Frege plagiarized the Stoics

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> *Plagiarism*: 'the practice of taking someone else's work or ideas and passing them off as one's own.'[1]

I. Introduction

In this essay, I propose that Frege plagiarized the Stoics on a large scale in his work on the philosophy of logic and language as written mainly between 1890 and his death in 1925 (much of which was only published posthumously), and possibly earlier. I use 'plagiarize' merely as a descriptive term. The essay is not concerned with finger pointing or casting moral judgement. This is left to those who feel so inclined. The point is rather to demonstrate that there are numerous and extensive parallels in both formulation and content between the Stoics and Frege, so plentiful that one would be hard pressed to brush them off as coincidence. These parallels include several that appear to occur in no other modern works that were published before Frege's own and were accessible to him. Additionally, a cluster of corroborating historical data is adduced to support the suggestion.

Once it is understood that Frege draws from the Stoics, and where in his work and by which channels he does so, some elements of his philosophy of language can be given new readings in the light of his—unacknowledged—contemplation and absorption of Stoic logic. Conversely, the comparison with Frege's philosophy makes it possible to shed new light on some issues in Stoic logic and philosophy of language. In this way, this piece also contributes on a small scale to the philosophical interpretation of Frege's work and of Stoic philosophy, and, as such, is not merely of historical interest. (Such issues include the logical structure of commands and questions, emotional elements in assertion-like contents, the treatment of what Frege calls mock thoughts, the logical status of indexicals, the reduction of causal statements, language regimentation, including Frege's 'Hilfssprache', and more.) In addition, I seek to provide a wider philosophical audience with the groundbreaking, original, but

[1] *The Oxford Dictionary of Phrase and Fable* (Knowles 2006).

widely neglected philosophical and logical work of the Stoics. I thus simultaneously offer an introduction to Stoic philosophy of logic and language. For those acquainted with Stoic philosophy but not Frege's, on the other hand, I provide a glimpse into Frege's work on logic and language.

No knowledge of Greek, Latin, or German is required of the reader. Some terms and phrases in these languages are given in brackets and footnotes as evidence, but they are inessential for the understanding of any major point. Greek expressions in the main text are transliterated. Translations are my own, except where noted otherwise. For the texts and editions used and cited in brackets by and of Frege, the Stoics and some others, sigla and abbreviations are introduced at their first occurrence. At the end of the paper, an alphabetical list of the sigla is added for ease of reference. It is in the nature of the project that there are copious comparisons. I hope to have succeeded in staying away from the terribly tedious by leading the reader through a number of philosophically fascinating issues that are still of contemporary relevance and providing a fresh perspective on various philosophical questions.

II. Historical background

I begin, however, with a light-hearted look at the historical background and at how this paper took shape. When I wrote my 2006 *Stanford Encyclopaedia* entry on ancient logic, I had just co-taught a graduate class on Frege's philosophy of logic and language, and noticed so many parallels to the Stoics that I recklessly included the sentence, 'The many close similarities between [the Stoic] Chrysippus' philosophical logic and that of Gottlob Frege are especially striking', planning to follow this lead at a later point. But I got side-tracked. Still, it seems, my sentence did not go unnoticed. In 2009, a German article appeared in which three German professors—a classicist and two historians of philosophy—contend that Frege had been influenced in his work by his knowledge of Stoic logic. These three men pooled their resources and embarked on some detective work that led them to fascinating revelations: revelations of a kind that had never crossed my mind.[2] Their article culminates with two photographic images. The first is a photo of the 350th anniversary celebration at the University of Jena in 1908 (from the Universitätsarchiv Jena).

[2] Gabriel, Hülser, and Schlotter 2009. A useful little summary is available in English in *The Bulletin of Symbolic Logic* (Wille 2010).

And this is a magnification of a tiny part of that photo.

You see here on the right the famous logician Gottlob Frege and on the left the famous classicist Rudolf Hirzel—although, as the authors hasten to mention, with an aisle between them. ‘So what?’ you may say, ‘I have sat next to a famous classicist, and still do not know much about Stoic logic’. But there is more in the German article. Hirzel is not just *any* famous classicist. He was a specialist on various aspects of Stoic philosophy who published,

in 1879, an article entitled *De logica Stoicorum* ('On the logic of the Stoics'). And there is even more: For *twenty-four* years (1889 to 1913), Hirzel lived in Frege's house, renting the upper floor. *And* they shared an acquaintance, the philologically trained philosopher Rudolf Christoph Eucken, Professor at Jena from 1874–1920, who, believe it or not, lived across the street from Frege. Moreover, both Hirzel and Frege were introverts. Therefore, the three German professors conclude, Hirzel and Frege must have talked to each other. And I agree that even two introverts, if they live floor to ceiling for twenty-four years, are likely to have exchanged a word or two. ('The tap is leaking.' 'Oh. I'll see to it.') Not so clear is *how* the three German professors imagine that elements of Stoic philosophy entered Frege's mind via Hirzel. No decisive evidence, in the form of, say, letters or diary entries, has yet surfaced, so we have here an open question.

The discursive ARGUMENT OF THE THREE PROFESSORS is in large part based on conjecture and it is not entirely compelling. Here is a brief summary of their reasoning, with my assessment in brackets. Details will be provided elsewhere.

(i) Frege knew the classicist Hirzel. (This is correct.)

(a) They talked to each other. (This is almost certainly correct.)

(b) Frege got a sentence from Plato's *Hippias Major* from Hirzel. (This is possible, if irrelevant.)

(ii) Frege was impacted by Stoic logic. (This is correct.)

(iii) The points of impact happened after 1889. (This is possible.)

(iv) The points of impact that happened after 1889 are the following:

(a) Stoic *lekta* led to Frege's 'Gedanken' (this is incorrect as it stands);

(b) The Stoic notion of predicate impacted Frege's notion of predicates as function (this is likely incorrect).

(v) Frege learned about the elements of Stoic logic mentioned in (iv) from the 'middleman' Hirzel. (There is no evidence.)

(vi) Hirzel was qualified in matters of Stoic logic. (This is incorrect. Hirzel was somewhat qualified in matters of Stoic epistemology, which the Stoics classified as *logikē*, yes. For qualifications in Stoic logic proper, or *dialektikē*, there is no evidence. His *De logica Stoicorum* is not about logic in the sense in which Frege would have understood the word.)

(vii) Frege learnt about the Stoics from Hirzel in a busy discussion circle which also included Eucken. (There is no evidence of such a discussion circle.)[3]

The authors produce no evidence that Hirzel ever talked to Frege about Stoic logic, or that he talked to him at all about philosophical issues, beyond *perhaps* alerting him to a passage in Plato's *Hippias Major*. Unless and until written evidence is unearthed that confirms conversations between Frege and Hirzel about Stoic logic, in Frege's understanding of the word, I take it as unproven that Hirzel was a 'qualified middle man' ('qualifizierter Mittelsmann')[4] between the Stoics and Frege.

THE ALTERNATIVE I SUGGEST contends that Stoic logic had a much wider-ranging impact on Frege. Imagine Frege had a choice between conversing with an introverted classicist who had, as far as we know, no understanding of logic in the sense in which Frege takes it ('logic proper'), and conversing with a group of brilliant logicians, logicians who could not talk back, at that. My guess is that he would have preferred the logicians. The assumption that Frege conversed with the Stoics (more) directly is, I suggest, a more promising assumption than that he conversed with them via discussion with Hirzel. In other words, if Stoic logic had an impact on Frege's logic, then this impact would have come primarily from books containing Stoic logic that Frege himself read—in his study, as is apt for an introvert.

Here is my hypothesis: Frege helped himself generously to elements of Stoic logic as they were presented in the first volume of Carl Prantl's monumental four-volume *Geschichte der Logik im Abendland* (*History of Western Logic*), published in 1855.[5] This volume contains a very long chapter on Stoic logic proper (401–96). More than half of the chapter consists of tightly printed footnotes that present a major part of the—then known—extant Stoic testimony on logic in the original Greek and Latin sources. The main text offers a relentlessly deprecating summary-cum-paraphrase of Stoic logic, based on the texts quoted in the footnotes. These ninety-five pages remained for almost a century and a half the best comprehensive source for Stoic logic in any language (and the only one in a work on the history of logic), despite the fact that Prantl tells us on every other page how idiotic

[3] Dathe 1995 suggests repeatedly that Frege and Eucken must have had philosophical conversations. He does not adduce any piece of evidence for this hypothesis.

[4] Gabriel, Hülser, and Schlotter 2009: 383.

[5] *Geschichte der Logik im Abendland* (München 1855–70, 4 vols, vol. I, published in 1855).

('blödsinnig') and piffling ('läppisch') the Stoic theory was. Some details will establish the *historical* plausibility of this assumption.

Hirzel's intercession would not have been required for Frege to learn about the existence and importance of Prantl's work. It was generally well known.[6] Sigwart for example, in his preface (p. VI) to the first edition of the 1873 first volume of his widely known *Logik*, acknowledges 'Prantl's terrific work' ('grossartiges Werk') in a breath with Trendelenburg, Ueberweg, and Mill, and no others, fifteen years before Hirzel moved into Frege's house. Sigwart also mentions Prantl at least five times in that volume. We know that at some point Frege took notes or excerpts from that very work.[7] One also needs to remember that the number of German books on logic to appear per year was relatively small, and that Frege seems to have read widely on logic—he even read Wilhelm Wundt's *Logik*.[8] Note further that Frege's colleague Eucken draws attention to Prantl's work in his lectures on logic in Jena in 1880-2 (Kreiser 2001: 290).

Frege would have had easy access to Prantl's work. Besides a bookstore and the Jena University Library, there would have been Hirzel's library as a possible source. Not only was Hirzel's father, Solomon Hirzel, the publisher of the work, but Hirzel himself refers to Prantl's section on the Stoics in his 1879 paper, and there are at least nine references to Prantl's first volume in his 1882 Cicero tome. So we can assume Hirzel read parts of it and most probably owned a copy.[9] Presumably, Hirzel would have understood very little of Prantl, who, although he loathed the Stoics, had at least the good sense to represent their views in many parts correctly, before labelling them inane.

Frege, on the other hand, was *in the best possible position of perhaps all German philosophers and logicians* at the time to comprehend and appreciate the extraordinary treasure trove Prantl lays bare. First, Frege knew and could read both Greek and Latin. Kreiser 2001: 38-43, especially the figure on p.42, suggest that at school Frege had ten years of intensive Latin courses and eight years of intensive Greek courses. Generally, this would have been part of the education at a German 'Gymnasium' in the mid-nineteenth century and

[6] See *e.g.* Zeller 1856–68, as noted by Gabriel, Hülser, and Schlotter 2009: 381; Trendelenburg 1862: (I) 33, 311; Ueberweg 1871: 19.

[7] *Cf.* Scholz's catalogue of Frege's 'Nachlass' as published at the end of Veraart 1976.

[8] *Ibid.*

[9] Despite the title, in his 1879 paper Hirzel is not concerned with any questions of logic, but with the question of whether it was the Stoics who introduced the word 'logic' (*logikē*) for a philosophical discipline—an application with a vastly wider range than that of the 'logic' that Frege was interested in, which would correspond more closely to what the Stoic called dialectic (*dialektikē*). In his 1882 volume, he also does not cite Prantl in any context of logic proper.

the norm for any professor in the humanities. For ancient Greek, compare for example Sigwart's above-mentioned *Logik*, which amply quotes Greek in footnotes without translation. For Latin, consider that publications in Latin were not unusual in the mid-nineteenth century. (Recall that Hirzel's *De logica Stoicorum* is written in Latin.) Frege's own writings show frequent sprinkles of Latin, many examples of and allusions to Classics, as well as a verse from Homer quoted in Greek, just so, as an example of onomatopoeia, and casual reference to the Greek *spiritus lenis*.[10] Frege's work also shows that he was acquainted with many aspects of Aristotelian logic.[11] Second, as those versed in Stoic logic know, it was in the first instance a propositional logic, a kind of logic barely understood by anyone in the nineteenth century before Frege's *Begriffsschrift*.

So it would be astounding if Frege (i) had not known of Prantl's work, (ii) had not had access to it, and (iii) did not have what it takes to understand Prantl's long chapter on Stoic logic with its many primary sources. Moreover (iv), even a cursory reading of parts of that chapter would have been bound to pique his interest.[12] Any perusing of that chapter would also have directed Frege to book 7 of Diogenes Laertius' *Vitae Philosophorum* (*Lives of the Philosophers*, composed *c.* the second century CE), which, in forty paragraphs (D.L. 7.43–83), contains perhaps our most valuable continuous source of Stoic logic, a detailed, historically reliable, summary of all its main aspects, hereafter referred to as the *Summary*. Much of it is found in Prantl's footnotes, but one should not rule out the possibility that Frege had independent knowledge of this work. At the time, it was well-known and widely available, including in two German translations.[13]

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[10] Latin: See SB, CP 164 for a Latin quote from Leibniz; and the footnote OCN, PW 79: '"*Omnia una sunt*", a Latinist would say, if not deterred by his feeling for the language […]'. BLC, PW 9–10 strongly suggest Frege read Leibniz in Latin, with several lines quoted in Latin, plus three work titles. He uses classical Latin examples as well: Cicero, Cato, *etc.* Greek: Frege quotes Greek expressions in Greek letters: PWLB, PW 139 he writes: 'man vergleiche dazu den homerischen Vers (*Odyss*. IX,71): τριχθά τε καὶ τετραχθὰ διέσχισεν ἴς ἀνέμοιο'. He quotes the entire verse in Greek and seems to assume that the reader, too, knows Homer in Greek. *Cf.* also the Greek πρῶτον ψεῦδος in DPE, PW 62, 64. There are references to Homer in *e.g. SB* (Odysseus), and in PWLB, PW 129 ('Scylla has six heads'). Further, we find in GG IX 45 a casual reference to the ancient Greek accent *spiritus lenis*: 'the smooth breathing, designating the value-range of a function, and a sign to play the role of the definite article in language' ('der Spiritus lenis zur Bezeichnung des Werthverlaufs einer Function und ein Zeichen, das den bestimmten Artikel der Sprache vertreten soll', GG IX 9).

[11] *E.g.* in BS.

[12] Even without further textual evidence, we can assume that Frege would have had an interest in Prantl's work. Philosophy was far more interwoven with its history than it is in Anglo-Saxon analytical philosophy nowadays.

[13] Snell and Snell 1806 and Borheck 1807.

In brief, my hypothesis then is this: Frege learned about Stoic logic from Prantl's *History of Western Logic*, which he may or may not have borrowed from Hirzel, but is likely to have known (of) before the Hirzels became his lodgers. Additionally, Frege may have read the *Summary* on Stoic logic in Diogenes Laertius, possibly in one of its widely available German translations. At this point, I deliberately leave several questions open. Why am I so certain that Frege drew on Prantl, rather than primary texts or some other source? How, more specifically, did the content of Prantl's chapter on Stoic logic become incorporated into Frege's work? Was it intentional? Would it really be plagiarism? Naturally, these questions can only be considered after the textual evidence has been unrolled, which is what comes next.

III. Textual evidence

The text-based argument in section III is strictly accumulative. No single textual parallel validates the thesis of plagiarism. It is by accruing passage by passage, sentence by sentence, phrase by phrase, the Stoic elements in Frege's oeuvre, organizing them by (Stoic) topic, and considering their philosophical significance (and adding to this the historical data provided above) that a compelling case is built.[14] Taking in the result requires patience on the part of the reader. Those who are less interested in the philosophical implications of the parallels can directly consult the tables with synopses added for each topic in order to facilitate absorption of the semblances at a glance.

In view of the various historical data given above, the investigation is almost entirely restricted to Stoic passages and Stoic doctrine on logic and language as found in Prantl and in the *Summary*. Here I add three further reasons, based on the assumption—to be corroborated below—that Frege had knowledge of Stoic logic. *First*, it is vastly more likely that Frege obtained his knowledge of Stoic logic from one text, rather than from browsing through the dozens of Greek and Latin works with testimonies on Stoic logic that Prantl brings together. (Of the hundreds of Stoic logical works, not one has survived in its entirety and we are almost completely dependent on later ancient sources.) *Second*, virtually all parallels between Stoics

[14] In the end, in order to get the full picture the reader would have to read Prantl, including the Greek and Latin footnotes, followed by all of Frege's later *philosophical* (as opposed to mathematical and purely logical) works. What becomes apparent when doing so is that—other than epistemic arguments, comments, and observations on psychological matters, remarks on post-medieval logic, contrasts with Aristotle, and academic disputes with contemporaries—there is very little in Frege that does not have an analogue of some sort in the Stoics as reported by Prantl.

and Frege are present in Prantl, and some important elements of Stoic logic without parallels in Frege are missing in Prantl. The main examples are: the important fragmentary papyrus of Chrysippus' *Logical Investigations* (PHerc 307), which was first published in 1873; the long passage in Alexander's *Prior Analytics* commentary on negation (*An. Pr.* 402–05), and a number of further passages in the Aristotle commentators, in some grammarians, and in scholia. *Third*, there are *several* misunderstandings or distortions of Stoic logic in Prantl which *do* have parallels in Frege.[15]

III.1. Content

The following agreement between Frege and the Stoics lies at the very bottom of the many similarities: that the *contents* of our thinking and communicating are imperceptible, incorporeal entities that we can all share, and which we express in language. The Stoic term for their contents was *lekta*, customarily translated as 'sayables' (what can be said), but also allowing the translation 'thinkables' (what can be thought). Most of what Frege considered content, he called 'sense' ('Sinn'). I disregard all metaphysical issues concerning these contents, beyond mentioning the following well-known facts: both the Stoics and Frege distinguished between (i) the things in the external world, (ii) our presentations or impressions based on these things, (iii) linguistic expressions, including sentences, as a special subclass of the things in the external world, (iv) the incorporeal content, including assertoric content, and (v) our mentally entertaining such content.[16] For both, content is most closely connected with linguistic expressions: with the linguistic expressions we say or

[15] It is my view that in the later works by Frege, in particular the latest publications and the unpublished work on logic and language, we often find in different texts slightly different views on individual issues, presumably due to Frege trying out various ways of developing and expressing his theory. It seems wrong to me to try to provide in all such cases complex and contorted textual interpretations for no reason but to show that all things Frege wrote in his later years are mutually consistent.

[16] STOICS: (i) *tugchanonta*, (ii) *phantasiai*, (iii) *logoi*, *ekphorai*, (iv) *ta sēmainonta*, *lekta*, with *axiōmata* as subclass, (v) *logikai phantasiai* (*cf.* D.L. 7.56, 57, 67; S.E. *M.* 8.11, 12, 70; *Epist*. 117.13; Prantl 415–21) FREGE: (i) 'Dinge in der Aussenwelt' (T69), (ii) 'Vorstellungen' (*Thoughts*, passim), (iii) 'Sätze', 'sprachliche Ausdrücke' (T60), (iv) 'Sinn', with 'Gedanken' as subclass (SB, *Thoughts*, *Logik*), (v) 'Denken', 'Fassen der Gedanken' (T74–75). Historians of logic have remarked on these parallels (Mates 1961: 19–26, esp. 20, if not entirely accurately; Bochenski 1956: 127; Gabriel, Hülser, and Schlotter 2009: 375–77). A close comparison of Prantl 1855: 416–17 with Frege's *Gedanke* or *Logik* may be interesting, but is not my topic here.

express the content.[17] In what follows, I concentrate on how either party treats the fundamental philosophical issue of *how meaning or content is related to linguistic expressions*. The Stoics and Frege each distinguish many different kinds of contents. As a rough structuring guide, I follow the detailed Stoic classification of contents as it is found in Diogenes Laertius' report of Stoic logic and which is largely followed by Prantl.

III.1.1. Incomplete content

For both Frege and the Stoics, a fundamental distinction is that between complete contents and incomplete contents.[18] Incomplete contents require completion. It appears that incomplete contents are not contents in the true sense: 'incomplete' produces a *contradictio in adjecto*.

FREGE says about (simple or basic) thoughts that, 'The sentence expressing such a thought is composed of a proper name […] and a predicative part, which corresponds to the unsaturated part of the thought' (*Introduction to Logic* (IL) PW 187).[19] By contrast, a 'thought […] needs no completion' (CT37, CP 391).[20]

The STOICS call the incomplete contents *ellipē* (incomplete, lacking). The Greek *ellipē* and—to a lesser degree—Prantl's German translation ('mangelhaft' = 'deficient') match Frege's terms for incompleteness ('unvollständig', 'ergänzungsbedürftig'). The Stoics, too, consider one-place-predicates as the paradigm case of incomplete content—'predicates are classified as incomplete *lekta*' (D.L. 7.63)—and they define their most common simple assertibles (*katēgorikon axiōma*) as 'composed from a *ptōsis* [roughly, the content of a proper name or noun] and a predicate'. The incomplete contents are said to have an unfinished

[17] *Cf.* Frege, *Logik* NS 142–43, PW 131, 'Das eigentliche Ausdrucksmittel für den Gedanken ist der Satz'; T61, 'der Satz drücke einen Gedanken aus'. STOICS: D.L. 7.57, λόγος ἀεί σημαντικός (a linguistic expressions always signifies), προφέρονται αἱ φωναί λέγεται δέ τά πράγματα, ἀ δή καὶ λεκτά τυγχάνει (what is said are the things, which are also called sayables (*lekta*)); see also the rest of section III.

[18] This similarity is pointed out by Kneale and Kneale 1962: 500 and repeated by Gabriel, Hülser, and Schlotter 2009: 384–85.

[19] 'Der Satz, der einen solchen Gedanken ausdrückt, besteht aus einem Eigennamen […] und einem prädikativen Teile, der dem ungesättigten Teile des Gedankens entspricht' ('Einleitung in die Logik' [Einleitung] NS 203). *Cf.* NS 129 Ausführungen über Sinn und Bedeutung: *ungesättigt, ergänzungsbedürftig*; *prädikativer Teil eines Gedankens*, *Einleitung in die Logik* (EidL), NS 203, PW 187; in *Begriff und Funktion* (FC 6, CP 140): 'eine Funktion "unvollständig, ergänzungsbedürftig oder ungesättigt zu nennen"'. In *Concept and Object*—CO 197, CP 187, n. 11: unsaturated; CO 205, CP 193: doubly unsaturated.

[20] 'Gedanke […] bedarf […] keiner Ergänzung' (CT37, LU 73).

(*anapartiston*) expression (*ekphora*). The Stoic example is 'writes', expression of which (on its own) is said to elicit the question 'who?' (D.L. 7.63, Prantl 438–39, n. 111). (We see later that this points to a difference in understanding between Frege and the Stoics of the kind of incompleteness.)[21]

INCOMPLETE CONTENTS	
STOIC incomplete (*ellipē*) (D.L. 7.43, S.E. *M.* 8.70, Prantl 418, n. 55).	FREGEAN unsaturated ('ungesättigt') (EidL, NS 203, PW 192), incomplete ('unvollständig') (FC 6, CP 140).
Requires completion (D.L. 7.63, Prantl 439).	Requires supplementation (*e.g.* CT37, CP 390, implied).
Unary predicate (*katēgorēma*) as main example of incomplete content (D.L. 7.63, Prantl 439).	Unary predicate as main example of incomplete content (EidL, NS 203, PW 187).
Predicate as function (suggested by D.L. 7.69–70, see Bobzien and Shogry, forthcoming; Gabriel, Hülser, and Schlotter 2009).	Predicate as function (Begr 15–19).
Doubly unsaturated or binary predicates (see Bobzien and Shogry, forthcoming).	Doubly unsaturated or binary predicates (EidL, NS 209, CO, CP 193).

