Multi-objective optimisation in the retail banking industry with stochastic discrete-event simulation

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B. Eng Industrial

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To my mother, father and (little) brother.
Because
where would I be?
– who would I be?
– what would life be?
without the craziness.

Declaration

I, the undersigned, hereby declare that the work contained in this final year project is my own original work and that I have not previously in its entirety or in part submitted it at any university for a degree.

. . . .
Signature

. . . .
Date

ECSA Exit Level Outcomes Reference

Outcome	**Reference**	
	Sections	Pages
1. Problem solving: Demonstrate competence to identify, assess, formulate and solve convergent and divergent engineering problems creatively and innovatively.	*All*	*All*
5. Engineering methods, skills and tools, including information technology: Demonstrate competence to use appropriate engineering methods, skills and tools, including those based on information technology.	*2, 3, 4, 5, 6 & 7*	*8 – 58*
6. Professional and technical communication: Demonstrate competence to communicate effectively, both orally and in writing, with engineering audiences and the community at large.	*All*	*All*
9. Independent learning ability: Demonstrate competence to engage in independent learning through well developed learning skills.	*2, 3, 4 & 5*	*8 – 32*
10. Engineering professionalism: Demonstrate critical awareness of the need to act professionally and ethically and to exercise judgment and take responsibility within own limits of competence.	*8.3 & 8.4*	*57 – 58*

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Thank you to Delyno du Toit for his willingness to answer those questions he was not contractually bound to avoid.

Thank you also to my friends for their interest, encouragement and support throughout the year. It meant more to me than these two sentences can describe.

Annelie and Stephanie need special mention. Your friendships mean the world to me. The mining and manufacturing meccas of Mpumalanga and the Eastern Cape, respectively, are lucky to have you two as residents in 2012.

To the industrial engineering class of 2011: it was fun hanging in Birga with you! You know some pretty awesome people are involved when the words "fun" and "Birga" are used in the same sentence.

Abstract

Cash management is a multi-objective optimisation problem which aims to maximise the service level provided to customers at minimum cost. The topic of du Toit (2011)'s masters thesis was automated teller machice (ATM) cash management for a specific South African retail bank. Focus was placed on an ATM network which primarily provides cash to blue collar laborers in the rural Eastern Cape. The aim of this final year project is to refine the work done by du Toit through the specific investigation into the effect of applying a combination of the vehicle routing problem (VRP) and continuous review policy for inventory management to the retail banking industry. A decision support system (DSS) in the form of a stochastic, discrete-event simulation model is developed. 90 different scenarios are experimented with using the DSS. Results show that the application of the VRP consistently yields high service levels at low cost when compared to two other routing approaches: first-in-first-out routing and direct replenishment. It is concluded that use of the VRP is especially beneficial when the bank has substantial control over transportation cost. The principal recommendation is therefore that cost control should be maximised to fully exploit the advantages obtainable from effective cash management. Finally, it is argued that the benefits to be gained from effective cash management (higher service levels at lower cost) can lead to the improvement of the lives of many a South African wage earner. These benefits could also lead to an increased profit margin – life is all about choices.

Opsomming

Kontantbestuur is multi-doelstelling optimeringsprobleem waarvan die doelwit is om die diensvlak wat aan kliënte gelewer word, te maksimeer, terwyl koste minimeer word. Die onderwerp van du Toit (2011) se meesters tesis was outomatiese tellermasjien (OTM) kontantbestuur vir 'n spesifieke Suid-Afrikaanse kleinhandelbank. Fokus is geplaas op 'n OTM netwerk wat hoofsaaklik kontant aan arbeiders in die landelike Oos-Kaap voorsien. Die doel van hierdie finale jaar projek is om du Toit se werk te verfyn deur spesifiek ondersoek in te stel na die effek wat die toepassing wat 'n kombinasie van die voertuigskeduleringsprobleem en die deurlopende hersieningsbeleid vir voorraadbestuur sal hê. 'n Stogastiese, diskrete-gebeurtenis simulasie model is ontwikkel om as besluitnemings ondersteuningstelsel te dien. 90 verskillende eksperimente is met die simulasie model voltooi. Resultate toon dat die toepassing van die voertuigskeduleringsprobleem deurlopend hoë diensvlakke teen vergelykende lae koste tot gevolg het. Die voertuigskeduleringsprobleem word vergelyk met twee ander skeduleringstegnieke: eerste-in-eerste-uit skedulering en direkte aanvulling. Die gevolgtrekking word gemaak dat gebruik van die voertuigskeduleringsprobleem veral voordelig is wanneer die bank aansienlike beheer oor vervoerkoste het. Die hoof aanbeveling is daarom dat kostebeheer gemaksimeer behoort te word om ten volle munt te slaan uit die voordele wat moontlik gemaak word deur effektiewe kontantbestuur. Ten slotte word daar aangevoer dat die voordele wat sal volg uit effektiewe kontantbestuur (hoër diensvlakke teen laer koste), die lewens van vele Suid-Afrikaanse loonwerkers kan verbeter. Dié voordele kan ook lei tot 'n vergrote winsmarge – die lewe is vol keuses.

Contents

List of Figures

List of Tables

Nomenclature

Acronyms

ATM	Automated teller machine.
BKP	Bounded knapsack problem.
CIT	Cash-in-transit.
CVRP	Capacitated vehicle routing problem.
DCVRP	Distance-constrained capacitated vehicle routing problem.
DP	Dynamic programming.
DVRP	Distance-constrained vehicle routing problem.
EOD	End of day.
EOM	End of month.
FIFO	First-in-first-out.
IP	Integer programming problem.
KP	Knapsack problem.
LP	Linear programming problem.
MCDM	Multiple criteria decision making.
MNAR	Minimum number of ATMs on a route.

MOO	Multi-objective optimisation.
MOOP	Multi-objective optimisation problem.
NSP	Near shortest path.
OTM	Outomatiese tellermasjien (Afrikaans for ATM).
TSP	Travelling salesperson problem.
VRPB	Vehicle routing problem with backhauls.
VRPPD	Vehicle routing problem with pickup and delivery.
VRPTW	Vehicle routing problem with time windows.
VRP	Vehicle routing problem.

Greek Symbols

e	Euler's number.

Roman Symbols

C_C	Total capacity cost.
c_{km}	Cost per kilometer.
q_i	Delivery size.
p_i	Denomination value.
d_{ij}	Distance between stations i and j.
C_D	Total distance cost.
D_i	Distance covered by vehicle i in a month.
C_{Di}	Distance cost per vehicle.
r_e	Effective daily interest rate.
f	Number of free kilometers available per month.

b_j	Number of identical copies of item type j available.
u_j	Variable indicating the number of alternative i included in the solution.
x_{ij}	Integer variable denoting the number of times an arc d_{ij} is traversed by the optimal solution.
I_r	Cash on hand at time of replenishment.
I_j	Cash level at the end of day j.
G	Item set for the knapsack problem.
A_K	Capacity value of the knapsack.
$\mathbf{S}_{ATM}$	Subset of the station set including ATMs that are traversed by the optimal solution.
$k(\mathbf{S}_{ATM})$	Minimum number of vehicles required to serve al customers in vertex set $\mathbf{S}_{ATM} \subseteq \mathbf{V}$.
$f_i(\mathbf{x})$	Set of objective functions of a MOOP.
S_{MOO}	MOO solution space.
r_n	Nominal annual interest rate.
n_{ATM}	Number of ATMs.
n_K	Number of items with profit p_j and weight w_j available for ‘filling’ the knapsack.
m	Number of months in the cost calculation period.
y_i	Number of notes of denomination value p_i.
n_{MOO}	Number of MOO objective functions.
n_{TSP}	Number of points covered by the TSP.
n_r	Number of replenishment events during cost calculation period.

K	Number of vehicles.
$\mathbf{x}$	n_{MOO}-vector of decision variables.
C_O	Opportunity cost.
p_j	Profit of knapsack item.
C_R	Total rebanking cost accrued.
c_R	Rebanking cost per R 100.
s	Reorder point when using the (s, S) policy for inventory management.
S	Reorder quantity when using the (s, S) policy for inventory management.
T_r	Time required to replenish one ATM.
$\mathbf{Z}_i$	Set of objective function values serving as the solution space for a MOOP.
T_{route}	Time required to complete a determined route.
$\mathbf{X}_i$	Set of scenarios decided upon for the simulation study.
$\mathbf{V}$	The station set.
A_V	Delivery vehicle capacity.
C_V	Monthly cost associated with using one CIT vehicle.
v	Vehicle speed measured in kilometers per hour.
w_j	Weight of knapsack item.

Terminology

Cash out	Any event where the cash level in an ATM is such that customer demand cannot be met.
Count house	Distribution center for a network of ATMs.

Direct replenishment	Routing method where vehicles are dispatched directly to ATMs as ATMs register orders.
Even-note-picking	Denomination dispensing algorithm used in industry. Notes are picked in a way which evens out inventory levels of the different notes available.
Least-note-picking	Denomination dispensing algorithm used in industry. Notes are picked in a way which minimises the number of notes dispensed.
Near shortest path routing	Routing method where the aim is to minimise the distance vehicles travel using the VRP.
Pareto principle	Principle stating that 20% of causes are responsible for 80% of effects.
Replenishment efficiency	Number of ATMs replenished using a single route. Also referred to as routing efficiency.
Routing efficiency	Number of ATMs replenished using a single route. Also referred to as replenishment efficiency.
Routing point	Minimum number of ATMs that need to have registered an order before a route will be determined.
Service level	The proportion of the total number of customers satisfactorily served to the total number of customers requiring service.
Speed of service	Time elapsed between the moment an ATM registers an order and its replenishment.
(s, S) policy	Inventory management policy where s refers to the reorder level and S to the reorder quantity.

Chapter 1

Introduction

The aim of this final year project is to apply engineering methods, skills and tools to refine work done by du Toit (2011) on the determination of the optimal cash deployment strategy for a South African retail bank. More specifically, a combination of operations research techniques will be used to develop a decision support system which will aid decision making regarding this multi-objective optimisation problem. This chapter will set out the problem statement and provide documentation of the literature study done on work relating to du Toit (2011)'s thesis. The project objectives and methodology will be laid out. Finally, the structure of the report will be detailed.

1.1 Problem statement

ATMs (automated teller machines) form a key part of retail banking service provision. For many clients ATM withdrawals are the only way to obtain the cash necessary for everyday living. If an ATM is out of cash, these clients face a predicament.

"Out of cash" might refer to an ATM that has absolutely no cash left, but the term may also refer to an ATM that has run out of certain denominations and can therefore not provide a specific amount. An ATM that has run out of R 50 notes, for example, would be unable to dispense multiples of R 50. If a specific ATM or the ATMs operated by a particular bank, is out of cash (or is unable to dispense the exact amount required by the client) on a regular basis, a real possibility exists that customers would switch to a bank better able to meet their cash requirements.

It is thus important, from the perspective of a consumer bank, to ensure that cash levels within its ATMs are sufficient for as large a proportion of time as possible. For now, let the 'proportion of time an ATM is not out of cash' be the service level of an ATM. To provide the highest possible service level to customers, it is necessary to have some cash in an ATM, but preferable that all the denominations are in stock, at all times.

Providing a 100% service level for one ATM would be simple enough. However, no bank has only one ATM and providing very a high service level to an entire network of ATMs is far more complicated.

An ATM network consists of a number of ATMs (the ATM network in question is made up of 18), each with its own stochastic, seasonal customer demand profile, and a count house from where ATMs are replenished. Several cash-in-transit (CIT) vehicles service a network at costs negotiated with the CIT company. The cost structure negotiated and the resulting cost to the bank are dependent on the total distance covered by CIT vehicles.

The ATM network at hand is situated in the rural Eastern Cape. Distances between these ATMs are great (see Table A.2 in Appendix A) and the costs associated with covering these distances can therefore be significant. This was the reason for researching this specific ATM network.

Figure 1.1 (the student's handiwork) illustrates the essence of the problem in a simple, lighthearted fashion. Note the rolling hills of the countryside in which the ATM network is located.

