REVIEW

Current challenges in modelling vibrational fatigue and fracture of structures: a review

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Abstract

Fatigue damage is a concern in the engineering applications particularly for metal structures. The design phase of a structure considers factors that can prevent or delay the fatigue and fracture failures and increase its working life. This paper compiled some of the past efforts to share the modelling challenges. It provides an overview on the existing research complexities in the area of fatigue and fracture modelling. This paper reviews the previous research work under five prominent challenges: assessing fatigue damage accurately under the vibration-based loads, complications in fatigue and fracture life estimation, intricacy in fatigue crack propagation, quantification of cracks and stochastic response of structure under thermal environment. In the conclusion, the authors have suggested new directions of work that still require comprehensive research efforts to bridge the existing gap in the current academic domain due to the highlighted challenges.

1 Introduction

In engineering world, crack occurrence and propagation in the material components and the structures are serious issues to address. Failure in early detection has consequences leading to catastrophic damage. The catastrophic failure is cost to bear as the machineries are expensive and not easily replaceable rather to be repaired. For this reason, academicians have performed extensive research to find solutions and minimize the crack growth.

Engineering materials, especially metals, develop cracks and fracture during their service time. In most of the applications, such as automotive, aircrafts and power plant, operate under high temperatures. Such structures and components of the machine develop crack more prevalent under the combined effect of thermal and mechanical loads and experience fatigue. Although the knowledge of the fracture and fatigue was confirmed in 1754 BC [1], the scientific society faced many challenges while considering the continuous changing needs of the people. A number of review articles published related to fatigue and fracture: thermal loadings fracture of shape memory alloys [2], multiaxial vibration of linear and nonlinear systems [3], variable amplitude fatigue [4], interpretive technical history of fracture mechanics [1], role of dynamic response parameters in damage prediction [5], cumulative fatigue damage mechanisms and quantifying parameters [6], stability and significance of residual stress during fatigue [7]. The existing reviews somehow managed to explain the details about the micromechanics of fatigue and fracture and the macroscopic behaviour of the structure distinctly separate. However, these details are unable to address a good number of modelling challenges which can come across if the mentioned mechanics and behaviour will be correlated.

These challenges are still unaddressed, especially when the loads are no more singular and quasi-static in nature. The couple loads dynamic in nature involve mechanical and thermal external disturbances, and this duality in time domain creates complexity in analytical mathematics to solve the real phenomenon on structure with exactness.

This paper compiled some of the past efforts to share the aforesaid challenges. It provides an overview on the existing research complexities in the area of fatigue and fracture modelling. This paper reviews the previous research work under five prominent challenges: assessing fatigue damage

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accurately under the vibration-based loads, complications in fatigue and fracture life estimation, intricacy in fatigue crack propagation, quantification of cracks and stochastic response of structure under thermal environment. In the conclusion, the authors have suggested new directions of work that still require comprehensive research efforts to bridge the existing gap in the current academic domain due to the highlighted challenges.

2 Challenge-1: assessing fatigue damage accurately under the vibration-based loads

Thermomechanical vibrations have been concerned for the engineering applications. In the growing advancement of technology, these applications exposed to the mechanical loads under thermal conditions and has tendered the scientists and engineers to investigate the dynamic behaviour of structures and their components [8]. The vibration of the components influences the durability and reliability. Generally, vibration is analysed in four domains and each domain offers identifiable data on the operational conditions and characteristics of the vibration [9]. Researchers apply different theories to study the characteristics of vibration: higher-order shear deformation theory [10–14], non-local elastic theory [15, 16], modified couple stress theory [17], generalized differential quadrature method [18], Bubnov–Galerkin method [19], thermal-energy method [20], finite element analysis [21, 22] among many others. Most of these methods are, however, purely analysing the vibrational characteristic behaviour of the structure without concerning the fatigue damage. Assessment of fatigue damage under such condition is still a concern and challenge. Further the current investigations on thermomechanical vibration are more focused on different materials: materials like graphene sheet [23, 24], functional graded nano-plate [25], composite beams [26] and nano-composite plate [27]. Primarily, it is on application-based studies rather than emphasizing in accurate measurement of the fatigue.

Warminska et al. [28] demonstrated that structural response is directed by the level of thermal loading and distribution. A large increase in temperature can shift the vibration centre and it affects the fundamental frequency. Yang and Shen [29] studied in similar way confirmed the vibration response as a result of thermal effect, material composition, boundary condition and loading mode. A free vibration analysis in cylindrical bodies highlighted the frequency vibration depends on mode, thickness and curvature of structure [30]. Shen and Wang [31] and Ghayesh et al. [32] investigated on nonlinear vibration. They noticed the increase in temperature minimized the natural frequency but has marginal effect on nonlinear-to-linear frequency ratio. Julien et al. [33] and Kitipornchai et al. [34] examined probabilistic random vibration. They concluded that the linear frequency dispersion is an influence of change in temperature. Rezaee et al. [35] studied vibration analysis of nano-wires and Ubertini [36], and Bao et al. [37] studied vibrations of bridge structures. A few of the articles reported fatigue due to vibration [38–40] and predominantly based on applications. These suggest the assessment of fatigue damage is on weak foundation due to lack of analytical relations and arguments, especially when the structures are operated under thermomechanical loads.

Modal analysis is a common method to predict the structural health based on experimentally measured vibrations [41]. This method is more often used for extracting defects and faults from structural applications, which are too complex to model analytically and numerically, for example modal analysis of exhaust manifold in hot condition [42], the turbine blade at complex thermomechanical loads [43], high-speed machine-like servo-hydraulic [44], jet engine fan [45], extracting full-field dynamic strain subjected to arbitrary excitations on a wind turbine rotor [46], etc. Modal analysis can also determine the dynamic stress intensity factor [47]. Similarly, it is also utilized in different other approaches. Jezequel [48] used a hybrid method of modal synthesis using the vibration tests. The method approved to be helping in studying the structuring modification. Yam et al. [49] experimentally studied and suggested that modal strain analysis can be effectively utilized to study structural dynamic and fatigue life estimation. Haider and Nayfeh [50] investigated the modal interaction in the nonlinear forced vibrations of a heated plate. Lamb and Rouillard [51] stated that system fatigue characteristic can be obtained by monitoring its modal parameters. Braghin et al. [52] modal control technique is new to the mechanical and structural engineers due to it modal coordinate representations. Nevertheless, modal analysis has limited over the applied boundary conditions which compromises the real-case scenario.

Thermomechanical vibration examinations are important to establish failure possibilities and crack propagation before catastrophic failure. In the past, researches used the system dynamic response and predicted the possible faults, defect and their conceivable locations [8]. Mathan and Prased [53] studied the dynamic response of the piping system at various temperatures using finite element analysis. They performed FE simulation with the help of modal and harmonic analysis. Experiments and numerical simulations were also performed to study the dynamic and acoustic response of clamped rectangular plate and of a composite structure [54, 55]. The flexural responses of a thermoelastic thin beam were also studied under thermal and mechanical load [56]. Shen et al. [57] studied Reissner–Mindlin plate resting elastic foundations. Beside the mentioned, researchers evaluated the vibration modes using video measurements [58]. It is well

established that the academics studied vibration behaviour of structures of different sizes, thicknesses and layers, i.e. sandwich plates [59–61]. Furthermore, the study of vibration in the finite element method becomes familiar among the researchers due to cost-effective and ease of evaluating the data. Nevertheless, the data obtained from experiments are always far from an agreement with the simulation results due to practical compromises [62].