My focus in section III will be almost entirely on complete or saturated contents, and it is with these that I start.

III.1.2. Complete contents

The first noteworthy similarity with regard to complete contents is that both Frege and the Stoics maintain that there are multiple kinds of complete contents that *are on a par*. For Frege, not every sentence that has a sense is a thought (T61, CP 61). A thought is *at the same level* as commands, requests, *etc.* (SB38, CP 167, '*auf derselben Stufe*', my italics). Prantl writes about the Stoics that they '[…] distinguish besides the proper *axiōma* a number of sentences as *co-ordinated* kinds, namely […]' (Prantl 441, my italics).[22] This co-ordination is

[21] Gabriel, Hülser, and Schlotter 2009 claim (384–85) that Frege's notions of unsaturatedness and the incompleteness of predicates (as opposed to Frege's understanding of predicates as functions in Begr) goes back to Stoic logic, and that (378) Chrysippus considered some predicates 'almost as Fregean functions'.

[22] 'indem [die Stoiker] von dem eigentlichen *axiōma* […] noch eine Mehrzahl von Sätzen als coordinierte Arten unterscheiden, nemlich […]'. The Aristotelian Prantl does not distinguish between complete *lekta* and the sentences that express these.

confirmed both by the relevant *Summary* passages (D.L. 7.63, 65–68, Prantl 441, n. 115) and by the other two sources for Stoic complete contents that Prantl quotes, *i.e.* a list of accounts in Sextus and a comparison with Aristotle in Ammonius (S.E. *M.* 8.70–73; Ammon. *Int*. 2.9–6.3, FDS 897; Prantl 441 ns 115, 117).

Such a coordination of different complete contents is remarkable, since the customary view is—and was at Frege's time—that there is one kind of complete content, something like a common propositional content, that can be combined with *all* different kinds of *force*, or that is part of *all* different kinds of *speech acts*.[23] A multiplication of complete contents is generally considered to be an unnecessary multiplication of entities, and both Frege and the Stoics have been admonished for their lavish ontology.[24]

In several of his works, FREGE mentions a plurality of complete contents, leaving their exact number unspecified. He itemizes explicitly Thoughts ('Gedanken'), two kinds of questions, commands, wishes (via 'Wunschsätze'), requests, apparent thoughts, and some that are more-than-thoughts.[25] To all the specified complete contents that we find in Frege, the STOICS have corresponding kinds, and, additionally, a few more (see D.L. 7.65–68; S.E. *M*. 8.70–74; Ammon. *Int.* 2.9–6.3, FDS 897; Prantl 442–43, with the mentioned texts in

[23] *E.g.* for Bolzano 1837: (I) 88, propositions ('Sätze an sich') include commands. (The issue of whether command sentences, interrogative sentences, *etc.* have truth-values had been mentioned in a number of logic texts between Aristotle's time and that of Frege. The issue here concerns complete contents, not sentences.)

[24] *Cf.* for example, Dummett, 1981a: 307: 'In "Ueber Sinn und Bedeutung" [Frege] […] regards the difference between assertoric, interrogative, imperative and optative sentences as a difference in their sense rather than in the force attaching to them. Thus he says that, just as assertoric sentences express thoughts, so interrogatives express questions, imperatives commands, and optatives wishes (SB38–39, FBB 53–54). This view we may regard as *definitely wrong* […]'; FPL 308: 'Frege makes a certain modification of this view in his "Der Gedanke" […]. There he still thinks that an imperative expresses a command, considered as something parallel to a thought; but he now thinks that a sentential interrogative expresses the same thought as the corresponding assertoric sentence, and differs from it only in the force attached to it (T62, CP 355; N143–44, CP 373–74). This parallel is thus in itself significant'. There are parallels to both these views in Prantl and his Stoic sources (see below). Bronowski 2019: 394–97 argues that *all* Stoic complete contents contain an *axiōma*. This does not sit well with their definition of command contents and their acceptance of addresses as complete contents. It may well be true that some kinds of Stoic command contents in some sense contain complete contents.

[25] SB38–39: '*Command*, *request* […] stand on the same level as thoughts; […] the case is similar for […] *questions*. ('Befehl, Bitte: stehen […] mit Gedanken auf derselben Stufe; ähnlich Frage'); T62: command [sentences] ('Befehlssätze'), wish-expressing sentences ('Wunschsätze') and request-expressing sentences ('Bittsätze') have sense ('Sinn'); N145–46: the sense of an interrogative sentence ('Sinn eines Fragesatzes') is a thought ('Gedanke'); *Logik* NS 140 = PWLB PW129: 'sentences expressing wishes, questions, requests, and commands […] assertoric sentences' ('Wunsch-, Frage-, Aufforderungs-, Befehlssätze […] Behauptungssätze'), 'truth is only ascribed to the sense ('Sinn') of assertoric sentences'.

footnotes 115–117). We do not know their exact number. In the following, I compare each of the individual kinds of complete contents that Frege mentions with their Stoic counterparts.

Here we have one of the reasons why Prantl (rather than Hirzel or individual Stoic texts) is more likely to be Frege's source: Prantl considers the Stoic quasi-*axiōmata* and the Stoic more-than-*axiōmata* to be two *different* kinds of complete content (Prantl 442–43: *homoion axiōma* 442, *pleonazein* 443), although closer reading of the texts (D.L. 7.65–68; S.E. *M*. 8.70–74; Ammon. *Int*. 2.9–6.3) makes it clear that they were the same Stoic kind of complete content.

COMPLETE CONTENTS	
STOIC complete assertible (*autoteles axiōma*, D.L. 7.73; 'vollständig', Prantl 438, 440).	FREGEAN saturated, complete ('gesättigt', 'vollständig').
——— as co-ordinated kinds ('co-ordinirte Arten', Prantl 441).	——— at the same level (SB38–39, CP 167).
——— include primarily *axiōmata*/assertibles, sentence questions, word questions, commands, wishes or requests (*euktikon*), quasi-*axiōmata*, more-than-*axiōmata*, and a few others (Prantl 441–43).	——— include thoughts ('Gedanken'), sentence questions, word questions, commands, wishes (via a 'Wunschsätze'), requests ('Bitten'), apparent thoughts, and some that are more-than-thoughts (SB38–39, CP 38–39; T61–62, CP 355; N145–46, CP 375; *Logik*, NS 140–41 = PWLB PW 129, not a good translation).
Assertibles are contrasted with other non-assertible complete sayables in order to bring out the nature of the assertibles (in particular their being true or false) (D.L. 7.66, 68; *cf.* S.E. *M*. 8.70–74; Prantl 442–43).	'In order to bring out more precisely what I mean by "a thought" I shall distinguish various kinds of sentences' ('Um das, was ich einen Gedanken nennen will, schärfer herauszuarbeiten, unterscheide ich Arten von Sätzen') (T62).

III.1.2.1. Assertoric content: Stoic assertibles and Fregean thoughts

One of Frege's main logical achievements is the distinction between expressing and asserting a thought. The Stoic contents that are closest to Frege's thoughts ('Gedanken') are their assertibles (*axiōmata*). As a generic term for both I use 'assertoric content'. Both Stoics and Frege have as their primary interest their *assertoric* complete contents.[26] Both the Stoics and Frege struggle somewhat with explaining exactly what assertoric contents are.

[26] Frege in (T62, CP 355), see above. The vast majority of Chrysippus' logical works are about assertibles of one kind or another (D.L. 7.190–98, Prantl 405–08). In the *Summary*, three sections are about non-assertoric complete contents (D.L. 7.66–68), fifteen about assertoric complete contents (65,

STOIC ASSERTIBLES are listed as being on a par with the other Stoic complete contents (Prantl 441; D.L. 7.66–68; S.E. *M.* 8.70–74). For *all* complete contents, the Stoics held that we do three different things simultaneously when we perform the corresponding speech act. First, we *utter* (*propherein*) or express/articulate (*ekpherein*) a meaningful or 'content-ful' sound, the sentence or complete phrase (*logos*) (D.L. 7.59; *cf.* S.E. *PH* 1.73; Prantl 415, n. 46; 416, n. 47). This is a physical entity. Second, we *say* the incorporeal content (*legein*, D.L. 7.59). Third, we *assert*, or *command*, or *ask*, *etc.*, the content—depending on what sort of content it is (D.L. 7.66, 67, 68; Prantl 441, n. 115). So, in the case of the Stoic *assertibles*, we *utter* or *formulate* an assertoric sentence (*apophantikos logos*, *axiōmatikē ekphora*, D.L. 7.67, Prantl n. 115);[27] we *say* the assertible, that is, we express the assertoric content (D.L. 7.66, Prantl n. 115); and we *assert* the assertible, that is, we make an assertion (D.L. 7.66, Prantl n. 115).[28] We can then distinguish the following five distinct elements in the Stoic theory of assertibles: (i) the incorporeal assertible (*axiōma*); (ii) the assertoric sentence with which it is standardly expressed (*logos apophantikē*); (iii) the uttering (*proferein*) of the meaningful sound and the formulation (*ekphora*) of that sentence/speech; and concomitant with the utterance, (iv) the saying (*legein*) of the assertible; and (v) the asserting (*apophainometha*) of the assertible.

The Stoics offered two accounts of 'assertible'. First, an assertible is defined as a complete content that can be asserted in itself (D.L. 7.65; Gell. 16.8.4; Prantl 438, 'vollständig' for 'complete').[29] Second, assertibles are said to be that which is either true or false (D.L. 7.65; Prantl 441, 442), *i.e.* that which satisfies bivalence (and the semantic *tertium non datur*, assuming that the 'or' is exclusive).[30] A variant of this second account reads that assertibles are the complete contents by saying which we say something true or something false (S.E. *M.* 8.73; Prantl 441). Either way, the bearers of truth-value are the incorporeal

68–82). The Sextus passage also uses the non-assertoric complete contents as a foil for the assertoric ones (S.E. *M.* 8.70–74).

[27] This is the standard case. The Stoics did *not* claim that there is a one-to-one correspondence between assertoric sentences and assertibles. See below.

[28] It appears that *which one we do doing which* (whether we assert by saying or say by asserting) is immaterial. The *Summary* has 'saying the assertible we assert it' (D.L. 7.66; similar S.E. *M.* 8.73); Sextus once has 'asserting the assertible we say it' (S.E. *M.* 8.71).

[29] The exact force of 'in itself' is debated (Frede 1974, Bobzien 1986), but this need not concern us here. *Cf.* also Borheck's translation, 'Ein Axiom aber ist […] eine an sich vollkommene Sache' ('the *axiōma* is a thing complete in itself').

[30] Barnes 2007: 4–5 argues that this was not a definition of *axiōma*. D.L. 7.65 (Prantl 442, n. 116) presents it in the manner of a definition. For our present purposes, it suffices that it provides a necessary and sufficient condition for *axiōmata*.

assertibles, *not* the assertoric sentences by which they are expressed ('The True (*to alēthes*) lies in the *lekton* (context shows this is the assertible *lekton*) and is incorporeal', Prantl 417, *cf.* 421; S.E. *PH* 2.81). Finally, the Stoics explain the choice of the Greek term for assertibles (*axiōma*) as originating from our acknowledgement of [the truth of] what we say: 'Someone who says 'it is day' is believed to acknowledge [as true] that it is day, and when it is day, that assertible is true, if not, it is false' (D.L. 7.65, *axiousthai*; Prantl 442, n. 116). So the fact that we give acknowledgement to an assertible when we say it is a feature that characterizes assertibles. The Stoics distinguish between having a logical presentation (*logikē phantasia*) of an *axiōma* (S.E. *M.* 8.70; Prantl 418, 419) and the giving of assent or assenting (*sugkatathesis*, D.L. 7.49, 51; Prantl 418, 419; Zeller 1852, 1883) to that presentation (of an *axiōma*) (D.L. 7.49). They also have the epistemic notion of the grasping (*katalepsis*, D.L. 7.49, 52) of a reliably true (logical) presentation (*kataleptikē phantasia*), *i.e.* one that presents a true *axiōma*.

Compare all this with FREGE'S various attempts at defining and explaining what a *thought* ('Gedanke') is. He maintains that not every sentence that has a sense is a thought (T61, CP 354) and that a thought is at the same level as commands, request, *etc.* (SB38, CP 167, 'auf derselben Stufe'). Thoughts are expressed in assertoric sentences, and, as Frege puts it, 'the imperceptible thought is dressed in the perceptible garb of the sentence' (T61, CP 354).[31] It is the thought, not the sentence, that is the bearer of truth-value, or that which is either true or false (T60, 61, CP 353–54; CT37: '[…] ein Gedanke […], nämlich etwas von dem gilt: es ist entweder wahr oder falsch, ein Drittes gibt es nicht'; also 38, *tertium non datur*, EidL, NS 202). Frege writes 'We express acknowledgement ('Anerkennung') of truth [of the thought] in the form of an assertoric sentence' (T63, CP 356).[32] He emphasizes that an assertoric sentence—when spoken sincerely[33]—contains (a) its content ('Inhalt'), which is the thought ('Gedanke') and (b) the assertion (of the thought) ('Behauptung') (T62, CP 355). He explicates that it is with the *saying* of the assertoric sentence that we both *convey* ('mitteilen') or express ('ausdrücken') the thought and *assert* ('behaupten') (as true) the thought (T62, CP 355). Finally, in *Negation*, we read that the thought does not require a supplementation in order to obtain, but is complete in itself ('in sich vollständig', N155, CP

[31] 'Der an sich unsinnliche Gedanke kleidet sich in das sinnliche Gewand des Satzes' (T61, LU 33).

[32] 'In der Form des Behauptungssatzes sprechen wir die Anerkennung der Wahrheit aus' (T63, LU 35).

[33] 'An assertoric sentence contains both thought and assertion [only] when we speak sincerely' (T62).

386).[34] Frege also makes the *epistemological* distinction between the 'grasping of the thought' ('Fassen des Gedankens') and the 'acknowledgement of the truth' ('Anerkennung der Wahrheit eines Gedankens') (T62, CP 356). This may correspond to the Stoic distinction between rational presentations (D.L. 7.49, 7.51, *phantasiai logikai*) and assenting (*sugkatathesis*, Prantl 419), or the acknowledgement mentioned in D.L. 7.65.[35]

ASSERTORIC CONTENTS	
STOICS: Assertibles (*axiōmata*) …	FREGE: Thoughts ('Gedanken') …
… are on a par with other complete contents like commands and questions (D.L. 7.65–68; Prantl 441).	… are on a par with other complete contents like commands and questions (SB38, CP 167).
… are expressed in assertoric sentences (or have an assertoric formulation) (*ekphora axiōmatikē*) (D.L. 7.67).	… are expressed in assertoric sentences (T61, CP 354).
When we *utter* an assertoric sentence we *say the assertible* (express assertoric content) and we *assert the assertible* (make an assertion) (D.L. 7.57; D.L. 7.66; S.E. *M.* 8.71; Prantl 441).	It is with the *uttering* ('aussprechen') of the assertoric sentence that we both *express ('ausdrücken') the thought* and *assert it* (as true) or *make an assertion* (T62, CP 355–56).[36] An assertoric sentence *contains the thought* (T62, CP 355). An assertoric sentence *contains the assertion* of the thought (T62, CP 355).
An assertible is a *complete* content that can be asserted *in itself* (D.L. 7.65).	A thought does not require a supplementation in order to obtain, but is *complete in itself* (N69).
… are that which is *either true or false* (D.L. 7.65) … are the complete contents by saying which we say something true or something false (S.E. *M.* 8.73; Prantl 441–42).	'I call a thought something for which the question of truth can arise at all. So I count what is false among thoughts no less than what is true' (T60–61, CP 353–54). Every thought is *either true or false, tertium non datur* (CT37, 38, CP 391, 392 IL = EidL, NS 202 = PW 186; FGII398, CP 329; IL, PW 186; LM, PW 198).

[34] 'Ein Gedanke ist nämlich vollständig und gesättigt, bedarf um bestehen zu können keiner Ergänzung' (CT, CP 391).

[35] Gabriel, Hülser, and Schlotter 2009: 382–83 argue that there is a parallel between, on the one hand, Frege's 'Fassen des Gedankens' and 'Akt der Zustimmung', and, on the other, the Stoic *katalēptikē phantasia* and *katalēpsis*. However, this is mistaken. A *katalēptikē phantasia* is veridical. Frege's 'Akt der Zustimmung', even though described as an act by which the thought is acknowledged as true, is not veridical. (At least this is the general view. For a dissenting interpretation of Frege see Kremer 2000.)

[36] T62, CP 356: 'die Anerkennung der Wahrheit eines Gedankens—[ist] das Urteilen'. (T62, CP 356): 'In der Form des Behauptungssatzes sprechen wir die Anerkennung der Wahrheit aus'.

The Greek term for assertibles (*axiōma*) originates from our acknowledgement of [the truth of] what we say (D.L. 7.65).	We express acknowledgement of truth [of the thought] in the form of an assertoric sentence (T63, CP 356).
Grasping a complete content (in a rational presentation, *phantasia logikē*, D.L. 7.49, 51) versus acknowledgement of the content (*sugkatathesis* or *axiousthai*)	Grasping a thought ('Fassen eines Gedankens') versus acknowledgement of the truth of a thought ('Anerkennung der Wahrheit eines Gedankens') (T62, CP 355–56).

Of these parallels, philosophically the most noteworthy are (i) the distinction between *saying or conveying* an assertoric content, on the one hand, and *asserting* it, on the other; and (ii) the account of an assertoric content as that which is (precisely) either true or false, that is, as something that has built in both bivalence and the semantic *tertium non datur*.[37] (There is an important difference between Stoic assertoric contents and those of Frege: Stoic assertions can change their truth-value over time—thus time of utterance is a contextual factor. When it is day, 'it is day' is true, when it is night, 'it is day' is false. Important though it is, this point is not obvious in the *Summary*, nor is it emphasized in Prantl.[38])

III.1.2.2. Commands

The second significant complete contents are the *command-contents*. THE STOICS introduced the rudiments of a logic of commands and of hybrids that combine assertibles with command-contents. Chrysippus, for instance, in the *Logical Investigations* (his only, and only partially, surviving work),[39] considered whether sentences of the form 'do *x*, since *q*!' express a conditional command, or whether the whole sentence expresses a complex command. This important papyrus fragment is not mentioned in Prantl. There is little in Frege that suggests he considered a logic of commands. He did however give commands *some* thought, and what we find has parallels in Stoic logic as it is reported by Prantl and the *Summary*.

The *Summary* has this: 'A command (*prostaktikon*) is a [complete] content by *saying which we command*'. It follows an imperative sentence that is meant to provide an example (D.L. 7.67; Prantl 441). Sextus has 'They say that some [of the complete contents] are

[37] The principle of bivalence is the semantic principle that every proposition is either true or false. By the semantic tertium non datur I mean the principle that no proposition is neither true nor false.

[38] For a thorough treatment of the question how Stoic *axiōmata* differ from propositions as understood in Frege and much of 20th century logic, see Bobzien 1986, 11-39.

[39] See *e.g.* Barnes 1986.

commands, [namely] those with which we are commanding when we say them' (S.E. *M*. 8.71; Prantl 441). Shortly after in Sextus' list, we read that of the complete contents only the assertibles are either true or false, *i.e.* have a truth-value (S.E. *M*. 8.73; Prantl 441). Since what we say, we say using a sentence (*logos*) (D.L. 7.59, see above), it is implied that the content of an imperative sentence is that saying which we (make a) command. This is confirmed by a passage from Plutarch (not in Prantl) which makes explicit how imperative sentences and commands relate to each other in the Stoic view: the content of the imperative sentence 'do not steal' is 'not to steal' (Plut. *St. Rep*. 1037D–E). The Stoic choice of the expression *prostaktikon* emphasizes that they thought of commands as a *content* that is available to us to command (*prostassō*/*prostattō*) with.[40]

In both SB and his later work, FREGE maintains that commands are on a par with thoughts and do not contain thoughts. In SB he states that an imperative (*i.e.* imperative sentence) does not have a 'Bedeutung' but only a 'Sinn'. This 'Sinn' of an imperative sentence is said to be a command ('Befehl').[41] In (T62, CP 355), Frege states that an imperative sentence has a 'Sinn', but that the 'Sinn' is not of the kind that could have a truth-value. This is presented as the reason why the 'Sinne' of imperative sentences are not called thoughts[42]—which is in line with Frege's definition of thought (see above). In sum, for Frege imperative sentences have a 'Sinn' but no 'Bedeutung'. The 'Sinn' is a command. It cannot have a truth-value and hence it is not a thought. (Frege uses 'command' ('Befehl') both for the 'Sinn' of an imperatival sentence and for the 'Bedeutung' (SB38, CP 167) and/or act of commanding—an ambiguity he is aware of for the parallel case of the thought (*Logic* PW 137: 'It would be just as wrong to identify a thought with an act of thinking';[43] *cf.* T62, n. 3).) Note that the above-quoted passages are virtually all Frege seems ever to have written about

[40] The Stoics indicate the status of a complete content by the use of neuter adjectival noun expressions formed from a verb and ending with *-tikon* (-τικόν). Probably *lekton* is understood.

[41] SB38–39: 'A subordinate clause with "that" after "command", "ask", "forbid", would appear in direct speech as an imperative [*i.e.* imperative sentence]. Such a sentence has no meaning but only a sense. […] The meaning of such a clause is therefore not a truth-value but a command, a request, and so forth'. ('Der Nebensatz mit "dass" nach "befehlen", "bitten", "verbieten" würde in gerader Rede als Imperativ [*i.e.* Imperativsatz] erscheinen. Ein solcher hat keine Bedeutung, sondern nur einen Sinn. […] Die Bedeutung eines solchen Satzes ist also nicht ein Wahrheitswert, sondern ein Befehl, eine Bitte, u.dgl.')

[42] T62, CP 355, LU 34: 'We should not wish to deny sense to a command, but this sense is not such that the question of truth could arise for it. Therefore I shall not call the sense of a command a thought'. ('Einem Befehlssatze wird man einen Sinn nicht absprechen wollen; aber dieser Sinn ist nicht derart, dass Wahrheit bei ihm in Frage kommen könnte. Darum werde ich den Sinn eines Befehlssatzes nicht Gedanken nennen.')

[43] *Logik* NS 148: 'Ebensowenig ist der Gedanke eine Denktat'.

commands (all that is in print), which makes the parallels (see table below) the more striking.[44]

COMMANDS	
STOIC commandable (*prostaktikon*).	FREGEAN command ('Befehl').
The content (*lekton*) of a command sentence is a command-content (D.L. 7.67, implied; Plut. *St. Rep*. 1037D–E, implied).	The content ('Sinn') of a command sentence is a command-content (T62, CP 355).
Command-contents are complete contents that are on a par with assertibles (D.L. 7.65–66).	Command-contents are complete contents that are on a par with thoughts (SB38, CP 167).
Command-contents cannot be true or false and therefore are not assertibles (D.L. 7.68; Plut. *St. Rep*. 1037D–E, implied).	The sense of a command sentence (*i.e.* the command-content) cannot be true or false and therefore is not a thought (SB38, CP 167) (T62, CP 355).
By saying a command-content (which we do by uttering a command-sentence) we give a command (D.L. 7.67).	(We have a parallel for *assertions* in (T62–63, CP 355–56): By means of (uttering) an assertoric sentence, we enunciate/pronounce ('sprechen aus') the recognition of truth (= the assertion) of the thought that we express with the sentence.)