Due to fact that retail banks have full control over inventory levels in ATMs (unlike, for example, a soft drink distributor delivering stock to outlets controlling their own inventory levels), providing 100% service levels to an entire ATM network would be ambitious, but achievable if the transportation and inventory costs of having all denominations in stock at all times could be ignored. Costs associated with delivery delays, transportation and inventory (amongst other ATM operating costs) are, however, very real and cannot be disregarded. Cash transit costs involve not only fuel and labour but also rigorous security and high risk. To a bank, cash in an ATM is not earning interest and is therefore adding to existing inventory costs. In the competitive consumer banking industry, unreasonably high costs – such as the costs associated with maintaining a perfect service level for an ATM network – are unaffordable.

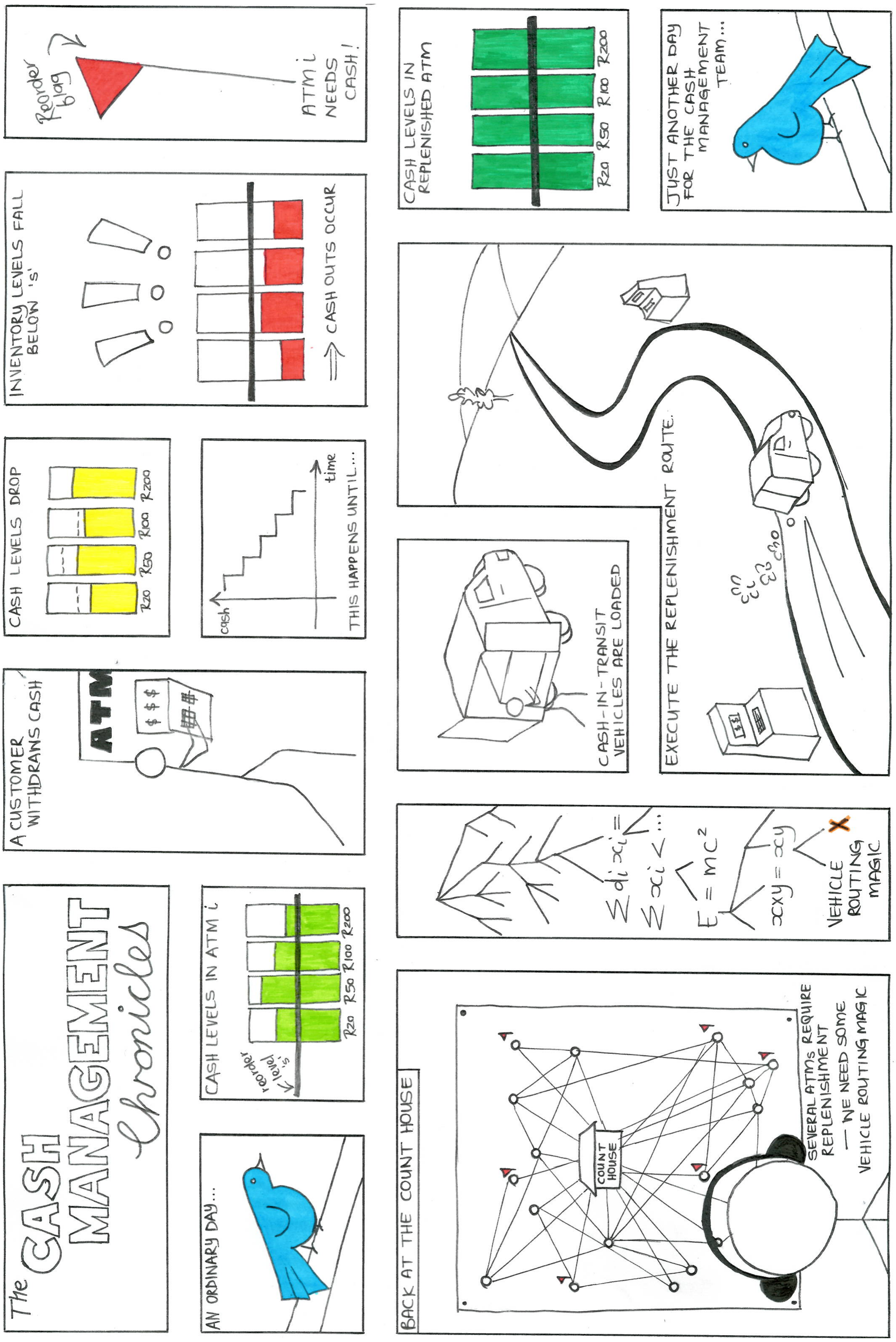

Figure 1.1: A graphic illustration of the essence of the research problem.

Taking these costs into account, the problem is thus: how can the service level of the researched ATM network, operated by the particular bank, be maximised at minimum cost using industrial engineering techniques?

1.2 Research rationale and motivation

The problem stated above is a simplification of a problem researched by du Toit (2011). His work forms the basis for this project and will now be discussed, after which details on the research rationale and motivation will be provided.

1.2.1 Delyno du Toit: ATM cash management for a South African retail bank

"ATM cash management for a South African retail bank", du Toit (2011)'s thesis, resulted from his career at a South African retail bank. The cash management strategy in du Toit's thesis is developed for a network of ATMs operated by the retail bank. The network is located in the Eastern Cape and consists of 18 ATMs and one count house. A cash-in-transit security company handles cash distribution in the region. At the time the thesis was written the cash management strategy used by the bank relied heavily on experience and personal judgement.

As a result, the thesis focuses primarily on prediction methods and inventory management models. Secondary focus is placed on routing techniques found in the operations research field; du Toit only investigated the travelling salesperson problem , comparing it to direct replenishment. Simulation is used to compare different inventory management models, routing techniques and combinations thereof.

Du Toit concludes that these techniques could achieve significant savings in the retail banking environment. The Holt-Winters prediction method and the TSP for vehicle routing are recommended. Du Toit does not make definite conclusions about the inventory models investigated and suggests further research in these areas.

1.2.2 Research rationale and motivation: Refining du Toit's work

Delyno du Toit completed his masters thesis for graduation in March 2011 with the study leader. The main focus of his work was to determine if the application of industrial engineering techniques to the retail banking would lead to significant cost savings.

He focused mainly on demand forecasting with little attention paid to vehicle routing and inventory management.

The study leader was not convinced that du Toit's work provided satisfactory indication of the effects of vehicle routing and inventory management as components of a cash management strategy. In order to refine the work done by du Toit, the study leader specifically required that attention be paid to the application of the vehicle routing problem (instead of the TSP) and the continuous review policy for inventory management. It was specified that the refinement must be done using discrete-event simulation. The work done to refine du Toit's work must follow a multi-objective optimisation (MOO) approach as the stated problem is a MOO problem.

1.3 Literature on cash management in the retail banking industry

Due to the high importance of security in the banking industry, it is difficult to find related work in the open literature. The student had access to du Toit (2011)'s work (with the clear instruction to "not leave it lying around") due to the fact that this final year project serves as a refinement of the work done in it. There are two other works openly available, which are discussed below.

1.3.1 A decision support model for the cash replenishment process in South African retail banking

The first application of industrial engineering techniques in the South African retail banking environment document in the open literature is attributed to Adendorff (1999).

Having investigated a South African retail bank branch for three months, Adendorff determined the nature of the demand distribution. From there an appropriate forecasting model was developed. After evaluating the existing order policies of the branch, a decision support model for the improvement of the cash replenishment process was developed.

Adendorff concludes that significant savings can be achieved through the application of industrial engineering techniques in the retail banking industry.

The focus Adendorff places on factors defining the South African retail banking environment, such as the crime situation, are worth noting.

1.3.2 The optimal cash deployment strategy

Wagner (2007) develops a conceptual framework for finding the optimal cash deployment strategy for a network of ATMs. For the purpose of the thesis, 'optimal' is equivalent to 'minimum-cost'. In determining a minimum-cost scheduling and replenishment strategy, logistics costs (holding cost plus the cost of rent), inventory policies and replenishment vehicle routing are taken into account.

Conceptually, Wagner assumes the existence of a perfect forecasting model, effectively eliminating the actual stochastic nature of demand. Focus is placed on the development of primarily deterministic scientific and simulation models.

Due to the algorithmic complexity of the inventory routing problem applied to a network of ATMs, confirmation about the effects of its application could not be given. The combined use of the Wagner-Whitin algorithm as inventory policy and the travelling salesperson problem for vehicle routing yielded the best result. Wagner concludes that substantial cost savings can be achieved using optimisation.

The work done by Wagner has focus similar to that of this project. There are, however, significant differences: firstly, the final year project is a MOO problem aiming to maximise service level, whilst minimising cost. Furthermore, it will aim to account for stochastic, seasonal demand, true the real-world situation. The effect of using the vehicle routing problem (VRP) instead of the TSP will also be investigated.

1.4 Project objectives

The primary objective of the final year project is to refine the work done by du Toit (2011). This refinement must at least achieve the following secondary objectives:

1. Provide an indication of the effect of applying the VRP to the retail banking industry.
    - None of the work discussed thus far applied the VRP. The reader will note in Chapter 4 that none of the literature available on the VRP applies it to retail banking. This is ground breaking stuff. Are you excited?

2. Supply a clear indication on how the continuous review policy should be applied for inventory management.

3. Deliver a decision support model with which a MOO set of solutions can be determined.

Figure 1.2 (as demonstrated by Bekker (2011a)) illustrates how such a decision support model functions.

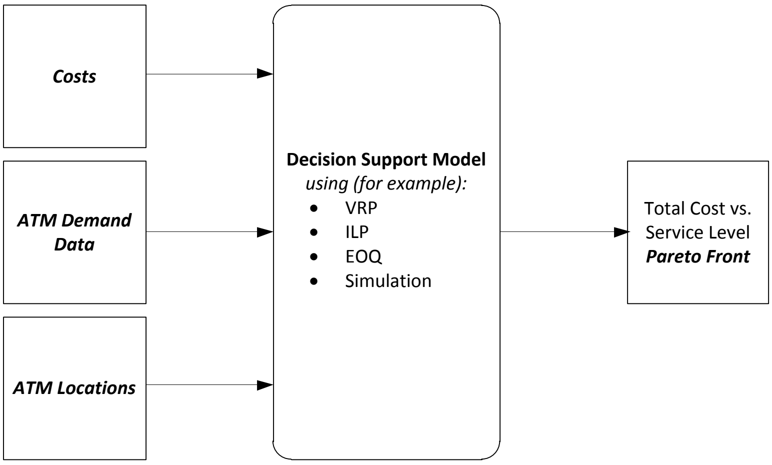

Figure 1.2: A graphic illustration of the decision support model to be delivered.

The three secondary objectives are compulsory refinements. The student was free to identify and make additional refinements. A discussion on additional refinements that were decided upon follows.

Wagner (2007) and du Toit (2011) both used simulation modelling. All indications are that both ignored some details associated with dispensing cash: the inventory level in an ATM does not simply drop R 250 when a customer demands R 250, but the number of R 200 and R 50 notes that are available becomes less. Wagner (2007) worked with the total number of banknotes withdrawn over a period, whereas du Toit (2011) used total daily withdrawal amounts.

The student argues that the fact that meeting customer demand requires the availability of a specific combination of notes for every withdrawal event must be taken into account. To do so, an algorithm must be determined according to which the cash amount demanded by a customer is made up.

As the final year project progressed, the student decided to do an elementary sensitivity analysis on the cost structure of the problem. This is an additional refinement on du Toit (2011)'s work.