3 Challenge-2: complications in fatigue and fracture life estimation

Fatigue is a failure phenomenon in materials that involves cyclic load with developing cracks [63, 64]. Fatigue normally occurs when a repeated or fluctuating load applied, which may either or both tensile, compressive or torsional. There are three stages in fatigue life [65]: crack initiation, crack propagation and failure stage. The crack initiates where there are discontinuities in the crystal structure of the material. The development of the discontinuity actually strengthens the structure through plastic deformation but lost its ability after reaching deformation limit and thereafter developed minute crack [62]. The slow and steadily growth of a crack threatens the structural integrity. The final stage of crack development is fatigue failure. The fatigue life is dependent upon the load history, residual stresses, properties of the material, manufacturing defects, grain size, design geometry, environmental conditions, etc. Studies on the thermomechanical fatigue [66–73] are limited to some applications and need comprehensive assessment, in particular, for fatigue life estimation. The following section discusses the fatigue life estimation methods to highlight the unaddressed challenges.

3.1 Life estimation methods

Knowing the life of the component before being used is the most important aspect to avoid fatigue or catastrophic failure. There are number of methods and techniques to predict the fatigue life. Approaches like frequency domain method [74, 75], modal approach [40, 47, 72], accumulative approach [76], thermal fracture model [77], continuum mechanics model [78] and intrinsic dissipation model [79] are predominant. The foremost and well-known method was formulated by Paris and Erdogan [80] given in Eq. (1).

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C\Delta K^m \tag{1}$$

where ΔK the stress intensity range, C and m are the material constants. The fatigue life is then determined by integration as in Eq. (2).

$$N = \int_{a_o}^{a_f} \frac{\mathrm{d}a}{C\Delta K^m} \tag{2}$$

Damir et al. [81] proved that modal analysis is a great potential for fatigue evaluation and characterization of the mechanical components. This investigation determined the capability of experimental modal analysis as a non-destructive tool. Braceesi et al. [74] used the modal frequency domain approach to evaluate the virtual fatigue life in a nonlinear system. They considered the random loads to estimate the component life considering the dynamic behaviour and state of stress. The fatigue damage due to the stress time history was acquired by augmenting the Gaussian damage (D_g). However, in real situation damage is non-Gaussian. The use of corrective coefficient of non-Gaussianity, λ_{ng} to estimate the non-Gaussian damage, D_{ng} is given by Eq. (3).

$$D_{ng} = \lambda_{ng} D_g \tag{3}$$

Using the Goodman symmetric law, the functional relation for the corrective coefficient is developed in the form of Eq. (4).

$$\lambda_{ng} = f(\mu, \sigma, \mathrm{Sk}, \mathrm{Ku}, m) \tag{4}$$

where m is the material constant of Wöhler's curve, μ is mean value, Sk is skewness and Ku is kurtosis. The Gaussian damage curve in given Eq. (5) was used to analyse the fatigue damage.

$$\lambda_{ng} = e^{\frac{m^{\frac{3}{2}}}{\pi}} \left(\frac{(\mathrm{Ku} - 3)}{5} - \frac{\mathrm{Sk}^2}{4} \right) \tag{5}$$

In another papers of Braccesi [75, 82], a modal approach was used to evaluate the frequency domain stress recovery and fatigue damage of mechanical components. The stress tensor of the model was obtained from the power spectral density matrix. In Dirlik's method, the fatigue damage D was calculated by the relation of Eq. (6).

$$D = \frac{T.n_t}{C} \int_0^\infty (\Delta\sigma)^k f_\sigma(\Delta\sigma) d(\Delta\sigma) \tag{6}$$

where T is the duration of the random signal application, n_t is the number of cycles per unit time, $(\Delta\sigma)^k$ probability density function of the stress range and $f_\sigma(\Delta\sigma)$ is frequency domain criteria. The other method of calculating fatigue damage is Bands method given by the relation of Eq. (7).

$$D = \frac{K.\alpha^{\frac{1}{\beta}}.f_{\mathrm{ref}}}{\left(\int G_{\mathrm{ref}}(f)\mathrm{d}f\right)^{1/2\beta}} \tag{7}$$

where the functions K parameters α and β are from the Wöhler' curve, f_{ref} is reference frequency and $G_{\mathrm{ref}}(f)$ is the ratio function.

In non-local-based fatigue life estimation, the dispersal of stress/strain in the locality of the crack tip is estimated [83]. The non-local damage parameter is the integration of all local damage parameter in the material domain. The local parameters are however estimated from the classical mechanics without considering the material homogeneity. The non-local concept considered the complexity of the geometry structure by analysing the influence of inhomogeneity stress field [84]. This approach is termed as critical distance method. The local parameter is estimated by a point, a line, an area or volume. The local parameter is calculated by:

$$\sigma_{\text{eff}}(t, r_o) = \frac{1}{\hat{w}(r_o)} \int_A \sigma_n(t, r)w(r - r_o)\mathrm{d}A \tag{8}$$

$$\hat{w}(r_o) = \int_A w(r - r_o)\mathrm{d}A \tag{9}$$

where t is the time, r is the unit vector and r_o is the reference point in the crack tip and A is the critical plane area. The weight function is presented in Eq. (10).

$$w(r) = \exp^{-\left(\frac{2r}{l_c}\right)^2} \tag{10}$$

where $r = |r - r_o|$ and l_c is the critical length.

In the point method, the fatigue failure occurs because of the presence of stress/strain to some critical distance from the crack tip. It is considered as non-local since the selected point is not from the crack-tip region. The fatigue life is calculated by reducing the stress/strain into equivalent stress/strain parameters. The critical distance is calculated [83] by Eq. (11).

$$D_c = \frac{1}{\pi} \left(\frac{\Delta K_{I,\text{th}}}{\Delta \sigma_{af}}\right)^2 \tag{11}$$

where $\Delta K_{I,\text{th}}$ is a range of threshold value of stress intensity factor and $\Delta \sigma_{af}$ is the range of fatigue limit.

The line method assumes that fatigue failure is due to the presence of stress over the line. The average of the stress is calculated as:

$$\sigma_{af} = \frac{1}{D} \int_0^D \sigma_y w(x)\mathrm{d}x \tag{12}$$

where D is the distance from the notch point and $w(x)$ is the weight function.

The area method assumes that failure is due to the damage parameter over some area. The average failure function is calculated by:

$$\overline{R}_\sigma = \frac{1}{d_0^2} \int_A R_\sigma(\sigma_n, \tau_{ns})\mathrm{d}A \tag{13}$$

where $\overline{R}_\sigma$ is the average failure function, σ_n, τ_{ns} are normal and shear stresses on the physical plane, the critical distance d_0 in area method is calculated by Eq. (14).

$$d_0 = \frac{2}{\pi} \left(\frac{K_{I,\text{th}}}{\sigma_{af}}\right)^2 \tag{14}$$

The volumetric method assumes that fatigue failure is due to the damage parameter over a critical volume. Saintier et al. [85] proposed a non-local energy-based fatigue life calculation subjected to multiaxial variable amplitude loading. They anticipated two hypotheses of volumetric damage parameters. The volumetric damage parameters are expressed as:

$$\overline{\omega}_{g_{\text{eq, dam}}}(C_i) = \frac{1}{V^*(C_i)} \int_{v^*(C_i)} W_{g_{\text{eq,dam}}}\mathrm{d}V \tag{15}$$

where $\overline{\omega}_{g_{\text{eq, dam}}}$ is the non-local damage parameter, V^* is the volume influencing fatigue crack initiation, C_i is the potential critical point and $W_{g_{\text{eq,dam}}}$ is damaging strain energy density. The final energy damage parameters S–N curve equation is given as:

$$\overline{\omega}_{g_{\text{eq, dam}}}(N) = \frac{\left(C/N^\gamma\right)^2 - (\sigma^*)^2}{E} \tag{16}$$

where E is Young modulus, σ^* is threshold stress, N is number of cycles, C and γ are material constant.