Today, the view that commands do not have propositional content is a view that is taken quite seriously by philosophers. In particular, Jennifer Hornsby's work shows the lasting significance of the Stoic and Fregean non-propositionalism (Hornsby 2016). (The difference between commands and utterances with propositional content may also be supported by the fact that some animals appear to understand commands, but it may be doubtful whether they understand propositions.)

III.1.2.3. Sentence questions

Both parties distinguish between what Frege calls word-questions and sentence-questions (D.L. 7.66; Frege: T62, CP 355; SB39, CP 167; PWLB, PW 138–39; N143–45, CP 373–75). FREGE considers *sentence-questions* in SB, T, PWLB, and N. Between SB and the three later works, his view appears to have changed somewhat.

[44] Frege's view that logic unfolds the meaning of 'true' combined with his view of commands entails that for him there can be no logic of commands. Stoic logic, as that (at the level of *lekta*) which is either true or false or neither is not bound by such a constraint.

In (SB39, CP 167), in his consideration of subordinate sentences or clauses (SB 36 'Betrachtung der Nebensätze') regarding his 'Sinn'/'Bedeutung' distinction, Frege writes 'The case is similar for the dependent question in phrases such as "doubt whether", "not to know what". It is easy to see that here also the words are to be taken to have their indirect meaning. Dependent clauses expressing questions […]'.[45] And a little later: '*i.e.* not a truth-value but a thought, a command, a request, *a question*' (SB39, italics mine).[46] This passage, taken along with its immediate context, implies several things. (i) Frege distinguishes interrogative sentences ('Fragesätze') and questions. (ii) Questions are the sense ('Sinn') of interrogative sentences. More precisely, as can be seen from the context, sentence-questions are the sense ('Sinn'), or complete content, of sentence-question interrogative sentences.[47] (iii) Questions are on a par with thoughts, but are not thoughts. (iv) It is implied that by saying an interrogative sentence we ask a question. The most pertinent passage is in *Thoughts* (T62, CP 355–56, emphasis mine).

> In order to bring out more precisely what I mean by 'a thought', I shall distinguish various kinds of sentences. […] Propositional questions are a different matter. We expect to hear 'yes' or 'no'. The answer 'yes' means the same as an assertoric sentence, for *in saying 'yes' the speaker presents as true the thought that was already completely contained in the interrogative sentence*. This is how a propositional question can be formed from any assertoric sentence. […] An interrogative sentence and an assertoric one contain the same thought; but the assertoric sentence contains something else as well, namely assertion. *The interrogative sentence contains something more too, namely a request [i.e. to respond]*. […] We have already performed the first act [*i.e.* the grasp of a thought] when we form a propositional question.[48]

[45] (Writing 'expressing' for 'expression' in CP.) SB39: 'Ähnlich ist es bei der abhängigen Frage in Wendungen wie "zweifeln, ob", "nicht wissen, was". Dass auch hier die Wörter in ihrer ungeraden Bedeutung zu nehmen sind, ist leicht zu sehen. Die abhängigen Fragesätze […]'.

[46] 'd.h. nicht ein Wahrheitswert, sondern ein Gedanke, ein Befehl, eine Bitte, *eine Frage*' (italics mine).

[47] Frege seems not to be completely consistent in his use of 'question' ('Frage') and 'interrogative sentence' ('Fragesatz'), but the context leaves no doubt that he distinguishes between them as indicated.

[48] 'Um das, was ich einen Gedanken nennen will, schärfer herauszuarbeiten, unterscheide ich Arten von Sätzen. […] Anders ist es bei den Satzfragen. Wir erwarten "ja" zu hören oder "nein". Die Antwort "ja" besagt dasselbe wie ein Behauptungssatz; denn *durch sie wird der Gedanke als wahr hingestellt, der im Fragesatz schon vollständig enthalten ist*. So kann man zu jedem Behauptungssatz

Logic (PW 129, emphasis mine): *Truth* is *only ascribed to the 'Sinn' of 'Behauptungssätzen'*.

> No one would deny that our predicate [*i.e.* 'true'] is, for the most part, ascribed to sentences. We are not, however, concerned with *sentences* expressing wishes, *questions*, requests and commands, but only with assertoric sentences […] In the cases which alone concern logic the *sense of an assertoric sentence* is either true or false, and then we have what we call a *thought proper*.[49]

And later in the same text (*Logic* PW 138–39, emphasis mine):

> We express the *same thought* in the question 'Is oxygen condensable?' and in the sentence 'Oxygen is condensable', *joining it in the one case with a request and in the other with an assertion.*[50]

In these two passages, the content of a sentence-question sentence ('interrogative sentence' henceforth, for brevity) contains two connected things: a thought (or assertoric content) joined with a request for an answer. The expected answer is 'yes' or 'no'. Thus the thought that is put forward in a question is not put forward as true or as false. This is only done with the answer 'yes' or 'no' (T62, CP 355). In fact, Frege introduces interrogative sentences and assertoric sentences together *in order to* bring out by means of an assertoric sentence the combination of thought and assertion of the thought. So also in (N144–45, CP 373–75), for example:

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eine Satzfrage bilden. […] Fragesatz und Behauptungssatz enthalten denselben Gedanken; aber der Behauptungssatz enthält noch etwas mehr, nämlich eben die Behauptung. *Auch der Fragesatz enthält etwas mehr, nämlich eine Aufforderung [i.e. zu antworten].* […] Indem wir eine Satzfrage bilden, haben wir die erste Tat [*i.e.* das Fassen des Gedankens] schon vollbracht' (T62, LU 34–35, emphasis mine). Cf. also PW 7-8 = NG 8.

[49] 'Am meisten legt man wohl unser Prädikat [*i.e.* 'wahr'] Sätzen bei; jedoch sind die Wunsch-, *Frage-*, Aufforderungs-, Befehls**sätze** auszuschliessen und nur die Behauptungssätze kommen in Betracht […] In den für die Logik allein in Betracht kommenden Fällen ist *der Sinn eines Behauptungssatzes* entweder wahr oder falsch, und wir nennen ihn dann einen *eigentlichen Gedanken*' (*Logik* NS 140–41, emphasis mine).

[50] 'In der Frage "ist Sauerstoffgas kondensierbar?" und in dem Satze "Sauerstoffgas ist kondensierbar" haben wir *denselben Gedanken* ausgedrückt, *einmal mit einer Aufforderung [i.e. zu antworten], das andere Mal mit einer Behauptung verbunden*' (*Logik* NS 150, emphasis mine).

> […] since the sense of an *interrogative sentence* is always also inherent in the assertoric sentence that gives an answer to *the question* […] In any case, we need a short term for what can be the *sense of an interrogative sentence*. I call this a *thought.* (N145, CP 374–75, emphasis mine)[51]

The key difference between SB, on the one hand, and the later T, PWLB, and N, on the other, is as follows: the distinction between the sense of an assertion-sentence as the thought and the sense of a question-sentence as a question has been replaced in *Thoughts* by the distinction between, in the interrogative sentence, the combination of a thought as its sense with a request for an answer as its force ('Kraft'), and, in the assertoric sentence, the combination of a thought as its sense with an assertion as its force ('Kraft'). The word 'question' is now avoided for the *content* of an interrogative sentence. The distinction between interrogative sentence and (sentence) question is still made (in N144, CP 373–74), possibly for the combination of the sense with the force of an interrogative sentence. The precise details of the change from SB and how it occurred may well be more complex, but for present purposes this representation of the difference suffices.

THE STOICS describe sentence questions thus: 'an assertible […] (quoted above); a sentence question (*erōtēma*) is a complete sayable like an assertible but demands an answer, for example 'is it day?' (literally '? It is day', with '?' for the question particle '*ara g*'). This is neither true nor false, hence 'it is day' is an assertible, but 'is it day?' is a sentence question' (D.L. 7.66; Prantl 441, n. 115; *cf.* D.L. 7.68; S.E. *M.* 8.66).[52] They contrast assertoric content and sentence questions with each other in order to bring out the nature of assertoric content (D.L. 7.66, 68). The juxtaposition with *axiōmata* implies that the Stoics distinguish between *interrogative sentence* and *question*. By direct analogy with what the Stoics say about the other complete content (D.L. 7.66), we expect 'the interrogative sentences express the (sentence) question' and 'saying the interrogative sentence we ask the question'.

[51] '[...] da der Sinn eines *Fragesatzes* immer auch in dem Behauptungssatze steckt, in dem die Antwort auf *die Frage* gegeben wird […] Jedenfalls bedarf man einer kurzen Bezeichnung dessen, was *Sinn eines Fragesatzes* sein kann. Ich nenne es *Gedanken*' (N145, LU 55–56, emphasis mine).

[52] That sentence questions are neither true nor false is implied also in D.L. 7.68. That an answer of yes or no is requested is also implied by S.E. *M.* 8.66.

SENTENCE QUESTIONS	
STOIC question (*erōtēma*).	FREGEAN sentence question ('Satzfrage') or question ('Frage').
Interrogative sentence and sentence-question are different things (implied D.L. 7.66–68).	Interogative sentence ('Fragesatz', interrogative sentence) and question are different things (S39B, CP 167; N144, CP 374[53]).
The sentence-question is a complete content (D.L. 7.65–66).	The sense of an interrogative sentence is complete (T62, CP 355).
The content of an interrogative sentence is a sentence-question (D.L. 7.66, implied by context).	The sense of an interrogative sentence is a (sentence-)question (SB39, CP 167, implied).
A sentence-question is like an assertible, but requests an answer (D.L. 7.66).	An interrogative sentence contains a thought joined with the request for an answer (T62, CP 355); (*Logik* NS 150, *Logic* PW 138–39 'joined').
The answer requested is 'yes' or 'no' (D.L. 7.66; S.E. *M.* 8.66, implied).	The expected answer is 'yes' or 'no' (T62).[54]
Sentence questions are neither true nor false (D.L. 7.66; expressly D.L. 7.68).	'A thought put forward in a question is not put forward as true or as false' (PWLB) (T62, CP 355 implied).
Sentence questions and assertibles are contrasted with each other in order to bring out the nature of the assertibles (in particular their being true or false) (D.L. 7.66, 68).	Sentence questions and thought (or thought plus assertoric force) are contrasted in order to bring out the nature of assertions: *i.e.* the putting forward of the thoughts as true or false (T62, CP 355) (N143–45, CP 373–75).
Sentence questions are on a par with assertibles, but are not assertibles (D.L. 7.66, implied).	Sentence questions are on a par with thoughts but are not thoughts (SB38–39, CP 167, implied).

The table shows that the overlap is extensive. The only clear difference appears to be that, in his later work, FREGE holds (i) that sentence-questions have a thought as *content* ('Inhalt'), but that in the question the thought is without the element of truth-value (N144–47, CP 373–75); [55] and (ii) that interrogative sentences *contain* (*enthalten*) a thought (T62, CP 355); and (iii) that the same thought is expressed in a question or interrogative sentence ('Frage') and in the corresponding assertoric sentence, but without the force of assertion ('[…] in der Frage

[53] 'Irgendeinen Sinn muss der Fragesatz doch wohl haben, wenn er überhaupt eine Frage enthalten soll'.

[54] Implied by what Sextus says about word questions (*pusma*), S.E. *M.* 8.66. Implied by 'expected', (T62, CP 355).

[55] In *Negation*, Frege explicitly and repeatedly distinguishes between the sentence-question ('Frage'), whose *content* the thought is and the interrogative sentence ('Fragesatz'), which *expresses* that thought (N144-147).

kann man denselben Gedanken ausdrücken wie im Behauptungssatze, aber ohne Behauptung […] behauptender Kraft', CT38, CP 391, *cf. Logik* NS 150 = *Logic* PW 138–39). Since a Fregean thought is either true or false, it follows that for Frege now a question (as that which is expressed with an interrogative sentence) stands in *some* relation to the thought's truth-value, except that, since neither the thought nor its negation is asserted, that truth-value is not acknowledged.

STOIC questions are never said to *contain* an assertible (nor are the corresponding interrogative sentences). But recall here my central hypothesis that Frege acquainted himself with Stoic logic via Prantl. The difference then becomes far less obvious: Prantl writes that at least some Stoics intended a reduction of all non-assertoric complete contents—and this includes sentence questions—to assertibles (Prantl 442–43). This is in parallel with the later Frege. Prantl then adds that—like assertibles—these non-assertoric contents are capable of *containing* ('enthalten') the element of the true and false (Prantl 443, *cf.* n. 117). This may be in line with Frege in CT38, CP 391; it differs from T62, CP 355; N145–6, CP 374–75; Logic, NS 150 = PW138–39. Prantl bases his claims on Ammonius (Ammon. *Int*. 2.26–3.6), whom he quotes in n. 117. Ammonius lists five Aristotelian kinds of sentence or speech (*logos*) and adds that the Stoics had five more, which he also lists. Then he adds that 'all these can take falsehood and truth *and can be subsumed under the assertoric*'. Prantl states that the quoted sentence can only be Stoic (Prantl 443, n. 117). He misses that the context strongly suggests the opposite, namely that this is Ammonius' addition, since it is Ammonius' objective to show that the Stoic ten kinds reduce to the Aristotelian five. Prantl then generalizes this point to all Stoic non-assertoric complete contents.[56] The table on sentence questions can thus be supplemented.

PRANTL: The Stoic non-assertoric complete contents can be reduced to the assertibles. They are capable of containing ('enthalten') the elements of the true and the false (Prantl 443, with n. 117).	FREGE: A sentence question has a thought as content (N144–47); an interrogative sentence (sentence that expresses a sentence question) contains ('enthält') a thought (T62, CP 355). Implied: the thought expressed with an interrogative sentence has a truth-value.

[56] '[…] einige Stoiker wenigstens eine Zurückführung der übrigen Sätze auf das axiōma beabsichtigt zu haben, insoferne nemlich erstere ebenfalls fähig seien, das Moment des Wahren und Falschen in sich zu enthalten […]' (Prantl 442–43). The Stoics may have held that an oath contains an assertible, but is neither true nor false because of the additional element of swearing. This would come close to Frege's view that sentences with an emotive particle contain ('uneigentliche') thoughts, that is thoughts without a truth-value (see *Ecl*. 1.28.17–19 with Barnes 1999: 201). There is no reason to think that Frege knew the Stobaeus passage.

So, virtually everything we find in Prantl and in the *Summary* about sentence questions has parallels in some of Frege's work.

Again, the positions of the Stoics and Frege that a question does *not* contain an assertoric content, or alternatively that there are questions that contain an assertoric content but without having an alethic status, may have a philosophical advantage. On such a view, one can *meaningfully* ask questions, even if there is no assertible, nothing that is precisely either true or false, that fully corresponds to one's question. These kinds of position are taken up in some recent research on questions, and have applications in, for example, theories of vagueness. One can meaningfully ask, 'Is Sam tall?', even if one holds the view that (it is possible that), in that—non-fictional—context, 'Sam is tall' has no settled (or definite or determinate) truth-value, and hence is not an assertoric content (*axiōma*, thought). Of course, since in Frege's radical view on vagueness, vague predicates have no *Bedeutung*, he would have had neither need nor use for this kind of deliberation.

III.1.2.4. Quasi-assertibles, the expression of emotion, and apparent or mock thoughts

For comparison with Frege, perhaps the most fascinating of Stoic complete contents are the so-called similar-to-assertibles or, as they are often translated, quasi-assertibles (*homoion axiōmati*, Prantl 442; D.L. 7.67 = Prantl 441, n. 115; Ammon. *Int*. 2.26–3.6 = Prantl 442, n. 117) and those that are *more than assertibles* (*pleiona ē axiōmata*, S.E. *M.* 8.73). 'Quasi-assertibles' and 'more-than-assertibles' are in fact two Stoic ways of referring to the same kind of complete assertible. Prantl, however, presents them separately (Prantl 441, 443). This is relevant, since Frege discusses two kinds of complete contents that, each in their own way, show remarkable similarities to Prantl's report from, and representation of, the Stoic position.

(i) EMOTIVISM:

The STOIC definition in the *Summary* reads: 'a quasi-assertible is linguistically expressed in the way assertibles are (*tēn ekphoran ekēon axiōmatikēn*), but it falls outside the genus of assertibles due to the addition of some particle or some emotion' (D.L. 7.67; Prantl 441, n. 115). The examples are '(Gee), the Parthenon is beautiful'[57] and '*How* the cowherd resembles

[57] ὅμοιον δ᾽ἐστὶν ἀξιώματι ὃ τὴν ἐκφορὰν ἔχον ἀξιωματικὴν παρά τινος μορίου πλεονασμὸν ἢ πάθος ἔξω πίπτει τοῦ γένους τῶν ἀξιωμάτων with the examples καλός γ᾽ὁ παρθενών. ὡς Πριαμίδῃσιν

Priam's sons'. Another source mentions that the additional particle for a quasi-assertible is '*how*' (*ōs*) (Ammon. *Int.* 2.26–3.6; Prantl 441, n. 115, 443, n. 117). This suggests that the Stoics use the particle *how* deliberately as a linguistic *indicator* that an assertoric sentence expresses a quasi-assertible, in line with their general method of language regimentation.[58] (In Greek, the grammatical mood of a sentence does not guarantee that the sentence is used to express a Stoic complete content of a particular kind.)[59] So, the Stoic quasi-assertibles have the grammatical form of an assertoric sentence, either exactly or with an additional particle (*ge* or *hōs*). They are very similar to assertibles, more so than any other complete content. The emotive element either has a lexical correlate in the sentence expressing the quasi-assertible or it has a correlate in how the sentence is expressed, presumably the intonation used. We seem to have examples of both.

The distinction the Stoics are after is philosophically significant. It is between assertible contents and contents that are expressed in sentences of very similar, or identical, form, but that are considered not assertible because they contain an additional element of emotion. They are, in *some* sense, *more than an assertible*. The emotive element is additional. This emotive element is *part of the content*, and, or so the Stoics believe, someone who says such an emotion-infused content does not make an assertion. Moreover, *qua* being part of the content, this element of emotion is something all humans can, in principle, share in. When we *say* such quasi-assertibles, we do something other than asserting. Some later sources suggest, for example, that what we do is marvel or admire (Prantl 442; Simpl. *in Cat*. 406.20–26; Prantl 443, n. 117).[60] Compare an assertion that 'She is strong' with 'How strong she is!', or 'Wow, she is strong' or 'She is *strong*' (with an intonation of wonder or similar). When we marvel, the content of our assertoric sentence is not truth-evaluable. Unlike an assertible, a quasi-assertibles is neither true nor false (D.L. 7.67–68). In Sextus, the quasi-assertibles are described as 'more-than-assertibles' (S.E. *M.* 8.73; Prantl 441, n. 115) and are explicitly said not to be assertibles. Context implies that they have no truth-value. It is not an asserting (of a content that indicates recognition of the truth of what we say), but a

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ἐμφερὴς ὁ βουκόλος (D.L. 7.67). It is not clear whether the *ge* counts as such a particle, and hence whether and how to translate the *ge*. Is this LSJ *s.v.* II.4? If so, maybe as 'gee', 'gosh', or 'golly'.

[58] The regimentation requires putting particles, as far as is grammatically felicitous, towards the beginning of sentences that express complete contents. See Bobzien and Shogry, forthcoming, Atherton and Blank 2003: 314–16, and Barnes, Bobzien, and Mignucci 1999: 96–97; also Frede 1974.

[59] Barnes 1999: 200; Bobzien and Shogry, forthcoming; also Prantl 442, in a woolly way.

[60] *E.g.* Simpl. *in Cat*. 406.20–26, cited in Prantl 443, n. 117. Further sources, which are hard to access, have been collected as 899, 900, 900A, 900B in Hülser 1987, frgs FDS 1118–22.

marvelling—an expression of an emotional perspective on an aspect of the world. Perhaps something surpasses our expectation to a point that motivates us to express our surprise—to marvel out loud, as it were. So what I say when I say 'Wow, she is strong' is not truth-evaluable, although it is *similar to* the truth-evaluable 'She is strong'. The Stoics did *not* think of quasi-assertibles as a *combination* of an assertible with an emotive element, such that by saying them we would (i) assert them and (ii) express an emotion. The Stoics thus maintain an *emotivist* position, not specifically for moral statements but for the content of declarative sentences with which, by means of the content that elicits an emotion, that emotion is expressed.

FREGE, too, considers assertoric sentences that express an element of emotion.[61] In the *Gedanke* he writes:

> An assertoric sentence often contains over and above a thought and assertion, a third component […] meant to act on the feeling and mood of the listener. Words like 'alas' and 'luckily' belong here.
> (T63, CP 356)

And a little later,

> Thus the content of a sentence often goes beyond the thought expressed by it.
> (T64, CP 357)[62]

Neither adverbial expressions like 'alas' nor exclamative particles like 'oh' are *necessary*. In his posthumous *Logik*, Frege writes:

> We can substitute words like 'oh' and 'unfortunately' for [an emotional] tone of voice without altering the thought.
> (PWLB, PW 140)[63]

[61] This parallel is also mentioned by Gabriel, Hülser, and Schlotter 2009: 385.

[62] 'Ein Behauptungssatz enthält außer einem Gedanken und der Behauptung oft noch ein Drittes […]. Das soll nicht selten auf das Gefühl, die Stimmung des Hörers wirken oder seine Einbildungskraft anregen. Wörter wie "leider", "gottlob" gehören hierher' (T63). 'So überragt der Inhalt eines Satzes nicht selten den in ihm ausgedrückten Gedanken' (T64).

[63] 'Diesen Klang der Stimme kann man auch durch Wörter wie "ach", "leider" ersetzen, ohne am Gedanken etwas zu ändern' (*Logik* NS 152).

Just like the Stoics, Frege suggests that the *content* of such sentences goes beyond that of an assertoric content (a thought, for Frege), and that what goes beyond 'does not belong to the thought' (T63, CP 357). And just like the Stoics, Frege suggests that that which goes beyond can be expressed either by intonation alone, or by certain linguistic indicators of emotion such as 'oh'.

Frege and the Stoics *differ* on the issue of whether what is expressed by such 'emotive' sentences is truth-evaluable. The Stoics are clear that the answer is 'no' (see above). Frege (in T, N, and PWLB) thinks of the content expressed as a *composite*, including a thought, an assertion, and something extra (T62–63, CP 357). And by way of the thought, what is expressed is truth-evaluable and the truth-value is independent of the emotive element (PWLB, PW 140) ('If someone announces the news of a death in a sad tone of voice without actually being sad, the thought expressed is still true').[64] (Note the difference to Fregean sentence questions. For Frege, the sentence 'Is she strong?' contains a thought but not an assertion. In contrast, 'Wow, she is strong' contains both a thought and an assertion.) The difference between Frege and the Stoics concerning truth-evaluability does not conflict with the hypothesis that Frege draws on Prantl for Stoic thought. For, once more, Frege and *Prantl's* Stoics are in agreement. Prantl misreads Ammonius' testimony on Stoic complete contents (see previous section on questions) and writes that Stoic complete contents that are not assertibles are still 'able to contain in themselves the True and the False'.[65] This is very much like what Frege writes.

