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A NON-ASSOCIATIVE MACRO-ELEMENT MODEL FOR VERTICAL PLATE ANCHORS IN CLAY

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Abstract. This work proposes a new plastic hardening, non-associative macro-element model to predict the behaviour of anchors in clay for floating offshore structures during keying and up to the peak load. Building on available models for anchors, a non-associated plastic potential is introduced to improve prediction of anchor trajectory and loss of embedment at peak conditions for a large range of padeye offsets and different pull-out directions. The proposed model also includes a displacement-hardening rule to simulate the force and displacement mobilisation at the early stages of the keying process. The model is challenged and validated against different sets of numerical and centrifuge data. This extensive validation process revealed that two of the four newly introduced model parameters assume a constant value for the range of simulated cases. This suggests that only two of the newly introduced parameters may need to be calibrated for the use of the proposed macro-element model in practice.

Keywords: Plate anchor; Macro-element model; Pull-out resistance; Trajectory; Floating offshore structures.

1 Introduction

The offshore renewable energy sector is moving towards the installation of floating wind production plants in deeper water sites, as well as pioneering new floating wave energy converter devices. In this context, floating structures are fixed to the seabed through mooring lines connected to an anchoring system, which differs from fixed foundation solutions such as monopiles typically used in water depths of up to 60m. Among different anchoring solutions, plate anchors provide considerable vertical holding capacity through a large embedded plate, which is commonly referred as fluke (Tian et al. 2015). Plate anchors can be installed by dragging into the seabed using drag-in vertically loaded plate anchors (VLAs) (e.g. Murff et al. 2005; Liu 2012; Aubeny and Chi 2014), or can be installed with the fluke in vertical position by an external mandrel, using suction (e.g. suction-embedded plate anchors – SEPLA, Dove et al. 1998; Wilde et al. 2001; Han et al. 2016) or free-fall gravity (e.g. dynamically embedded plate anchors – DEPLA, O'Loughlin et al. 2014). In plate anchors, the mooring line attached to the padeye is tensioned after the anchor is installed, making the anchor rotate from its initial position and become approximately normal to the load applied by the mooring line (Dove et al. 1998). This process of rotation is called keying, during which the anchor experiences vertical motion, resulting in loss of embedment (Gaudin et al. 2015). The major concern associated with offshore plate anchors is the anchor trajectory prediction during keying and the operational loading (Gaudin et al. 2006; Song et al. 2009; Yang et al. 2012), which enables accurate determination of current embedment and hence anchor's capacity.

Several researchers have carried out centrifuge tests and numerical analyses to assess the keying behaviour of plate anchors in clay (Song et al. 2006; Song et al. 2008; Hu and Song 2008; Song et al. 2009; Wang et al. 2011; Beemer and Aubeny 2012; Cassidy et al. 2012; Zhao and Liu 2014; Tian et al. 2015). Numerical modelling of anchor keying is a challenging and time-consuming 3-D problem which requires the careful handling of large mesh deformation associated with the considerable anchor displacements, as well as complex solutions for modelling the plate-soil-chain interactions. Centrifuge modelling, on the other hand, also faces considerable challenges in enabling extended assessments of plate-soil-chain behaviour due to the complexity of sample preparation and testing. In the interest of simplicity, macro-element modelling techniques can represent a valuable tool for exploring the mechanical response of soil-anchor interaction problems at very low computational costs while accounting for all the key problem variables (e.g. anchor geometry, embedment, soil properties, loading conditions). Macro-element models have already been successfully employed to predict the mechanical response of shallow foundation behaviour (e.g. Nova and Montrasio 1991; Cremer et al. 2001), spudcans (Martin and Houlsby 2001) and plate anchors (e.g. Cassidy et al. 2012), among others.

Two macro-element models for plate anchors are available in the literature (Cassidy et al. 2012; Yang et al. 2012). Chain and SEPLA Plasticity Analysis – CASPA (Cassidy et al. 2012) is based on a classical rigid plasticity theory developed to predict forces, rotation and displacements of the plate anchor during keying and up to the peak load. The model incorporates the chain solution by Neubecker and Randolph (1995). This model can predict the ultimate resistance of the anchor, the rotation and anchor displacement in the first stage of the keying process and the influence of different padeye offsets. However, the simplified nature of the employed macro-element model, especially in relation to rigid plasticity and the assumption of an associated flow rule governing the development and direction of plastic displacements, resulted in incorrect predictions of the anchor trajectory, including excessive initial backward and upward movements and downward trajectories (i.e. re-embedment of the anchor) in the medium-large displacement domain for many padeye offset values. Similarly, Yang et al. (2012) employed a rigid plasticity approach with an associated plastic potential and a chain solution to predict load capacity and anchor trajectory, but accounting for the presence of a hinged flap. The presence of the flap resulted in reduced backward movement, slightly more embedment loss and higher ultimate capacity.