1.5 Research methodology

To reach primary and secondary project objectives, a certain approach must be followed. The approach used is dictated in part by the research rationale. The methodology followed comprises of five main phases:

1. Do a literature study on related work available to gain a full understanding of the problem (as documented in Sections 1.3.1 and 1.3.2).
2. Do a literature study on techniques and methods required to solve the problem: MOO, discrete-event simulation, the VRP, inventory management and integer programming.
3. Develop a simulation model allowing for decision making regarding the VRP and continuous review policy. Such a model should be able to compare different routing approaches for a network of ATMs with stochastic, seasonal demand. Setting up different continuous review policy scenarios should also be possible. Finally, the simulation model should dispense notes according to the algorithm developed by the student. This requires the incorporation of various operations research and programming techniques (as discussed in the chapters to follow).
4. Set up experiments which will illustrate the effects of the VRP and continuous review policy on the system.
5. Analyse simulation experiment results and draw conclusions about the systems and the variables that affect it.

The section that follows shows that these phases form the backbone of this report.

1.6 Structure of the report

Chapter 2 will provide information on MOO, discrete-event simulation and will break down the project objective functions.

In Chapter 3 emphasis will fall on inventory management: existing models, the current inventory situation and the suggested inventory model will be discussed.

Vehicle routing techniques found in the field of operations research will be the focus of Chapter 4 where the VRP will be concisely compared to the TSP. The suggested vehicle routing model will be formulated.

Chapter 5 will show the development of an algorithm with which notes can be dispensed to meet customer demand.

An overview of the experiments drawn up for the simulation study will be given in Chapter 6.

The results of said simulation study will be analysed and discussed in Chapter 7.

Chapter 8 will serve as summary and conclusion to the final year project report.

1.7 Conclusion: Introduction

This chapter stated the MOO problem to be addressed by the final year project: maximising the service level of an ATM network at minimum cost. It was emphasised that the project serves as a refinement of work done by du Toit (2011). "ATM cash management for a South African retail bank" was therefore reviewed as part of the research rationale and motivation. Next, literature regarding cash management in the retail banking industry was outlined. The primary and secondary project objectives were identified and the research methodology was subsequently discussed. Finally, the structure of the report was laid out. Chapter 2 will provide information on MOO and discrete-event simulation as a MOO problem solving tool. The two project objective functions will be defined and detailed.

Chapter 2

Multi-objective optimisation

The previous chapter was the introduction to the final year project report. It stated the problem, laid out the research rationale and motivation, and reviewed literature on cash management. The project objectives were introduced and the research methodology developed. This chapter will provide information about basic multi-objective optimisation concepts. Discrete-event modelling will be discussed as it can be seen as a MOO problem solving tool. Finally, the two objective functions for the project are broken down.

2.1 MOO principles

As stated in Section 1.1, the question to be answered in this project is: how can the service level of an ATM network be maximised at minimum cost. Two conflicting objectives must thus be considered. This is usually the case when it is impossible to tie all decision making criteria into a single trade-off function (Zeleny, 1974). Problems of this nature are referred to as multi-objective optimisation (MOO) problems, multiple criteria decision making (MCDM) problems (Thiele *et al.*, 2009) or vector optimisation problems (Ravindran, 2008), to name a few.

Optimal solutions to individual objectives of the MOO problem (MOOP) do not occur at the same alternative (Ravindran, 2008). The aim of solving a MOO problem is therefore not to find the optimal solution (as it does not exist) but rather a set of solutions where all objectives are at their “best” possible values under the given conditions (Zeleny, 1974). Such a set is referred to as efficient, nondominated, noninferior or Pareto optimal solutions (Thiele *et al.*, 2009). If $\mathbf{x}$ denotes an n-vector of decision variables and $f_i(\mathbf{x}), i = 1, \ldots, k$ represents the k objective functions of a MOOP, Ravindran (2008) describes the Pareto optimal set of solutions as: "A solution $\mathbf{x}^\circ \in S_{MOO}$ to a MCDM problem is said to be efficient if $f_k(\mathbf{x}) > f_k(\mathbf{x}^\circ)$ for some $\mathbf{x} \in S_{MOO}$ implies that $f_i(\mathbf{x}) < f_i(\mathbf{x}^\circ)$ for at least one other index i."

2.2 Simulation as a MOO problem solving tool

Simulation provides a method of analysing real-world systems using software designed to imitate system characteristics. It is most applicable when studying complex systems (Kelton *et al.*, 2004). Simulation aids decision making, improves stake holders' understanding of the system and allows decision makers to explore possibilities. It can also help in diagnosing problems, identifying constraints and specifying system requirements (Banks *et al.*, 1998).

Kelton *et al.* (2010) provides an introduction to basic simulation concepts. The book distinguishes between static and dynamic simulation models, continuous-change versus discrete-change dynamic models and differentiates between deterministic and stochastic simulation models. Focus falls on dynamic, discrete-change, stochastic simulation models built using the Simio software package.

For more information on simulation, the reader is referred to Kelton *et al.* (2004), Kelton *et al.* (2010) and Banks *et al.* (1998).

Dynamic, discrete-change, stochastic simulation will be used for the problem at hand. The problem is 'dynamic' because the system changes with time. Even though the system changes with time, it does not change continuously but rather due to the occurrence of specific events, thus: 'discrete-change'. Finally, inputs in the simulation model are not certain, but stochastic. On the recommendation of the study leader Arena is used for building the simulation model. Kelton *et al.* (2004) introduces simulation concepts, focusing on modelling dynamic, discrete-change, stochastic systems in Arena. The Arena work package, operations modelling and statistical considerations are detailed.

Banks *et al.* (1998) suggest the simulation process illustrated in Figure 2.1. The steps in a simulation study correspond well with the execution of the final year project. The relationship between the simulation process and the project (and the resulting project report) will now be discussed. A simulation study starts with the formulation of the problem, after which project objectives are decided upon. Chapter 1 contains the problem statement as well as primary and secondary project objectives. Model conceptualisation is the process of mathematically formulating the model and is detailed in below as well as in Chapters 3, 4 and 5. The thesis written by du Toit (2011) (discussed in Chapter 1) along with the data he used for writing it make up a part of 'Data Collection'. A personal interview with him as well as questions asked and answered via email correspondence make up a further part of 'Data Collection'. Details on the conceptualisation (not contained in Chapters 3, 4 and 5) and translation of the model can be found in Appendices A and B. Model verification was done throughout model implementation and translation. The validation of the simulation model is discussed in Appendix C.

Once a working model exists, experiments can be designed and run. The experimental design is discussed in Chapter 6. Chapter 6 also provides introductory information on the production runs. As was mentioned in Chapter 1, the results of the 45 initial experiments led to an additional cost structure being drawn up. 45 more experiments were run using the adjusted cost structure. The answer to the "More runs?" question asked by Banks *et al.* (1998), was thus "Yes". Finally Banks *et al.* (1998) suggest that the model be documented thoroughly after which the results can be reported (results are discussed in Chapter 7). This report serves as partial documentation of the model; meeting agendas, minutes and explanatory notes serve as further documentation. Comments accompanying programming code (written in VBA for Arena) are a further addition to the model documentation. The implementation step can be ignored for the purposes of this project, but the results will be discussed with representatives of the retail bank for their interest. If deemed fit to do so, the retail bank can then implement the suggestions made by the student or changes of their own based on the results of the simulation study.

The ATM network in question is nothing if not a complex system. For seasonally variable demand unique to the ATM, the machine dispenses denominations according to a specified algorithm. Depending on the inventory level in an ATM, stock might have to be added. This applies to the entire network of ATMs. Given that some ATMs require replenishment and others not, an optimal vehicle routing needs to be deter-

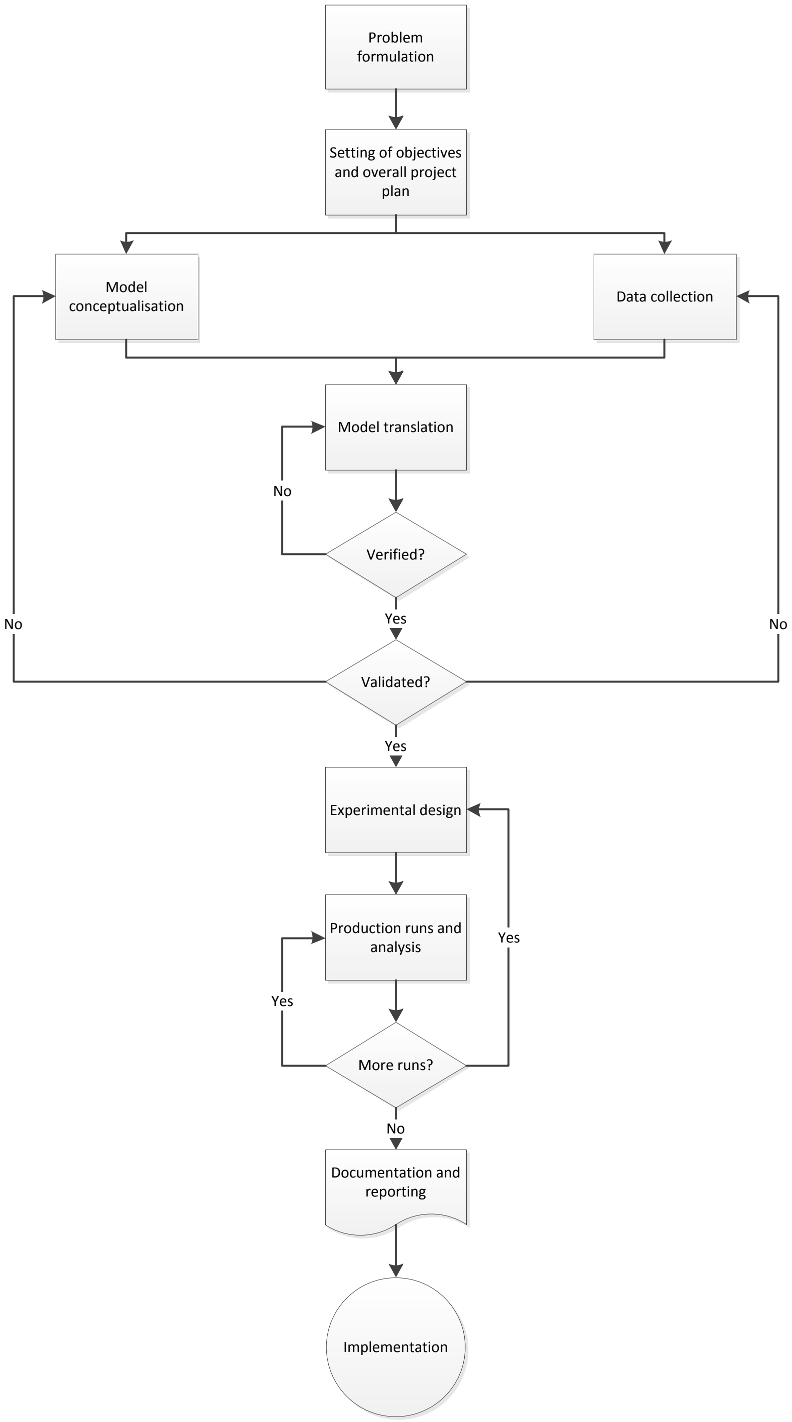

Figure 2.1: Steps in a simulation study.

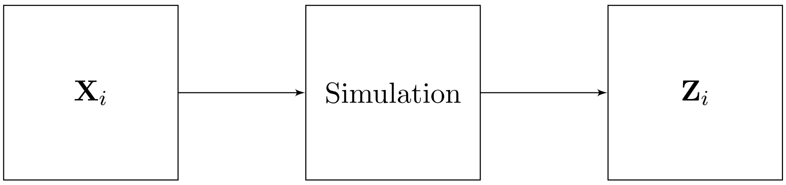

Figure 2.2: Simulation as a MOO decision support system.