Another technique of evaluating fatigue damage is the accumulative approach. This technique is known as Palmgren–Miner rule which is a linear damage accumulation rule. It states that failure occurs when the fatigue damage D is equal to unity.

$$D = \sum \frac{n_i}{N_i} = 1 \tag{17}$$

Palmgren–Miner rule is, however, not suitable for nonlinear damage. Proso et al. [38] proposed the damage accumulation correction rule for structural nonlinearity. The corrected damage accumulation is estimated by the following relation:

$$D_c = \sum_i D_i R^{n_i} = 1 \tag{18}$$

where R is the constant correction factor and n is the rate of nonlinearity which is obtained experimentally. The fatigue damage is calculated by Eq. (19).

$$D = 1 = \sum \frac{n_i}{N_i} D_i = \sum \frac{t_i}{T_i} \tag{19}$$

where t_i and T_i are, respectively, the time of excitation and fatigue lifetime. There are several factors, which are responsible for nonlinear damage mechanisms. The effects of overloading and geometrical discontinuities are predominately studied in the existing literature [86–88]. The nonlinear nature of fatigue damage due to overloading and under loading creates irregular local stress amplitude near the crack tip. However, damage evolution due to overload is also dependent on the nature of the microstructure which can be improved by the heat treatment processes [87]. The geometrical discontinuities affect the fatigue life due to stress concentration. These discontinuities are inevitable in the structures mainly due to design requirements. However, researchers do relate their influence with microstructural voids, corrosion pits, riveting joints, wear and tear [89–91]. Therefore, these discontinuities can lead a nonlinear damage progression.

There are other approaches to fatigue life estimation modelling found in articles: thermal fracture modelling method for functionally grade plate [77], continuum mechanics modelling for fracture analysis [78] and intrinsic dissipation modelling for high-cycle fatigue life prediction [78]. Susmel and Taylor [92] attempted to use a critical distance method to estimate the finite life of notch components based on the modified Wöhler curve method under variable amplitude loading. In similar work of using a critical plane approach, Gates and Fatemi [93] studied the multiaxial amplitude fatigue life of un-notch specimens. The shortcoming of the critical distance method is averaging the local parameters which may theoretically not precisely measured. Other researchers studied the lifetime by the simulation method. Riedler et al. [94] simulated the lifetime of thermomechanical loaded components which investigated the transferability of the simulation models to the real components. Khan et al. [94–96] studied the remaining life prediction method based on the natural frequency. High-cycle and low-cycle fatigue life prediction model was also presented for Nickel-based single-crystal blade [97]. Researcher applied the thermal-fatigue life assessment on real applications [22, 39, 98–101]. Kamaya and Kawakubo [102] studied the effect of loading sequence on fatigue life. They found fatigue life is longer in the case of high-low loading than low–high loading sequences. Besides, their studied found crack mouth opening effect is the main cause in reducing fatigue life whereas stress intensification, damage interaction and damage sequence effect demonstrated less impact on fatigue life. Several life extension techniques have been proposed in the literature. They have made a fair impact on the existing fatigue life estimation approaches. Ayatollahi et al. [103] studied crack growth retardation technique by introducing a drilling hole along the crack flanks. Their numerical analysis results revealed that the presence of holes around the crack tip reduces the stress concentration and

the stress intensity factor. In other papers of same authors [104, 105], crack growth retardation was applied with the help of a stop hole at the crack tip. These investigations demonstrated an extension in the overall fatigue life. All these studies specify despite numerous concept of fatigue life estimation, it is hypothetically demanding to locate the perfect explanation and amplification.

4 Challenge-3: intricacy in fatigue crack propagation

A fatigue crack growth curve is a graphical representation of the stress intensity factor range and cracks growth rate [63, 64]. The region I describe the crack initiation which is termed as near-threshold zone below which the fatigue crack does not happen. Region II is the intermediary zone where there is linear and steady crack propagation occur appropriated for Paris and Walker models. The abnormal fatigue crack growth occurs in the region III due to the phenomenon of plasticity that leads it to the catastrophic failure. The crack growth rate rises rapidly as the maximum stress intensity factor reaches the fracture toughness, as shown in Fig. 1. However, the crack growth rate may not be same for all the materials.

The simplest form of fatigue crack growth analysis is those which are subjected to constant amplitude loading because no loading history has to be considered [64]. The complexity of the crack growth evaluation arises due to the variable amplitude loading where the existing loading interaction effects. Chowdhury et al. [64] proposed a framework to select the fatigue crack growth method for life cycle assessment. Their paper discussed the four fatigue crack models, their advantages and highlighted the model's limitations. Zang et al. [106] explored the
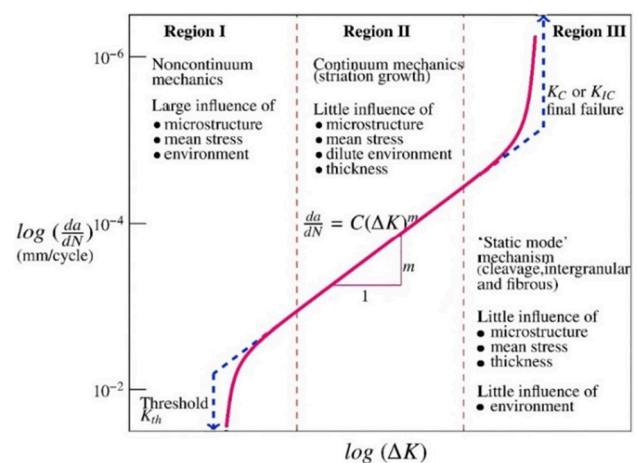

Fig. 1 Fatigue crack growth curve [64]

judgement criterion of the dominant factor of creep-fatigue. They considered the interaction of creep-fatigue at different intensity factor ranges at different temperatures with dwell time under various loading condition. Their studied highlighted that both time-dependent (creep-fatigue) and cycle-dependent (fatigue) involves in the crack growth process as shown in Eq. (20). The crack

