

Received April 16, 2020, accepted April 27, 2020, date of publication April 30, 2020, date of current version May 14, 2020.

Digital Object Identifier 10.1109/ACCESS.2020.2991590

# Condition-Based Maintenance Policy for Systems With a Non-Homogeneous Degradation Process

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This work was supported in part by the National Natural Science Foundation of China under Grant 61873096 and Grant 61673181, and in part by the Guangdong Basic and Applied Basic Research Foundation under Grant 2020A1515011057.

**ABSTRACT** This paper investigates a condition-based maintenance policy for systems subject to a non-homogeneous degradation process. A nonhomogeneous degradation process occurs as a result of deterioration nature and the environmental effect. In the first step, it develops two maintenance models, which consider the constant inspection interval and non-periodic inspection interval. The paper then optimizes a maintenance policy with monotone preventive replacement thresholds. The optimal maintenance decision is shown as a control-limit policy, where the optimal preventive replacement threshold is monotonically decreasing with system age. An illustrative example is presented to show the effectiveness of the proposed maintenance models. The result indicates that preventive replacement can significantly improve the effectiveness of the maintenance policy and sustain system operation.


**INDEX TERMS** Condition-based maintenance, non-homogeneous degradation, control-limit policy, preventive replacement, time-dependent drift.

## I. INTRODUCTION

With the advances of sensing technologies, system states now can be revealed at a much lower cost, which facilitates the development of condition-based maintenance (CBM). To date, CBM has received considerable attention owing to its effectiveness in preventing system failure and reducing operating cost [1]. As CBM utilizes in-situ information to estimate and predict the state of a system, maintenance actions are implemented only when necessary. The advantages of CBM over the traditional age-based preventive maintenance have been demonstrated in both academic research and industrial applications [2]–[4].

Research for CBM can be classified into two streams. One focuses on discrete deterioration processes, whereas the other on continuous degradation processes. The Markov chain is commonly used to model the discrete deterioration process [5]. Numerous studies have been conducted on Markov-chain-based degradation processes. For example, [6] proposed a hidden Markov model to assess the bearing performance. Reference [7] integrated random shocks into a physics-based Markov chain to assess

system reliability. However, the Markov chain model suffers several drawbacks. The state space of a Markov chain model is unable to fully describe the continuous degradation process and the classification of system states may be arbitrary. In such scenarios, using a continuous stochastic process to model the degradation process may be more appropriate. The development of sensing techniques facilitates the application of continuous degradation models in characterizing the physical deterioration [8]–[10]. Another advantage of using a continuous stochastic process to model the degradation process is that it provides the flexibility in describing the failure-generating mechanisms [11]–[13]. Stochastic processes such as Wiener process, Gamma process and inverse Gaussian process, are widely used in characterizing degradation processes, thanks to their feasible mathematical properties and clear physical interpretations. The property of independent degradation increments makes these stochastic models extremely attractive. Reference [8] developed a CBM model for a system subject to an inverse Gaussian process and unit heterogeneity. Reference [14] established an imperfect maintenance model for systems with a Wiener process. Reference [15] investigated the impact of unit heterogeneity on maintenance decisions with a Gamma process.

The associate editor coordinating the review of this manuscript and approving it for publication was Yu Liu .

Despite the popularity in degradation modeling, an implicit assumption of the aforementioned stochastic models is that the degradation process is homogeneous, which, however, fails to capture the influence of environment variations [16], [17]. Due to the varying environment, non-homogeneous degradation processes exist extensively in reality. Actually, for most systems such as civil infrastructures and airplanes, which are designed to operate for decades, the assumption that a system operates under a stationary environment is too restrictive and therefore unrealistic [18]–[21]. More often than not, the degradation process may accelerate as a result of cumulative damage/degradation. Reference [22] reported that the drift coefficient in an inertial navigation platform and fatigue-crack length in 2017-T4 aluminum alloy clearly show a nonlinear characteristic in the degradation process. Reference [23] observed a non-homogeneous deterioration process in large format lithium-ion batteries. Reference [24] observed that the degradation rate of a Micro-Electro Mechanical System (MEMS) increases rapidly after exposure to large shocks. More examples of the non-homogeneous degradation phenomenon can be found extensively in engineered systems, such as jet engine, machines in manufacturing system and wind turbines [25], [26].

For systems operating under dynamic environments, it is more appropriate to adopt a non-homogeneous stochastic process to model the degradation process. Reference [26] and [27] reported the use of the non-homogeneous Gamma process in degradation modeling and maintenance decision making. Reference [28] investigated the failure time of the non-stationary Gamma process. Reference [29] proposed an age-and state-dependent Markov model to describe non-stationary degradation processes, where the transition rate is determined by both the system age and system state.

Among the stochastic process-based models, the Wiener process with linear drift has received considerable attention in modeling the degradation process in recent years. The Wiener process has been widely used to model degradation processes for systems such as LED light, crack growth in railway track and immune system of human body, to name a few [30]–[32]. One of the advantages of the Wiener process with linear drift is that its first-passage-time (FPT) can be formulated analytically and follows an inverse Gaussian distribution [33]. However, for a non-homogeneous Wiener process with nonlinear drift, to obtain a closed form of the FPT is somewhat complicated [34]–[36]. From a mathematical and application point of view, two difficulties impede the use of the non-homogeneous Wiener process in degradation modeling. For one thing, the distribution of FPT is achieved by solving the Fokker-Planck-Kolmogorov (FPK) equation with absorbing barriers, which, however, is difficult or even impossible to obtain for a general non-homogeneous Wiener process [37]. For another, numerically calculating the FPT requires a massive computation time and memory storage space, which makes it unappealing for real-time decision making such as online fault diagnostics and prognostics [38].

With respect to CBM models for non-homogeneous degradation processes, the research is quite limited in literature. There are several studies that investigated CBM for systems subject to degradation processes and shocks, e.g., [39], [40]. But the degradation processes investigated in their works are different from ours in nature. We consider a non-homogeneous degradation process that the degradation rate is varying with time, while the others consider the combination of a homogeneous degradation and external shocks. Reference [41] developed a control limit maintenance policy for a system subject to non-stationary degradation, where the Markov chain model is employed to characterize the transition of system states. Reference [42] established a physics-based model to describe the degradation process of fatigue crack growth, where the system is subject to varying environment and uncertain condition monitor. Reference [43] developed an optimal replacement policy for a continuously degrading system subject to partially observed environments. A partially observable Markov decision process (POMDP) is formulated to obtain the optimal maintenance decision.

