}_{R}, \mathbf{j}_{R}, \mathbf{k}_{R}\right\}$ are defined in the same way, but having the origin at a recovery platform position.

The transformations between ECEF and local frames, $C_{\mathbf{q}_{\mathrm{L}}^{\mathrm{E}}}$ and $C_{\mathbf{q}_{\mathrm{R}}^{\mathrm{E}}}$, are time-invariant and of straightforward computation [Mat17]. To determine the position and velocity relative to the LP frame, its origin and the contribution of the Earth's rotation must be accounted for as follows:

$$
\left\{\begin{array}{l}
\mathbf{r}_{\mathrm{L}}(t)=C_{\mathbf{q}_{\mathrm{L}}^{\mathrm{E}}} C_{\mathbf{q}_{\mathrm{E}}^{\mathrm{I}}}(t)\left[\mathbf{r}_{\mathrm{I}}(t)-\mathbf{r}_{\mathrm{I}}(0)\right]  \tag{8.2}\\
\mathbf{v}_{\mathrm{L}}(t)=C_{\mathbf{q}_{\mathrm{L}}^{\mathrm{E}}} C_{\mathbf{q}_{\mathrm{E}}^{\mathrm{I}}}(t)\left[\mathbf{v}_{\mathrm{I}}(t)-\mathbf{\Omega}_{\mathrm{I}} \times \mathbf{r}_{\mathrm{I}}(t)\right]
\end{array}\right.
$$

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with $C_{\mathbf{q}_{\mathrm{E}}^{\mathrm{I}}}(t)$ given by Eq. (8.1), and in the same way for the RP frame. The latter frame is defined based on the location of the recovery platform, which is well-known when the mission starts.

Then, the vehicle's body-fixed reference frame is fixed to its Centre of Gravity (CG) and has basis vectors $\left\{\mathbf{i}_{\mathrm{B}}, \mathbf{j}_{\mathrm{B}}, \mathbf{k}_{\mathrm{B}}\right\}$. Vector $\mathbf{i}_{\mathrm{B}}$ lies along the vehicle's longitudinal axis and $\mathbf{j}_{\mathrm{B}}$ is defined so as to remain perpendicular to the pitch plane and have a positive pitch angle. The relationships between LP, RP and body-fixed frames are illustrated in Fig. 8.2. Note that the pitch plane changes slightly throughout the trajectory as a result of Earth's rotation.


Figure 8.2: Relationships between local (launch and recovery pad) and body-fixed reference frames

Following the above definition, roll, pitch and yaw angles $\{\phi(t), \theta(t), \psi(t)\}$ represent the orientation of the body-fixed frame with respect to the LP frame. Hence, upon launch, i.e. $\{\phi(0), \theta(0), \psi(0)\}=\left\{\pi, \frac{\pi}{2}, 0\right\}$ radians, the inertial orientation of the vehicle's body is given by:

$$
C_{\mathbf{q}_{\mathbf{B}}^{I}}(0)=\left[\begin{array}{ccc}
0 & 0 & 1  \tag{8.3}\\
0 & -1 & 0 \\
1 & 0 & 0
\end{array}\right] C_{\mathbf{q}_{\mathrm{L}}^{\mathrm{L}}} C_{\mathbf{q}_{\mathrm{E}}^{\mathrm{E}}}(0)
$$

and its initial angular velocity corresponds to $\omega_{\mathrm{B}}(0)=C_{\mathbf{q}_{\mathrm{B}}^{\mathrm{I}}}(0) \boldsymbol{\Omega}_{\mathrm{I}}$. Note that an initial roll angle of $\pi$ is assumed because $\mathbf{j}_{\mathrm{L}}$ and $\mathbf{j}_{\mathrm{B}}$ are pointing in opposite directions, and that the offset between launch pad and the vehicle's CG can be neglected for practical computations.

For the computation of the aerodynamic characteristics, a Velocity Reference Frame (VRF) is defined using vectors $\left\{\mathbf{i}_{\mathrm{V}}, \mathbf{j}_{\mathrm{V}}, \mathbf{k}_{\mathrm{V}}\right\}$. This frame is also fixed to the vehicle's CG, but now with $\mathbf{i}_{\mathrm{V}}$ directed along the air-relative velocity vector $\mathbf{v}_{\text {air }}(t)$. A vector rotation from the body-fixed to VRF, $C_{\mathbf{q}_{\mathbf{V}}^{\mathrm{B}}}(t)$, can be represented by two aerodynamic angles, the angle of attack $\alpha(t)$ and sideslip $\beta(t)$ (also depicted in Fig. 8.2), as follows:

$$
C_{\mathbf{q}_{\mathrm{V}}^{\mathrm{B}}}(t)=\left[\begin{array}{ccc}
\cos \alpha(t) \cos \beta(t) & \sin \beta(t) & \sin \alpha(t) \cos \beta(t)  \tag{8.4}\\
-\cos \alpha(t) \sin \beta(t) & \cos \beta(t) & -\sin \alpha(t) \sin \beta(t) \\
-\sin \alpha(t) & 0 & \cos \alpha(t)
\end{array}\right]
$$

The gravity model adopted in this study is the 2008 Earth Gravitational Model (EGM) [PHKF08], which is based on a $120^{\text {th }}$ order spherical harmonic approximation of the gravity field and implemented in [Mat17]. It contains a function $\mathbf{g}_{\text {EGM }}$ that computes the gravity acceleration in the ECEF frame. Hence, the corresponding vector in inertial coordinates is given by:

$$
\begin{equation*}
\mathbf{g}_{\mathrm{I}}(t)=C_{\left.{\mathbf{\mathbf { q } _ { \mathrm { I } } ^ { \mathrm { I } }}}(t) \mathbf{g}_{\mathrm{EGM}}\left(C_{\mathbf{q}_{\mathrm{E}}^{\mathrm{I}}}(t) \mathbf{r}_{\mathrm{I}}(t)\right),{ }^{2}\right)} \tag{8.5}
\end{equation*}
$$

The atmosphere model adopted is also available in [Mat17] and implements the mathematical representation of the 1976 Committee On Extension to the Standard Atmosphere (COESA) [COE76]. This representation provides, as a function of altitude, the air density $\rho(h(t))$ and the speed of sound $a(h(t))$.

Finally, wind gusts are included by adding a velocity field with Northern and Eastern components, $w_{\text {NRN }}(h(t))$ and $w_{\text {ERN }}(h(t))$ relative to launch pad, hence the wind perturbation with respect to the ECEF frame corresponds to:

$$
\mathbf{w}_{\mathrm{E}}(t)=C_{\mathbf{q}_{\mathrm{E}}^{\mathrm{L}}}\left[\begin{array}{cc}
\cos \chi & \sin \chi  \tag{8.6}\\
\sin \chi & -\cos \chi \\
0 & 0
\end{array}\right]\left[\begin{array}{l}
w_{\mathrm{NRN}}(h(t)) \\
w_{\mathrm{ERN}}(h(t))
\end{array}\right]
$$

where $w_{\text {NRN }}(h(t))$ and $w_{\text {ERN }}(h(t))$ can be generated, for example, using a traditional noisecolouring Dryden filter.

With such a filter, wind disturbance velocity $v_{\mathrm{w}}$ is modelled by colouring a white noise signal $n_{\mathrm{w}}$ through the following transfer function:

$$
\begin{equation*}
G_{\mathrm{wind}}(\mathrm{~s})=\frac{v_{\mathrm{w}}(\mathrm{~s})}{n_{\mathrm{w}}(\mathrm{~s})}=\frac{\sqrt{\frac{2}{\pi} \frac{V_{\mathrm{w}}(h)}{l_{\mathrm{w}}(h)} \sigma_{\mathrm{w}}^{2}(h)}}{\mathrm{s}+\frac{V_{\mathrm{w}}(h)}{l_{\mathrm{w}}(h)}} \tag{8.7}
\end{equation*}
$$

where the turbulence length scale $l_{\mathrm{w}}(h)$ and standard deviation $\sigma_{\mathrm{w}}(h)$ are tabulated as a function of altitude for light, moderate and severe levels of wind [Joh93], and $V_{\mathrm{w}}(h)$ represents the vehicle's airspeed with respect to a steady-state vertical wind profile. In this equation, the dependence of altitude on time has been dropped for clarity. For further information on the application of Dryden filters to launcher flight control, the reader is referred to $\left[\mathrm{SBM}^{+} 16, \mathrm{NMS}^{+} 19\right]$.

### 8.1.2 Equations of motion

The dynamic equations that describe the motion of a vehicle in space are summarised below and complete derivations can be found in [Gre70]. These equations are based on the initial states $\left\{\mathbf{r}_{\mathrm{I}}(0), \mathbf{v}_{\mathrm{I}}(0), \mathbf{q}_{\mathrm{B}}^{\mathrm{I}}(0), \omega_{\mathrm{B}}(0)\right\}$ identified in Sec. 8.1.1 and on the assumption that effects related

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to moving masses (including "tail-wags-dog" moment and rocket jet damping) are negligible for trajectory assessment. In addition, it suffices to account for the impact of time-varying mass through the mass-depletion dynamics of Eq. (8.28) [Eke98].

The vehicle's translational motion (i.e. acceleration $\ddot{\mathbf{r}}_{\mathbf{I}}(t)$ and velocity $\left.\mathbf{v}_{\mathbf{I}}(t)\right)$ is described in the ECI frame by:

$$
\begin{equation*}
\ddot{\mathbf{r}}_{\mathrm{I}}(t)=\dot{\mathbf{v}}_{\mathbf{I}}(t)=\mathbf{g}_{\mathrm{I}}(t)+\frac{1}{m(t)}\left[\mathbf{F}_{\text {aero,I }}(t)+\mathbf{F}_{\mathrm{TVC}, \mathrm{I}}(t)+\mathbf{F}_{\mathrm{fin}, \mathrm{I}}(t)+\mathbf{F}_{\mathrm{thr}, \mathrm{I}}(t)\right] \tag{8.8}
\end{equation*}
$$

where $\mathbf{F}_{\text {aero, }}(t)$ represents the aerodynamic force of the vehicle's body (Sec. 8.1.3) expressed in the ECI frame, $\mathbf{F}_{\text {TVC,I }}(t), \mathbf{F}_{\text {fin,I }}(t)$ and $\mathbf{F}_{\text {thr,I }}(t)$ represent control forces (Sec. 8.1.4 to 8.1.6), $m(t)$ is the total mass of the vehicle (Sec. 8.1.7) and $\mathbf{g}_{\mathrm{I}}(t)$ is given by Eq. (8.5).

In addition, the rotational dynamics are described in the body-fixed frame by:

$$
\begin{align*}
\dot{\omega}_{\mathrm{B}}(t)=J^{-1}(t)\left[\mathbf{M}_{\mathrm{aero}, \mathrm{~B}}(t)+\mathbf{M}_{\mathrm{TVC}, \mathrm{~B}}(t)+\mathbf{M}_{\mathrm{fin}, \mathrm{~B}}(t)\right. & +\mathbf{M}_{\mathrm{thr}, \mathrm{~B}}(t) \\
& \left.-\omega_{\mathrm{B}}(t) \times J(t) \omega_{\mathrm{B}}(t)-\dot{J}(t) \omega_{\mathrm{B}}(t)\right] \tag{8.9}
\end{align*}
$$

Equivalently to the forces, $\mathbf{M}_{\text {aero, } \mathrm{B}}(t), \mathbf{M}_{\mathrm{TVC}, \mathrm{B}}(t), \mathbf{M}_{\mathrm{fin}, \mathrm{B}}(t)$ and $\mathbf{M}_{\mathrm{thr}, \mathrm{B}}(t)$ represent aerodynamic and control moments written in the body axes, $J(t)$ is the inertia tensor of the vehicle (Sec. 8.1.7) and $\dot{J}(t)$ its time-derivative.

Finally, the orientation of the vehicle's body axes in the ECI frame is propagated by the quaternion-based kinematics equation:

$$
\dot{\mathbf{q}}_{\mathrm{B}}^{\mathrm{I}}(t)=\frac{1}{2}\left[\begin{array}{ccc}
q_{4}(t) & -q_{3}(t) & q_{2}(t)  \tag{8.10}\\
q_{3}(t) & q_{4}(t) & -q_{1}(t) \\
-q_{2}(t) & q_{1}(t) & q_{4}(t) \\
-q_{1}(t) & -q_{2}(t) & -q_{3}(t)
\end{array}\right] \omega_{\mathrm{B}}(t)
$$

where the quaternion $\mathbf{q}_{\mathrm{B}}^{\mathrm{I}}(t)=\left[q_{1}(t) q_{2}(t) q_{3}(t) q_{4}(t)\right]^{\mathrm{T}}$ is defined to contain the scalar part, $q_{4}(t)$, as its last component. This vector and its associated DCM $C_{\mathbf{q}_{\mathrm{B}}}(t)$ are also essential for the computation of the forces and moments in Eq. (8.8) and (8.9).

The equations of motion can be easily augmented to include effects such as structural flexibility and propellant sloshing. However, in this chapter, only rigid-body motion is considered.

### 8.1.3 Aerodynamics characteristics

Aerodynamic forces and moments generated by the vehicle's main body depend on its external shape, as well on the instantaneous dynamic pressure. Assuming that the Earth's atmosphere rotates with the planet without slippage and shearing, dynamic pressure is given by:

$$
\begin{equation*}
Q(t)=\frac{1}{2} \rho(t) V^{2}(t) \tag{8.11}
\end{equation*}
$$

where $V(t)=\left\|\mathbf{v}_{\text {air }}(t)\right\|$ and $\mathbf{v}_{\text {air }}(t)=\left[v_{\text {air, }}(t) v_{\text {air, } \mathrm{y}}(t) v_{\mathrm{air}, \mathrm{z}}(t)\right]^{\mathrm{T}}$ is the air-relative velocity vector at the vehicle's CG written in the body-fixed frame. This vector accounts for the vehicle's inertial velocity $\mathbf{v}_{\mathrm{I}}(t)$, Earth's rotation $\boldsymbol{\Omega}_{\mathrm{I}}$ and wind gusts $\mathbf{w}_{\mathrm{E}}(t)$ as follows:

$$
\begin{equation*}
\mathbf{v}_{\mathrm{air}}(t)=C_{\mathbf{q}_{\mathrm{B}}^{\mathrm{I}}}(t)\left[\mathbf{v}_{\mathrm{I}}(t)-\mathbf{\Omega}_{\mathrm{I}} \times \mathbf{r}_{\mathrm{I}}(t)-C_{\mathbf{q}_{\mathrm{I}}^{\mathrm{E}}}(t) \mathbf{w}_{\mathrm{E}}(t)\right] \tag{8.12}
\end{equation*}
$$

Moreover, the local velocity at an arbitrary location $\mathbf{x}_{j}(t)$ along the launcher's body is also affected by its rotational rate:

$$
\begin{equation*}
\mathbf{v}_{\mathrm{air}}^{j}(t)=\mathbf{v}_{\mathrm{air}}(t)+\omega_{\mathrm{B}}(t) \times\left[\mathbf{x}_{j}(t)-\mathbf{x}_{\mathrm{CG}}(t)\right] \tag{8.13}
\end{equation*}
$$

where $\mathbf{x}_{j}(t)$ and the CG position, represented as $\mathbf{x}_{\mathrm{CG}}(t)=\left[x_{\mathrm{CG}}(t) y_{\mathrm{CG}}(t) z_{\mathrm{CG}}(t)\right]^{\mathrm{T}}$, are measured with respect to the same reference point in the body-fixed frame. The two aerodynamic angles at the Centre of Pressure $(\mathrm{CP})$, where aerodynamic forces are applied, i.e. at $\mathbf{x}_{j}(t)=\mathbf{x}_{\mathrm{CP}}(t)$, can then be defined as follows:

$$
\begin{align*}
\alpha^{\mathrm{CP}}(t) & =\arctan _{2} \frac{v_{\mathrm{air}, \mathrm{z}}^{\mathrm{CP}}(t)}{v_{\mathrm{air}, \mathrm{x}}^{\mathrm{CP}}(t)}  \tag{8.14}\\
\beta^{\mathrm{CP}}(t) & =\arcsin \frac{v_{\mathrm{air}, \mathrm{y}}^{\mathrm{CP}}(t)}{\left\|\mathbf{v}_{\mathrm{air}}^{\mathrm{CP}}(t)\right\|} \tag{8.15}
\end{align*}
$$

The vehicle has a generic axisymmetric shape that is representative of Europe's lightweight VEGA launcher [Bia08]. Having in mind its axisymmetry, the forces are expressed in the air-relative velocity reference frame as:

$$
\mathbf{F}_{\mathrm{aero}, \mathrm{~V}}(t)=-Q(t) S_{\mathrm{ref}}\left[\begin{array}{c}
C_{D}\left(\alpha_{\mathrm{eff}}(t), M(t)\right)  \tag{8.16}\\
0 \\
C_{L}\left(\alpha_{\mathrm{eff}}(t), M(t)\right)
\end{array}\right]
$$

where $S_{\text {ref }}$ is a reference aerodynamic area and $\left\{C_{D}, C_{L}\right\}$ are the drag and lift coefficients, respectively. These coefficients are estimated as functions of the effective angle of attack $\alpha_{\text {eff }}(t)$ and Mach number $M(t)=V(t) / a(t)$. The former is defined based on the so-called aeroballistic coordinates [Zip07] as follows:

$$
\begin{equation*}
\alpha_{\mathrm{eff}}(t)=\arccos \left(\cos \alpha^{\mathrm{CP}}(t) \cos \beta^{\mathrm{CP}}(t)\right) \approx \sqrt{\left(\alpha^{\mathrm{CP}}(t)\right)^{2}+\left(\beta^{\mathrm{CP}}(t)\right)^{2}} \tag{8.17}
\end{equation*}
$$

The aerodynamic force of Eq. (8.16) is thus written in the ECI frame as:

$$
\begin{equation*}
\mathbf{F}_{\mathrm{aero}, \mathrm{I}}(t)=C_{\mathbf{q}_{\mathrm{I}}^{\mathrm{B}}}(t) C_{\mathbf{q}_{\mathrm{B}}^{\mathrm{V}}}(t) \mathbf{F}_{\mathrm{aero}, \mathrm{~V}}(t) \tag{8.18}
\end{equation*}
$$

In addition, an aerodynamic moment around the vehicle's CG is produced due to the offset between this point and the CP. Hence, this moment is directly expressed in the body-fixed frame as:

$$
\begin{equation*}
\mathbf{M}_{\mathrm{aero}, \mathrm{~B}}(t)=\left[\mathbf{x}_{\mathrm{CP}}(t)-\mathbf{x}_{\mathrm{CG}}(t)\right] \times C_{\mathbf{q}_{\mathrm{B}}^{\mathrm{V}}}(t) \mathbf{F}_{\mathrm{aero}, \mathrm{~V}}(t) \tag{8.19}
\end{equation*}
$$

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Similar to drag and lift, the CP position, represented as $\mathbf{x}_{\mathrm{CP}}(t)=\left[x_{\mathrm{CP}}(t) 00\right]^{\mathrm{T}}$, is estimated as a function of $\alpha_{\mathrm{eff}}(t)$ and $M(t)$, while the evolution of $\mathbf{x}_{\mathrm{CG}}(t)$ is computed in Sec. 8.1.7.

Aerodynamic coefficients were kindly provided by VEGA's manufacturer, AVIO, using the full vehicle configuration up to a 10 degree angle of attack and for the post-separation configuration sparsely from 0 to 180 degrees angles of attack. These coefficients are then linearly interpolated. $x_{\mathrm{CP}}$ is not defined for $\alpha_{\mathrm{eff}}(t)$ equal to 0 and 180 degrees, hence it is linearly extrapolated from the adjacent interval. The variation of $C_{D}, C_{L}$ and $x_{\mathrm{CP}}$ for these two cases with respect to $\alpha_{\text {eff }}$ and $M$ is illustrated in Fig. 8.3 and 8.4 respectively.


Figure 8.3: Aerodynamic coefficients for full vehicle configuration


Figure 8.4: Aerodynamic coefficients for first stage configuration

For simplicity, aerodynamic coefficients are assumed to be independent of the level of thrust, which is a very rough approximation for retro-propulsive flight, where there are complicated interactions between the engine plume and the oncoming flow. However, the goal of this work is not to provide a high-level fidelity industrial simulator, but to show how a reusable launcher benchmark can be built. Moreover, the limited validity of the assumptions does not invalidate the modelling approach and the adopted model can be easily upgraded.

The proneness of the vehicle to generate aerodynamic loads is conventionally quantified using the following parameter:

$$
\begin{equation*}
\mu_{\alpha}(t)=\left[x_{\mathrm{CP}}(t)-x_{\mathrm{CG}}(t)\right] \frac{S_{\mathrm{ref}} C_{N_{\alpha}}(t)}{J_{N}(t)} Q(t) \tag{8.20}
\end{equation*}
$$

which provides an affine relationship between the angle of attack and the load-induced angular acceleration (see Appendix C). In this equation, $J_{N}(t)$ is the lateral Moment of Inertia (MoI) component and the normal load gradient is given by:

$$
\begin{equation*}
C_{N_{\alpha}}(t)=\frac{\partial C_{N}(\alpha(t), M(t))}{\partial \alpha(t)} \tag{8.21}
\end{equation*}
$$

with:

$$
\begin{equation*}
C_{N}(\alpha(t), M(t))=C_{L}(\alpha(t), M(t)) \cos \alpha(t)+C_{D}(\alpha(t), M(t)) \sin \alpha(t) \tag{8.22}
\end{equation*}
$$

In addition, $Q_{\alpha}(t)=Q(t) \alpha_{\text {eff }}(t)$ is an extremely useful indicator as it directly assesses the impact of trajectory and attitude on the induced loads. For physically meaningful results, $\alpha_{\text {eff }}(t)$ must always be reduced to the first quadrant, i.e. $\alpha_{\mathrm{eff}}(t) \in\left[0, \frac{\pi}{2}\right]$.

Finally, it is also essential to have an idea of the thermal environment encountered by the RLV throughout L\&R. A simple way to achieve this is by analysing the heat flux at the vehicle's stagnation point [RA93]. Given a reference nose radius $R_{\text {ref }}$, the maximum heat rate can be approximated by the Sutton Graves equation:

$$
\begin{equation*}
Q_{H}(t)=k_{H} \sqrt{\frac{\rho(t)}{R_{\mathrm{ref}}}} V^{3}(t) \tag{8.23}
\end{equation*}
$$

with $k_{H} \approx 1.74 \times 10^{-4}$ for Earth.

### 8.1.4 TVC system

The vehicle's ascent and descent trajectories are controlled by adjusting the magnitude and direction of the thrust vector generated by its rocket engine. This adjustment is achieved via two TVC actuators that deflect the engine's nozzle by $\left\{\beta_{\mathrm{TVC}, \mathrm{y}}(t), \beta_{\mathrm{TVC}, \mathrm{z}}(t)\right\}$ along the body $\mathbf{j}_{B}$ and $\mathbf{k}_{\mathrm{B}}$ axes respectively.

With this in mind, the TVC-generated force becomes:

$$
\begin{align*}
\mathbf{F}_{\mathrm{TVC}, \mathrm{~B}}(t) & =T_{\mathrm{ref}}(t)\left[\begin{array}{c}
\cos \beta_{\mathrm{TVC}, \mathrm{y}}(t) \cos \beta_{\mathrm{TVC}, \mathrm{z}}(t) \\
\cos \beta_{\mathrm{TVC}, \mathrm{y}}(t) \sin \beta_{\mathrm{TVC}, \mathrm{z}}(t) \\
-\sin \beta_{\mathrm{TVC}, \mathrm{y}}(t)
\end{array}\right]  \tag{8.24}\\
\mathbf{F}_{\mathrm{TVC}, \mathrm{I}}(t) & =C_{\mathbf{q}_{\mathrm{I}}^{\mathrm{B}}}(t) \mathbf{F}_{\mathrm{TVC}, \mathrm{~B}}(t) \tag{8.25}
\end{align*}
$$

and, with $\mathbf{x}_{P V P}=\left[\begin{array}{lll}x_{P V P} & 0 & 0\end{array}\right]^{\mathrm{T}}$ representing the TVC pivot position, the moment around the CG is given by:

$$
\begin{equation*}
\mathbf{M}_{\mathrm{TVC}, \mathrm{~B}}(t)=\left[\mathbf{x}_{\mathrm{PVP}}-\mathbf{x}_{\mathrm{CG}}(t)\right] \times \mathbf{F}_{\mathrm{TVC}, \mathrm{~B}}(t) \tag{8.26}
\end{equation*}
$$

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The required thrust magnitude $T_{\text {ref }}(t)$ and direction are commanded by the guidance system (Sec. 8.2), with the latter using vehicle attitude reference angles $\left\{\phi_{\text {ref }}(t), \theta_{\text {ref }}(t), \psi_{\text {ref }}(t)\right\}$ as a surrogate.

Similar to Eq. (8.20), the TVC effectiveness in counteracting aerodynamic loads is measured through the coefficient:

$$
\begin{equation*}
\mu_{\mathrm{c}}(t)=\left[x_{\mathrm{CG}}(t)-x_{\mathrm{PVP}}\right] \frac{T_{\mathrm{ref}}(t)}{J_{N}(t)} \tag{8.27}
\end{equation*}
$$

which is naturally driven by the actual thrust level $T_{\text {ref }}(t)$. This coefficient establishes an affine approximation between in-plane TVC deflection and control-induced angular acceleration (see Appendix C).

The generation of thrust then causes the depletion of propellant. In this case, assuming negligible engine back-pressure losses, the mass-depletion dynamics is given by the rocket equation [Gre70]:

$$
\begin{equation*}
\dot{m}(t)=-\frac{1}{I_{\mathrm{sp}} g_{0}} T_{\mathrm{ref}}(t) \tag{8.28}
\end{equation*}
$$

where $I_{\mathrm{sp}}$ is the specific impulse of the engine and $g_{0} \approx 9.81 \mathrm{~m} / \mathrm{s}^{2}$ is the gravitational acceleration at the Earth's surface.

Recovering the launch vehicle requires the use of a re-ignitable and throttleable rocket engine. In this vehicle, a fictional liquid engine using highly-refined kerosene (RP-1) as fuel and liquid oxygen (LOx) as oxidizer is adopted. This type of engine is common among many launcher manufacturers [IHJH99], including SpaceX [CS04]. Its main characteristics are summarised in Table 8.1. In this table, the required initial propellant masses have been determined based on the reference mission under analysis and on information from the VEGA launcher [Bia08].

Table 8.1: RLV rocket engine characteristics

| Parameter | Value |
| :--- | :---: |
| Sea-level specific impulse (s) | 282 |
| Oxidizer/fuel mass ratio | 2.56 |
| Oxidizer/fuel density ratio | 1.42 |
| Initial fuel mass (kg) | 25913 |
| Initial oxidizer mass (kg) | 66337 |

In order for the tanks to meet VEGA's dimensions and initial MCI properties while ensuring that the remaining propellant after launch is enough for a powered descent, this results in a structural mass lower than the original one and in propellant densities higher than the actual LOx/RP-1 engine. Nevertheless, the same density ratio of Table 8.1 is kept so as to have a meaningful representation of the CG travel throughout the burn. Further details on the launcher's MCI evolution are provided in Sec. 8.1.7.

### 8.1.5 Fins

Planar fins are also included in the RLV model to ensure enough control authority under low (or zero) TVC effectiveness (recall Eq. (8.27)). This will be particularly critical throughout the descent flight, and thus they should be ideally placed above the vehicle's centre of pressure during this phase for improved stability. Here, only two pairs of fins are considered as depicted in Fig. 8.5, but the generalisation in case additional surfaces are exploited for improved controllability is straightforward.

One pair of fins is assigned to pitch motion control using deflections $\left\{\beta_{\mathrm{fin}, 1}(t), \beta_{\mathrm{fin}, 2}(t)\right\}$, the other pair to yaw control via $\left\{\beta_{\mathrm{fin}, 3}(t), \beta_{\mathrm{fin}, 4}(t)\right\}$. It is assumed that any roll perturbation is rejected by the attitude control system (see Sec. 8.3) so that the two pairs always remain in the trajectory yaw and pitch planes, respectively.

It is further assumed that, due to the reduced fin area compared to the RLV body, their axial force contribution is negligible so that only the normal component is accounted for. When flow separation is neglected, this contribution has a sinusoidal dependence on the fin angle of attack given by:

$$
\begin{equation*}
C_{\mathrm{f}}\left(\alpha_{\mathrm{fin}}(t)\right)=\bar{C}_{\mathrm{f}} \sin \alpha_{\mathrm{fin}}(t) \tag{8.29}
\end{equation*}
$$

where $\bar{C}_{\mathrm{f}}$ is the maximum normal fin force coefficient and $\alpha_{\mathrm{fin}}(t)$ is the local angle of attack. The impact of these assumptions will be verified at a later stage through the consideration of aerodynamic uncertainties in the model.

The $i^{\text {th }}$ fin's angle of attack and its associated force in the RLV body-fixed frame $\mathbf{F}_{\text {fin }, i}(t)$ are then defined in the pitch plane as:

$$
\left\{\begin{array}{l}
\alpha_{\mathrm{fin}, i}(t)=\beta_{\mathrm{fin}, i}(t)-\alpha^{\mathrm{fin}}(t)  \tag{8.30}\\
\mathbf{F}_{\mathrm{fin}, i}(t)=Q(t) S_{\mathrm{fin}} C_{\mathrm{f}}\left(\alpha_{\mathrm{fin}, i}(t)\right)\left[-\sin \beta_{\mathrm{fin}, i}(t) \quad 0 \quad c \quad \cos \beta_{\mathrm{fin}, i}(t)\right]^{\mathrm{T}}, i=\{1,2\}
\end{array}\right.
$$

and in the yaw plane as:

$$
\left\{\begin{array}{l}
\alpha_{\mathrm{fin}, i}(t)=-\beta_{\mathrm{fn}, i}(t)-\beta^{\mathrm{fin}}(t)  \tag{8.31}\\
\mathbf{F}_{\mathrm{fin}, i}(t)=Q(t) S_{\mathrm{fin}} C_{\mathrm{f}}\left(\alpha_{\mathrm{fin}, i}(t)\right)\left[\begin{array}{lll}
\sin \beta_{\mathrm{fin}, i}(t) & \cos \beta_{\mathrm{fin}, i}(t) & 0
\end{array}\right]^{\mathrm{T}}, i=\{3,4\}
\end{array}\right.
$$

where $S_{\mathrm{fin}}$ is the projected area of one fin, and $\alpha^{\mathrm{fin}}(t)$ and $\beta^{\mathrm{fin}}(t)$ represent the vehicle's aerodynamic angles at the fins' location, computed as in Sec. 8.1.3. With this in mind, the fin-generated force in the ECI frame and the moment in the body frame correspond to:

$$
\begin{align*}
\mathbf{F}_{\mathrm{fin}, \mathrm{I}}(t) & =C_{\mathbf{q}_{\mathbf{1}}^{\mathrm{B}}}(t) \sum_{i=1}^{4} \mathbf{F}_{\mathrm{fin}, i}(t)  \tag{8.32}\\
\mathbf{M}_{\mathrm{fin}, \mathrm{~B}}(t) & =\sum_{i=1}^{4}\left[\mathbf{x}_{\mathrm{fin}, i}-\mathbf{x}_{\mathrm{CG}}(t)\right] \times \mathbf{F}_{\mathrm{fin}, i}(t) \tag{8.33}
\end{align*}
$$

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where the offset between each fin's CP and attachment point $\mathbf{x}_{\mathrm{fin}, i}$ is negligible compared to the distance between the latter and the vehicle's CG.

Once again, the effectiveness of the fins in generating control moments can be quantified. Assuming equal fin deflections within the same plane, it is given by:

$$
\begin{equation*}
\mu_{\mathrm{f}}(t)=2\left[x_{\mathrm{fin}}-x_{\mathrm{CG}}(t)\right] \frac{Q(t) S_{\mathrm{fin}} C_{\mathrm{f}_{\alpha}}(t)}{J_{N}(t)} \tag{8.34}
\end{equation*}
$$

where $x_{\mathrm{fin}}$ is the longitudinal position of the fins and $C_{\mathrm{f}_{\alpha}}(t)=\bar{C}_{\mathrm{f}} \cos \alpha_{\mathrm{fin}}(t)$ is the normal fin force gradient, with $\alpha_{\text {fin }}(t)$ computed from Eq. (8.30) for the pitch plane, or from Eq. (8.31) for the yaw plane.

Because $\alpha_{\mathrm{fin}}(t)$ has a dependence on the vehicle's aerodynamic angles, the fins will also have an impact on the natural aerodynamics of the vehicle and its load-proneness parameter has to be generalised accordingly:

$$
\begin{equation*}
\mu_{\alpha^{\prime}}(t)=\mu_{\alpha}(t)+\mu_{\mathrm{f}}(t) \cos \beta_{\mathrm{fin}, 0}(t) \tag{8.35}
\end{equation*}
$$

where $\beta_{\text {fin }, 0}(t)$ represents the in-plane fin deflection required to trim the vehicle (see Appendix C).
For the rest of this part, $\bar{C}_{\mathrm{f}}$ has been fixed to 6 , which is a reasonable value among conventional symmetric airfoils, and $S_{\text {fin }}$ has been set to $0.54 \mathrm{~m}^{2}$ based on preliminary stability analyses (see Sec. 8.3.1).