Based on the above developments, an improved macro-element model for plate anchors is proposed in this paper. The new model is characterised by a non-associated plastic potential while a plastic hardening rule is also implemented. These two additional ingredients allow a controlled simulation of plastic displacements since the early stage of keying (through the inclusion of the plastic hardening rule) and enable a more accurate prediction of the anchor trajectory across the whole displacement domain and for a large range of padeye offsets (through the inclusion of the non-associated plastic potential). While these improvements come at the expense of four additional model parameters (three for the plastic potential and one for the hardening rule), the application of the model to different datasets of experimental and numerical studies for variable anchor geometries and initial conditions demonstrates that only one parameter related to the plastic potential needs to be calibrated in practice (alongside the parameter related to the hardening rule) and they may be used to capture the effect of the installation procedure. Still, this paper presents a set of parameters that produces satisfactory results for most analysed cases and therefore can be used as a starting point in the model calibration procedure.

2 Geometry and definitions

The geometry of a generic plate anchor and the definition of the involved force and displacement components are represented in two-dimensions in Fig. 1a, and a sketch is displayed in Fig. 1b. The overall in-plane length of the anchor's fluke is denoted by *B*, while the position (eccentricity) of the padeye with respect to the centre of the fluke is defined by e_p and e_n in the direction parallel and perpendicular to the fluke, respectively. The chain is connected to the padeye of the anchor exerting a force T_a with an inclination θ_a, while the inclination of the chain at the mudline is denoted by θ_0. The pulling force of the chain results in a combination of loads to the anchor: normal (*V*), sliding (*H*) and rotational (*M*) forces which are considered to be applied at the centre of the fluke (i.e. *B*/2 from each end of the anchor). Imposing force equilibrium conditions to the anchor, the following relationships can be obtained:

$$V = T_a \sin(\beta + \pi/2 - \theta_a) - W' \sin\beta \tag{1}$$

$$H = T_a \cos(\beta + \pi/2 - \theta_a) - W' \cos\beta \tag{2}$$

$$M = T_a \; [e_n \cos(\beta + \pi/2 - \theta_a) + e_p \sin(\beta + \pi/2 - \theta_a) \tag{3}$$

where β is the current inclination of the anchor from the vertical direction and W' is the effective anchor weight. The incremental tangential and normal displacement (with respect to the fluke direction) and rotation of the anchor at the centre of the fluke are defined by δu, δw and $\delta\beta$. The horizontal and vertical displacement increments, defined by δx and δz, are linked to the incremental displacement by:

$$\delta x = \cos(\beta)\, \delta w - \sin(\beta)\, \delta u \tag{4}$$

$$\delta z = \sin(\beta)\, \delta w + \cos\,(\beta)\, \delta u \tag{5}$$

3 Modelling framework

3.1 General

Following previous developments (i.e. Cassidy et al. 2012), this model considers a macro-element model for the mechanical response of the anchor (shown in black in Fig. 1a) interacting with the chain (shown in grey in Fig. 1a) according to the solution proposed by Neubecker and Randolph (1995). The macro-element model for the anchor is based on the theory of incremental plasticity. A schematic 2-D illustration of the surfaces and modelling strategy for the mechanical response of the plate anchor is provided in Fig. 2 in the normalised V/V_M versus H/H_M force plane (imposing M/M_M =0). The terms V_M, H_M and M_M are the normal, sliding and rotational capacities when acting independently on the anchor. These capacities are commonly defined through the capacity factors N_v, N_h and N_m, with:

$$N_v = V_M/(L\ B\ s_u) \tag{6}$$

$$N_h = H_M/(L\ B\ s_u) \tag{7}$$

$$N_m = M_M/(L\ B^2\ s_u) \tag{8}$$

The current chain load state (H, V, M) lies on the loading surface (f in Fig. 2) which can change in size during keying or operational loading. The strength surface (F) represents the maximum capacity of the anchors, and in turn the maximum size of the loading surface. The direction of the plastic deformation is defined by the normal to the plastic potential (g) passing through the current chain load state.

3.2 Macro-element modelling of plate anchor

Loading and strength surfaces. The loading surface (f), expressed in Eq. 9, and the strength surface (F) follow a similar format to that suggested by Bransby and O'Neill (1999) and applied to drag anchors (Elkhatib and Randolph 2005; Aubeny and Chi 2010) and to plate anchors (Murff et al. 2005; Cassidy et al. 2012; Yang et al. 2012). In its original form, Eq. 9 included an exponent 1/p to the sum of the second and third terms (involving the moment and horizontal loads). However, the aforementioned references showed that p assumes values close to the unity, thus it has been supressed in the present study:

$$f = \left(\frac{V}{V_M}\right)^q + \left(\frac{|M|}{M_M}\right)^m + \left(\frac{|H|}{H_M}\right)^n - \rho_c = 0 \quad (9)$$

where m, n and q are the terms defining the shape of the three-dimensional surface in the V/V_M-H/H_M-M/M_M space and ρ_c is the hardening parameter which varies between 0 and 1 and defines the size of the loading surface. The strength surface (F) is obtained by imposing ρ_c =1. The capacities V_M, H_M and M_M are dependent on the capacity factors N_v, N_h and N_m, as per Eqs. (6-8).