The aforementioned CBM models focus on a discrete degradation model or a homogeneous degradation model. Little research, however, has been devoted to CBM models that model non-homogeneous (non-stationary) degradation processes, despite its importance and necessity in practice. In this study, we aim to fulfill this gap by providing a CBM model for a non-homogeneously degraded system. The system is subject to continuous degradation, modeled by a Wiener process with time-dependent drift and diffusion. The system degradation level is revealed by inspection. At the first step, two CBM policies are developed with respect to the inspection interval. One is the widely-used inspection-replacement policy with periodic inspection, where the system is replaced when it is found failed at inspection. The other policy is implemented under decreasing inspection, where the inspection interval is geometrically decreasing to diminish the effect of increasing degradation rate. In addition, we develop a CBM model with monotone preventive replacement threshold. Optimal maintenance decisions are achieved by minimizing the long-run cost rate.

The remainder of this paper is organized as follows. Section II presents the non-homogeneous Wiener degradation process. Section III develops an inspection-replacement CBM policy, where CBM models with constant inspection intervals and geometrically decreasing inspection intervals are established respectively. Section IV develops a CBM model with monotone preventive replacement thresholds, where the optimal maintenance policy is shown as a control-limit policy. Section V offers an illustrative example. Finally, Section VI concludes the paper and proposes further research.

## II. NON-HOMOGENEOUS WIENER DEGRADATION PROCESS

Consider a system subject to a non-homogeneous Wiener degradation process. The system degradation level  $X(t)$

evolves according to the stochastic differential equation

$$dX(t) = \mu(t)dt + \sigma(t)dB(t) \quad (1)$$

where  $\mu(t)$  is the time-dependent drift coefficient,  $\sigma(t)$  is the time-dependent diffusion coefficient, and  $B(t)$  is the standard Brown motion with independent and identical increment,  $B(t) - B(s) \sim N(0, t - s)$ . Taking integral of Eq (1), the degradation level is given by

$$X(t) = x_0 + \int_0^t \mu(s)ds + \int_0^t \sigma(s)dB(s) \quad (2)$$

where  $x_0$  is the initial degradation level. For the non-homogeneous Wiener process, the independent increment property no longer holds, as opposed to the traditional Wiener process with linear drift. We consider the case that the degradation process is influenced by the operating environment. According to Nelson's Cumulative Exposure model, both the drift coefficient and the diffusion coefficient are impacted by the current environment but not the history of the degradation process [44]. Particularly, a proportionality relationship exists between  $\mu(t)$  and  $\sigma^2(t)$ ,  $\mu(t)/\sigma^2(t) = \gamma$  [34], [37], [45], where  $\gamma$  is a known parameter associated to a specific environment. Compared with the homogeneous Wiener process, the increment of degradation of Eq (1) is dependent on time  $t$ . Nonetheless, it is shown that the degradation process defined in Eq (1) is a Wiener process with a time-dependent mean value function and diffusion [45]. Let  $\Delta X(t) = X(t + \Delta t) - X(t)$ . Clearly from Eq (2),  $\Delta X(t)$  follows the normal distribution. The expectation and the variance of  $\Delta X(t)$  can be obtained by

$$E[\Delta X(t)] = \int_0^{t+\Delta t} \mu(s)ds - \int_0^t \mu(s)ds = M(t + \Delta t) - M(t)$$

and

$$Var[\Delta X(t)] = \int_t^{t+\Delta t} \sigma^2(s)ds = Z(t + \Delta t) - Z(t)$$

respectively, where  $M(t) = \int_0^t \mu(s)ds$  and  $Z(t) = \int_0^t \sigma^2(s)ds$ .

The system fails when the degradation level hits the failure threshold  $l$  for the first time. The FPT of the system is given as

$$T_f = \inf \{t : X(t) \geq l\} \quad (3)$$

If the system is subject to a stationary Wiener process, it is well known that the FPT follows an inverse Gaussian distribution [46]. For systems degrading with time-dependent drift and diffusion, the cumulative distribution function (cdf) of FPT is expressed as [45]

$$F_{T_f}(t|x_0) = \frac{1}{2} \left\{ 1 + erf(\eta_1) + \exp \left[ \frac{2(l - x_0)M(t)}{Z(t)} \right] (1 + erf(\eta_2)) \right\} \quad (4)$$

where

$$\eta_1 = \frac{x_0 + M(t) - l}{\sqrt{Z(t)}}$$

$$\eta_2 = \frac{x_0 - M(t) - l}{\sqrt{Z(t)}}$$

$erf(\cdot)$  is the error function with

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-s^2} ds$$

If the drift and diffusion coefficients are invariant of time (homogeneous Wiener process), then Eq (4) can be reduced to an inverse Gaussian distribution,

$$F_{T_f}(t|x_0) = \Phi \left( \frac{\mu t + x_0 - l}{\sigma \sqrt{t}} \right) + \exp \left( \frac{2\mu(l - x_0)}{\sigma^2} \right) \Phi \left( \frac{-\mu t + x_0 - l}{\sigma \sqrt{t}} \right) \quad (5)$$

where  $\Phi(\cdot)$  is the cdf of the standard normal distribution,  $\Phi(x) = 1/2 (1 + erf(x/\sqrt{2}))$ .

In reality, most systems may experience an accelerated degradation process, as a result of the external shocks and environmental effects. In other words, the mean degradation rate  $\mu(t)$  increases with time  $t$ ,  $d\mu(t)/dt > 0$ . In this paper, we focus on the accelerated degradation process and develop maintenance models accordingly.

### III. INSPECTION-REPLACEMENT POLICY

Continuous monitoring is often too costly or even impossible for some systems. Discrete inspection is therefore used here to reveal the degradation level of the system. We assume that the inspection is perfect in the sense that the degradation level can be accurately revealed by inspection. The system is failed when the degradation level exceeds the failure threshold. The system failure is not self-announcing and can only be discovered at inspection, which is referred to as a hidden failure [10], [47]–[49]. Denote  $\{\delta_i : i = 1, 2, \dots\}$  as the inspection epoch. At each inspection, if the system is failed, corrective replacement is carried out. The cost items include inspection cost  $c_i$ , corrective replacement cost  $c_r$  and downtime cost. The downtime cost is incurred during the period of system inactivity, from failure occurrence till the next replacement time, at a cost of  $c_d$  per unit time. Optimal maintenance models are achieved by minimizing the long-run cost rate. In the following, two maintenance models are formulated. One is with a constant inspection interval, where periodic inspection is implemented over the operation horizon. The other is with a geometric inspection interval, where the inspection intervals decrease geometrically.