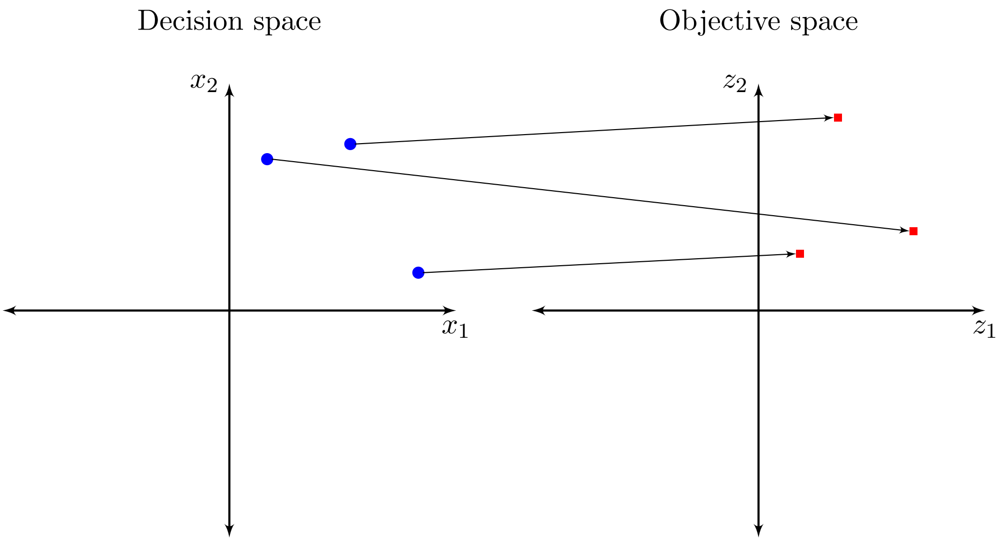

Figure 2.3: MOO mapping.

mined. Additionally, this is a MOO problem and not only one, but two optimisation objectives need be considered. The problem yields itself very well to simulation.

In essence simulation enables simulation analysts to evaluate the effects of changes in certain variables on the complex system studied. The analyst decides on a set of scenarios ($\mathbf{X}_i$) for the simulation study. These scenarios are combinations of variables chosen from the decision space and serve as an input to the simulation model. For each scenario, the simulation model yields a unique response. A specific reorder level or routing method would, for example, produce a certain response. These responses make up $\mathbf{Z}_i$ a set of objective function values serving as the solution space for the MOOP. From the solution space, a Pareto front containing efficient solutions can be obtained. Figure 2.2 shows this process: scenarios serve as input to the simulation model which yields the solution space. Figure 2.3 shows how scenarios chosen for the simulation study map onto the responses produced by the simulation model.

2.3 Simulation modelling: service level versus total cost

As stated in Section 1.1, the question to be answered in this project is: how can the service level of an ATM network be maximised at minimum cost. This is a multi-objective optimisation problem with two goal functions: (1) maximise service level and (2) minimise total cost.

Due to the nature of multi-objective optimisation problems, plotting service level against total cost for a large number of experiments should yield a Pareto front. This front represents optimal scenarios. Valuable conclusions about variables affecting the system can be drawn from such a plot and the resulting Pareto front. In order to plot the values of these goal functions, service level and total cost need to be defined. These definitions follow.

2.3.1 Total service level

For this model, a *cash out* will be defined as any situation where a customer cannot be served due to a shortage of cash. An ATM might for example have more than R 70, but in denominations with which a R 70 combination of notes cannot be formed – this would count as a cash out.

Service level for this problem is the ratio of the total number of customers satisfactorily served to the total number of customers requiring service. Service level is expressed in Equation 2.1.

$$Service\ level = \frac{Total\ customers\ requiring\ service - Total\ number\ of\ cash\ outs}{Total\ customers\ requiring\ service}. \tag{2.1}$$

From Equation 2.1, the first objective function for the MOOP to be modelled is

$$\text{maximise} \quad \frac{Total\ customers\ requiring\ service - Total\ number\ of\ cash\ outs}{Total\ customers\ requiring\ service}. \tag{2.2}$$

2.3.2 Total cost

Costs that need to be considered for the model include cash transportation, handling and storage. These costs are all logistics related expenses.

Wagner (2007) recommended work done by Daganzo (2005) as “the most detailed and comprehensive framework for the classification and analysis of logistics costs to date”. Wagner (2007)’s summary of Daganzo (2005)’s work is shown in Figure 2.4.

Not all the elements taken into account in Daganzo (2005)’s analysis are applicable to the current problem. The following elements do not have to be taken into consideration:

- **Overcoming “Time” – Holding – Rent:** There is no indication in du Toit’s work that the cash in ATMs is insured or protected. The cost associated with renting facilities are not influenced by either the inventory or routing models.
- **Overcoming “Time” – Waiting – Loss of value:** Lost interest will be considered as ”Cost of capital”.
- **Overcoming “Distance” - Transportation - Mode:** Only the scenario where dedicated vehicles are used will be considered.
- **Overcoming “Distance” – Handling – Ordering:** It is assumed that there is no special ordering cost involved with orders from ATMs to the counthouse. Costs involved with ordering are primarily administrative and therefore “fixed” irrespective of the inventory and routing models in use.

Costs which have to be taken into consideration are:

- **Overcoming “Time” - Waiting - Cost of capital:** Cash kept in ATMs earns no interest. This should be taken into account. A nominal yearly rate of 6% is assumed. For the purposes of this study, the cost of capital will be referred to as “opportunity cost”.
- **Overcoming “Distance” - Transportation - Capacity:** Increasing capacity (adding an extra vehicle) will impact total cost significantly. A dedicated vehicle costs R 78 710 per month.
- **Overcoming “Distance” - Transportation - Distance:** The first 4500 km covered by a vehicle in a month is free. After that every kilometer costs R 3.18.

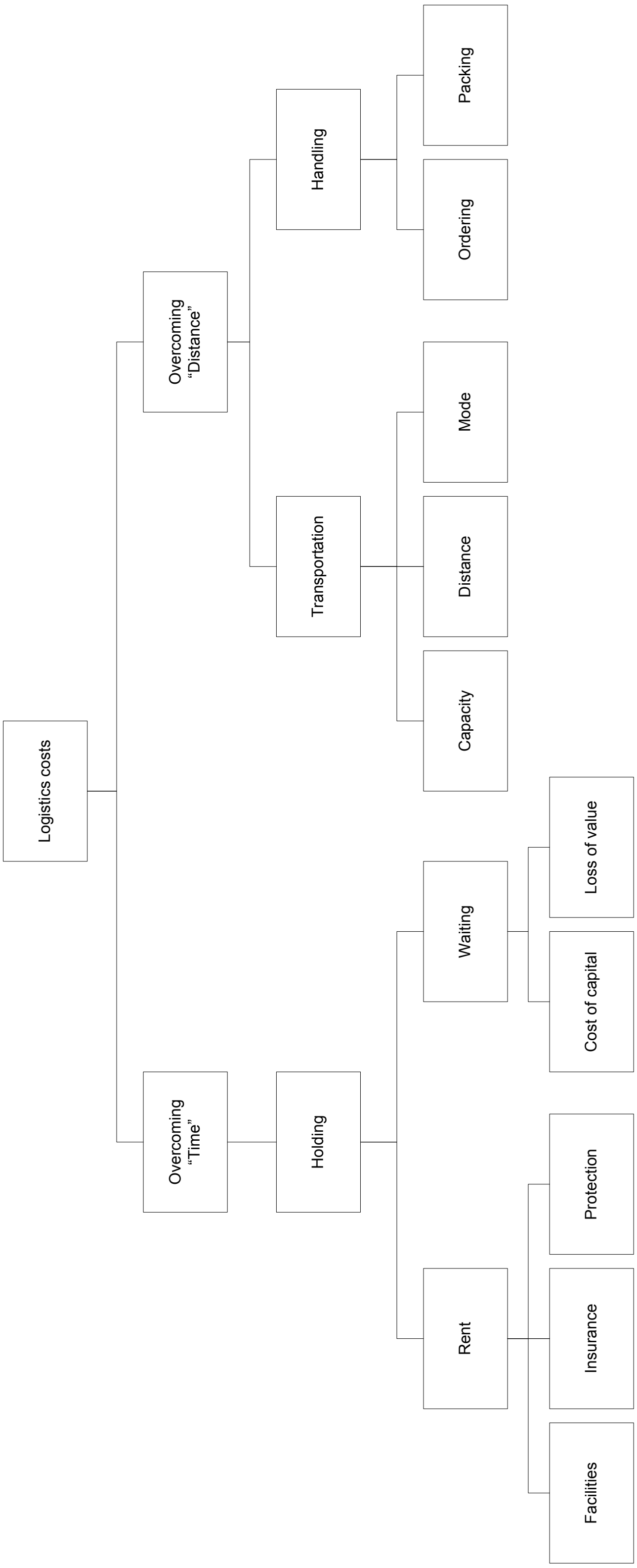

Figure 2.4: Logistics cost breakdown.

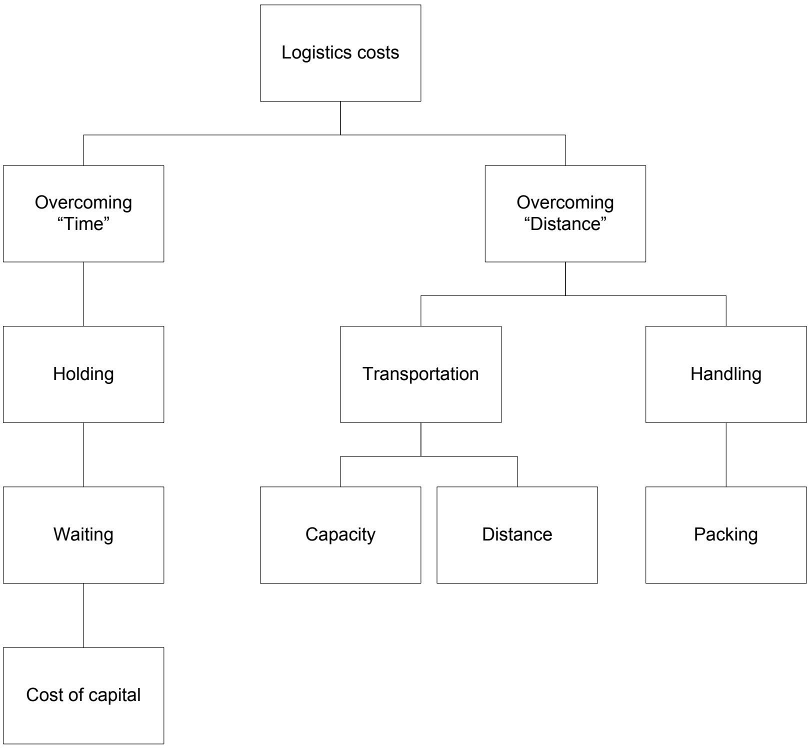

Figure 2.5: Logistics cost breakdown adjusted for ATM network model.

- **Overcoming "Distance" - Handling - Packing:** The only packing cost applicable is that of rebanking which occurs when notes are left in an ATM at the time the ATM is replenished. A rebanking cost of R 0.21 per R 100 rebanked needs to be taken into consideration.

Figure 2.5 shows a logistics costs breakdown adjusted to suit the project. From this breakdown, the second objective function is

$$\text{Minimise} \qquad C_O + C_C + C_D + C_R. \tag{2.3}$$

The opportunity cost C_O is calculated by multiplying the effective daily interest rate r_e with the cash left I_j in an ATM at the end of a day j (du Toit, 2011). Opportunity cost is calculated on a daily basis from the first day of the month until month end EOM and then summed from month i until the end of the cost calculation period at month m. This is shown in Equation 2.4.

$$C_O = \sum_{i=1}^{m} \sum_{j=1}^{EOM} r_e \times I_j. \tag{2.4}$$

Equation 2.5 shows the calculation of the effective daily interest rate r_e for a continuously compounded annual nominal rate r_n.

$$r_e = e^{r_n/365} - 1. \tag{2.5}$$

Capacity cost C_C is a function of the number of vehicles employed (K) and the monthly cost associated with using a vehicle C_V. m denotes the number of months for which cost is calculated. Capacity cost is also referred to as "vehicle cost" throughout the project report. The capacity cost denoted by Equation 2.6.

$$C_C = \sum_{i=1}^{m} K \times C_V. \tag{2.6}$$

Furthermore, distance cost C_D is the sum of the distance costs per vehicle C_{Di} with $i = 1 \ldots K$. C_{Di} is a function of the distance travelled by each vehicles D_i and dependent on f the number of free kilometers available, and c_{km} the cost per kilometer. Distance cost is calculated using Equations 2.7 and 2.8.

$$C_{Di}(D) = \begin{cases} 0 & \text{if } D_i \leq f, \\ (D_i - f) \times c_{km} & \quad \text{if } D_i > f, \qquad \text{for} \qquad i = 1, \ldots, K. \end{cases} \tag{2.7}$$

$$C_D(i) = \sum_{i=1}^{K} C_{Di}. \tag{2.8}$$

Finally, rebanking cost C_R depends on m the number of ATMs, I_r the cash left in each ATM at the time of replenishment and c_R the rebanking cost per R 100. Equation 2.9 shows the calculation of C_R.

$$C_R = \frac{\sum_{r=1}^{n_r} I_r \times c_R}{100} \tag{2.9}$$

where n_r is the total number of replenishment events that occurred during the cost calculation period.