### 8.1.6 Cold gas thrusters

In addition, cold gas thrusters are included for controllability under zero main engine thrust and low fin effectiveness (recall Eq. (8.34)), which is the case at high altitudes where air density is very low.

Similar to the fins, a different pair of thrusters is assigned to pitch and yaw control. The force generated in the ECI frame is given by:

$$
\begin{equation*}
\mathbf{F}_{\mathrm{thr}, \mathrm{I}}(t)=C_{\mathbf{q}_{\mathrm{I}}^{\mathrm{B}}}(t) \bar{T}_{\mathrm{thr}}\left(\beta_{\mathrm{thr}, \mathrm{y}}(t) \mathbf{k}_{\mathrm{B}}-\beta_{\mathrm{thr}, \mathrm{Z}}(t) \mathbf{j}_{\mathrm{B}}\right) \tag{8.36}
\end{equation*}
$$

where $\bar{T}_{\text {thr }}$ is the maximum thruster force and $\left\{\beta_{\mathrm{thr}, \mathrm{y}}(t), \beta_{\mathrm{thr}, \mathrm{Z}}(t)\right\}$ are attitude commands about the body $\mathbf{j}_{\mathrm{B}}$ and $\mathbf{k}_{\mathrm{B}}$ axes, normalised between $[-1,1]$. Equivalently, the corresponding moment in the body-fixed frame is:

$$
\begin{equation*}
\mathbf{M}_{\mathrm{thr}, \mathrm{~B}}(t)=\bar{T}_{\mathrm{thr}}\left(\left[\mathbf{x}_{\mathrm{thr}, \mathrm{y}}-\mathbf{x}_{\mathrm{CG}}(t)\right] \times \beta_{\mathrm{thr}, \mathrm{y}}(t) \mathbf{k}_{\mathrm{B}}-\left[\mathbf{x}_{\mathrm{thr}, \mathrm{Z}}-\mathbf{x}_{\mathrm{CG}}(t)\right] \times \beta_{\mathrm{thr}, \mathrm{z}}(t) \mathbf{j}_{\mathrm{B}}\right) \tag{8.37}
\end{equation*}
$$

In this equation, $\mathbf{x}_{\mathrm{thr}, \mathrm{y}}$ and $\mathbf{x}_{\mathrm{thr}, \mathrm{z}}$ represent the position of the thruster that is triggered for pitch and yaw control, which has to be adjusted in accordance with the sign of $\beta_{\mathrm{thr}, \mathrm{y}}(t)$ and $\beta_{\mathrm{thr}, \mathrm{z}}(t)$ since commands in opposite directions trigger thrusters in opposite sides of the vehicle. For simplicity, thruster commands are assumed to be continuous.

Control effectiveness of cold gas thrusters can also be quantified as follows:

$$
\begin{equation*}
\mu_{\mathrm{t}}(t)=\left[x_{\mathrm{thr}}-x_{\mathrm{CG}}(t)\right] \frac{\overline{\mathrm{T}}_{\mathrm{thr}}}{J_{N}(t)} \tag{8.38}
\end{equation*}
$$

where $x_{\text {thr }}$ is the longitudinal position of the thrusters (here assumed equal to $x_{\text {fin }}$ for simplicity). In addition, $\bar{T}_{\text {thr }}$ is assumed constant and equal to 400 N , which is a reasonable value among conventional actuators. This assumption is not restrictive since the subsequent G\&C design/analysis is able to explicitly account for thrust variations throughout the flight (which cannot be neglected in the case of the main engine).

It is also important to note that, although there is a mass budget and depletion associated to cold gas thrusters, it is assumed to be negligible compared to that of the main engine.

### 8.1.7 Mass, CG \& inertia evolution

In line with all the considerations above, the launcher configuration adopted is detailed in Fig. 8.5, showing the full vehicle on the left and the reusable first stage on the right. Since this thesis is focused on the ascent and descent flight of the first stage only, all the other vehicle bodies (e.g. upper stages) will be referred to as PayLoad (PL) from the first stage's perspective.

The vehicle is assumed to have an axisymmetric shape and a uniform material (dry) density, with more mass allocated to the bottom and middle sections to account for the weight of the main engine module, retractable landing gear and inter-tank adapter. Moreover, fuel and oxidizer masses are modelled as cylinders in end burn. Other types of burn include centrifugal and centripetal burn, see [Eke98].

The MCI properties of the dry first stage and payload are summarised in Table 8.2. In this table, and for the remainder of this section, heights are measured with respect to the first stage base (i.e. 0.66 m above the nozzle exit) and MoI is relative to the CG of the corresponding body.

Table 8.2: RLV structural characteristics

|  | Dry first stage | Payload total |
| :--- | :---: | :---: |
| Mass $(\mathrm{kg})$ | 2750 | 43000 |
| CG Height $(\mathrm{m})$ | 4.60 | 12.91 |
| MoI Axial $\left(\mathrm{kg} \mathrm{m}^{2}\right)$ | 3981 | 44000 |
| MoI Lateral $\left(\mathrm{kg} \mathrm{m}^{2}\right)$ | 40267 | $3 \times 10^{6}$ |

Based on this configuration, the vehicle's mass is broken down into structural mass and time-dependent propellant mass, which is updated via Eq. (8.28) during engine burn, as follows:

$$
\begin{equation*}
m(t)=m_{\text {prop }}(t)+m_{\text {dry }}+m_{\mathrm{PL}} \tag{8.39}
\end{equation*}
$$

with propellant mass defined as $m_{\text {prop }}(t)=m_{\text {fuel }}(t)+m_{\text {oxid }}(t)$, and $m_{\text {dry }}$ and $m_{\text {PL }}$ given in Table 8.2. Note that $m_{\text {PL }}$ must be set to 0 after separation.


Figure 8.5: Generic RLV and first stage configuration

In order to compute the change over time of fuel and oxidizer masses, $m_{\text {fuel }}(t)$ and $m_{\text {oxid }}(t)$, as well as their level on the corresponding tank, $h_{\text {fuel }}(t)$ and $h_{\text {oxid }}(t)$, it is useful to normalise the propellant mass with respect to the initial values of Table 8.1:

$$
\begin{equation*}
\nu(t)=\frac{m_{\text {prop }}(t)}{m_{\text {prop }}(0)} \tag{8.40}
\end{equation*}
$$

and:

$$
\begin{equation*}
m_{\#}(t)=\nu(t) m_{\#}(0), \quad h_{\#}(t)=\nu(t) d_{\mathrm{tk}, \#} \tag{8.41}
\end{equation*}
$$

where $\#=\{$ fuel, oxid $\}, d_{\mathrm{tk}, \text { fuel }}(=3.30 \mathrm{~m})$ and $d_{\mathrm{tk}, \text { oxid }}(=5.97 \mathrm{~m})$ are the tank depths depicted in Fig. 8.5. The ratio between propellant burnt and its initial value is also quantified by:

$$
\begin{equation*}
\frac{m_{\text {burnt }}(t)}{m_{\text {prop }}(0)}=1-\nu(t) \tag{8.42}
\end{equation*}
$$

Due to propellant mass and level variations, the total vehicle CG and MoI vary substantially throughout the flight. In the nominal case, the former lies along the body longitudinal axis, $\mathrm{x}_{\mathrm{CG}}(t)=\left[\begin{array}{lll}x_{\mathrm{CG}}(t) & 0 & 0\end{array}\right]^{\mathrm{T}}$, and is computed as:

$$
\begin{align*}
x_{\mathrm{CG}}(t)=\frac{1}{m(t)}\left[m_{\text {fuel }}(t)( \right. & \left.h_{\text {tk,fuel }}+\frac{h_{\text {fuel }}(t)}{2}\right) \\
& \left.+m_{\text {oxid }}(t)\left(h_{\mathrm{tk}, \text { oxid }}+\frac{h_{\text {oxid }}(t)}{2}\right)+m_{\mathrm{dry}} h_{\mathrm{dry}}+m_{\mathrm{PL}} h_{\mathrm{PL}}\right] \tag{8.43}
\end{align*}
$$

where $h_{\mathrm{tk}, \text { fuel }}(=1.2 \mathrm{~m})$ and $h_{\mathrm{tk}, \text { oxid }}(=5.4 \mathrm{~m})$ are the tank heights provided in Fig. 8.5 and $h_{\mathrm{dry}}$ and $h_{\mathrm{PL}}$ are given in Table 8.2.

Based on the same assumptions, the inertia tensor is diagonal in nominal conditions and can be expressed as $J(t)=\operatorname{diag}\left[J_{A}(t), J_{N}(t), J_{N}(t)\right]$. The axial component corresponds directly to:

$$
\begin{equation*}
J_{A}(t)=\frac{1}{2} m_{\mathrm{prop}}(t) r_{\mathrm{tk}}^{2}+J_{A, \mathrm{dry}}+J_{A, \mathrm{PL}} \tag{8.44}
\end{equation*}
$$

in which $r_{\text {tk }}(=1.4 \mathrm{~m})$ is the tank radius. The lateral contributions of propellant masses relative to their CG are given by:

$$
\begin{equation*}
J_{N, \#}(t)=\frac{1}{12} m_{\#}(t)\left(3 r_{\mathrm{tk}}^{2}+h_{\#}^{2}(t)\right) \tag{8.45}
\end{equation*}
$$

with $\#=\{$ fuel, oxid $\}$, and all the contributions are converted to vehicle's CG coordinates using the parallel axis theorem as follows:

$$
\begin{align*}
& J_{N}(t)=J_{N, \text { fuel }}(t)+J_{N, \text { oxid }}(t)+J_{N, \mathrm{dry}}+J_{N, \mathrm{PL}}+m_{\text {fuel }}(t)\left(h_{\mathrm{tk}, \text { fuel }}+\frac{h_{\mathrm{fuel}}(t)}{2}-x_{\mathrm{CG}}(t)\right)^{2} \\
& +m_{\text {oxid }}(t)\left(h_{\mathrm{tk}, \text { oxid }}+\frac{h_{\text {oxid }}(t)}{2}-x_{\mathrm{CG}}(t)\right)^{2}+m_{\mathrm{dry}}\left(h_{\mathrm{dry}}-x_{\mathrm{CG}}(t)\right)^{2}+m_{\mathrm{PL}}\left(h_{\mathrm{PL}}-x_{\mathrm{CG}}(t)\right)^{2} \tag{8.46}
\end{align*}
$$

Once again, $m_{\mathrm{PL}}, J_{A, \text { PL }}$ and $J_{N, \mathrm{PL}}$ are set to 0 after separation and $x_{\mathrm{CG}}(t)$ is computed via Eq. (8.43).

### 8.2 RLV guidance approach

This section provides a description of the guidance techniques considered in this benchmark for the coupled assessment of RLVs. First, a general introduction of the reference mission profile and booster-back recovery strategies is presented, followed by the presentation of the baseline landing algorithm currently implemented.

### 8.2.1 L\&R mission profiles

As mentioned in Chapter 1, several studies [BH09, BGF14, TBLG15, SSBD17, DSE ${ }^{+}$17] have addressed the problem of RLV performance optimisation, but they are mostly focused on the application of Multi-Disciplinary Optimisation (MDO) methods to determine L\&R trajectories that allow delivering the highest payload while fulfilling competing mission and aerothermal load requirements. In opposition, the present thesis is not focused on the optimisation of payload capabilities, but rather on analysing the practical feasibility of different Descent \& Landing (D\&L) trajectories, together with the impact of different G\&C choices on the aerothermal loads encountered during the flight as well as on recovery performance.

For the above reason, the launch mission profile and the vehicle configuration described here remain fixed throughout the study and only the recovery trajectory is modified. The ascent profile corresponds to that of a $1,100 \mathrm{~kg}$ satellite injection in a quasi-polar orbit at an altitude of 800 km using an expendable launcher from the European Space Centre in French Guiana (same as $\left[\mathrm{SBM}^{+} 16, \mathrm{NMS}^{+} 19\right]$ ). Concerning its recovery, two distinct strategies are addressed and discussed below: DRL and RTLS, see Fig. 8.6.


Figure 8.6: Recovery mission profiles

The most straightforward booster-back recovery strategy is known as Down-Range Landing (DRL). In this scenario, the idea is for the reusable stage to land close to its unpropelled impact site, therefore minimising the propellant required for the landing. However, launches typically take place in the direction of the sea due to safety reasons, thus a sea-going recovery platform needs to be placed at the landing point and then bring the stage back. This approach has been successfully employed by SpaceX [Bla16], which uses a 91 m by 52 m drone ship as recovery pad.

From Fig. 8.6a, DRL missions start naturally with lift-off and ascent of the first stage (points 1 and 2 in the figure). This part of the mission is typically flown with open-loop guidance,
hence dispersions due to system uncertainties and environmental perturbations tend to grow. These dispersions are then compensated for by the exo-atmospheric stages in order to ensure an accurate satellite injection. Open-loop guidance commands are provided in terms of reference attitude angles $\left\{\theta_{\text {ref }}(t), \psi_{\text {ref }}(t)\right\}$ relative to the LP frame and thrust magnitude $T_{\text {ref }}(t)$.

The reference profiles adopted in this thesis are depicted in Fig. 8.7, where the distinct launch phases of pitch over and gravity turn have been highlighted. The thrust profile (bottom plot) is representative of a (non-throttleable) solid rocket engine and could be optimised for the present liquid-propellant launcher, with important consequences on the staging altitude and Mach number. Nonetheless, as mentioned above, the launch trajectory was kept the same for simplicity. Similar to the simplifications made in the aerodynamics model, it does not invalidate G\&C design/analysis for the recovery phase and opens the door to more detailed analyses, featuring a more realistic ascent thrust profile.


Figure 8.7: Ascent attitude and thrust references

After 110 seconds of flight, the first stage cuts off its engine and separates two seconds after (points 3 and 4 of Fig. 8.6a). The second stage then ignites its engine and proceeds the flight towards the payload's destination orbit. In the meantime (point 5), the first stage continues its exo-atmospheric motion in the direction of the recovery platform with approximately constant dispersions but with increasing velocity due to the action of gravity.

Then, at a pre-specified altitude $h_{\mathrm{s}}$, the first stage re-ignites its engine for the recovery burn (point 6), which must be able to counteract dispersions (point 7) and bring the booster from its current position and velocity to a soft touchdown at the drone ship (point 8).

The ability to cope with dispersions during recovery leads to the need for closed-loop guidance techniques, where guidance commands are computed in real-time to correct the trajectory based on onboard measurements. All the computations are made in the RP frame and the command thrust vector in this frame is then converted back to $\left\{\theta_{\text {ref }}(t), \psi_{\text {ref }}(t)\right\}$ and $T_{\text {ref }}(t)$.

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Furthermore, in order to more efficiently meet aerothermal loads or propulsion system requirements, sometimes it is convenient to explicitly split the recovery into three phases: [i] a re-entry burn aimed at decelerating the booster, [ii] a second engine cut-off and coast phase and [iii] a landing burn that ensures a precise touchdown. The duration of [i] and [ii] relative to the total recovery phase will be denoted $t_{1}$ and $t_{2}$ respectively.

In addition, as depicted in Fig. 8.6a, the reusable stage needs to undergo a slow but significant change in attitude between points 4 and 6 . This flip-over manoeuvre is simulated by a change in pitch with constant rate executed using fins and cold gas thrusters.

As an alternative recovery solution, the reusable booster can use its main engine not only for deceleration and landing, but also to deliver an additional burn that brings it to a recovery pad close to the launch site. The additional firing naturally leads to a more demanding propellant consumption, which can be nonetheless paid for by avoiding the use of a sea-going platform and all the associated infrastructure and operational costs. This recovery strategy is known as Return To Launch Site (RTLS) and is illustrated in Fig. 8.6b.

Until separation (i.e. point 4), the RTLS and DRL missions have the same profile, which is therefore omitted in Fig. 8.6b for clarity. From this point forward, as introduced above, the reusable stage needs to perform a rapid flip-over manoeuvre (point 5) followed by the boostback burn (point 6) in the direction of the launch site.

Following the boostback cut-off (point 7), the rest of the recovery (from point 8 onwards) is similar to DRL: an exo-atmospheric flight where the booster flips over at a slower rate, a closed-loop recovery stage that starts at $h_{\mathrm{s}}$ (and is achieved with one or two distinct burns), and finally a soft touchdown. In the RTLS scenario, the flip-over manoeuvres are again executed by fin and thruster induced pitch variations with constant rate. In addition, for this study, the boostback burn is fixed to an open-loop firing with magnitude $T_{\text {ref }}=600 \mathrm{kN}$, direction $\left\{\theta_{\text {ref }}, \psi_{\text {ref }}\right\}=\{180,0\}$ degrees and duration 30 seconds.

Finally, it is important to mention that the implementation of guidance laws for powered D\&L is independent of the recovery strategy - since all the computations are made in the RP frame, the only difference lies on the definition of this frame and of the corresponding transformation $C_{\mathbf{q}_{R}^{\mathrm{R}}}$. The baseline technique adopted in this study is presented in the following subsection.

### 8.2.2 Baseline recovery guidance

The same closed-loop guidance law studied in the first part of the thesis, i.e. Constrained Terminal Velocity Guidance (CTVG), is adopted as baseline in this second part. The main strength of this technique lies in its simplicity, which makes it extremely easy to implement and provides a rough idea of recovery flight mechanics very quickly.

As introduced in Sec. 2.1, CTVG is based on the compensation of zero-effort-miss and zero-effort-velocity vectors, $\mathbf{Z E M}(t)$ and $\mathbf{Z E V}(t)$, which quantify the position and velocity error
at the end-of-mission if no corrective manoeuvres are made after time $t$. Using these coordinates, the commanded thrust vector is given in the RP frame by:

$$
\mathbf{T}_{\mathrm{CTV}}(t)=\hat{m}(t)\left[\begin{array}{ll}
k_{r} & k_{v}
\end{array}\right]\left[\begin{array}{l}
\frac{\mathbf{Z E M}(t)}{\left(t_{f}-t\right)^{2}}  \tag{8.47}\\
\frac{\mathbf{Z E V}(t)}{t_{f}-t}
\end{array}\right]
$$

where $\hat{m}(t)$ represents the estimated mass of the vehicle and $t_{f}$ is the end-of-mission (i.e. touchdown) time. Optimal values of $\{6,-2\}$ for the two gains $\left\{k_{r}, k_{v}\right\}$ were shown to be the standard values derived by recasting the problem as a fuel-optimal trajectory generation problem with constrained boundary position and velocity and assuming a uniform and well-known gravity field (recall the discussion on Chapter 4). This was an invalid approximation for the Phobos D\&L case, but it is valid for an RLV on Earth.

The estimation of $\mathbf{Z E M}(t)$ and $\mathbf{Z E V}(t)$ involves propagating the equations of motion from $t$ to $t_{f}$, which can become computationally challenging without some approximations. The ability of the guidance law to accurately enforce the boundary conditions (i.e. to minimise touchdown errors) naturally depends on the conservativeness of these approximations. This section seeks the simplest possible law and hence, neglecting: [i] gravity variations during descent, [ii] mass variations due to propellant consumption, [iii] aerodynamic forces, and [iv] non-inertial effects of the RP frame, then the zero-effort errors can be computed using Eq. (2.11).

This calculation requires the specification of terminal conditions $\mathbf{r}_{f}=\mathbf{v}_{f}=[0 ; 0 ; 0]$ for a soft landing in the RP frame, as well as position, velocity and gravity acceleration estimates, $\hat{\mathbf{r}}(t), \hat{\mathbf{v}}(t)$ and $\hat{\mathbf{g}}(t)$. These estimates are computed applying similar rotations to Eq. (8.2) and Eq. (8.5). Although the vehicle's landing gear has to be able to withstand a (small) non-zero touchdown velocity, $\mathbf{v}_{f}$ is set to $0 \mathrm{~m} / \mathrm{s}$ in order to be as conservative as possible.

Due to its simplicity, the computational time required by the CTVG algorithm is extremely low, so guidance commands can be updated at the same frequency as the simulation, $f_{\text {gui }}=f_{\text {sim }}$, starting at a pre-specified altitude $h_{\mathrm{s}}$. Initial values adopted for the verification of the algorithm are summarised in Table 8.3. The simulation frequency was set to 10 Hz since it was verified to provide enough accuracy for trajectory assessment while minimising simulation runtime.

Because of the simplifications and algorithmic framework, the inherent capabilities of CTVG are rather limited. The most relevant limitations lie in its inability to explicitly enforce path constraints (i.e. only boundary states can be constrained) and account for mass-depletion dynamics, i.e. Eq. (8.28). Path constraints may be applied to the states (e.g. subsurface flight avoidance) as well as to the control inputs (e.g. lower and upper throttling magnitude and rate limits) and both of them are critical for RLV recovery.

It is also noted that, although it is possible to use two burns for recovery (defined by $t_{1}$ and $t_{2}$ ), these burns are not explicitly accounted for with the CTVG approach. These limitations motivate the need for more sophisticated algorithms, such as the one proposed in Chapter 9.

Table 8.3: Initial CTVG algorithm parameters

| Parameter | Value |
| :--- | :---: |
| $h_{s}(\mathrm{~km})$ | 25 |
| $t_{f}(\mathrm{~s})$ | 380 |
| $t_{1}(\%)$ | 0 |
| $t_{2}(\%)$ | 0 |
| $f_{\text {sim }}(\mathrm{Hz})$ | 10 |
| $f_{\text {gui }}(\mathrm{Hz})$ | 10 |

### 8.3 RLV control approach

This section exemplifies the manner attitude control algorithms can be designed and integrated in the RLV benchmark. In an axisymmetric launcher with quasi-zero roll rate, the pitch and yaw motions are often assumed uncoupled and the task of attitude control design and analysis can be performed in a single plane. A model often employed for preliminary launcher calculations (e.g. [Orr10, KSB12, NMBR16]) is used for this demonstration.

The model representing the rotational dynamics under the presence of aerodynamic effects [Gre70] is given in the frequency-domain for a certain instant of time $t$ as:

$$
\begin{equation*}
\frac{\theta(\mathrm{s})}{m_{\mathrm{ctr}}(\mathrm{~s})}=\frac{1}{\mathrm{~s}^{2}-\mu_{\alpha^{\prime}}(t)} \tag{8.48}
\end{equation*}
$$

In the equation, $\theta(\mathrm{s})$ represents pitch deviations relative to a certain trim state, $m_{\mathrm{ctr}}(\mathrm{s})$ represents specific pitch control moment divided by the lateral moment of inertia and $\mu_{\alpha^{\prime}}(t)$ is the loadproneness parameter introduced in Eq. (8.35).

The high-level objective of any launcher's attitude control system is to stabilise its dynamics and track commands $\theta_{\text {ref }}(\mathrm{s})$ provided by the guidance module, which can be achieved classically using a Proportional-Derivative (PD) controller. The architecture of the adopted PD controller in closed-loop with the linear system of Eq. (8.48) is depicted in Fig. 8.8, with the PD gains $k_{\mathrm{p}}(t)$ and $k_{\mathrm{d}}(t)$ shown as time-varying due to $\mu_{\alpha^{\prime}}(t)$. Note that a first-order derivative filter with time constant $\sigma_{\mathrm{d}}=0.2$ seconds is used to calculate the reference pitch rate command. In the nonlinear simulator, angular errors are measured based on the orientation of the body-fixed frame with respect to the LP frame (recall Sec. 8.1.1), which requires the knowledge of $C_{\mathbf{q}_{\mathrm{E}}^{\mathrm{L}}}$, $C_{\mathbf{q}_{\mathrm{I}}^{\mathrm{E}}}(t)$ and $C_{\mathbf{q}_{\mathrm{B}}^{\mathrm{I}}}(t)$.

Figure 8.8 also shows a "control allocation" block that relates the specific control moment $m_{\mathrm{ctr}}(\mathrm{s})$ to the actual inputs: $\beta_{\mathrm{TVC}, \mathrm{y}}(\mathrm{s}), \beta_{\text {fin, }}(\mathrm{s})$ and $\beta_{\mathrm{thr}, \mathrm{y}}(\mathrm{s})$, with $\beta_{\text {fin }, \mathrm{y}}(\mathrm{s})=\beta_{\text {fin }, 1}(\mathrm{~s})=\beta_{\text {fin }, 2}(\mathrm{~s})$ for the pitch channel. Using the control effectiveness coefficients defined in Eq. (8.27), (8.34) and (8.38), this relation is conveniently expressed as:

$$
m_{\mathrm{ctr}}(\mathrm{~s})=\Gamma_{\mathrm{ctr}}(t)\left[\begin{array}{c}
\beta_{\mathrm{TVC}, \mathrm{y}}(\mathrm{~s})  \tag{8.49}\\
\beta_{\mathrm{fin}, \mathrm{y}}(\mathrm{~s}) \\
\beta_{\mathrm{thr}, \mathrm{y}}(\mathrm{~s})
\end{array}\right]
$$

where:

$$
\Gamma_{\mathrm{ctr}}(t)=-\left[\begin{array}{lll}
\mu_{\mathrm{c}}(t) \cos \beta_{\mathrm{TVC}, \mathrm{y} 0}(t) & \mu_{\mathrm{f}}(t) \cos \beta_{\mathrm{fin}, \mathrm{y} 0}(t) & \mu_{\mathrm{t}}(t) \tag{8.50}
\end{array}\right]
$$

and $\beta_{\mathrm{TVC}, \mathrm{y} 0}(t)$ and $\beta_{\mathrm{fin}, \mathrm{y} 0}(t)$ represent TVC and fin trim deflections. The role of control allocation is to compute the inverse of this (overdetermined) equation in a way that is suitable for the different phases of L\&R flight.


Figure 8.8: Closed-loop attitude control model (pitch channel)

### 8.3.1 Preliminary control design

Equation (8.48) is time (and trajectory) dependent through $\mu_{\alpha^{\prime}}(t)$, which determines its poles: they are placed at $\pm \sqrt{\mu_{\alpha^{\prime}}(t)}$ when $\mu_{\alpha^{\prime}}(t)>0$ and at $\pm j \sqrt{\mu_{\alpha^{\prime}}(t)}$ otherwise. Since $\mu_{\alpha^{\prime}}(t)$ is a combination of the aerodynamic contributions of $\mu_{\alpha}(t)$ and $\mu_{\mathrm{f}}(t)$, Eq. (8.20) and (8.34), it is also interesting to look at their individual effects, as well as at the contribution of $\mu_{\mathrm{c}}(t)$ and $\mu_{\mathrm{t}}(t)$, Eq. (8.27) and (8.38).

Figure 8.9a shows the estimates of $\sqrt{\mu_{*}(t)} \operatorname{sgn} \mu_{*}(t)$, with $*=\left\{\alpha, \mathrm{f}, \alpha^{\prime}\right.$, c $\}$, throughout the ascent of Fig. 8.7 followed by a closed-loop DRL recovery. In the case of $\mu_{\alpha^{\prime}}(t)$ and $\mu_{\alpha}(t)$, this corresponds to the natural frequency of the poles with and without fins; positive values indicate open-loop instability and non-positive values indicate marginal stability. The figure was obtained in an iterative manner since changes on control parameters affect the trajectory, while changes on the trajectory affect the effectiveness levels and consequently the control parameters.

The evolution of the $\mu_{\alpha}(t)$ indicator shows that an RLV with no fins would be open-loop unstable for the first half of the trajectory (while the launcher is flying with the nose pointing forward, until around 200 seconds) and for most of the second half. Indeed, only a few narrow areas of marginal stability between the time regions of [200, 270] and [320, 350] seconds are seen in Fig. 8.9a. They can be related to a combination of $C_{N_{\alpha}}(t)<0$ and $x_{\mathrm{CP}}(t)>x_{\mathrm{CG}}(t)$. The magnitude of the $\mu_{\mathrm{f}}(t)$ indicator follows that of $\mu_{\alpha}(t)$ due to their common dependence on $Q(t) / J_{N}(t)$, but its sign has a more monotonic variation due to $C_{\mathrm{f}_{\alpha}}(t)$ being strictly positive for the first half and negative for the second.


Figure 8.9: Stability and control indicators of DRL mission

From $\mu_{\alpha^{\prime}}(t)$ in Fig. 8.9a, it is seen that the chosen fins have very little impact during launch, but they do provide a significant stability improvement for recovery and landing with $\mu_{\alpha^{\prime}}(t) \ll \mu_{\alpha}(t)$ - the most challenging area is around 300 seconds and coincides with re-entry and first stage re-ignition as will be seen later. In addition, $\mu_{\alpha^{\prime}}(t)$ is considerably inferior to $\mu_{\mathrm{c}}(t)$ or $\mu_{\mathrm{f}}(t)$ (in absolute value) during the atmospheric flight, so that a margin for stability and control is ensured. This is different during the exo-atmospheric flight phase, for which cold gas thrusters with effectiveness $\mu_{\mathrm{t}}(t)$ have to be employed.

The closed-loop transfer function of Fig. 8.8 is then given by:

$$
\begin{equation*}
\frac{\theta(\mathrm{s})}{\theta_{\mathrm{ref}}(\mathrm{~s})}=\frac{\left(k_{\mathrm{d}}(t)+\sigma_{\mathrm{d}} k_{\mathrm{p}}(t)\right) \mathrm{s}+k_{\mathrm{p}}(t)}{\left(\sigma_{\mathrm{d}} \mathrm{~s}+1\right)\left(\mathrm{s}^{2}+k_{\mathrm{d}}(t) \mathrm{s}+k_{\mathrm{p}}(t)-\mu_{\alpha^{\prime}}(t)\right)} \tag{8.51}
\end{equation*}
$$

and can be easily employed to choose the gains $k_{\mathrm{p}}(t)$ and $k_{\mathrm{d}}(t)$ via pole-placement. Here, this is carried out so as to have a constant natural frequency and damping ratio of the 2nd order term in the denominator throughout the trajectory (with the other term solely defined by $\sigma_{\mathrm{d}}$ ). Once again, this is a simplification for illustration purposes and does not represent the industrial state-of-practice, where the required attitude tracking performance changes with the phase of flight. For example, tracking performance should be less demanding at high dynamic pressure $\left[\mathrm{NMS}^{+} 19\right]$ and more demanding for landing. The control gains required for a constant natural frequency of $4 \mathrm{rad} / \mathrm{s}$ and damping ratio of 0.8 are depicted in Fig. 8.9b.

As mentioned before, the same process is followed for yaw attitude control design. There are however two main differences that must be pointed out. The first one is related to the fact that, as there is no flip-over manoeuvre in the yaw channel, there is no sign reversal of the fin effectiveness $\mu_{\mathrm{f}}(t)$ (recall Fig. 8.9a) to be taken into account for the computation of $\beta_{\mathrm{fin}, \mathrm{Z}}$ from

Eq. (8.49). The second remark is that fins in the yaw plane will also be employed to compensate for any roll rate perturbation $\dot{\phi}$. This is achieved by adding extra differential deflections as follows:

$$
\left\{\begin{array}{l}
\beta_{\mathrm{fin}, 3}=\beta_{\mathrm{fin}, \mathrm{z}}+k_{\phi} \dot{\phi}  \tag{8.52}\\
\beta_{\mathrm{fin}, 4}=\beta_{\mathrm{fin}, \mathrm{z}}-k_{\phi} \dot{\phi}
\end{array}\right.
$$

Here, $k_{\phi}$ is assumed constant and equal to 0.1 seconds for the sake of simplicity. Although fin effectiveness is not uniform throughout the trajectory, this roll-rate stabilisation strategy is proven very successful for the present case study.

### 8.3.2 Control allocation

The control moment $m_{\mathrm{ctr}}(t)$ has now to be allocated between the three RLV effectors: TVC, planar fins and cold gas thrusters. The most general control allocation method uses the weighted pseudo-inverse of $\Gamma_{\text {ctr }}$ [BB02], given by:

$$
\begin{equation*}
\Gamma_{\mathrm{ctr}}^{+}=W_{\Gamma}^{-1} \Gamma_{\mathrm{ctr}}^{\mathrm{T}}\left(\Gamma_{\mathrm{ctr}} W_{\Gamma}^{-1} \Gamma_{\mathrm{ctr}}^{\mathrm{T}}\right)^{-1} \tag{8.53}
\end{equation*}
$$

where matrix $W_{\Gamma}$ is applied for effector regularisation, to invert Eq. (8.49). This method, however, is very sensitive to numerical issues related to the conditioning of matrix $\Gamma_{\text {ctr }}$ and the choice of $W_{\Gamma}$ renders the design and validation process more complex. Hence, in this thesis, a simpler algorithm based on [MMP09] is proposed.

The main idea of this algorithm is to try allocating $m_{\text {ctr }}(t)$ entirely to a primary effector $\epsilon_{1}$ and only employ a secondary effector $\epsilon_{2}$ if the maximum authority of the primary effector $\bar{\beta}_{\epsilon_{1}}$ were to be exceeded. Following the notation above and the definition of Eq. (8.49), control inputs are first computed as:

$$
\begin{equation*}
\beta_{\epsilon_{1}}(t)=-\frac{m_{\mathrm{ctr}}(t)}{\mu_{\epsilon_{1}}(t)}, \quad \beta_{\epsilon_{2}}(t)=0 \tag{8.54}
\end{equation*}
$$

Then, if the maximum authority of $\epsilon_{1}$ is exceeded, i.e. if $\left|\beta_{\epsilon_{1}}(t)\right|>\bar{\beta}_{\epsilon_{1}}, \beta_{\epsilon_{1}}(t)$ is constrained and $\beta_{\epsilon_{2}}(t)$ is updated as follows:

$$
\begin{equation*}
\beta_{\epsilon_{1}}(t)=\bar{\beta}_{\epsilon_{1}} \operatorname{sgn} \beta_{\epsilon_{1}}(t), \quad \beta_{\epsilon_{2}}(t)=-\frac{m_{\operatorname{ctr}}(t)+\mu_{\epsilon_{1}}(t) \beta_{\epsilon_{1}}(t)}{\mu_{\epsilon_{2}}(t)} \tag{8.55}
\end{equation*}
$$

where the numerator of $\beta_{\epsilon_{2}}(t)$ corresponds to the difference between the commanded moment and the one achievable with $\epsilon_{1}$.