EMOTIONAL CONTENT IN ASSERTORIC SENTENCES	
STOIC 'more-than-assertibles' (*pleiona ē axiōmata*) (D.L. 7.67; Prantl 441, n. 115).	FREGEAN assertoric sentences containing more than thought and assertion (T63, CP 356).
Emotional element expressed either by additional particle or by tone of voice (D.L. 7.67; Prantl 441, n. 115).	Emotion-eliciting element expressed either by additional particle or adverb, or by intonation or emphasis (T63, CP 356; PWLB, PW 140).

[64] 'Wenn jemand eine wahre Todesnachricht mit einer traurigen Stimme ausspricht, ohne wirklich traurig zu sein, so ist der ausgedrückte Gedanke dennoch wahr' (*Logik* NS 152). Frege's *test* for truth evaluability is epistemic (Logic 152, PW 140); the Stoics would presumably reject the test.

[65] '[…] fähig seien, das Moment des Wahren und Falschen in sich zu enthalten' (443, read together with n. 117, 'δεκτικά ὄντα ψεύδους τε καὶ ἀλήθειας […] aus dem Munde stoischer Anschauungen'.

Go beyond assertibles by an emotional element (are not assertibles) (D.L. 7.67; Prantl 441, n. 115).	Go beyond the thought cum assertion by an emotional (emotion-eliciting) element (T63, CP 356).
Stoics: are neither true nor false (emotivism) (implied D.L. 7.67; S.E. *M.* 8.73; Prantl 441, n. 115).	(See below.)
Prantl: contain ('enthalten') in them the True and the False (Prantl 443, based on Ammon. *Int.* 2.26–3.6; Prantl 443, n. 117).	Contain a thought and assertion, and hence are true or false (T62, CP 356–57 implied—perhaps unclear; PWLB, PW 140 unclear).

There is a difference between the Stoics and Frege regarding the emotive element. For the Stoics, it regards the emotion that the speaker (has or pretends to have and) expresses. For Frege, it regards the emotions the speaker intends to elicit in the listeners. Emotive language of course fulfils both functions.

(ii) MOCK THOUGHTS:

Interestingly, Frege also discusses cases of complete contents that are very similar to thoughts, and of which he says in at least *some* of his writings that they are neither true nor false. These are Frege's infamous 'Scheingedanken': apparent thoughts or mock thoughts.[66] These mock thoughts display the feature of the Stoic quasi-assertibles which Frege's emotion-laden thoughts lack, *i.e.* the absence of alethic values.

In his PWLB, Frege suggests that mock thoughts (i) are expressed by/in assertoric sentences; (ii) are neither true nor false (*cf.* Prantl 443 'because of the less simple form only similar to the true, but not themselves true');[67] and (iii) are not thoughts, but (iv) only apparent thoughts. He also says (v) that when we express a mock thought in an assertoric sentence, we do not have an *assertion* but, rather, an *apparent assertion*. The similarity of Frege's apparent thoughts to Stoic quasi-assertibles is evident (all in *Logik* NS 141–42 = PWLB, PW 130). In some other places, Frege propounds the view that contents such as those he discusses in *Logik* (NS 141–42 = PWLB, PW 130) are thoughts but have no 'Bedeutung', and, when expressed, we do not have assertions but only pseudo/mock-assertions (*e.g.* Letter to Russell, 28.12.1902, PMC 152; 13.11.1904, PMC 165). (Generally, Frege situates mock thought mostly in fiction. The Stoics employ examples from literature for their quasi-

[66] PWLB, PW 130: 'the writer […] has his eye on appearances ('Schein')'. A 'Schein' *x* is what looks like an *x* but is not an *x*.

[67] '[…] wegen der weniger einfachen Form dem Wahren bloss ähnlich, nicht aber selbst wahr'.

assertibles, but they have non-fictional uses in mind. However, this is not obvious from Prantl.)

APPARENT ASSERTORIC CONTENT	
STOIC quasi-assertibles (*homoion axiōmati*) (D.L. 7.67; Prantl 441, n. 115; Ammon. *Int.* 2.9–6.3; Prantl 441, n. 117).	FREGEAN mock thought ('Scheingedanke') (*Logik* NS 141–42 = PWLB, PW 130).
Said by using assertoric sentences (D.L. 7.67; Prantl 441, n. 115).	Expressed in/by assertoric sentences (*Logik* NS 141 = PWLB, PW 130).
Neither true nor false (Implied D.L. 7.67; S.E. *M.* 8.73; Prantl 441, n. 115).	Neither true nor false (*Logik* NS 141–42 = PWLB, PW 130, implied Letters to Russell from 28.12.1902, PMC 152; 13.11.1904, PMC 165).
Not assertibles and do not contain assertibles (D.L. 7.65–68, implied by context).	Not ('eigentliche') thoughts, do not contain thoughts (implied, *Logik* NS 142 = PWLB, PW 130).
Similar to assertibles (D.L. 7.67, implied by name).	Apparent thoughts and apparent assertions (*Logik* NS 141–42 = PWLB, PW 130).

Frege's two cases do not fully correspond to Prantl's two mentions of Stoic quasi-assertibles and more-than-assertibles. Rather, there are two Fregean kinds of complete contents, each of which exemplifies several features of the Stoic quasi-assertibles and similar-to-assertibles, as Prantl reports them. Frege's mock thoughts have more in common with Prantl's quasi-assertibles (443) and Frege's more-than-thoughts with Prantl's more-than-assertibles (441).

III.1.2.5. Word questions: two kinds of completeness of content

The Stoics and Frege agree that, like sentence questions, word questions request an answer. STOICS: inquiries (*pusmata*) are that by saying which we inquire (*punthanometha*), *e.g.* 'where does Dion live?' (D.L. 7.66; S.E. *M.* 8.71–72; Prantl 441). FREGE: 'In a word-question we utter an incomplete sentence, which is meant to be given a true sense just by means of the completion for which we are asking' (T62, CP 355).[68] The texts on word questions do, however, reveal a difference in how the two parties look at complete contents. For the STOICS, word questions express a complete content (D.L. 7.65–66). For FREGE, word-

[68] 'In einer Wortfrage sprechen wir einen unvollständigen Satz aus, der erst durch die Ergänzung, zu der wir auffordern, einen wahren [= genuine] Sinn erhalten soll' (LU 34–35).

questions do not express a genuine complete sense (T62, CP 355). Nor do exclamations, since one cannot form a corresponding sentence question (T62, CP 355). This exemplifies the use of different criteria by the Stoics and Frege for what counts as complete. For Frege, 'who has the key?' has no corresponding (complete) 'Sinn', even if the context of utterance is taken into account. All there is is the *'Sinn' fragment*, or function, '[…] has the key'. By contrast, for Frege a command such as 'close the door!' provides the addressee in its context, which presumably is relevant for making it express a Fregean complete sense. For a word question, the context of utterance does not provide the answer and thus leaves the thought incomplete. For THE STOICS, we may assume that a word question is *a complete move in the language game*. There is nothing missing. A second Stoic kind of complete sayable that confirms this are addresses (*prosagoreutika*, D.L. 7.66–67). An example is 'most magnificent son of Atreus, leader of men, Agamemnon' (D.L. 7.67). Addresses are explicitly counted among the complete sayables. The criteria for completeness of Frege and Stoics are thus *de facto* quite different. Both these notions of completeness are critical in philosophy of language.

III.1.2.6. Complete contents expressed with indexicals

So both Frege and the Stoics expressly consider the case in which the content of an *assertoric sentence* goes beyond that of an *assertoric content* (T64, CP 357–58; *Logik* NS 150 = PWLB, PW 139; S.E. *M.* 8.70–73; Prantl n. 115; Prantl 443). Both comment on the fact that the opposite can also happen: 'that the mere wording does not suffice for the expression of the' assertoric content, to borrow Frege's formulation (T64, CP 358; *Logik* NS 150–51 = PWLB, PW 139**)**. Examples are sentences such as 'I am cold ('ich friere')' (*Logik* NS 146, PW 134) or 'this one is walking' (D.L. 7.70). More specifically, both had to show how their theory of incorporeal content can cope with the fact that such sentences alone are insufficient to express a complete content. It is for this purpose, it seems, that each introduced a basic theory of indexicals and context sensitivity.

Sentences like the examples above immediately raise a number of philosophical questions. They can be used to express different content in different contexts. Still, speakers uttering them usually succeed in unambiguously expressing a complete content. What is this content? How do they succeed in using such sentences? What are the truth-conditions for such content? How do we mention such contents? How is the content of, for example, 'I am cold' when Sam says it related to the content of 'Sam is cold'? Do the contents have the same

semantic value? Are they the same assertoric contents? Both the Stoics and Frege appear to have considered most of these questions.

THE STOICS' main emphasis is on sentences with *demonstratives*, like 'this one'. They are aware that a sentence like 'this one is walking' expresses different things in different contexts. They called such sentences *deictic sentences* (S.E. *M.* 8.96, *ta kata deixin ekpheromena*; Prantl 444, n. 119) and held that, in order for them to express anything, they must be accompanied by an act of pointing. A (simple) deictic assertible is said to be composed of a predicate and a deictic referent (*deiktikē ptōsis*, D.L. 7.70; Prantl 444). They called the assertibles expressed in this way *deictic assertibles* (*katagoreutikon*, *ibid*.). The deictic referent is what is at the receiving end of the pointing. The pointing need not be done with a finger or hand, but apparently can be a nodding with the chin, when saying 'I' (*egō*) (Gal. *PHP* 2.2). A successful assertion of a deictic assertible requires that the speaker combines the deictic sentence with a pointing at the deictic referent. The standard way of *mentioning* a deictic assertible was: 'this one, pointing at Dion, is asleep' (Alex. *An. Pr.* 177.25–178.1; Prantl 464–65: 'wenn bei dem Aussprechen des Satzes "wenn Dion gestorben ist, ist dieser gestorben" zugleich mit dem Finger auf Dion hingezeigt werde').[69]

The most noteworthy aspect of the Stoic theory of indexicals is that they are very clear that pairs of sentences like the following express two different contents:

(1) Dio walks.
(2) This one (with a pointing at Dio) walks.

These express two different assertibles. First, their truth-values can differ. 'Dio walks' is true when Dio walks. By contrast, 'this one walks', with a pointing at Dio, is true when that which is pointed at walks (S.E. *M.* 8.100; *cf.* Prantl 465). Its truth-conditions are expressed without use of a proper name. Second, the assertibles expressed by (1) and (2) have different existence conditions. For (2), pointing at Dio, to exist, the non-verbally indicated object must exist, so that it can be pointed at. By contrast, since in (1) a proper name refers to an object, we can say something about the object even when it no longer exists (Alex. *An. Pr.* 177.25–178.1; Prantl 464–65, n. 166). The deictic assertibles have stricter requirements for assertibility than those with proper names. For their assertion, it must be possible to point at

[69] The details of the Stoic theory of *deixis* are controversial. Luckily, they are not required here. I also disregard any irrelevant metaphysical details.

the deictic referent. In (1), by contrast, the proper name has some content (the *orthos ptōsis*) that warrants assertibility, even when the referent of the proper name has ceased to exist. When Dion is dead, (1) is false and cannot be true (*ibid*.). Our texts imply that, for the Stoics, even when Dio is alive, 'this one is dead' (with 'this one' for Dio) cannot be asserted without Dio being in pointing distance.

FREGE tackles the very same problem as the Stoics in his T, when he says that often 'the sentence alone is not enough for the expression of a thought', and this is particularly so when the sentence contains an indexical. We learn that 'the complete expression of the thought' may require 'finger pointing, hand gestures, and glances'. This is just what the Stoics suggest. Frege also remarks that sentences with indexicals express different thoughts when used by different people, in which case one thought can be true, the other false (all in T64, CP 358). The case Frege investigates in detail is a sentence that involves the first person pronoun 'I' rather than a demonstrative. Frege considers the question whether the sentence

(3) Dr Gustav Lauben was wounded

expresses the same thought as the sentence

(4) I was wounded (when said by Dr Gustav Lauben) (T64-5, CP 358).

His verdict is 'no': (3) and (4) express different thoughts. The similarity to the Stoics is striking. Where the Stoic argument is primarily metaphysical, Frege's is epistemic: someone who doesn't know that the person uttering (4) was Gustav Lauben cannot know that (3) and (4) concern the same person (T65, CP 358–59). We can safely assume that the thought expressed by (4) cannot be expressed when Dr Lauben is not present. [70]

Frege is silent on the question of whether there can be situations in which what is expressed with (3) and what is expressed with (4) do not share a truth-value. He expressly states that, as long as the referent of different *proper names* is the same, thoughts that differ only with regard to those *names* will have the same truth-value (T66, CP 359). He is also

[70] In some of his earlier work, Frege writes that one can obtain the same thought if a third person uses a name instead of the first person using 'I' (*Logic* PW 134-5). It is quite possible that in this passage Frege has the specific circumstances in mind in which a third person is present. If this is so, there is no significant difference to Frege's view as expressed in *Thoughts*.

clear that different sentences can express the thought expressed by (4). For example, he writes that Gustav Lauben may say

(5) The one who is now speaking to you was wounded

('by doing this he makes the conditions accompanying his utterance serve toward the expression of a thought',[71] T66, CP 360).[72] Perhaps the same thought is *still* expressed if someone else present says 'the one who just spoke was wounded'.

Another question on which Frege is silent is whether there can be situations in which the thought expressed by (3) can be asserted but the thought expressed by (4) cannot. Unlike Stoic assertibles, Fregean thoughts are defined as being eternal. So the Stoic view cannot be Frege's. However, the fact that a thought is eternal does not entail that it can be *expressed* or *asserted* at all times. In fact, it is compatible with what we know of the later Frege that he then believed that sentences of type (4) cannot be expressed by using a proper name or definite description in its subject place: the Lauben example strongly suggests this. It seems then that for the late Frege there are situations in which the thought that corresponds to an indexical sentence cannot be expressed. (Suggested by T65–66, CP 358–60 and by (4) and (5) above in particular.) Compare also Key Sentences on Logic 9 (PW 174): 'the sentence "this table is round" is the expression of a thought only if the words "this table" are not empty sounds but designate something specific for me' (Kernsätze zur Logik 10 (NS 189)).[73] This suggests that, eternity notwithstanding, when it is not possible to indicate the subject of the thought via indexical expressions (plus gesture, where required), the thought cannot be asserted. The eternity of thoughts would also be compatible with the assumption that indexical thoughts are, as it were, silenced for good when the possibility no longer exists of expressing them via indexical expressions (plus gesture, where required). Metaphysical considerations aside, this would come close to what the Stoics suggested.

[71] 'wobei er die sein Sprechen begleitenden Umstände dem Gendankenausdrücke dienstbar macht' (LU 40).

[72] This fact suggests that Frege's mysterious unconveyable 'I'-thoughts are not those expressed by (4) and (5), but are a third kind of thoughts that are of minor interest for logic.

[73] '[…] ist der Satz "dieser Tisch ist rund" nur dann Ausdruck eines Gedankens, wenn die Worte "dieser Tisch" mir etwas Bestimmtes bezeichnen, nicht leere Worte sind'.

ASSERTORIC CONTEXTS THAT ARE EXPRESSED WITH INDEXICALS	
STOICS: There are assertoric sentences that are, on their own, insufficient to express an *axiōma* and need to be supplemented by *deixis*, gesture (D.L. 7.70; Alex. *An. Pr.* 177.25–178.1; Prantl 464–65).	FREGE: There are assertoric sentences that are, on their own, insufficient to express a thought and need to be supplemented by finger pointings ('Fingerzeige'), gestures, glances (T64, T65, CP 358).
We *mention* such sentences by supplementing them with a phrase, 'pointing at *x*' where *x* is the referent of the demonstrative (Alex. *An. Pr.* 177.25–178.1; Prantl 464–65). Such pointings are not restricted to finger pointings, but also include *e.g.* gesturing with one's chin.	We *mention* such sentences by supplementing them with a phrase (T66, CP 360), 'by doing this he makes the conditions accompanying his utterance serve toward the expression of a thought' (T64, CP 358): Such 'Umstände' can include '*Pointing the finger, hand gestures, glances*', ('Fingerzeige, Handbewegungen, Blicke').
The truth-conditions for such assertoric content require that the predicate holds of that which is pointed at (S.E. *M.* 8.100; implied Prantl 464–65).	(The truth-conditions for such assertoric content are not given. Context implies that they have a truth-value.)
Discuss two sentences that differ only in that their subject expression is once a name or definite description, once an indexical expressed together with *deixis* and which *express different axiōmata* (Alex. *An. Pr.* 177.25–178.1; Prantl 464–65).	Discusses two sentences that differ only in that their subject expression is once a name or definite description, once an indexical expressed together with *deixis* and which *express different thoughts* (T64–66, CP 358–59).
Argue that these express different assertibles (*ibid.*).	Argues that these express different thoughts (T64–66, CP 358–59).
In the normal cases, *i.e.* when they can both be expressed, these have the same truth-value (Alex. *An. Pr.* 177.25–178.1; Prantl 464–65, implied).	In normal cases, *i.e.* when they can both be expressed, these have the same truth-value (T65–66, CP 358–59, implied).
There are situations in which the assertible that corresponds to an indexical sentence with *deixis* cannot be expressed (Alex. *An. Pr.* 177.25–178.1; Prantl 464–65).	Conjecture: There are situations in which the thought that corresponds to an indexical sentence cannot be expressed. (Suggested by T64–66, CP 358–60 and by (4) and (5) above in particular and Kernsätze zur Logik 10.)

III.1.3. Propositional logic and (more) language regimentation

The second main subject area in which parallels between Frege and the Stoics abound is that of the elements of propositional logic. As with our first area, some of these parallels have been noted before, in particular the definition of material implication. However, nobody has yet exposed the vast extent to which there is overlap in terminology, choice of topics, and

theory. This section is structured by the different kinds of molecular assertoric contents (propositions from here on, for brevity) and any closely related notions.

Unlike the predominant contemporary classification of atomic and molecular propositions, THE STOICS distinguished *simple and non-simple assertibles*. The simple ones are atomic assertibles, their negations (*literals*, in contemporary jargon), and any negations of simple assertibles. In his late published essay triad, Frege deals with negation in one essay and with what he calls compound thoughts, which are analogous to Stoic non-simple assertibles, in another. Simple negations are not compound thoughts. For comparison, on the one hand, Russell introduces negation in his *Principles of Mathematics* in terms of the conditional ('implication', Russell 1903, 16–17) and in 'Mathematical Logic as Based on the Theory of Types' he introduces negation and disjunction in one breath, and then defines the conditional in terms of negation and disjunction (Russell 1908, 244–45). On the other hand, standard logic texts, all in the Aristotelian tradition, tend to introduce affirmation/negation or assertion/denial in term-logical form and then, after presenting Aristotelian syllogistic, briefly mention hypothetical and disjunctive syllogisms, also generally in term-logical garb (if A is B, then C is D; *a* is either F or G).[74] So what the Stoics and Frege have in common is the combination of non-term-logically defined assertoric contents with the introduction and treatment of simple negations independently of their compound propositions.

III.1.3.1.1. Negation

Frege's *Negation* (N) and Prantl's discussion of Stoic negation both show clearly that the notion of negation that classical propositional logic takes for granted was not at all intuitive at a time when Aristotelian logic set the standard (Arist. *De Int.*, esp. chapters 5–6). While Aristotle distinguishes between assertion and denial,[75] Stoic and Fregean negations have the same assertoric force as Stoic and Fregean affirmations (*e.g.* D.L. 7.69; Prantl 444, n. 120; Frege: N154, CP 384; *Logik* NS 164 = PW 152; EidL, NS 201 = PW 185; *Kurze Übersicht*, NS 214=PW 197). Both Prantl's criticism of the Stoics (Prantl 444–45, 449–50) and Frege's defence of his negation (N152–55, CP 382–86) have to be read in this wider context.

A STOIC negation (*apophatikon*) is an assertible (D.L. 7.69; Prantl 444). It is syntactically defined iteratively as formed from an assertible and a negation part 'not' (D.L. 7.69; Prantl 445, n. 121). An example for a negation is 'not: it is day' (D.L. 7.69; Prantl 444,

[74] *Cf. e.g.* Ulrici 1852, who goes on to deny the existence of hypothetical propositions.

[75] This is a position that has regained some popularity with Ian Rumfitt's contemporary uptake of Aristotle's view in Rumfitt 2000.

n. 119; Prantl has the Apuleius passage, 445, n. 121). Stoic language regimentation requires the use of the negation particle *ouchi* ('not:' or 'it is not the case that') as an indicator of scope to be placed at the beginning of the sentence that expresses the negation (Apul. *Herm.* 191.6–11, *cui negativa particula praeponitur*; Prantl 445; also S.E. *M.* 8.90).

The *NEGATION PART* is sometimes called 'negative' (*apophasis*, S.E. *M.* 8.89, 103), sometimes 'negation' or 'negative [part]' (*apophatikon*, Alex. *An. Pr.* 402; *FDS* 921, *apophatikon* [*morion*]). The term 'apophatikon' is also used for the whole negative assertible. Presumably in one case 'assertible' (*axiōma*) is understood, in the other 'part(icle)' ('morion') (see Apuleius' *negativa particula* at *Herm.* 191.6–11; Prantl 445, n. 121). *STOIC NEGATIONS CAN BE OF* simple or *NON-SIMPLE ASSERTIBLES*. A Stoic example of the negation of a non-simple assertible is: 'Not [both] Plato is dead and Plato is alive'. These were called *negation* of a conjunction (D.L. 7.80 plus context, S.E. *PH* 2.182 plus context, 226 plus context, all in Prantl 473–74, n. 182). This contrasts with the Peripatetic term 'negative conjunction'.[76] The *TRUTH-CONDITIONS* of the negation can be gauged from a passage that tells us that 'when "it is day" is true, "not: it is day" is false, and vice versa' (S.E. *M.* 8.103; *cf.* Prantl 449–450). So, Stoic negation is truth-functional. There is no difference in *'FORCE'* between Stoic affirmations and negations. They are each assertoric contents that can be asserted (are *apophantikos*): the Stoic definition of *axiōma* as assertible covers negations.