2.4 Conclusion: Multi-objective optimisation

This chapter highlighted some important MOO principles, after which simulation was discussed as a MOO problem solving tool. The objective functions for this project – maximise service level; minimise cost – were broken down last.

The following chapter will concentrate on inventory management.

Chapter 3

Inventory management

The previous chapter focused on multi-objective optimisation and discrete-event simulation as a MOO problem solving tool, detailing the simulation process. The simulation process starts by identifying the problem and setting project objectives. Once this is done, model conceptualisation can begin. Model conceptualisation begins in Chapter 2 where the MOO objective functions for the research project were laid out.

Model conceptualisation continues in this chapter with the discussion of inventory management. As mentioned in Chapter 1 the refinement that is the aim of this project must at least investigate the effect of the continuous review policy for inventory management. Subsequently this chapter will outline different existing inventory management models. After which, the inventory situation on hand will be described. Finally, the continuous review policy for inventory management is discussed and the proposed inventory management model is mathematically formulated.

3.1 Inventory management models

Inventory control models can primarily be divided in two categories according to the nature of the demand to be met: deterministic inventory models and stochastic inventory models.

Deterministic inventory models deal with certain demand. These models include the economic order quantity formula (EOQ) and Walter-Whitin's time-varying model (Ravindran, 2008).

Dealing with uncertain demand, stochastic models include the news vendor problem and the (s, S) policy (Winston, 2004). Ravindran (2008) mentions stochastic multi-echelon inventory models. Bellman (1957) discusses a stochastic dynamic programming approach referred to as the "the optimal inventory equation".

3.2 Inventory situation in question

The inventory situation in the Eastern Cape can be described as a stochastic, multi-echelon system. Inventory levels in the count house can be described as multi-period, whilst inventory levels in ATMs can be managed as either single- or multi-period. Notes can be added to the cash remaining in an ATM during replenishment (this is called a "cash-add" system and would be multi-period management) or the remaining notes can be removed and replaced with newly packed canisters (a single-period inventory management technique referred to as "cash-swap"). Retail banking is a harsh industry and there are no backorders allowed.

The scope of this project does not cover the entire multi-echelon inventory system but focuses only on inventory management of the cash in ATMs, assuming infinite availability of notes at the count house.

3.3 Formulation of inventory management model

As part of the refinement of du Toit (2011)'s work, various scenarios relating to the continuous review (s, S) policy for inventory management must be investigated. The (s, S) policy involves continuously monitoring stock levels within the ATMs. As soon as the inventory level in ATM i drops below the reorder point s, an order is placed which would bring inventory position to S (Axsäter, 2000). Figure 3.1 shows the primary characteristics of the continuous review policy.

The inventory management model developed uses the cash-swap approach described above. Effectively, this means that order size is equal to a fixed S, as the inventory position would become zero when the remaining notes are removed. Once all remaining notes were removed, inventory level increases by said fixed S, bringing inventory levels to 100% every time an ATM is replenished. Figure 3.2 shows the (s, S) policy to be implemented for the simulation model.

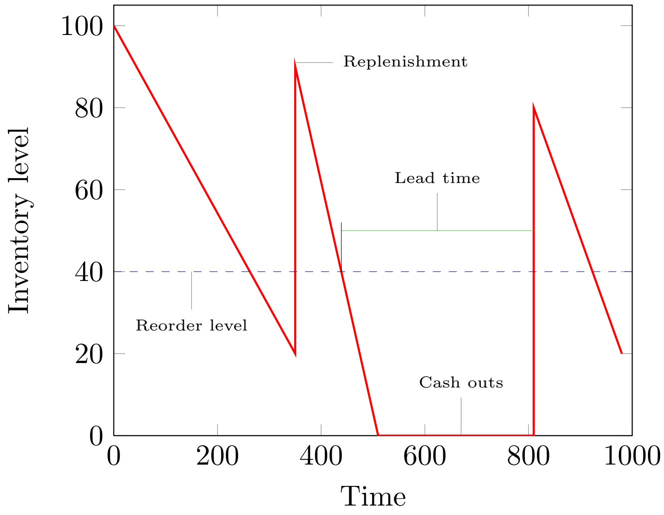

Figure 3.1: Some characteristics of the (s, S) inventory management process.

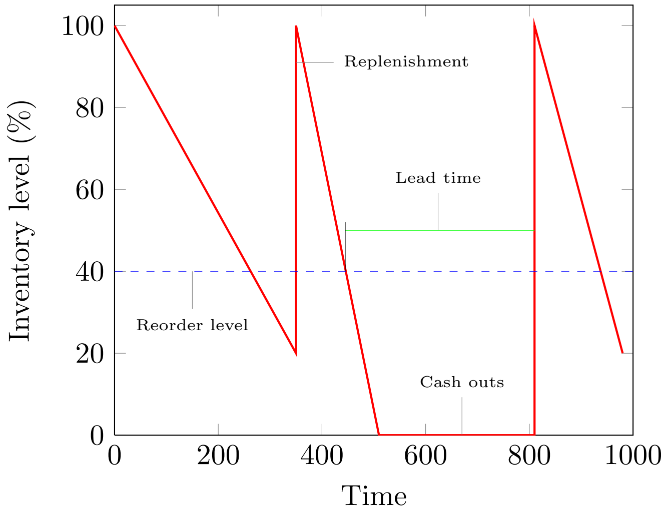

Figure 3.2: The (s, S) inventory management process to be implemented.

Order size S is based on operational data provided by du Toit (2011): canisters containing cash can at most contain 2500 notes. Unfortunately, dispensing problems occur when canisters are filled to the brim. To prevent such errors, newly packed canisters are filled with 2000 notes each. Additionally du Toit (2011) noted that canisters filled with R 200 notes tend to run empty first. For this reason, the retail bank packs two of the five canisters available in an ATM with R 200 notes. Order size can be expressed as

$$S = \sum_{i=1}^{4} p_i \times y_i \tag{3.1}$$

$$y_i = 2000 \qquad \text{for } i = 1, 2, 3, \tag{3.2}$$

$$y_4 = 4000, \tag{3.3}$$

$$p_1 = 20, \tag{3.4}$$

$$p_2 = 50, \tag{3.5}$$

$$p_3 = 100, \tag{3.6}$$

$$p_4 = 200. \tag{3.7}$$

where y_i denotes the number of notes of denomination value p_i. The numerical value of S is R 1 140 000.

For this simulation study S will not be varied; only changes to s will be investigated. The reorder point s can be either the total monetary value of cash in the ATM, such as R 300 000, or the total number of notes left in the ATM. The reorder point can also be the number of notes left of a certain denomination: an order could be triggered once there are less than 500 R 50 notes available. The variations in s that were experimented with are discussed in Chapter 6.

3.4 Conclusion: Inventory management

This chapter provided an overview of various existing inventory management models. The inventory situation in the Eastern Cape was then briefly described. Finally, the (s, S) policy was formulated as the proposed inventory management model.

The following chapter will focus on vehicle routing: the TSP and VRP will be compared, literature on the VRP will be reviewed, the vehicle situation described and a VRP formulation proposed.

Chapter 4

Vehicle routing

The previous chapter briefly mentioned various existing inventory management models, described the inventory situation on hand and discussed the continuous review (s, S) policy.

As mentioned in Chapter 1 the refinement that is the aim of this project must investigate the effect of using the VRP as a vehicle routing approach. Taking into account that du Toit (2011) and Wagner (2007) both used the TSP, this chapter will start by comparing the TSP to the VRP to highlight some major differences between the two methods. Some VRP case studies available in the open literature will then be reviewed. Next, the vehicle routing situation at hand is described, after which the formulation of the suggested vehicle routing model is discussed. Finally an overview of methods for solving the VRP will be provided.

4.1 Existing vehicle routing methods

Several routing techniques exist. The one used by the retail bank is logic based on experience: count house employees select routes based on the routes preferred by the CIT vehicle drivers in the past. The field of operations research offers several mathematical alternatives to this tried and trusted method. Two of these alternatives are the travelling salesperson problem (TSP) and the vehicle routing problem (VRP).

4.1.1 The travelling salesperson problem

Wagner (2007) and du Toit (2011) both experimented with the TSP as a vehicle routing scenario; both concluding that using the travelling salesperson routing method yields better results than other techniques experimented with. Wagner compared the TSP to inventory routing (about which his results were inconclusive) whilst du Toit compared it to direct replenishment.

In short the travelling salesperson problem is a combinatorial optimisation problem formulated to minimise the distance required to cover n_{TSP} points once (Winston, 2004). No capacity constraints are imposed and a solution is found for only one transporter (it is assumed that one traveller can service all points). Once capacity or other additional constraints need to be considered in the mathematical formulation, using the VRP becomes necessary (Toth & Vigo, 2002).

4.1.2 The vehicle routing problem

Vehicle routing problems deal with the distribution of goods between supply points and customers. According to Toth & Vigo (2002) "the solution of the VRP calls for the determination of a set of routes, each performed by a single vehicle that starts and ends at its own depot, such that all requirements of the customers are fulfilled, all the operational constraints are satisfied, and the global transportation cost is minimised."

Main categories of the vehicle routing problem include (Toth & Vigo, 2002):

- *Capacitated and distance-constrained VRPs* which deal with cases where the constraints imposed are vehicle capacity (the capacitated VRP) and maximum route length (the distance-constrained VRP), respectively. The distance-constrained capacitated VRP (DCVRP) deals with the case where both capacity and distance constraints need to be considered.

- *The VRP with time windows (VRPTW)* which builds on the capacitated VRP (CVRP). In addition to the imposed capacity constraints, every customer is associated with a time interval (called a time window) during which delivery to the customer must take place.

- *The VRP with backhauls (VRPB)* which once again builds on the CVRP. In addition to capacity constraints, the problem is complicated by the fact that customers are divided into backhaul and linehaul customers. The latter require product deliveries, whilst products need to be picked up from the former.

- *The VRP with pickup and delivery (VRPPD)* differs from the VRPB in that each customer is associated with both delivery and pickup quantities.

Although the vehicle routing problem is essentially a generalisation of the TSP, it is much more difficult to solve in practice (Laporte, 2009). Neither du Toit (2011) nor Wagner (2007) experimented with the VRP. In fact, no implementation of the VRP in the retail banking industry could be found in the open literature.

4.1.3 Vehicle routing applications in open literature

Even though literature on vehicle routing in the banking industry is limited, information on vehicle routing in other industries is not as difficult to come by. Solutions found in these industries are significant because at the end of the day the transportation of cash is not that different from the transportation of other commodities. After all, the cases discussed all have efficient, cost-effective transportation of commodities as a common aim.