Hardening rule. As previously mentioned in this paper, the inclusion of a hardening rule aims to provide a controlled simulation of the early stages of keying, especially in the force-displacement domain. According to the theory of incremental plasticity, changes in size of the loading surface are linked to plastic displacements experienced by the anchor. The relationship between the loading surface's hardening term ρ_c and the plastic displacements u, w and β follows the finite form proposed by Nova and Montrasio (1991):

$$\rho_c = 1 - \exp\left\{-R_0\,[w^2 + u^2 + (B\,\beta)^2]^{1/2}\right\} \quad (10)$$

where R_0 is the non-dimensional hardening parameter. The hardening variable ρ_c is null at zero displacement and asymptotically approaches failure conditions ($\rho_c = 1$) for large value of displacements. It should be noted that Nova and Montrasio (1991) included non-dimensional weighting parameters for the displacement u and rotation β in its original form. However, a parametric analysis carried out in this study (omitted for the sake of brevity) has revealed that these additional parameters would have very little influence in the model predictions.

Plastic potential and flow rule. It is assumed that the plastic potential is characterised by an expression similar to the loading and failure surfaces, but its shape is modified through the introduction of scaling factors ξ, χ and ω for the normal, sliding and rotational capacities (V_M, H_M and M_M , respectively). An expression similar to the proposed by Nova and Montrasio (1991) has been selected as it provides an independent modification of the skewing on the V/V_M, H/H_M and M/M_M axis, which allows an independent calibration, as shown later in this manuscript. A slight modification to the power of the horizontal loads, if compared to the loading and strength surfaces in Eq. (9), has been introduced following the authors' experience in using the model and the better agreement achieved for all simulations carried out in the following of this paper.

$$g = \left(\frac{V}{V_M/\xi}\right)^q + \left(\frac{|M|}{M_M/\omega}\right)^m + \left(\frac{|H|}{H_M/\chi}\right)^m - \rho_g = 0 \tag{11}$$

where the parameter ρ_g defines the current size of the plastic potential but its numerical value has no practical relevance since only the derivatives of g are of interest for the determination of the vector of anchor incremental displacements $\boldsymbol{\delta q}$ (δw, δu and $\delta\beta$):

$$\boldsymbol{\delta q} = \begin{pmatrix} \delta w \\ \delta u \\ B\delta\beta \end{pmatrix} = \lambda \begin{pmatrix} \partial g/\partial V \\ \partial g/\partial H \\ \partial g/\partial(M/B) \end{pmatrix} \tag{12}$$

where λ is the plastic multiplier. Satisfaction of the consistency condition expressed in equation (13) ensures that the current load state ($\boldsymbol{Q} = [V, H, M]^{\mathrm{T}}$) lies always on the loading surface:

$$df(\boldsymbol{Q}, \rho_c) = \frac{\partial f}{\partial \boldsymbol{Q}} d\boldsymbol{Q} + \frac{\partial f}{\partial \rho_c} d\rho_c = 0 \tag{13}$$

By substituting the derivatives of Eq. (10) into Eq. (13) and then introducing Eq. (12), the following expression for the plastic multiplier can be obtained:

$$\lambda = \frac{-\left(\frac{\partial f}{\partial V}\dot{V} + \frac{\partial f}{\partial H}\dot{H} + \frac{\partial f}{\partial(M/B)}\dot{M}\right)}{\frac{\partial f}{\partial \rho_c}\left(\frac{\partial \rho_c}{\partial w}\frac{\partial g}{\partial V} + \frac{\partial \rho_c}{\partial u}\frac{\partial g}{\partial H} + \frac{\partial \rho_c}{\partial(B\beta)}\frac{\partial g}{\partial(M/B)}\right)} \tag{14}$$

3.3 Chain solution

The chain solution proposed by Neubecker and Randolph (1995) is included to relate the angle of pull at the mudline (θ_0) to the chain angle at the padeye (θ_a):

$$e^{\mu(\theta_a-\theta_0)}(\cos\theta_0+\mu\sin\theta_a)-\cos\theta_a-\mu\sin\theta_a=E_n d_{bar} N_c\left(s_{u0}z_p+\frac{k_{su}{z_p}^2}{2}\right)\left(\frac{1+\mu^2}{T_a}\right) \quad (15)$$

where d_{bar} is the diameter of the chain; E_n is a multiplier giving the effective chain width in the normal direction to the chain; N_c is the bearing capacity factor for the chain; z_p is the current vertical depth of the padeye; s_{u0} and k_{su} are the soil shear strength at the mudline and the rate of increase with depth; and μ is the friction between the soil and the chain.

3.4 Model parameters and calculation procedure

The proposed macro-element model requires the definition of (i) the geometry and properties of the anchor and the chain, (ii) the soil conditions and (iii) the calibration of the parameters for the anchor models. All these quantities are summarised in Table 1, which also provides the values adopted in the following parametric analysis and in the simulation exercises against different sets of experimental or numerical data, which considered uniform seabed profiles characterised by an undrained shear strength either constant or increasing linearly with depth ($s_u = s_{u0} + kz$, where s_{u0} is the undrained shear strength at mudline level, k is the strength gradient and z is the depth of the anchor centroid). The section in which the simulation exercises were carried out are indicated in square brackets. While 10 macro-element parameters are introduced, the following calibration procedure and model validation will show that most of them can be either determined from the literature or lie within a very narrow range, leaving just one of the plastic potential parameters (ω) to be calibrated.