FREGE discusses negations in detail in his *Negation* (*Die Verneinung*) (N), and in shorter form in *Logik* (NS 161–62, PW 149–50), EidL (NS 201, PW 185), Kurze Übersicht (NS 214, PW 198), and at the beginning of CT. Negations are assertoric contents formed from an assertoric content and a negation [function] (N155, CP 386): the negation of a thought is a thought (N156, CP 387) and the negation [function] requires supplementation with a thought (N155, CP 386; CT37 CP 391). The account of negation is thus iterative. In *Negation*, Frege introduces and uses the operator 'die Verneinung von …' (N155–57, CP 385–89). In CT, Frege frequently expresses a negation by 'not *A*' (so explained at *e.g.* CT40, CP 394 and used *passim* in CT for sentence schemata: CT40, 41, 42, 44, 45, 46, 48, 49, 50, 51 CP 394, 395, 396, 398, 399, 400, 401, 403, 404, 405, 406). Either way, the negator stands at the beginning of the negation—just as the negation stroke ('Verneinungsstrich') in Begr. Frege uses the term negation ('Verneinung') both for the negation function and for the complete negative thought (*e.g.* N155, 157, CP 386, 388–89). Frege offers 'not [A and B]' as an example of a *NEGATION OF A COMPOSITE ASSERTORIC CONTENT*. He calls such composite

[76] *Cf.* Bobzien 2014, section 2.1.

contents negations (CT40, CP 393) and *not* negative conjunctions; the latter is common in nineteenth-century logic.

Regarding *TRUTH-CONDITIONS*, he writes that of two assertoric contents *A* and the negation of *A*, one is true and one is false: Of the two thoughts *A* and the negation of *A*, always one and only one is true (N157, CP 389).

Negation belongs to the content, not to the force of a judgement ('Urtheil'): a negation is asserted, not an affirmative content denied (Begr 4, §4: 'Verneinung haftet am Inhalte', Begr 10 §7: assertion of negation (not denial) 10; N153–55, CP 383–86). There is no difference in *'FORCE'* between affirmations and negations. They are each assertoric contents that can be asserted ('behauptet') (*Logik* NS 161 = PWLB, PW 149; EidL NS 201 = PW 185; N154, CP 384–85).

Both Frege and Stoics have the negator at the level of content (Frege N155, CT37; S.E. *M.* 8.89, 90). Frege explicitly distinguishes from this the word that expresses the negator at the linguistic level (N155, CT37). In ancient non-Stoic sources that do not distinguish between linguistic expressions and their content, the distinction can be blurred (Apul. *Herm.* 191.6–11). Frege uses 'Verneinung' (for the negation function) and 'Verneinungswort' for the part of the sentence that expresses the 'Verneinung' (N67).

NEGATION	
STOIC negation (*apophatikon*).	FREGEAN negation ('Verneinung').
The word *apophatikon* is used both for the negation particle and for the negative assertible (D.L. 7.69 (assertible); Plut. *Quaest. Convivales* 8.9.3 (assertible); Prantl 449 ('Verneinung': used for both); S.E. *M.* 8.90 (assertible); Alex *An. Pr.* 402 (negator)).	The word *Verneinung* is used both for the negation function and for the negative thought (N155, 157, CP 386, 388–89).
The negation is an assertible (D.L. 7.69; Prantl 444).	The negation of a thought is a thought (N156, CP 387).
It is formed from an assertible and a negation part 'not' (D.L. 7.69; Prantl 445 n. 121).	The negation [function] requires supplementation with a thought (N155, CP 386; CT37, CP 391).
The linguistic expression of the negation part is prefixed to the assertion it negates (Apul. *Herm.* 191.6–11; Prantl 445, n. 121).	The linguistic expression of the negation is at the front of the sentence expressing the negation. (N155–57, CP 385–89, 'die Verneinung von'; 'Nicht( )'. So announced (CT40, CP 394) and used *passim* in CT for sentence schemata.

The definition of negation is iterative.	The definition of negation is iterative.
A Stoic example for the negation of a non-simple assertible is 'Not [both] Plato is dead and Plato is alive'. Such assertibles were called *negation* of a conjunction (D.L. 7.80; S.E. *PH* 2.182; Prantl 473–74, n. 182).	Frege's example of the negation of a compound thought is 'not[A and B]'. This is called '*negation* of a compound thought (of the first kind)' (CT40, CP 393, italics mine).
When 'it is day' is true, 'not: it is day' is false, and vice versa (S.E. *M.* 8.103; *cf.* Prantl 449–50).	Of the two thoughts *A* and the negation of *A*, always one and only one is true (N157, CP 389).
The force of negations is assertoric (entailed by the definition of *axiōma*) (D.L. 7.68–70 with 65).	The force of negations is assertoric (Begr 4, 10, *Logik* NS 161 = PWLB, PW 149; EidL, NS 201 = PW 185; N154, CP 384–85).

III.1.3.1.2. Contradictories

One essential role of the negation is in the definition of contradictories (*antikeimenon*). For THE STOICS, the notion of contradictoriness (or of pairs of complementary literals) is central to their propositional logic (Bobzien 2019). Syntactically, they define contradictory assertibles as 'those in which the one surpasses the other by a negation [particle]' (S.E. *M.* 8.89; Prantl 449). Semantically, they say that 'two assertibles are contradictories (*antikeimena*) with respect to truth and falsehood when one is the negation of the other (D.L. 7.73; Prantl 450 'that of two contradictory opposites only one can be true' ('dass von zwei contradictorischen Gegentheilen nur das eine wahr sein […] könne'). Prantl translates the Stoic *antikeimenon* also as 'sprachlicher Widerspruch' (Prantl 460)—again, he ignores the fact that Stoic assertibles are not linguistic entities.

Like the Stoics, FREGE maintains that the contradictory of a thought is composed of that thought and the negation [function] (N155, CP 386), and holds that of contradictories one is true, the other false: 'For every thought there is a contradictory thought so that a thought is declared false by the acknowledgement of its contrary as true' (N154, CP 385). 'The sentence that expresses the contradictory thought is formed by means of a negation word from the expression of the original thought' (N67).[77]

At first blush, the Stoic formulation appears logically neater, in that it captures the symmetry of contradictoriness. It has, however, the less neat consequence that every negation

[77] 'Der den widersprechenden Gedanken ausdrückende Satz wird mittels eines Verneinungswortes aus dem Ausdrucke des ursprünglichen Gedankens gebildet' (N154).

has two contradictories (see below).[78] Once again, PRANTL'S—*inaccurate—representation of the Stoics* parallels Frege. Overinterpreting a passage in Simplicius (Simpl. *in Cat.* 403.32–33; Prantl 449, n. 134), he claims that '[…] the Stoics teach expressly that exclusively only the affirmative and the negative [assertible] stand mutually in the relation of contradictoriness' (Prantl 449).[79] Frege expresses the symmetry more explicitly:

> [T]he only difference is that we have the opposite thought. So to each thought there corresponds an opposite. Here we have a symmetrical relation: If the first thought is the opposite of the second, then the second is the opposite of the first.[80] (*Logic* PW 149)

The Stoics would agree.

CONTRADICTORIES	
STOIC contradictories (*antikeimena*, D.L. 7.73; S.E. *M.* 8.89; Prantl 449 'das Contradictorische'; Prantl 460 'sprachlicher Widerspruch').	FREGEAN contradictories ('widersprüchliche Gedanken') ('widersprechende Gedanken', N154).
[The Stoics] say that contradictory [assertibles] are those in which the one surpasses the other by a negation/negator (S.E. *M.* 8.89; Prantl 449, n. 133).	The thought that is contradictory to another thought seems composed of that [thought] and the negation [function] (N155, CP 386).
The relation between contradictories is 'umkehrbar'/symmetric—follows from definition.	The relation between contradictories is 'umkehrbar'/symmetric (*Logik NS* 161 = PWLB, PW 149).
Stoics: For every affirmation there is precisely one contradictory. Prantl, reporting the Stoics incorrectly: For every *axiōma* there is precisely one contradictory (Prantl 449).	For every thought there is a contradictory (*Logik* NS 161 = PWLB, PW 149).

[78] Not-p had both p and not-not-p as contradictories. See below on double negation.

[79] '[…] die Stoiker ausdrücklich lehren, dass ausschliesslich nur das bejahende und das verneinende Urtheil gegenseitig in diesem [*i.e.* contradictorischen] Verhältnisse stehen' (Prantl 449). Simplicius says that 'the Stoics believe that only the negations are contradictories to the affirmations', which from an Aristotelian perspective suggests—to Simplicius and Prantl—that *e.g.* 'Mieze is a non-horse' is not the contradictory to 'Mieze is a horse'. Alex. *An. Pr.* 402 and Apul. *Herm.* 191.6–11 suggest that the Stoic distinction was between a negation, which does not presuppose the existence of the referent of the subject expression, and an affirmation of the form 'a is *not* F', which presupposes the existence of the referent. Nothing follows about the question whether negations can have negations as contradictories.

[80] 'nur der Gedanke ist der entgegengesetzte. So gibt es zu jedem Gedanken einen entgegengesetzten. Wir haben hier eine umkehrbare Beziehung: Wenn der erste Gedanke dem zweiten entgegengesetzt ist, so ist auch der zweite dem ersten entgegengesetzt' (*Logik* NS 161).

Two assertibles are contradictories with respect to truth and falsehood when one is the negation (*apophatikon*) of the other (D.L. 7.73; Prantl 449, n. 133).	For every thought there is a contradictory thought so that a thought is declared false by the acknowledgement of its contrary as true (N154, CP 385).

III.1.3.1.3. Double negation

Double negation was anything but intuitive in antiquity, as well as at Prantl's and Frege's time, as is exemplified by both Lucian's and Prantl's ridiculing of Stoic double negation (Lucian, *Gallus* 11; FDS 930; Prantl 444, 'a truly insurmountable nonsense') and in Frege's long paragraph in which he desperately searches for metaphors (N157, CP 388–89).

Both the Stoic and the Fregean definition of negation allow the formation of a negation by 'prefixing' a negator (negation part, negation functor) to a negation. Both spell this option out explicitly.

THE STOICS have a special term for the negation of a negation, the übernegation (*huperapophatikon*) (D.L. 7.69; Prantl 444, n. 120). They define it as 'the negation of a negation' (*ibid*.) and say that it is a kind of negation. The übernegation is thus itself a negation, and by implication an assertible. Semantically, we learn by example that the assertible '"not: not: it is day" posits [the assertible] "it is day"' (D.L. 7.69, following emendation; Prantl 444, n. 120; Lucian, *Gallus*).[81] Prantl writes: 'in which two negations cancel each other out and bring about an affirmation' ('in welchem zwei Negationen sich aufheben und eine Affirmation bewirken', Prantl 444). Thus, by the Stoic definition of contradictories as well as by the Stoic truth conditions for negation (above), if an übernegation is true, the negation from which it was formed is false and vice versa.

FREGE introduces the double negation ('doppelte Verneinung') in his *Negation*, describing it as the negation of the negation (N156–57, CP 388–89). In CT he writes: 'But since 'not (not B)' has the same sense as 'B' […] ' ('Da aber 'nicht (nicht B)' denselben Sinn hat wie 'B' […]', CT44, CP 399). He states that of a negation and the negation of that negation, one and only one is true (N157, CP 389). Already in BS156 §18 we read, '*Duplex negatio affirmat*. The denial of denial is affirmation' (emphasis omitted).[82] '*Duplex negatio affirmat*' can be found in logic texts of Frege's era, and thus does not imply a Stoic impact. Rather, the relevant points here are Frege's specific view and wording. Shortly after

[81] Confirmed by Lucian, *Gallus* 11: αἱ δύο ἀπόφασεις μίαν καταφάσιν ἀποτελοῦσιν.

[82] Begr44 §18: '*duplex negatio affirmat*. Die Verneinung der Verneinung ist Bejahung' (emphasis omitted).

introducing the symmetry of contradictories, Frege writes 'We could declare it false by inserting a second "not" […] And from this it follows *that two negatives cancel one another out*' (*Logic* PW 149, emphasis mine).[83]

The combination of incorporeal assertoric content and their definitions of negation and contradictories leave both Frege and the Stoics facing the following awkwardness. On the one hand, it is a property of an assertoric content that it is a negation or that it is an affirmation, and a double negation is a negation of a negation, and hence itself a negation. If one adds to this the syntactic definitions of contradictories (above), including the symmetry relation of contradictories, it oddly results that affirmations have one contradictory, and negations have two. On the other hand, a double negation ¬¬A and the corresponding affirmation A seem to be considered logically equivalent. So if the relation of contradictoriness is considered to hold between the equivalence classes that result from the *duplex negatio affirmat*, then each such class has one contradictory. But neither Frege nor the Stoics say this. (Frege's treatment of double negation is also in tension with his claim that thoughts have 'building blocks' which somehow mirror the words that compose the sentences which express them.)

DOUBLE NEGATION	
STOIC übernegation (*huperapophantikon*).	FREGEAN double negation ('doppelte Verneinung').
An übernegation is the negation of a negation (D.L. 7.69; Prantl 444). It is a negation (*ibid*. entailed by context, *eidos de toutou*). Entailed by its definition and by context: an übernegation is an assertible (*ibid*.).	One can call the negation of the negation of … double negation (N157, CP 388–89). *The negation of a thought* … can serve as supplement of the negation [function] … . So I obtain the negation of the negation [of the thought] … which again, *is a thought* (N156, CP 387).
[The assertible] 'not: not: it is day' posits 'it is day' (D.L. 7.69; Prantl 444, n.120). '[I]n welchem zwei Negationen sich aufheben und eine Affirmation bewirken' (Prantl 444).	But since 'not (not B)' has the same sense as 'B', […] (CT44, CP 399). The denial of denial is affirmation (Begr 44 §18).

[83] 'Man könnte das Fürfalscherklären mit einem zweiten eingeschobenen "nicht" bewirken […] Und es ergibt sich so, *dass die doppelte Verneinung sich aufhebt*' (*Logik* NS 161, emphasis mine).

(Entailed by the truth-conditions of negation and by the semantic account of contradictories: either the negation or the negation of the negation is true.)	Always one and only one is true … of the negation and the negation of the negation. (N157, CP 389). The double negation that dresses a thought does not change the truth-value of the thought (N157, CP 389).
It appears that negations have two contradictories.	It appears that negations have two contradictories.

III.1.3.1.4. Other negative assertoric contents

Beyond negations (*apophatikon*), THE STOICS and Prantl each mention, one after the other, in the same sentence, two other kinds of negative assertibles. One is the *eliminating assertible* (*arnētikon* [*axiōma*]),[84] defined as constituted from an eliminating part (*morion*) and a predicate, with 'nobody walks' as illustration (D.L. 7.70; Prantl 444). The other is a *privative assertible* (*sterētikon*), defined as constituted from a privative part (*morion*) and what is potentially an assertible, with 'Unkind is this one' as illustration. Prantl writes 'that which negates seems solely to be classified according to the respective negative linguistic expression, […] or a universally negating word, *e.g.* "nobody" (*arnētikon*), or a word that is composed with the *α privativum* (*sterētikon*) […]' (Prantl 444).[85]

In his PWLB, FREGE mentions analogues to precisely these two kinds of negative content, and like the Stoics and Prantl, in the same sentence and in the same order: 'We have other signs for negation like "no", and we often use the prefix "un" as, for example, in "unsatisfactory"' (PWLP, PW 150).[86] As the Stoics think that privative sentences express assertibles but do not think they express negations (entailed by the definitions D.L. 7.69–70), so Frege does not think that the sense ('Sinn') that is expressed by sentences with privatives, like 'This man is *un*happy' are negations:[87] we do have a thought, but it is not a negation.

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[84] A common translation is 'denying', but this has the wrong connotations, since it can be seen as a force indicator, which it is not. LSJ has ἀρνητικός, ή, όν, denying, negative, μόριον ἀξιώματος Chrysipp. *Stoic*. 2.66, *cf.* Alex. Aphr. *in Metaph.* 333.26; φαντασίαι Numen. *ap. Eus*. PE 14.8; ἐπίρρημα Eust. 211.37. *Adv*. ἀρνητικῶς Porph. *in Cat.*136.27, Simp. *Phys.* 812.17, *Sch. Ar. Ra.* 1455. None of those passages forces a translation of the word family of denial rather than negation.

[85] '[…] erscheint das verneinende bloss nach den jeweiligen negativen Sprach-Ausdrücken eingetheilt, […] oder ein allgemein verneinendes Wort, z.B. "Niemand" (*arnētikon*), oder ein mit dem α privativum zusammengesetztes Wort (*sterētikon*) […]'.

[86] 'Wir haben für die Verneinung auch andere Zeichen wie "kein" und die Vorsatzsilbe "un" in manchen Fällen, wie z.B. in "ungenügend"' (*Logik* NS 162).

[87] 'For this reason the sentences "This man is not unhappy" and "This man is happy" do not have the same *sense*' (*Logic* PW 150, emphasis mine, 'Daher haben denn auch die Sätze "Dieses Haus ist nicht

OTHER NEGATIVE ASSERTORIC CONTENTS	
THE STOICS have two further negative assertoric contents, mentioned (in the *Summary* D.L. 7.70 and in Prantl 444) in one sentence: first *eliminating* (*i.e.* universal negative) contents, second *privative* contents.	FREGE mentions two further negative assertoric contents in one sentence, first (universal negative) contents, second privative contents.
These are complete contents and assertibles (implied by context in D.L. and Prantl).	These are thoughts (*Logik* NS 162, PW 150).
Stoic eliminating assertibles (*arnētikon* [*axiōma*]) are defined as constituted from an eliminating part and a predicate (D.L. 7.70). The eliminating part is *oudeis* (nobody, no-), *i.e.* universal. Prantl 444: 'allgemein verneinendes Wort'.	Frege: 'We have other signs for negation like "no" […]' (PWLP, PW 150).
Privative assertibles (*sterētikon*) are defined as constituted from a privative part and what is potentially an assertible. As an example we get 'Unkind is this one' (D.L. 7.70). Context implies that these are not negations.	'[…] the prefix "un" as, for example, in "unsatisfactory"' (PWLP, PW 150). Implied: 'This man is *un*happy' is not a negation (PWLP, PW 150, emphasis mine).

III.1.3.2. Assertoric contents with binary connectives (compound propositions)

Partly for its entertainment value, partly because it shows what Aristotelian prejudice Frege would have encountered as the norm, here is a quote from Prantl ranting about the semantics of the Stoic non-simple assertibles:

> This unscientific and inane treatment emerges even more clearly where for the 'non-simple' judgments, too, principles are established for what is true and what is false; and there could hardly be anything that has come about in the field of humanities or human thought in general, that could even approximate in worthlessness and arrogant

unschön", "Dieses Haus ist schön" nicht denselben *Sinn*', *Logik* NS 162, emphasis mine; the translator changed the example since 'unbeautiful' is rare in English.). By contrast, for Frege, 'This man is not not happy' and 'This man is happy' would have the same sense.

> twaddle this Stoic drivel about the conditional, disjunctive, causal and similar judgements.[88] (Prantl 453)

Now to the comparison with Frege: I start with some similarity factors that cut across different kinds of molecular propositions.

(i) THE DEFINITIONS:

STOIC non-simple assertibles are assertibles which consist of two or more assertibles or of the same assertible taken twice (or more often) (D.L. 7.68; Prantl 445). Stoic examples of non-simple assertibles of the latter kind are 'if it is day, it is day' (S.E. *M.* 8.95; Prantl 445) and 'if A then A' ('wenn A, dann A') in Prantl (Prantl 456). FREGE, in his 'Compound thoughts' ('Gedankengefüge', CT), systematically introduces different kinds of *compositions-of-thoughts (thought compounds)*, or as the title is usually translated, of *compound thoughts*. He makes the same sort of distinction as the Stoics. Compound thoughts are thoughts in which two or more thoughts are composed into one—new—thought, or in which one thought is compounded with itself (CT37, CP 390: two; CT50–51, CP 406: more than two; CT 49–50, CP 404–05: one with itself). In EidL PW 188 (NS 204) Frege chooses a careful formulation that allows for the thoughts in a compound thought to be identical: 'each of which expresses a thought' ('von denen jeder einen Gedanken ausdrückt'). Frege's examples for the latter include 'A or A', [(not A) and A]. (Frege CT37, CP 391: any compound thought is itself a thought. CT49–50, CP 404–05: 'cases where a thought is compounded with itself rather than with some different thought […] 'A or A' […] '(not A) and A''.[89] (He also has if A, A, see below.) This kind of connection of a proposition with itself is not common in the Aristotelian traditions in which the Stoics and Frege find themselves: Alexander of Aphrodisias ridicules it; Frege feels the need to justify it (CT50, CP 405).)

(ii) THE DEFINITIONS ARE ITERATIVE:

[88] 'Noch stärker nun tritt diese unwissenschaftliche und verstandlose Behandlungsweise da hervor, wo auch für die "nicht einfachen" Urtheile Grundsätze aufgestellt werden, was wahr und was falsch sei; und es dürfte wohl kaum je irgend im Gebiete der Litteratur oder der menschlichen Geistesthätigkeit überhaupt Etwas aufgetreten sein, was an Nichtswürdigkeit und arrogantem Blödsinne diesem stoischen Geschwätze über die hypothetischen, disjunctiven, causalen und dergleichen Urtheile auch nur gleichkäme'.

[89] 'Fälle […] in denen nicht verschiedene Gedanken, sondern ein Gedanke mit sich selbst gefügt ist […] "A oder A" […] "[(nicht A) und A]"' (CT49–50, LU 88).

All STOIC non-simple definitions are defined in such a way that their components can themselves be simple or non-simple or mixed (*i.e.* one is simple, the other non-simple) (see definitions below). This fact is stated explicitly (S.E. *M.* 8.124). Prantl's example is 'When the first is and also the second, the third is' ('Wenn das Erste und zugleich das Zweite ist, ist das Dritte', Prantl 480, with n. 190, S.E. *M.* 8.234–36). FREGE also provides an explicit informal iterative account: every composite thought (thought compound) is a thought *and can be used in further composite thoughts* (thought compounds), *e.g.* '(A and B) and C' (CT50, CP 405–06), not [not A and [B and C]] (CT51, CP 406). So, BOTH the Stoics and Frege choose accounts that permit molecular propositions as elements of molecular propositions, and put no limit on the complexity and length of propositions (*e.g.* S.E. *M.* 8.124), and both provide examples. Frege (in CT51, CP 406) uses the same form of example that we find for the Stoics in Prantl (Prantl 480): 'if [A and B] then C'.

(iii) KINDS OF NON-SIMPLE PROPOSITIONS:

The exact number of *kinds of non-simple assertibles* likely varied among the Stoics. Among a few others, they discuss conjunction, exclusive and inclusive disjunction, the material conditional and a stronger conditional, and a causal proposition, but no biconditional (D.L. 7.71–74 Prantl 447–48, ns 125–128, 461, n. 160). Prantl lists conditional ('hypothetisch'), copulative, disjunctive, causal, and comparing judgements (Prantl 462). Frege, too, discusses or mentions conjunction, exclusive and inclusive disjunction, the material conditional, and a causal proposition, but no biconditional.[90]

(iv) CONNECTIVES AT THE LEVEL OF CONTENT

As in the case of the negator, in both Stoic logic and that of Frege there is a *tension with regard to the connectives*. Are logical binary connectives linguistic items or are they something at the level of thought that is expressed by corresponding linguistic items? For both Frege and the Stoics, there is evidence for the second option. FREGE says that there is something in the realm of sense that corresponds to the (linguistic) 'and' and which is doubly unsaturated ('und was dem "und" im Gebiete des Sinnes entspricht, muss zwiefach ungesättigt sein' CT39, CP 393). The term he uses is 'the compounding' ('das Fügende'), CT40, 41, 42, 43 (as in 'compound thought', 'Gedankengefüge'). The Stoics say throughout

[90] Russell, for instance has a biconditional (Russell 1908: 245; *cf.* Russell and Whitehead 1910: 120, definition *4.01).

that the connectives (*sundesmoi*, Prantl 445: 'Conjunctionen') connect *axiōmata* (D.L. 7.71–74; S.E. *M.* 8.95; Prantl 445, n. 122). Where the *Summary* lists parts of speech (*merē logou*), *sundesmos* is also defined as a part of *speech* that connects parts of speech (D.L. 7.58); this is attributed to Diogenes of Babylon and probably comes from his work on (spoken) language that had been mentioned just before.