A suitable definition of primary and secondary effectors naturally changes throughout the trajectory according to their effectiveness coefficients. The choice implemented in this study is summarised in Table 8.4.

As the table shows, 6 modes have been defined, and for each a maximum authority value of the primary effector can be assigned. Note that there are a couple of modes where no secondary effector is necessary. In addition, the few seconds prior to PL separation are performed in open-loop so as to avoid instability caused by the rapid change of inertia.

Table 8.4: Control allocation modes

|  | Primary <br> effector | Secondary <br> effector |
| :--- | :---: | :---: |
| From lift-off to engine cut-off $(t=110 \mathrm{~s})$ | TVC | - |
| From previous to PL separation $(t=112 \mathrm{~s})$ | Open-loop | - |
| From previous to drop of $\left\|\mu_{\mathrm{f}}(t)\right\|$ | Fins (35 deg max) | Thrusters |
| From previous to recovery of $\left\|\mu_{\mathrm{f}}(t)\right\|$ | Thrusters | - |
| From previous to $t_{f}$ with: low thrust | Fins (35 deg max $)$ | TVC |
|  | high thrust | TVC $(3$ deg max $)$ | Fins | TV |
| :--- |

### 8.4 Coupled assessment

The assessment of coupled flight mechanics and G\&C laws is demonstrated in this section in two ways. First, a detailed comparison of DRL and RTLS missions using CTVG recovery and a perfect attitude control is given. Then, the impact of attitude control on stability and performance of DRL missions is assessed.

### 8.4.1 Recovery analysis

The aim of this subsection is to provide a quantitative comparison between DRL and RTLS recovery strategies. Since the main differences between these missions are mostly related to their trajectory, perfect attitude control dynamics will be assumed for simplicity. This means that attitude angles are exactly what they are commanded to be, $\{\theta(t), \psi(t)\}=\left\{\theta_{\text {ref }}(t), \psi_{\text {ref }}(t)\right\}$, and that all the aerodynamic moments generated by the vehicle are compensated for, i.e. that $\mathbf{M}_{\text {aero }}(t)+\mathbf{M}_{\mathrm{TVC}}(t)+\mathbf{M}_{\mathrm{fin}}(t)+\mathbf{M}_{\mathrm{thr}}(t)=0$. This is the standard approach used when developing and assessing guidance schemes.

The two L\&R trajectories obtained with CTVG triggered at $h_{\mathrm{s}}=25 \mathrm{~km}$ are depicted in Fig. 8.10. Although sharing the same launch profile, the distinction between DRL using the sea-going recovery platform with final time $t_{f}=390$ seconds and RTLS with $t_{f}=500$ seconds is clear in the figure.

Detailed results of the simulations are plotted in Fig. 8.11 for DRL and Fig. 8.12 for RTLS. The phases of launch (from lift-off to separation), exo-atmospheric flight (from separation to recovery burn) and recovery (from recovery burn to touchdown) are distinguished in every plot using dash-dotted, dashed and continuous lines respectively, and a thorough analysis of the plots is provided subsequently. The figures show the most relevant flight performance indicators, also gathered in Table 8.5. These indicators include total propellant consumption $m_{\text {burnt }}$ given by Eq. (8.42), maximum aerothermal loads, $Q$ and $Q_{H}$, vertical and horizontal touchdown velocity, $\left\|\mathbf{v}_{\mathbf{R}_{z}}\left(t_{f}\right)\right\|$ and $\left\|\mathbf{v}_{\mathbf{R}_{x, y}}\left(t_{f}\right)\right\|$, as well as position error $\left\|\mathbf{r}_{\mathbf{R}_{x, y}}\left(t_{f}\right)\right\|$.


Figure 8.10: DRL and RTLS trajectories (image generated with Google Earth)

The top-left plot of Fig. 8.11 shows the evolution of vertical and horizontal velocity as a function of altitude (in the vertical axis) for the DRL scenario. During launch, velocity increases in both vertical and horizontal directions and separation occurs when the latter reaches its maximum value at an approximate altitude of 51 km . From that point, the vehicle continues to ascend until its vertical velocity becomes zero at an approximate altitude of 74 km and then plunges downwards due to the action of gravity. In the meantime, horizontal velocity is dissipated as a consequence of aerodynamic forces, which become more intense as altitude decreases (and air density increases). Then, at an altitude of 25 km , recovery guidance is activated and the commanded burn brings both components to zero at the landing point. It is important to notice that horizontal velocity converges to this value significantly before their counterpart, which is critical to ensure a vertical landing.

The bottom-left plot illustrates the evolution of dynamic pressure (Eq. (8.11)) and heat flux (Eq. (8.23)) as a function of the vehicle's Mach number. During launch, velocity increases and air density decreases, which causes $Q$ and $Q_{H}$ to tend to zero at lift-off and at maximum altitude ( 74 km , where $M \approx 5.3$ ) and to have a peak value in-between. These indicators then increase abruptly once the RLV starts to descend (at $M \approx 4.9$ ) and re-enters the atmosphere. At this point, recovery guidance is activated in order to manage the second peak values of these indicators and bring them to zero at the landing point. Maximum values of $Q$ and $Q_{H}$ during L\&R are registered in Table 8.5 (it is important to note that the ascent peak of $Q$ is higher than the descent either with DRL or RTLS, but the descent peak of $Q_{H}$ with DRL is the highest).

The uppermost plot on the right-hand side of Fig. 8.11 shows the reference pitch and yaw angles $\left\{\theta_{\text {ref }}, \psi_{\text {ref }}\right\}$ as well as the total angle of attack $\alpha_{\text {eff }}$ over mission time. The reference attitude angles were shown in Fig. 8.7 for the launch phase (i.e. up to approximately 110 seconds).


Figure 8.11: Flight mechanics results of DRL mission

In terms of the angle of attack, it remains close to zero during this phase, with a maximum value under 5 degrees around pitch over (at 10 seconds).

Subsequently (dashed phase), the reference pitch angle follows a constant-rate manoeuvre to flip over the booster during the exo-atmospheric flight while demanding a constant yaw angle. This constant-rate flip over manoeuvre causes the angle of attack to go from zero to 180 degrees, which implies that the velocity vector becomes aligned with the booster's base. And finally during the recovery, the reference attitude angles are computed by the guidance algorithm, which results in $\theta_{\text {ref }}$ converging to 90 degrees at touchdown (ensuring a vertical landing), $\psi_{\text {ref }}$ remaining close to zero due to little aerodynamic couplings with the pitch motion and $\alpha_{\text {eff }}$ close to 180 degrees.

The second right-hand plot illustrates the evolution of the thrust vector magnitude $T_{\text {ref }}$ and aerodynamic moment $\left\|\mathbf{M}_{\text {aero }}\right\|$. Similar to the previous plot, $T_{\text {ref }}$ is given by Fig. 8.7 during
the launch phase and computed by the guidance algorithm during recovery. The recovery burn is more intense at the beginning (where zero-effort errors are larger) and close to touchdown (where $t \rightarrow t_{f}$ ). The aerodynamic moment to be compensated, as expected, is more demanding in zones of high dynamic pressure and angle of attack.

The third right-hand plot of Fig. 8.11 shows the evolution of the vehicle's total mass using logarithmic scale for clarity. Here, three zones of mass variation can be identified: [i] launch burn, where most of the propellant is depleted, [ii] separation (sudden drop of $m_{\text {PL }}$ at 112 seconds of flight), and [iii] recovery burn for re-entry and landing. Propellant consumption during L\&R is also registered in Table 8.5 with respect to its initial mass. In this scenario, while $90.30 \%$ of propellant is depleted during launch, only $4.98 \%$ is required for the recovery, which leaves a margin of about $4.7 \%$.

Finally, the bottom-right plot illustrates the longitudinal travel of CP and CG relative to the booster's base throughout the flight. While the former is governed by the aerodynamic environment encountered, the latter follows the depletion of mass as per Eq. (8.43). Hence, the same three zones of launch (with CG moving forward), separation (sudden drop of $m_{\mathrm{PL}} h_{\mathrm{PL}}$ ) and recovery (CG moves backward, although not visible in the plot as mass variation is quite small) can be distinguished. In any case, the RLV is inherently unstable during powered fight since the CP is located in front of the CG during ascent and behind during descent.

Regarding the RTLS results provided in Fig. 8.12, the main difference with respect to the previous scenario lies on the boostback burn required to bring the stage back to the launch site. In terms of velocity (top-left plot), this burn results in the inversion of the horizontal component and the reduction of its magnitude to approximately half of its value at separation. In addition, there is a stronger interplay between vertical and horizontal components, also visible in Fig. 8.10, caused by the close interactions between trajectory and aerodynamics that are not taken into account by the CTVG algorithm. This was one of the drawbacks identified before for this type of guidance. The velocity reduction is then translated into less demanding aerothermal recovery indicators than for the DRL trajectory, as evidenced by the bottom-left plot and in Table 8.5.

In terms of attitude (top-right plot of Fig. 8.12), the main difference is related to the rapid constant-rate flip over manoeuvre, followed by a 30 seconds period with constant pitch $\theta_{\text {ref }}=180$ degrees during which the boostback burn takes place (recall Fig. 8.6b) and by a second constant-rate manoeuvre prior to the recovery burn. The flip over manoeuvre also causes the total angle of attack to follow the pitch variation, but the former angle returns to zero as soon as the horizontal velocity is inverted by the boostback burn. Once the recovery burn is activated, the observations are similar to the DRL approach, although the coupling between pitch and yaw motion is slightly more intense.

The thrust reference associated with the boostback burn is clearly visible in the second right-hand plot. The velocity reduction this burn induces is reflected into less intense thrust commands during recovery compared to the DRL scenario, but larger aerodynamic moments

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Figure 8.12: Flight mechanics results of RTLS mission

Table 8.5: Nominal L\&R performance indicators

| Indicator | Launch | DRL | RTLS |
| :--- | :---: | :---: | :---: |
| $m_{\text {burnt }}\left(t_{f}\right) / m_{\text {prop }}(0)(\%)$ | 90.3 | 4.98 | 9.60 |
| $\max Q(\mathrm{kPa})$ | 53.0 | 44.8 | 11.2 |
| $\max Q_{H}\left(\mathrm{~kW} / \mathrm{cm}^{2}\right)$ | 51.7 | 89.1 | 15.3 |
| $\left\\|\mathbf{v}_{\mathrm{R}_{z}}\left(t_{f}\right)\right\\|(\mathrm{m} / \mathrm{s})$ | - | 0.39 | 1.34 |
| $\left\\|\mathbf{v}_{\mathrm{R}_{x, y}}\left(t_{f}\right)\right\\|(\mathrm{m} / \mathrm{s})$ | - | 1.24 | 1.19 |
| $\left\\|\mathbf{r}_{\mathrm{R}_{x, y}}\left(t_{f}\right)\right\\|(\mathrm{m})$ | - | 0.01 | 0.08 |

due to higher angles of attack. Also as a result of this burn, an additional mass-depletion zone is visible in the third right-hand plot. The evolution of CP and CG in the bottom-right plot is also slightly different in comparison with the DRL case, but the overall observations are the same.

Despite the less intense effort during recovery, the boostback burn makes the RTLS approach a significantly more demanding trajectory in terms of propellant consumption (see Table 8.5). In fact, $9.6 \%$ of propellant is now required for the recovery, leaving a margin of only $0.1 \%$. This very tight margin also motivates the need for more performing recovery guidance algorithms, such as the one presented in Chapter 9.

In addition to the indicators already covered, Table 8.5 includes vertical and horizontal touchdown velocity, as well as position error, which are naturally of critical relevance for recovery assessment. For the two cases analysed, all these values are well within an adequate range. It is highlighted that these indicators should be ideally as small as possible but, in practice, their combined optimisation is an extremely challenging activity. The reason behind this is that a choice of recovery guidance tuning parameters that minimises propellant consumption is likely to subject the vehicle to higher aerothermal loads [LSL16, WG17], and vice-versa.

Finally, to analyse the sensitivity of the proposed guidance approach to wind, 1000 runs of the DRL recovery were performed with gusts perturbing both ascent/descent phases (and with no other perturbations). These gusts are introduced using two orthogonal components $w_{\text {NRN }}$ and $w_{\text {ERN }}$ (recall Eq. (8.6)) and modelled by uncorrelated Dryden filters. The final landing trajectories and touchdown position/velocity dispersions obtained with severe wind are provided in Fig. 8.13a and 8.13b, respectively.


Figure 8.13: Impact of wind on DRL trajectory (1000 runs)
These figures show that all the wind-induced dispersions could be shrunk towards successful pinpoint soft landings. As highlighted in Fig. 8.13b, the touchdown position has a $3 \sigma$ dispersion of 0.1 m around the recovery platform's centre and less than $1 \%$ of the cases landed with velocity higher than $3.5 \mathrm{~m} / \mathrm{s}$.

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### 8.4.2 Controllability analysis

The impact of attitude control on L\&R trajectories is now addressed by closing the loop with the controller introduced in Sec. 8.3 and using the time-scheduled control gains of Fig. 8.9b. This section focuses on DRL missions since they were shown to be the most demanding ones in terms of aerothermal loads (recall Table 8.5). Simulation results are depicted in Fig. 8.14. An animation of this simulation can also be accessed in http://doi.org/10.13140/RG.2.2.35780.37760.


Figure 8.14: Flight mechanics results of DRL mission with closed-loop control
The two left-hand side plots in Fig. 8.14 show the same information of Fig. 8.11 (i.e. velocity components vs. altitude and aerothermal indicators vs. Mach), but without distinguishing now the phases of flight. It can be seen that general trends and peak values with respect to the previous perfect control trajectory are extremely similar. The main differences take place close to touchdown, where the evolution of vertical and horizontal velocity becomes slightly more elaborate. This happens because descent guidance and attitude control are tightly coupled: actuator deflections required for attitude control generate lateral forces as a side effect, which then cause landing trajectory and guidance commands to change.

The uppermost plot on the right-hand side of Fig. 8.14 shows now the evolution of commanded and actual attitude angles. It is clear that the pitch and yaw commands are successfully tracked (with larger errors during re-entry and first stage re-ignition) while roll is kept approximately constant throughout the mission. It is also interesting to verify that the commands are quite different from those in Fig. 8.11 due to the aforementioned guidance-control coupling.

The required actuator deflections are provided in the second right-hand plot for TVC and in the last plot for the fins. The main critical events are also identified in both plots. Cold gas thrusters are only active during the designated area and not shown for the sake of conciseness. Evolution of the main engine thrust magnitude and vehicle MCI properties are similar to those of Fig. 8.11 and therefore also omitted.

As these two last right-hand plots show, attitude is controlled using TVC only (with maximum deflection of 2 degrees) until engine cut-off. This is followed by a few seconds of open-loop control (recall Table 8.4) and by fin activation to compensate for perturbations related to PL separation and high-speed aerodynamics. There is then a period of thruster control only where fin effectiveness is reduced due to the high altitude. Finally, fins are reactivated for precise control during D\&L, and are also aided by the TVC when its effectiveness is higher, at the start and end of recovery burn.

Naturally, the majority of control activity takes place in the trajectory pitch plane (through $\beta_{\mathrm{TVC}, \mathrm{y}}, \beta_{\mathrm{fin}, 1}$ and $\beta_{\mathrm{fin}, 2}$ ) and the remaining deflections are mostly employed for small corrections. It is important to highlight that, while $\beta_{\mathrm{fin}, 1}=\beta_{\mathrm{fin}, 2}$, the same does not hold for $\beta_{\mathrm{fin}, 3}$ and $\beta_{\mathrm{fin}, 4}$ due to their differential deflection for roll stabilisation given by Eq. (8.52). This deflection is highlighted in Fig. 8.15 and is in the order of $10^{-3}$ degrees, even at high altitudes (between 200 and 250 seconds).


Figure 8.15: Differential fin deflection for roll stabilisation. Yaw control offset has been deducted.

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The information provided by these simulations is also crucial at the vehicle's sizing stage as it allows to evaluate the control authority that is required to ensure its controllability over the flight and under different levels of wind perturbations. For instance, one can compute trim conditions of the lateral motion throughout the trajectory based on the equilibrium of moments with respect to the body $\mathbf{j}_{\mathrm{B}}$ or $\mathbf{k}_{\mathrm{B}}$ axes.

The aforementioned conditions correspond to the solution of Eq. (C.5) from Appendix C. Wind perturbations of velocity $v_{\mathrm{w}}$ in the vehicle's normal axis are introduced via Angle of Attack (AoA) perturbations as follows:

$$
\begin{equation*}
\tan \alpha=\frac{v_{\mathrm{air}, \mathrm{Z}}^{\mathrm{CP}}-v_{\mathrm{w}}}{v_{\mathrm{air}, \mathrm{x}}^{\mathrm{CP}}} \approx \tan \alpha_{0}-\frac{v_{\mathrm{w}}}{V \cos \alpha_{0}} \tag{8.56}
\end{equation*}
$$

in which $\alpha_{0}$ is the value taken from the simulation with no wind. The impact of such perturbations changes greatly throughout the trajectory and this can be quantified using the $Q_{\alpha}$ indicator introduced in Sec. 8.1.3. The time responses of this indicator during the simulation of Fig. 8.14 and for different values of $v_{\mathrm{w}} \in[-15,45] \mathrm{m} / \mathrm{s}$ around that trajectory are provided (using different colours) in Fig. 8.16.


Figure 8.16: Impact of wind on DRL $Q_{\alpha}$ with closed-loop control

As this figure shows, there is one main $Q_{\alpha}$ peak during launch and another one during recovery. These peaks are worsened when $v_{\mathrm{w}} \neq 0$ in accordance with Eq. (8.56). The impact of wind is higher for launch, but the nominal $Q_{\alpha}$ peak is more demanding for recovery, both of them caused by the fact that $\alpha_{0}$ is also higher at recovery than at launch.

Trim curves are then plotted in Fig. 8.17a and 8.17b for two extreme conditions during recovery: booster re-ignition (at 288 seconds) and maximum $Q_{\alpha}$ (at 335 seconds), respectively. Each plot shows solutions of Eq. (C.5) for fin deflections $\left|\beta_{\text {fin }, \mathrm{y}}\right| \leq 40$ degrees ( $x$ axis), TVC deflections $\left|\beta_{\mathrm{TVC}, \mathrm{y}}\right| \leq 15$ degrees ( $y$ axis) and wind perturbations $v_{\mathrm{w}}$.


Figure 8.17: Attitude (TVC vs. fin deflection) trim curves at extreme flight conditions
From Fig. 8.17a, it is possible to observe that, in spite of the dynamic disturbance induced by the booster re-ignition at 288 seconds, controlling the vehicle is not particularly demanding nor affected by wind. This happens because the aerodynamic stress encountered at this point is fairly small due to low values of both $Q$ and $\alpha$. In fact, for the configurations under analysis, the vehicle can always be trimmed using individual fin or TVC deflections under 1 degree.

This situation is naturally opposed to the point of maximum $Q_{\alpha}$, depicted in Fig. 8.17b. The first conclusion to be drawn from here is that the vehicle could not be controlled without using fins, as the trim curves do not intersect the $\beta_{\text {fin, }}=0$ coordinate. In contrast, without using TVC, trim with no wind requires a fin deflection of about 22 deg (point ${ }^{(1)}$, the lower-left point in the inner rectangle of the figure). This equilibrium value is very close to the one observed in Fig. 8.14 at 335 seconds, where the vehicle is slowly pitching up in preparation for landing. The required fin deflection could however be alleviated for example to 16.4 deg, allowing an additional TVC deflection of 10 deg (point (2), the upper-left point going along the line from (1)).

Introducing wind then makes the vehicle's controllability more challenging. Suppose a maximum authority of 30 deg is allocated to the fins. In case there is wind, fins alone allow to compensate for speeds up to $16 \mathrm{~m} / \mathrm{s}$ (point (3), the lower-right point) or, if they are fixed to their previous value and TVC is used, up to $12 \mathrm{~m} / \mathrm{s}$ (point (4), above along the line from (3). However, if an efficient allocation of both fins and TVC is employed, the present configuration allows to accommodate wind speeds of $29 \mathrm{~m} / \mathrm{s}$ (point (5), upper-right point in the rectangle), which represent rather severe gusts.

In addition to wind, controllability is also affected when the vehicle's properties do not match those assumed at design stage. To demonstrate this effect, the previous analysis is repeated considering multiplicative perturbations in the aerodynamic effectiveness of the vehicle and its fins, $\mu_{\alpha}$ and $\mu_{\mathrm{f}}$. These perturbations cover potential mismatches in terms of CP and CG travel, as well as of reference area and load coefficient of the vehicle and fins.

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Trim curves at maximum recovery $Q_{\alpha}$ are now computed for two corner-cases and illustrated in Fig. 8.18. Figure 8.18a shows a case where $\mu_{\mathrm{f}}$ is $30 \%$ lower and $\mu_{\alpha}$ is the same as before, which accounts for a reduced ability of the fins to counteract loads. Figure 8.18b depicts a configuration with $\mu_{\alpha} 30 \%$ higher and same $\mu_{\mathrm{f}}$, simulating an increased proneness of the vehicle to generate aerodynamic loads. The numbers of the marked points remain the same as before for ease of explanation.


Figure 8.18: Attitude trim curves at maximum recovery $Q_{\alpha}$ with corner-case RLV properties

As expected, both cases lead to a more challenging controllability of the vehicle, as indicated by the shift of the curves to the right (compare e.g. the values of the black line at the $x$ axis), but its attitude trim capability remains ensured over a very wide range of wind speeds. For instance, with full fin and TVC deflections (point (5), the maximum velocity that can be accommodated is reduced from 29 to $25 \mathrm{~m} / \mathrm{s}$ in the first case and to $23 \mathrm{~m} / \mathrm{s}$ in the second. Despite being generally verified, this conclusion does not hold for every control combination because, in addition to control effectiveness, fin effectiveness contributes to the vehicle aerodynamics as well (recall the generalisation of $\mu_{\alpha^{\prime}}$ in Eq. (8.35)). As an example, the wind velocity tolerated in point (4) increases from $12 \mathrm{~m} / \mathrm{s}$ in Fig. 8.17b to $15 \mathrm{~m} / \mathrm{s}$ in Fig. 8.18a.

The control and allocation laws presented in this chapter represent a baseline against which more sophisticated techniques will be compared in Chapter 10. As anticipated by the present analysis, the ability to account for the impact of wind and AoA will be critical for the development of high-performance attitude controllers.


## Improving Guidance via Onboard Convex Optimisation

The objective of this chapter is to improve the recovery guidance algorithm proposed with the Reusable Launch Vehicle (RLV) benchmark in Sec. 8.2.2, which will be achieved through the Computational Guidance and Control paradigm introduced in Sec. 2.1.3.

In particular, special emphasis is placed on the build-up of knowledge from simpler Descent \& Landing (D\&L) techniques such as Constrained Terminal Velocity Guidance (CTVG) towards the development of a novel onboard algorithm coined Descending over Extended envelopes using Successive ConvExificatioN-baseD Optimisation (DESCENDO). The main strength of this approach is that, as the main parameters on which CTVG depends are intrinsically representative of the physics of the recovery problem, it allows to have a quick understanding of the acceptable flight performance, which can then be leveraged for improvements.

The DESCENDO algorithm follows similar lines to [SABJH16, JMBS17], but it is specifically tailored to the extended flight envelope encountered by reusable launchers. Furthermore, the proposed algorithm is implemented in a closed-loop fashion (in opposition to an offline setting, recall Sec. 2.1.2) and verified using the complete reusable launcher recovery scenarios of Sec. 8.2.1, which is something that the launcher control community has not yet formally investigated.

With the aforementioned approach in mind, this chapter begins with the baseline performance assessment driving the first improvements in Sec. 9.1. Subsequently, the DESCENDO algorithm is developed in Sec. 9.2 and the results of its application are provided in Sec. 9.3. The proposed approach has been presented together with the RLV benchmark in [SMB19a] and further detailed in [SMB19b].

In order to expedite the simulations, a coarser RLV aerodynamic coefficient set is employed. Hence, the Launch \& Recovery ( $L \& R$ ) performance indicators provided in this chapter are slightly different from those of Chapter 8, but the trends and conclusions established here have been verified via simulation to be independent of the aerodynamic fidelity level.

### 9.1 Baseline guidance performance assessment

Before tackling the improvement of performance through a more sophisticated guidance algorithm (Sec. 9.2), the purpose of this section is to analyse the impact of different guidance parameters and the compromises that can be achieved. To do so, an analysis approach using CTVG is proposed in Sec. 9.1.1 and its results discussed in Sec. 9.1.2.

### 9.1.1 The trade-off map approach revisited

The baseline performance assessment is strongly based on the trade-off map approach proposed in Sec. 4.2. It relies on systematic simulation of the nonlinear benchmark model over a guidance parameter grid to generate trade-off maps that enable a clear quantification of candidate choices.

Performance trade-off maps are generated by overlapping contour plots associated with key D\&L metrics, which may represent either nominal or dispersed (e.g. standard deviation) values. For the present study, these indicators include:

- $m_{\text {rec }}=m\left(t_{\text {sep }}\right)-m\left(t_{f}\right)$, propellant mass required for recovery. This value is the most direct mission performance indicator since decreasing the required propellant from separation $\left(t_{\text {sep }}\right)$ to touchdown $\left(t_{f}\right)$ enables an increase of payload mass and subsequent reduction of the launch cost per kilogram of payload;
- $Q_{\text {max }}=\left.\max Q(t)\right|_{t \in\left[t_{\text {sep }}, t_{f}\right]}$, maximum dynamic pressure encountered during recovery, to which aerodynamic loads are proportional. A higher dynamic pressure therefore requires a more structurally robust vehicle's body to withstand the associated loads and typically leads to an increase of its dry mass;
- $v_{f}=\left\|\mathbf{v}\left(t_{f}\right)\right\|$, touchdown velocity norm, which needs to be sustained by the vehicle's landing gear. Similar to the above, higher touchdown velocities will require reinforced mechanisms, which tend to increase the dry mass.

In the first two indicators, the separation time is taken to be $t_{\text {sep }}=112$ seconds.
Another mission-critical indicator is the maximum heat flux (proportional to $V^{3}(t)$ ). This indicator was verified to follow similar trends to the maximum dynamic pressure (proportional to $V^{2}(t)$, see Eq. (8.11)) and therefore it is not shown in the trade-off maps for the sake of conciseness.

Finally, it is important to recall that the two main parameters on which the CTVG technique depends, $h_{s}$ and $t_{f}$, are in fact intrinsically representative of the physics of the reusable $\mathrm{L} \& \mathrm{R}$ problem. In other words, performance indicators change depending on the guidance solution utilised, but their general trends remain comparable. Hence, CTVG trade-off maps over the parameter space of $h_{s}$ and $t_{f}$ become extremely valuable since they can be generated very quickly and the understanding they provide regarding the performance impact of those parameters remains valid for other guidance algorithms.

### 9.1.2 Results and discussion

Trade-off maps for the Down-Range Landing (DRL) and Return To Launch Site (RTLS) missions are depicted in Fig. 9.1a and 9.1b, respectively. They both show contours of the previous indicators over the parameter space of start altitude $h_{s} \in[5,40] \mathrm{km}$ and final time $t_{f} \in[360,420]$ seconds. In addition, the optimal (minimum) value of each of the three indicators is highlighted using the symbol $\circ$ (with the colour corresponding to the appropriate indicator). Note that these optimal values lie on the axes of the figures.


Figure 9.1: CTVG trade-off maps for $h_{s}$ and $t_{f}$. $*$ indicates initial guidance choice, and $\{\square, \triangle\}$ different optimisation choices.

Starting with DRL recovery, Fig. 9.1a, the trade-offs between propellant mass $m_{\mathrm{rec}}$ (black), dynamic pressure $Q_{\max }$ (cyan) and touchdown speed $v_{f}$ (red) are clearly perceptible. For example: [i] the minimum $m_{\text {rec }}$ (of 2.65 ton, located in the top-left corner) leads to a very high $Q_{\max }$ because it is associated with a very low start altitude, [ii] the minimum $Q_{\text {max }}$ (of 25.3 kPa , on the top-right side) is related to a high $m_{\text {rec }}$, as physically expected, and [iii] the minimum $v_{f}$ (of $0.27 \mathrm{~m} / \mathrm{s}$, located towards the mid-bottom of the left axis) requires more demanding guidance commands - mostly via a reduced final time (recall Eq. (8.47)) and therefore increased $m_{\text {rec }}$ and $Q_{\text {max }}$.

In other words, a single guidance choice cannot simultaneously attain all three optima, but is the result of a performance compromise. Suitable compromises and guidance choices can however be easily identified and quantified using the trade-off map.

As an illustration, consider the initial choice taken from Table 8.3 of $\left\{h_{s}, t_{f}\right\}=\{25.0,380\}$ (indicated by the symbol $*$ in Fig. 9.1a). This choice leads to $m_{\mathrm{rec}}=5.26$ ton, $Q_{\max }=83.7 \mathrm{kPa}$ and $v_{f}=1.62 \mathrm{~m} / \mathrm{s}$. Now assume that the vehicle's landing gear has a stronger structural resistance and is able to withstand speeds $25 \%$ higher, i.e. up to $2 \mathrm{~m} / \mathrm{s}$ (meaning that it does not need to slow down as much throughout the descent). In this case, keeping the same value of
$Q_{\text {max }}$, the required propellant can be reduced to 4.95 ton by choosing $\left\{h_{s}, t_{f}\right\}=\{24.7,370\}$ (indicated by the symbol $\square$ ). This choice translates to initiating the manoeuvre slightly later and finishing 10 seconds earlier. If, in addition to that, the vehicle is able to sustain a dynamic pressure of 60 kPa (closer to the 53 kPa encountered during launch), the optimal choice becomes $\left\{h_{s}, t_{f}\right\}=\{28.9,383\}($ marked by $\triangle)$ and the required propellant increases to 5.30 ton.

It is interesting to note that, although the latter guidance choice $(\triangle)$ and the initial one (*) have comparable propellant requirements, they are considerably different in terms of trajectory and dynamic pressure, as evidenced by Fig. 9.2a and 9.2b, respectively. This consideration proves that it is not enough to rely on a single indicator for a fair comparison of D\&L guidance approaches.


Figure 9.2: DRL results with different guidance choices. Markers show increments of 20 seconds.

The trade-off map for RTLS recovery is provided in Fig. 9.1b. Performance trends are roughly similar to the DRL case, although values of $m_{\text {rec }}$ are now globally higher and values of $Q_{\text {max }}$ lower. This effect is caused by the additional firing needed to bring the stage back, which additionally reduces the magnitude of its horizontal velocity and thus dynamic pressure. From the analysis of Sec. 8.4.1, it is also known that only about 8.9 ton of propellant is available after launch and therefore not all the parameter space shown in the trade-off map is feasible.

Conflicting guidance choices that optimise each single indicator for the RTLS case can again be identified: [i] minimum $m_{\text {rec }}$ (of 7.71 ton, bottom-left corner) leads to a high $Q_{\max }$, [ii] minimum $Q_{\text {max }}$ (of 9.31 kPa , top-right corner) is related to a high $m_{\text {rec }}$, and [iii] minimum $v_{f}$ (of $1.19 \mathrm{~m} / \mathrm{s}$, bottom-right corner) lies in one of the unfeasible areas mentioned above. The same trade-off exercise carried out for DRL recovery is repeated next.

The initial choice $(*)$ from Table 8.3 of $\left\{h_{s}, t_{f}\right\}=\{25.0,380\}$ leads to $m_{\text {rec }}=8.55$ ton, $Q_{\text {max }}=31.8 \mathrm{kPa}$ and $v_{f}=1.85 \mathrm{~m} / \mathrm{s}$. Now considering that the vehicle's constraints are exactly the same as for the $\triangle$ point before, i.e. 60 kPa dynamic pressure and $2 \mathrm{~m} / \mathrm{s}$ touchdown speed,
revising the guidance choice to $\left\{h_{s}, t_{f}\right\}=\{18.2,381\}$ leads to an alleviation of the propellant requirement to 8.37 ton (also marked in Fig. 9.1b by $\triangle$ ).

### 9.2 The DESCENDO algorithm

The $\operatorname{DESCENDO}$ algorithm has been designed as a middle-ground between computational efficiency and trajectory optimality. It is built-up from the understanding obtained from the previous trade-off analysis, as well as from concepts of convex optimisation-based guidance (introduced in Sec. 2.1.4.3 and revisited here in Sec. 9.2.1). The implementation strategy and mathematical details of DESCENDO are provided in Sec. 9.2.2 and 9.2.3, respectively.

### 9.2.1 Convex optimisation-based guidance revisited

The proposed algorithm relies extensively on the lossless and successive convexification techniques, introduced in Sec. 2.1.4.3, which represent the foundation of convex optimisation-based guidance. More specifically, it is based on the work of Szmuk et al. [SABJH16] and Jerez et al. [JMBS17], with a few fundamental and critical differences.