growth rates of fatigue, creep and creep-fatigue interaction are presented in Fig. 2a–e. The crack growth rates were compared for 0-s with 90-s dwell time at temperatures 650 °C, 670 °C, 690 °C, 710 °C and 750 °C. The graphs indicated that fatigue item dominated the crack growth rates at lower temperatures and creep item has little influence in the whole stress intensity range. However,
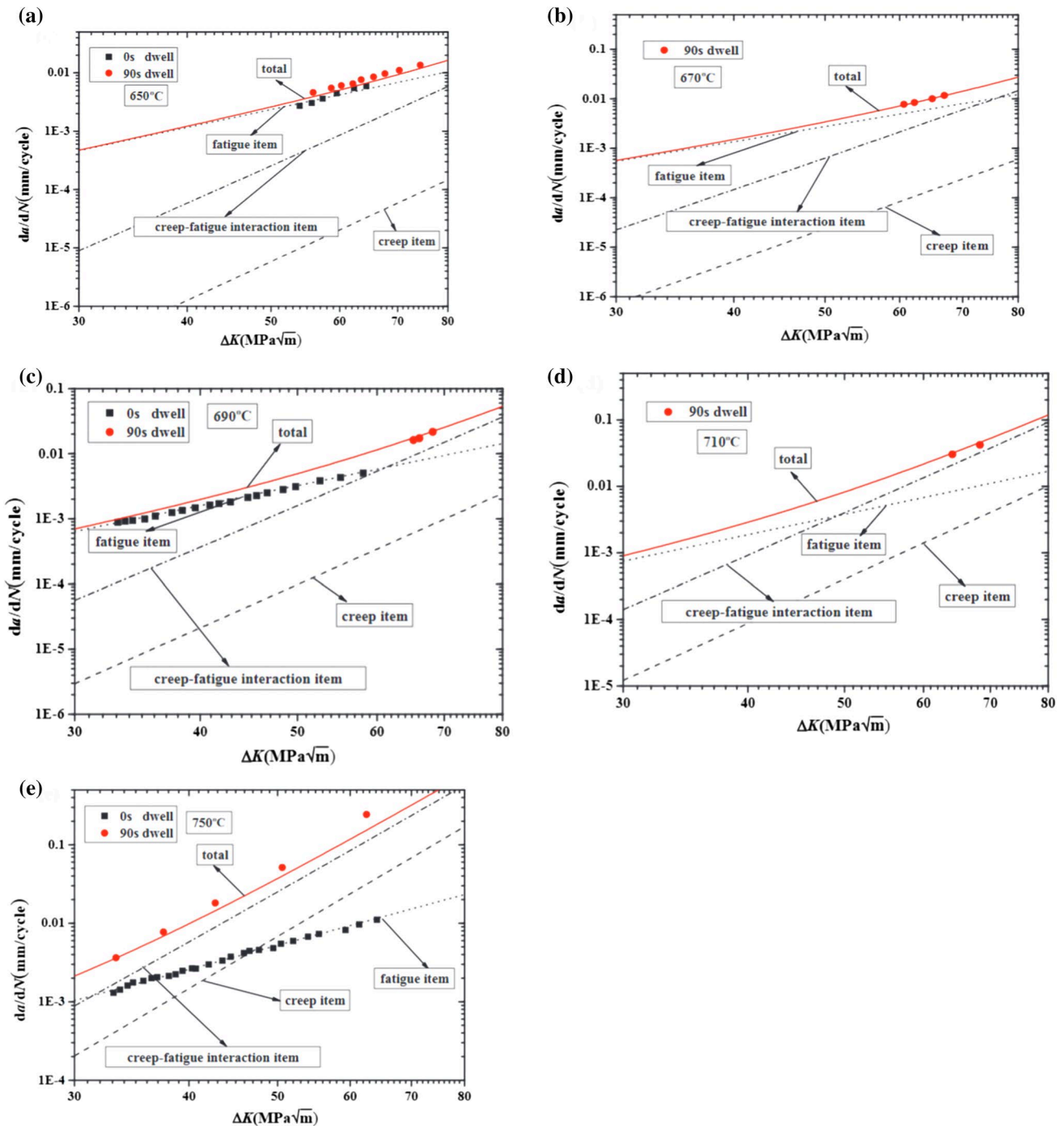
Fig. 2 Crack growth of creep-fatigue interaction for nickel-based superalloy at a **a** 650 °C, **b** 670 °C, **c** 690 °C, **d** 710 °C and e750 °C. with 90 s dwell time [106]
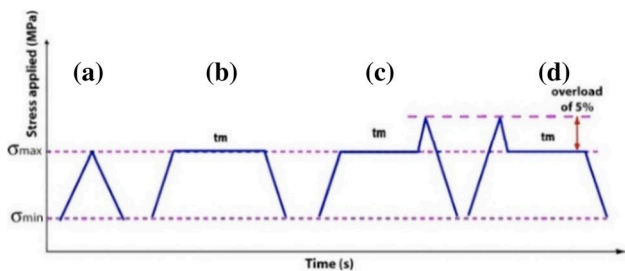
Fig. 3 Shape of analysed cycles **a** without hold time, **b** with a hold time, **c** and **d** in overloads test [110]

Fig. 4 Crack growth rate evolution as a function of crack length increases at a frequency of 5 Hz for AM1 single-crystal super alloy [110]

the creep item dominates the crack growth at higher temperatures as shown in Fig. 2e.

$$\frac{\mathrm{d}a}{\mathrm{d}N} = \left(\frac{\mathrm{d}a}{\mathrm{d}N}\right)_{\text{cycle - dependent}} + \left(\frac{\mathrm{d}a}{\mathrm{d}N}\right)_{\text{time - dependent}} \qquad (20)$$

Crack growth tests are normally performed in isothermal situations at a different temperature to illustrate the material behaviour [107]. Fatigue crack growth rate increases with the rise in temperature. Lu et al. [108] studied the hold-time effect of elevated temperature crack growth behaviour of strengthening super alloys. This research applied the fracture mechanics parameters to correlate the crack behaviour at a different temperatures. The result showed that the time-dependent creep damage mechanisms control the cracking path. Narashimhachary et al. [109] found that there is a difference in the stress intensity solutions for a rectangular edge-cracked plate emphasizing the importance of using correct boundary conditions. Bouvard et al. [110] formulated a phenomenological model to predict the crack growth in crystal super alloys at elevated temperatures. Fatigue and creep-fatigue tests were taken out to inspect the consequences of time on crack growth rates. A different form of loading conditions was performed to examine the crack growth process for (a) without hold time (b) hold time, (c) and (d) at overloads as shown in Fig. 3.

It is remarkable that the two regimes in Fig. 4 are also viewed from the fracture surface from scanning electron micrographs as shown in Fig. 5. The four zones are the pre-crack, short crack regime, long crack regime and failure zone. Sankararaman et al. [111] presented a method to quantify the uncertainty in fatigue crack growth prognosis which are applied to structure with complex geometry and subjected to variable amplitude multiaxial loading. The model modified the Paris law to include the retardation effects which is shown in Fig. 6. The sources of the uncertainty mentioned the physical variability like loading, initial flaw size and data uncertainty. They also
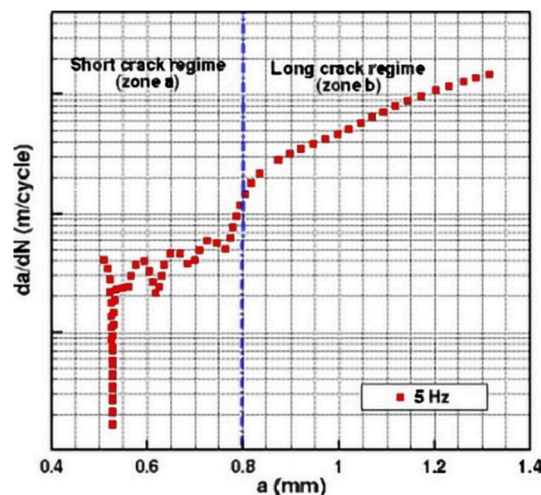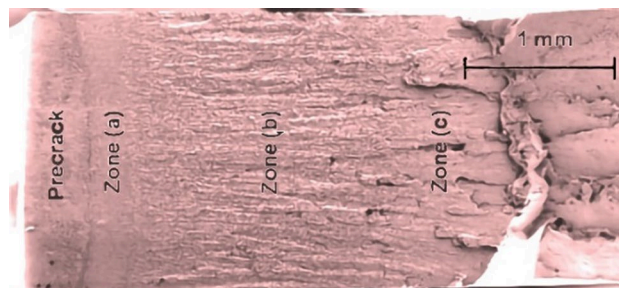

Fig. 5 Crack scanning electron micrographs of the fracture surface from the specimen tested at 5 Hz for AM1 single-crystal super alloy [110]
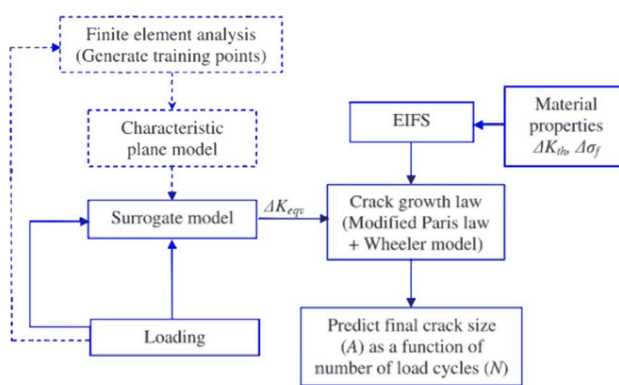

Fig. 6 Crack propagation analysis [111]

discussed model uncertainty parameters like crack growth law, calculation of stress intensity factor, etc.