The calculation procedure involves the following stages:

1) DEFINITION OF INITIAL CONDITIONS: The initial conditions of the simulation procedure consider a vertical orientation of the chain at the padeye after installation, by imposing an initial value of θ_a = 90°. The chain inclination at seabed level θ_0 is set to comply with the desired boundary conditions of the analysed problem. The chain solution in Eq. 15 is then employed to determine the initial value of the chain force T_a. Using the force-equilibrium equations (1) to (3), the initial value of the load components H, V and M on the plate anchors are determined

and in turn the initial value of the hardening parameters ρ_c can be defined through Eq. (9). The initial values of the displacements u and w are also updated using Eq. (10) where it is simplistically imposed no rotation of the plate (β=0°) and that the ratio $u/w=H/V$ is valid for the initial conditions. This is a rudimentary estimation with insignificant influence on the model simulations as the displacements under these initial conditions are negligible.

2) SIMULATION OF KEYING AND LOADING. The simulations are carried out imposing dynamic step increments to the monotonically-increasing hardening parameter ρ_c from the initial value determined in stage 1 above to the asymptotic value of 1. The problem is then solved by using a 4th order Runge-Kutta method to simultaneously solve the incremental form of force-equilibrium (Eq. 1 to 3), the consistency condition (Eq. 13), the three expressions for the plastic potential defined by Eq. (12), the chain solution in Eq. (15) and the displacement relationships in Eqs. (4) and (5) to find the values of the problem variables: load components (H, V, M); the displacements with respect to the fluke directions (u, w, β); the absolute displacement of the anchor (x and z); and the inclination and force at the padeye (θ_a and T_a, respectively).

4 Effect of new modelling features

The CASPA model, proposed by Cassidy et al. (2012), can be considered the predecessor of this model with the limitation of being formulated within a rigid plasticity framework and associated flow rule. Compared to CASPA, the proposed model introduces four new model parameters (ξ, χ, ω related to the shape of the plastic potential, and R_0 governing the hardening rule) whose influence will be analysed in this section. For this exercise, the same anchor and soil properties as in Cassidy et al. (2012) are assumed (Table 1). The initial orientation of the anchor is vertical (β=0°) while the imposed chain angle – and therefore the direction of the monotonic load – at the mudline is constant with θ_0 = 45°.

4.1 Plastic potential parameters

The variation of the value of the three plastic potential related parameters permits to skew the shape of the plastic potential (g) and modify the direction of the normal vectors to the plastic potential surfaces. The inclusion of these model parameters provides an additional degree of freedom to control the incremental displacements and the overall plate anchor trajectory during loading. Fig. 3 presents the independent influence of each of the three model parameters on the normalised total chain load (T_a/LBs_{ui}, with s_{ui} being the soil undrained shear strength at the initial anchor depth) and anchor rotation (α = 90° – β) versus the normalised padeye travel distance (d/B), and on the overall anchor trajectory. The anchor trajectory is represented in the normalised vertical displacement (z/B) versus the normalised horizontal displacement (x/B) plot. The padeye travel distance is defined by:

$$d = \int \sqrt{\delta x_P{}^2 + \delta z_P{}^2} \tag{16}$$

where δx_p and δz_p are the incremental horizontal and vertical displacements of the padeye, respectively.

Four values were adopted in the sensitivity analysis of the parameter ξ (0.5, 1.0, 1.5 and 2.0), which controls the intercept of the plastic potential on the V/V_M axis. This parameter seems to affect the load-displacement, rotation and trajectory of the anchor at large displacements only (Fig 3a, b, and c). The peak load (Fig. 3a) slightly decreases when ξ increases from 0.5 to 2.0 whereas the post-peak drop is faster for lower values of ξ. Anchor rotation (Fig. 3b) is not affected up to $d/B \approx 0.7$, beyond which lower values of ξ make the anchor rotate more rapidly up to its final orientation. The most important effect of ξ, though, is the capacity to modify the anchor trajectory (Fig. 3c). For higher values of ξ, the anchor moves upwards and no longer embeds again after a certain point, as observed in the model with associated plastic potential (ξ = 1). This feature would not be possible without the inclusion of a non-associative plastic potential.

For the parameter χ controlling the skewing on the H/H_M direction of the plastic potential surface, the values adopted were 0.5, 1.0, 1.2 and 1.5. As opposed to ξ, the parameter χ affects the anchor behaviour from the early stages of keying. Fig. 3d shows that the peak load decreases by 10% and the padeye travel distance at the peak load increases by 75% when χ increases from 0.5 to 1.5. In terms of anchor rotation, it can be observed from Fig. 3e that the anchor rotates more rapidly for lower values of χ. The increase in the padeye travel distance to peak load can be attributed to the increase in both vertical and horizontal displacements with increasing χ (see Fig. 3f). Within the analysed range, the sole variation of the values of χ does not prevent the re-embedment of anchor into the seabed at large displacements, which does not seem realistic given that the anchor chain is pulled upwards at an angle of 45°.