The latter reference [JMBS17] is mostly focused on maximising computational efficiency. This algorithm is ideal for real-time execution, but it relies on an extensive simplification of the equations of motion including disregarding aerodynamic forces, which play a decisive role in the vehicle's recovery trajectory.

On the other hand, reference [SABJH16] takes a completely different approach to [JMBS17] and focuses on maximising the optimality of the solution by employing successive convexifications to account for aerodynamic effects (as well as engine back-pressure losses). It is also noted that the two approaches have only been verified for low altitude and velocity, and that the assumption of constant air density made in [SABJH16] is not physically representative for RLVs.

With these considerations in mind, the objective of the proposed algorithm is to attain a middle-ground between the efficiency and optimality of the two aforementioned approaches that is suitable for the extended flight envelope encountered by RLVs. This algorithm is termed Descending over Extended envelopes using Successive ConvExificatioN-baseD Optimisation (DESCENDO).

One additional difference with respect to Szmuk et al. [SABJH16] is that the final time $t_{f}$ is not an optimisation variable and needs to be specified. This choice is made in order to provide a common comparative framework with the baseline approach of Sec. 8.2.2.

Moreover, this development relies on a perfect attitude control assumption, which is a common practice for designing and assessing guidance schemes since attitude can be changed quickly compared to the trajectory. The impact of attitude control on the loads experienced by the vehicle and its minimisation using robust Load Relief (LR) algorithms will be addressed in Chapter 10.

### 9.2.2 The DESCENDO algorithm implementation

The DESCENDO guidance algorithm is schematised in Fig. 9.3 and further detailed in Sec. 9.2.3. It has been implemented in MATLAB using the CVX library [GB14] to formulate the convex problems, and the ECOS routine [DCB13] to solve them. Both tools are freely available.


Figure 9.3: The DESCENDO guidance algorithm
The algorithm consists of two Second-Order Cone Programming (SOCP) stages, see Fig. 9.3: [i] SOCP 9.1, which allows to find a discrete trajectory and acceleration profile not accounting with aerodynamic effects, and [ii] SOCP 9.2, where successive convexifications are iteratively applied to define a convex approximation of those effects - for solution optimality purposes.

At each simulation instance (determined by the simulation rate $f_{\text {sim }}$ ), a commanded thrust vector in the Recovery Pad (RP) frame $\mathbf{T}_{\mathrm{CVX}}(t)$ is computed from the most recent guidance solution via linear interpolation. That solution is stored as an onboard look-up table and updated only when an SOCP step is executed and a feasible solution is found.

SOCP 9.1 is triggered at every guidance step, which is determined by the guidance update frequency $f_{\text {gui }}$ and by a pre-specified altitude $h_{s}$. To enable its formulation, trajectory and optimisation variables are first discretised into $N$ uniformly-spaced points, ranging from the current instant of time $t$ to touchdown time $t_{f}$. The discretisation interval between two consecutive points corresponds to:

$$
\begin{equation*}
T_{\mathrm{S}}=\frac{t_{f}-t}{N-1} \tag{9.1}
\end{equation*}
$$

and, since $T_{\mathrm{S}} \rightarrow 0$ as $t \rightarrow t_{f}$, the accuracy of the discretisation becomes more refined towards the end. The execution of SOCP 9.2 is determined by two additional variables, $h_{\mathrm{P}}$ and $N_{\mathrm{P}}$ (not to be confused with $h_{s}$ and $N$ ), which are shown in Fig. 9.3 and will be detailed in Sec. 9.2.3.

The accuracy of $D E S C E N D O$ increases with the guidance update frequency $f_{\text {gui }}$ and with the number of points $N$, at the expense of a higher computational load - for this reason, $f_{\text {gui }}$ is typically smaller than the simulation rate $f_{\text {sim }}$. It is important to note that a higher $f_{\text {gui }}$ does not increase the accuracy of the SOCPs themselves, but executing them more frequently minimises errors introduced by the trajectory discretisation and aerodynamic forces.
$N$ could be adjusted over time, in order to have a larger number of points in earlier (longer) trajectories. In practice, varying $N$ is more demanding as it involves the implementation and validation of different guidance modes. Since this preliminary study is focused on the behaviour of the algorithm itself, this extra effort will be avoided by keeping $N$ constant.

### 9.2.3 The DESCENDO optimisation problems

The formulation of SOCP 9.1 is similar to SOCP 2.1, with a few important additions. It employs the same change of variables of Eq. (2.54) and (2.55), cost function of Eq. (2.56), and boundary conditions, with the exception of an inequality relaxation of the final vertical velocity (i.e. $\mathbf{v}_{z}[N] \leq \mathbf{v}_{f_{z}}$ ).

The discrete dynamics equations are also the same, with Eq. (2.57) representing the translational motion of Eq. (8.8), Eq. (2.58) describing the mass-depletion dynamics of Eq. (8.28), and $T_{\mathrm{S}}$ given by Eq. (9.1). As mentioned before, recovery calculations are made in the RP reference frame where non-inertial effects are neglected.

This optimisation problem relies also on the same lossless convexification constraints on $\mathbf{w}[k]$ and $\sigma[k]$ given by Eq. (2.59) and (2.60), as well as on the control limitations $\theta_{\max }, T_{\min }$ and $T_{\max }$, imposed by Eq. (2.61) and (2.62).

It is important to notice that, at this point, aerodynamic effects are not yet included. In addition, constraints (2.61) and (2.62) are only enforced during the burn periods defined by a
pre-specified set $\mathcal{T}_{P}$, which depends on the relative duration of re-entry and landing burn, $t_{1}$ and $t_{2}$ (recall Sec. 8.2.1). Outside these periods, the applied thrust acceleration is set to zero.

The maximum thrust magnitude rate is also bounded to $\dot{T}_{\max }$ using a forward discretisation scheme for the differentiation of $\sigma[k]$ and a constraint equivalent to Eq. (2.62):

$$
\begin{equation*}
\sigma[k]-T_{\mathrm{S}} \frac{\dot{T}_{\mathrm{max}}}{\hat{m}(t)} \leq \sigma[k+1] \leq \sigma[k]+T_{\mathrm{S}} \frac{\dot{T}_{\max }}{\hat{m}(t)} \tag{9.2}
\end{equation*}
$$

This constraint was proven critical to the reduction of control chattering without having a noticeable impact on the tightness of Eq. (2.60).

The SOCP 9.1 optimisation problem subject to the constraints introduced above is then formulated as:

## SOCP 9.1

$$
\max _{\mathbf{a}, \sigma} z[N], \quad \text { subject to: }
$$

Boundary conditions

$$
\begin{aligned}
& z[1]=\ln \hat{m}(t), \quad \mathbf{r}[1]=\hat{\mathbf{r}}(t), \quad \mathbf{v}[1]=\hat{\mathbf{v}}(t), \quad \mathbf{a}[1]=\hat{\mathbf{a}}(t) \\
& \mathbf{r}[N]=\mathbf{r}_{f}, \quad \mathbf{v}_{x, y}[N]=\mathbf{v}_{f_{x, y},}, \quad \mathbf{v}_{z}[N] \leq \mathbf{v}_{f_{z}}, \quad \mathbf{a}_{x, y}[N]=0_{2 \times 1}, \quad \mathbf{a}_{z}[N] \geq 0
\end{aligned}
$$

$$
\text { Dynamics equations, } \quad \forall k \in[1, \cdots, N-1]
$$

$$
\mathbf{r}[k+1]=\mathbf{r}[k]+T_{\mathrm{S}} \mathbf{v}[k]+\frac{T_{\mathrm{S}}^{2}}{3}\left(\mathbf{w}[k]+\frac{\mathbf{w}[k+1]}{2}\right)
$$

$$
\mathbf{v}[k+1]=\mathbf{v}[k]+\frac{T_{\mathrm{S}}}{2}(\mathbf{w}[k]+\mathbf{w}[k+1])
$$

$$
z[k+1]=z[k]-\frac{1}{I_{\mathrm{sp}} g_{0}} \frac{T_{\mathrm{S}}}{2}(\sigma[k]+\sigma[k+1])
$$

$$
\underline{\text { Surrogate variables, }}, \forall k \in[1, \cdots, N]
$$

$$
\mathbf{w}[k]=\mathbf{a}[k]+\hat{\mathbf{g}}(t)
$$

$$
\|\mathbf{a}[k]\| \leq \sigma[k]
$$

Control constraints, $\quad \forall k \in[1, \cdots, N-1]$

$$
\begin{cases}\mathbf{a}_{z}[k] \geq \frac{\left\|\mathbf{a}_{x, y}[k]\right\|}{\tan \theta_{\max }}, \frac{T_{\min }}{\hat{m}(t)} \leq \sigma[k] \leq \frac{T_{\max }}{\hat{m}(t)}, & \text { if } T_{\mathrm{S}}(k-1) \in \mathcal{T}_{P} \\ \mathbf{a}[k]=0_{3 \times 1}, & \text { otherwise }\end{cases}
$$

Control rate constraints, $\forall k \in[1, \cdots, N-1]$
$\sigma[k]-T_{\mathrm{S}} \frac{\dot{T}_{\max }}{\hat{m}(t)} \leq \sigma[k+1] \leq \sigma[k]+T_{\mathrm{S}} \frac{\dot{T}_{\text {max }}}{\hat{m}(t)}$
The main limitation of this formulation lies on its inability to account for aerodynamic forces. Without any knowledge of the deceleration caused by these forces, the algorithm overestimates the amount of propellant needed to slow the vehicle down for landing. But most importantly, it
is highlighted that, because of this discrepancy, earlier SOCP 9.1 solutions can only be found if subsurface flight avoidance (or any other flight path constraint) is not enforced. Moreover, the algorithm is able to recover more effectively from an earlier solution where all boundary conditions are met (even if with subsurface flight) than from one without subsurface flight. This recovery will be highlighted in Sec. 9.3.

To overcome the aforementioned limitations, the DESCENDO algorithm then introduces the successive convexification procedure of [LL14, MSA16]. This procedure involves solving a second (iteratively more refined) SOCP in which the solution of the previous problem is employed to define a convex approximation of the aerodynamic effects.

The successive convexification loop is executed $N_{\mathrm{P}}$ times per guidance step, and the solution of SOCP 9.1 (if feasible) is used for the first iteration. This approach results in $1+N_{\mathrm{P}} \mathrm{SOCPs}$ (i.e. 1 SOCP 9.1 and $N_{P}$ SOCPs 9.2) being solved at each guidance step. If SOCP 9.1 is not feasible, the guidance solution is not updated and the interpolation uses the solution obtained in the previous guidance step.

Aerodynamic effects in the refined SOCP 9.2 are approximated by augmenting the surrogate acceleration vector with a velocity-dependent term defined as:

$$
\begin{equation*}
\mathbf{w}[k]=\mathbf{a}[k]+\hat{\mathbf{g}}(t)-d_{i}^{*}[k] \mathbf{v}[k] \tag{9.3}
\end{equation*}
$$

where $d_{i}^{*}[k]$ is a template computed before the SOCP, and therefore its complexity does not affect the efficiency of the problem's solution. Since the most significant aerodynamic effect during recovery is the deceleration due to drag, which is parallel to the velocity vector, $d_{i}^{*}[k]$ is defined as:

$$
\begin{equation*}
d_{i}^{*}[k]=\frac{1}{2} \rho_{i}^{*}[k] S_{\mathrm{ref}} C_{D i}^{*}[k] \frac{\left\|\mathbf{v}_{i}^{*}[k]\right\|}{\exp z_{i}^{*}[k]} \tag{9.4}
\end{equation*}
$$

In this equation, $\mathbf{v}_{i}^{*}[k]$ and $z_{i}^{*}[k]$ are given directly by the solution of the previous SOCP $i^{\text {th }}$ iteration, and $\rho_{i}^{*}[k]$ and $C_{D i}^{*}[k]$ can be estimated as functions of $\mathbf{v}_{i}^{*}[k]$ and $\mathbf{r}_{i}^{*}[k]$. For this scenario however, defining $\rho_{i}^{*}[k]$ as a linearly-spaced vector from $\hat{\rho}(t)$ to $\rho_{0}\left(\rho_{0} \approx 1.23 \mathrm{~kg} / \mathrm{m}^{3}\right.$ is the density at sea level) and $C_{D i}^{*}[k]=\left.C_{D}\right|_{\alpha=\pi}$ as a constant was verified to yield acceptable results.

In addition to the aerodynamic template, the iterative process includes a quadratic condition that facilitates the algorithm's convergence by bounding the deviation between guidance solutions found in two consecutive iterations. This condition is known as a Trust Region Constraint (TRC), see also Fig. 9.3, and defined as:

$$
\begin{equation*}
\left\|\mathbf{a}[k]-\mathbf{a}_{i}^{*}[k]\right\| \leq \eta_{\mathbf{a}}[k] \tag{9.5}
\end{equation*}
$$

where $\mathbf{a}_{i}^{*}[k]$ is the thrust acceleration template determined by the previous SOCP $i^{\text {th }}$ iteration. The TRC is enforced by minimising $\eta_{\mathbf{a}}[k]$, hence the SOCP objective function needs to be augmented with the point-wise sum of this vector, weighted by $w_{\eta_{\mathrm{a}}}$. A smaller value of $w_{\eta_{\mathrm{a}}}$ will be reflected in a larger variation between solutions, and vice-versa.

The formulation of the refined optimisation problem featuring the acceleration of Eq. (9.3), the TRC of Eq. (9.5) and the augmented objective function is provided in SOCP 9.2:

## SOCP 9.2

$\max _{\mathbf{a}, \sigma} z[N]-w_{\eta_{\mathbf{a}}} \sum_{k=1}^{N} \eta_{\mathbf{a}}[k], \quad$ subject to:
Boundary conditions
$z[1]=\ln \hat{m}(t), \quad \mathbf{r}[1]=\hat{\mathbf{r}}(t), \quad \mathbf{v}[1]=\hat{\mathbf{v}}(t), \quad \mathbf{a}[1]=\hat{\mathbf{a}}(t)$
$\mathbf{r}[N]=\mathbf{r}_{f}, \quad \mathbf{v}[N]=\mathbf{v}_{f}, \quad \mathbf{a}_{x, y}[N]=0_{2 \times 1}, \quad \mathbf{a}_{z}[N] \geq 0$
Dynamics equations, $\forall k \in[1, \cdots, N-1]$
$\mathbf{r}[k+1]=\mathbf{r}[k]+T_{\mathrm{S}} \mathbf{v}[k]+\frac{T_{\mathrm{S}}^{2}}{3}\left(\mathbf{w}[k]+\frac{\mathbf{w}[k+1]}{2}\right)$
$\mathbf{v}[k+1]=\mathbf{v}[k]+\frac{T_{\mathrm{S}}}{2}(\mathbf{w}[k]+\mathbf{w}[k+1])$
$z[k+1]=z[k]-\frac{1}{I_{\mathrm{sp}} g_{0}} \frac{T_{\mathrm{S}}}{2}(\sigma[k]+\sigma[k+1])$
Surrogate variables, $\forall k \in[1, \cdots, N]$
$\mathbf{w}[k]=\mathbf{a}[k]+\hat{\mathbf{g}}(t)-d_{i}^{*}[k] \mathbf{v}[k]$
$\|\mathbf{a}[k]\| \leq \sigma[k]$
Trust region constraints, $\quad \forall k \in[1, \cdots, N]$
$\left\|\mathbf{a}[k]-\mathbf{a}_{i}^{*}[k]\right\| \leq \eta_{\mathbf{a}}[k]$
Flight path constraints, $\quad \forall k \in[1, \cdots, N-1]$
$\mathbf{r}_{z}[k] \geq \frac{\hat{\mathbf{r}}_{z}(t)}{\left\|\hat{\mathbf{r}}_{x, y}(t)\right\|}\left\|\mathbf{r}_{x, y}[k]\right\|$
Control constraints, $\forall k \in[1, \cdots, N-1]$

$$
\begin{cases}\mathbf{a}_{z}[k] \geq \frac{\left\|\mathbf{a}_{x, y}[k]\right\|}{\tan \theta_{\max }}, \frac{T_{\min }}{\hat{m}(t)} \leq \sigma[k] \leq \frac{T_{\max }}{\hat{m}(t)}, & \text { if } T_{\mathrm{S}}(k-1) \in \mathcal{T}_{P} \\ \mathbf{a}[k]=0_{3 \times 1}, & \text { otherwise }\end{cases}
$$

Control rate constraints, $\quad \forall k \in[1, \cdots, N-1]$
$\sigma[k]-T_{\mathrm{S}} \frac{\dot{T}_{\text {max }}}{\hat{m}(t)} \leq \sigma[k+1] \leq \sigma[k]+T_{\mathrm{S}} \frac{\dot{T}_{\text {max }}}{\hat{m}(t)}$
Comparing this problem with SOCP 9.1, two increasingly stringent specifications are made: $[i]$ the inequality relaxation of the final vertical velocity $\mathbf{v}_{z}[N]$ is dropped, and [ii] the following flight path constraint is introduced:

$$
\begin{equation*}
\mathbf{r}_{z}[k] \geq \frac{\hat{\mathbf{r}}_{z}(t)}{\left\|\hat{\mathbf{r}}_{x, y}(t)\right\|}\left\|\mathbf{r}_{x, y}[k]\right\| \tag{9.6}
\end{equation*}
$$

In addition to subsurface flight avoidance, this constraint ensures that the recovery trajectory remains in the interior of a shrinking cone with vertex at the landing point and with the vehicle's current position $\hat{\mathbf{r}}(t)$ on its surface.

It is also important to note that, as evidenced in Fig. 9.3, SOCP 9.2 is only solved while the vehicle is higher than a pre-specified altitude $h_{\mathrm{P}}$. The reason for this choice is related to the fact that velocity is significantly smaller at low altitudes and thus aerodynamic forces become less intense. Therefore, disregarding their impact here introduces less error while reducing the overall computational time.

### 9.3 DESCENDO results and discussion

The initial parameters adopted for the verification of the DESCENDO algorithm are listed in Table 9.1. The start altitude $\left(h_{s}\right)$ and final time $\left(t_{f}\right)$ are maintained for consistency the same as for the baseline guidance in Sec. 8.2.2. The thrust control constraints are set to $T_{\max }=600 \mathrm{kN}$ and $\dot{T}_{\text {max }}=1 \mathrm{kN} / \mathrm{s}$, while $T_{\text {min }}$ is set to zero without loss of generality.

Additionally, values of $h_{\mathrm{P}}, \theta_{\max }$ and $w_{\eta_{\mathrm{w}}}$ are stipulated as a result of a pattern search aimed at minimising $Q_{\max }$ while keeping computational times as low as possible. It is remarked that testing optimisation-based algorithms in closed-loop is particularly challenging due to the drastic influence of the choice of $f_{\text {gui }}$ and $N$. These parameters have been kept constant throughout the present study, but a thorough assessment of their impact would be crucial prior to any real-world application.

Table 9.1: Initial $D E S C E N D O$ algorithm parameters

| Parameter | Value |
| :--- | :---: |
| $h_{s}(\mathrm{~km})$ | 25 |
| $t_{f}(\mathrm{~s})$ | 380 |
| $t_{1}(\%)$ | 0 |
| $t_{2}(\%)$ | 0 |
| $f_{\text {sim }}(\mathrm{Hz})$ | 10 |
| $f_{\text {gui }}(\mathrm{Hz})$ | 1 |
| $N$ | 20 |
| $N$ |  |$\quad$| Parameter | Value |
| :--- | :--- |
| $N_{\mathrm{P}}$ | $\{0,1,2\}$ |
| $h_{\mathrm{P}}(\mathrm{km})$ | 2 |
| $\theta_{\max }(\mathrm{deg})$ | $65(\mathrm{DRL}), 70(\mathrm{RTLS})$ |
| $T_{\min }(\mathrm{kN})$ | 0 |
| $T_{\max }(\mathrm{kN})$ | 600 |
| $\dot{T}_{\max }(\mathrm{kN} / \mathrm{s})$ | 1 |
| $w_{\eta_{\mathrm{a}}}$ | 0.0001 |

Figure 9.4 gathers a set of closed-loop DRL simulations using $D E S C E N D O$ and its three columns (Fig. 9.4a, b and c) correspond to solutions with $N_{\mathrm{P}}=\{0,1,2\}$. The main purpose of this analysis is to demonstrate the positive impact of having merely 1 or 2 successive convexification loops, rather than showing how accurate the algorithm can be for a larger number (which, in practice, is only limited by the available computational power).

The rows of Fig. 9.4 represent trajectory in the RP frame (similar to Fig. 9.2a), remaining propellant mass, thrust magnitude, and angle with respect to the vertical direction. Furthermore, each plot shows the CTVG baseline in red (which is the same for all the values of $N_{\mathrm{P}}$ ), the DESCENDO result in black and its intermediate solutions (i.e. those used for interpolation, recall Fig. 9.3) in gray scale. All these solutions start from their current states in the black line and become more accurate towards touchdown as $T_{\mathrm{S}} \rightarrow 0$ (with the same number of points $N$ spanning over shorter distances).


Figure 9.4: DRL recovery using $\operatorname{DESCENDO}$ and intermediate solutions with increasing $N_{\mathrm{P}}$. The solutions 30 seconds into the recovery are indicated by magenta lines.

Figure 9.4a shows that a successful recovery is already achieved with $N_{\mathrm{P}}=0$. This outcome can be interpreted as the result of an algorithm in which aerodynamic effects are not considered, such as [JMBS17]. However, without any knowledge of the deceleration caused by aerodynamic forces, the algorithm provides inadequate guidance solutions earlier in the descent, which is demonstrated by the large dispersion of intermediate trajectory and mass estimates. It is important to note that lighter colours are associated with solutions earlier in the descent.

Although these estimates are not satisfactory since they assume subsurface flight (as explained before) and often require more propellant than the CTVG baseline (second plot), they converge to an acceptable result relatively quickly. As an indication of this convergence, the intermediate solutions 30 seconds into the recovery are depicted in Fig. 9.4 with magenta lines and show that there is already a good match with the actual, final trajectory.

The situation improves significantly as soon as the successive convexification loop is introduced and repeated. A progressive decrease of dispersions is visible in Fig. 9.4b $\left(N_{\mathrm{P}}=1\right)$ and Fig. 9.4c $\left(N_{\mathrm{P}}=2\right)$, and flight path constraints are successfully met by all the intermediate guidance solutions. In addition, for $N_{P}=2$, a propellant saving of $6.9 \%$ is achieved in comparison to the CTVG baseline, and thrust magnitude and vertical direction profiles (the two bottom rows of plots) become less abrupt than for $\left(N_{P}=0\right)$ (notice the time range between 310 and 350 seconds), while meeting the bounds of Table 9.1.

The exact same conclusions can be drawn for RTLS recovery, depicted in Fig. 9.5, except that since the remaining propellant after the boostback burn is considerably lower than in the DRL case, the impact of the DESCENDO algorithm becomes clearer.

Continuing with the RTLS analysis, on the one hand, many of the initial $D E S C E N D O$ solutions with $N_{\mathrm{P}}=0$, Fig. 9.5a, show larger dispersions (many of them requiring more propellant than what is actually available) and even degenerate into high control chattering close to touchdown (third plot from the top on the left). On the other hand, for $N_{\mathrm{P}}=2$ (Fig. 9.5c), a propellant saving of $16.4 \%$ is achieved in comparison to the CTVG baseline (and most of the initial solutions align with the final one). This value is not as high as for $N_{\mathrm{P}}=1$ (Fig. 9.5b) because it is achieved using a much smoother thrust profile, which once again indicates that a single indicator may not be enough for a fair $\mathrm{D} \& \mathrm{~L}$ comparison.

In order to have a better understanding of the convergence and computational efficiency of the algorithm, Fig. 9.6 illustrates the optimal value of the cost function of Eq. (2.56), in the top plot, and the cumulative ECOS runtime, in the bottom plot, as functions of the intermediate guidance solutions for the RTLS scenario with $N_{P}=\{0,1,2\}$. The cost function corresponds to:

$$
\begin{equation*}
z[N]=\ln \hat{m}\left(t_{f}\right) \tag{9.7}
\end{equation*}
$$

The top plot of Fig. 9.6 confirms that, although all the cases end up with similar objective function values, its accurate prediction takes place much later in the descent for $N_{\mathrm{P}}=0$ (without successive convexification of aerodynamic effects). Solutions for $N_{\mathrm{P}}=1$ and $N_{\mathrm{P}}=2$ are relatively similar, which demonstrates the quick convergence of the algorithm for the present case study, with $N_{\mathrm{P}}=2$ slightly under $N_{\mathrm{P}}=1$ as anticipated.

In terms of efficiency, the computational time naturally increases with $N_{\mathrm{P}}$, but it only takes a total of $0.6,1.3$ or 2.2 seconds to compute the 100 seconds of recovery using $N_{\mathrm{P}}=\{0,1,2\}$, as evidenced in the bottom plot of Fig. 9.6. These values account for core solver time only and a small overhead needs to be added for problem parsing, but this is often negligible, especially if an embeddable solver such as FORCES [JMBS17] or CVXGEN [MB12] is employed.


Figure 9.5: RTLS recovery using $D E S C E N D O$ and intermediate solutions with increasing $N_{\mathrm{P}}$


Figure 9.6: Comparison of $\operatorname{DESCENDO}$ objective function and solver time during RTLS recovery

It is interesting to note that the execution speed increases significantly around the $70^{\text {th }}$ run for $N_{\mathrm{P}}>0$. This increase is due to the successive convexification switch-off for $h(t) \leq h_{\mathrm{P}}$ (where smaller velocities lead to less intense aerodynamic forces) and allows to reduce the overall runtime by about $30 \%$ with almost no optimality compromise (as seen in the top plot). Such a saving is not relevant for a real-world implementation since the system will have to be capable of coping with the successive convexification loops at higher altitudes, but it is very useful for preliminary mission feasibility studies where many simulations need to be performed.

In order to obtain a thorough understanding of the flight envelope encountered by the vehicle, results for the same simulation of Fig. 9.5c are provided in Fig. 9.7. For a detailed interpretation of this figure, the reader is referred to the discussion on Fig. 8.12.


Figure 9.7: Flight mechanics results of RTLS mission using $D E S C E N D O$ and $N_{P}=2$

As an overview, the recovery metrics achieved with this approach are $m_{\mathrm{rec}}=8.06$ ton, $Q_{\max }=50.1 \mathrm{kPa}$ and $v_{f}=1.63 \mathrm{~m} / \mathrm{s}$. Comparing these results with those of the baseline CTVG case (specifically, the choice set at the end of Sec. 9.1.2, i.e. $m_{\mathrm{rec}}=8.37$ ton, $Q_{\max }=60 \mathrm{kPa}$ and $v_{f}=2 \mathrm{~m} / \mathrm{s}$ ), it is clear that the $D E S C E N D O$ algorithm enables a significant improvement for all the indicators (most noticeably a simultaneous decrease of 310 kg propellant mass and $16 \%$ dynamic pressure), which was exactly the main motivation behind its development.

## Recovery using two burns

The same trade-off maps approach of Sec. 9.1.1 could be repeated to further improve these results by exploiting the parameter space of start altitude and final time. Alternatively, it is also interesting to utilise similar trade-off maps but to analyse the performance impact of explicitly splitting the recovery into two separate burns. To do so, Fig. 9.8a and 9.8 b show DRL and RTLS maps for the parameter space of $t_{1}$ (relative duration of the first burn with respect to the total recovery phase) and $t_{2}$ (relative duration of the pause in-between burns). The single burn performance indicators (extracted from the simulation of Fig. 9.4c and 9.5c, respectively) are also provided in the captions for comparison.

(a) DRL. Note: $\left\{m_{\text {rec }}, Q_{\text {max }}, v_{f}\right\}=\{5.05,144,1.49\}$
(b) RTLS. Note: $\left\{m_{\text {rec }}, Q_{\text {max }}, v_{f}\right\}=\{8.06,50.1,1.63\}$ with single burn.

Figure 9.8: $D E S C E N D O$ trade-off maps for $t_{1}$ and $t_{2}$

For the DRL case (Fig. 9.8a), results are shown for $t_{1}$ and $t_{2}$ up to $35 \%$. Although there is a large area of unfeasible solutions (above the $v_{f}=1.8 \mathrm{~m} / \mathrm{s}$ contour line, notice the abrupt transition to $v_{f}=30 \mathrm{~m} / \mathrm{s}$ ) as well as a few peaks of $Q_{\max }$ (most notably around $\left\{t_{1}, t_{2}\right\}=$ $\{10,10\} \%$ ), considerable gains can still be achieved. For instance, see the point marked by $\triangle$, having a first burn of $t_{1}=10 \%$ followed by a pause of $t_{2}=20 \%$ (before the second burn) enables to bring $m_{\text {rec }}$ and $Q_{\max }$ from 5.05 down to 3.13 ton and from 144 to 136 kPa , respectively, while only increasing the touchdown speed $v_{f}$ from 1.49 to $1.58 \mathrm{~m} / \mathrm{s}$.

Similar observations can be made for RTLS recovery (Fig. 9.8b), in which the parameter space $t_{1}+t_{2} \leq 80 \%$ (i.e. allowing at least $20 \%$ of time for the second burn) was investigated. This map facilitates understanding the trade-off between propellant mass and dynamic pressure in the areas where $v_{f}$ assumes feasible values (note that there is again a large region of unfeasibility, with $v_{f}$ above $\left.30 \mathrm{~m} / \mathrm{s}\right)$. As an example, indicated again by $\triangle$, keeping the same $v_{f} \approx 1.63 \mathrm{~m} / \mathrm{s}$, $Q_{\text {max }}$ can be reduced from 50.1 to 45.4 kPa by increasing $m_{\text {rec }}$ from 8.06 to 8.10 ton while choosing $\left\{t_{1}, t_{2}\right\}=\{20,20\} \%$.

To complement this result, the corresponding CTVG and DESCENDO simulation using two burns is depicted (following the same lines of Fig. 9.5) in Fig. 9.9.


Figure 9.9: RTLS recovery and intermediate solutions using two burns $\left\{t_{1}, t_{2}\right\}=\{20,20\} \%$

This figure confirms the successful implementation of the DESCENDO algorithm, with the burn periods explicitly accounted for at all the intermediate guidance solutions (in opposition to the CTVG baseline, in which the distinct burns are not enforced a priori). The pause is reflected in a constant propellant mass, zero thrust and undefined angle during the $t_{2}=20 \%$ of pause in-between burns (see the time range between 300 and 320 seconds in the bottom three plots).

# 10 

## Improving Control via Active Load Relief

The guidance improvement proposed in Chapter 9 relied on the assumption that attitude commands are perfectly followed so that attitude control has no impact on the trajectory. The present chapter, on the other hand, is focused on the impact of attitude control on the loads experienced by Reusable Launch Vehicles (RLVs) and on how these loads can be minimised through proper control design.

FeedBack (FB)-based design principles for load management are rooted on the work of Hoelker [Hoe61], who established a set of minimum error/drift/load control conditions, although it is well-known that achieving an acceptable compromise between these competing conditions over the flight and for different launcher configurations is not straightforward [LR68]. More advanced methods have been proposed based on total Angle of Attack (AoA) estimation [Boe89], the adoption of a Light Detection And Ranging (LiDAR) sensor for forward-looking wind information [Mar02, $\left.\mathrm{BSG}^{+} 03\right]$, and the use of adaptive control augmentation to recover performance in off-nominal conditions [OV12]. Nonetheless, even though analogous developments have taken place, and are well-established for operational deployment, in the fields of aeronautics [CFDS17, THS18, FJD19, OPV19] and wind energy [KBA ${ }^{+}$15, TJ16], the reliance on conventional control design principles remains the state-of-practice for launcher Load Relief (LR) [Suz04, WDW08].

The objective of this chapter is to showcase a simple way to reconcile more conventional launcher control design approaches with the benefits of anticipating the contribution of wind on the load experienced by the vehicle. This is achieved first by augmenting a feedback-only architecture $\left[\mathrm{SBM}^{+} 16, \mathrm{NMS}^{+} 19\right]$ with a channel that provides information of wind disturbances, and then designing an observer to estimate those disturbances using robust control techniques (Sec. 2.2), here termed Robust Wind Disturbance Observer (rWDO). As seen in Sec. 8.3.2, most of the recovery phase is unpropelled, so descent attitude control (and therefore LR) relies
extensively on the use of fins. Conversely, the use of fins during launch (where thrust level is very high) is not industrial practice, yet it will also be shown that further LR improvements are possible by doing so.

This chapter begins with a review of the RLV model in Sec. 10.1, followed by an analysis of achievable LR capabilities and consequent performance trade-offs in Sec. 10.2. The rWDO design approach is then presented in Sec. 10.3 and applied to ascent flight using Thrust Vector Control (TVC) alone and then combined fins/TVC in Sec. 10.4, as well as to descent flight in Sec. 10.5. This work has also been presented in [SMB19c].

### 10.1 Linear RLV model

For this part of the study, the Linear Time-Invariant (LTI) model of Eq. (8.48) is replaced by a more sophisticated one. The model of the vehicle in the pitch plane is depicted in Fig. 10.1. Since the present chapter is dedicated to the study of atmospheric flight only, the use of cold gas thrusters is not accounted for. In addition, the main focus is on the impact of uncertainties and wind disturbances on flight performance, hence the effects of actuator, sensor, bending and sloshing dynamics are not considered for the sake of simplicity.