Yao et al. [112] studied the effect of thermal-induced damage on the dynamic fracture toughness. The notch

specimen of two mortars were heat treated and tested with a split Hopkinson pressure bar (SHPB). X-ray computer tomography (CT) was used to quantify the thermally induced micro-crack and changes in chemical in terms of CT value. They observed that there was a decrease in fracture toughness due to the heat treatment. They found fracture damage is a combined effect of micro-crack and deterioration in binding property. Kidane and Shukla [113] also investigated the effect on a thermal and loading rate on the fracture toughness of Ti/TiB using a three-point bend specimen. Unlike Yao et al. [112], they used the modified SHPB apparatus with an induction coil as a heating system. The fracture toughness was found to be sensitive to temperature as shown in Fig. 7. Fracture initiation toughness was noticed to be strain rate sensitive and higher for dynamic loading as compared to the quasi-static conditions. Verlesen and Peirs [114] investigated the fracture behaviour of Ti6A114V under quasi-static and high strain rate conditions. The fracture behaviour was illustrated from Johnson–Cook damage initiation criterion combined with energy-based ductile damage law. The process of void nucleation and crack growth was unexpectedly culminated by strain localization. The strain rate effect was experienced caused by thermal softening which support the strain localization.

In past studies, fatigue crack growth was tested under the isothermal situation. Lansinger et al. [107] attempted to test the crack growth rate in the combined condition of thermal and mechanical loading. The tests used a temperature loading frequency of 1 Hz with cooling rates of 3 to 5 °C/min. They discovered that the combined test was in good agreement with the isothermal crack propagation at the same loading frequency. Estimation of fatigue crack growth was also performed in complex loading conditions [76]. You and Lee [115] evaluated the crack behaviour of steel in mixed-mode I and mo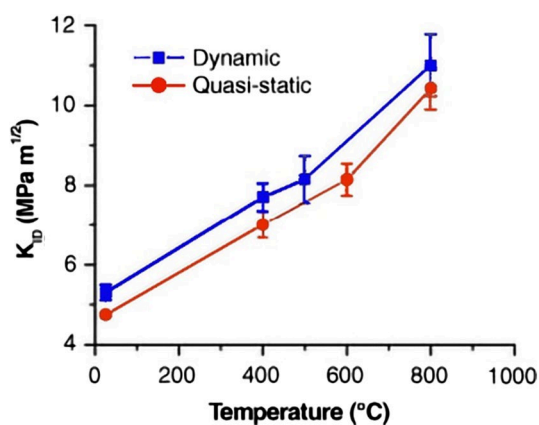de II loadings. Their studies revealed the accelerated crack growth by the addition of cyclic mode II. The result of mixed-mode crack growth was affected by the collaboration of plastic deformation zone, load ratio and increase in contact area on the rough crack surface.

Fatigue crack growth rate at elevated temperature has studied extensively in recent past. These studies concluded that the crack propagation is caused by several factors like environmental, stress-assisted grain boundary, oxygen embrittlement [116], stress intensity factor with the effect of frequency, load ratio and temperature [117, 118]. Wen et al. [119] analysed the finite deformation under small creep condition. The crack growth rate was discovered to be more sensitive to the finite element size under cyclic loading than monotonic loading conditions. Tvergaard and Needleman [120] studied the brittle–ductile transition in dynamic crack growth by considering the thermal softening and ductile failure. They established the transition from cleavage crack at a lower temperature to ductile cracked growth at a higher temperature. Nishioka et al. [121] investigated the effect of specimen size on dynamic crack propagation of double cantilever beam. The propagating crack was observed through high-speed photography. The investigation reported the values of crack arrest toughness were larger for shorter specimen than a longer specimen. Thus, it can be concluded that the complexities in estimating crack growth are due to the inconsistent crack propagation pattern.

5 Challenge-4: quantification of cracks

Quantification of cracks is the measurement of crack magnitude. There is no such single equation which satisfies different crack problems and their concerning boundary conditions. However, in most cases, the researcher attempted to quantify cracks by calculating the state of the stress/strain field, residual stress and the displacement data. In many situations, the stress intensity factor (SIF) is used to predict the stress state near the crack tip [122–125]. In dynamic loading, SIF is reinstated by the dynamic stress intensity factor (DSIF) [126–133]. In thermal loading condition, several literatures [123, 134–148] used thermal stress intensity factor (TSIF) to determine the thermal stress state. Kim et al. [122] stated that the stress intensity factor due to the thermal loads is the function not only to immediate boundary conditions but also of the preceding history of the boundary temperature. Stress intensity factor is nonetheless dependent on the state of stress/strain, displacement and temperature fields which are the potential factors to quantify cracks. Henceforth, quantifying cracks are perplexed especially in complex geometrical structures. Consequently, the study of cracks includes classification of remote stress, orientations of the obtained cracks and respective material properties.

Fig. 7 Effect of temperature on the fracture initiation toughness of FGM graded in the crack direction under quasi-static and dynamic loading [113]

Defining the stress intensity factor is vital to estimate accurate crack propagation. Different stress intensity factors are defined in the literature based on the boundary conditions. Tran et al. [149] conducted a modal analysis to compute stress intensity factor. Modal stress intensity factor was extracted from stationary crack and computed the dynamic stress intensity factor using the extended finite element method. In similar work, Galenne et al. [150] offered a modal approach to linear fracture mechanics. Modal stress intensity factor is extracted from either the crack-tip displacement method or by the energy method. The general equation for energy release rate G for the crack in dynamic loading was given as [149, 150].

$$G = -\frac{d}{dl}\left(W^{el}(u) + W^{kin}(\dot{u}) - P(u)\right) \tag{21}$$

where W^{el} is the elastic energy, W^{kin} is the kinetic energy, P is the external power forces, u is the displacement and $\dot{u}$ is the time derivative. Ni and Zhang [151] defined the dynamic stress intensity factor under the ultrasonic loading. They claimed to have the equivalent with the effective stress intensity factor values as in traditional fatigue tests. Calcada et al. [63, 152] developed an efficient crack analysis using modal stress intensity factor and their research methodology flowchart is shown in Fig. 8. The total stress intensity factor for dynamic loading is given in Eq. (22):

$$K_{total} = K_{sta} + K_{dyn} \tag{22}$$

where $K_{sta}(t) = C_n\sqrt{\pi a}.\sigma_{sta}$

$K_{dyn}(t) = C_n\sqrt{\pi a}.\sigma_{dyn}$

The total value of K through time is expressed as

$$K_{total}(t) = \begin{cases} K_{sta} + \sum_j K_j.Y_j(t) & \text{if} K_{sta} + \sum_j K_j.Y_j(t) \geq 0 \\ 0 & \text{if} K_{sta} + \sum_j K_j.Y_j(t) \leq 0 \end{cases} \tag{23}$$