The parameter ω, controlling the skewing of the plastic potential in the M/M_M direction has qualitatively similar but opposite effect to χ. The increase of ω from 0.65 to 2.0 increases the peak load by 11% (Fig. 3g) and increases rotation significantly (Fig. 3 h), but decreases the anchor displacement (Fig. 3i), including a 77% reduction to the displacement at peak load (Fig. 3g). In terms of anchor trajectory (Fig. 3i), ω is not able to modify the direction of travel of the anchor, which seems to be parallel irrespective of the value adopted for this parameter.

4.2 Hardening parameter R_0

Four values were selected (0.5, 1.0, 1.5 and 2.5) to show the influence of the hardening parameter R_0 on the response of a plate anchor. The main effect of the inclusion of R_0 can be observed in the normalised chain load versus padeye travel distance plot presented in Fig. 4a. The increase of the hardening parameter R_0 provides larger initial stiffness but produces negligible effect on the peak loads. The influence of this parameter on the anchor rotation (Fig.4b) and trajectory is also negligible (Fig. 4c).

4.3 Discussion on chain load path and incremental displacements

While the introduced modelling features provide additional capability and versatility to the macro-element model, they do not seem to considerably influence the load path experienced by the anchor as shown in Fig. 5, where comparison for a CASPA type model and the new displacement hardening, non-associated flow model is presented. Beyond the fact that for the latter model the load path originates inside the strength surface, the main difference between the cases lies in the direction of the plastic incremental displacement vectors. It is the control of such directions, not limited to be normal to the strength surface, that provides the additional model capabilities for predicting the anchor trajectory, evolution of its rotation, peak response and post-peak drops.

5 MODEL PREDICTIONS

5.1 Introduction

For validation purposes, it is desirable that the model is calibrated and challenged against an extensive set of data covering wide range of loading and geometrical conditions (e.g. padeye eccentricities, anchor geometry or initial embedment). While extensive field or centrifuge testing program of this type are not available, results from numerical analyses, such as the three dimensional large-deformation finite-element (LDFE) analyses by Tian et al. (2015), offer a comprehensive set of data. In the numerical studies by Tian et al. (2015) the soil was modelled as elastic-perfectly plastic Tresca material with undrained shear strength s_u increasing with depth. The elastic behaviour of the soil was modelled by a Young's modulus E = $500s_u$ and a Poisson's ratio v = 0.49 (to model near undrained conditions). Seven padeye offsets e_p varying from 0 to $0.5B$ for two conditions of padeye eccentricities e_n = 2.5 and 5m (e_n/B = 0.5 and 1.0) will be first employed to calibrate and challenge the macro-element model. In this process, further comparison with an associated flow rule hypothesis will be carried out to demonstrate the importance of the proposed model addition. The newly proposed model will be subsequently challenged against a wider set of LDFE analyses and centrifuge data from the literature, as listed in Table 1.

5.2 Calibration

Calibration of 10 model parameters is required: 3 normalised capacities (N_v, N_h and N_m) governing size of the failure surface, 3 shape parameters of the loading and failure surfaces (m, n, q), 3 parameters defining the plastic potential surface (ξ, χ and ω) and 1 hardening parameter (R_0), as listed in Table 1.

The normalised capacities (N_v, N_h and N_m) defined in Eqs (6), (7) and (8) have been subject of previous literature studies. From an extensive set of finite-element analyses, Wang et al. (2011) suggested a value of 14 for N_v which was also applied by Cassidy et al. (2012) for rectangular anchors. Similarly, Elkhatib (2006) has numerically derived a value of 1.73 for the moment capacity N_m. However, these simulations did not take into account the effect of the shank, therefore a value of 2 has been adopted, as suggested by Cassidy et al. (2012). The value of parameter N_h is influenced by both anchor geometry and the roughness of the anchor material. Cassidy et al. (2012) reported a range of values varying between 2.78 and 3.41 and adopted a value of 3, which is also adopted in the present study.

The shape parameters for the loading and failure surfaces have also been selected according to the recommendation of the literature. Elkhatib (2006) found a value of q = 4.0 for square and rectangular anchors. The parameter n was found to vary between 3.72 (square anchors, Elkhatib 2006) and 5.31 (strip anchors, Elkhatib and Randolph 2005; Bransby and O'Neill 1999). For rectangular anchors, a value of 4.2 was used by Cassidy et al. (2012). For the sake of simplicity, an integer value was selected herein (n = 4) with minimal influence on the model predictions. Finally, the parameter m was found to vary between 1.07 and 2.58 (Elkhatib 2006 and Elkhatib and Randolph 2005, respectively) therefore an integer value of m = 2 is selected here.