The perturbed pitch dynamics are described using the model derived in Appendix C by linearising the nonlinear RLV dynamics. The reader is referred to this appendix for details on the model, but the main equations are repeated below for convenience. The linearised perturbations are represented by the LTI model $G_{\mathrm{RLV}}(\mathrm{s})$, which is defined as:

$$
\left[\begin{array}{llll}
\dot{\theta} & \ddot{\theta} & \dot{z} & \ddot{z}
\end{array}\right]^{\mathrm{T}}=A_{\mathrm{RLV}}\left[\begin{array}{llll}
\theta & \dot{\theta} & z & \dot{z}
\end{array}\right]^{\mathrm{T}}+B_{\mathrm{RLV}}\left[\begin{array}{c}
\mathbf{u}  \tag{10.1}\\
\hline v_{\mathrm{w}}
\end{array}\right]
$$

where the perturbation notation " $\delta$ " is dropped for improved readability, $\theta, \dot{\theta}$ and $\ddot{\theta}$ represent pitch angle and first/second-order derivatives, $z, \dot{z}$ and $\ddot{z}$ are the lateral drift and derivatives. In this equation, the vector $\mathbf{u}=\left[\begin{array}{ll}\beta_{\text {TVC }} & \beta_{\text {fin }}\end{array}\right]^{\mathrm{T}}$ gathers TVC and fin control inputs, $v_{\mathrm{w}}$ is the wind disturbance speed and matrices $A_{\mathrm{RLV}}$ and $B_{\mathrm{RLV}}$ are given by:

$$
\begin{align*}
& A_{\mathrm{RLV}}=\left[\begin{array}{cccc}
0 & 1 & 0 & 0 \\
\mu_{\alpha^{\prime}} & -\frac{l_{\alpha} \mu_{\alpha}+l_{\mathrm{f}} \mu_{\mathrm{f}} \cos \beta_{\mathrm{fin}, 0}}{V \cos \alpha_{0}} & 0 & \frac{\mu_{\alpha^{\prime}}}{V \cos \alpha_{0}} \\
0 & 0 & 0 & 1 \\
-\frac{N_{\alpha^{\prime}}}{m}-a_{0} & \frac{l_{\alpha} N_{\alpha}+l_{\mathrm{f}} N_{\mathrm{f}} \cos \beta_{\mathrm{fin}, 0}}{m V \cos \alpha_{0}}+u_{0} & 0 & -\frac{N_{\alpha^{\prime}}}{m V \cos \alpha_{0}}
\end{array}\right]  \tag{10.2}\\
& B_{\mathrm{RLV}}=\left[\begin{array}{cc|c}
0 & 0 & 0 \\
-\mu_{\mathrm{c}} \cos \beta_{\mathrm{TVC}, 0} & -\mu_{\mathrm{f}} \cos \beta_{\mathrm{fin}, 0} & -\frac{\mu_{\alpha^{\prime}}}{V \cos \alpha_{0}} \\
0 & 0 & 0 \\
-\frac{T}{m} \cos \beta_{\mathrm{TVC}, 0} & \frac{N_{\mathrm{f}}}{m} \cos \beta_{\mathrm{fin}, 0} & \frac{N_{\alpha^{\prime}}}{m V \cos \alpha_{0}}
\end{array}\right] \tag{10.3}
\end{align*}
$$

with all the coefficients introduced in Eq. (C.3), (C.27), (C.28) and (C.29).
The aerodynamic load generated by the vehicle is directly proportional to the angle of attack $\alpha$, hence the following equation is also necessary to provide a point-mass load indication at its Centre of Gravity (CG):

$$
\begin{equation*}
\alpha=\theta+\frac{\dot{z}-v_{\mathrm{w}}}{V \cos \alpha_{0}} \tag{10.4}
\end{equation*}
$$



Figure 10.1: RLV model diagram
Nominal values of all the required variables are imported from the Down-Range Landing (DRL) simulation of Sec. 8.4.2 for two flight time instances (potential load cases), one during ascent and the other during descent, see Table 10.1 in the next page. In addition, parametric uncertainties are introduced in the previous state-space matrices and a Linear Fractional Transformation (LFT) form (Sec. 2.2.1.1) is obtained, similar to $\left[\mathrm{SBM}^{+} 16\right]$. These parametric uncertainties are encapsulated in the uncertainty block:

$$
\begin{align*}
& \Delta_{\mathrm{RLV}}(\mathrm{~s})=\operatorname{diag}\left[\mathbb{I}_{6 \times 6} \delta C_{N_{\alpha}}, \mathbb{I}_{6 \times 6} \delta C_{\mathrm{f}_{\alpha}} \cos \beta_{\mathrm{fin}}, \mathbb{I}_{4 \times 4} \delta Q, \mathbb{I}_{2 \times 2} \delta T \cos \beta_{\mathrm{TVC}},\right.  \tag{10.5}\\
&\left.\mathbb{I}_{8 \times 8} \delta V \cos \alpha, \delta a_{0}, \mathbb{I}_{6 \times 6} \delta x_{\mathrm{CP}}, \mathbb{I}_{9 \times 9} \delta x_{\mathrm{CG}}, \mathbb{I}_{6 \times 6} \delta m, \mathbb{I}_{3 \times 3} \delta J_{N}\right]
\end{align*}
$$

As indicated in Table 10.1, an uncertainty range of $20 \%$ is assumed for aerodynamic-related parameters, $2 \%$ for Mass, CG \& Inertia (MCI) and $10 \%$ for the remaining variables. The thrust force $T$, although commanded externally by the guidance subsystem as $T_{\text {ref }}$ (recall Sec. 8.2), is more conveniently modelled as an uncertain parameter. Its uncertainty range of $10 \%$ then accounts for guidance variations with respect to the nominal trajectory as well as mismatches between $T$ and $T_{\text {ref }}$ caused by uncertainties in the propulsion system.

Table 10.1: Model parameters and uncertainty ranges at ascent and descent design points

| Variable | Symbol | Units | Nominal <br> ascent | Nominal <br> descent | Uncert. |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Normal RLV force gradient | $C_{N_{\alpha}}$ | - | 2.007 | -2.963 | $20 \%$ |
| Normal fin force gradient | $C_{\mathrm{f}_{\alpha}} \cos \beta_{\mathrm{fin}, 0}$ | - | 6.000 | -5.583 | $20 \%$ |
| Dynamic pressure | $Q$ | kPa | 52.48 | 7.835 | $20 \%$ |
| Normal thrust force | $T \cos \beta_{\mathrm{TVC}, 0}$ | kN | 2386 | 49.69 | $10 \%$ |
| Trim acceleration | $a_{0}$ | $\mathrm{~m} / \mathrm{s}^{2}$ | 7.122 | 9.331 | $10 \%$ |
| Longit. velocity | $V \cos \alpha_{0}$ | $\mathrm{~m} / \mathrm{s}$ | 571.4 | -175.0 | $10 \%$ |
| Longit. CP coordinate | $x_{\mathrm{CP}}$ | m | 18.36 | 5.317 | $10 \%$ |
| Longit. CG coordinate | $x_{\mathrm{CG}}$ | m | 8.919 | 4.452 | $2 \%$ |
| Total RLV mass | $m$ | kg | $93.5 \times 10^{3}$ | $8.49 \times 10^{3}$ | $2 \%$ |
| Normal RLV MoI | $J_{N}$ | $\mathrm{~kg} \cdot \mathrm{~m}^{2}$ | $46.0 \times 10^{5}$ | $0.64 \times 10^{5}$ | $2 \%$ |
| Reference RLV area | $S_{\mathrm{ref}}$ | $\mathrm{m}{ }^{2}$ | 7.14 | 7.14 | - |
| Reference fin area | $S_{\mathrm{fin}}$ | $\mathrm{m}^{2}$ | 0.54 | 0.54 | - |
| Fin pivot coordinate | $x_{\mathrm{fin}}$ | m | 11.1 | 11.1 | - |
| TVC pivot coordinate | $x_{\mathrm{PVP}}$ | m | 0.96 | 0.96 | - |

### 10.2 Achievable load relief performance

Launcher missions impose a challenging set of requirements to the flight control system [ $\left.\mathrm{SBM}^{+} 16\right]$ :

- Stabilising the vehicle and ensuring adequate gain and phase margins in nominal and dispersed conditions;
- Tracking attitude commands with an error that converges to zero;
- Attenuating induced aerodynamic loads and drift from the reference trajectory;
- Minimising control actuation;
- Rejecting disturbances such as wind gusts and internal dynamics.

These requirements are well-known to be competing: for example, load minimisation involves keeping the angle of attack small by pitching into the wind field, which necessarily causes attitude errors to grow. Hence, a successful control design must be able to exploit the underlying trade-offs while operating as close to the limits of performance as possible. This interplay is illustrated by the Pareto front in Fig. 10.2a. The achievable performance is determined both by the sophistication of the adopted controller and by the physical limitations of the system (e.g. an attitude correction using TVC will always generate drift as a result of its side-force).

Control design becomes even more challenging because the wide flight envelope variation over the mission makes the limits of performance change and requires different control priorities.

For instance, LR is critical in regions of high dynamic pressure but not near touchdown, where the priority is to minimise tracking errors. The evolution of dynamic pressure and Mach number along the trajectory used in this work is shown in Fig. 10.2b. This plot is the same as that of Fig. 8.14, but is repeated here to identify the two flight instances introduced in the previous section. The ascent design point coincides with the launch peak of dynamic pressure and the other one with the recovery $Q_{\alpha}$ peak (recall Fig. 8.16).


Figure 10.2: Main drivers of RLV control requirements

### 10.2.1 Proposed control functionalities

In this chapter, two control functionalities are proposed to enlarge the limits of performance (i.e. illustrated by the shift from the dashed line to the solid line in Fig. 10.2a).

The first functionality consists of augmenting a feedback-only architecture with a channel that provides information of wind disturbances and employ an observer to estimate those disturbances. This allows to enhance wind rejection without compromising nominal stability and tracking properties. The second functionality relies on the use of fins to compensate for the TVC side-force and therefore enabling further improvements in LR. The use of fins is mandatory in descent flight but it is not a common practice during launch. As it will be shown, significant gains (with relatively minor costs) are possible by also using fins in this phase.

To achieve this, consider first the closed-loop block diagram of Fig. 10.3a. This diagram features the guidance subsystem and an LFT block composed of $G_{\mathrm{RLV}}(\mathrm{s})$ and $\Delta_{\mathrm{RLV}}(\mathrm{s})$, introduced before in Sec. 10.1. In addition, a derivative filter $H_{\text {deriv }}(\mathrm{s})$ is included to differentiate pitch angle commands, as well as a Dryden filter $G_{\text {wind }}(\mathrm{s})$ to simulate the frequency content of wind disturbances $v_{\mathrm{w}}$ by colouring a white noise signal $n_{\mathrm{w}}$. The derivative filter is the same used in Sec. 8.3 and the Dryden filter is derived using Eq. (8.7) for the two design points as follows:

$$
\begin{equation*}
G_{\mathrm{wind}}^{\uparrow}(\mathrm{s})=\frac{3.54}{\mathrm{~s}+0.32}, \quad G_{\mathrm{wind}}^{\downarrow}(\mathrm{s})=\frac{2.36}{\mathrm{~s}+0.15} \tag{10.6}
\end{equation*}
$$

where the superscripts $\uparrow$ and $\downarrow$ refer to ascent and descent, respectively.


Figure 10.3: Closed-loop attitude control architectures with LR augmentation
The attitude control loop is closed using a conventional static feedback controller $K_{\text {FB }}$. Comparing Fig. 10.3a with Fig. 8.8, it can be seen that $K_{\mathrm{FB}}$ is already responsible for the control allocation between TVC and fins, and includes an extra drift rate feedback for drift and AoA minimisation (recall Eq. (10.4)). This controller is tuned using structured $\mathcal{H}_{\infty}$ optimisation (Sec. 2.2.2.2) with constraints on the transfer functions $\theta_{\mathrm{ref}} \rightarrow\left\{\theta_{\mathrm{e}}, \theta, \dot{z}, \beta_{\mathrm{TVC}}\right\}$ and $n_{\mathrm{w}} \rightarrow\left\{\theta_{\mathrm{e}}, \alpha\right\}$. The design weights used in the optimisation will be shown later on in Sec. 10.4.

Assuming that perfect wind knowledge is available, then this information could be employed in a feedforward manner, through $K_{\text {LR }}$ (see Fig. 10.3a), to anticipate and compensate for the effect of wind. However, in practice the wind needs to be either measured or estimated. Wind measurements can be obtained with a LiDAR sensor at the expense of additional instrumentation complexity [Mar02, $\left.\mathrm{BSG}^{+} 03\right]$. Alternatively, an observer $L_{\mathrm{w}}(\mathrm{s})$ can be included, see Fig. 10.3b, to provide wind disturbance estimates $\hat{v}_{\mathrm{w}}$. Section 10.3 is dedicated to the design of the observer $L_{\mathrm{w}}(\mathrm{s})$, but the impact of $K_{\mathrm{LR}}$ on the achievable performance is first analysed in Sec. 10.2.2.

### 10.2.2 Impact of wind anticipation

This analysis is based on the ascent design point with no fin control, for which the following (FB-only) controller was designed (note that only 3 gains are used now):

$$
K_{\mathrm{FB}}^{\uparrow}=\left[\begin{array}{ccc}
-5.64 & -1.60 & 4.3 \times 10^{-4}  \tag{10.7}\\
0 & 0 & 0
\end{array}\right]
$$

The impact of $K_{\mathrm{LR}}$ can be appreciated by evaluating the system's response to a step signal in $n_{\mathrm{w}}$. In this case, the states deviate from equilibrium in a monotonic manner, hence their value after a certain interval of time can be employed as a good indicator. The values of the most relevant variables after 6 seconds are shown in Fig. 10.4a using different colours. Here, $K_{\text {LR }}$ represents a single gain from $v_{\mathrm{w}}$ to $\beta_{\mathrm{TVC}}$, ranging from 0 (i.e. FB-only) to -0.015 . In practice, more complex FB controllers can be used, e.g dynamical ones, but to facilitate the understanding this simple case is chosen (especially since the conclusions are not invalidated by this assumption). The figure includes the indicators in nominal conditions $\left(\Delta_{\text {RLV }}=0\right)$ using solid lines and the worst corner-case from the LFT using dashed lines.

(a) Impact of varying $K_{\mathrm{LR}}$


Figure 10.4: Wind step indicators

From Fig. 10.4a, distinct values of $K_{\text {LR }}$ for minimum attitude error, drift rate and AoA (i.e. load) can be identified. These minima are analogous to the well-known Hoelker conditions [Hoe61], but with the key difference that $K_{\text {LR }}$ does not enter the feedback loop and thus nominal stability/tracking properties are not affected. This figure also reflects the performance compromises discussed above.

The performance compromises are then evidenced by the radar plots of Fig. 10.4b to 10.4d, in which the FB-only indicators are compared to those obtained using the various minimum conditions. These radar plots show that each minimum condition optimises the corresponding indicator in both nominal and worst cases at the expense of degrading one of the others. Of special relevance for this work, Fig. 10.4d demonstrates that a direct trade-off between load relief and tracking error minimisation can be achieved.

### 10.3 Robust WDO design

This section is focused on the synthesis of the Wind Disturbance Observer (WDO) $L_{\mathrm{w}}(\mathrm{s})$, introduced in Fig. 10.3b. As depicted in that figure, $L_{\mathrm{w}}(\mathrm{s})$ uses as inputs the feedback variables $\mathbf{y}=\left[\begin{array}{lll}\theta & \dot{\theta} & \dot{z}\end{array}\right]^{\mathrm{T}}$ and the control inputs $\mathbf{u}=\left[\begin{array}{ll}\beta_{\mathrm{TVC}} & \beta_{\mathrm{fin}}\end{array}\right]^{\mathrm{T}}$ to provide wind disturbance estimates $\hat{v}_{\mathrm{w}}$. The ascent point is again adopted for exemplification purposes in this section.

### 10.3.1 Problem formulation

A robust WDO (here termed rWDO) must fulfil the following requirements:

- $\hat{v}_{\mathrm{w}}$ shall tend to zero in the absence of wind so that nominal stability/tracking properties of the system are not altered;
- $\hat{v}_{\mathrm{w}}$ shall tend to $v_{\mathrm{w}}$ so that the wind is accurately estimated
- The former two requirements shall hold for all the allowable control inputs $\mathbf{u}$ and uncertainties $\Delta_{\text {RLV }}$ of Eq. (10.5).

More formally, the design problem consists of finding an observer $L_{\mathrm{w}}(\mathrm{s})$ such that:

$$
\hat{v}_{\mathrm{w}}(\mathrm{~s})=L_{\mathrm{w}}(\mathrm{~s})\left[\begin{array}{l}
\mathbf{y}(\mathrm{s})  \tag{10.8}\\
\mathbf{u}(\mathrm{s})
\end{array}\right] \approx\left\{\begin{array}{ll}
0, & v_{\mathrm{w}}(\mathrm{~s})=0 \\
v_{\mathrm{w}}(\mathrm{~s}), & v_{\mathrm{w}}(\mathrm{~s}) \neq 0
\end{array} \quad \forall\left\{\mathbf{u}(\mathrm{~s}), \Delta_{\mathrm{RLV}}(\mathrm{~s})\right\}\right.
$$

As a tentative solution, it is possible to estimate the wind by solving the equations of motion with respect to $v_{\mathrm{w}}$. For instance, inverting the pitch dynamics (second row of Eq. (10.1)) yields the initial guess:

$$
L_{\mathrm{w}}^{0}(\mathrm{~s})=\frac{V \cos \alpha_{0}}{\mu_{\alpha^{\prime}}}\left[\begin{array}{lll}
\mu_{\alpha^{\prime}} & -\frac{\mathrm{s}}{\sigma_{\mathrm{d}}^{\mathrm{s}}+1}-\frac{l_{\alpha} \mu_{\alpha}+l_{\mathrm{f}} \mu_{\mathrm{f}} \cos \beta_{\mathrm{fn}, 0}}{V \cos \alpha_{0}} & \frac{\mu_{\alpha^{\prime}}}{V \cos \alpha_{0}} \tag{10.9}
\end{array} \Gamma_{\mathrm{ctr}}\right]
$$

in which $\sigma_{\mathrm{d}}=0.05$ seconds is employed to make the differentiation proper and $\Gamma_{\mathrm{ctr}}$ is the control moment effectiveness matrix of Eq. (8.50) without the last column (relative to the cold gas thrusters input). The zero of Eq. (10.9) determines the frequency at which wind anticipation starts.

This approach, however, presents severe robustness problems. To clearly understand them, it is convenient to reformulate the design problem using the robust control framework, see Fig. 10.5.


Figure 10.5: Closed-loop model for rWDO design

The block diagram in Fig. 10.5a is obtained by rearranging the closed-loop interconnections of Fig. 10.3b so that the requirements of Eq. (10.8) can be assessed. This involves adding the output (error) signal $w_{\mathrm{e}}=v_{\mathrm{w}}-\hat{v}_{\mathrm{w}}$ to quantify the observation mismatch. In addition, the impact of guidance commands is accounted for through a new input signal $\mathbf{u}_{\text {pert }}$ acting as a perturbation on the control inputs, while the impact of wind is assessed through the same input $n_{\mathrm{w}}$ as before. It is also noted that the feedback controller $K_{\mathrm{FB}}$ must be included when performing the rWDO design, otherwise $G_{\text {RLV }}(\mathrm{s})$ and the whole closed-loop could not be stabilised.

The input/output singular values of Fig. 10.5a, with $L_{\mathrm{w}}^{0}(\mathrm{~s})$ from Eq. (10.9) as WDO, are depicted in Fig. 10.6 using solid black lines. This figure shows that $L_{\mathrm{w}}^{0}(\mathrm{~s})$ is capable of ensuring a small observation error in the face of wind (right plot), i.e. the solid black line is below 0 dB . But, as seen respectively in the left and mid plots, this is not the case in the presence of TVC and fin perturbations, particularly at frequencies higher than $1 \mathrm{rad} / \mathrm{s}$. On the other hand, at lower frequencies, an unnecessary roll-off is provided in the three channels.

This result indicates that there is potential to improve the observer by reshaping these frequency responses, for which $\mathcal{H}_{\infty}$ optimisation (Sec. 2.2.2.1) is extremely suitable, as shown next.


Figure 10.6: Multi-channel closed-loop singular values and requirements for rWDO design

### 10.3.2 $\mathcal{H}_{\infty}$ WDO synthesis

To tackle this problem, Fig. 10.5a is further rearranged into the interconnections of Fig. 10.5b, in which the same inputs/outputs, uncertainty block $\Delta_{\text {RLV }}(\mathrm{s})$ and WDO $L_{\mathrm{w}}(\mathrm{s})$ can be identified. All the other elements are encapsulated in a generalised plant $P(\mathrm{~s})$. As shown in Fig. 10.5b, this system is input/output weighted by $W_{\mathrm{u}}, W_{\mathrm{n}}$ and $W_{\mathrm{e}}(\mathrm{s})$, which specify the desired design requirements, forming the augmented plant $M(\mathrm{~s})$. Similar to Sec. 2.2.2.1, the $\mathcal{H}_{\infty}$ problem consists of finding an observer $L_{\mathrm{w}}^{*}(\mathrm{~s})$ that minimises the $\mathcal{H}_{\infty}$-norm of $M(\mathrm{~s})$ :

$$
\begin{equation*}
\min _{L_{\mathrm{w}}(\mathrm{~s})} \gamma=\|M(\mathrm{~s})\|_{\infty} \tag{10.10}
\end{equation*}
$$

In practice, $W_{\mathrm{u}}$ and $W_{\mathrm{n}}$ are applied to define the expected range of $\mathbf{u}_{\text {pert }}$ and $n_{\mathrm{w}}$, and $W_{\mathrm{e}}(\mathrm{s})$ normalises the system so that the Maximum Singular Value (MSV) of $M(\mathrm{~s})$ is bounded by $W_{\mathrm{e}}^{-1}(\mathrm{~s})$ if $\gamma<1$.

Expected TVC and fin deflections are set to 1 and 20 degrees, respectively, based on the results of Fig. 8.14. The range of $n_{\mathrm{w}}$ is defined to 3 to account for $3 \sigma$ wind dispersions. Furthermore, the input weights $W_{\mathrm{u}}$ and $W_{\mathrm{n}}$ are constant so as to minimise the order of the system, but frequency-dependent functions can also be considered. Hence, they are given by:

$$
W_{\mathrm{u}}=\frac{\pi}{180}\left[\begin{array}{cc}
1 & 0  \tag{10.11}\\
0 & 20
\end{array}\right], \quad W_{\mathrm{n}}=3
$$

The advantage of having the initial guess $L_{\mathrm{w}}^{0}(\mathrm{~s})$ is related to the insight it provides when choosing a reasonable $W_{\mathrm{e}}^{-1}(\mathrm{~s})$ specification (see Fig. 10.6). More specifically, $W_{\mathrm{e}}(\mathrm{s})$ is tailored to have: [i] a maximum value of $1 / 0.6$ so that the worst-case observation error is approximately the same as $L_{\mathrm{w}}^{0}(\mathrm{~s})$, [ii] a zero at $80 \mathrm{rad} / \mathrm{s}$ so that the observation bandwidth is also the same, and [iii] a pole $10^{4}$ times faster to make the transfer function proper. Using the definition of Eq. (5.4), these considerations lead to the following weight:

$$
\begin{equation*}
W_{\mathrm{e}}(\mathrm{~s})=\frac{1}{0.6} \frac{\mathrm{~s}+80}{\mathrm{~s}+80 \times 10^{4}} \times 10^{4} \tag{10.12}
\end{equation*}
$$

With the above choice of weights, the $\mathcal{H}_{\infty}$ synthesis problem yielded a stable 5th-order observer with the fastest pole below $90 \mathrm{rad} / \mathrm{s}$ and an optimal performance of $\gamma=0.43$. The frequency-dependent requirement imposed by $W_{\mathrm{e}}^{-1}(\mathrm{~s})$ and the singular values attained with the latter observer are included in Fig. 10.6 using black dashed and red lines.

Comparing the $\mathcal{H}_{\infty}$ design with the initial guess (the solid black lines in Fig. 10.6), it is clear that the optimised observer is able to effectively reject input perturbations (particularly at higher frequencies) at the expense of a larger steady-state error, which is bounded to $W_{\mathrm{e}}^{-1}(0) / W_{\mathrm{n}}=20 \%$. To verify whether this behaviour holds in the face of launcher uncertainties, $\mu$ analysis (see Sec. 2.2.3.1) can now be employed.

The top plot of Fig. 10.7 depicts the bounds of $\mu$ for the system of Fig. 10.3b with $K_{\mathrm{LR}}^{*}=-1.04 \times 10^{-2}$ for load minimisation (recall Fig. 10.4a) and with the $\mathcal{H}_{\infty}$-designed observer in red (solid line for upper bound and dashed for lower bound). The $\mu$ sensitivities are shown in the bottom plot using a different colour for each uncertainty of Table 10.1. In the top plot, a peak of $\mu>1$ indicating lack of Robust Stability (RS) can be seen for the $\mathcal{H}_{\infty}$ design. From the bottom plot, it is possible to recognise the thrust uncertainty $\left(\delta T \cos \beta_{\mathrm{TVC}, 0}\right)$ as the main responsible for this degradation.


Figure 10.7: Robust stability upper and lower bounds (top plot) and sensitivities (bottom plot)

### 10.3.3 $D-K$ iteration WDO synthesis

In this subsection, a new observer is designed using the $D-K$ iteration algorithm to prevent the lack of RS. By employing a combination of $\mathcal{H}_{\infty}$ synthesis and $\mu$ analysis (recall Sec. 2.2.2.1), the $D-K$ iteration allows to recover the loss of robustness introduced by the $\mathcal{H}_{\infty}$-designed observer. The weight functions are kept the same as for the latter design, but critically, only the thrust uncertainty is considered in the LFT model used for $D-K$ synthesis. This is done in order to minimise the complexity of the problem driven by the $\mu$ analysis results.

Following this approach, the $D-K$ synthesis problem yielded an observer of the same order as before, with the fastest pole also below $90 \mathrm{rad} / \mathrm{s}$ and with an optimal performance of $\gamma=0.58$. The bounds of $\mu$ obtained with this new observer are also shown in the top plot of Fig. 10.7 (blue lines), indicating that $\mu$ is now smaller than 1 for all the frequencies. This means that accounting for the impact of thrust changes in the synthesis process is enough to design an observer that is robust in the face of all the uncertainties $\Delta_{\mathrm{RLV}}(\mathrm{s})$.

To clarify how the three WDOs ( $L_{\mathrm{w}}^{0}(\mathrm{~s})$, the $\mathcal{H}_{\infty}$ and $D-K$ designs) differ from each other, their MSV are provided in Fig. 10.8a and the closed-loop pole dispersion generated with 500 random $\Delta_{\text {RLV }}(\mathrm{s})$ samples are depicted in Fig. 10.8b.


Figure 10.8: WDO design evolution in the frequency-domain
The initial guess $L_{\mathrm{w}}^{0}(\mathrm{~s})$ provides the desired wind anticipation action between 1 and $100 \mathrm{rad} / \mathrm{s}$ (Fig. 10.8a), but results in most configurations being unstable (notice the black positive poles in Fig. 10.8b). The $\mathcal{H}_{\infty}$-designed WDO (in red) keeps the steady-state gain of the initial guess (see Fig. 10.8a) but adds a high-frequency roll-off that enables the rejection of input perturbations, at the expense of a higher order system. Although it stabilises most of the cases, a few of them remain unstable, as observed in Fig. 10.8b and identified by the peak of $\mu>1$ in Fig. 10.7.

A robust disturbance observation is only ensured by the $D-K$ design (blue colour), which effectively shrinks the pole dispersion to remain inside the left complex half-plane. This is
achieved by relying less on low-frequency information and more on higher frequencies (as indicated by the arrows in Fig. 10.8a), while maintaining the roll-off of the $\mathcal{H}_{\infty}$ design. The reduced confidence in low-frequency information when the thrust uncertainty is considered for the $D-K$ design is consistent with the fact that this variable determines mostly the slower dynamics of the system $\left[\mathrm{SBM}^{+} 16\right]$. The application of the rWDO design approach to ascent and descent flight is further detailed in Sec. 10.4 and 10.5, respectively.

### 10.4 Application to ascent flight

The application and benefits of the functionalities proposed in Sec. 10.2.1 for ascent flight are demonstrated in this section. To do so, four controllers are designed following different strategies combining the two proposed functionalities (i.e. the rWDO and the use of fins):

1. In black, the TVC-only controller with the 3 non-zero gains of Eq. (10.7) and using $K_{\mathrm{LR}}^{\uparrow}=\left[\begin{array}{ll}0 & 0\end{array}\right]^{\mathrm{T}}$, i.e. with FB-only control.
2. In blue, a TVC-only controller augmented with the rWDO designed (using the $D-K$ iteration) in Sec. 10.3.3, in which $K_{\mathrm{FB}}^{\uparrow}$ and $K_{\mathrm{LR}}^{\uparrow}$ are jointly tuned using structured $\mathcal{H}_{\infty}$ optimisation. This is achieved by employing a tighter tuning requirement on the $n_{\mathrm{w}} \rightarrow \alpha$ channel (and dropping the $n_{\mathrm{w}} \rightarrow \theta_{\mathrm{e}}$ requirement). The feasibility of applying this tighter weight could be identified thanks to the LR performance analysis of Sec. 10.2.2. The LR controller adds 1 extra gain ( $\hat{v}_{\mathrm{w}}$ to $\beta_{\mathrm{TVC}}$ ) for a total of 4 gains to be tuned.
3. In red, a combined fins/TVC controller with $K_{\mathrm{LR}}^{\uparrow}=\left[\begin{array}{ll}0 & 0\end{array}\right]^{\mathrm{T}}$ and with $K_{\mathrm{FB}}^{\uparrow}$ tuned using structured $\mathcal{H}_{\infty}$ optimisation. As anticipated in Sec. 10.2.1, fins can be used for load minimisation by counteracting the TVC side-force, which is enforced by augmenting the tuning requirements of the first controller with new constraints on the $\theta_{\text {ref }} \rightarrow\left\{\alpha, \beta_{\text {fin }}\right\}$ channels. The FB controller with combined fins/TVC has 6 gains to be tuned (twice as the first controller).
4. In green, a combined fins/TVC controller augmented with the rWDO designed in Sec. 10.3.3, in which $K_{\mathrm{FB}}^{\uparrow}$ and $K_{\mathrm{LR}}^{\uparrow}$ are jointly tuned using structured $\mathcal{H}_{\infty}$ optimisation. Representing a combination of the last two strategies, this controller is generated using their respective $n_{\mathrm{w}} \rightarrow\left\{\alpha, \beta_{\text {fin }}\right\}$ and $\theta_{\text {ref }} \rightarrow\left\{\alpha, \beta_{\text {fin }}\right\}$ requirements concurrently. The LR controller adds 2 extra gains ( $\hat{v}_{\mathrm{w}}$ to $\beta_{\mathrm{TVC}}$ and to $\beta_{\mathrm{fin}}$ ) to be tuned. The final gains of this controller are the following:

$$
K_{\mathrm{FB}}^{\uparrow}=\left[\begin{array}{ccc}
-5.76 & -2.72 & 4.8 \times 10^{-4}  \tag{10.13}\\
-0.46 & 0.26 & 9.8 \times 10^{-4}
\end{array}\right], \quad K_{\mathrm{LR}}^{\uparrow}=-\left[\begin{array}{c}
9.75 \\
0.24
\end{array}\right] \times 10^{-3}
$$

The indicated controller colours are used in the figures of Sec. 10.4.1 and 10.4.2, where comparative analyses of frequency and time-domain properties are provided. The outcome of this
comparison and the number of tunable gains is then summarised in Table 10.2 and a simple verification using the nonlinear benchmark is shown in Sec. 10.4.3.

Before that, in order to clarify how load relief is influenced by the tuning requirements mentioned above, Fig. 10.9 shows the closed-loop singular values obtained with the first two controllers (black and blue), together with the corresponding design weights in dashed and solid magenta lines, respectively.


Figure 10.9: Multi-channel closed-loop singular values and requirements for ascent control design without fins. Note that, in the left column, the dashed and solid magenta lines are overlapped, as well as the black and blue lines.

The improved load relief capability of the blue controller (compared to the black one) is then verified by the reduction of the maximum singular value of the $n_{\mathrm{w}} \rightarrow \alpha$ channel (fourth plot on the right-hand side of Fig. 10.9) at the expense of an increased steady-state attitude error (first plot on the right-hand side). As expected, these changes do not impact the tracking response, which is confirmed by the overlapped black and blue lines on the left-hand side of the figure.

### 10.4.1 Frequency-domain analysis

In the frequency-domain, the controllers are compared in nominal conditions using Nichols charts and in the face of uncertainties using $\mu$ analysis (Sec. 2.2.3). Since Nichols chart analysis is a Single-Input Single-Output (SISO) tool, the closed-loop of Fig. 10.3b is first transformed as depicted in Fig. 10.10. This single-loop transformation uses the pitch control moment
$m_{\text {ctr }}=\Gamma_{\text {ctr }} \mathbf{u}$. The obtained Nichols charts are provided in Fig. 10.11a, where the resulting Gain Margin (GM) and Phase Margin (PM) are also indicated. The outcome of $\mu$ analysis is presented in Fig. 10.11b and is equivalent to that of the top plot of Fig. 10.7.