Kabayashi and Ramulu [131] evaluated DSIF for unsymmetrical dynamic. Rokach and Łabędzki [128] proposed the method to determine the dynamic stress intensity factor for a four-point bend impact test. Wang et al. [129] established a dynamic stress intensity factor applying the interaction integral method. The integral method was executed without crack tip enrichment for both homogeneous and nonhomogeneous material. The relationship between interaction integral and stress intensity factor is given in equation below [129, 153, 154]:

$$I = \frac{2}{E_{tip}^*}\left(K_I K_I^{aux} + K_{II} K_{II}^{aux}\right) \tag{24}$$

where $E_{tip}^* = \begin{cases} E_{tip} & \text{for plane stress} \\ \frac{E_{tip}}{1-v_{tip}^2} & \text{for plane strain} \end{cases}$

Moreover, Huang et al. [125, 153] used the domain-independent interaction integral method to evaluate the DSIF for crack with nonhomogeneous materials. The relationship is given in Eq. (25).

$$I = \frac{2}{E_{tip}^* \cosh^2\left(\pi \in^{tip}\right)}\left(K_I K_I^{aux} + K_{II} K_{II}^{aux}\right) \tag{25}$$

Dynamic stress intensity factor is also derived through finite element and J-integral methods [130]. A formula was developed from the path-independent J-integral by taking the effect of inertia. Aloui et al. [127] studied the dynamic vibration behaviour of a cracked plate subjected to sinusoidal loading and analysed the stress field and dynamic stress intensity factor. Rokach [132] numerical investigation on the accuracy in the determination of dynamic stress intensity. Wen et al. [155] studied the dynamic behaviour of stationary cracks using dual boundary methods and determined the dynamic stress intensity factor through crack opening displacement.

Evaluation of crack in the thermal environment is vastly studied. Han and wang [156] investigated the thermal shock resistance of multiple cracking in functional graded material (FGM). Thermal shock resistance was determined based on the stress-based and fracture mechanics-based criteria. Zhang et al. [153] also examined the thermal shock resistance of FGM in a mixed-mode crack. The modelled considered both strength and fracture theories pondering the failure criteria of maximum local tensile stress, maximum thermal stress intensity and maximum hoops stress criterion. The transient temperature field for finite element equation is expressed in Eq. (26).

$$[K_1] + [K_2]\{T(x,y,t)\} + [K_3]\{\dot{T}(x,y,t)\} = \{P(x,y,t)\} \tag{26}$$

where $[K_1]$ is the heat conduction matrix, $[K_2]$ is the convection matrix and $[K_3]$ is the heat capacity matrix. Petrova and Schmauder [148] developed a mathematical modelling and evaluated the TSIF of FGM for an interface crack. The parametric analysis showed the dependence of the TSIF at the crack tip location and its orientation. Stress intensity factor under thermal stress was analysed for the interfacial crack of bimaterial [42, 121, 135]. The transient SIFs are observed to be a greater value than in the transient state. Nemat and Noda [135] analysed TSIF for functionally gradient material. They observed that TSIF decreased dramatically for selected nonhomogeneous parameters. Transient thermal stress intensity factor was investigated for a hollow cylinder with a circumferential crack [141, 142]. Xue et al. [141] evaluated based on the heat conduction model. It was

Fig. 8 Flowchart for the application of the proposed methodology [63]
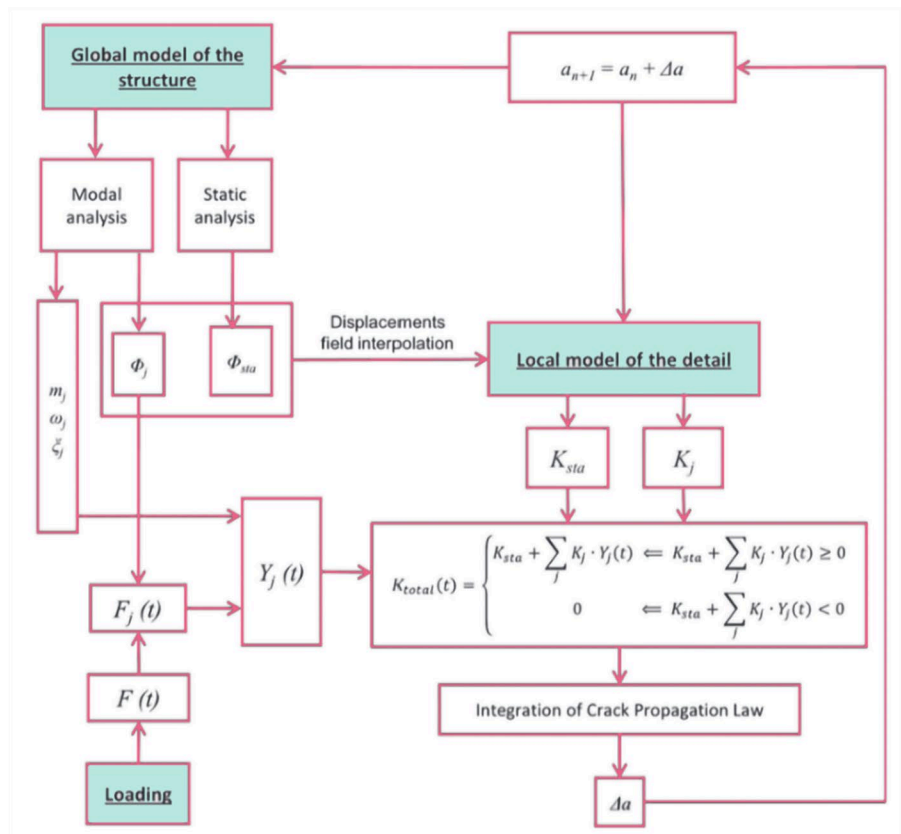
(a) With submodeling
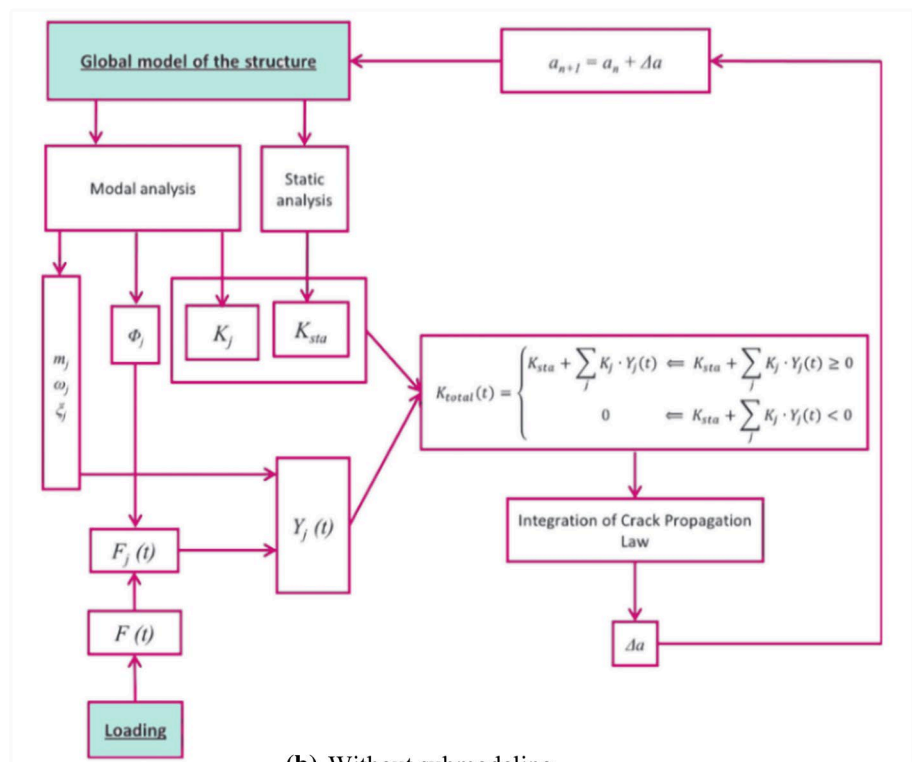
(b) Without submodeling

found that the thermal stress at the crack tip was different for different parameter values. Zhang and Li [142] studied the generalized fractional heat conduction and found a striking difference in thermal stress intensity factor with the Fourier heat conduction model.