(v) TERMINOLOGY

For the component assertibles of a non-simple assertible, the Stoics use 'the first', 'the second', 'the third' (*etc.*) or 'A'', 'B'', 'Γ'' (*etc.*), which are the Greek ordinal numerals. In CT *passim* (and in EidL, NS 202 = PW 186), Frege uses 'the first' (or 'the first thought'), 'the second', 'the third' (*etc.*), and schematically 'A', 'B', 'C' (*etc.*). The use of 'the first', 'the second' seems to be absent in his earlier works.

COMPOSITE ASSERTORIC CONTENT (COMPOSITE PROPOSITIONS)	
STOIC non-simple assertibles (*ouch hapla axiōmata*).	FREGEAN compound thoughts (literally thought compound, *Gedankengefüge*).
Non-simple assertibles are those which consist of two (or more) assertibles or of the same assertible taken twice (or more times) (D.L. 7.68).	Compound thoughts are those where two (CT37, CP 390) [or more (CT51, CP 406)] thoughts [or the same thought taken twice (CT50, CP 404–05)] are composed into one [new] thought.
Can be combined from simple, non-simple, or mixed (*i.e.* one simple one non-simple) assertible (S.E. *M.* 8.124; *cf.* Prantl 480, n. 190).	'In this way compound thoughts containing three thoughts can originate. […] So too it will be possible to find examples of compound thoughts containing four, five, or more thoughts' (CT51, CP 406).[91]
Non-simple assertibles include conjunction, inclusive and exclusive disjunction, conditional (D.L. 7.71–74).	Compound thoughts include conjunction, inclusive and exclusive disjunction, conditional (CT *passim*).
Also discussed: causal conditionals (D.L. 7.71, 74; Prantl 447, 457).	Also discussed: causal conditional (SB48, CP 175).
As schematic letters for component assertibles, the Stoics and Prantl use 'the first', 'the second', 'the third', or 'A', 'B', 'C' (D.L. 7.80–81; Prantl 471-74 with notes).	Frege in CT uses 'the first [thought]', 'the second [thought]' and 'A', 'B', 'C' for component thoughts (CT *passim*).

[91] 'So können Gedankengefüge entstehen, die drei Gedanken enthalten […] So wird man auch Beispiele von Gedankengefügen finden können, die vier, fünf oder mehr Gedanken enthalten' (CT51, LU 90).

The same assertible can be taken twice. (D.L. 7.68) Example: if A, then A ('wenn A, dann A', Prantl 456).	The same thought can be taken twice (CT50, CP 404–05). Example, among others: if A, then A ('wenn A, so A', CT50, CP 404–05).
Prantl's example from Sextus of a combination of a simple and a non-simple assertible: If the first and the second, then also the third (Prantl 480, S.E. *M.* 8.23–25).	Frege's examples of a combination of simple and compound thoughts: if [B and C] then A, not [not A and [B and C]] (CT51, CP 406).
Connectives are variably said to connect propositions (D.L. 7.71–72; S.E. *M.* 8.95) and (in a work on language by Diogenes of Babylon) to connect parts of speech, D.L. 7.58). The Stoic term is 'those which connect' (*sundesmoi*).	Connective expressions have analogues at the level of thought, which, like predicates, are unsaturated, but doubly so (CT37, 39, CP 391, 393). Frege's term is 'the compounding', '*das Fügende*', CT40, 41, 42, 43 (as in '*Gedankengefüge*').

III.1.3.2.1. Conjunction

STOIC conjunctive assertibles (*sumpeplegmena*) are syntactically defined as non-simple assertibles that are constructed by conjunctive connectives. An example is 'both it is day and it is light' (D.L. 7.72).[92] From the definition of non-simple assertibles we know that the connectives conjoin two *axiōmata*. (Prantl 447 writes 'The copulative judgment, *to sumpeplegmenon*, is the one brought about by 'and' or 'both—and' (*kai*, *kai—kai*)'.)[93] The thus connected assertibles are called 'those [*axiōmata*] in the conjunction' (*e.g.* S.E. *PH* 2.58, Galen, *Inst. Log*. 6.6). The truth-conditions are truth-functional. A conjunction is true when all its conjuncts are true, and otherwise false (S.E. *M.* 8.125; Prantl 459, n. 155). (Prantl 459, 'In fact, as regards the copulative judgment, *the sumpeplegmenon*, it has come down to us that it was true (*hugies*) if all the components connected by 'both' or 'both—and' correspond to the truth, but false if only one among them is false'.)[94] Some sources have been interpreted as suggesting that conjunctions could have more than two conjuncts but all reliable Stoic

[92] συμπεπλεγμένον δέ ἐστιν ἀξίωμα ὃ ὑπό τινων συμπλεκτικῶν συνδέσμων συμπέπλεκται, οἷον 'καὶ ἡμέρα ἐστὶ καὶ φῶς ἐστι' (D.L. 7.72).

[93] 'Das copulative Urtheil, to sumpeplegmenon, ist das durch "Und" oder "Sowohl—als auch" (*kai, kai—kai*) bewirkte'.

[94] 'Und zwar ist uns in Betreff des copulativen Urtheiles, des sumpeplegmenon, überliefert, dass dasselbe als richtig (hugies) galt, wenn sämmtliche durch "sowohl-als auch" verbundenen Glieder desselben der Wahrheit entsprechen, als falsch aber, wenn auch nur Eines unter jenen falsch ist'.

sources can be read as suggesting that Stoic conjunctions had precisely two conjuncts (so also Prantl 447).[95] Of these, one, of course, could be non-simple (S.E. *M.* 8.124) and so itself a conjunction. FREGE, in his *Einleitung in die Logik*, writes this about his conjunction ('Kondukt, Verein'):

> If a whole is composed of two sentences connected by 'and', each of which expresses a thought, then the sense of the whole is also to be construed as a thought, for this sense is either true or false; it is true if each component thought is true, and false in every other case—hence when at least one of the two component thoughts is false. [96] If we call the thought of the whole the conjunction of the two component thoughts, […][97] (EidL, PW 188)

A conjunction thus contains several thoughts (itself and component conjunct(s)) and is truth-functional. Elsewhere Frege emphasizes that the conjoining word ('Bindewort')[98] 'and' combines whole sentences that express thoughts (CT74, CP 392) and that what corresponds to the word 'and' in the realm of sense is doubly unsaturated.[99]

Both in ancient Greek and in German (as in English) the conjunctive connective can combine what is expressed by noun phrases or by predicate phrases, and in Aristotle and Peripatetic texts we find it used in that way in the context of logic. Both the Stoics and Frege choose the use that combines complete contents.

The early Stoics appear to have regimented the use of the conjunctive connective so that either sentence that expresses a component assertible has 'and' prefixed to it (*kai* […] *kai* ---). This, together with similar choices for the other non-simple assertibles, gave the Stoics

[95] That the early Stoics had only conjunctions with two conjuncts is confirmed also by S.E. *M.* 8.124, and by the fact that their syllogistic did not require conjunctions with more than two conjuncts, and that Plutarch's riddle can be solved more easily on this assumption (Bobzien 2019; Bobzien 2011).

[96] This sentence is very close to Prantl 459.

[97] 'Wenn ein Ganzes aus zwei durch 'und' verbundenen Sätzen besteht, von denen jeder einen Gedanken ausdrückt, so ist auch der Sinn des Ganzen als ein Gedanke aufzufassen, denn dieser Sinn ist entweder wahr oder falsch; wahr nämlich, wenn jeder der beiden Teil-Gedanken wahr ist, falsch in jedem anderen Falle -, also wenn mindestens einer der beiden Teilgedanken falsch ist. Nennen wir diesen Gedanken des Ganzen das Kondukt von den beiden Teilgedanken […]' (EidL NS 204–05).

[98] 'Bindewort' in Borheck 1807: 304–05, translating *sundesmos* for all Stoic non-simple assertibles in the *Summary*. Prantl has 'Conjunction'. Both are standard grammatical terms.

[99] '[…] "und". Dieses Wort wird hier in besonderer Weise gebraucht. Es kommt hier nur in Betracht als Bindewort zwischen *eigentlichen Sätzen*. Eigentlich nenne ich einen Satz welcher einen Gedanken ausdrückt' (CT74, CP 392); 'und was dem "und" im Gebiete des Sinnes entspricht, muss zwiefach ungesättigt sein' (CT75, CP 393).

the means for a simple bracketing system similar to Polish notation. It appears that Prantl did not pick up on this point. Since this use is grammatical but not that frequent in ancient Greek, we can assume that the first 'and' (*kai*) was dropped by those not aware of the *logical* function of the first 'and'. Frege does suggest such a use of 'und' (or 'sowohl, als auch'), but uses brackets in CT *passim*.

CONJUNCTION	
STOIC conjunction or conjunctive assertible (*to sumpeplegmenon*).	FREGEAN conjunction (*Kondukt*, EidL, NS 204 = PW 188.)
A conjunction connects assertibles, which are called 'those in a conjunction'. (There is no term for conjuncts.)	In a *Kondukt*, thoughts, called part-thoughts, are connected. (EidL, NS 204 = PW 188. There is no term for conjuncts.)
The conjuncts are conjoined by conjunctive connectives (*sumpeplegmena sundesma*) 'and' or 'both—and' (*kai*, *kai—kai*) (Prantl 447, S.E. *M.* 8.124, D.L. 7.72).	The part-thoughts are connected with that which corresponds to the word 'and' (EidL, NS 204–05 = PW 188, CT74–75, CP 392–93).
The connective 'and' (*kai*) connects assertibles, and that is, connects what is expressed by whole assertoric sentences.	The connective 'and' ('und') combines whole sentences that express thoughts (CT74, CP 392).
Truth-conditions: a conjunction is true when all its conjuncts are true, otherwise false, *i.e.* when at least one of those [conjuncts] is false (S.E. *M.* 8.125; Prantl 459).	Truth-conditions: a 'Kondukt' is true when both its part-thoughts are true, in any other case false, *i.e.* when at least one of the two part-thoughts is false (EidL, NS 204, PW 188).
Some formulations imply that the connectives (*sundesma*) are not linguistic items, but are at the level of assertibles.	There exists something in the realm of sense that corresponds to the word 'and'. It is doubly unsaturated (*e.g.* CT75, CP 393).
The early Stoic canonical view appears to restrict the conjuncts in a conjunction to two (D.L. 7.72; Prantl 447, n. 127).	Conjunctions have two conjuncts (CT *passim*).

III.1.3.2.2. Disjunction

Both the Stoics and Frege distinguish between inclusive and exclusive disjunction. The STOIC primary disjunction, the *diezeugmenon*, is exclusive (D.L. 7.72; Galen, *Inst. Log.* 3.3; Prantl 447–48, 460). It is formed with the disjunctive connectives (D.L. 7.72) 'either […] or […]'. These connectives indicate that one of the component assertibles is false (S.E. *PH* 2.191,

Prantl 460).[100] Prantl writes: '[…] is effected by the conjunction 'or' or 'either—or', and the inner sense of this composition is the relation of *a mutual exclusion*' (447–48, emphasis mine) and 'a disjunctive judgment is true, if between the two disjunctive constituents a complete opposite obtains that effects mutual elimination' (Prantl 460 with Prantl's reference to Prantl page 604, in particular to what Prantl says about Galen *Inst. Log.* on that page, i.e. on page 604; cf. also Gellius 5.11.8).[101] Stoic exclusive disjunction is non-truth-functional. However, the way in which Prantl presents it, this can easily escape notice. The inclusive disjunction (*paradiezeugmenon*) plays no role in early Stoic syllogistic. Its truth conditions are truth-functional. They require only that not all disjuncts are true (Prantl 521–22, 604; Gellius 16.8.14). There is not enough evidence to determine what the—regimented—syntax of Stoic inclusive disjunction was.

FREGE explains the truth-functional inclusive and exclusive 'or' in his *Begriffsschrift*. He writes,

> Now the words 'or' and 'either—or' are used in two ways […] Of the two uses for the expression 'A or B', the first, in which the coexistence of A and B is not excluded, is the more important; and we shall use the word 'or' with this meaning. Perhaps it is appropriate to make this distinction between 'or' and 'either—or' that only the latter shall have the secondary meaning *of mutual exclusion*.[102]
> (Begr 11§7 = BS 121–22, emphasis mine)

Frege's use of 'the non-exclusive "oder"' ('das nicht ausschliessende "oder"', CT42, CP 396) suggests that the distinction was still relevant in the 1920s and the exclusive 'or' still the norm. The suggestion of using a two-part connective for the exclusive disjunction matches the Stoics.

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[100] The word rendered 'indicate', literally 'announces' (ἐπαγγέλλεται), is commonly used by the Stoics to express the semantics of an expression.

[101] '[…] ist durch die Conjunction 'oder' oder 'entweder-oder' […] bedingt, und der innere Sinn dieser Zusammensetzung ist das Verhältnis *einer wechselseitigen Ausschliessung*' (447–48); 'ein disjunctives Urtheil sei wahr, wenn zwischen den in ihm disjungirten Gliedern ein vollständiger, gegenseitige Vernichtung bewirkender Gegensatz […] bestehe' (460, emphasis mine).

[102] 'Die Wörter "oder" und "entweder—oder" werden nun in zweifacher Weise gebraucht […] Von den beiden Gebrauchsweisen des Ausdruckes "A oder B" ist die erstere, bei der das Zusammenbestehen von A und B nicht ausgeschlossen ist, die wichtigere, und wir werden das Wort "oder" in dieser Bedeutung gebrauchen. Vielleicht ist es angemessen zwischen "oder" und "entweder—oder" den Unterschied zu machen, dass nur das Letztere die Nebenbedeutung *des sich gegenseitig Ausschliessens* hat' (BS 11 §7, emphasis mine).

DISJUNCTION	
STOICS distinguish exclusive and inclusive disjunction (*to diezeugmenon, to paradiezeugmenon*, D.L. 7.72; Gellius 16.8.11–14; Prantl 447–48 and 521, n. 21).	FREGE distinguishes exclusive and inclusive disjunction (Begr 11 §7 = BS 121–22).
The disjunctions are formed with the disjunctive connective 'either […] or' or 'or' (D.L. 7.72, Gellius 5.11.8, Gal. *Inst. Log*. 3.3, Prantl 448).	The 'or' and 'either […] or' are used for inclusive and exclusive disjunction. Frege recommends language regimentation (Begr 11 = BS 121–22).
The inclusive 'or' is truth-functional.	The inclusive 'or' is truth-functional (also CT42, CP 396).
The exclusive disjunction requires mutual exclusion (*mache*) of the disjuncts, and one disjunct being false (S.E. *PH* 2.191, Prantl 460). Prantl's presentation: 'mutual exclusion' ('Wechselseitige Ausschliessung', Prantl 447–48) compatible with truth-functional reading.	Exclusive disjunction is regarded as truth-functional. Mutual exclusion ('[Das] sich gegenseitige Ausschliessen', Begr 11).
Exclusive disjunction is primary and more important.	Inclusive disjunction is primary and more important.

Here we have another case in which Prantl's presentation of Stoic logic is closer to Frege than to Stoic logic itself. However, Frege's disjunction need not have been impacted by the Stoic one via Prantl. The distinction between inclusive and exclusive disjunctions was a commonplace at Frege's time. However, again, we often find the Peripatetic syntax that (does not have the 'either' and) combines the predicates rather than entire sentences (Mill, Sigwart, Ulrici, *etc.*). So disjunction is added here only because we cannot rule out the possibility that Frege was inspired by the Stoics here as well.

III.1.3.2.3. Conditional

It is generally known that Fregeo-Russellian classical logic includes virtually the same truth-functional definition for the material conditional as that which Philo and some Stoics adopted for their assertibles, and the Stoic discussion of conditionals and their truth-conditions has been connected with Frege's logic in the past.[103] My focus will be on parallels between Frege

[103] Łukasiewicz 1935: 125, Mates 1962: 46–47, Kneale 1962: 531.

and the Stoic view as presented in Prantl and the *Summary*. I juxtapose the most substantial similarites case by case.

(i) THE DEFINITION AND SYNTAX OF CONDITIONALS:

THE STOICS define the conditional assertible (*to sunēmmenon*) syntactically as a non-simple *axiōma* in which two *axiōmata* are connected with the connective 'if' (*ei*). The component *axiōma* after the 'if' is called antecedent (*ēgoumenon*) or 'the first'; the other component *axiōma* is called the consequent (*lēgon*) or 'the second' (D.L. 7.71; S.E. *M.* 8.109ff; Prantl 446-47). For 'the first' and 'the second' we also find 'A', 'B' in some manuscripts.[104] So each conditional consists of at least two assertibles, and at least three if the component assertibles differ.[105] A conditional indicates a relation of consequence (*akolouthia*, D.L. 7.71, *akolouthein*; 'Verhältniss einer Folge', Prantl 447);[106] that is, it indicates that the second (assertible) follows from the first. Standard examples are 'if it is day, it is light' and 'if it is day, it is day' (D.L. 7.71; S.E. *M.* 8.109ff; Prantl n. 125).

In the posthumous *Einleitung in die Logik* (EidL), FREGE calls conditionals 'hypothetische Gedanken' (EidL, NS 205 = PW 188–89). In *Compound Thoughts* (CT45, CP 400) he calls them 'hypothetische Gedankengefüge', announces that he will use the linguistic form 'If B, then A' ('Wenn B, so A'), and says that its consequent ('Folge') is the sense (or thought content) of 'A', and its antecedent or condition ('Bedingung') the sense of 'B'.[107] In addition to the schematic 'A' and 'B', he also uses the metalinguistic 'the first [thought]' and 'the second [thought]' to refer to these (*ibid.*). A little later (CT47, CP 402), he writes that 'in a hypothetical compound thought we can distinguish three thoughts, namely the antecedent, the consequent, and the thought composed from the two'. In the *Einleitung in die Logik*, Frege distinguishes between hypothetical sentence, consequent sentence, and antecedent sentence, on the one hand, and hypothetical thought, consequent, and antecedent expressed

[104] 'First' and 'second' and 'A' and 'B' are used in inference schemata where reference to the (schematic) component assertibles has not yet been made with other letters. Some later texts have A′, B′, *etc.* for A, B, *etc.*

[105] Prantl 446–47: 'Das hypothetische Urtheil […] ist jenes nicht einfache Urtheil, in welchem die Verknüpfung durch "Wenn" (ei) bewerkstelligt ist, mag hiebei Ein [*sic*] einfaches Urtheil zweimal oder zwei verschiedene einfache Urtheile gesetzt sein; der grammatische Vordersatz heisst hgoumenon, der grammatische Nachsatz lhgon' (reference in n. 125 to D.L. 7.71, S.E. *M.* 8.109ff.).

[106] '[…] der innere Sinn dieser Verknüpfung ist das Verhältniss einer Folge, einer *akolouthia*'.

[107] 'Statt "Gedankengefüge sechster Art" sage ich auch "hypothetisches Gedankengefüge" und nenne den *ersten Gedanken* "Folge", den *zweiten* "Bedingung" im hypothetischen Gedankengefüge' (CT45, CP 400).

by these, on the other (EidL, NS 203, 205 = PW 187, 188–89).[108] His use of 'Bedingung' and 'Folge' corresponds to Prantl's 'Folgeverhältnis' and to the Stoic *akolouthia*. Both Frege (EidL, NS 201 = PW 185) and Prantl (453) use 'hypothetical judgement' ('hypothetisches Urtheil'). This was the common expression for conditionals at the time, and need not indicate any influence. Note also the *terminological similarities* for the component contents. In CT44–45, CP 398–400 Frege uses throughout the expressions 'the first thought' and 'the second (thought)' for the two atomic thoughts in the composite thought. This corresponds to the Stoic use of 'the first (*i.e. axiōma*)' and 'the second (*i.e. axiōma*)'. And where in Stoic sources we sometimes find 'the first' and sometimes alpha ('A') (where this is a way of saying the first), Frege says (CT43, CP 398) about the fifth kind of composite thought, 'Given that 'A' expresses the first thought and 'B' expresses the second' (CT43, LU 80, 'Wenn 'A' den ersten Gedanken, 'B' den zweiten Gedanken ausdrückt'. *Cf.* also EidL, NS 205 = PW 189, 'erster Gedanke, zweiter Gedanke, hypothetischer Gedanke').

(ii) THE TRUTH CONDITIONS:

The Stoic account of the conditional introduced by Philo, which Prantl discusses at length since he finds it most abhorrent, is analogous to the one that Frege considers to be correct but persistently misunderstood. Where PRANTL bemoans 'the merely formal relationship of the combination of the True and the False' (454, also 455, *i.e.* the truth-functionality), and 'that it is two judgements that are put side by side' (453), which leaves only the 'Debris of the hypothetical judgment' (*ibid.*),[109] FREGE (in EidL, NS 201–03 = PW 185–87) emphasizes that

[108] '[…] können wir den *hypothetischen Satz* nennen, dessen Folgesatz der Ausdruck des ersten Gedankens, und dessen Bedingungssatz der Ausdruck des zweiten Gedankens ist' (EidL NS 205, PW 188, emphasis mine); '[…] den *hypothetischen Gedanken* […], dessen Folge der erste Gedanke, und dessen Bedingung der zweite Gedanke ist' (EidL NS 205, PW 188, emphasis mine).

'[…] das Entgegengesetzte eines Kondukts von dem Entgegengesetzten eines ersten Gedankens und von einem zweiten Gedanken ist […] das was ich mit dem Bedingungsstriche ausdrücke. Der Satz des ersten Gedankens ist wieder der Folgesatz, der des zweiten der Bedingungssatz. *Den ganzen Satz aber, der ausdrückt das Entgegengesetzte eines Kondukts von dem Entgegengesetzten eines ersten Gedankens und von einem zweiten Gedanken, können wir den hypothetischen Satz nennen, dessen Folgesatz der Ausdruck des ersten Gedankens, und dessen Bedingungssatz der Ausdruck des zweiten Gedankens ist*' (EidL NS 205, PW 188, emphasis mine).

'Kondukt von dem Entgegengesetzten des ersten Gedankens und von dem zweiten Gedanken […] Entgegengesetztes des Konduktes von dem Entgegengesetzten des ersten Gedankens und von dem zweiten Gedanken: […] Dies ist der *hypothetische Gedanke,* dessen Folge der erste Gedanke und dessen Bedingung der zweite Gedanke ist' (EidL NS 205, PW 189, emphasis mine).