#### A. CONSTANT INSPECTION INTERVAL

We first construct a maintenance model with constant inspection interval since periodic inspection is easy to implement for engineers and is most widely used in practice. The inspection epoch is denoted as  $\delta_i = i\delta$ , where  $\delta$  is a base inspection interval. At each inspection, corrective replacement is carried out if the system is failed; otherwise, the system is left as it be. The objective is to minimize the long-run cost rate by searching an optimal inspection interval  $\delta$ . As corrective replacement brings the system back to an as-good-as-new

state, the renewal cycle theorem can be used to calculate the long-run cost rate [50]. A renewal cycle is defined as the interval between two consecutive replacements or from system installation to the first replacement. Let  $C(t)$  be the total cost until time  $t$ . Based on the renewal cycle theorem, the long-run cost rate  $\kappa$  can be formulated as

$$\kappa(\delta) = \lim_{t \rightarrow \infty} \frac{C(t)}{t} = \frac{c_c + c_i E[N_I^{(\delta)}] + c_d E[T_d^{(\delta)}]}{E[\tau^{(\delta)}]} \quad (6)$$

where  $\tau^{(\delta)}$  is the length of a renewal cycle,  $N_I^{(\delta)}$  is the number of inspections in a renewal cycle, and  $T_d^{(\delta)}$  is the system downtime, under the maintenance strategy with constant inspection interval. In the following, we will suppress the subscript  $(\delta)$  for notational convenience. Clearly, the number of inspections in a renewal cycle,  $N_I$ , depends on the failure time and the inspection interval  $\delta$ . The probability of  $N_I = i$  is expressed by

$$P(N_I = i) = P((i-1)\delta < T_f < i\delta) = F_{T_f}(i\delta) - F_{T_f}((i-1)\delta) \quad (7)$$

The expected number of inspections in a renewal cycle can be readily obtained as

$$E[N_I] = \sum_{i=1}^{\infty} iP(N_I = i) = \sum_{i=1}^{\infty} i(F_{T_f}(i\delta) - F_{T_f}((i-1)\delta)) = \sum_{i=1}^{\infty} R(i\delta) \quad (8)$$

The expected system downtime can be achieved as

$$E[T_d] = \sum_{i=1}^{\infty} E[T_d | T_f = t] dF_{T_f}(t) dt = \sum_{i=1}^{\infty} \int_{(i-1)\delta}^{i\delta} (i\delta - t) dF_{T_f}(t) dt \quad (9)$$

As the system failure is not self-announcing, a renewal cycle always ends at inspection. Thus, the length of a renewal cycle is given as

$$E[\tau] = E[N_I] \delta = \delta \sum_{i=1}^{\infty} R(i\delta) \quad (10)$$

Integrating Eq (7-10), the long-run cost rate can be readily obtained as

$$\kappa(\delta) = \frac{c_i}{\delta} + \frac{c_c + c_d \sum_{i=1}^{\infty} \int_{(i-1)\delta}^{i\delta} (i\delta - t) dF_{T_f}(t) dt}{\delta \sum_{i=1}^{\infty} R(i\delta)} \quad (11)$$

The optimal maintenance policy is achieved by minimizing the long-run cost rate,

$$\delta^* = \arg \min_{\delta} \kappa(\delta) \quad (12)$$

## B. GEOMETRIC INSPECTION INTERVAL

Due to the non-homogeneity of the degradation process, maintenance policy with a constant inspection interval may not be so effective to prevent system failure and reduce operating cost. Instead, we focus on a maintenance model with varying inspection intervals. In this model, to balance the accelerated degradation process, the system is inspected with geometrically decreasing inspection intervals. The logic is as follows: as the system is subject to accelerated degradation, it is expected to inspect the system more frequently in later stage. Specifically, the  $i$ th inspection interval length is  $\alpha^{i-1}\delta$ , where  $\alpha$  is a scale parameter, scaling the decreasing rate of inspection intervals. Within a renewal cycle, for  $i = 1, 2, \dots$ , the inspection epoch is expressed by

$$\delta_i = \delta + \alpha\delta + \alpha^2\delta \dots + \alpha^{i-1}\delta = \frac{\delta(1-\alpha^i)}{1-\alpha} \quad (13)$$

Under the maintenance policy with a geometric inspection interval, the objective is to determine the optimal  $(\alpha, \delta)$ , so as to minimize the long-run cost rate. Based on the renewal cycle theorem, the long-run cost rate is given as

$$\begin{aligned} \kappa(\delta, \alpha) &= \frac{c_i}{\delta} \\ &+ \frac{c_c + c_d \sum_{i=1}^{\infty} \int_{\delta(1-\alpha^{i-1})/(1-\alpha)}^{\delta(1-\alpha^i)/(1-\alpha)} \left( \frac{\delta(1-\alpha^i)}{1-\alpha} - t \right) dF_{T_f}(t) dt}{\delta \sum_{i=1}^{\infty} \frac{(1-\alpha^i)}{1-\alpha} \left( F_{T_f} \left( \frac{\delta(1-\alpha^i)}{1-\alpha} \right) - F_{T_f} \left( \frac{\delta(1-\alpha^{i-1})}{1-\alpha} \right) \right)} \end{aligned} \quad (14)$$

Eq (14) can be obtained by integrating the expected number of inspection and downtime. Detailed derivation of Eq (14) is shown in the Appendix. The optimal solution is achieved by minimizing the long-run cost rate,

$$(\alpha^*, \delta^*) = \arg \min_{\alpha, \delta} \kappa(\alpha, \delta; 0 < \alpha < 1) \quad (15)$$

## IV. CBM POLICY WITH MONOTONE PREVENTIVE MAINTENANCE THRESHOLDS

In this section, we formulate the maintenance model into a Markov decision process framework, where the optimal maintenance policy is proved as a control-limit policy with monotone preventive maintenance thresholds.

For systems with an accelerated degradation process, it is natural to replace the system at a smaller degradation level with the increase of inspection numbers, so as to counteract the influence of degradation acceleration. This is because the mean degradation rate increases with time for an accelerated degradation process, which indicates an increasing degradation increment with the increase of inspection numbers. Therefore, with an increase of inspection numbers, the system should be replaced at a smaller degradation level to avoid failure. A maintenance policy with varying preventive replacement thresholds is proposed in this section. We focus on the control limit policy as it is most commonly used

in maintenance practice. The control limit policy works in such a way that the system is preventively replaced whenever the inspected degradation level exceeds a preventive replacement threshold [21], [51], [52]. Although both corrective replacement and preventive replacement restore the system to the as-good-as-new state, they differ in nature as corrective replacement is often implemented at an unplanned time and at a more deteriorated state; as a result, additional cost may be incurred for the potential damages caused by failure. The cost items include inspection cost  $c_i$ , preventive replacement cost  $c_p$ , corrective replacement cost  $c_r$  and downtime cost. Obviously we have  $c_i < c_p < c_r$ . While implementing a periodic inspection to detect the degradation level, the optimal policy can be formulated into a Markov decision process. Denote  $(k, X_k)$  as the system state for maintenance decision, where  $k = 0, 1, 2 \dots$  represents the inspection epoch, and  $X_k$  is the observed degradation level at the  $k$ th inspection. A maintenance policy is defined as a set of actions  $a : B \rightarrow A$ , where  $B$  and  $A$  are the sets of system state before and after maintenance actions, respectively,  $a(k, X_k)$  is the action taken in state  $(k, X_k) \in B$ .  $a(k, X_k) = 1$  denotes a preventive replacement, whereas  $a(k, X_k) = 0$  denotes that the system is left as it be. The long-run cost rate  $\kappa$  is expressed as

$$\kappa = \inf_{a, \delta} E \left[ \lim_{K \rightarrow \infty} \frac{1}{K\delta} \sum_{k=1}^K c_i + c_p I \{a(k, X_k) = 1\} + (c_c + c_d T_d) I \{a(k, X_k) = 0, G(k, X_k) = 1\} \right] \quad (16)$$

where  $G(k, X_k) = 1$  denotes the event that the system fails between the  $k$ th and  $(k + 1)$ th inspections, starting with the state  $(k, X_k)$ .