Figure 10.10: Loop transformation for Nichols chart generation


Figure 10.11: Frequency-domain comparison of ascent control functionalities

The first two controllers (both without including fin control, and respectively without and with rWDO) have the same gain and phase margins, as their responses overlap in Fig. 10.11a. This is because the observation error is zero in nominal conditions and the rWDO does not affect closed-loop stability. This is no longer the case in the face of uncertainties, as illustrated in Fig. 10.11b, where the presence of the rWDO in the second controller leads to an RS degradation (particularly around PM frequencies) due to delay added by the observer for wind estimation.

This blue plot is the same as that of Fig. 10.7, and is repeated here for comparison of designs. Despite the RS degradation, the peak of $\mu=0.53$ indicates that a very reasonable robustness margin is guaranteed to cover for effects that are not accounted for by the model (e.g. actuator and bending dynamics). This degradation is expected since the introduction of the WDO leads to a performance improvement that must be traded-off with robustness. Nonetheless, since the
$\mu$ peak is still about $50 \%$ from the limit, this shows that improvements can be achieved with these techniques (without an unacceptable loss of robustness).

The red controller (fin control, without rWDO) yields a completely different frequency response, with smaller GM but larger PM, as shown by the Nichols chart. As it will be seen in Sec. 10.4.2, these differences are reflected in an improved command tracking response. In terms of RS, $\mu$ analysis identifies a slight degradation with respect to the TVC-only controller (in black), but much larger margins compared to the one without fins but with rWDO (in blue).

Finally, the inclusion of the rWDO with the fins (green lines) leads to smaller phase and RS margins. Nonetheless, the rWDO is critical for LR (as it will be shown) and both phase and RS margins obtained with rWDO using fin control are superior than without fins (compare the green vs. blue lines).

### 10.4.2 Time-domain analysis

In the time-domain, the four closed-loop LTI systems are subjected to three different tests: [i] a 0.5 deg step command in $\theta_{\text {ref }}$, [ii] a low-frequency, high-amplitude gust in $v_{\mathrm{w}}$ (maximum peak of $33 \mathrm{~m} / \mathrm{s}$ ), with a shape that is often used in the validation of launcher control systems, and [iii] high-frequency, low-amplitude gusts, generated by 100 white noise signals in $n_{\mathrm{w}}$ with different seeds. Each test is repeated for the $2^{10}$ corner-cases covering the 10 parametric uncertainties. Results of the three tests are provided in Fig. 10.12, which depict both nominal and dispersed responses of $\theta, \dot{z}, \alpha, \beta_{\mathrm{TVC}}, \beta_{\text {fin }}$ (from top to bottom), and observation error $w_{\mathrm{e}}$ when applicable. The latter plot also shows the actual wind gust $v_{\mathrm{w}}$ in purple.

In terms of command tracking (Fig. 10.12a), the first two controllers exhibit similar responses, and the main difference is a reduction of drift rate dispersion at the expense of higher TVC actuation with the blue controller. This small difference is caused by the rWDO, whose error is zero in nominal conditions but grows in the presence of uncertainties (see last row of plots). The benefits of the rWDO become more evident in the face of both Low-Frequency (LF) and High-Frequency (HF) wind gusts (Fig. 10.12b and 10.12c), where the induced load/AoA (third row) and drift (second row) are drastically minimised at the expense of a larger pitch angle (first row), as predicted in Sec. 10.2.2. In the case of HF gusts, the load dispersion is alleviated by approximately $50 \%$. The adequate operation of the rWDO is also confirmed by noting that the observation errors are significantly smaller than the actual wind, particularly for the LF gust.

With the introduction of fin control (red controller), the tracking response improves profoundly as the drift caused by the TVC side-force is effectively compensated (second row of Fig. 10.12a), which minimises the overshoot in pitch and AoA. This compensation is also useful for wind rejection as it reduces its impact on pitch and drift (first two rows of Fig. 10.12b and 10.12 c ), but the induced AoA remains equivalent to the first controller (no fin control, without rWDO). The load relief capabilities of the blue controller (no fin control, with rWDO) are only recovered when the rWDO is employed in combination with TVC and fins (green
controller). These two controllers have very similar AoA, pitch and observation error responses in the presence of wind. However, the latter controller sees the required TVC actuation reduced (fourth row) by using minimal fin deflections (barely noticeable on the fifth row). This comes at the expense of larger tracking overshoots (Fig. 10.12a) compared to the red controller, but not as large as with the blue controller.
——No fin control, FB-only —— No fin control, rWDO-LR ——With fin control, FB-only ——With fin control, rWDO-LR


Figure 10.12: MC comparison of ascent control functionalities using a step command, LF and HF wind fields

Table 10.2: Overview of ascent control functionalities. Legend:
++ Major improvement, + Minor improvement, - Minor degradation, -- Major degradation

| Fin <br> control | rWDO <br> load relief | Tunable <br> gains | Phase <br> margin | Robust <br> stability | Command <br> tracking | Wind <br> rejection |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No | No | 3 | Baseline for comparison |  |  |  |  |
| No | Yes | $3+1$ |  | -- |  | + |  |
| Yes | No | 6 | ++ |  | ++ |  |  |
| Yes | Yes | $6+2$ | + | - | + | ++ |  |

Table 10.2 provides an overview of the observations made throughout this section. The main objective of improving wind rejection while also enhancing phase margin and command tracking is effectively achieved with the introduction of fin control and wind disturbance observation, at the expense of a small decrease of RS. The insight provided by $\mu$ analysis is key when specifying the rWDO requirements - smaller observation errors will require higher observation gains in the system (recall Fig. 10.8a), which will result in reduced RS. Nonetheless, as demonstrated, if the observer design is performed appropriately, this degradation is still within acceptable bounds.

### 10.4.3 Verification and discussion

In order to verify the achievable performance using the nonlinear RLV simulator, a simple Monte-Carlo (MC) campaign of the ascent phase with 100 severe wind seeds was carried out. The $Q_{\alpha}$ and attitude error responses produced with and without wind anticipation for pitch and yaw control are compared in Fig. 10.13. The MC $Q_{\alpha}$ check of Fig. 10.13a is a widely-employed procedure to verify whether the structural integrity limits of the vehicle are not exceeded.


Figure 10.13: Ascent $Q_{\alpha}$ and attitude errors of 100 MC runs with severe wind and LR active between [30, 70] s

The black lines correspond to results obtained with a TVC and FB-only controller (i.e. as the black controller before) that was tuned and scheduled for distinct points along the launch trajectory. In blue, the same controller is augmented with the LR gain between [30, 70] seconds area (where the dynamic pressure is higher), which was tuned as in Sec. 10.2.2 for the various trajectory points. The achievable performance of the blue controller becomes clear from Fig. 10.13a, with a $Q_{\alpha}$ reduction of more than $50 \%$ when the LR augmentation is active. This improvement comes at the expense of much larger pitch and yaw errors, Fig. 10.13b, although they remain bounded during the same period.

In practice, as discussed throughout the present chapter, a successful control tuning will most likely rely on a more balanced compromise between $Q_{\alpha}$ and error minimisation.

### 10.5 Application to descent flight

In this section, the generalisation of the proposed wind disturbance observation to the descent design point is showcased through the following strategies:

1. In black, a combined fins/TVC controller with $K_{\mathrm{LR}}^{\downarrow}=\left[\begin{array}{ll}0 & 0\end{array}\right]^{\mathrm{T}}$ and with $K_{\mathrm{FB}}^{\downarrow}$ tuned using structured $\mathcal{H}_{\infty}$ optimisation. Using constraints on the $\theta_{\text {ref }} \rightarrow\left\{\theta_{\mathrm{e}}, \theta, \beta_{\mathrm{TVC}}, \beta_{\mathrm{fin}}\right\}$ and $n_{\mathrm{w}} \rightarrow\left\{\theta_{\mathrm{e}}, \alpha\right\}$ channels, the following FB-only controller is obtained:

$$
K_{\mathrm{FB}}^{\downarrow}=\left[\begin{array}{ccc}
-0.44 & -3.0 \times 10^{-3} & 2.5 \times 10^{-3}  \tag{10.14}\\
1.28 & 1.31 & 3.9 \times 10^{-3}
\end{array}\right]
$$

2. In green, the controller of Eq. (10.14) augmented with a rWDO, which is generated by repeating the procedure of Sec. 10.3 for the descent design point. In opposition to the ascent case, the $\mathcal{H}_{\infty}$ rWDO design (Sec. 10.3.2) proved to be robust enough since the thrust level is much smaller (recall Table 10.1) and therefore no $D-K$ iteration was necessary. $K_{\mathrm{LR}}^{\downarrow}$ is chosen, as in Sec. 10.2.2, to be a single gain from $\hat{v}_{\mathrm{w}}$ to $\beta_{\text {fin }}$ that minimises the attitude tracking error. In opposition to Sec. 10.4 , no joint tuning of $K_{\mathrm{FB}}^{\downarrow}$ and $K_{\mathrm{LR}}^{\downarrow}$ is performed in this section.
3. In blue, a controller similar to the previous one, but in which $K_{\mathrm{LR}}^{\downarrow}$ is chosen to minimise the aerodynamic load.

The frequency and time-domain analyses of the three descent controllers are presented in Sec. 10.5.1 and 10.5.2, respectively.

### 10.5.1 Frequency-domain analysis

Having the same feedback controller $K_{\mathrm{FB}}^{\downarrow}$, the three descent designs exhibit the same behaviour in nominal conditions, hence their frequency-domain comparison is only carried out in terms of $\mu$ analysis. The outcome of this analysis is provided in Fig. 10.14.


Figure 10.14: Robust stability upper and lower bounds of descent control functionalities

The $\mu$ analysis shows that, similar to Sec. 10.4.1, the introduction of the rWDO leads to an RS degradation when load is minimised (blue controller), but to an improvement when error is minimised (green controller). This is consistent with the previous observation that a better tracking performance is tightly related to better stability margins. Compared to the ascent case, the peaks of degradation are similar (maximum is $\mu=0.49$ ), but take place for lower frequencies due to a value of dynamic pressure $85 \%$ smaller at the descent design point (recall Fig. 10.2b).

### 10.5.2 Time-domain analysis

The time-domain verification of the descent controllers is exactly the same as described in Sec. 10.4.2. Results of the three tests in nominal and dispersed conditions are depicted in Fig. 10.15.

Focusing on command tracking first (Fig. 10.15a), the three cases have the same nominal response, but dispersions are smaller with the minimum error controller (green) and larger with the minimum load controller (blue), as anticipated in Sec. 10.5.1. In descent flight, the controllers prioritise the use of fins with respect to TVC, which is the same for the three cases as the rWDO only acts on the fin channel. The level of rWDO errors (last row) is similar to the ascent case during the three tests, since its design requirements were kept the same.

The trade-off between error and load alleviation using disturbance observation then becomes evident with the LF and HF wind gusts (Fig. 10.15b and 10.15c, respectively). Both figures confirm that, compared to the FB-only controller, pitch and drift errors are minimised by the green controller at the expense of larger AoA deviations in nominal and dispersed conditions, and vice-versa for the blue controller. As an example, following the $33 \mathrm{~m} / \mathrm{s}$ gust (Fig. 10.15b), the latter controller effectively brings the steady-state AoA from 2.4 deg to zero (third row) while only worsening the pitch angle from -1.3 to -3.2 deg (first row).

The above observations highlight the benefits of the proposed WDO augmentation, introduced in Sec. 10.2. That is, control priorities can be adjusted along the mission via $K_{\text {LR }}$ (e.g. load relief at high dynamic pressure, minimum error near touchdown), which does not affect the system's nominal stability/tracking properties and therefore does not require retuning the feedback loop.


Figure 10.15: MC comparison of descent control functionalities using a step command, LF and HF wind fields


## Analysing the Impact of Aeroelastic Loads

Up to the present chapter, Guidance \& Control (G\&C) analyses have been carried out assuming (time-varying) rigid-body and point-mass loads, which means that local angles of attack and aerodynamic loads are uniform along the vehicle. In fact, flight mechanics and aeroelasticity are often treated separately in the traditional Verification \& Validation (V\&V) approach. However, as the focus on weight minimisation of launchers tends to lead towards more flexible structures, the frequency separation between flexible and rigid-mode motion becomes smaller and the above approach looses accuracy.

Hence, accurate load prediction requires accounting for the combined impact of distributed aerodynamics and structural flexibility. An accurate prediction is critical not only to ensure the structural integrity of the launcher, but also to enable the development of more sophisticated load relief systems.

In this thesis, aeroelastic effects are included using a modeling approach known as MultiBody (MB) dynamics. As it will be detailed in Sec. 11.1, aeroelastic modelling via MB is particularly valuable because it enables an integrated multi-physics approach and allows: [i] more accurate load predictions in the preliminary design phase, and [ii] faster design iterations between trajectory, structure and flight control. Following the motivation to this type of approach, Sec. 11.2 describes the development of the Reusable Launch Vehicle (RLV) MB model, while Sec. 11.3 concludes the chapter with a detailed aeroelastic load analysis.

### 11.1 Aeroelastic modelling via multibody dynamics

Although aeroelasticity of stationary wings is considered a mature research topic, the field of aeroelasticity in general is still evolving [Kal14, IML18, ISM19].

The most common and simple way to capture structural flexibility effects in flight mechanics models for control design and analysis is to augment the equations of motion (Sec. 8.1.2) with elastic degrees-of-freedom (see [OJWM09, MG16] and references therein). This can be achieved via modal superposition, where it is assumed that elastic deflections can be represented as a linear combination of the $k$ most impactful orthogonal vibration modes.

In this case, a vector $\mathbf{s}(x, t)$ containing the translation of a set of nodes along the RLV longitudinal axis and a vector $\xi(x, t)$ with the corresponding rotations are written as:

$$
\begin{align*}
& \mathbf{s}(x, t)=\Phi(x) \eta(t)  \tag{11.1}\\
& \xi(x, t)=\dot{\Phi}(x) \eta(t) \tag{11.2}
\end{align*}
$$

where $\Phi(x)$ is a matrix consisting of modal column shape vectors, $\dot{\Phi}(x)$ its spatial derivatives and $\eta(t)$ is a vector with the associated generalised coordinates. Since the modes are orthogonal, modal mass and stiffness are defined by diagonal matrices $\bar{M}_{\mathrm{s}}$ and $\bar{K}_{\mathrm{s}}$, which can be related to the modal frequency $\omega_{\mathrm{f}}$ through Rayleight's quotient $\omega_{\mathrm{f}}^{2}=\bar{K}_{\mathrm{s}} \bar{M}_{\mathrm{s}}^{-1}$. In addition, it is common practice to model structural damping as a diagonal fraction $\zeta_{f}$ of the stiffness and, following this assumption, the dynamics of $\eta(t)$ are described by:

$$
\begin{equation*}
\ddot{\eta}(t)+2 \zeta_{\mathrm{f}} \omega_{\mathrm{f}} \dot{\eta}(t)+\omega_{\mathrm{f}}^{2} \eta(t)=\bar{M}_{\mathrm{s}}^{-1} \Phi^{\mathrm{T}}(x) F_{\mathrm{s}}(t) \tag{11.3}
\end{equation*}
$$

The shape vectors contained in $\Phi(x)$ correspond to the nodal displacement eigenvectors of the above equation and $F_{\mathrm{s}}(t)$ represents the force excitation applied to the structure.

In practice, $\bar{M}_{\mathrm{s}}, \bar{K}_{\mathrm{s}}$ and $\Phi(x)$ are time-varying because they depend on the amount of propellant at each instant of time. They can be computed offline (for different amounts of propellant) using Finite Element Analysis (FEA) software or analytically under a few simplifying assumptions $\left[\mathrm{SKC}^{+} 17\right]$. As an illustration of the former, the normalised shapes of the first three bending modes of the full RLV with empty tanks are depicted in Fig. 11.1.

An example of the modal superposition approach is available in $\left[\mathrm{SBM}^{+} 16, \mathrm{NMS}^{+} 19\right]$ and a more evolved derivation featuring the coupling between flexibility and aerodynamics (as well as propellant sloshing) can be found in [Orr10, Orr11]. Essentially, in the aeroelastic case, the local Angle of Attack (AoA) at a certain node $j$ in the launcher's body is given by:

$$
\begin{equation*}
\alpha^{j}=\theta+\frac{\dot{z}-v_{\mathrm{w}}}{V \cos \alpha_{0}}-\frac{l_{j}}{V \cos \alpha_{0}} \dot{\theta}+\sum_{i=1}^{k}\left(\dot{\phi}_{i j} \eta_{i}-\frac{1}{V \cos \alpha_{0}} \phi_{i j} \dot{\eta}_{i}\right) \tag{11.4}
\end{equation*}
$$

where the first two terms correspond to Eq. (10.4), the third term accounts for the pitch damping effect and the last term describes the aeroelastic coupling. The pitch damping is a non-steady low-frequency aerodynamic effect (due to $\dot{\theta}$ ) which is often neglected in preliminary studies (recall Sec. 8.3). In the aeroelastic term, $\eta_{i}$ and $\dot{\eta}_{i}$ represent the modal coordinate and time-derivative of mode $i$, and $\phi_{i j}$ and $\dot{\phi}_{i j}$ represent the normalised displacement and slope of mode $i$ at node $j$ (recall Eq. (11.1)).


Figure 11.1: First three normalised bending modes of the full vehicle with empty tanks

The most accurate aeroelastic modelling approach, on the other hand, is to complement a dedicated flight mechanics simulator with high-fidelity FEA and Computational Fluid Dynamics (CFD) tools. Although this approach yields very accurate results, it is extremely demanding and involves large models associated with very high computational times [VS11]. Hence, this approach is not compatible with the vehicle's preliminary design phase (where various choices of shape and subsystems have not yet been fixed) nor with complicated iterations between highly interconnected disciplines (recall Fig. 1.5).

In some cases, e.g. [KS04] for aircraft and [TMW09] for launchers, the integration between the different disciplines relies on a MultiBody modelling system, since large flexible structures can be effectively modelled as distributed systems [MG16]. However, the inherent properties of the MB approach can also be employed for the development of solutions that allow to largely reduce computational times without a significant compromise in terms of accuracy. This is achieved by replacing the CFD solver with simpler aerodynamic models, such as the quasi-steady model of Eq. (11.4), and by relying on MB dynamics approximations to avoid extensive FEA simulations.

MB dynamics studies require a very rigorous theoretical understanding (see, e.g. [HR13]) but, once the theoretical formalism is understood, it can be very efficiently implemented using physical modelling software suites such as MODELICA or MATLAB's Simscape Multibody (formerly SimMechanics), in which degrees-of-freedom are introduced via an abstraction to joints and frame transforms. In contrast to causal signal-based approaches (such as Simulink),
these toolboxes represent subsystems using non-causal connections where an energy balance is applied and described by a set of hybrid differential algebraic equations [Loo08]. These equations can then be self-mapped into explicit ordinary differential equations and subsequently solved numerically.

Thanks to their physical description, these tools offer the possibility to [i] make quick changes in model configuration and instrumentation, [ii] be interfaced with different physical domains (such as electrical, hydraulic and power), and [iii] provide dedicated visualisation libraries for bodies and motions. All these features are of vital importance to achieve a reduction of the launcher design efforts mentioned above, i.e. by integrating distinct domains in a single multiphysics framework that can be quickly built and analysed, the number of iterations between discipline experts using their own dedicated tools can potentially be reduced.

Notable multibody developments in the literature include: [Loo08] (aircraft modelling using MODELICA), [Acq16] (launcher modelling using MODELICA) and [Kal14] (aircraft modelling using Simscape Multibody). In this chapter a MB launcher model is developed using Simscape Multibody. For further information on Simscape functions or modelling principles, the reader is referred to [Mat19].

Following this approach, the inhomogeneous structure of the launcher is discretised into a number of simple beam elements with distinct geometrical and structural properties and connected by joints, as illustrated in Fig. 11.2. The dynamics of the joints are chosen to reflect the characteristics main flexible modes of the launcher, as it will be detailed throughout the next section.


Figure 11.2: Multibody discretisation of the flexible vehicle structure

The MB model will be equivalent to $G_{\text {RLV }}(\mathrm{s})$ from Chapter 10, but it will account for structural flexibility and its coupling with distributed aerodynamics. Hence, it is based on the following assumptions: [i] launcher is axisymmetric so that only pitch plane and in-plane bending motions are considered (i.e. the other degrees-of-freedom are constrained), [ii] aerodynamics are computed using the quasi-steady model of Eq. (11.4) and [iii] trajectory-related (velocity, dynamic pressure, etc.) and Mass, CG \& Inertia (MCI) conditions are frozen in time (same as
$\left.G_{\text {RLV }}(\mathrm{s})\right)$ to avoid the added complexity of having to interpolate the beam properties [TMW09]. As before, the following examination focuses on the state of the vehicle at the point of maximum launch dynamic pressure, which is imported from the RLV benchmark.

### 11.2 Multibody model development

The architecture of the MB RLV model is schematised in Fig. 11.3 and its building blocks are introduced below. In this figure, the green lines indicate Simscape's physical modelling interconnections, while the black lines represent Simulink's interface signals. The latter signals correspond to the control input $\mathbf{u}=\left[\begin{array}{ll}\beta_{\mathrm{TVC}} & \beta_{\mathrm{fin}}\end{array}\right]^{\mathrm{T}}$, wind disturbance $v_{\mathrm{w}}$ and output vector $\mathbf{y}=\left[\begin{array}{lllll}\theta & \dot{\theta} & z & \dot{z} & \alpha\end{array}\right]^{\mathrm{T}}$ of $G_{\mathrm{RLV}}(\mathrm{s})$ from Chapter 10.


Figure 11.3: Multibody model interconnections
Referring to Fig. 11.3, the "Sensor at CG" subsystem determines the values of $\mathbf{y}$ at the vehicle's Centre of Gravity (CG) with respect to a local trajectory frame, $\left\{x_{L}, z_{L}\right\}$ in Fig. 10.1, in body-fixed coordinates. The local frame is defined in the "Frames" block based on the trim
pitch angle. This frame is also employed by the "Rigid-body motion" and "Gravity force" blocks. The latter block computes the gravity component normal to the body axis and injects it as an external force at the CG. The rigid-body motion is then simulated by one revolute joint and one prismatic joint at the CG, respectively providing degrees-of-freedom for pitch and drift.

In addition, attached to the CG, there is the "RLV Structure" subsystem. This is the most complex subsystem because it is responsible for distributed aerodynamics and structural computations and will be treated in more detail during the following subsections. As it will be seen, the former computations require the knowledge of $v_{\mathrm{w}}$ and of the local frame. Through a series of rigid transforms, attachments for fins and TVC Pivot Point (PVP) are also defined in the structure.

Thrust and fin forces are then injected at the corresponding attachment points. While the thrust force depends only on $\beta_{\mathrm{TVC}}$ and $T \cos \beta_{\mathrm{TVC}, 0}$ (recall Sec. 10.1), the fin force depends on $\beta_{\mathrm{fin}}$ and $C_{\mathrm{f}_{\alpha}} \cos \beta_{\mathrm{fin}, 0}$, as well as on the local aerodynamics at the fin's location. Local aerodynamics calculations will be further discussed in Sec. 11.2.1.

The control deflections $\beta_{\mathrm{TVC}}$ and $\beta_{\mathrm{fin}}$ are also used to drive the Thrust Vector Control (TVC) and fin motion through independent revolute joints in the associated subsystems of Fig. 11.3. TVC motion is modelled using a truncated cone in the "Nozzle" block, which is assumed to have no mass in order to be consistent with the negligible moving masses assumption of Sec. 8.1. Equivalently, the fins are modelled by four massless elliptical lofts, of which only two are actuated for pitch control (recall Sec. 8.3). Since all these elements are massless, they do not affect the dynamics of the system, but they are extremely useful for visualisation purposes, see Fig. 11.2.

Coming back to Fig. 11.2, each one of the vehicle's beam element contributes to its aerodynamics in a distributed manner. The implementation of this distributed RLV structure is described next.

### 11.2.1 Implementation of distributed aerodynamics

The internal architecture of a single beam element $j$ is depicted in Fig. 11.4 and divided into MCI (on the top) and aerodynamics components (on the bottom).

As shown in the figure, the MCI components include a rigid-body block and two translations defining the location of the element's CG with respect to the adjacent elements' interfaces $j-1$ and $j+1$. The rigid-body block allows to specify the shape and MCI properties of the beam element. While the former is simply chosen so as to meet the geometry of Fig. 11.2, the definition of local MCI properties will be detailed in Sec. 11.2.2.

If such elements are connected in series through the red dashed lines without any degree-of-freedom in-between, a distributed rigid-body structure is obtained. The distributed forces applied in this structure are kept in dynamic equilibrium by its internal forces (e.g. shear, axial, torsion and bending moment), which are directly computed by Simscape.


Figure 11.4: Beam element architecture. The orange blocks represent Simulink-Simscape conversion operations.

Regarding the aerodynamics components, the outcome is to inject an external normal load force $N^{j}$ at the element's CG. This force is calculated as follows:

$$
\begin{equation*}
N^{j}=N_{\alpha}^{j} \alpha^{j} \tag{11.5}
\end{equation*}
$$

where $N_{\alpha}^{j}$ is the local load gradient of element $j$, which will be further discussed in Sec. 11.2.2, and $\alpha^{j}$ is the local AoA. Note that the coefficients $N_{\alpha}^{j}$ do not account for the dynamic flow propagation along the launcher nor the effects of elastic deformation on that flow. The local AoA computation represents the MB equivalent to Eq. (11.4). Therefore, the corresponding subsystem in Fig. 11.4 involves:

- Measuring the pitch and drift motion of element $j$ with respect to the local frame, which is performed using a "sensor" block similar to the one of Fig. 11.3. If any flexible motion occurs in the model, elastic deformations are automatically included in the measurements.
- Accounting for the wind disturbance $v_{\mathrm{w}}$. Although this approach allows to consider a distributed wind profile (with a distinct velocity at every beam element), the wind velocity is assumed constant along the launcher body for the sake of simplicity.
- Accounting for the pitch damping effect caused by the element's velocity $l_{j} \dot{\theta}$ about the vehicle's CG. The offset $l_{j}$ is sensed by the "Longitudinal displacement" block in Fig. 11.4.


### 11.2.2 Definition of beam properties

As it was introduced in Sec. 11.1, in the MB approach, the inhomogeneous structure of the launcher is modelled as a series of uniform beam elements with distinct mass and load gradient. Integrating these distributed elements then entails defining spatial distributions for the MCI and aerodynamics characteristics of the vehicle.

The beam elements are assumed to be identical cylinders, as depicted in Fig. 11.2, with length $l=2 \mathrm{~m}$ and radius $r=1.5 \mathrm{~m}$. The centroid and CG of such an element $j$ with respect to the launcher's base, $x^{j}$, is given by:

$$
\begin{equation*}
x^{j}=l\left(j-\frac{1}{2}\right), \quad j=\{1, \ldots, n\} \tag{11.6}
\end{equation*}
$$

with $n=15$, and the normal moment of inertia per unit of mass, $J_{0}$, corresponds to:

$$
\begin{equation*}
J_{0}=\frac{1}{12}\left(3 r^{2}+l^{2}\right) \tag{11.7}
\end{equation*}
$$

Since this study is based on a fictional launcher, a cursory mass distribution, $m^{j}$, with $j=\{1, \ldots, n\}$, is adopted. The local masses must add to the total values of rigid-body mass, CG and normal inertia, which are expressed by the following three conditions:

$$
\begin{align*}
\sum_{j=1}^{n} m^{j} & =m  \tag{11.8}\\
\sum_{j=1}^{n} m^{j} x^{j} & =m x_{\mathrm{CG}}  \tag{11.9}\\
\sum_{j=1}^{n} m^{j}\left[J_{0}+\left(x^{j}-x_{\mathrm{CG}}\right)^{2}\right] & =J_{N} \tag{11.10}
\end{align*}
$$

In these conditions, $m, x_{\mathrm{CG}}$ and $J_{N}$ are taken from the ascent design point of Sec. 10.1, and $x^{j}$ and $J_{0}$ are given by Eq. (11.6) and (11.7). Note that the masses of fuel and oxidizer in the tanks are directly captured by the values of $x_{\mathrm{CG}}$ and $J_{N}$.

The values of $m^{j}$ are then defined by solving a linear least-squares problem minimising the equality errors of Eq. (11.8) to (11.10), and subject to the following constraints:

$$
\begin{equation*}
200 \mathrm{~kg} \leq m^{j} \leq 14000 \mathrm{~kg} \forall j, \quad \sum_{j=1}^{6} m^{j} \leq \frac{2}{3} m \tag{11.11}
\end{equation*}
$$

These constraints specify lower and upper bounds for $m^{j}$, and require that the first stage $(j=\{1, \ldots, 6\})$ accounts for less than $2 / 3$ of the total mass. The obtained mass distribution is depicted in Fig, 11.5a, where the $x$ axis denotes the longitudinal stations along the launcher and the $n=15$ bars the corresponding beam elements (with length $l=2 \mathrm{~m}$ ). In this figure, the contributions of first stage, second stage and payload to the total mass are clearly visible. For a more detailed mass distribution example, the reader is referred to [MG16].


Figure 11.5: Distributed launcher mass and load gradient

Due to the unavailability of detailed aerodynamic analyses, a similar approach is followed to obtain a reasonable normal load distribution, $N_{\alpha}^{j}$, with $j=\{1, \ldots, n\}$. Local loads are applied at the element' centroid, $x^{j}$, and must equal the total load gradient when integrated, i.e.:

$$
\begin{equation*}
\sum_{j=1}^{n} N_{\alpha}^{j}=N_{\alpha} \tag{11.12}
\end{equation*}
$$

For more realistic values of $N_{\alpha}^{j}$, the load distribution is parameterised as proposed in [VS11] for the same vehicle of [Orr10], which leads to the following three conditions:

$$
\begin{align*}
\sum_{j=1}^{5} N_{\alpha}^{j}+1.1 \sum_{j=14}^{15} N_{\alpha}^{j} & =N_{\alpha}  \tag{11.13}\\
N_{\alpha}^{6}+0.1 \sum_{j=14}^{15} N_{\alpha}^{j} & =0  \tag{11.14}\\
\sum_{j=7}^{13} N_{\alpha}^{j}-0.2 \sum_{j=14}^{15} N_{\alpha}^{j} & =0 \tag{11.15}
\end{align*}
$$

These conditions impose that the main contributions to the total load come from the nose $(j=\{14,15\})$ and base $(j=\{1, \ldots, 5\})$ of the launcher, and that there is a negative contribution around the fin and inter-stage location $(j=6)$. Equation (11.12) is then recovered by summing Eq. (11.13) to (11.15). Furthermore, the load distribution must ensure that the net load torque about the rigid-body Centre of Pressure (CP) is null, which translates into:

$$
\begin{equation*}
\sum_{j=1}^{n} N_{\alpha}^{j}\left(x^{j}-x_{\mathrm{CP}}\right)=0 \tag{11.16}
\end{equation*}
$$

As before, $N_{\alpha}$ and $x_{\mathrm{CP}}$ are defined in Sec. 10.1. The values of $N_{\alpha}^{j}$ were obtained by solving a linear least-squares problem minimising the equality errors of Eq. (11.13) to (11.16), and subject to the following linear constraints:

$$
\begin{equation*}
N_{\alpha}^{1} \geq N_{\alpha}^{2} \geq N_{\alpha}^{3} \geq N_{\alpha}^{4} \geq N_{\alpha}^{5}, \quad N_{\alpha}^{14} \geq N_{\alpha}^{15} \tag{11.17}
\end{equation*}
$$

which are employed to ensure a smooth load distribution along the vehicle. The resulting distribution is provided in Fig. 11.5b as a function of the element's station. Note that, with this approach, once the distributed RLV structure is implemented, any change of input data (due to a change of trajectory or trajectory point) is fully automated.

In order to verify the choice, and Simscape implementation, of the distributed mass and load gradient, the outcomes of the MB model are now compared to the responses obtained with the Linear Time-Invariant (LTI) system of Chapter 10. To do so, the MB model is formed by rigidly connecting the $n$ elements defined above and this rigid-MB model is then employed (instead of $G_{\text {RLV }}(\mathrm{s})$ ) in the closed-loop design/analysis interconnections of Fig. 10.3. The expectation is that the results between the LTI $G_{\text {RLV }}(\mathrm{s})$ model and the rigid-MB will be very similar.

Simulation results using the same 0.5 degree step command input and High-Frequency (HF) wind gusts of Sec. 10.4 are depicted in Fig. 11.6a and b, respectively. Both LTI and MB models have been closed with the same controller - the TVC-only, feedback-only control system defined by Eq. (10.7) and $K_{\mathrm{LR}}^{\uparrow}=\left[\begin{array}{ll}0 & 0\end{array}\right]^{\mathrm{T}}$.


Figure 11.6: Comparison of closed-loop responses obtained with LTI and MB models
Figures 11.6 a and b show the time responses of $\theta$ (blue), $\dot{\theta}$ (red) and $\dot{z}$ (green) obtained with the LTI model using dashed lines, with the MB model using solid lines and the difference between the two models using dash-dotted lines. The correct behaviour of the MB model is then corroborated by the overlapping dashed and solid lines of the same colour, as well as by the quasi-zero values of the dash-dotted lines. Moreover, this small difference (in both figures) is mostly caused by the ability of the MB model to account for the pitch damping effect at every
beam element (last point in Sec. 11.2.1), while the LTI model only considers that effect at the CP and fin locations (through the terms $l_{\alpha} \dot{\theta}$ and $l_{\mathrm{f}} \dot{\theta}$ ).