[109] 'das bloss formale Verhältniss der Combination von Wahr und Falsch' (Prantl 454); 'dass eben "*zwei*"_*Urtheile* es sind, welche nebeneinandergestellt werden' (Prantl 453); 'Trümmer des hypothetischen Urtheiles übrig' (*ibid.*).

two thoughts are connected in the hypothetical thought. Compare Frege's EidL, PW 186–87: 'People probably feel the lack of an inner connection between the thoughts: we find it hard to accept that it is only the truth or falsity of the thoughts that is to be taken into account, that their content doesn't really come into it all'.[110] Prantl's complaint that the Philonian-type conditional disregards the causal relation, genus and species relations, and the like (Prantl 455) exemplifies the kind of criticism to which Frege responds in CT when stating that his (Philonian or material) conditional is a useful tool for logic (CT 45, 46). Here are the passages about the truth-conditions from Prantl and Frege. They speak for themselves.

PRANTL writes:

> And thus first the merely formal relationship of the combination of the True and the False was explained, resulting in
>
> antecedent true, consequent true, *e.g.* 'When it is day, the sun shines'
>
> antecedent false, consequent false, *e.g.* 'When the earth flies, the earth has wings'
>
> antecedent false, consequent true, *e.g.* 'When the earth flies, the earth exists'
>
> antecedent true, consequent false, *e.g.* 'When the earth exists, the earth flies'
>
> or 'When it is day, it is night'
>
> of these four combinations, only the fourth was called incorrect (*mochtēron*), and it was in particular Philo (see above fn. 8) who maintained this view of the hypothetical judgment, and hence also defined the correct hypothetical judgment—*to hugies sunēmmenon*—as the one which does not transition from a true antecedent to a false consequent.[111]

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[110] 'Man vermisst wahrscheinlich eine innere Verbindung zwischen den Gedanken; es will nicht recht einleuchten, dass von dem Gedanken nur in Betracht kommen soll, ob er wahr oder falsch ist, gar nicht eigentlich der Gedankeninhalt selbst' (EidL NS 202–03).

[111] 'Und so wurde denn nun auch zunächst das bloss formale Verhältniss der Combination von Wahr und Falsch […] auseinandergesetzt, und es ergab sich:[141])

Vordersatz	wahr,	Nachsatz	wahr,	z.b.	"Wenn es Tag ist, scheint die Sonne"
″	falsch,	″	falsch,	″	"Wenn die Erde fliegt, hat die Erde Flügel"
″	falsch,	″	wahr,	″	"Wenn die Erde fliegt, existirt die Erde"
″	wahr,	″	falsch,	″	"Wenn die Erde existirt, fliegt die Erde"
				oder	"Wenn es Tag ist, ist es Nacht".[111])

Von diesen vier Combinationen nun wurde bloss die vierte als eine unrichtige (*mochthēron*) bezeichnet[142]), und zwar war es besonders Philo (s. oben Anm.8), welcher diese Auffassung des

(Prantl 454, *cf.* S.E. *PH* 2.105; D.L. 7.81)

This passage is essentially a free translation of S.E. *PH* 2.105, which is Prantl 454, n. 141. Prantl's n. 142 has the continuation of the text—'Of these only that with a true antecedent and a false consequent is false (incorrect), they say, but/and the others are true (sound)'—as well as the parallel in S.E. *M.* 8.449. N. 143 adds: 'Philo says that correct is the conditional which does not have a true antecedent and a false consequent' (S.E. *PH* 2.110).

FREGE, PW 186, writes:

> with two thoughts, only four cases are possible:
>
> 1. the first is true and likewise the second.
> 2. the first is true, the second false.
> 3. the first is false, the second is true.
> 4. both are false.
>
> Now, when the third of these cases does not obtain, then the connection which I have signified with the 'Bedingungsstrich' exists. The sentence that expresses the first thought is the consequent sentence; the sentence which expresses the second thought is the antecedent sentence.[112]
>
> (EidL, NS 202 = PW 186, my translation)

In Begr 5 §5 (BS 115) we have the almost identical:

> If A and B stand for assertible contents […], there are the following four possibilities: (1) A is affirmed and B is affirmed; (2) A is affirmed and B is denied […] [The

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hypothetischen Urtheiles vertrat und daher auch das richtige hypothetische Urtheil—to hygies sunēmmenon—als dasjenige definirte, welches nicht von einem wahren Vordersatze zu einem falschen Nachsatze übergehe[143])' (Prantl 454).

[112] Frege EidL NS 202: 'Wenn man zwei Gedanken hat, so sind nur vier Fälle möglich:

1. der erste ist wahr und desgleichen der zweite;
2. der erste ist wahr, der zweite falsch;
3. der erste ist falsch, der zweite ist wahr;
4. beide sind falsch.

Wenn nun der *dritte* dieser Fälle *nicht* stattfindet, so besteht die Beziehung, die ich durch den *Bedingungsstrich* bezeichnet habe. Der Satz, der den ersten Gedanken ausdrückt, ist der Folgesatz; der Satz, der den zweiten Gedanken ausdrückt, ist der Bedingungssatz'.

> 'Bedingungsstrich'] stands for the judgment that the third of these possibilities does not occur, but one of the other three does.[113] (emphasis omitted)

Note the following differences which are in line with Frege's later philosophy: in EidL (i) the antecedent gets first place, the consequent second place; (ii) Frege uses 'der erste', 'der zweite' for 'B', 'A', and 'der erste (Gedanke), der zweite (Gedanke)' throughout; [114] (iii) 'wahr' and 'falsch' for 'bejaht' and 'verneint'; and (iv) he adds the names for antecedent and consequent. All four changes have parallels in Prantl and the Stoics. In (CT, CP 399), finally, we find a sentence very similar to the one Prantl adds after the four truth-functional possibilities (Prantl 454), where Frege introduces the truth conditions for hypothetical composite thought: they are '[…] false if and only if the [consequent, 'Folge'] is false, but the [antecedent, 'Bedingung'] is true'.[115]

(iii) THE INTERDEFINABILITY OF THE CONDITIONAL IN TERMS OF NEGATION AND CONJUNCTION:

THE STOICS are aware of the interdefinability of the Philonian conditional in terms of negation and conjunction. Evidence shows that they rephrase the Philonian conditional as the negation of a conjunction with the antecedent as first conjunct and the negation of the consequent as second conjunct (Cic. *Fat.* 15–16). It is likely that they used this alternative formulation in order to retain both Chrysippus' and Philo's conditional in their logic. The Cicero passage suggests that if the Chrysippean conditional 'if A, ¬B' is true then ¬(A∧B) is true, but not vice versa (*ibid.*), and that in certain specific cases only the 'Philonian conditional' comes out true, and is hence appropriate to use.[116] Interestingly, the *Summary*

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[113] 'Wenn A und B beurtheilbare Inhalte bedeuten, so giebt es folgende vier Möglichkeiten: 1) A wird bejaht und B wird bejaht; 2) A wird bejaht und B wird verneint; […] [Der Bedingungsstrich] bedeutet nun das Urtheil, dass die dritte dieser Möglichkeiten nicht stattfinde, sondern eine der drei andern' (Begr 5 §5) .

[114] CT 44, CP 399: Frege's use of 'der erste Gedanke', 'der zweite Gedanke': 'Daraus folgt, dass ein Gefüge sechster Art eines *ersten Gedankens* mit *einem zweiten* dann und nur dann falsch ist, wenn der *erste Gedanke* falsch, der zweite aber wahr ist. Ein solches Gedankengefüge ist also wahr, wenn der *erste Gedanke* wahr ist, einerlei, ob der *zweite Gedanke* wahr oder falsch ist. Ein solches Gedankengefüge ist auch wahr, wenn *der zweite Gedanke* falsch ist, einerlei, ob *der erste Gedanke* wahr oder falsch ist. […]' (emphasis mine).

[115] '[…] dann und nur dann falsch ist, wenn der erste Gedanke falsch, der zweite aber wahr ist'. The text continues '[…] true when the consequent is true, and true when the antecedent is false, whether the consequent is true or false' (CT44, CP 399).

[116] *Cf. e.g.* Bobzien 1998: 156–67. The passage of Cicero's *De Fato* is not in Prantl (I believe), but several other passages of that work with substantial logical content are. So it would be evident for

sports a version of the Sorites paradox in the form A_1,¬(A_1∧¬A_2), ¬(A_2∧¬A_3), […] ¬(A_{n-1}∧¬A_n), A_n (D.L. 7.82, Prantl 54, n. 94, referred to in ns 210, 213, 216).

Similarly, FREGE regards a hypothetical thought as interdefinable with a negation of a conjunction of the antecedent with the negation of the consequent. He treats 'if B, then A' and 'not ((not A) and B)' as two ways of expressing the same hypothetical thought (CT45, CP 400), after having just on the previous page said that 'B and (not A)' expresses the same thought as '(not A) and B' (CT44, CP 399). He produces truth-conditions for the expression with negation and conjunction: a hypothetical thought-compound 'not ((not A) and B)' is false if and only if the first thought is false, but the second true; true when the first is true, whether the second is true or false; and true when the second thought is false, whether or not the first is true or false (CT44, CP 399).[117] And eerily close to the Stoic and Prantl's description of the Philonian conditional: the hypothetical connective of A and B is the contradictory of the conjunction of A and the contradictory of B (NS 216, *Kurze Übersicht* = PW 200).[118]

(iv) CONDITIONALS OF THE FORM 'IF A, A':

As an example of a non-simple assertible in which the same assertible is taken twice, THE STOICS offer 'if it is day, it is day' (*e.g.* D.L. 7.69; S.E. *PH* 2.112; Prantl 456, n. 148). Prantl implies that, for the Stoics, conditionals of the form 'Si A est, A est' are true (456).[119] Prantl provides further Greek Stoic examples, including one case that adds that the conditional is

someone reading Prantl that *De Fato* deals with various logical questions. Might Frege have read Cicero's text as a result of studying Prantl? Without independent evidence, we have no compelling reason to assume he did.

[117] (CT45, CP 400) 'Statt "Gedankengefüge sechster Art" sage ich auch "hypothetisches Gedankengefüge" und nenne den *ersten Gedanken* "Folge", den zweiten "Bedingung" im hypothetischen Gedankengefüge [nicht ((nicht A) und B)]. Demnach ist ein hypothetisches Gedankengefüge wahr, wenn die Folge wahr ist. Auch ist ein hypothetisches Gedankengefüge wahr, wenn die Bedingung falsch ist, einerlei, ob die Folge wahr oder falsch ist. -- Wir können dafür auch schreiben "Wenn B, so A"'.

[118] NS 216 (Kurze Uebersicht): 'Nun ist die hypothetische Verbindung von A und B das Entgegengesetzte des Vereins von A und vom Entgegengesetzten von B'. The terms are all Frege's own, rather than coming from the logical tradition. The only difference to the Stoics description is that, instead of 'negation of the conjunction of', Frege has 'the contradictory of the conjunction of'. (*Cf.* also 'Den ganzen Satz aber, der ausdrückt das Entgegengesetzte eines Kondukts von dem Entgegengesetzten eines ersten Gedankens und von einem zweiten Gedanken, können wir den hypothetischen Satz nennen, dessen Folgesatz der Ausdruck des ersten Gedankens, und dessen Bedingungssatz der Ausdruck des zweiten Gedankens ist' (EidL NS 205, PW 188).)

[119] Un-Stoically, perhaps inspired by the 'is' (*estin*) in 'day is' (*ēmera estin*), Prantl seems to use A as a term variable here, similar to above.

true (447, n. 125). Generally, the Stoics consider all conditionals of the form 'if A, A' true (S.E. *M*. 8.281, 8.466; Prantl 447, n. 125). This is also remarked on by Prantl (461). In particular, Prantl argues that all such assertibles (*diphoroumena*) are analytically true, because they satisfy the Stoic criterion that a conditional is true if the contradictory of the consequent is in opposition to the antecedent.[120]

FREGE states that a compound thought in which a thought A is compounded with itself and which is expressed by 'if A, then A' ('wenn A, so A') is true. As reason for the truth, he adduces that compound thoughts that are the contradictory/opposite/negation of these, *i.e.* those expressed by '[(not A) and A]', are false, since of two thoughts of which one is the negation of the other, one is always false, and, hence, so is the compound thought (CT50–51, CP 405). This reason is closely related to that of Prantl just mentioned.

CONDITIONAL ASSERTORIC CONTENTS	
STOICS: conditional assertible (*sunēmmenon axiōma*) (D.L. 7.71; S.E. *M*. 8.109; Prantl 447). PRANTL: hypothetical judgement (*hypothetisches Urtheil*) (447, 453).	FREGE: hypothetical thought (*hypothetischer Gedanke*) (EidL, NS 205 = PW 189), hypothetical thought compound (*hypothetisches Gedankengefüge*) (CT45, CP 400).
TERMINOLOGY: If …, --- (*ei*, *eiper*) (Prantl *wenn* and *wenn/dann*). Antecedent (*ēgoumenon*), consequent (*lēgon*) (Prantl, *Vordersatz*, *Nachsatz*). Relation of consequence (*akolouthia*, *Verhältniss einer Folge*, *Folgeverhältn*is, Prantl 447).	TERMINOLOGY: If …, then --- (*Wenn …, so* ---). Antecedent (*Bedingung*), consequent (*Folge*). Relation of antecedent (*Bedingung*) and consequent (*Folge*) (CT45, CP 400, EidL, NS 205 = PW 188).
TRUTH-FUNCTIONALITY 'the merely formal relation of the combination of true and false' (*das bloss formale Verhältniss der Combination von Wahr und Falsch*) (Prantl 454, also 455). Not a causal relation (Prantl 455).	TRUTH-FUNCTIONALITY 'that it is only the truth or falsity of the thoughts that is to be taken into account' (EidL, PW 187) (EidL, NS 202–03, *dass von dem Gedanken nur in Betracht kommen soll, ob er wahr oder falsch ist*). Not a causal relation (Begr 5–6).
Truth-functionality of the (Philonian) conditional *axiōma* (S.E. *PH* 2.105; Prantl 454).	Truth-functionality of the hypothetical thought (Begr 5–6; EidL, NS 202–03 = PW 186–87; CT44, CP 399).

[120] 'fällt theilweise dem Sprachlichen anheim, da es ja dann wahr ist, wenn das contradictorische (d.h. sprachliche) Gegentheil des Nachsatzes einen Gegensatz zum Vordersatze bildet' (Prantl 461).

List of four combinations: true/true, false/false, false/true, true/false (Prantl 454). The correct/true conditional is the one that does not have a true antecedent and a false consequent (S.E. *PH* 2.110; Prantl 454, n. 143).	List of four combinations: true/true, true/false, false/true, false/false (EidL, NS 202 = PW 186; Begr 5–6; CT44 CP 399). 'false iff the consequent is false, but the antecedent is true' (CT45, CP 399).
Of these four combinations, only the fourth was called incorrect (*mochtēron*) (Prantl 454).	'false if and only if the [consequent, *Folge*] is false, the [antecedent, *Bedingung*] false (CT44, CP 399).
INTERDEFINABILITY The Philonian conditional *axiōma* can (and should) be rephrased as a negated conjunction with the antecedent and the negation of the consequent as conjuncts (Cic. *Fat.* 15–17, with D.L. 7.82 for example).	INTERDEFINABILITY Frege considers the hypothetical thought compound expressible both in hypothetical form, and as the negation of a conjunction in which the antecedent and the negation of the consequent are the conjuncts (CT44–45, CP 399–400); or the contradictory of a conjunction with the antecedent and contradictory of the consequent as conjuncts (*Kurze Übersicht*, NS 216 = PW 200).
LOGICAL TRUTH OF 'IF A, A' Conditionals of the form 'If A, A' are said to be true (S.E. *M.* 8.281, 8.466; Prantl 456). They are analytically true, since the contradictory of the consequent is incompatible with the antecedent (Prantl 461).	LOGICAL TRUTH OF 'IF A, A' Conditionals of the form 'If A, then A' are said to be true (CT50–51, CP 405). Such a compound thought is (analytically) true, because the conjunctive thought that is its contradictory is always false, since it has a thought (the antecedent) and its negation (the consequent) as conjuncts (CT50–51, CP 405).

III.1.3.2.4. Assertoric contents expressed by sentences with 'because'

Finally, there is the unusual parsing of the content of sentences with 'because' clauses, or *causal content*.

Among their non-simple assertibles, THE STOICS list para-conditionals (*parasunnēmena*). These are assertibles that are *para-connected*[121] by the because-connective and that have an assertible as antecedent and another as consequent. The example is 'because it is day, it is light'. The truth-conditions are reductive. They combine the truth-conditions of the corresponding conditional with the truth of the first assertible in the para-conditional: 'The connective indicates that the second assertible follows from the first and that the first

[121] It is unclear how '*para*' should be translated. There was also a para-disjunction, the inclusive truth-functional disjunction (see above). This suggests that '*para*' did not indicate a logical property, but rather a non-simple content of secondary importance (presumably for Stoic logic).

holds' (D.L. 7.71) (*cf.* Prantl 447; Simpl. *in Cael.*, Prantl 386; 'the older Peripatetics'; 'follows from', *akolouthein* is the—generic—truth-condition for conditionals).[122]

> A para-conditional is true if its antecedent is true and the consequent follows from it, for example "if it is day, the sun is above the earth". It is false when it either has a false antecedent or the consequent does not follow from it, for example "since it is night, Dio is walking" when said while it is day.
> (D.L. 7.74; *cf.* Prantl 457).

The non-truth-functional (Chrysippean) truth-criterion for the Stoic conditional immediately precedes that for the para-conditional. This non-truth-functionality would thus likely have been inherited by the para-conditional.

Compare this with what FREGE writes in *Sense and Reference* (SB):

> [In] the sentence 'because ice is less dense than water, it floats on water' we have [the thoughts]: 1. Ice is less dense than water; 2. If anything is less dense than water, it floats on water; 3. Ice floats on water. The third thought, however, need not be explicitly introduced, since it is contained in the remaining two.
> (SB48, CP 175)

Frege adds that, as a result, the 'because'-clause cannot be substituted *salva veritate* by one expressing a different content with the same truth-value (*ibid*.).

This is not exactly the same as the Stoic account, but the underlying principle seems to be. In both cases, we have the *reduction* of the content of a sentence with a causal clause to two sentences and, resulting from this, the truth-conditions of that content. First, in either case, the content of the 'because'-sentence is constructed from three assertoric contents: a conditional and the antecedent and the consequent of the causal clause, and thus three assertoric contents from which the causal content is constructed. (In the Stoic case this is implied.) The key difference is that in his 2. Frege uses a—universal—conditional of the form 'if something is *F*, it is *G*'. Second, in either case the content of the sentence is taken to

[122] D.L. 7.71: παρασυνημμένον μέν ἐστιν […] ἀξίωμα ὃ ὑπὸ τοῦ 'ἐπεί' συνδέσμου παρασυνῆπται ἀρχόμενον ἀπ᾽ ἀξιώματος καὶ λῆγον εἰς ἀξίωμα, οἷον 'ἐπεὶ ἡμέρα ἐστί, φῶς ἐστιν'. ἐπαγγέλλεται δ᾽ ὁ σύνδεσμος ἀκολουθεῖν τε τὸ δεύτερον τῷ πρώτῳ καὶ τὸ πρῶτον ὑφεστάναι. As mentioned earlier, the word translated as 'indicate' is commonly used by the Stoics to express the semantics of an expression.

be reducible to the conjunction of a related conditional and the antecedent of the causal clause. Third, since in both cases the relevant conditional is not truth-functional—if for different reasons—in neither case can we substitute the antecedent *salva veritate*, and the content of 'because'-sentences is not truth-functional. Fourth, in each case, we can surmise, the motivation for the analysis of the 'because'-sentences is to reduce the complex sentences to simpler elements of the author's logical system, to make them logically treatable.

CAUSAL ASSERTORIC CONTENTS	
STOIC para-conditional (*parasunnēmenon*) (D.L. 7.71, 73–74; Prantl 447).	FREGEAN compound sentence with 'because' ('Satzgefüge mit "weil"') (SB48, CP 175).
Expression that identifies the kind of content: 'because' (*epei*) (D.L. 7.71; Prantl 447, n. 126).	Expression that identifies the kind of content: 'because' ('weil') (SB48, CP 175).
Examples: '*because* it is day it is light' (D.L. 7.71; Prantl 447, n. 126), '*because* it is day, the sun is above the earth' (D.L. 7.74; Prantl 457, n. 150).	Example: '*because* ice is less dense than water, it floats on water' (SB48, CP 175).
Constructed from three assertibles: one related conditional and antecedent and consequent of the para-conditional (D.L. 7.71; Prantl 447). The context, a classification of assertibles, entails that the conditional is an assertible.	Contains three thoughts: one related conditional and the thoughts expressed by the antecedent and consequent sentences of the because-sentence '[In] the sentence "because ice is less dense than water, it floats on water" we have [the thoughts] 1. Ice is less dense than water; 2. If anything is less dense than water, it floats on water 3. Ice floats on water' (SB48, CP 175).
The content is reducible to the (conjunction of) related conditional and antecedent of the para-conditional (D.L. 7.71; Prantl 447).	The content is reducible to (the conjunction of) the related conditional and the thought expressed by the antecedent of the because-sentence (SB48, CP 175).
Since the related conditional is not truth-functional, the because-sentence is not truth-functional (D.L. 7.73–74, implied).	Since the related conditional is not truth-functional, the because-sentence is not truth-functional (SB48, CP 175).

The next section investigates how, considered comparatively, the Stoics and Frege dealt with quantified or universal conditionals of the form 'if something is *F*, it is *G*'.

III.1.4. First-order logic: universality and language regimentation

FREGE: The last comparison concerns Frege's posthumously published remarks on Logical Generality (LG),[123] which may have been intended as a fourth part of his logical investigations. In this short piece, Frege moves from propositional logic to elements of first-order logic. Here Frege provides another case in which we have 'different expressions for the same […] thought', this time three ways of expressing a universal thought. Here are his three sentences:[124]

(6) All men are mortal.

(7) Every man is mortal.

(8) If something a man is, is it mortal.[125] (German word-order retained.)

(Note that Frege's claim that (6) and (7) express the same thought as (8) does not reflect the logic of his time. In his *The Principles of Mathematics* (1903), Russell treats (6) and (8) as logically different. 'All men' is considered to have a special reference (*cf.* §59), whereas (8) is used as an example for his *formal implication*, which he discusses in ch. 3, and which appears to have nothing to do with the universal statement (6).)

The syntactically infelicitous (8) is a literal rendering from the German. The relevance of the choice of literal translation becomes evident below. Frege advocates (8) over (6) and (7) as best suited to expressing universality: 'In [(8)] we have the form of a conditional *sentence* and the *indefinitely signifying* sentence parts "something" and "it". These contain the expression of universality' (LA NS 280, LG, PW 259). They express only *one* thought (CT46–47, CP 402–03).

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[123] *Logische Allgemeinheit*, LA, NS 278–81, written 1923 or later; English trans. *Logical Generality*, LG, PW 258–62.

[124] Frege's double quotation marks indicate the linguistic expressions of thoughts.