Following the procedure of Markov decision process, the optimality equation can be formulated as

$$V(k, X_k) = \begin{cases} \min\{c_p + V(0, 0), \\ W(k, X_k) + E[V(k + 1, X_{k+1})]\}, & X_k < l \\ c_c + V(0, 0), & X_k \geq l \end{cases} \quad (17)$$

where  $V(k, X_k)$  is the value function starting with the state  $(k, X_k)$ ,  $W(k, X_k) = E[c_d T_d | (k, X_k)]$  is the expected downtime cost incurred during period  $(k\delta, (k + 1)\delta)$ , and  $E[V(k + 1, X_{k+1})]$  is the expected cost-to-go after the  $k$ th inspection. For notational simplicity, we suppress the dependence of  $W(k, X_k)$  and  $E[V(k + 1, X_{k+1})]$  on the current state  $(k, X_k)$ . The rationale behind the optimality equation is that if the system has failed at inspection ( $X_k \geq l$ ), then corrective replacement is carried out. Otherwise, the system is either preventively replaced or left as it be, depending on which is more cost-effective. The value function  $V(k, x_k)$  denotes the expected cumulative maintenance cost at the  $k$ th inspection epoch given the system state  $x_k$ .  $V(0, 0)$  denotes the maintenance cost given the system state 0 at the beginning the of the operation period. In other word,  $V(0, 0)$  stands for the expected maintenance cost when the initial system state

is 0. Note that the inspection cost is considered separately, which is not incorporated in Eq (17). Actually, with the inspection interval  $\delta$ , the cost rate due to inspection can be denoted as  $c_i/\delta$ .

In the following, we will investigate the stochastic property of the degradation process, which paves the way to obtain the optimal maintenance policy. The insights are summarized in Lemma 1 and Lemma 2.

*Lemma 1:*  $\langle X_{k+1} | X_k \rangle$  is stochastically non-decreasing in  $X_k$  for all  $k$ , i.e.,  $\langle X_{k+1} | X_k = x_1 \rangle < \langle X_{k+1} | X_k = x_2 \rangle$ , for  $x_1 < x_2$ .

*Lemma 2:*  $\langle X_{k+1} | X_k \rangle$  is stochastically non-decreasing in  $k$  for all  $X_k$ , i.e.,  $\langle X_{k_1+1} | X_{k_1} = x \rangle < \langle X_{k_2+1} | X_{k_2} = x \rangle$ , for  $k_1 < k_2$ .

Detailed proofs of lemmas and the theorems of this paper are provided in the Appendix. Lemma 1 implies that system state tends to be larger with a larger observed degradation level in a stochastic sense. Lemma 2 indicates that for an accelerated degradation process, the system is likely to degrade faster at later stage, with larger degradation increments. On the basis of Lemma 1 and 2, the following result in terms of the expected downtime cost  $W(k, X_k)$  can be obtained.

*Lemma 3:* The expected downtime cost  $W(k, X_k)$  is non-decreasing in  $k$  and  $X_k$ .

Lemma 3 is intuitive. The expected downtime cost is determined by the conditional failure probability at current system state. If the system has a larger observed degradation level, it is likely to fail at an earlier time before the next inspection. In addition, given the system state at inspection, the system tends to fail earlier at later stage, due to the acceleration of the degradation process. The monotone property of Lemma 3 directly leads to the following theorem.

*Theorem 1:* The value function,  $V(k, X_k)$ , is a non-decreasing function in inspection time  $k$  and degradation level  $X_k$ .

Theorem 1 establishes the foundation to explore the structural property of the optimal maintenance policy. In the following, we will show that for an accelerated degradation process, the optimal maintenance policy is a non-increasing monotone control limit policy.

*Theorem 2:* At any inspection epoch  $k$ , the optimal maintenance policy is to replace the system when the degradation level exceeds the control limit  $\zeta_k^*$ . Equivalently,  $a(k, X_k) = 1$  if  $X_k > \zeta_k^*$ ; otherwise  $a(k, X_k) = 0$ . In addition,  $\zeta_k^*$  is non-increasing in  $k$ .

Theorem 2 shows that for an accelerated degradation process, the preventive maintenance threshold is monotonically decreasing (non-increasing) with the inspection number  $k$ , and vice versa for a decelerated degradation process. The monotone non-increasing control limit policy is intuitive. Since the degradation process accelerates with the elapsed time, maintenance policy should be more conservative so as to reduce the failure risk. The optimal maintenance strategy can be achieved via existing algorithms such as policy iteration or value iteration policy [53]. The structural property

of the monotone control limit can be employed to reduce the computational burden for systems with large state space. In this paper policy iteration is adopted to compute the value function and determine the optimal maintenance policy. In addition, the continuous degradation process is discretized before performing the algorithm.

Now that we are able to obtain the optimal maintenance decisions for any given inspection interval  $\delta$ , we proceed to minimize the long-run cost rate for varying  $\delta$ . From Eq (17), the optimal inspection interval  $\delta^*$  can be achieved as

$$\delta^* = \arg \min_{\delta} \frac{c_i}{\delta} + E \left[ \lim_{K \rightarrow \infty} \frac{1}{K\delta} \sum_{k=1}^K c_p I \{a(k, X_k) = 1\} + (c_c + c_d T_d) I \{a(k, X_k) = 0, G(k, X_k) = 1\} \right] \quad (18)$$

which turns out to be a one-dimensional optimization problem. A simple algorithm such as exhaustive search can be adopted to obtain the  $\delta^*$ . Denote

$$H(\delta) = E_a \left[ \lim_{K \rightarrow \infty} \frac{1}{K\delta} \sum_{k=1}^K c_p I \{a(k, X_k) = 1\} + (c_c + c_d T_d) I \{a(k, X_k) = 0, G(k, X_k) = 1\} \right]$$

If  $\delta$  approaches infinity, then no maintenance action is implemented, which implies that the long-run cost rate  $\frac{c_i}{\delta} \rightarrow 0$ . On the other hand, if  $\delta$  approaches 0, then the system turns out to be under continuous monitoring, where unexpected failures can always be prevented. The long-run cost rate is computed within a horizon that is long enough to approximate the infinite horizon. It may not be easy to explicitly determine the horizon that is long enough. That depends on the parameters of the degradation process and maintenance actions. We believe that a horizon is long enough when enlarging the horizon has little influence on the maintenance cost.