### 11.2.3 Implementation of structural elasticity

Once the distributed launcher structure is implemented, its flexibility can be modelled by changing the way beam elements are connected.

There are essentially two MB approaches to capture deformations that are small, linear and elastic: [i] the lumped-parameter method, in which rigid-body elements are coupled by joints with internal mechanics that collectively account for the deformation, and [ii] the FEA-import method, in which the flexible body is treated as the superposition of distinct deformation modes imported from a FEA model. Both methods are extensively described and compared in [MSWW17]. The former is generally slightly less accurate than the FEA-import method because the bending moment at a particular element does not depend on the neighbouring deflection angles; however, due to the unavailability of high-fidelity FEA data, the approach adopted in this thesis is the lumped-parameter method.

In accordance with this method, the discrete elements are connected as illustrated in Fig. 11.7. This figure shows the beam elements of Fig. $11.4(n=15$ in total) with the associated input/output signals, and the connection joints ( $n-1$ in total) which correspond to the blue crosses in the bottom diagram of Fig. 11.2. Each connection contains one revolute joint with internal mechanics and two transform blocks that specify the deformation frame.


Figure 11.7: Flexible vehicle structure interconnections
The internal mechanics of each joint $j$ is defined by the generation of a restorative and dissipative torque $\tau^{j}(t)$ :

$$
\begin{equation*}
\tau^{j}(t)=-a_{\mathrm{r}}^{j} \xi^{j}(t)-b_{\mathrm{r}}^{j} \dot{\xi}^{j}(t), \quad j=\{1, \ldots, n-1\} \tag{11.18}
\end{equation*}
$$

In this equation, $\xi^{j}(t)$ and $\dot{\xi}^{j}(t)$ are the joint's rotational state and derivative, and $a_{\mathrm{r}}^{j}$ and $b_{\mathrm{r}}^{j}$ represent its stiffness and damping coefficients.

The flexible behaviour of the structure is then determined by the $a_{\mathrm{r}}^{j}$ and $b_{\mathrm{r}}^{j}$ coefficients. For homogeneous beams, the value of $a_{\mathrm{r}}^{j}$ can be found analytically [PHEL09] based on its Young's modulus, cross-section shape and discretisation level, and the value of $b_{\mathrm{r}}^{j}$ can be empirically set by matching the lumped-parameter deformations to reliable benchmark data [MSWW17].

In this thesis, as a cursory example, these coefficients are assumed uniform along the launcher and selected as follows:

- The stiffness coefficient is chosen so that the frequency of the first resonant MB mode matches the corresponding FEA estimate ( $\omega_{\mathrm{f}}=4.4 \mathrm{~Hz}$ );
- The damping coefficient is heuristically set to the smallest value that results in a numerically stable MB model. It is important to note that the loss of numerical stability for quasi-zero damping coefficients represents an inherent limitation of this multibody approach.

The adopted coefficients are provided in Table 11.1. For an example of non-uniform launcher stiffness distribution, the reader is referred to [MG16].

The determination of the resonant modes is carried out based on the open-loop pitch rate Power Spectral Density (PSD) at the vehicle's CG. This PSD is determined by injecting the control deflections associated to the simulation of Fig. 11.6a (i.e. the case of the rigid-MB model) into the flexible-MB model.

The open-loop PSD produced with the selected values of $a_{\mathrm{r}}^{j}$ and $b_{\mathrm{r}}^{j}$ is depicted in Fig. 11.8 using a blue dashed line, in which the frequency peaks of the first three bending modes (not to be confused with the joint number $j$ ) are clearly visible. The other lines of Fig. 11.8 are subsequently described in Sec. 11.2.4.


Figure 11.8: Bending motion tuning in the frequency-domain. PSDs are computed with respect to the vehicle's CG using a step response.

### 11.2.4 Inclusion of bending filters

When the same control loop is closed with the flexible-MB model, the system becomes severely unstable due to the high power associated to the bending modes. This situation was expected based on state-of-practice knowledge of launcher control and on the fact that the PSD of the
first mode is higher than the steady-state dynamics of the system (i.e. the magnitude of the blue dashed line in Fig. 11.8 is higher at 4.4 Hz than at lower frequencies).

In order to cope with the structural flexibility, the controller has to be augmented with bending filters, which must provide two actions: [i] attenuate the amplification of high-frequency internal dynamics and stabilise the system, and [ii] notch the remaining resonances in the system. The first action can be achieved using a 2nd order Low-Pass Filter (LPF) to ensure the required attenuation. The transfer function of this filter, $H_{\mathrm{LPF}}(\mathrm{s})$, is defined as:

$$
\begin{equation*}
H_{\mathrm{LPF}}(\mathrm{~s})=\frac{\omega_{\mathrm{L}}^{2}}{\mathrm{~s}^{2}+2 \zeta_{\mathrm{L}} \omega_{\mathrm{L}} \mathrm{~s}+\omega_{\mathrm{L}}^{2}} \tag{11.19}
\end{equation*}
$$

where $\omega_{\mathrm{L}}$ represents the roll-off frequency and $\zeta_{\mathrm{L}}$ is its damping ratio.
The notch filter, $H_{\text {notch }}(\mathrm{s})$, in its simplest form, can be parameterised as follows [NMBR18]:

$$
\begin{equation*}
H_{\mathrm{notch}}(\mathrm{~s})=\frac{\mathrm{s}^{2}+\epsilon_{\mathrm{N}} \mathrm{~s}+\omega_{\mathrm{N}}^{2}}{\mathrm{~s}^{2}+\epsilon_{\mathrm{N}} / \kappa_{\mathrm{N}} \mathrm{~s}+\omega_{\mathrm{N}}^{2}} \tag{11.20}
\end{equation*}
$$

In this transfer function, $\omega_{N}$ represents the central notch frequency, $\kappa_{N}$ is the gain applied at that frequency and $\epsilon_{\mathrm{N}}$ defines the selectivity of the filter. Smaller values of $\kappa_{\mathrm{N}}$ provide more attenuation but introduce a larger phase lag into the system. The value of $\epsilon_{\mathrm{N}}$ must be large enough to cover the relevant frequency band but not too large in order not to affect other dynamics. With the augmented controller, the bending filters of Eq. (11.19) and (11.20) are then applied to both deflections, TVC and fins, as follows:

$$
\mathbf{u}=H_{\mathrm{LPF}}(\mathrm{~s}) H_{\mathrm{notch}}(\mathrm{~s}) \mathbb{I}_{2 \times 2}\left[\begin{array}{c}
\beta_{\mathrm{TVC}}  \tag{11.21}\\
\beta_{\mathrm{fin}}
\end{array}\right]
$$

In this thesis, the parameters that describe $H_{\text {LPF }}(\mathrm{s})$ and $H_{\text {notch }}(\mathrm{s})$ were tuned manually, based on the PSD of Fig. 11.8 (e.g. $\omega_{\mathrm{N}}$ was set to the first resonant frequency $\omega_{\mathrm{f}}=4.4 \mathrm{~Hz}$, and $\omega_{\mathrm{L}}$ was chosen sufficiently higher). All the adopted values are registered (together with the lumped-parameter coefficients) in Table 11.1, and the obtained pitch rate PSD with the bending filters is depicted in Fig. 11.8 using a blue solid line.

Table 11.1: Bending motion parameters

| MB joints, $\forall j$ | Value |
| :--- | :---: |
| $a_{\mathrm{r}}^{j}(\mathrm{kN} . \mathrm{m} / \mathrm{rad})$ | $146 \times 10^{4}$ |
| $b_{\mathrm{r}}^{j}(\mathrm{kN} . \mathrm{m} . \mathrm{s} / \mathrm{rad})$ | 100 |


| $H_{\mathrm{LPF}}(\mathrm{s})$ | Value |
| :--- | :---: |
| $\zeta_{\mathrm{L}}$ | 0.8 |
| $\omega_{\mathrm{L}}(\mathrm{rad} / \mathrm{s})$ | 40 |


| $H_{\text {notch }}(\mathrm{s})$ | Value |
| :--- | :---: |
| $\epsilon_{\mathrm{N}}$ | 6.0 |
| $\kappa_{\mathrm{N}}(\mathrm{dB})$ | -4.4 |
| $\omega_{\mathrm{N}}(\mathrm{rad} / \mathrm{s})$ | 27.6 |

From Fig. 11.8, it can be observed that applying the filters of Eq. 11.21 (from the dashed to the solid blue line) results in a significant attenuation of the bending modes, particularly the first one, as required. The transfer function gain of the bending filters is also shown in Fig. 11.8
with red colour: $H_{\text {LPF }}$ (s) using a dashed line and the combination $H_{\text {LPF }}(\mathrm{s}) H_{\text {notch }}$ (s) using a solid line, with the additional 4.4 dB attenuation at the first bending mode.

For a clearer understanding of the importance of the notch filter $H_{\text {notch }}$ (s), Fig. 11.9a and b illustrate the elastic displacement of the launcher respectively without and with that filter (but both with the LPF, as otherwise the system is unstable). The plots show the closed-loop response to a step command at $t=1$ second (same as Fig. 11.6a) and different colours refer to the displacement of different elements along the launcher, as indicated by the colour bar.


Figure 11.9: Impact of notching the first bending mode on the elastic displacement. The displacement is computed with respect to the vehicle's CG using a step response.

As expected, in both cases, the step command induces an oscillation starting at 1 second, which increases towards the nose of the launcher where it reaches a maximum deflection of about 6 cm . However, while this oscillation persists for several seconds in Fig. 11.9a, the introduction of the notch filter in Fig. 11.9b ensures its rejection soon after the step command is injected.

As mentioned before, the aim of this section was to exemplify the implementation of bending filters in their simplest form but, in practice, more complex filters are often necessary to achieve a better flight performance. The development of such filters, together with their systematic tuning in conjunction with rigid-body gains, is thoroughly addressed in [NMBR18].

### 11.3 Results and discussion

In this section, the MB launcher model is employed for two dedicated analyses: first (Sec. 11.3.1) to compare the accuracy of rigid-body and aeroelastic load predictions, and then (Sec. 11.3.2) to verify whether the superior Load Relief (LR) capability of the control functionalities developed in Chapter 10 also holds in the aeroelastic case.

### 11.3.1 Analysis using feedback-only controller

A more detailed analysis of the closed-loop MB model with the same inputs used for Fig. 11.6 (and Sec. 10.4) is now provided. The distributed aeroelastic loads obtained using the TVC-only, feedback-only control system, Eq. (10.7) and $K_{\mathrm{LR}}^{\uparrow}=\left[\begin{array}{ll}0 & 0\end{array}\right]^{\mathrm{T}}$, in series with the bending filters of Eq. (11.21), are depicted in Fig. 11.10. The distributed loads represent the local AoA (in absolute value) as a function of launcher station ( $x$ axis) and time ( $y$ axis).

The top row of the figure, Fig. 11.10a and b, corresponds to the MB model with the $n$ beam elements rigidly connected, and the bottom, Fig. 11.10c and d, to the flexible vehicle structure developed in Sec. 11.2.3. The left-hand side column of the figure, Fig. 11.10a and c, depicts the responses to the 0.5 degree step command input, and the right-hand side, Fig. 11.10b and d, to a 30 seconds HF wind field.


Figure 11.10: Distributed load comparison using feedback-only controller. Top and bottom rows show rigid-body and aeroelastic predictions. Left and right columns show responses to a step command and HF wind field.

The top row represents rigid-body predictions, which take into account the distributed aerodynamics of Sec. 11.2.1 but not yet the effect of structural elasticity. In this case, at each instant of time, the AoA is approximately constant along the launcher's body, thus the step response and wind disturbance results are equivalent to the nominal "no fin control, FB-only" AoA responses of Fig. 10.12a and c, respectively. Note that the AoA is not exactly the same along the launcher due to the pitch damping effect but, as anticipated by the analysis of Fig. 11.6, differences are so small that they cannot be perceived in Fig. 11.10.

The impact of structural elasticity is then evidenced in the bottom row of Fig. 11.10 by the presence of oscillations (recall Fig. 11.9) and AoA variations along the launcher's body, caused by the elastic deformation. In order to appreciate the variations of AoA, the differences between rigid-body and aeroelastic predictions are illustrated in Fig. 11.11. This figure shows the difference between the bottom and top row of Fig. 11.10, with the step command on the left (Fig. 11.11a) and wind gusts on the right (Fig. 11.11b).


Figure 11.11: Distributed load underestimation in the absence of aeroelastic effects

The AoA differences of Fig. 11.11 confirm that, depending on the RLV station, rigid-body predictions of aerodynamic loads may be both underestimated (yellow peaks) or overestimated (blue peaks). The prediction error (in absolute value) tends to increase towards the nose of the launcher (same as Fig. 11.9) as well as when the rigid-body AoA is already high, reaching a maximum error of about $20 \%$ in Fig. 11.11a.

The fact that distributed loads may be severely underestimated using rigid-body models entails that conclusions extracted from traditional point-mass (centralised) $Q_{\alpha}$ tests along the trajectory (such as Fig. 10.13a) may result in limited reliability. As showcased in this chapter, the MB approach provides much more reliable and insightful load information without the need for complicated and time-consuming FEA and CFD iterations.

### 11.3.2 Analysis using controller with rWDO and fins

The same approach is now followed to assess the distributed loads generated if the load relief functionalities developed throughout Chapter 10 are employed instead of the baseline controller. Hence, the same tests carried out in the previous section are repeated with the combined fins/TVC controller augmented with the Robust Wind Disturbance Observer (rWDO), defined by Eq. (10.13), also in series with the bending filters of Eq. (11.21). The results are depicted in Fig. 11.12, which presents the same layout and axis/colour scale of Fig. 11.10 for ease of comparison.


Figure 11.12: Distributed load comparison using controller with rWDO and fins. Top and bottom rows show rigid-body and aeroelastic predictions. Left and right columns show responses to a step command and HF wind field.

Comparing Fig. 11.12 with 11.10 , it is evident that the rWDO (plus fins) controller results in much smaller aerodynamic loads for both step and HF disturbance cases (note that the yellow and green peaks present in Fig. 11.10 have practically disappeared).

This conclusion was already expected for rigid-body predictions, i.e. from the top row of Fig. 11.10 to 11.12 , since it corresponds to the AoA improvement observed in Fig. 10.12a and c, from "no fin control, FB-only" to "with fin control, rWDO-LR". But the MB approach allows to easily extend these conclusions to the aeroelastic case (bottom row of Fig. 11.12): although the elastic oscillations lead to an increase of AoA variations (same as in Sec. 11.3.1), the LR superiority of the controller with rWDO and fins still holds in the presence of aeroelastic effects (comparing the bottom rows of Fig. 11.10 and 11.12).

To further appreciate this outcome, a few additional details are provided below. Figure 11.13 shows the control deflections (TVC on the top and fins on the bottom) associated with the various wind simulations.


Figure 11.13: Wind response actuator signals related to Fig. 11.10b, 11.10d, 11.12b and 11.12d

In this figure, the actuator signals using the rigid-body model with feedback-only (related to Fig. 11.10b) are depicted in black and are naturally characterised by zero fin deflections. The aeroelastic model (Fig. 11.10d) then results in additional TVC oscillations, in green, around the rigid-body response, which are caused by the aeroelastic effects. The rigid-body model with rWDO and fins (related to Fig. 11.12b), on the other hand, requires larger TVC and fin deflections, depicted in red, to successfully alleviate the AoA (recall Fig. 10.12c). Finally, as before, the aeroelastic model (Fig. 11.12d) leads to small control oscillations around the rigid-body case, which are represented in Fig. 11.13 using blue lines.

The aeroelastic LR improvement achieved with rWDO and fins is then illustrated, as a function of the RLV station, in Fig. 11.14. For each element, the blue bar corresponds to the largest positive difference between Fig. 11.10d and Fig. 11.12d over the 30 seconds wind response, and the yellow bar represents the largest negative difference. In other words, the blue bars indicate the maximum AoA relief achieved over time (the more positive the better) and the yellow bars the local AoA degradation (the less negative the better).


Figure 11.14: Distributed wind load variation achieved with rWDO and fins

The superiority of the LR functionality in the presence of aeroelastic effects is again confirmed by the fact that the blue bars are significantly bigger than the yellow ones. This means that, although the local AoA worsens under certain circumstances (on average by about 0.27 degrees), it is globally outweighed by an achievable load alleviation of about 0.70 degrees. Similar to the elastic oscillations, the intensity of these two phenomena grows in the direction of the launcher's nose.

The occasional AoA degradations observed in Fig. 11.14 are most likely induced by a combination of elastic motion with the phase lag introduced by the rWDO. Recall that, as highlighted in Sec. 11.2.4, the rigid-body gains and rWDO were designed independently of the bending filters and vice-versa, which leaves room for performance improvement. The proposed MB approach is also particularly suitable for joint rigid/flexible robust control synthesis because MB models are highly compatible with the Linear Fractional Transformation (LFT) representation of robust control used in references $\left[\mathrm{PAL}^{+} 16, \mathrm{PCH}^{+} 18\right]$.


## Conclusions of Application II

The optimisation of launcher flight performance while meeting tight aerodynamic and thermal load constraints is a challenging problem. Significant benefits can be achieved by jointly addressing the two tasks of dimensioning (vehicle and components) and design (guidance and control) but, in order to do this, a complete revision of the current process for launchers must be performed to manage the more complex process of reusable launchers.

In this second part of the thesis, a flight mechanics benchmark of an RLV was developed to study and address the critical coupling between guidance and control. It incorporates the main critical components for studying such effects, although some standard simplifying assumptions were employed in the interest of reducing the complexity to focus on the aims of the thesis. Despite these assumptions, the validity and relevance of the benchmark was demonstrated in Chapter 8 using traditional G\&C techniques. This integrated G\&C approach allowed to perform preliminary assessments of different RLV recovery strategies using a baseline Constrained Terminal Velocity Guidance (CTVG) law and of the controllability challenges encountered, while highlighting the existing room for improvement.

In terms of guidance, the potential for improvement was exploited throughout Chapter 9 in two ways: first via a trade-off analysis of the physics of the powered descent and landing problem, and then by developing and implementing a more sophisticated algorithm coined Descending over Extended envelopes using Successive ConvExificatioN-baseD Optimisation (DESCENDO).

Regarding the trade-off analysis, improvements were possible because the proposed method enabled a quick and clear mapping of the impact of different parameters and of the achievable compromises in terms of recovery propellant, dynamic pressure and/or touchdown speed. Nonetheless, major improvements do require the application of more sophisticated guidance techniques. The $D E S C E N D O$ algorithm addressed this need. It is a convex optimisation-based path planning strategy developed to provide a balance between computational efficiency and trajectory optimality that is suitable for the extended flight envelope encountered by RLVs.

The effectiveness of $D E S C E N D O$ and its superiority over the baseline CTVG were then verified using the reusable launcher benchmark. But the major significance of the $D E S C E N D O$ algorithm, and of the trade-off map method, lies in the fact that they enable a less conservative exploitation of the parameter trade-off space. The reduced conservativeness, in turn, supports a deeper understanding of reusable flight mechanics, as well as G\&C margin policies related to the physical limits of performance.

While this development was carried out assuming that launcher attitude commands were perfectly tracked, subsequently Chapter 10 focused on the impact of attitude control on the loads experienced by RLVs and on how these loads could be minimised through proper launcher control design. Two complementary flight control functionalities for enhanced Load Relief (LR) were proposed.

The first functionality consisted of augmenting a feedback-only controller with a disturbance observer for onboard wind anticipation. It was validated via an achievable performance analysis, which showed that this approach allows control priorities (e.g. minimum load vs. minimum error) to be adjusted along the mission without retuning the stabilisation/tracking feedback loop. It was also shown that the introduction of a wind observer results in reduced robust stability but, if the observer design is performed appropriately, this reduction is still within acceptable bounds. The second functionality introduced fin control in ascent flight, which significantly improved attitude tracking by counteracting the drift caused by the TVC side-force.

Based on the presented analyses, the best LR performance was achieved by combining fin control with the wind disturbance observer. Several disturbance observers were successfully designed and verified using robust control techniques - using one point during the ascent and another one during the descent. Despite this local illustration, the proposed framework allows to easily repeat the procedure for other points, as well as to address other effects not considered in this thesis (e.g. actuator, sensor, bending and sloshing dynamics).

Finally, the reusable launcher benchmark was extended with a MultiBody (MB) model that captures the effects of distributed aerodynamics and structural flexibility. As demonstrated in Chapter 11, this modelling paradigm provides an extremely insightful tool for aeroelastic launcher load prediction. As an example, this tool was employed to highlight the performance impact of bending filter design, as well as to confirm the superiority of the controller using wind disturbance observation and fins even in the presence of aeroelastic effects.

In this thesis, the adopted launcher system only included an aeroelastic model for analysis purposes, but the same framework can be extended to MB-based control synthesis, including joint rigid/flexible design. This capability, together with the developed coupled benchmark, the proposed $D E S C E N D O$ algorithm and LR functionalities, establish the fundamental building blocks to tackle the complex process of aeroelastic reusable launcher load management.


## Conclusions \& Recommendations

This chapter concludes the thesis. First, the conclusions drawn in Chapter 7 and 12, for Application I and II, respectively, are reviewed in Sec. 13.1 while highlighting the thesis' contributions listed in Sec. 1.5. Subsequently, based on the encountered room for improvement, the most interesting future lines of research are recommended and briefly discussed in Sec. 13.2.

### 13.1 Review of conclusions

This thesis investigated the benefits that specific Guidance \& Control (G\&C) techniques can bring for improved exploration and access to space. Despite the strengths of the investigated techniques, there is still a big gap between their academic development and industrial application. By tailoring and applying these techniques to two case studies that are well-representative of those strengths, the high-level outcome of the thesis was a Technology Readiness Level (TRL) increase of modelling, design and analysis approaches for planetary landers and reusable launchers.

The first application case, fuel-optimal Descent \& Landing (D\&L) on small planetary bodies, was motivated by the need to improve the effectiveness of state-of-practice $\mathrm{D} \& \mathrm{~L}$ solutions. To facilitate such an improvement, a reconciliation of G\&C architectures for $\mathbf{D} \& L$ was proposed and distinct methods based on open and closed-loop guidance with and without control compensation were analysed. Special attention was dedicated to the integration of legacy industry knowledge with sophisticated robust control design techniques that allow to account for the effects of variable and uncertain environments (such as Phobos' gravity field) in a systematic manner. This capability in turn paves the way for more reliable planetary landers.

Building up on this knowledge, a methodology for performance quantification and optimisation of guidance laws was then developed. This methodology employs a systematic
evaluation of high-fidelity models to generate performance trade-off maps of Constrained Terminal Velocity Guidance (CTVG) tuning selections. The trade-off maps can be generated before the mission and applied once the spacecraft approaches the target body, having in consideration the actual propellant available and without the need for expensive computations. It was demonstrated that, for a landing on Phobos, propellant consumption savings of around $40 \%$ could be achieved compared to state-of-practice tuning solutions, derived under the assumption of simplified and well-known gravity fields.

An alternative D\&L approach was then investigated, relying on a simpler (open-loop) guidance law but augmenting it with a robust control compensator. This marked the earliest application of robust control synthesis techniques to spacecraft orbital control, as they were well-established for attitude control only. The effectiveness of this approach lied in the use of structured $\mathcal{H}_{\infty}$ optimisation which, in addition to its ability to specify the controller structure and take advantage of legacy knowledge, enables the synthesis of controllers that are valid for multiple plants. Design challenges related to the use of non-smooth optimisation were investigated and, despite these challenges, this approach was successfully transferred to Airbus and its teams are currently employing structured $\mathcal{H}_{\infty}$ in the preparation of future missions.

In addition to control design, the thesis showed how robust control techniques can be effective to validate modelling choices and complement state-of-practice verification \& validation methods of space systems. On the one hand, critical uncertainty combinations can be computed via $\mu$ analysis and employed to complement and enhance conventional MonteCarlo campaigns. On the other hand, closed-loop sensitivities allow to quantify the impact of each uncertain parameter, which is key to develop the simplest mathematical model that still captures the most relevant physical phenomena of the real system.

Using $\mu$ and IQC analysis, it was proven that the developed control compensators are robust in the presence of gravitational uncertainties and nonlinear issues and that, for a landing of Phobos, the former effects are indeed the most critical ones. From a safety-oriented point of view, the ability to provide this kind of formal proofs of robustness makes robust control techniques more attractive than other recent alternatives such as adaptive or intelligent control.

The second application case, Reusable Launch Vehicle (RLV) G\&C with active load relief, was aimed at the optimisation of launcher development and operation costs, which is notably demanding due to fundamental couplings between flight mechanics and closed-loop algorithms. In order to study and address these critical couplings, a novel benchmark/framework for RLV G\&C design was developed in this part of the thesis. Despite relying on a few standard simplifying assumptions in the interest of reducing complexity, the coupled benchmark allowed the assessment of different RLV recovery strategies using a baseline CTVG law and of the controllability challenges encountered.

Following this assessment, D\&L performance could be improved first by using the trade-off map method proposed in the first part of the thesis, and then by developing an onboard
convex optimisation-based recovery guidance algorithm coined Descending over Extended envelopes using Successive ConvExificatioN-baseD Optimisation (DESCENDO). The DESCENDO algorithm employs the principles of lossless and successive convexification to provide a balance between computational efficiency and trajectory optimality that is specifically tailored to the extended flight envelope encountered by RLVs. These principles were implemented and verified in a closed-loop fashion, which is particularly novel in the literature. The trade-off map method and the $D E S C E N D O$ algorithm enable a less conservative exploitation of the design parameter space, as well as of G\&C margins related to the physical limits of performance.

The limits of performance were subsequently enlarged thanks to the development of two control functionalities for enhanced launcher Load Relief (LR). The first functionality consisted of augmenting a feedback-only controller with a disturbance observer for onboard wind anticipation, designed and verified using robust control tools. This marked the earliest application of robust wind disturbance observation to improve the control of launcher aerodynamic loads. The second functionality introduced fin control in both descent and ascent flight. The use of fins during launch is not industrial practice, but the thesis showed that further LR improvements are possible by doing so. Wind observation paves the way for an increase of wind resilience and operational availability, together with a decrease of structural mass and launch costs. One additional strength of this approach is related to the possibility of control priorities to be adjusted along the mission without having to alter the stabilisation/tracking properties.

To conclude the thesis, the RLV benchmark was extended with a MultiBody (MB) model that captures the effects of distributed aerodynamics and structural elasticity. The accurate modelling of these effects is becoming ever more important, as the next-generation of lighter and more flexible launchers tends to have a smaller separation between flexible and rigid-body frequencies. Moreover, the MB approach allows the integration of distinct domains in a single multi-physics framework that can be quickly built and analysed. Hence, the proposed approach enables more accurate load predictions and faster design iterations between trajectory, structure and flight control.

The MB launcher model was then employed to confirm the superiority of the LR control functionalities even in the presence of aeroelastic effects, and to compare the accuracy of rigid-body and aeroelastic load predictions. This comparison showed that distributed loads may be severely underestimated using rigid-body models, and therefore insights from the MB approach can be used to revise the traditional point-mass (centralised) $Q_{\alpha}$ verification along the trajectory.

### 13.2 Recommendations for future work

This section discusses possible lines for future research on the most interesting technical aspects encountered throughout the thesis.

## Coping with nonlinear effects on structured $\mathcal{H}_{\infty}$ optimisation

The main limitation of the first part of the thesis was the inability of structured $\mathcal{H}_{\infty}$ to capture more performing guidance tuning solutions identified using the trade-off maps of Chapter 4 (although the state-of-practice tuning selections could be successfully recovered). This was caused by the inherent loss of the propagation of nonlinear dynamics throughout the D\&L trajectory. Hence, the applicability of structured $\mathcal{H}_{\infty}$ optimisation to problems where nonlinear effects are critical remains an open research question. This applicability can be tackled either by adopting a different type of formulation for the guidance problem or by modifying the behaviour of the structured $\mathcal{H}_{\infty}$ algorithm itself.

## Advancement of convexification-based D\&L algorithms

As it was mentioned before, the amount of research on convexification-based guidance (and computational G\&C in general) has increased exponentially in the past couple of years, which opens up various possibilities to improve the $D E S C E N D O$ algorithm developed in Chapter 9.

As a first step, motivated by [SA18], it would be recommended to incorporate attitude kinematics and free final-time in the optimisation problem. Moreover, the versatility of successive convexification enables the formulation of additional continuous or state-triggered non-convex constraints $\left[\mathrm{RSM}^{+} 19\right]$, including dynamic pressure and heat flux. Finally, although the application of convexification-based guidance to Phobos D\&L was not investigated (as the focus was on strategies that are more likely to be adopted in the short-term future), high-order gravitational harmonics can be handled via successive convexification, so this would also represent an extremely interesting line of research.

## Load relief with wind measurements and preview

Chapter 10 demonstrated the benefits of anticipating the effects of wind by augmenting a state-of-practice launcher control architecture with a simple disturbance observer. Following this demonstration, further LR improvements can be achieved in two ways: [i] replacing the observer with a Light Detection And Ranging (LiDAR) sensor that provides real-time wind measurements, and [ii] employing estimates or measurements of the disturbance ahead of the plant with enough lead time, which is known as preview control.

The use of a LiDAR sensor has the advantage of avoiding the small robust stability degradation caused by the wind observation delay, but involves an additional instrumentation complexity. This option was first proposed in [Mar02] but did not become industrial practice due to the conservative character of space industry, motivating the present disturbance observer development as a more gradual approach. Preview control enables a more accurate wind anticipation (due to the use of past wind information) and it is inclusively compatible with onboard convex optimisation algorithms $\left[\mathrm{CAW}^{+} 15\right]$. The application of preview control has been investigated in the field of aeronautics (e.g. [THS18, FJD19]), but not launchers.

## Flight testing using the $E A G L E$ demonstrator

As introduced in Sec. 1.4, the second part of the thesis accounts for the possibility/potential to verify the developed G\&C elements using the EAGLE demonstrator of Fig. 1.6. Having in mind the TRL of the algorithms and of the demonstrator itself, the most interesting element that is suitable for flight testing would be the robust wind disturbance observation functionalities.

## Multibody-based launcher control design

In this thesis, the adopted launcher system only included an aeroelastic model for analysis purposes, yet significant gains can be expected if such a model is employed for control design as well. The MB launcher modelling approach presented in Chapter 11 is particularly suitable for this purpose, not only because it offers the possibility to make quick changes in model configuration, but also because it is highly compatible with the LFT representation of robust control. Recent developments of MB-based robust control synthesis in other areas include $\left[\mathrm{PAL}^{+} 16\right]$, which addresses the integrated design and control of a satellite with uncertain flexible appendages, and $\left[\mathrm{PCH}^{+} 18\right]$, where the disturbance propagation between microvibration sources and sensitive instruments is tackled.

## Impact of additional dynamical RLV perturbations

The final recommendation is to model and study the impact of critical dynamical RLV effects that were not considered in the thesis. Structural flexibility was only analysed using one point during the ascent, so the analysis should be repeated for other points during ascent as well as descent, where aeroelastic couplings are expected to be stronger since the launcher is flying with quasi-emptied propellant tanks. Also tightly related to level of propellant in the tanks, the impact of sloshing dynamics was not yet investigated and should be addressed. In addition, actuators and sensors were assumed ideal in the second part of the thesis, but their inherent limitations should also be considered as they can have deep G\&C design implications. The main driving implications are expected to be related to engine throttling magnitude and rate constraints, and to sensor disturbances induced by the flexible RLV structure.

Although the above effects have not been addressed, they can be systematically managed using the robust control framework thanks to its key capabilities (recall Table 2.2). In other words, the same robust control approaches exploited throughout the thesis can be used to design for incrementally more challenging dynamical models and sizing cases.

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## List of Publications

During the PhD period, a total of 5 first-authored journals, 6 first-authored conferences, 2 co-authored journals and 4 co-authored conferences were published, together with 2 invited presentations at international workshops. They are listed below for each application case (i.e. Phobos D\&L and reusable launcher G\&C). The last four publications were also produced during this period, but are not included in the thesis as they were part of collaborations within the TASC research group ${ }^{1}$.

## Application I: Fuel-optimal descent \& landing on small planetary bodies

## Journals

- P. Simplício, A. Marcos, E. Joffre, M. Zamaro, and N. Silva, "Systematic Performanceoriented Guidance Tuning for Descent \& Landing on Small Planetary Bodies," Acta Astronautica, 158, May 2019 [SMJ+ 19];
- P. Simplício, A. Marcos, E. Joffre, M. Zamaro, and N. Silva, "Synthesis and Analysis of Robust Control Compensators for Space Descent \& Landing," International Journal of Robust and Nonlinear Control, 28 (13), Sep. 2018 [SMJ ${ }^{+}$18c];
- P. Simplício, A. Marcos, E. Joffre, M. Zamaro, and N. Silva, "Review of Guidance Techniques for Landing on Small Bodies," Progress in Aerospace Sciences, 103, Nov. 2018 [SMJ ${ }^{+}$18a];
- E. Joffre, M. Zamaro, N. Silva, A. Marcos, and P. Simplício, "Trajectory Design and Guidance for Landing on Phobos," Acta Astronautica, 151, Oct. 2018 [JZS ${ }^{+}$18].