It follows that only the parameters for the plastic potential (ξ, χ, ω) and the hardening parameter (R_0) need to be directly calibrated when using this model. The numerical results for the intermediate padeye offset value of e_p/B = 0.3 and e_n/B = 0.5 have been chosen for the calibration process, while the other available data will be used for independent challenging of the model. The calibration is carried out by comparing and fitting the three experimental plots: anchor inclination (α) vs vertical displacement (z/B) (Fig. 6(a)), normalised chain load ($T_a/(LBs_u)$) vs vertical displacement (z/B) (Fig. 6b), and vertical (z/B) vs horizontal displacement (x/B) (Fig. 6c). Each parameter is calibrated to capture the respective behavioural feature as discussed in sections 4.1 and 4.2 above: ξ is adjusted to fit the anchor trajectory in the horizontal versus horizontal displacement plot (Fig. 6c); χ and ω are adjusted to match the length of the plateau in the force displacement plot (Fig. 6a) and the evolution of anchor rotation (Fig.6b); R_0 is adjusted to match the initial part of the force displacement curve (Fig 6a).The parameters' values of ξ = 1.6, χ = 1.1, ω = 1.5 and R_0 = 2.5 provide an overall good fit of the LDFE results as shown in Fig. 6. These values can be used as an initial attempt for other sets of numerical and experimental data, as will be shown later in this study.

5.3 Model simulations

Effect of padeye eccentricity and offset

The model performance is initially evaluated against the results from three-dimensional LDFE analyses carried out by Tian et al. (2015) for a padeye eccentricity of e_n/B=0.5 and seven distinct padeye offsets (e_p/B = 0, 0.05, 0.1, 0.2, 0.3, 0.4 and 0.5), all subjected to monotonic vertical load. Comparison of LDFE results (Fig. 7a, 7d and 7g) with model simulations are presented for macro-element model with (Fig. 7b, 7e, 7h) and without (Fig. 7c, 7f, 7i) the new features (non-associated flow rule and displacement hardening). To ensure fairness of such comparison, an independent calibration of the associated plastic potential model has been carried out in order to obtain the best fit with the FE results: values of m = 4, n = 3 and q = 4.5 have been imposed to the strength surface formulation.

Fig. 7c shows the rotational behaviour predicted by the macro-element model with associated plastic potential and no hardening rule. The model shows significantly higher vertical displacement than predicted by LDFE simulations (Fig. 7a). Furthermore, an underestimation of the final anchor inclination can be observed. On the other hand, with the inclusion of a non-associated plastic potential and a strain-hardening rule (Fig. 7b), the macro-element model simulations exhibit good agreement for all seven padeye offsets e_p/B. Both initial rotation and final orientation of the anchor are well captured.

Analysis of the anchor trajectories demonstrates also the superior capabilities of the new model (Fig. 7e), in comparison with the non-associative model (Fig. 7f), to predict the overall anchor trajectory given by the LDFE analyses (Fig. 7d). Although the initial backward movement of the anchor is slightly overpredicted by the new model for high eccentricities (i.e. e_p/B>0.1) both horizontal and vertical movements seem reproduced quite well. The absence of displacement hardening and non-associated flow rule results in much larger displacements than the LDFE simulations (Fig. 7f).

Analysing the predicted force-displacement trends, both versions of the macro-element model successfully predict the peak load. However, the new model (Fig. 7h) offers an improved agreement with the LDFE results (Fig. 7g) in terms of vertical displacement at the peak conditions when compared with the non-associative rigid model (Fig. 7i). This modelling capability may be important for field applications when an assessment of the anchor movement at its full load capacity is required. While some discrepancies can be observed between the LDFE and new macro-element predicted post-peak behaviours, this is a post-failure behavioural feature which would be less relevant for field applications.

The simulations were also conducted for a higher padeye eccentricity (e_n/B=1.0) and showed good agreement with LDFE analyses of Tian et al. (2015). Whereas the anchor inclination (Fig. 8a and 8b) is well reproduced by the new macro-element model simulations, tendency to slightly underestimate the peak load between 0.7% and 12.4%, depending on the padeye offset (Fig. 8d and 8e), and overestimate the movement in both horizontal and vertical directions (Fig. 8c and 8d) is also observed – and accentuated for e_n/B=1.0 in comparison with e_n/B=0.5.

5.4 Further validation

In this section, the proposed macro-element model is further challenged against other sets of numerical and experimental data from the literature to show the model capabilities for different loading and geometrical conditions. The same model parameters as previously calibrated in the section 'Calibration' are assumed here, with the exception of the parameter ω, which will be adjusted to reproduce the observed pre-failure behaviour (i.e. padeye travel distance to peak load) of the plate anchors.