*Remark 1:* Generally, we cannot have the monotone thresholds under geometric inspection. The monotone threshold derives from the monotonicity of the value function at inspections, which is due to the acceleration or deceleration of the degradation process. However, when the inspection interval is varying, we fail to guarantee a monotone value function. We can have the geometric inspection and the monotone thresholds under the cases (1) accelerated degradation process and increasing inspection interval; (2) decelerated degradation process and decreasing inspection interval. A decelerated process may not be so common as an accelerated degradation process for real systems. However, some systems have been reported to present a decelerated degradation process, e.g., a surfactant-extracted MCM-41-type mesoporous silica [54].

## V. AN ILLUSTRATIVE EXAMPLE

This section illustrates the performance of the proposed maintenance models. The degradation process of the system is

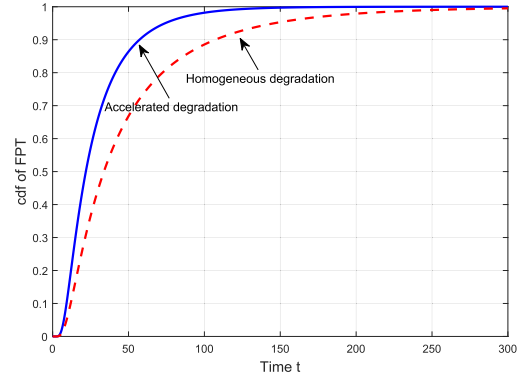


FIGURE 1. cdf of FPT for accelerated and homogeneous degradation process.

assumed to follow the Wiener process with power-law time-dependent drift and diffusion coefficient. Let  $\mu(t) = b_{\mu} t^{\beta}$  and  $\sigma^2(t) = b_{\sigma} t^{\beta}$ , where  $\beta$  is a known constant, depending on the operating environment. The parameters are  $b_{\mu} = 0.1$ ,  $b_{\sigma} = 0.1$  and  $\beta$ , so that we have  $\mu(t) = 0.1t^{\beta}$ ,  $\sigma^2(t) = 0.5t^{\beta}$ , the ratio  $\gamma = 0.2$ , the mean of the degradation process  $M(t) = \int_0^t \mu(s) ds = 1/12t^{1.2}$  and  $Z(t) = \int_0^t \sigma^2(s) ds = 5/12t^{1.2}$ . The initial degradation level is  $x_0 = 0$ . The system fails when its degradation level reaches the failure threshold  $l = 5$ . According to Eq (4), the cdf of FPT is given as

$$F_{T_f}(t|x_0) = \Phi \left( \frac{1/12t^{1.2} - 5}{\sqrt{5/12t^{1.2}}} \right) + e^2 \cdot \Phi \left( \frac{-1/12t^{1.2} - 5}{\sqrt{5/12t^{1.2}}} \right) \quad (19)$$

By differentiating Eq (19), the probability density function (pdf) of FPT can be obtained as

$$f_{T_f}(t|x_0) = \frac{5 \times 1.2^{3/2}}{\sqrt{\pi} t^{1.6}} \exp \left[ -\frac{(-1/12t^{1.2} + 5)^2}{5/6t^{1.2}} \right] \quad (20)$$

To investigate how the failure probability varies with the operating time, we plot the cdf and pdf of FPT, as is shown in Fig. 1 and Fig. 2. In addition, the cdf of the homogeneous degradation process is shown in Fig. 1 for comparison, with the parameters  $\mu(t) = 0.1$  and  $\sigma^2(t) = 0.5$ . It appears that the system with an accelerated degradation process always has a higher cdf, which implies that the system with accelerated degradation is more likely to fail.

A sensitivity analysis is conducted to investigate the impact of parameters on  $F_{T_f}(t|x_0)$  and  $f_{T_f}(t|x_0)$ . We are interested in the effect of  $\beta$  as it dominates the nonlinearity of the degradation process. Fig. 3 and Fig.4 show the results of the sensitivity analysis. Obviously,  $\beta$  has a significant impact on the cdf and pdf of FPT. When  $\beta$  increases from 0.1 to 1, the time for  $F_{T_f}$  to reach 1 decreases from 200 to 20 approximately. The results indicate that an increased  $\beta$  will significantly accelerate the degradation process and reduce system lifetime. As such, reliability engineers and managers are suggested to pay more effort to determine an accurate value of  $\beta$ .

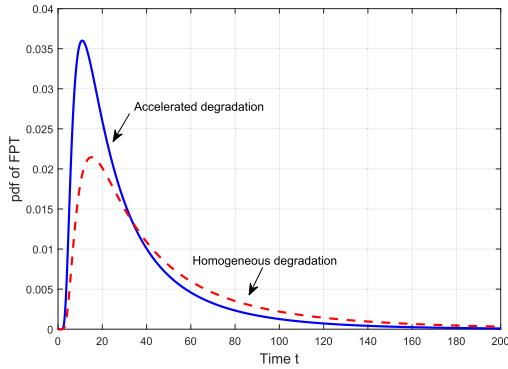


FIGURE 2. pdf of FPT for accelerated and homogeneous degradation process.

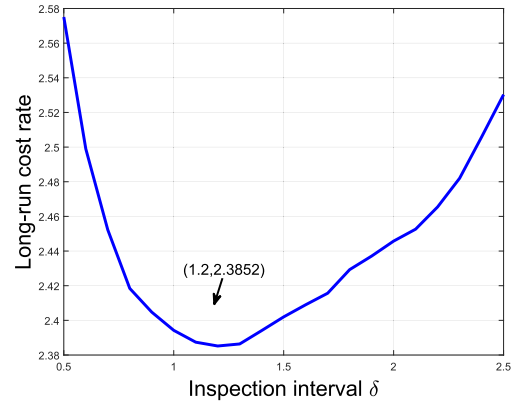


FIGURE 5. Long-run cost rate with respect to inspection interval.

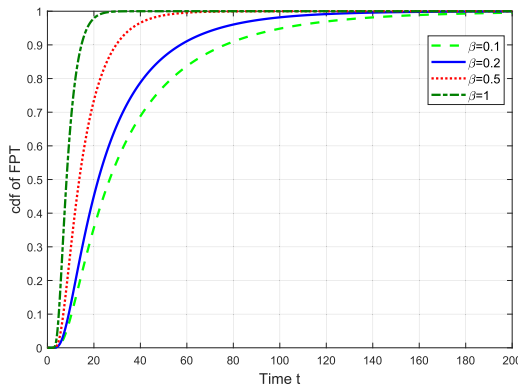


FIGURE 3. Sensitivity analysis of cdf on  $\beta$ .