[^1]
## Conferences

- P. Simplício, A. Marcos, E. Joffre, M. Zamaro, and N. Silva, "Robust Control Compensation for Space Descent \& Landing," The 16th European Control Conference, Limassol (Cyprus), 12-16 Jun. 2018 [SMJ ${ }^{+}$18b], invited for publication in the European Journal of Control;
- P. Simplício, A. Marcos, E. Joffre, M. Zamaro, and N. Silva, "Parameterised Laws for Robust Guidance and Control of Planetary Landers," The 4 th CEAS Specialist Conference on GNC, Warsaw (Poland), 25-27 Apr. 2017 [SMJ $\left.{ }^{+} 17 \mathrm{~b}\right]$;
- P. Simplício, A. Marcos, E. Joffre, M. Zamaro, and N. Silva, "A Systematic Performanceoriented Tuning for Space Exploration Descent \& Landing Guidance," The 7th European Conference for Aeronautics and Space Sciences, Milan (Italy), 3-6 Jul. 2017 [SMJ+ 17a], invited for publication in Acta Astronautica;
- E. Joffre, M. Zamaro, N. Silva, A. Marcos, P. Simplício, and B. Richardson, "Landing on Small Bodies Trajectory Design, Robust Nonlinear Guidance and Control," The 27th AAS/AIAA Space Flight Mechanics Meeting, San Antonio (TX), 5-9 Feb. 2017 [JZS+ ${ }^{+}$17];
- E. Joffre, M. Zamaro, N. Silva, A. Marcos, P. Simplício, and B. Richardson, "Results of new Guidance and Control Strategies for Landing on Small Bodies," The 10th ESA Conference on Guidance, Navigation © Control Systems, Salzburg (Austria), 29 May-2 Jun. 2017.


## Invited presentations

- "Robust Guidance and Control for Space Descent and Landing," International Workshop on Robust Modelling, Design $\xi^{\mathcal{Z}}$ Analysis, University of Bristol (UK), 18-19 Sep. 2017.


## Application II: Reusable launcher guidance \& control with active load relief

## Journals

- P. Simplício, A. Marcos, and S. Bennani, "Reusable Launchers: Development of a Coupled Flight Mechanics, Guidance and Control Benchmark," Journal of Spacecraft and Rockets, Articles in Advance, Sep. 2019 [SMB19d];
- P. Simplício, A. Marcos, and S. Bennani, "Guidance of Reusable Launchers: Improving Descent and Landing Performance," Journal of Guidance, Control and Dynamics, 42 (10), Oct. 2019 [SMB19b].


## Conferences

- P. Simplício, A. Marcos, and S. Bennani, "New Control Functionalities for Launcher Load Relief in Ascent and Descent Flight," The 8th European Conference for Aeronautics and Space Sciences, Madrid (Spain), 1-4 Jul. 2019 [SMB19c], winner of the Best Student Paper Award;
- P. Simplício, A. Marcos, and S. Bennani, "A Reusable Launcher Benchmark with Advanced Recovery Guidance," The 5th CEAS Specialist Conference on GNC, Milan (Italy), 3-5 Apr. 2019 [SMB19a].


## Invited presentations

- "Coupled Flight Mechanics \& GNC of Reusable Launchers with Convex Optimisationbased Guidance," 12th ESA Workshop on Avionics, Data, Control and Software Systems, European Space Agency (Netherlands), 16-18 Oct. 2018.


## Other publications during the PhD period but not included in this thesis

## Journals

- D. Navarro-Tapia, A. Marcos, P. Simplício, S. Bennani, and C. Roux, "Legacy Recovery and Robust Augmentation Structured Design for the VEGA Launcher," International Journal of Robust and Nonlinear Control, 29 (11), Jun. 2019 [NMS+19].


## Conferences

- P. Simplício, D. Navarro-Tapia, A. Iannelli, and A. Marcos, "From Standard to Structured Robust Control Design: Application to Aircraft Automatic Glide-slope Approach," The 9th IFAC Symposium on Robust Control Design, Florianopolis (Brazil), 3-5 Sep. 2018 [SNIM18];
- A. Iannelli, P. Simplício, D. Navarro-Tapia, and A. Marcos, "LFT Modelling and $\mu$ Analysis of the Aircraft Landing Benchmark," The 20th IFAC World Congress, Toulouse (France), 9-14 Jul. 2017;
- D. Navarro-Tapia, P. Simplício, A. Iannelli, and A. Marcos, "Robust Flare Control Design using Structured H-infinity Synthesis: A Civilian Aircraft Landing Challenge," The 20th IFAC World Congress, Toulouse (France), 9-14 Jul. 2017.



## Mars-Phobos System Parameters

This appendix contains the most relevant parameters of the Mars-Phobos system introduced in Sec. 3.1. Table B. 1 provides an overview of the system's physical and orbital properties. Table B. 2 gathers the nominal values of the normalised coefficients for the spherical harmonics series expansion of Phobos' gravity field.

Table B.1: Physical and orbital properties of Mars (around the Sun) and Phobos (around Mars). Source: [ZB15]

| Property | Mars | Phobos |
| :--- | :---: | :---: |
| Mass (kg) | $6.42 \times 10^{23}$ | $1.07 \times 10^{16}$ |
| Mean dimension (km) | $3.39 \times 10^{3}$ sphere | $13.1 \times 11.1 \times 9.3$ ellipsoid |
| Revolution period (days) | 687 | 0.32 |
| Rotation period (days) | 1.03 | 0.32 |
| Semi-major axis (km) | $2.28 \times 10^{8}$ | $9.38 \times 10^{3}$ |
| Eccentricity (-) | 0.0934 | 0.0156 |
| Inclination (deg) | 1.85 | 1.07 |

Table B.2: Normalised coefficients for the series expansion of Phobos' gravity field. Source: [CR89]

| Order $n$ | Degree $m$ | $C_{n, m}$ coefficient | $S_{n, m}$ coefficient |
| :---: | :---: | :---: | :---: |
| 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 |
| 2 | 0 | -0.04698 | 0 |
| 2 | 1 | 0.00136 | 0.00138 |
| 2 | 2 | 0.02276 | -0.000202 |
| 3 | 0 | 0.00293 | 0 |
| 3 | 1 | -0.00309 | 0.00181 |
| 3 | 2 | -0.00847 | -0.000655 |
| 3 | 3 | 0.00224 | -0.01392 |
| 4 | 0 | 0.00762 | 0 |
| 4 | 1 | 0.00347 | -0.000776 |
| 4 | 2 | -0.00288 | -0.00112 |
| 4 | 4 | -0.0028 | 0.00337 |
| 4 | -0.0012 | -0.000622 |  |



## Reusable Launcher Model Linearisation

The study of the combined use of Thrust Vector Control (TVC) and fins during descent flight is particularly novel in launcher control literature. Descent flight control of a Vertical Take-off and Vertical Landing (VTVL) vehicle has been addressed in [Boe99], but only TVC was considered. Although flight dynamics models are nonlinear, models for control design and analysis are often linearised. Hence, a linear representation of the more general Reusable Launch Vehicle (RLV) dynamics in ascent and descent flight is derived in this appendix.

This derivation is based on the axisymmetric launcher of Fig. 8.5 with quasi-zero roll rate, for which pitch and yaw motions can be assumed uncoupled and the task of attitude control design and analysis can be performed in a single plane. The model of the vehicle in the pitch plane is provided in Fig. 10.1, reproduced below for ease of readability.


Figure 10.1: RLV model diagram (repeated from page 169)

Since this work is mostly dedicated to the atmospheric flight phase, the use of cold gas thrusters is not accounted for in the model. In addition, the main focus is on the impact of uncertainties and wind disturbances on flight performance, hence the effects of actuator, sensor, bending and sloshing dynamics are not considered for the sake of simplicity.

## Nonlinear equations of motion

The nonlinear equations of motion under analysis are obtained by writing Eq. (8.8) and (8.9) in the vehicle's body axes, depicted in Fig. 10.1 as $\{x, z\}$. Here, $\theta, \dot{\theta}$ and $\ddot{\theta}$ represent pitch angle and first/second-order derivatives, and $z, \dot{z}$ and $\ddot{z}$ are the lateral drift and derivatives.

Referring to Fig. 10.1, the sum of forces in the $z$ axis yields:

$$
\begin{equation*}
m(\ddot{z}-\dot{\theta} \dot{x})=-Q S_{\mathrm{ref}} C_{N}\left(\alpha^{\mathrm{CP}}\right)+2 Q S_{\mathrm{fin}} C_{\mathrm{f}}\left(\alpha_{\mathrm{fin}}\right) \cos \beta_{\mathrm{fin}}-T \sin \beta_{\mathrm{TVC}}+m g \cos \theta \tag{C.1}
\end{equation*}
$$

with all the variables introduced in Sec. 8.1. Note also that the dependence on time has been omitted for improved readability, as well as the subscript "y" for pitch-plane fin and TVC deflections, $\beta_{\text {fin }}$ and $\beta_{\text {TVC }}$.

Equivalently, the sum of lateral moments corresponds to:

$$
\begin{equation*}
J_{N} \ddot{\theta}=l_{\alpha} Q S_{\mathrm{ref}} C_{N}\left(\alpha^{\mathrm{CP}}\right)-2 l_{\mathrm{f}} Q S_{\mathrm{fin}} C_{\mathrm{f}}\left(\alpha_{\mathrm{fin}}\right) \cos \beta_{\mathrm{fin}}-l_{\mathrm{c}} T \sin \beta_{\mathrm{TVC}} \tag{C.2}
\end{equation*}
$$

where:

$$
\begin{equation*}
l_{\alpha}=x_{\mathrm{CP}}-x_{\mathrm{CG}}, \quad l_{\mathrm{f}}=x_{\mathrm{fin}}-x_{\mathrm{CG}}, \quad l_{\mathrm{c}}=x_{\mathrm{CG}}-x_{\mathrm{PVP}} \tag{C.3}
\end{equation*}
$$

## Steady-state solutions

The steady-state solutions of Eq. (C.1) and (C.2) consist in the set of Angle of Attack (AoA), pitch, fin and TVC angles ( $\alpha_{0}, \theta_{0}, \beta_{\mathrm{fin}, 0}$ and $\beta_{\mathrm{TVC}, 0}$ ) required to trim the vehicle, i.e. to cancel out its acceleration. For the translational motion, $\ddot{z}=0$ gives:

$$
\begin{equation*}
-m q_{0} V \cos \alpha_{0}=-Q S_{\mathrm{ref}} C_{N}\left(\alpha_{0}^{\mathrm{CP}}\right)+2 Q S_{\mathrm{fin}} C_{\mathrm{f}}\left(\alpha_{\mathrm{fin}, 0}\right) \cos \beta_{\mathrm{fin}, 0}-T \sin \beta_{\mathrm{TVC}, 0}+m g \cos \theta_{0} \tag{C.4}
\end{equation*}
$$

where $q_{0}$ represents the trim pitch rate and $V \cos \alpha_{0}$ is the longitudinal velocity (i.e. along the $x$ axis). In terms of rotational motion, $\ddot{\theta}=0$ results in:

$$
\begin{equation*}
0=l_{\alpha} Q S_{\mathrm{ref}} C_{N}\left(\alpha_{0}^{\mathrm{CP}}\right)-2 l_{\mathrm{f}} Q S_{\mathrm{fin}} C_{\mathrm{f}}\left(\alpha_{\mathrm{fin}, 0}\right) \cos \beta_{\mathrm{fin}, 0}-l_{\mathrm{c}} T \sin \beta_{\mathrm{TVC}, 0} \tag{C.5}
\end{equation*}
$$

## Perturbed equations of motion

In order to linearise the equations of motion, each term is first replaced by the corresponding steady-state plus a perturbation, e.g. $\nu=\nu_{0}+\delta \nu$. Applying this transformation to Eq. (C.1) yields:

$$
\begin{align*}
& m\left(\delta \ddot{z}-q_{0} V \cos \alpha_{0}-q_{0} \delta \dot{x}-V \cos \alpha_{0} \delta \dot{\theta}-\delta \dot{\theta} \delta \dot{x}\right)=-Q S_{\mathrm{ref}} C_{N}\left(\alpha_{0}^{\mathrm{CP}}+\delta \alpha^{\mathrm{CP}}\right) \\
& \quad+2 Q S_{\mathrm{fin}} C_{\mathrm{f}}\left(\alpha_{\mathrm{fin}, 0}+\delta \alpha_{\mathrm{fin}}\right) \cos \left(\beta_{\mathrm{fin}, 0}+\delta \beta_{\mathrm{fin}}\right)-T \sin \left(\beta_{\mathrm{TVC}, 0}+\delta \beta_{\mathrm{TVC}}\right)+m g \cos \left(\theta_{0}+\delta \theta\right) \tag{C.6}
\end{align*}
$$

and Eq. (C.2) becomes:

$$
\begin{align*}
J_{N} \delta \ddot{\theta}=l_{\alpha} Q & S_{\mathrm{ref}} C_{N}\left(\alpha_{0}^{\mathrm{CP}}+\delta \alpha^{\mathrm{CP}}\right) \\
& -2 l_{\mathrm{f}} Q S_{\mathrm{fin}} C_{\mathrm{f}}\left(\alpha_{\mathrm{fin}, 0}+\delta \alpha_{\mathrm{fin}}\right) \cos \left(\beta_{\mathrm{fin}, 0}+\delta \beta_{\mathrm{fin}}\right)-l_{\mathrm{c}} T \sin \left(\beta_{\mathrm{TVC}, 0}+\delta \beta_{\mathrm{TVC}}\right) \tag{C.7}
\end{align*}
$$

## Aerodynamics linearisation

Aerodynamic forces are linearised around the steady-state AoA using the corresponding force gradient coefficient. The RLV normal force, applied at its Centre of Pressure (CP), is then approximated by:

$$
\begin{equation*}
C_{N}\left(\alpha_{0}^{\mathrm{CP}}+\delta \alpha^{\mathrm{CP}}\right) \approx C_{N}\left(\alpha_{0}^{\mathrm{CP}}\right)+C_{N_{\alpha}} \delta \alpha^{\mathrm{CP}} \tag{C.8}
\end{equation*}
$$

Regarding the fins, it is convenient to recall Eq. (8.30), i.e.:

$$
\begin{equation*}
\alpha_{\mathrm{fin}}=\beta_{\mathrm{fin}}-\alpha^{\mathrm{fin}} \tag{C.9}
\end{equation*}
$$

which states that the fin AoA $\alpha_{\text {fin }}$ depends on the fin deflection $\beta_{\text {fin }}$ and on the vehicle's AoA at the fin's location $\alpha^{\mathrm{fin}}$. Based on Eq. (C.9), the force generated by the fins is linearised as follows:

$$
\begin{align*}
C_{\mathrm{f}}\left(\alpha_{\mathrm{fin}, 0}+\delta \alpha_{\mathrm{fin}}\right) & \approx C_{\mathrm{f}}\left(\alpha_{\mathrm{fin}, 0}\right)+C_{\mathrm{f}_{\alpha}} \delta \alpha_{\mathrm{fin}} \\
& \approx C_{\mathrm{f}}\left(\alpha_{\mathrm{fin}, 0}\right)+C_{\mathrm{f}_{\alpha}} \delta \beta_{\mathrm{fin}}-C_{\mathrm{f}_{\alpha}} \delta \alpha^{\text {fin }} \tag{C.10}
\end{align*}
$$

## Small angle approximations

The linearisation of the model relies on a set of well-known approximations for functions of a small perturbation $\delta \nu$, including:

$$
\begin{align*}
\delta \nu \delta \nu & \approx 0  \tag{C.11}\\
\cos \delta \nu & \approx 1  \tag{C.12}\\
\sin \delta \nu & \approx \delta \nu  \tag{C.13}\\
\cos \left(\nu_{0}+\delta \nu\right) & \approx \cos \nu_{0}-\sin \nu_{0} \delta \nu  \tag{C.14}\\
\sin \left(\nu_{0}+\delta \nu\right) & \approx \sin \nu_{0}+\cos \nu_{0} \delta \nu \tag{C.15}
\end{align*}
$$

Multiplying Eq. (C.10) by (C.14) with $\nu=\beta_{\text {fin }}$, it is also useful to consider the following relationship:

$$
\begin{align*}
& C_{\mathrm{f}}\left(\alpha_{\mathrm{fin}, 0}+\delta \alpha_{\mathrm{fin}}\right) \cos \left(\beta_{\mathrm{fin}, 0}+\delta \beta_{\mathrm{fin}}\right) \approx \\
& \quad \approx C_{\mathrm{f}}\left(\alpha_{\mathrm{fin}, 0}\right) \cos \beta_{\mathrm{fin}, 0}+\left[C_{\mathrm{f}_{\alpha}} \cos \beta_{\mathrm{fin}, 0}-C_{\mathrm{f}}\left(\alpha_{\mathrm{fin}, 0}\right) \sin \beta_{\mathrm{fin}, 0}\right] \delta \beta_{\mathrm{fin}}-C_{\mathrm{f}_{\alpha}} \cos \beta_{\mathrm{fin}, 0} \delta \alpha^{\mathrm{fin}} \tag{C.16}
\end{align*}
$$

Using the approximations of Eq. (C.8), (C.11) and (C.14) to (C.16), as well as Eq. (C.4) to remove the steady-state terms, the translational dynamics of Eq. (C.6) can now be recast as:

$$
\begin{array}{r}
m\left(\delta \ddot{z}-q_{0} V \cos \overline{\alpha_{0}}-q_{0} \delta \dot{x}-V \cos \alpha_{0} \delta \dot{\theta}-\delta \dot{\theta} \dot{\delta} \dot{\dot{x}} \approx 0\right)=-Q S_{\mathrm{ref}}\left\{\underline{\left.C_{\mathcal{N}}\left(\alpha_{0}^{\mathrm{CP}}\right)+C_{N_{\alpha}} \delta \alpha^{\mathrm{CP}}\right\}}\right. \\
+2 Q S_{\mathrm{fin}}\left\{\underline{C_{\mathrm{f}}\left(\alpha_{\mathrm{fin}, 0}\right) \cos \beta_{\mathrm{fin}, 0}}+\left[C_{\mathrm{f}_{\alpha}} \cos \beta_{\mathrm{fin}, 0}-C_{\mathrm{f}}\left(\alpha_{\mathrm{fin}, 0}\right) \sin \beta_{\mathrm{fin}, 0}\right] \delta \beta_{\mathrm{fin}}-C_{\mathrm{f}_{\alpha}} \cos \beta_{\mathrm{fin}, 0} \delta \alpha^{\mathrm{fin}}\right\} \\
-T\left\{\underline{\left.\sin \beta_{\mathrm{TVC}, 0}+\cos \beta_{\mathrm{TVC}, 0} \delta \beta_{\mathrm{TVC}}\right\}+m g\left\{\cos \theta_{0}-\sin \theta_{0} \delta \theta\right\} \quad \text { (C.17) }}\right. \tag{C.17}
\end{array}
$$

Equivalently, using the approximations of Eq. (C.8), (C.15) and (C.16), with Eq. (C.5) to eliminate the steady-state terms, the rotational dynamics of Eq. (C.7) becomes:

$$
\begin{align*}
& J_{N} \delta \ddot{\theta}=l_{\alpha} Q S_{\mathrm{ref}}\left\{\underline{C_{N}\left(\alpha_{0}^{\mathrm{CP}}\right)}+C_{N_{\alpha}} \delta \alpha^{\mathrm{CP}}\right\} \\
& -2 l_{\mathrm{f}} Q S_{\mathrm{fin}}\left\{\underline{\left.C_{\mathrm{f}}\left(\alpha_{\mathrm{ffn}, 0}\right) \cos \beta_{\mathrm{fin}, 0}+\left[C_{\mathrm{f}_{\alpha}} \cos \beta_{\mathrm{fin}, 0}-C_{\mathrm{f}}\left(\alpha_{\mathrm{fin}, 0}\right) \sin \beta_{\mathrm{fin}, 0}\right] \delta \beta_{\mathrm{fin}}-C_{\mathrm{f}_{\alpha}} \cos \beta_{\mathrm{fin}, 0} \delta \alpha^{\mathrm{fin}}\right\}} \begin{array}{rl} 
& \quad l_{\mathrm{c}} T\left\{\underline{\sin \beta_{\mathrm{TVC}, 0}}+\cos \beta_{\mathrm{TVC}, 0} \delta \beta_{\mathrm{TVC}}\right\} \quad \text { (C.18) }
\end{array}\right.
\end{align*}
$$

## Angle of attack linearisation

In addition to the aerodynamic forces, the AoA is affected by pitch and drift variations, $\delta \theta$ and $\delta \dot{z}$, as well as by wind gusts with velocity $v_{\mathrm{w}}$ (assumed constant along the launcher body). Using a small angle approximation for the inverse tangent of Eq. (8.14), AoA perturbations at the vehicle's Centre of Gravity (CG) correspond to:

$$
\begin{equation*}
\delta \alpha \approx \delta \theta+\frac{\delta \dot{z}-v_{\mathrm{w}}}{V \cos \alpha_{0}} \tag{C.19}
\end{equation*}
$$

At other locations, local velocity variations also depend on the effect of the vehicle's rotation, and therefore AoA perturbations at the CP and fin's location are respectively given by:

$$
\begin{equation*}
\delta \alpha^{\mathrm{CP}} \approx \delta \alpha-\frac{l_{\alpha}}{V \cos \alpha_{0}} \delta \dot{\theta}, \quad \delta \alpha^{\mathrm{fin}} \approx \delta \alpha-\frac{l_{\mathrm{f}}}{V \cos \alpha_{0}} \delta \dot{\theta} \tag{C.20}
\end{equation*}
$$

Replacing Eq. (C.19) in the local perturbations $\delta \alpha^{\mathrm{CP}}$ and $\delta \alpha^{\text {fin }}$ of Eq. (C.20) and the latter in Eq. (C.17), the translational dynamics is re-written as:

$$
\begin{align*}
& m\left(\delta \ddot{z}-q_{0} \delta \dot{x}-V \cos \alpha_{0} \delta \dot{\theta}\right)=-Q S_{\mathrm{ref}} C_{N_{\alpha}}\{ \left.\delta \theta+\frac{\delta \dot{z}}{V \cos \alpha_{0}}-\frac{v_{\mathrm{w}}}{V \cos \alpha_{0}}-\frac{l_{\alpha}}{V \cos \alpha_{0}} \delta \dot{\theta}\right\} \\
&-2 Q S_{\mathrm{fin}} C_{\mathrm{f}_{\alpha}} \cos \beta_{\mathrm{fin}, 0}\left\{\delta \theta+\frac{\delta \dot{z}}{V \cos \alpha_{0}}-\frac{v_{\mathrm{w}}}{V \cos \alpha_{0}}-\frac{l_{\mathrm{f}}}{V \cos \alpha_{0}} \delta \dot{\theta}\right\} \\
&+2 Q S_{\mathrm{fin}}\left[C_{\mathrm{f}_{\alpha}} \cos \beta_{\mathrm{fn}, 0}-C_{\mathrm{f}}\left(\alpha_{\mathrm{fin}, 0}\right) \sin \beta_{\mathrm{fin}, 0}\right] \delta \beta_{\mathrm{fin}} \\
&-T \cos \beta_{\mathrm{TVC}, 0} \delta \beta_{\mathrm{TVC}}-m g \sin \theta_{0} \delta \theta \tag{C.21}
\end{align*}
$$

Following the same procedure, the rotational dynamics of Eq. (C.18) becomes:

$$
\begin{align*}
& J_{N} \delta \ddot{\theta}=l_{\alpha} Q S_{\mathrm{ref}} C_{N_{\alpha}}\left\{\delta \theta+\frac{\delta \dot{z}}{V \cos \alpha_{0}}-\frac{v_{\mathrm{w}}}{V \cos \alpha_{0}}-\frac{l_{\alpha}}{V \cos \alpha_{0}} \delta \dot{\theta}\right\} \\
& +2 l_{\mathrm{f}} Q S_{\mathrm{fin}} C_{\mathrm{f}_{\alpha}} \cos \beta_{\mathrm{fin}, 0}\left\{\delta \theta+\frac{\delta \dot{z}}{V \cos \alpha_{0}}-\frac{v_{\mathrm{w}}}{V \cos \alpha_{0}}-\frac{l_{\mathrm{f}}}{V \cos \alpha_{0}} \delta \dot{\theta}\right\} \\
& \\
& \quad-2 l_{\mathrm{f}} Q S_{\mathrm{fin}}\left[C_{\mathrm{f}_{\alpha}} \cos \beta_{\mathrm{fin}, 0}-C_{\mathrm{f}}\left(\alpha_{\mathrm{fin}, 0}\right) \sin \beta_{\mathrm{fin}, 0}\right] \delta \beta_{\mathrm{fin}}  \tag{C.22}\\
& \quad-l_{\mathrm{c}} T \cos \beta_{\mathrm{TVC}, 0} \delta \beta_{\mathrm{TVC}}
\end{align*}
$$

## Final assumptions

Finally, the following two assumptions are also applied:

$$
\begin{align*}
q_{0} \delta \dot{x} & \approx 0  \tag{C.23}\\
C_{\mathrm{f}}\left(\alpha_{\mathrm{fin}, 0}\right) \sin \beta_{\mathrm{fin}, 0} & \approx 0 \tag{C.24}
\end{align*}
$$

The first assumption, Eq. (C.23), is very realistic since $q_{0}$ and $\delta \dot{x}$ are very small (particularly compared to $\delta \dot{z}$ ) and allows to uncouple the longitudinal and lateral dynamics. The second one, Eq. (C.24), is not genuinely necessary for the linearisation as that term is already linear, but it is also realistic because both $C_{\mathrm{f}}\left(\alpha_{\mathrm{fin}, 0}\right)$ (recall Eq. (8.29)) and $\sin \beta_{\mathrm{fin}, 0}$ are small in atmospheric flight.

In previous works on ascent flight control $\left[\mathrm{SBM}^{+} 16, \mathrm{NMS}^{+} 19\right]$, it was further assumed that trim angles are close to zero, i.e. $\cos \alpha_{0} \approx \cos \beta_{\mathrm{TVC}, 0} \approx 1$, but this simplification is not applicable in the general RLV case.

## Linear equations of motion

With the assumptions of Eq. (C.23) and (C.24), Eq. (C.21) can be further re-organised into:

$$
\begin{align*}
& m\left(\delta \ddot{z}-u_{0} \delta \dot{\theta}\right)=-N_{\alpha^{\prime}} \delta \theta-\frac{N_{\alpha^{\prime}}}{V \cos \alpha_{0}} \delta \dot{z}+\frac{N_{\alpha^{\prime}}}{V \cos \alpha_{0}} v_{\mathrm{w}}+\frac{l_{\alpha} N_{\alpha}+l_{\mathrm{f}} N_{\mathrm{f}} \cos \beta_{\mathrm{fin}, 0}}{V \cos \alpha_{0}} \delta \dot{\theta} \\
&+N_{\mathrm{f}} \cos \beta_{\mathrm{fin}, 0} \delta \beta_{\mathrm{fin}}-T \cos \beta_{\mathrm{TVC}, 0} \delta \beta_{\mathrm{TVC}}-m a_{0} \delta \theta \tag{C.25}
\end{align*}
$$

and Eq. (C.22) into:

$$
\begin{align*}
& \delta \ddot{\theta}=\mu_{\alpha^{\prime}} \delta \theta+\frac{\mu_{\alpha^{\prime}}}{V \cos \alpha_{0}} \delta \dot{z}-\frac{\mu_{\alpha^{\prime}}}{V \cos \alpha_{0}} v_{\mathrm{w}}-\frac{l_{\alpha} \mu_{\alpha}+l_{\mathrm{f}} \mu_{\mathrm{f}} \cos \beta_{\mathrm{fin}, 0}}{V \cos \alpha_{0}} \delta \dot{\theta} \\
&-\mu_{\mathrm{f}} \cos \beta_{\mathrm{fin}, 0} \delta \beta_{\mathrm{fin}}-\mu_{\mathrm{c}} \cos \beta_{\mathrm{TVC}, 0} \delta \beta_{\mathrm{TVC}} \tag{C.26}
\end{align*}
$$

To arrive at these two equations, a few additional variables are defined, which include trim velocity and acceleration:

$$
\begin{equation*}
u_{0}=V \cos \alpha_{0}, \quad a_{0}=g \sin \theta_{0} \tag{C.27}
\end{equation*}
$$

aerodynamic and fin force gradients:

$$
\begin{equation*}
N_{\alpha}=Q S_{\mathrm{ref}} C_{N_{\alpha}}, \quad N_{\mathrm{f}}=2 Q S_{\mathrm{fin}} C_{\mathrm{f}_{\alpha}}, \quad N_{\alpha^{\prime}}=N_{\alpha}+N_{\mathrm{f}} \cos \beta_{\mathrm{fin}, 0} \tag{C.28}
\end{equation*}
$$

and moment proneness coefficients:

$$
\begin{equation*}
\mu_{\alpha}=\frac{N_{\alpha}}{J_{N}} l_{\alpha}, \quad \mu_{\mathrm{f}}=\frac{N_{\mathrm{f}}}{J_{N}} l_{\mathrm{f}}, \quad \mu_{\mathrm{c}}=\frac{T}{J_{N}} l_{\mathrm{c}}, \quad \mu_{\alpha^{\prime}}=\mu_{\alpha}+\mu_{\mathrm{f}} \cos \beta_{\mathrm{fin}, 0} \tag{C.29}
\end{equation*}
$$

## State-space representation

The perturbed RLV dynamics of Eq. (C.25) and (C.26) can then be written in the state-space form as follows:

$$
\left[\begin{array}{llll}
\delta \dot{\theta} & \delta \ddot{\theta} & \delta \dot{z} & \delta \ddot{z}
\end{array}\right]^{\mathrm{T}}=A_{\mathrm{RLV}}\left[\begin{array}{llll}
\delta \theta & \delta \dot{\theta} & \delta z & \delta \dot{z}
\end{array}\right]^{\mathrm{T}}+B_{\mathrm{RLV}}\left[\begin{array}{c}
\mathbf{u}  \tag{C.30}\\
v_{\mathrm{w}}
\end{array}\right]
$$

where the vector $\mathbf{u}=\left[\begin{array}{ll}\delta \beta_{\mathrm{TVC}} & \delta \beta_{\text {fin }}\end{array}\right]^{\mathrm{T}}$ gathers TVC and fin control inputs, and with matrices $A_{\mathrm{RLV}}$ and $B_{\mathrm{RLV}}$ given by:

$$
\begin{align*}
& A_{\mathrm{RLV}}=\left[\begin{array}{cccc}
0 & 1 & 0 & 0 \\
\mu_{\alpha^{\prime}} & -\frac{l_{\alpha} \mu_{\alpha}+l_{\mathrm{f}} \mu_{\mathrm{f}} \cos \beta_{\mathrm{fin}, 0}}{V \cos \alpha_{0}} & 0 & \frac{\mu_{\alpha^{\prime}}}{V \cos \alpha_{0}} \\
0 & 0 & 0 & 1 \\
-\frac{N_{\alpha^{\prime}}}{m}-a_{0} & \frac{l_{\alpha} N_{\alpha}+l_{\mathrm{f}} N_{\mathrm{f}} \cos \beta_{\mathrm{fin}, 0}}{m V \cos \alpha_{0}}+u_{0} & 0 & -\frac{N_{\alpha^{\prime}}}{m V \cos \alpha_{0}}
\end{array}\right]  \tag{C.31}\\
& B_{\mathrm{RLV}}=\left[\begin{array}{cc|c}
0 & 0 & 0 \\
-\mu_{\mathrm{c}} \cos \beta_{\mathrm{TVC}, 0} & -\mu_{\mathrm{f}} \cos \beta_{\mathrm{fin}, 0} & -\frac{\mu_{\alpha^{\prime}}}{V \cos \alpha_{0}} \\
0 & 0 & 0 \\
-\frac{T}{m} \cos \beta_{\mathrm{TVC}, 0} & \frac{N_{\mathrm{f}}}{m} \cos \beta_{\mathrm{fin}, 0} & \frac{N_{\alpha^{\prime}}}{m V \cos \alpha_{0}}
\end{array}\right] \tag{C.32}
\end{align*}
$$

## Gravity turn case

It is important to highlight that during a gravity turn trajectory (which is the case for most of the launch phase, recall Fig. 8.7), the commanded pitch reference is such that the centripetal acceleration $q_{0} u_{0}$ is equalised by the gravity. In (and only in) these specific conditions, it is common practice [Gre70, Suz04, WDW08, OJWM09] to linearise the equations of motion using a local trajectory frame, $\left\{x_{L}, z_{L}\right\}$ in Fig. 10.1, as the term $u_{0}$ can be set to zero with the remaining trim acceleration on that frame given by:

$$
\begin{equation*}
a_{0}=\frac{T \cos \beta_{\mathrm{TVC}, 0}-A}{m} \tag{C.33}
\end{equation*}
$$

where $A$ represents the axial drag force component.
In practice, care must be taken when switching between the conventions of Eq. (C.27) and (C.33) essentially because the drift motion will be quantified with respect to different reference frames (i.e., body in the former case and trajectory in the latter).


[^0]:    ${ }^{1}$ Classical control theory is not able to provide an explicit consideration of plant uncertainties, stability/performance guarantee by design or frequency-wise insight on driving perturbations, but some ramifications such as Quantitative Feedback Theory (QFT) [Hor01, HRGS05] allow to exploit these features without the need for robust control techniques. However, the use of QFT in the literature is not widespread, mostly because it is more complex than classical techniques and still does not offer the ability to systematically manage multiple control requirements in a MIMO fashion (in opposition to robust control).

[^1]:    ${ }^{1}$ Website: https://www.tasc-group.com/