Comparison with LDFE and centrifuge test under vertical pull-out

Model capabilities are assessed here against numerical and centrifuge results in transparent soil on vertically installed plate anchors subjected to vertical monotonic ($\theta_0 = \theta_a = 90°$) pull-out at the mudline (Wang et al. 2011; Song et al. 2006, 2009). Anchor geometry and soil properties are presented in Table 1 and the results are presented in Fig. 9. It can be observed that LDFE and centrifuge test data exhibit different anchor capacities. Wang et al. (2011) suggest that remoulding of the soil during anchor installation and degradation of the undrained shear strength s_u is the reason for the lower experimental capacity in comparison with the LDFE result. The same study reports that T-bar tests showed a 25% reduction in resistance during extraction compared with during penetration. Therefore, the macro-element model simulations are carried out with the original undrained shear strength (s_u=18 kPa) to match the final capacity for LDFE data and with a degraded undrained shear strength reduced by 25% (s_u=13 kPa) to match the final capacity for the centrifuge test as shown in Figure 9b. The model parameters remain unchanged and assume the same value as in the previous challenging exercise (ξ = 1.6, χ = 1.1, ω = 1.5 and R_0 = 2.5). There is a clear practical advantage in assuming fixed values for these parameters and this paper shows that the adopted values can provide satisfactory simulations for several cases.

The comparisons of the rotational behaviour in Fig. 9a shows that the model simulation lies between centrifuge and LDFE results. Whereas reducing the undrained shear strength from 18 kPa to 13 kPa is not sufficient to match the experimental curve, the difference can be explained by the experimental set-up. The macro-element model simulation assumes that the anchor padeye is pulled at a constant angle θ_a of 90^o, which makes the anchor progressively rotate until becoming horizontal (perpendicular to the direction of pulling) at large displacements. In the centrifuge test, instead, the pulley was initially located just above the padeye but, as the anchor is being pulled, a misalignment between the padeye and pulley is created due to the horizontal displacement of the anchor padeye. This misalignment can explain the difference between centrifuge and macro-element (and LFDE) results as well as the residual final orientation of 10^o observed in the centrifuge test (Song et al. 2009; Wang et al. 2011). The results obtained from the model with associated flow rule and without hardening show that, for this model, the vertical displacement of the anchor is significantly overestimated (Fig. 9b) when compared to the centrifuge and LDFE data.

Comparison among force-displacement trends in Fig. 9c shows that the model-predicted anchor peak capacities agree well with centrifuge and LDFE data. However, the model predicts a lower displacement at mobilisation of the peak resistance. This slight difference may be explained by the assumption made for the calculation of chain displacement which, in the macro-element, is considered to be equal to the variation of the linear distance between the padeye and pulley neglecting any curvature of the chain after anchor installation. Conversely, the force-displacement curve for the associative model without a hardening rule (Fig. 9d) does not capture the non-linear response at the early stages of keying, while the peak resistance is mobilised after a slightly higher displacement if compared to the non-associative plastic potential case.

Comparison with LDFE and centrifuge testing under inclined pull-out

The performance of the new macro-element model was also verified through comparison with the results published by Cassidy et al. (2012) for monotonic inclined pull-out (θ_0 = 40°), as shown in Fig. 10. The parameter ω was calibrated against the rotation (Fig. 10a) and trajectory (Fig. 10b) plots from the LDFE analysis, and a value of 1.75 was found to be the best fit. The anchor rotation (Fig. 10a) predicted by the new model shows good agreement with LDFE results and reasonable agreement with centrifuge PIV. The new model's result shows that the anchor continues to rise vertically, as opposed to the model with associated plastic potential, which shows an anchor re-embedment beyond a certain point ($\alpha \approx 40°$). This can be explained by the change in the shape of the surface that governs the direction of displacements and rotations, as previously shown in Fig. 5. The deviation between the centrifuge PIV and LDFE results can be attributed to the experimental set-up (Cassidy et al. 2012), where the anchor is continuously pulled (with θ_0 increasing progressively) and not dragged with θ_0 remaining constant, as implied by LDFE models and by the proposed model.

In terms of anchor trajectory (Fig. 10b), the results from the new model also show good agreement with LDFE and centrifuge PIV. The initial vertical and backward motion is reduced in comparison with the model with associated plastic potential while the re-embedment at large displacements is avoided.

Effect of caisson extraction method in SEPLA's

Gaudin et al. (2006) carried out several centrifuge modelling tests on a square SEPLA in kaolin clay subjected to monotonic inclined pull-out (θ_0 = 45°) (see Table 1 for details), with various caisson extraction processes after suction installation of the anchor: reverse pumping versus vented pull-out extraction; short term versus long term pull-out (time allowed between end of installation and anchor pull-out). Three cases are presented in Fig. 11: PE-LT (reverse pumping extraction, long-term pull-out), VE-LT (vented extraction, long-term pull-out) and VE-ST (vented extraction, short-term pull-out).

The results presented in Fig. 11 (where s_{up} is the undrained shear strength at the estimated anchor embedment depth at the peak load) show that it is possible to fit the force-displacement curves for distinct retrieval methods by varying the parameter ω, while keeping the other plastic potential parameters unchanged with respect to the initial calibration. For comparison, the curve predicted using a model with associated plastic potential and without hardening rule is also shown in Fig. 11 and lies between the VE-LT and VE-ST experimental load-displacement curves.