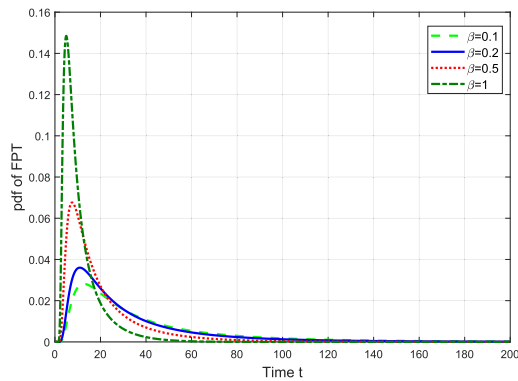


FIGURE 4. Sensitivity analysis of pdf on  $\beta$ .

**A. INSPECTION-REPLACEMENT CBM POLICY**

With the replacement model, the system is subject to discrete inspection, either periodically or non-periodically. The inspection cost is  $c_i = 0.3$ . Corrective replacement is carried out when the system is found failed at inspection; the corrective replacement cost is  $c_r = 10$ . During system inactivity, the system is charged with downtime cost per unit time,  $c_d = 5$ . For the case where periodic inspection is implemented, the long-run cost rate is plotted in Fig. 5. The optimal maintenance policy is achieved as  $\kappa^* = 2.3852$ , at the inspection interval  $\delta^* = 1.2$ .

In terms of the maintenance model with geometric inspection, Table 1 shows how the long-run cost rate varies with the base inspection interval  $\delta$  and the scale parameter  $\alpha$ . The optimal long-run cost rate is obtained as  $\kappa^* = 2.3852$  at  $\delta = 6$  and  $\alpha = 0.25$ . The optimal long-run cost rate achieved with geometric inspection is smaller than that of constant inspection interval, which implies the advantage of geometric inspection. On the other hand, as can be observed, the base inspection is much larger than the constant inspection interval, which indicates that inspection is unnecessary in the early state of system degradation. Actually, the constant inspection policy can be viewed as a special case of the geometric inspection policy, which can be easily achieved by letting  $\alpha = 0.25$ .

It can be found out that under geometric inspection, the inspection interval diminishes quickly to zero, while for periodic inspection the inspection interval remains at  $\alpha = 1.2$ . This is due to the nature of accelerated degradation. At the early stage of the degradation process, the system will not fail (or fail with an extremely small probability). Therefore, it is unnecessary to inspect the system at the early stage. However, with the increase of the operating time, the system will degrade fast (due to the acceleration effect), and more frequent inspections are expected to detect the failure. With the two parameters,  $\alpha$  and  $\alpha$ , geometric inspection is able to capture the degradation characteristics at the early and later stage. A large  $\alpha$  indicates that no inspection is implemented upon the system at the early stage, and a small  $\alpha$  implies that the system is frequently inspected at the later stage. By contrast, the periodic inspection fails to distinguish the difference of degradation, since it only has one parameter, the inspection interval  $\delta$ , which leads to a moderate inspection interval ( $\delta = 1.2$  in this example) to balance the degradation process at the early and later stage.

In addition, for illustration purpose, in Fig. 6 and Fig. 7, we plot the optimal long-run cost rate  $\kappa$  and the optimal scale parameter  $\alpha$ , with respect to different inspection intervals  $\delta$ . As shown in Fig. 6, the optimal long-run cost rate exhibits a unimodal trend with  $\delta$ . Fig 7 shows that the optimal scale parameter  $\alpha$  is decreasing with the inspection interval, which

TABLE 1. Long-run cost rate with geometric inspection.

		$\alpha$						
		0.1	0.15	0.2	0.25	0.3	0.35	0.4
$\delta$	5	11.7145	8.826	5.6537	3.3256	2.4865	2.4719	2.5682
	5.5	8.1602	5.2632	3.2373	2.4613	2.422	2.4965	2.5759
	6	5.2667	3.2585	2.4548	<b>2.3802</b>	2.4723	2.5291	2.6001
	6.5	3.4237	2.5504	2.3898	2.4626	2.5533	2.6383	2.7144
	7	2.6794	2.4503	2.506	2.6069	2.6866	2.7622	2.8553
	7.5	2.533	2.5517	2.6372	2.7352	2.8236	2.9134	3.0089
	8	2.5978	2.6846	2.7811	2.8751	2.9546	3.0491	3.1184

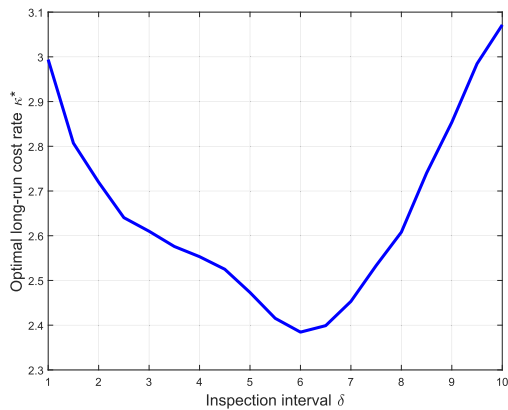


FIGURE 6. Optimal long-run cost rate with different  $\delta$ .

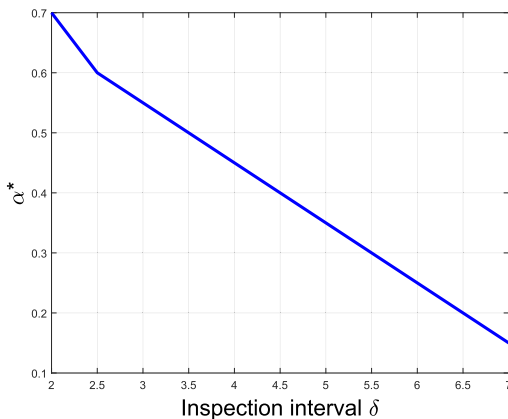


FIGURE 7. Optimal scale parameter  $\alpha$  with different  $\delta$ .

is due to the fact that the optimal decision is achieved by balancing the failure probability and the inspection frequency. A large inspection interval increases the probability of system failure and the downtime cost, while a small inspection interval increases the inspection cost.

**B. MAINTENANCE POLICY WITH MONOTONE PREVENTIVE MAINTENANCE THRESHOLDS**

For the maintenance policy with monotone preventive maintenance thresholds, the system is preventively replaced if the degradation level at inspection exceeds specific thresholds; the preventive replacement cost is  $c_p = 4$ . The optimal long-run cost rate is obtained as  $\kappa^* = 1.21$ , at the optimal inspection interval  $\delta^* = 2$ . Fig 8 presents the optimal preventive maintenance threshold with system age.