Dantzig & Ramser (1959) introduced the vehicle routing problem, proposing a mathematical formulation for the optimum routing of fuel trucks from a bulk supplier to gasoline stations. The proposed solution was named “the truck dispatching problem” and described as “a generalisation of the travelling-salesperson problem”. This generalisation involved imposing the condition that deliveries of size q_i be made at every station, with A_V the capacity of the delivery vehicle

$$A_V \ll \sum_i q_i \tag{4.1}$$

in contrast to the carrier capacity

$$A_V \gg \sum_i q_i \tag{4.2}$$

allowed by the the travelling salesperson problem. Dantzig & Ramser (1959)’s fuel transport problem essentially deals with the transportation of a single product (fuel) which is similar to the problem at hand: the transportation of cash.

Jacobsen & Madsen (1980) compared three different methods for determining the ideal location of and subsequent routing to and from newspaper transfer points. Newspapers are printed at a printing office. From the printing office, the newspapers are transported to transfer points from where final deliveries to sales points are made. Newspapers need to be delivered in a very short period of time and as a result large fleets are involved.

More recently Zeng *et al.* (2007) suggested two composite methods for solving the vehicle routing problem. The solution methods were applied to the routing of soft drink deliveries in Singapore. An adaption of the distance-constrained VRP was applied.

Still transporting liquids, Igbaria *et al.* (1996b) and Igbaria *et al.* (1996a) investigated a decision support model called FleetManager used for the transportation of milk in New Zealand. FleetManager combines vehicle routing with a user interface which allows schedulers to make adjustments according to their judgment: people possess qualitative knowledge (such as ease of access to a customer by a certain vehicle) essential to effective decision making which cannot be formulated mathematically. Igbaria *et al.* (1996b) and Igbaria *et al.* (1996a) conclude that the successful implementation of mathematical vehicle routing techniques cannot take place if no space is left for human intervention.

Fan *et al.* (2009) uses AnyLogic to model a multi-objective VRPTW based on real data of a consumer goods distribution center in the USA. The objectives considered are (1) minimise the total distance covered by all vehicles, (2) minimise the number of vehicles used and (3) maximise service punctuality. This MOO application to the VRP is very similar to the problem at hand.

As stated earlier, transporting cash parallels the shipping of other consumer goods. There are, however, some considerations distinguishing vehicle routing problems used for cash management from the cases examined above:

- Unlike the newspaper problem for which fixed routes can be determined due to the fact that the sales points to be serviced do not vary, the ATMs demanding replenishment change over time.

- Fixed routes are also not a feasible option as fixed routes increase the likelihood of vehicle heists taking place. Naturally, vehicle heists are to be avoided.

- With the network consisting of the count house and ATMs, the bank has full control over the inventory levels of the "sales points". This differs significantly from, for example, the distribution-center-shipping-to-a-store network. In the latter scenario, stores determine the inventory levels to be maintained. The distribution center then merely has the responsibility of transporting the goods requested when stores requiring replenishment have registered an order.

4.2 Vehicle routing situation

In the South African retail banking environment there are two main alternatives for cash transportation: scheduled cash-in-transit (CIT) vehicles or dedicated CIT vehicles. A scheduled CIT vehicle is controlled entirely by the CIT company providing the vehicle. A dedicated CIT vehicle is manned by a private CIT company but routed by the bank. Dedicated vehicles provide banks with greater control over cash deliveries (the date, time and place as well as the route of a delivery are determined by the bank) at greater cost (du Toit, 2011).

At the time du Toit (2011)'s thesis was written, the ATM network in question was divided in three service areas. The first area, consisting out of three of a total of 18 ATMs, was serviced by a scheduled vehicle. A dedicated CIT vehicle serviced the five ATMs in the second region, whilst a second dedicated vehicle delivered to the remaining ten ATMs. Deliveries were made on a *direct replenishment* basis. Direct replenishment implies that ATMs register replenishment requirements, after which vehicles respond to these requests one at a time, as soon as possible.

Further details include the distances covered during deliveries, primary costs associated with the delivery situation as well as the time linked to deliveries. These details can be found in the functional specification of the simulation model included in Appendix A.

Distances between ATMs in the network are symmetrical and shown in Table A.2, available in Appendix A. Delivery cost and transportation time detail can be found in the same appendix, in Table A.1.

4.3 Suggested vehicle routing model

For the purposes of this study, the ATM network will not be split up but serviced as a whole. The student reasons that serving the entire network will provide a better understanding of the differences between routing scenarios.

Due to the fact that the network under consideration is located in the Eastern Cape, large distances need to be covered between ATMs. Cash in transit vehicles need to be back at the count house by the end of the working day. The VRP is thus constrained by the daily working hours available. Time required to complete a route can be expressed as follows:

$$T_{route} = \sum_{i=0}^{n_{ATM}} d_{ij} \div v + (n_{ATM} \times T_r) \tag{4.3}$$

$$d_{ij} > 0, \tag{4.4}$$

$$i = 0, 1, \ldots, n_{ATM}, \tag{4.5}$$

$$j = i + 1. \tag{4.6}$$

where d_{ij} represents the distance (measured in kilometers) between the i^{th} station on the route and $j = i^{th} + 1$ station. n_{ATM} is the number of ATMs to be visited. The count house is at $i = 0$ and $j = n_{ATM} + 1$. Vehicle speed is measured in kilometers per hour and is denoted by v, whilst T_{route} and T_r respectively denote the time required to complete the determined route and the time required to replenish one ATM.

For the purposes of the problem, it is assumed the constraint on vehicle capacity is insignificant compared to the time constraint. In other words, a CIT truck would be able to carry all the cash required to service a route.

T_{route} is a function of distance. Note that the distances between stations are symmetric. The suggested VRP formulation is thus an adaption of the symmetric distance-constrained vehicle routing problem (DVRP) put forth by Toth & Vigo (2002):

$$\text{minimise} \quad \sum_{i \in \mathbf{V} \setminus \{\mathrm{m}\}} \sum_{j>i} d_{ij} x_{ij} \tag{4.7}$$

$$\text{subject to} \tag{4.8}$$

$$\sum_{h<i} x_{hi} + \sum_{j>i} x_{ij} = 1 \qquad \forall \qquad i \in \mathbf{V} \setminus \{0\}, \tag{4.9}$$

$$\sum_{j \in \mathbf{V}\{0\}} x_{0j} = 2K, \tag{4.10}$$

$$\sum_{i \in \mathbf{S}_{ATM}} \sum_{\substack{h<i \\ h \notin \mathbf{S}_{ATM}}} x_{hi} + \sum_{i \in \mathbf{S}_{ATM}} \sum_{\substack{j>i \\ j \notin \mathbf{S}_{ATM}}} x_{ij} \geq 2k(\mathbf{S}_{ATM}) \ldots$$

$$\forall \qquad \mathbf{S}_{ATM} \subseteq \mathbf{V} \setminus \{0\}, \mathbf{S}_{ATM} \neq \emptyset, \tag{4.11}$$

$$x_{0j} \in \{0,1\} \qquad \text{for} \qquad j \in \mathbf{V} \setminus \{0\}, j <> 5, \tag{4.12}$$

$$x_{0j} \in \{0,1,2\} \qquad \text{for} \qquad j = 5. \tag{4.13}$$

Here d_{ij} denotes the distance between stations i and j and x_{ij} is an integer variable denoting the number of times an arc d_{ij} is traversed by the optimal solution. K represents the number of vehicles required. $\mathbf{V} = \{0, \ldots, n\}$ is the vertex or station set. $\mathbf{V} = \{1, \ldots, n\}$ represents ATMs, whereas the 0^{th} vertex is the count house. $k(\mathbf{S}_{ATM})$ represents the minimum number of vehicles required to serve al customers in vertex set $\mathbf{S} \subseteq \mathbf{V}$.

Equation 4.9 enforces that every station must be entered and exited once (two routes must occur at every station), whilst Equation 4.10 imposes that all vehicles must depart and finish at the count house. These equations are called degree constraints (Toth & Vigo, 2002).

Equation 4.11 links the solution whilst at the same time enforcing capacity constraints and is called a capacity-cut constraint (Toth & Vigo, 2002). As stated above, the capacity constraint for this problem is time.

ATM 5 is located in a manner such that it is never optimal to include it in a route. It does, however, have to be replenished when inventory levels are below the reorder point. For this reason an exception is made for ATM 5: if need be, a single-customer route can be constructed to service this ATM as indicated by Equation 4.13. Single-routes will otherwise not be allowed (enforced by Equation 4.12).

4.4 Solving vehicle routing problems

Similar to the knapsack problem discussed in Chapter 5, exact algorithms for solving the VRP exist. These include branch-and-bound, branch-and-cut and set-covering-based algorithms (Toth & Vigo, 2002). The VRP can also be formulated as a dynamic programming problem (Laporte, 2009). Due to its computational complexity, heuristics and metaheuristics have been proposed for the VRP. Among the constructive method heuristics are the Clarke and Wright savings algorithm and sequential insertion heuristics. Two-phase methods include the sweep algorithm and the truncated branch-and-bound. Six main categories of metaheuristics have been successfully applied to the VRP: simulated annealing, deterministic annealing, Tabu search, genetic algorithms, ant systems and neural networks (Toth & Vigo, 2002).

The VRP in this problem was solved using a simple branch-and-bound based heuristic developed by the student.

4.5 Conclusion: Vehicle routing

This chapter looked at the difference between the TSP and VRP. Literature on the VRP was reviewed, after which the current vehicle situation was laid out. The suggested VRP formulation was then developed and finally methods for solving the VRP were outlined.

The following chapter will discuss the algorithm developed for dispensing notes.

Chapter 5

An algorithm for dispensing denominations

The previous chapter discussed literature on the VRP as well as the formulation suggested for this problem.

This chapter discusses the need for an algorithm according to which the combination of notes dispensed can be determined during an ATM transaction. A summary of the literature studied regarding integer programming as a possible solution area follows. The formulation of the suggested knapsack problem formulation is then detailed, after which algorithms commonly used to solve integer programming problems are investigated.

5.1 Dispensing denominations

South African ATMs typically do not carry R 10-notes. The ATMs of the retail bank in question all contain five cash canisters of which three are filled with equal numbers of R 20, R 50 and R 100 notes. The remaining two canisters contain R 200 notes. If, for example, a customer requests R 500, the ATM can dispense twenty-five R 20 notes, ten R 50 notes, five R 100 notes or a multitude other denomination combinations. As long as no denominations are out of stock, the only constraint is that the sum of the monetary values of the notes dispensed must equal the amount requested.

In industry two main algorithms exist (du Toit, 2011): *least-note-picking* and *even-note-picking*. These algorithms are not available to members of the general public.

Given that an ATM can never dispense fractions of notes (an ATM dispensing fractions of notes would indeed be quite problematic), the field of integer programming problems is investigated for possible algorithms.

5.2 The knapsack problem

Winston (2004) states that an *integer programming problem (IP)* can be described simply as a *linear programming problem (LP)* in which some or all of the variables are required to be non-negative integers. *Mixed integer programming* refers to those cases where only some of the variables are required to be integers. If all variables need be integers, the problem is referred to as a *pure integer programming* problem (Kellerer *et al.*, 2004).

Often a decision is the choice between two options: accepting or rejecting an alternative. Such decisions are called *binary decisions* (Kellerer *et al.*, 2004). An instance of binary decision making, the *knapsack problem* will subsequently be discussed. The *vehicle routing problem*, another instance of binary decision making, is detailed in Chapter 4.

Kellerer *et al.* (2004) define the knapsack problem (KP) as follows: "We are given an instance of the knapsack problem with item set G, consisting of n_K items with profit p_j and weight w_j, and the capacity value A_K. Then the objective is to select a subset of G such that the total profit of the selected items is maximized and the total weight does not exceed A_K."

The knapsack problem is viewed as a simple non-trivial integer programming problem as it has only one constraint (the capacity constraint) and only positive coefficients. Its general formulation is

$$\text{maximise} \sum_{j=1}^{n_K} p_j u_j \tag{5.1}$$

$$\text{subject to} \sum_{j=1}^{n_K} w_j u_j \leq A_K, \tag{5.2}$$

$$u_j \in \{0,1\}, \qquad j = 1, \ldots, n_K. \tag{5.3}$$

where $u_j \in \{0, 1\}$ is a *binary variable*. If $u_j = 1$ the alternative in question is accepted whereas $u_j = 0$ indicates that the alternative is rejected.