For the proposed non-associative hardening model, good agreement is observed for all the three curves, and the prediction of the initial part of the curve is much improved due to the addition of the hardening rule when compared with the model without such ingredient. Furthermore, the peak load is well captured and so is the vertical displacement at which the peak load is reached. However, it is worth noting that a hardening parameter R_0 = 0.3 was adopted in all three simulations, as opposed to R_0 = 2.5 in all other simulations carried out in this paper. This may be related to the soil disturbance caused during installation and extraction of the caisson, but also to some simplifying assumptions in the experimental determination of the initial loss of embedment as discussed in the original paper (Gaudin *et al.* 2006). The influence of installation and extraction processes is of interest for further research, as the relationship between such aspects and the parameters ω and R_0 could be further explored and rationalised.

6 Conclusions

An improved macro-element model for vertically installed plate anchors was proposed in this paper and compared to a previously published model. The proposed model includes (i) a non-associated plastic potential and (ii) a strain-hardening rule into the plasticity theory framework. Four new parameters (three for the plastic potential and one for the hardening rule) were included and calibrated with an LDFE analysis from Tian et al. (2015). The results of model simulations were compared with four other studies, including results from LDFE simulations and centrifuge tests with PIV measurement technique. The following conclusions can be drawn:

- The proposed macro-element model was shown to be an effective tool to predict the force-displacement, rotation and trajectory of plate anchors covering distinct anchor geometries, soil properties and loading conditions.
- The addition of a non-associated plastic potential is fundamental to predict the expected trajectory of the anchor during keying avoiding large initial backwards and upwards movements and allowing the control of the direction of anchor motion. It is also essential to predict the anchor vertical displacement at peak load.
- Validation of the model against four different sets of numerical and centrifuge data suggests that fixed values of the newly introduced parameters ξ = 1.6, χ = 1.1 can be assumed while only the parameter ω should be calibrated for practical use. Variation of the parameter ω was also found effective in capturing the influence of different anchors' installation methods.
- It is desirable to calibrate the model's parameters against a limited set of centrifuge experimental data, numerical simulations or field tests (one or more) in order to apply

the proposed macro-element model for design purposes. The calibration procedure could use the strategy identified in the model calibration section of this paper. Since the values of parameters adopted in this paper (ξ = 1.6, χ = 1.1, ω = 1.5 and R_0 = 2.5) seem to produce satisfactory simulations for most analysed cases, these may be used as starting point in the calibration process. Alternatively, it may be also conceivable that such values may be used for initial/outline assessment of expected anchor behaviour in the field, for example to assessing the effect of different anchor geometries, padeye eccentricities, load inclination and/or soil properties. Nevertheless, validation and challenging of the proposed model against real field data would be beneficial and boost the potential use of macro-element modelling in anchor design.

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LIST OF TABLES

Table 1 – Geometrical and modelling parameters of the proposed macro-element model and values adopted in parametric analyses and model simulations.

	Sym-bol	Description	Values adopted for the following simulation exercises:					
			Parametric analysis [s. 4]	Tian *et al.* 2015 [s. 5.3]	Song *et al* 2006,2009 [s.5.4.1]	Wang *et al.* 2011 [s. 5.4.1]	Cassidy *et al.* 2012 [s. 5.4.2]	Gaudin *et al.* 2006 [s. 5.4.3]
Anchor geometry and properties	B	Anchor height (m)	4.64	5.0	4.0	4.0	4.64	5.075
	L	Anchor width (m)	7.92	5.0	4.0	4.0	7.92	5.075
	e_n	Padeye normal eccentricity (m)	2.59	2.5-5.0	2.5	2.5	2.59	3.35
	e_p	Padeye offset (m)	0.492	0.0-2.5	0.0	0.0	0.492	0.0
	W'	Submerged anchor weight (kN)	416.25	331.25	396.90	396.90	416.25	199.80
Chain geometry and	d_{bar}	Bar diameter (m)	0.41	-	-	-	0.41	0.319
	E_n	Effective width multiplier	1.0	-	-	-	1.0	1.0
	μ	Friction of chain	0.1	-	-	-	0.1	0.1
	N_c	Bearing capacity of chain	7.6	-	-	-	7.6	7.6
Soil conditions	s_{u0}	Undrained shear strength at mudline (kPa)	1.0	1.5	13.0	18.0	1.0	0.1
	k_{su}	Strength gradient (kPa/m)	1.25	1.5	0.0	0.0	1.25	1.1
Parameters of anchor Macro-element model	m	Exponent (moment)	2	2	2	2	2	2
	n	Exponent (horizontal)	4	4	4	4	4	4
	q	Exponent (vertical)	4	4	4	4	4	4
	N_v	Normalised normal capacity factor	14	14	14	14	14	14
	N_h	Normalised sliding capacity factor	3	3	3	3	3	3
	N_m	Normalised rotational capacity factor	2	2	2	2	2	2
	ξ	Plastic potential parameter (vertical)	0.5-2.0	1.6	1.6	1.6	1.6	1.6
	χ	Plastic potential parameter (horizontal)	0.5-1.5	1.1	1.1	1.1	1.1	1.1
	ω	Plastic potential parameter (moment)	0.65-2.0	1.5	1.5	1.5	1.75	0.95-1.5
	R_0	Hardening parameter	0.5 – 2.5	2.5	2.5	2.5	2.5	0.3

LIST OF FIGURES