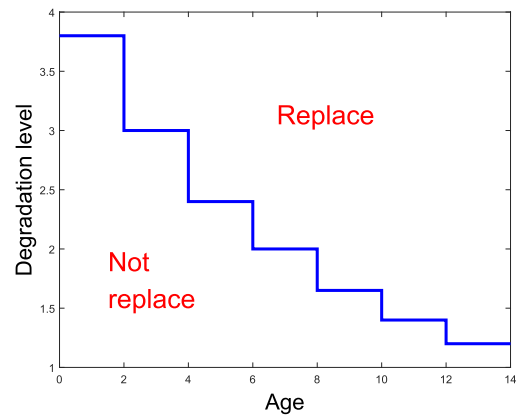


FIGURE 8. Optimal preventive maintenance threshold with system age.

We are interested to find out, compared with constant preventive replacement threshold, how much improvement can be achieved with the monotone preventive replacement thresholds. In Table 2, we show how the long-run cost rate varies with the inspection interval  $\delta$  and the preventive replacement threshold  $\zeta$ . It can be observed that, for CBM policy with constant preventive replacement threshold, the optimal maintenance decision is achieved at  $\delta^* = 1.7$  and  $\zeta^* = 1.6$ , with the long-run cost rate as  $\kappa^* = 1.35$ .

As can be observed, for CBM with constant preventive maintenance threshold, under the periodic inspection policy  $\delta^* = 1.7$ , the optimal preventive replacement threshold is a constant  $\zeta^* = 1.6$ , which is much smaller than the failure threshold  $l = 5$ . Part of the reason is due to the acceleration of the degradation process. In the presence of accelerated degradation, the degradation level will increase fast and may lead to system failure between two inspections. Therefore, a small preventive maintenance threshold is expected to prevent the system from failure. Another reason is that the cost of preventive replacement is much small than the corrective replacement cost and the downtime cost. For a small preventive replacement cost, it is economic beneficial to carry out preventive replacement frequently to avoid system failure, which lead to a small preventive maintenance threshold.

From the previous discussions, we can conclude that, among the three proposed maintenance polities, the preventive maintenance policy with monotone thresholds is the optimal one. In addition, the long-run cost rate is significantly reduced in presence of preventive maintenance, which implies the necessity of preventive maintenance for a system with a



TABLE 2. Variation of long-run cost rate with  $\delta$  and  $\zeta$ .

	$\zeta$										
	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3
1	1.69	1.56	1.46	1.42	1.39	1.38	1.39	1.41	1.44	1.48	1.51
1.1	1.65	1.52	1.44	1.39	1.36	1.37	1.38	1.4	1.44	1.47	1.51
1.2	1.61	1.49	1.42	1.38	1.37	1.37	1.39	1.42	1.45	1.49	1.52
1.3	1.56	1.46	1.4	1.37	1.36	1.37	1.39	1.41	1.45	1.48	1.52
1.4	1.53	1.44	1.39	1.37	1.36	1.37	1.4	1.43	1.46	1.5	1.54
$\delta$ 1.5	1.51	1.43	1.39	1.37	1.37	1.39	1.41	1.44	1.48	1.51	1.55
1.6	1.47	1.41	1.37	1.36	1.36	1.38	1.41	1.44	1.48	1.51	1.55
1.7	1.44	1.39	1.36	<b>1.35</b>	1.36	1.37	1.4	1.44	1.47	1.5	1.54
1.8	1.44	1.4	1.38	1.38	1.39	1.41	1.44	1.48	1.52	1.55	1.59
1.9	1.42	1.4	1.38	1.38	1.4	1.42	1.45	1.48	1.52	1.56	1.6
2	1.41	1.39	1.39	1.4	1.41	1.44	1.47	1.5	1.54	1.58	1.61

non-homogeneous degradation process. However, it is time consuming to obtain the optimal decision of the CBM policy with monotone preventive replacement thresholds. In the case where computational time is the major concern, a CBM policy with constant preventive replacement thresholds is a suitable option.

VI. CONCLUSION

In this paper, we investigate the CBM policies for systems with a non-homogeneous degradation process. In particular, three maintenance polices are proposed to minimize the long-run cost rate, among which the preventive maintenance policy with monotone thresholds provides the optimal decision. The results indicate that preventive replacement is of vital importance in reducing maintenance cost and sustaining system operation. Maintenance engineers and managers are suggested to pay more attention to the preventive replacement threshold and the inspection interval.

In future study, we can extend the geometric inspection policy to a condition-based inspection policy, where the next inspection time is scheduled by the observed degradation level and system age. In addition, a more sophisticated degradation process can be addressed, which relaxes the proportional relationship between the mean and the variance of the degradation process.

APPENDIX

A. DERIVATION OF EQ (14)

The probability that there are  $i$  inspections in a renewal cycle is given by

$$P(N_I = i) = F_{T_f}(\delta_i) - F_{T_f}(\delta_{i-1}) = F_{T_f}\left(\frac{\delta(1-\alpha^i)}{1-\alpha}\right) - F_{T_f}\left(\frac{\delta(1-\alpha^{i-1})}{1-\alpha}\right)$$

The expected number of inspections in a renewal cycle can be obtained as

$$E[N_I] = \sum_{i=1}^{\infty} iP(N_I = i) = \sum_{i=1}^{\infty} i \left( F_{T_f}\left(\frac{\delta(1-\alpha^i)}{1-\alpha}\right) - F_{T_f}\left(\frac{\delta(1-\alpha^{i-1})}{1-\alpha}\right) \right)$$

The expected downtime is given by

$$E[T_d] = \sum_{i=1}^{\infty} E[T_d|T_f = t] dF_{T_f}(t) = \sum_{i=1}^{\infty} \int_{\delta(1-\alpha^{i-1})/(1-\alpha)}^{\delta(1-\alpha^i)/(1-\alpha)} \left( \frac{\delta(1-\alpha^i)}{1-\alpha} - t \right) dF_{T_f}(t) dt$$

The length of a renewal cycle is determined by the number of inspections, with the expectation

$$E[\tau] = \sum_{i=1}^{\infty} E[\tau|N_I = i] P(N_I = i) = \sum_{i=1}^{\infty} \frac{\delta(1-\alpha^i)}{1-\alpha} \left( F_{T_f}\left(\frac{\delta(1-\alpha^i)}{1-\alpha}\right) - F_{T_f}\left(\frac{\delta(1-\alpha^{i-1})}{1-\alpha}\right) \right)$$

Integrating the above equations completes the proof.