Variations on the knapsack problem include the *bounded* and *unbounded knapsack problems* as well as the *multidimensional knapsack* problem. The bounded knapsack problem implies that there are b_j identical copies of item type j available. It is formulated as follows:

$$\text{maximise} \quad \sum_{j=1}^{n_K} p_j u_j \tag{5.4}$$

$$\text{subject to} \quad \sum_{j=1}^{n_K} w_j u_j \leq A_K, \tag{5.5}$$

$$0 \leq u_j \leq b_j, \qquad u_j \text{ integer}, \qquad j = 1, \ldots, n_K. \tag{5.6}$$

The unbounded knapsack problem implies that there are an unlimited amount of item j available. Knapsack problems with more than one constraint are referred to as multidimensional knapsack problems (Kellerer *et al.*, 2004).

5.3 Suggested algorithm for denomination dispensing

The bounded knapsack problem (BKP) lends itself beautifully to be used as an algorithm for denomination dispensing. The combination of notes dispensed will occasionally be constrained by the number of notes b_j of a certain denomination j that are available. Every denomination can be said to have a profit p_j equal to the monetary (or face) value of the note. There are, however, significant differences between the BKP and the suggested algorithm below.

$$\text{Minimise} \quad \sum_{j=1}^{4} u_j \tag{5.7}$$

$$\text{subject to} \quad \sum_{j=1}^{4} p_j u_j = A_K, \tag{5.8}$$

$$p_1 = 20, \tag{5.9}$$

$$p_2 = 50, \tag{5.10}$$

$$p_3 = 100, \tag{5.11}$$

$$p_4 = 200, \tag{5.12}$$

$$0 \leq u_j \leq b_j, \qquad u_j \text{ integer}, \qquad j = 1, 2, 3, 4. \tag{5.13}$$

According to the logic that dispensing as few notes as possible would lead to the fewest replenishment runs and subsequently to the lowest total cash management cost, the number of notes to be dispensed is minimised, instead of maximising profit to be achieved from a selection of items (as in the BKP). Given that a note is a note, irrespective of its denomination, every note dispensed effectively has the same weight: $w_j = 1$. Minimising the number of notes versus maximising the profit of the selected items, results in differences between Equations 5.4 and 5.7.

It is also not acceptable for an ATM to dispense less (or more) than the amount requested by the customer c. (Using the '$\leq$'-operator whilst minimising the number of notes dispensed, would result in the ATM never dispensing any notes.) Instead, the '='-operator is used in Equation 5.8.

Finally, there are only four types of notes. This is taken into account in Equation 5.13.

5.4 A numerical example of the denomination dispensing algorithm

The algorithm described above is an instance of least-note-picking denomination dispensing. The mathematical formulation laid out in Section 5.3 can easily be illustrated at the hand of a numerical example.

A customer demands R 350. Assuming that all denominations are in stock, the algorithm will dispense one R 200, one R 100 and one R 50 note. If, however, R 200 notes are out of stock, the algorithm will dispense three R 100 and one R 50 notes. If R 100 notes are out, one R 200 and three R 50 notes will be dispensed. For the case where both R 200 and R 100 notes are depleted, seven R 50 notes will be dispensed. If there are no R 50 notes, a cash out will be registered as it is impossible to make up R 350 from only R 200, R 100 and R 20 notes.

5.5 Algorithms for solving integer programming problems

Several techniques for solving IP problems exist. The most intuitive methods are the *greedy algorithm* and *linear programming relaxation* (Kellerer *et al.*, 2004). Even though both the methods are straightforward, these methods deliver solutions which may be far from the true optimum. To obtain the optimal solution to integer programming problems either *dynamic programming* or the *branch-and-bound* approach can be used (Kellerer *et al.*, 2004).

Dynamic programming (DP) is an optimisation technique developed by Richard Bellman in the early 1950's. Kasana & Kumar (2004) makes the remark that although dynamic programming is a beautifully simple technique, it lacks computational efficiency. As a result the branch-and-bound algorithm will be used for solving the knapsack problem in this project.

It is worth noting that several software packages capable of solving integer programming problems exist. Both the Lindo and Lingo packages have such capabilities. Microsoft Excel's *Solver* can also be used for solving IP problems.

5.6 Conclusion: An algorithm for dispensing denominations

This chapter discussed the necessity of clearly defining an algorithm according to which notes are dispensed. Given that partial notes should not be dispensed, integer programming was investigated as a solution area. The bounded knapsack problem was adapted to be used as the suggested algorithm. Finally, algorithms with which the BKP can be solved were mentioned.

The next chapter will provide information about the simulation model; its function as a decision support model and variables controlled during the simulation study.

Chapter 6

Simulation study

The previous chapters provided details on knowledge required for developing the simulation model. Details about the design, implementation and validation of the simulation model are provided in Appendices A, B and C respectively.

This chapter will provide insight into the simulation model developed for this project as well as its use as a decision support system. Variables that are controlled for the simulation study experiments will briefly be discussed. Lastly, information on the execution of these experiments will be provided.

6.1 The simulation model developed for the final year project

Chapter 1 specifies that du Toit (2011)'s work must be refined using discrete event simulation: a simulation model was to be built with which the effects of the VRP and (s, S) policy on an ATM network could be investigated.

The simulation model built for the project is made up of two main and several secondary processes. The first primary process is the arrival and servicing of customers. The ATM network network modelled consists of 18 ATMs. Customers arrive at each ATM according to a unique, seasonal arrival rate. After arrival, customers demand an amount drawn from a distribution which is also unique for every ATM. Each customer is served using the note dispensing algorithm discussed in Chapter 5. With each customer arrival and service, inventory levels in an ATM drops. Once inventory levels in an ATM fall below s the reorder point, a reorder flag is put up for said ATM, registering an order. The reorder point can easily be varied, so as to evaluate changes to it.

The second primary process is vehicle routing. The number of reorder flags are monitored periodically. At the beginning of each day all reorder flags are counted and if the number of flags is greater than the routing point, vehicles are routed. (The routing point is defined as the minimum number of reorder flags that have to be up before a replenishment route will be determined.) To be able to evaluate the effect of the VRP, there are three different methods according to which routes are determined. These methods are discussed in Section 6.3.1. Determining routes is only the first step in the replenishment process. After replenishment routes have been determined, vehicle availability must be taken into account: only vehicles that are at the count house can be loaded and sent on a route. The time required to complete a route must also be considered: a vehicle can only be sent out if there is enough time left in the day to complete a route. If the time available is too little, the route can only be completed the following day. Once these factors have been accounted for, available vehicles are loaded and sent out. Vehicles travel from the count house to all ATMs on an assigned route, replenishes the ATMs and then returns to the count house.

Secondary processes merely enable primary processes.

The simulation model supports MOO decision making as will be discussed in the section to follow.

6.2 A decision support model for vehicle routing in the retail banking industry

The aim of this project (as mentioned before) is to determine how the service level of an ATM network can be maximised, whilst keeping cost to a minimum. The nature of MOO problems is such that a Pareto optimal set of scenarios need to be found rather than a single optimal solution. Finding such a set requires changes to be made to the system so that the effects of these changes can be observed, evaluated and compared to one another.

Simulation modelling makes it possible to experiment with a model of a system rather than experimenting with the actual system. It enables the simulation analyst to ask what-if questions about the system (Bekker, 2011b).

Many what-if questions can be asked about the modelled ATM network. Naturally, all these questions can never be asked and answered. The Pareto principle is a concept well known to any fourth year Industrial engineering student. It states that 20% of causes are responsible for 80% of effects. The student hypothesises that, similarly, 80% of telling information about the system can be obtained by asking the most relevant 20% of the what-if questions.

Section 6.3 discusses the questions which the student deemed most important whereas Section 6.4 briefly looks at some questions that might also be asked.

Details of the simulation model are contained in the appendices: the functional specification can be found in Appendix A, key algorithms used for implementation are presented in Appendix B and results summaries are contained in Appendix D.

6.3 Controlled variables

There are many variables in the developed simulation model. Only the most significant ones were controlled during experiments. The student's arguments about the significance of these variables are discussed below.

6.3.1 Routing method

A secondary objective of the final year project was to evaluate the use of the VRP in the retail banking industry. In order to do this, the routing method should naturally be varied. An essential question to ask was: what routing alternatives should the VRP be compared to?

It was decided that the VRP (which is referred to as the near shortest path method) could effectively be compared to both an interpretation of the status quo and a second intuitive routing method: *First-in-first-out routing*. As discussed in Section 4.2, the routing method preferred by the bank is referred to as *direct replenishment* and dispatches vehicles on an one-ATM-at-a-time basis. Vehicles are dispatched directly to ATMs as the ATMs register orders.

First-in-first-out (FIFO) routing follows intuitively from direct replenishment as routes are determined based on the sequence in which ATMs register orders. Unlike the direct replenishment method, however, FIFO routes contain more than one ATM.

Finally, the near shortest path (NSP) method aims to minimise the length of routes.

6.3.2 Number of vehicles

The bank currently uses two dedicated CIT trucks for the ATM network in question. It seems sensible to investigate what the effect of adding or removing a vehicle would be. The student thus experimented with scenarios where one, two or three vehicles were available.

6.3.3 Routing point

Routing point refers to the minimum number of ATMs that need to have registered an order, before a route will be determined. It is largely dependent on the routing method: direct replenishment routes contain only one ATM and routes are subsequently created at a routing point equal to one. FIFO and near shortest path routing on the other hand have to contain more than one ATM and the routing point has to be greater or equal to two.

Whilst the student was developing the model, she varied the routing points for the FIFO and near shortest path method for testing purposes. She was curious as to what the effect of these alterations would be when made to the final model. Is it better to determine a route as soon as possible or wait just a bit longer?

The routing point is thus dependently varied based on the routing method, but for the two latter methods cases for routing points equal to two and three were set up.

6.3.4 Minimum number of ATMs on a route

The *minimum number of ATMs on a route* (MNAR) differs from the routing point in that the routing point refers to the number of ATMs that must have registered an order before a route can be determined, whereas the MNAR specifies a condition for route validity. If more ATMs require replenishment than can be handled with one route, the MNAR prevents routes that are too short to qualify as valid FIFO or near shortest path routes from being created.

Similar to the routing point, the minimum number of ATMs on a route is dependent on the routing method. If there are fewer than two ATMs on a route for the near shortest path and FIFO methods, these methods effectively yield direct replenishment routes. Note that an exception is made for ATM 5 when using the near shortest path routing method (as discussed in Section 4.3). Likewise, if a direct replenishment route has to contain more than one ATM, it is effectively no longer a direct replenishment route.

For the direct replenishment method, then, the MNAR is equal to one. For the other two methods, the MNAR is equal to two. The MNAR was not varied for these two methods because of an observation the student made whilst developing the simulation model: the distances between ATMs are often too large for routes containing more than two ATMs to meet all requirements.

6.3.5 Reorder point

The *reorder point* refers to the inventory level at which an ATM triggers an order. As discussed in Section 3.3, many variations are possible for defining the reorder point as either the cash value or the number of notes in an ATM can be seen as the primary measurement of inventory level.

For the purposes of this project, the cash value of notes left in an ATM will serve as measurement of the inventory level in the ATM. The reorder point for all ATMs in the model are the same. Reorder points were varied from R 150 000 to R 350 000 to R 550 000.

6.3.6 Cost structuring

The student ran 45 experiments set up for the vehicle cost structuring currently used by the bank. The results obtained from these experiments are discussed in Chapter 7. There were some patterns in these results which begged the question: what would happen if the cost structure was different?

The student then created an additional cost structure. This cost structure was meant to highlight variations that would take place due to big changes to the cost structure. It was decided to halve the cost of vehicles per month, whilst quadrupling the cost of kilometers falling outside of the free kilometers that make out part of the CIT service. The amount of free kilometers were reduced to compensate for the lower vehicle cost. The two cost scenarios are shown in Table 6.1. Forty-five additional experiments were run using the adjusted cost structure.