[125] LA, NS 279 = LG, PW 259. The example 'all men are mortal' is a standard example in logic. For parallels *cf.* Frege's 'Ausführungen über Sinn und Bedeutung' (NS 130), '"Alle gleichseitigen Dreiecke sind gleichwinklig" d.h.: "Wenn etwas gleichseitiges Dreieck ist, so ist es gleichwinkliges Dreieck" (*sic*)'. ' "All equilateral triangles are equiangular", *i.e.* "If anything is an equilateral triangle, then it is an equiangular triangle" ' (CSB, PW 119). Also CO197–98, CP 186-87: '"all mammals have red blood", "what is mammal has red blood", "if something is a mammal, then it has red blood" can all be said to say the same thing.' *Cf.* also CT46–47, CP 402: 'In dem Satzgefüge "Wenn jemand ein Mörder ist, so ist er ein Verbrecher" drückt weder der Bedingungssatz noch der Folgesatz für sich genommen einen Gedanken aus. […] weil das Wort "er" […] in dem aus dem Zusammenhange gelösten Satze ohne hinzukommnden Wink nichts bezeichnet […] Es ist sehr wesentlich, die beiden Fälle zu unterscheiden, die bei einem Satzgefüge von der Form "Wenn B, so A" vorkommen'. *Einleitung* NS 203–05 (PW 187–89), esp. 205 (PW 188–89) has another close parallel.

Frege adds that we can make a logical transition from this mode of expression to the particular ('Besonderem') by substituting the same proper name for the two indefinitely signifying sentence parts.[126]

(9) If Napoleon a man is, is Napoleon mortal.[127] (German word-order retained.)

Frege then introduces the idea of a 'Hilfssprache' or 'helping language', which 'is meant to serve as a bridge from the perceptible (*i.e.* language) to the imperceptible (*i.e.* thought)'. He applies this idea of a helping language to (9). Even after using 'Napoleon' instead of 'he' in (the consequent of) (9), (this repetition is thus part of the 'Hilfssprache')[128] one still cannot read off that the sentence expresses a thought composed of the two thoughts 'Napoleon is a man' and 'Napoleon is mortal', and 'in this deviation from what is language-related to what is thought-related, there is still a defect in the helping language' (LA NS 281 = LG, PW 261). To remove this defect, he replaces (9) by

(10) If Napoleon is a man, Napoleon is mortal. (In German (10) is *in*felicitous.)

THE STOICS[129] maintained that the following two sentences each have a different linguistic expression but mean the same, since both cover all individual cases.

(11) Man is a rational mortal living being.

(12) If something is a man, it is a rational mortal living being.

(S.E. *M.* 11.8–11, *cf.* Epictetus *Diss.* 2.20.2–3)[130]

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[126] This produces a hypothetical compound thought (PW 261, 'hypothetisches Gedankengefüge', NS 281). A hypothetical compound in which the same name occurs in the antecedent and consequent can be considered as a singular hypothetical thought ('singuläre[r] hypothetische[r] Gedanke'), EidL NS 205 = PW 188.

[127] LA NS 281 = PW 261; Also LA NS 280 = PW 260: 'If *a* a human being is, is *a* mortal' (German word-order retained).

[128] The repetition of the noun that has argument place in both sentences that express the component thoughts recurs in several Fregean passages (*e.g.* LM, NS 231 = PW 213–14: 'Wenn Cato ein Mensch ist, so ist Cato sterblich', derived from 'Wenn etwas ein Mensch ist, ist es sterblich').

[129] We can assume that this is Stoic since it makes a Stoic claim and uses Stoic terminology, and, more importantly, since later in the same passage Chrysippus' view is represented as maintaining a relation like that between (11) and (12) but with a disjunctive sentence as the consequent sentence (S.E. *M.* 11.11).

[130] 'for the one saying "Man is a mortal rational animal" says the same thing in meaning, though different in expression, as the one saying "if something is a man, it is a mortal rational animal"' (ὁ

(12) was said to be 'universal' (*katholikon*), since it encompasses all cases given in the antecedent. And since only conditionals with an indefinite pronoun and an anaphoric pronoun were called universal, one can infer that the universality was taken to be signified by these pronouns. This is confirmed by the fact that the Stoics named conditionals like (12) indefinite conditionals (Cic. *Fat.* 15).[131]

The Stoics indicated that from an indefinite conditional such as

(13) If someone is born in the sign of the dogstar, then he won't die at sea.

a legitimate logical transition can be made to

(14) If Fabius is born in the sign of the dogstar, then Fabius won't die at sea.[132]

(*Cf.* Prantl 456, 'It is distinctly clear from a passage in Cicero that this conception of the hypothetical judgment, which agrees with the doctrine of the categorical judgment, ensued precisely from Chrysippus'.[133] And in the footnote with the text passage: 'For if what is connected as follows is true "If someone is born in the sign of the Dog Star, he will not die at sea", then the following is also true "if Fabius is born in the sign of the Dog Star, Fabius will not die at sea"'.)

γὰρ εἰπὼν 'ἄνθρωπός ἐστι ζῷον λογικὸν θνητόν' τῷ εἰπόντι 'εἴ τί ἐστιν ἄνθρωπος, ἐκεῖνο ζῷόν ἐστι λογικὸν θνητόν' τῇ μὲν δυνάμει τὸ αὐτὸ λέγει, τῇ δὲ φωνῇ διάφορον) (S.E. *M.* 11.8). The following sentence (S.E. *M.* 11.9) leaves no doubt that the singular noun 'man' without an article is understood universally, as covering every man (*i.e.* human being). For *katholikon* for an indefinite conditional, see S.E. *M.* 1.86; *cf.* Epictetus *Diss.* 2.20.2–3; Plutarch *Comm. Not.* 1080c. The equivalence between the universal 'man' and 'all men' should have been familiar to Frege. For example Bolzano states this in his 1837: (I) 250.

[131] For the Stoic treatment of logical generality, see Bobzien and Shogry, forthcoming.

[132] Cic. *Fat.* 15; Prantl 456, n. 147. At least part of the example is Roman, possibly provided by Cicero in order to illustrate Chrysippus' view. But there can be no doubt that the Stoics accepted logical transitions of this kind. *Cf.* the sophism discussed by the Stoics: 'If someone is in Athens, he is not in Megara. If (a) man is in Athens, (a) man is not in Megara (Prantl 492, n. 213). (The 'a' is in brackets, since Greek has no indefinite articule and 'man' is thus syntactically ambiguous). This argument is *paradoxical* only if the general scheme of inference is accepted as valid with proper names, demonstratives, or descriptions. Hence, in line with *Fat.* 15, we assume that the step from (13) to (14) was accepted as valid.

[133] 'Dass aber diese mit der Lehre vom kategorischen Urtheile übereinstimmende Auffassung des Hypothetischen gerade von Chrysippus ausging, erhellt deutlich aus einer Stelle Ciceros'.

Like Frege, the Stoics introduced a helping language, a language that was meant to build a bridge from the corporeal linguistic expressions to the incorporeal contents—and thus a language that reflects the structure of the assertibles and of content generally. We have dozens of cases as evidence that this is what the Stoics did.[134] One relevant case here is that, in conditional sentences such as (14) which express instantiations of indefinite conditionals, the Stoics standardly used the same proper name *both* in the antecedent sentence *and* in the consequent sentence, although this is *not* standard Greek but, rather, atypical Greek.[135] Note how Frege does the very same thing with Napoleon—and this is not standard in German *either*. Note also that in Greek you can put the words in a clause in almost any order, since the syntax is determined largely by case markings. Hence, the Stoics were generally able to choose formulations in which the antecedent and consequent sentences are each syntactically identical to the standard formulations for the atomic assertibles that are the component of the conditional. So we see that Frege has the same sentence in *his* helping language that the Stoics use in their regimented language—except that the German is ungrammatical, whereas the Greek is grammatical. (The German needs a little more help.) If the language does not suitably reflect content, the language needs adjusting.

The more significant example of language regimentation is that from Frege's (6) and (7) to (8); and from the Stoic (11) to (12): from the expression of universality in a simple sentence to the expression in an indefinite conditional. The formulation with an indefinite conditional is a natural language analogue to the formalization with a universal quantifier in symbolic logic, 'For all *x*, if *x* is F, *x* is G'.[136] Both the Stoics and Frege advocate this natural language sentence form to express universality against an Aristotelian tradition. The goal is likely the same both times: the form retains the assumed correspondence between linguistic expression and content, and thus *reflects more accurately the structure of the imperceptible* assertibles *or* thoughts. *In particular, it reflects valid inference patterns that permit detachment* and allows them to be performed semi-automatically.

[134] *Cf. e.g.* Atherton and Blank 2003: 314–16, and Barnes, Bobzien, and Mignucci 1999: 96–97; Frede 1974.

[135] *Cf.* D.L. 7.77, 78, 80; S.E. *M.* 8. 246, 252, 254, 305, 308, 423; S.E. *PH* 2.105, 106, 141; Gellius 16.8.9; *Inst. Log.* 4.1; Simp. *Phys.* 1300; Alex. *An. Pr.* 345; Cic. *Fat*. 12. In Prantl, *e.g.* 454, 'die Erde […] die Erde' three times; 456, n. 147 'Fabio […] Fabio'; 466, n. 168: 'Dion […] Dion', and thus accessible to those familiar with Prantl.

[136] This does not mean that the Stoics and Frege unpacked these sentences in the same way. Quantifiers that range over variables were Frege's, not Stoic. *Cf.* Bobzien and Shogry, forthcom.

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THE STOICS: that which is universal (*to katholikon*).	FREGE: Universality/Generality (*Allgemeinheit*).
The following two sentences have different linguistic expression, but have the same meaning (S.E. *M.* 11.8–11). (11) Man is a rational mortal living being. (12) If something is a man, it is a rational mortal living being.	Three 'different expressions for the same […] thought' (LA NS 279, PW 259). (6) All men are mortal. (7) Every man is mortal. (8) If something a man is, is it mortal. (German word-order is retained in (8)).
(12) and (13) were called indefinite conditionals. They contain an indefinite pronoun ('something') and an anaphoric pronoun ('it'/'he'/'she') (Cic. *Fat.* 15). They were said to be universal (*katholikon*). The Stoics generally used (12) instead of (11).	In (8) we have the form of a conditional *sentence* and the indefinitely signifying sentence parts 'something' and 'it'. These contain the expression of universality. (8) is the one we should use (LA NS 280, PW 259–60).
One can make a legitimate logical transition from indefinite conditionals like (12) and (13) to conditionals that are non-simple assertibles like (14) by substituting the same proper name for the indefinitely signifying sentence parts. (13) If someone is born in the sign of the dogstar, then he won't die at sea. (14) If Fabius is born in the sign of the dogstar, then Fabius won't die at sea. (Cic. *Fat.* 15)	One can make a legitimate transition from the mode of expression (8) to the particular, by substituting the same proper name for the indefinitely signifying sentence parts (LA NS 281, PW 261). (8) If something a man is, is it mortal. (9) If Napoleon a man is, is Napoleon mortal.
Stoic logic provides ample evidence for language regimentation intended to bridge the gap between linguistic expression and the structured content expressed. The regimentation recommends formulations in which the antecedent sentence and consequent sentence do not differ from the sentences that express the simple assertibles used as antecedent and consequent in the conditional.	As a bridge from language to thought, a 'helping language' ('Hilfssprache') is used. The helping language offers formulations in which the antecedent sentence and consequent sentence do not differ from the sentences that express the 'Teilgedanken' of the conditional.

(i) It requires the same subject expression in the antecedent and consequent sentence. (Standard in Stoic logic and argumentation.) Nothing further is required, since (14) is already in the regimented language.	(i) It requires the use of the same subject expression in the antecedent and consequent sentences. (ii) Additionally, it removes the defect of (9) by replacing it with (10) If Napoleon is a man, Napoleon is mortal. (The sentence structure of (10) is infelicitous in German.)

Comparison with the Stoics here aids us in understanding Frege's helping language. It is not unusual that 'Hilfssprache' is translated and interpreted as 'meta-language', as opposed to the object language—presumably 'If Napoleon a man is, is he mortal' ('Wenn Napoleon ein Mensch ist, ist er sterblich'). But there is nothing meta-linguistic in Frege's 'Hilfssprache'. It is a regimented object language. Its purpose is to bring out the logical structure of the—incorporeal—thought in the medium of language (this being the only medium we have). It is intrinsically connected with Frege's logico-ontology, just as in the Stoic cases of language regimentation.

IV. Conclusion

The historical data strongly suggest that Frege knew Prantl's *History of Western Logic*. A textual comparison that results in well over a hundred parallels (I count 120, and this is without the parallels in epistemology and on inference) strongly suggests that Stoic logic had an impact on Frege. Common sense suggests that, if this is so, it is more likely that the impact of Stoic logic came via Prantl than from the dozens of individual Greek and Latin sources that Prantl amasses in his footnotes.[137] This is supported by the fact that in at least three or four cases (depending on how one individuates them), Frege's view corresponds to Prantl's representation or interpretation of the Stoics more closely than to the Stoic view itself (or to a different, more plausible interpretation). These include: Stoic non-assertible complete contents, in particular questions, contain the element of truth and falsehood; the treatment of

[137] I, for one, cannot see that Frege read his way through the edition of PHerc307 and the Greek volumes of the Aristotle commentators, and Galen (although S.E. is another possibility) to pick up the various fragmentary testimonies of Stoic logic sprinkled throughout, which we now have collected in SVF II and, in particular, in FDS.

more-than-assertibles and quasi-assertibles as two different kinds; complete contents that include emotional elements contain truth and falsehood; the presentation of the truth-conditions of the Philonian conditionals. Moreover, this view is supported by the fact that several important testimonies of Stoic logic that are not in Prantl have no parallels in Frege: *e.g.* elements from the *Logical Investigations*, about Stoic analysis, and about hypothetical arguments. If Stoic logic had an impact on Frege, Prantl is thus the likeliest source.

Where does this leave us with respect to plagiarism, which I so bumptiously included in the title of this essay? Did Frege take the work or ideas of the Stoics and pass them off as his own?[138] None of the similarities presented taken by itself is proof that he did. The sheer magnitude of the similarities makes it extremely unlikely that he did not.

What would be our alternatives? I see two. First, Frege could have come up with all these points himself, without any external influence. Second, Frege could have drawn on non-Stoic sources. Evidently, given the quantity of parallels proffered, the three options allow for all sorts of combination. There will be no way to prove for any specific point that Frege came up with it by himself, independently of Prantl's Stoics. (We would need proof that he could not have had access to Prantl, and, given our historical data, that would be extremely hard to come by.) In principle, there will be ways of showing that Frege drew on more recent non-Stoic sources. For some of the parallels I adduce there are similarities and overlap in nineteenth-century logic books with which Frege was familiar. One recurrent significant feature is that other sources may touch on the same topic but only Frege and the Stoics end up adopting effectively the same view on the topic, or views very close to each other. Other logicians talk at length about negation. But they do not use 'not' ('*nicht*') as an informal prefix in the schematic expression of negations. Other logicians may talk about sentences with 'no' ('*kein*') and 'un', but they do not do so in the same sentence, in the same order, and directly after defining negations proper. Other logicians talk about generality, but do they introduce examples of instances in which the subject term is the same in antecedent and consequent? And so forth. This is the pattern I found in many of the cases of Frege's contemporaries that I have examined: overlap in topic, yes, but only rarely in the position taken. A text that combines both the topic that Frege considers and the position he adopts, compared to one that does not take, and possibly explicitly dismisses, Frege's position, seems more likely to have been an influence. For a good number of the parallels there seems to be no source other than Stoic logic available at the time when Frege writes.

[138] *Cf.* the epigram of this paper.

Even if it were possible to show that sixty percent of the parallels I have adduced have equally close parallels in other works on logic that Frege knew (something I doubt can be shown), this would leave sufficient parallels for a claim of plagiarism of sorts (see below). Moreover, there is a further probabilistic factor. Prantl's chapter on the Stoics offers *in one chapter of one book* a possible source for a hundred or so parallels. We can assume that with such a number of parallels the likelihood is greater that the instances in Frege, or in any case many of them, come from one source rather than a broad scattering of sources.

We know that Frege read many works in which were discussed psychological, epistemological, and mathematical themes which he takes up in his own philosophical writings. There is an ample literature devoted to establishing individual examples of such connections, albeit not always successfully in my view (*e.g.* in the cases of Eucken and Hirzel). My focus in section III has been exclusively on topics that fall within the category of philosophical logic, broadly understood. Nineteenth century discussions of these topics are rare. The best sources are logic texts of the time: Sigwart 1873, Mill 1843, Ulrici 1852, Bolzano 1837, Boole 1854. De Morgan 1847, Ueberweg 1857, and Trendelenburg 1840, 1870.[3] So unless and until someone provides a study which sets out the required evidence, we may maintain that the proposed thesis of Stoic influence on Frege via Prantl stands.

We still need an answer to the question of how Prantl's Stoic logic became incorporated into Frege's work. Was it intentional? Was it really plagiarism? To start with, we can with certainty rule out one kind of plagiarism: that of the ignorant student or career-obsessed academic who lifts entire sentences or passages without understanding what they copy. Even though we often witness Frege battling with details of his theory, he only writes what he has thought through, understands, and approves of. We can also rule out a related kind of plagiarism, in which someone does understand what they take from their source, but nonetheless copies mechanically and verbatim. Whatever Frege took (assuming he did) is reproduced neither mindlessly nor mechanically. This leaves us with several somewhat more benign options.

(i) He incorporated elements of what he had read and studied from Prantl (and possibly from the *Summary*) when writing his own work. However, he did not do so deliberately but, rather, considered what he wrote as his own ideas. This is a familiar psychological phenomenon. In this case we would have a process of the –illegitimate– appropriation or assimilation of Stoic thought rather than *intentional* plagiarism.

(ii) He thought of the ideas of the ancients as being freely available to anyone to help themselves to and not subject to any copyright, and for this reason did not refer to the source on which he relied. We know that Frege was influenced by later philosophers in some of his thought and frequently felt no need to reference them (*e.g.* Dummett 1981b, Lotze; Schlotter 2006, Bruno Bauch; Dathe 1995, Eucken).

(iii) He incorporated elements of what he had read and studied from Prantl when writing his own work, and he knowingly omitted any mention of this fact for reasons other than those in (ii).

The reality could be any combination of (i), (ii), and (iii), and details could be spun out in many ways. It is neither in my interest to adjudicate between the three options or to spin out possible details. Nor—as I said at the beginning—~~Polish~~ is my interest in questions of accountability or culpability. If Dummett is correct that Frege's 'Kernsätze' 'form a series of comments by Frege upon Lotze's Introduction [to his *Logik*], or, more exactly, of remarks prompted by reflection upon it',[139] we have here one illustration of how Frege makes notes and interacts with the texts of other philosophers, when forming or rethinking his own ideas. This is consistent with all combinations of (i)–(iii). (Of course the *Kernsätze* were just an unpublished fragment.) The purpose of this paper has been accomplished if it establishes that there are similarities to such a colossal extent, in terminological distinctions, choice of topics, and content, that the probability that Frege did not substantially draw on Stoic logic is minute.

Yet what if, *against all odds*, Frege did, in fact, not draw on Stoic logic? Then we have the following immensely fascinating situation. Separated by over two millennia, we witness logicians who started (a) with the same general idea of content that—in some sense at least—exists independently of our saying or thinking it, and (b) with the same general conception of a propositional logic. These logicians were then confronted by the same set of problems: problems regarding how linguistic expressions can serve us to express and communicate that imperceptible content and can explain the complexity of content (especially as it is required for reasoning); how for this purpose natural language expressions may fall short in several ways: in particular how they may contain *too much* or *too little* or *the wrong expressions*, and how they may not provide the means to unambiguously express

[139] Dummett 1981b: 440, 1991: 66.

content of potentially unlimited complexity. In this case, independently of each other, both the Stoics and Frege would have thoroughly considered all four issues, and in doing so would have followed staggeringly similar pathways.[140]

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SIGLA, Ancient authors

Alex. *An. Pr.*	Alexander of Aphrodisias, *In Aristotelis Analyticorum priorum librum I commentarium*
Ammon. *Int.*	Ammonius, *In Aristotelis De interpretatione commentarius*
Apul. *Herm.*	Apuleius, *Peri Hermeneias*
Arist. *De Int.*	Aristotle, *De Interpretatione*
Cic. *Fat.*	Cicero, *De Fato*
Comm. Not.	Plutarch, *Against the Stoics on Common Notions*
Diss.	Epictetus, *Dissertationes*
D.L.	Diogenes Laertius, *Vitae Philosophorum*
Ecl.	Stobaeus, *Anthologii libri duo priores qui inscribi solent Eclogae physicae et ethicae*
Epist.	Seneca, *Epistulae morales ad Lucilium*
Gal. *Inst. Log.*	Galen, *Institutio logica*
Gal. *PHP*	Galen, *De Placitis Hippocratis et Platonis*
Gellius	Gellius, *Attic Nights*
Log. Inv.	Chrysippus, *Logical Investigations*
Plut. *St. Rep.*	Plutarch, *De Stoicorum Repugnantiis*
Plut. *Quaest.*	Plutarch, *Quaestiones Convivales*
S.E. *M.*	Sextus Empiricus, *Adversus Mathematicos*
S.E. *PH*	Sextus Empiricus, *Pyrroneion hypotyposeon*
Simpl. *in Cat.*	Simplicius, *In Aristotelis Categorias commentarium*

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Simpl. *in Cael.*Simplicius, *In Aristotelis De Caelo commentaria*

SIGLA, Frege, English texts

CN *Conceptual Notation and Other Works*
CP *Collected Papers on Mathematics, Logic, and Philosophy*
GG *Basic Laws of Arithmetic*
PMC *Philosophical and Mathematical Correspondence*
PW *Posthumous Writings*

BLC Boole's Logical Calculus and the Concept-script, in PW, 9–52
BS *Begriffsschrift*, in CN
CO On Concept and Object, in CP, 182–194
CSB Comments on Sense and Meaning, in PW, 118–125
CT Compound Thoughts, in CP, 390–406
DPE Dialogue with Pünjer on Existence, in PW, 53–67
FC Function and Concept, in CP, 137–156
FGII On the Foundations of Geometry: Second Series, in CP, 293–340
IL Introduction to Logic, in PW, 185–196
Logic Logic, in PW, 126-151
LG Logical Generality, in PW, 258–262
LM Logic in Mathematics, in PW, 203–250
N Negation, in CP, 373–389
OCN On the Concept of Number, in PW, 72–86
PWLB Logic, in PW, 126–151
SB On Sense and Meaning, in CP, 157–177
T Thoughts, in CP, 351–372

SIGLA, Frege, German texts

FBB *Funktion, Begriff, Bedeutung: fünf logische Studien*
LU *Logische Untersuchunten*
NS *Nachgelassene Schriften*
Begr *Begriffsschrift, eine der arithmetischen nachgebildete Formelsprache des reinen Denkens*

BG	Über Begriff und Gegenstand, reprinted in [FBB], 66-80.
Gedanke	Der Gedanke. Eine Logische Untersuchung, reprinted in [LU], 30-53.
Gedankengefüge	Logische Untersuchungen. Dritter Teil: Gedankengefüge, reprinted in [LU] 72-91.
Grundlagen	*Die Grundlagen der Arithmetik: eine logisch mathematische Untersuchung über den Begriff der Zahl*
Funktion und Begriff	Funktion und Begriff, reprinted in [FBB], 17-39.
SB	Ueber Sinn und Bedeutung, reprinted in [FBB], 40-65.
Verneinung	Die Verneinung. Eine Logische Untersuchung reprinted in [LU] 54-71.

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