B. PROOF OF LEMMA 1

The proof is intuitive. Let  $\Delta X_k = X_{k+1} - X_k$ . It is obvious that  $\Delta X_k \sim N(\Delta M_k, \Delta Z_k)$ , where  $\Delta M_k = M_{k+1} - M_k$  and  $\Delta Z_k = Z_{k+1} - Z_k$ . The probability

$$P\{X_{k+1} \leq \theta | X_k = x\} = \Phi\left(\frac{\theta - x - \Delta M_k}{\Delta Z_k}\right) = \Phi\left(\frac{\gamma(\theta - x)}{\Delta M_k} - \gamma\right)$$

increases for any  $\theta$ . Thus, for  $x_1 < x_2$ , we have

$$P\{X_{k+1} \leq \theta | X_k = x_1\} > P\{X_{k+1} \leq \theta | X_k = x_2\}$$

According to the definition of stochastic order, it can be readily obtained that  $\langle X_{k+1} | X_k = x_1 \rangle < \langle X_{k+1} | X_k = x_2 \rangle$ , which completes the proof.

C. PROOF OF LEMMA 2

As we consider the accelerated degradation process, we have  $M''(t) = \mu'(t) > 0$ , which implies that is a convex function of time  $t$ . Following the Jensen's inequality [55], which states that

$$\eta f(x_1) + (1-\eta)f(x_2) > f(\eta x_1 + (1-\eta)x_2)$$

for any convex function  $f(\cdot)$  and  $\eta \in (0, 1)$ , we have

$$(M_{k+2} + M_k) / 2 > M_{k+1}$$

which implies that

$$M_{k+2} - M_{k+1} > M_{k+1} - M_k \Leftrightarrow \Delta M_{k+1} - \Delta M_k > 0$$

It can be concluded that  $\Delta M_k$  increases with the inspection epoch  $k$ . Similarly we can conclude that  $\Delta Z_k$  increases with the inspection epoch  $k$ . Recall that

$$\begin{aligned} P\{X_{k+1} \leq \theta | X_k = x\} &= P\{X_{k+1} = X_k \leq \theta - x\} \\ &= \Phi\left(\frac{\theta - x - \Delta M_{k+1}}{\sqrt{\Delta Z_{k+1}}}\right) \end{aligned}$$

Denote

$$y = \frac{\theta - x - \Delta M_{k+1}}{\sqrt{\Delta Z_{k+1}}}$$

Since  $\Phi(y)$  increases with  $y$  and  $y$  decreases with  $\Delta M_k$  and  $\Delta Z_k$ , it can be concluded that for any  $k_1 < k_2$ ,

$$P\{X_{k_1+1} \leq \theta | X_{k_1} = x\} > P\{X_{k_2+1} \leq \theta | X_{k_2} = x\}$$

leading to

$$\langle X_{k_1+1} | X_{k_1} = x \rangle < \langle X_{k_2+1} | X_{k_2} = x \rangle$$

which concludes the proof.

#### D. PROOF OF LEMMA 3

Given that the system fails after  $k$ th inspection, i.e.,  $T_f \in (k\delta, (k+1)\delta)$ , the expected downtime cost  $W(k, X_k)$  can be denoted as  $W(k, X_k) = c_d E[(k+1)\delta - T_f]$ . Obviously,  $W(k, X_k)$  decreases with the failure time  $T_f$ . Conditioning on the state  $(k, X_k)$ , cdf of the failure time  $T_f$  can be expressed by

$$P(T_f < t | (k, X_k)) = P(X_t > l | (k, X_k))$$

Following Lemma 1 and Lemma 2, we can conclude that  $X_t$  is stochastically non-decreasing in  $k$  and  $X_k$ , for  $t > k\delta$ . Then we have

$$P(T_f < t | k_1, X_{k_1}) < P(T_f < t | k_2, X_{k_2}) \text{ for } k_1 < k_2$$

and

$$P(T_f < t | k, X_{k_1}) < P(T_f < t | k, X_{k_2}) \text{ for } X_{k_1} < X_{k_2}$$

which implies that  $T_f$  is stochastically non-increasing in  $k$  and  $X_k$ . As  $W(k, X_k)$  is decreasing with  $T_f$ , it can be concluded that  $W(k, X_k)$  is non-decreasing in  $k$  and  $X_k$ .

#### E. PROOF OF THEOREM 1

We will prove this theorem via mathematical induction. Let  $V^n(k, X_k)$  be the value function at the  $n$ th iteration of value iteration algorithm. We set  $V^0(k, X_k) = 0$  at  $n = 0$ , which obviously satisfies the property that  $V^0(k, X_k)$  is non-decreasing in  $k$  and  $X_k$ . Suppose that at the  $n$ th iteration, this property holds, i.e.,  $V^n(k, X_k)$  is a non-decreasing function in

$k$  and  $X_k$ . According to the optimality equation of Eq (20), we have

$$\begin{aligned} V^{n+1}(k, X_k) &= \begin{cases} \min\{c_p + V^n(0, 0), \\ W(k, X_k) + E[V^n(k+1, X_{k+1})]\}, & X_k < l \\ c_c + V^n(0, 0), & X_k \geq l \end{cases} \end{aligned}$$

Since  $V^n(k, X_k)$  is non-decreasing function in  $k$  and  $X_k$ , and  $\langle X_{k+1} | X_k \rangle$  is stochastically non-decreasing in  $k$  and  $X_k$ , we can conclude that  $E[V^n(k+1, X_{k+1})]$  is non-decreasing in  $k$  and  $X_k$ . Since the right-hand terms of Eq (21) are all non-decreasing in  $k$  and  $X_k$ , we can conclude that  $V^{n+1}(k, X_k)$  is a non-decreasing function in  $k$  and  $X_k$ , which completes the proof.

#### F. PROOF OF THEOREM 2

If the system has not failed at the  $k$ th inspection,  $X_k < l$ , the optimal maintenance decision can be expressed by

$$a(k, X_k) = \begin{cases} 1, & c_p + V(0, 0) < W(k, X_k) \\ & + E[V(k+1, X_{k+1})] \\ 0, & \text{Otherwise} \end{cases}$$

Since the term  $W(k, X_k) + E[V(k+1, X_{k+1})]$  is non-decreasing in  $k$  and  $X_k$  (from Theorem 1), if there exists a  $\zeta_k^*$  such that

$$c_p + V(0, 0) < W(k, \zeta_k^*) + E[V(k+1, X_{k+1})(k, \zeta_k^*)]$$

then the inequality always holds for any  $X_k > \zeta_k^*$ , which establishes the control limit policy, that the optimal maintenance decision is replacement when the degradation level exceeds the control limit  $\zeta_k^*$ . Similarly, if there exists  $j$  such that

$$c_p + V(0, 0) < W(j, X_j) + E[V(j+1, X_{j+1})]$$

then the inequality holds for any  $k > j$ , as  $W(k, X_k) + E[V(k+1, X_{k+1})]$  is non-decreasing in  $k$ . Hence, we should have  $\zeta_k^* < \zeta_j^*$ , for any  $k > j$ , which concludes that  $\zeta_k^*$  is non-increasing in  $k$ .

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