Different approaches were used for calculating the thermal stress intensity factor for a cracked cylinder [123, 137, 143, 145, 146]. Zhong and Huang [134] observed that TSIF was not only dependent on applied thermal loading but also dependent on mechanical loading and physical properties of the crack interior. Tanigawa and Komatsubara [140] evaluated the thermal stress of a rectangular plate. Fracture behaviour of the crack was examined with a non-uniform heat source. Their studied explained the variation in stress intensity depending on the kind of parameters like mechanical boundary conditions, thickness of the plate, orientation of the crack, surface friction, etc. Lee et al. [138, 147] studied the TSIF for interface crack under uniform heat flow. The finding showed the stress intensity factor was strongly dependent on material properties. Analysis of thermal stress intensity factor was also studied using different methods and approaches [122, 139, 157, 158]. Assessment of mixed-mode stress intensity factor under the thermal and mechanical loading [159].

Le and Gardin [160] used the analytical approach based on stress intensity factor and weight function to study the crack shape and kinetics of crack propagation under the thermal cyclic loadings. The result showed the crack decelerated at the deepest point and accelerated at the surface due to the compression state in the centre and tension near the surface. Sankraraman et al. [161] examined the influence of an initial flaw size of the crack. The stress intensity factor expressed in terms of the initial flaw size of crack and loadings. The size of the crack opening was, however, dependent on the geometry [162]. Another type of crack is a breathing-fatigue crack as discussed by Yan et al. [163]. The researcher proposed the procedure to identify the prevailing breathing crack and computed the crack quantitatively.

Crack is also analysed in different schemes. Huan Chen [164] considered partially insulated crack. Using the hyperbolic heat conduction theory as per Eq. (27), thermal stresses and thermal effect on the crack were examined.

They expressed that the conduction parameters, thermal conductivity and geometry size have a greater influence on the stress of the crack surface and temperature gradient.

$$a \cdot \nabla^2 T = \frac{\partial T}{\partial t} + \tau \frac{\partial^2 T}{\partial t^2} \qquad (27)$$

where a is thermal diffusivity and ∇^2 is Laplace differential operator. Ekhlakov et al. [165] presented thermoelastic crack analysis in FGM. They observed that the influence of the

thermomechanical loading in DSIF was much lower than the material gradation. Moreover, DSIF is dependent on the crack angle and orientation with correspond to the material gradation. Lee et al. [166] and Zhang et al. [167] similarly analysed the exponential gradation of FGM to develop the thermomechanical stress and displacement.

Nicholas et al. [168] estimated the crack propagation as the function of the phase angle between load and temperature. Their research acknowledged the contribution of load position and time effects. John et al. [169] experimentally worked out TSIF generated by temperature gradient around the crack and hence confirmed the thermal load induces crack growth. Park et al. [170] in a similar way categorized the thermal fatigue of an underfill cracking by applying the thermal stress and calculated the TSIF using the crack length correlated to life cycles. Kim and Stone [171] studied the utility of integral parameters for crack propagation under thermomechanical loading. Their research reported that the parameters are path-independent for deformation. Chu et al. [172] examined the dynamic crack path in a brittle material. The aim of their studied was to evaluate the transient heat transfer, dynamic stress evolution under thermomechanical couple load. Their experiment in disk specimen showed that the crack branching is within the large heating radius; however, it goes outside the heating area in case of smaller radius. Maletta et al. [173] analysed the crack-tip thermal and mechanical hysteresis. They tested shape memory alloys subjected to fatigue loads. Using infrared thermography, they reported a sudden rise in surface temperature at the initial stage. The SIF versus load curves appeared a nonlinear hysteretic behaviour reflecting the complex stress–strain hysteretic behaviour of SMAs. Merhy et al. [174] studied the crack growth characterization of aluminium alloy. The contribution of parameters like temperature, frequency and SIF on crack growth was analysed. Crack growth increased at high temperature with a load ratio on decreasing frequencies. Temperature effect on crack growth was found dependent on the applied frequencies.

6 Challenge-5: stochastic response of the structure under thermomechanical environment

A structure exposed to cyclic loads for a reasonable time may change its global temperature. The changes in temperature impact the amount of stress moderation which in turn alters the deformation of the structures. Khan et al. [175] studied the outcomes of heat generation in viscoelastic material under cyclic loading. The cyclic mechanical loading was applied in a polymeric beam with or without heat conduction. They found that the higher stress point produces more heat which hastened the stress relaxation and triggered

thermal gradient in the polymer beam. In the case of the heat conduction, a uniform temperature preceded to the instability of the beam by diminishing the overall stiffness as shown in Fig. 9. El-Kafrawy [176] and Bovsunovsky and Surace [177] confirmed that the presence of a crack in beam or frame reduces the dynamic response. Hasbroucq et al. [178] studied the elastic–plastic response under the thermomechanical loading where the elastic properties vary with the temperature. The result showed that the residual stress and strain fields were time-dependent. Kidane and Shukla [179] experimentally investigated the dynamic behaviour of Ti/TiB functionally graded material exposed to thermomechanical loading. The effect of the temperature on dynamic response was studied by infrared spot heaters. The stress–strain curve at a different temperature is illustrated in Fig. 10. At high temperature, the materials displayed thermal relaxing and caused a reduction in compressive strength and increase in failure strain.

Thermal effect on vibrating structure has been widely investigated [12, 18, 25, 32, 61, 180–186]. Cao et al. [181] investigated numerically the thermal vibration and thermal contraction of single-wall carbon nanotubes (CNT). They explained that the effect of thermal vibration is more distinct than quasi-static thermal expansion which causes the thermal contraction in both the radial and axial directions when the temperature is well below 800 K. Zhang et al. [184] also investigated the thermal effect on the beam. Their work focuses on the high-frequency response in thermal exposure using the energy flow analysis method. The study highlighted the increase in vibration response of the beam with the increase in temperature. In supplement, two temperature-dependent material properties, elastic modulus and thermal expansion coefficient were considered to verify the thermal effect as shown in Fig. 11.
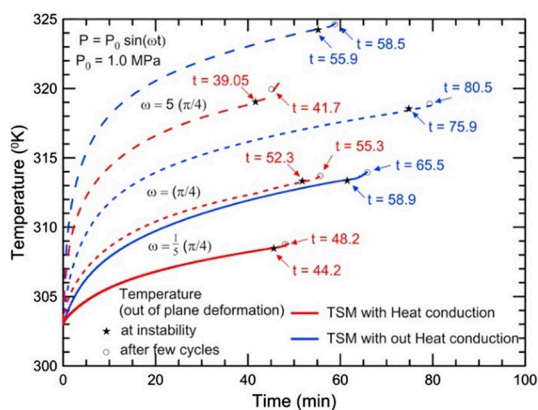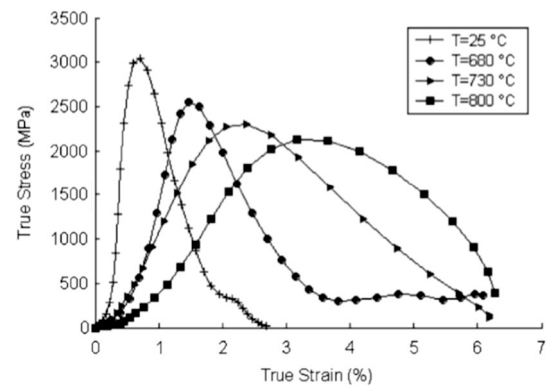

Fig. 10 True stress–strain curve as a function of time of Ti/TiB FGM [179]

Ebrahimi et al. [185] analysed the thermal effect on vibration behaviour of functionally graded beams with porosities. The influence of porosity volume fraction, material profile, temperature and boundary condition on frequencies was examined. The increase in porosity fraction increased the dimensionless frequencies. But at elevated temperatures, the increase in gradient index value decreased the frequency. The natural frequencies decreased with an increase in temperature and moving to zero at critical temperature. However, the frequencies increased afterward.

In another paper, Ebrahimi and Salari [186] studied the vibration behaviour of functional graded (FG) nanobeams under the effects of in-plane thermal loading. The parametric study showed that change in temperature, mode shape and boundary conditions influenced the fundamental frequencies. Liu et al. [187] investigated on temperature-dependent of material properties. They focused on thermal shock fracture in the principle of non-Fourier heat
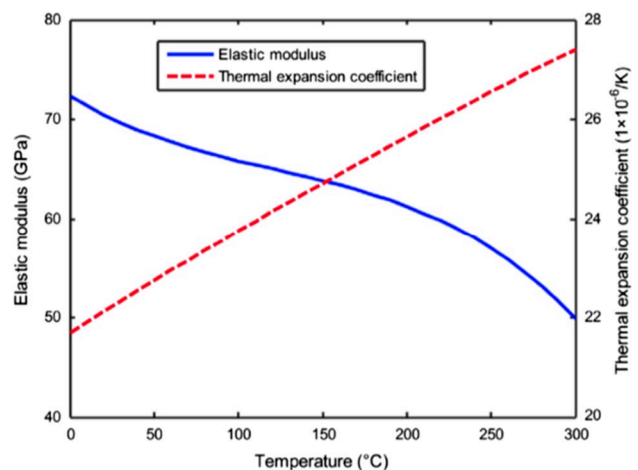

Fig. 9 Temperature increase at point 1 of the simply supported at 1.0 MPa with/without heat conduction at different frequencies of HDPE polymer [175]

Fig. 11 Temperature-dependent material properties for aluminium alloy [184]

conduction. The thermal stress intensity factor was found to be vastly receptive to the temperature-dependent material properties.

The variations of TSIF with shock time are presented in Fig. 12. Cui and Hu [180] studied on Euler–Bernoulli beam under uniform heating. They observed that there is a small influence of temperature-dependent material properties on the natural frequency at low temperature but developed greater influence at high temperature. Fazzolari [188] also analysed the modal characteristic of FG with temperature-dependent materials under thermal loading. They analysed the use of the higher-order equivalent single-plate theories to study the two-volume fractions: power-law function (P-FGM) and sigmoid function (S-FGM). The declining rate of dimensionless frequency of temperature-dependent FGM was found to be more than temperature-independent FGM at elevated temperatures. Silva et al. [189] performed the modal analysis at elevated temperatures. They studied the effect of non-uniform temperature distribution heated rectangular plate. The resonant frequency and displacement data were extracted by the pulse laser-digital image correlation method. The resonant frequencies were noticed to be higher in transverse heating while longitudinal heating did not have much effect. Finite element analysis suggested the results were an after-effect of the disparity in the curvature of the plate at elevated temperature in a steady state. In related studies, Kawamura et al. [190] performed numerical studies on nonhomogeneous material properties of a rectangular plate subjected to a varying temperature at one surface and other at zero temperature change. They observed the amplitude of deflection decreased with the decrease in young modulus.
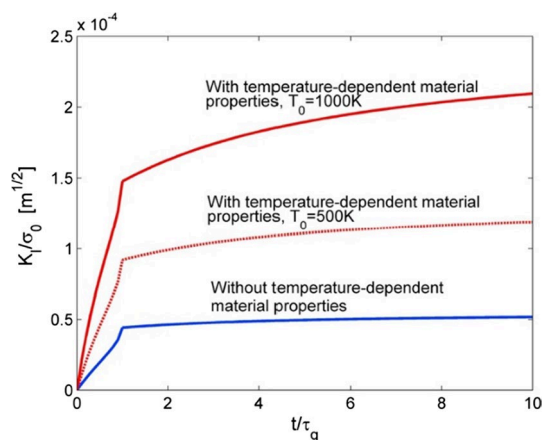

Fig. 12 TSIF for the surface crack of depth $d/l = 1$ [187]

7 Conclusion

This review emphasizes the existing research complexities in the area of fatigue and fracture modelling. Assessing fatigue damage accurately under vibration-based loads is still a challenge. Most of the available researches on thermomechanical vibrations are more or less focused on application-based studies. Researchers apply different theories to evaluate the characteristic of the complex geometry. However, modelling of thermomechanical vibrations become multifaceted when the geometry is complicated. Thus, the available research data for the fatigue damage assessment are still lacking of analytical reasoning and proper explanations concerning accurate fatigue measurement.

Complications in fatigue and fracture life estimation are another challenging obligation. The difficulties of fatigue life estimation are due to the inhibiting variables. Fatigue life is dependent on the loading histories, loading sequences, loading amplitude and frequency, operation environment, physical properties, etc. Different materials correspond to different fatigue life patterns allowing existing life estimation methods inadequate. A number of methods and techniques are available to predict the fatigue life. Nevertheless, the challenges are the precise measurement considering those variables which change independently according to the working conditions.

Crack propagation is not a new topic but still requires a comprehensive research to describe its governing mathematics with experimental justifications. Crack growth behaves in a pseudo-manner, especially in thermal condition. Many investigations have been complied about the nature of crack growth but fail to conclude with valid explanations. Some references, presented in this paper, were crack growth rate for supper alloys increases with the increase in temperature. Yet, there is no convincing reason why and how the crack will behave for different working conditions. It is learnt that fracture mechanics parameters correlated the crack behaviour at different temperature. In theory, crack growth and fracture behaviour at elevated temperature is still to address.

The thermal effect on dynamic response of structure has great concern in engineering applications. Structure expose to reasonable high temperatures can modify the material properties and increase crack propagation. This modification influences the overall deformation of the structure. The changes in temperature impact the amount of stress concentration thereby diminishing the global stiffness. Several studies were presented for structural vibration in above review. Moreover, thermomechanical vibration of structure relating to crack propagation and modal properties are still inadequate. It is well known that

the presence of crack reduces the dynamics response of structure. The existing studies are yet to input sufficient explanations about the dynamic behaviour of structure under distributed temperature across the structure. The structural stiffness can be influenced by material microstructure, temperature distribution and time of exposure. Different temperature can cause significant variations in the material properties. Existing studies are limited with respect to temperature change and compel extensive investigation in crack and uncrack conditions. The impact of thermal properties on structural response under varying temperatures needs to be investigated. Temperature-dependent properties like elastic modulus, thermal expansion and thermal conductivity changes with respect to temperature. The relationship of temperature-dependent properties with structural response in crack and uncrack conditions has not been established yet. There could be considerable changes in structural response due to vary in temperature-dependent properties. Dissimilar temperature distributions across the structure may also have diverse dynamic response trend. Subsequently, the modal properties will change corresponding to the changes in the temperature.

Nevertheless, a focus on developing accurate fatigue models, especially for complex geometries, is required in future. There are many factors still to be considered in modelling like overloading and geometrical discontinuities. The existing theories of fatigue and fracture also address without a proper consideration of temperature effect on structural dynamics. The change in temperature has impacted the mechanical properties which can relate to uncertain dynamics responses. Therefore, fatigue and fracture modelling under those criteria will be a good contribution in the existing academic literature.

Compliance with ethical standards

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