Cost component	**Status quo cost scenario**	**Adjusted cost scenario**
Cost of vehicle per month	R 78 710	R 39 355
Free kilometers	4 500	2 500
Cost of additional kilometers	R 3.18 per km	R 12.72 per km

Table 6.1: Status quo cost structure compared to additional adjusted cost structure.

6.4 Uncontrolled variables

Variables that were deemed less significant were not adjusted during experiments. Adjustments left uninvestigated include:

- The effect of changing the make up of notes inside an ATM: instead of filling two canisters with R 200 notes, rather fill three canisters with R 100 notes and so on.
- Using the number of notes in an ATM as measurement for inventory level: for example, registering an order when 2000 notes are left or when 750 R 200 notes remain.
- The algorithm used for dispensing notes could also be varied.

6.5 Simulation study experiments

Simulation study experiments represent different system scenarios. Each scenario is made up of a set of conditions (controlled variables are set to a predetermined value). A scenario would for instance use NSP routing, with two vehicles available, a routing point equal to two and a reorder point of R 350 000 whilst costs are determined using the status quo cost structure. The status quo cost structure might be used for a similar scenario which also uses NSP routing, with two available vehicles, a routing point of two ATMs but has a reorder point of R 550 000. 90 such scenarios were experimented with. The 90 scenarios are primarily categorised by the cost structure used. The first half of the experiments were done using the status quo cost structure, whereas the latter half experimented with the adjusted cost structure.

Conditions of all simulation study experiments are summarised in Tables 6.2 and 6.3. Table 6.2 shows the experiments for the status quo cost structure whereas Table 6.3 contains details of the adjusted cost scenario experiments.

	Routing method			Number of vehicles			Routing point			MNAR		Reorder point		
	Near shortest path	**First-in-first-out**	**Direct replenishment**	**One**	**Two**	**Three**	**One**	**Two**	**Three**	**One**	**Two**	**R150 000**	**R350 000**	**R 550 000**
Experiment 1	*			*				*			*	*		
Experiment 2	*			*				*			*		*	
Experiment 3	*			*				*			*			*
Experiment 4	*			*					*		*	*		
Experiment 5	*			*					*		*		*	
Experiment 6	*			*					*		*			*
Experiment 7	*				*			*			*	*		
Experiment 8	*				*			*			*		*	
Experiment 9	*				*			*			*			*
Experiment 10	*				*				*		*	*		
Experiment 11	*				*				*		*		*	
Experiment 12	*				*				*		*			*
Experiment 13	*					*		*			*	*		
Experiment 14	*					*		*			*		*	
Experiment 15	*					*		*			*			*
Experiment 16	*					*			*		*	*		
Experiment 17	*					*			*		*		*	
Experiment 18	*					*			*		*			*
Experiment 19		*		*				*			*	*		
Experiment 20		*		*				*			*		*	
Experiment 21		*		*				*			*			*
Experiment 22		*		*					*		*	*		
Experiment 23		*		*					*		*		*	
Experiment 24		*		*					*		*			*
Experiment 25		*			*			*			*	*		
Experiment 26		*			*			*			*		*	
Experiment 27		*			*			*			*			*
Experiment 28		*			*				*		*	*		
Experiment 29		*			*				*		*		*	
Experiment 30		*			*				*		*			*
Experiment 31		*				*		*			*	*		
Experiment 32		*				*		*			*		*	
Experiment 33		*				*		*			*			*
Experiment 34		*				*			*		*	*		
Experiment 35		*				*			*		*		*	
Experiment 36		*				*			*		*			*
Experiment 37			*	*			*			*		*		
Experiment 38			*	*			*			*			*	
Experiment 39			*	*			*			*				*
Experiment 40			*		*		*			*		*		
Experiment 41			*		*		*			*			*	
Experiment 42			*		*		*			*				*
Experiment 43			*			*	*			*		*		
Experiment 44			*			*	*			*			*	
Experiment 45			*			*	*			*				*

Table 6.2: Simulation model experiments with status quo cost structure.

	Routing method			**Number of vehicles**			**Routing point**			**MNAR**		**Reorder point**		
	Near shortest path	**First-in-first-out**	**Direct replenishment**	**One**	**Two**	**Three**	**One**	**Two**	**Three**	**One**	**Two**	**R150 000**	**R350 000**	**R 550 000**
Experiment 46	*			*				*			*	*		
Experiment 47	*			*				*			*		*	
Experiment 48	*			*				*			*			*
Experiment 49	*			*					*		*	*		
Experiment 50	*			*					*		*		*	
Experiment 51	*			*					*		*			*
Experiment 52	*				*			*			*	*		
Experiment 53	*				*			*			*		*	
Experiment 54	*				*			*			*			*
Experiment 55	*				*				*		*	*		
Experiment 56	*				*				*		*		*	
Experiment 57	*				*				*		*			*
Experiment 58	*					*		*			*	*		
Experiment 59	*					*		*			*		*	
Experiment 60	*					*		*			*			*
Experiment 61	*					*			*		*	*		
Experiment 62	*					*			*		*		*	
Experiment 63	*					*			*		*			*
Experiment 64		*		*				*			*	*		
Experiment 65		*		*				*			*		*	
Experiment 66		*		*				*			*			*
Experiment 67		*		*					*		*	*		
Experiment 68		*		*					*		*		*	
Experiment 69		*		*					*		*			*
Experiment 70		*			*			*			*	*		
Experiment 71		*			*			*			*		*	
Experiment 72		*			*			*			*			*
Experiment 73		*			*				*		*	*		
Experiment 74		*			*				*		*		*	
Experiment 75		*			*				*		*			*
Experiment 76		*				*		*			*	*		
Experiment 77		*				*		*			*		*	
Experiment 78		*				*		*			*			*
Experiment 79		*				*			*		*	*		
Experiment 80		*				*			*		*		*	
Experiment 81		*				*			*		*			*
Experiment 82			*	*			*			*		*		
Experiment 83			*	*			*			*			*	
Experiment 84			*	*			*			*				*
Experiment 85			*		*		*			*		*		
Experiment 86			*		*		*			*			*	
Experiment 87			*		*		*			*				*
Experiment 88			*			*	*			*		*		
Experiment 89			*			*	*			*			*	
Experiment 90			*			*	*			*				*

Table 6.3: Simulation model experiments with adjusted cost structure.

A single replication of the simulation model takes about 20 seconds to run. To achieve satisfactory confidence interval half-widths, the student decided on running 80 replications per experiment. As a result, every experiment had a simulation run time of about 25 minutes. Including setups and a few reruns, running 90 experiments would take about 40 hours on a single computer. Three computers were available for experiments, so the run time of scenarios for the final year project added up to around 13 hours (excluding experimental runs and reruns).

6.6 Conclusion: Simulation study

This chapter started by illustrating how the simulation model serves as a decision support system. Next, the variables controlled for simulation study experiments were laid out. Comments were then made about the execution of these experiments. Finally, details on simulation study experiments were summarised in Tables 6.2 and 6.3.

Chapter 7 will provide an analysis of simulation study results, whilst Chapter 8 will conclude the final year project report.

Chapter 7

Analysis of simulation study results

The previous chapter discussed the simulation model as a decision support model and briefly laid out how simulation study experiments were set up. Conditions of all simulation study experiments were summarised in Tables 6.2 and 6.3. It was explained that the results yielded by the first 45 experiments led to an additional 45 experiments. Table 6.2 showed the experiments for the status quo cost structure whereas Table 6.3 contained details of the adjusted cost scenario experiments.

This chapter will provide analysis of the results of these experiments. The discussion of the results is structured so as to illustrate the effects of the cost structure, number of available vehicles, routing method, routing point and reorder point. A section is devoted to each of these control variables. There is a significant difference between the results obtained using the status quo cost structure and the results of the adjusted cost structure experiments. Each control variable section therefore discusses results for status quo cost structure experiments and the adjusted cost scenario experiments separately, before general conclusions are drawn about the effect of the control variable.

It is important to note that there were a total of 90 experiments. These experiments can be categorised according to any of the control variables. As mentioned already, 45 experiments were run using each of the two cost structures. 30 experiments used a reorder point equal to R 150 000, another 30 were run with the reorder point set to R 350 000 and the remaining 30 used a reorder point of R 550 000. Similarly, 36 experiments used near-shortest-path-routing, 36 experiments used FIFO routing and for the remaining 18 experiments, vehicles were routed according to the direct replenishment approach. The plot sets presented all contain the same sets of data: all plots labelled "Status quo cost structure" contain the 45 experiments done using said structure, plots labelled "Adjusted cost structure" contain the 45 experiments done using this structure and plots labelled "Overall" show all 90 experiments. For each control variable, however experiments are colour coded to illustrate the effect of the control variable.

To further simplify this results analysis, two informal performance indicators need to be defined:

> **Routing efficiency** (or replenishment efficiency) refers to the number of ATMs that are replenished using a single route. If many ATMs are replenished, routing efficiency is high.
>
> **Speed of service** refers to the time elapsing between the moment an ATM registers an order and its replenishment. If little time elapsed, speed of service is high.

7.1 The effect of the cost structure

Figure ?? shows the difference between results for the status quo and the adjusted cost structures. The status quo cost structure resulted in three bundles of results. This is due to the fact that vehicle cost for this scenario is so large that adding a vehicle results in a giant step in cost. These steps made it especially difficult to analyse the effect of routing methods on the system.

The additional cost structure was thus created in such a manner that it would place greater emphasis on distance cost than the status quo. To shift the emphasis, vehicle cost in the new structure was reduced and the cost associated with distance covered increased greatly.

No adjustments were made to rebanking and opportunity costs. All differences observed between the two cost structures are a result of only the reduced vehicle and increased distance costs. There is no difference in operational performance measures

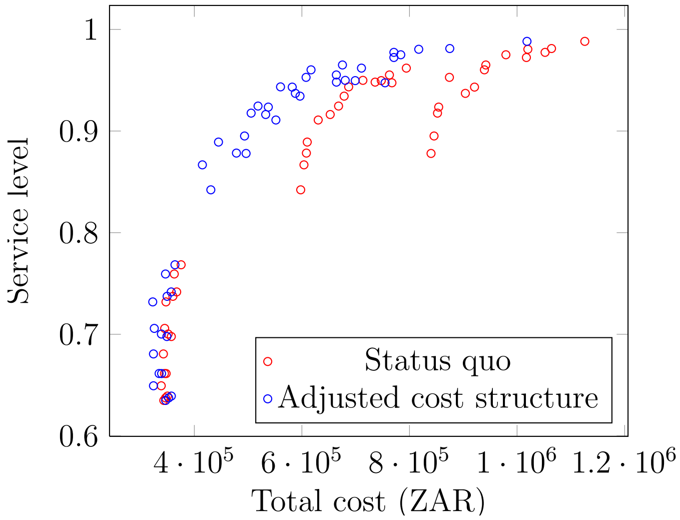

Figure 7.1: Overall difference between status quo and adjusted cost structures.

(such as service level, distance travelled, the number of routes determined, vehicle utilisation etc.) between the two cost scenarios.

There is more cost variation in the results obtained for the adjusted cost structure than for the status quo. Despite the higher levels of variation, the adjusted cost structure yields lower cost results than the status quo for the majority of the cases.

7.2 Varying the number of vehicles

Three different vehicle scenarios were experimented with (as explained in Chapter 6). One, two or three vehicles were available. The effect of the number of available vehicles will now be discussed.

7.2.1 Varying the number of vehicles: Status quo cost structure

The current cost structure has a large fixed vehicle cost component. Total cost jumps with every vehicle addition (see Figure 7.2). Service level also improves significantly for the cases where two or three vehicles are available. One vehicle is incapable of effectively servicing the entire ATM network.

The cost associated with operating only one vehicle varies little. This is due to the fact that rebanking cost and opportunity cost (which are variable components of total cost) is limited in a sense: due to the low service level achieved for all cases, there

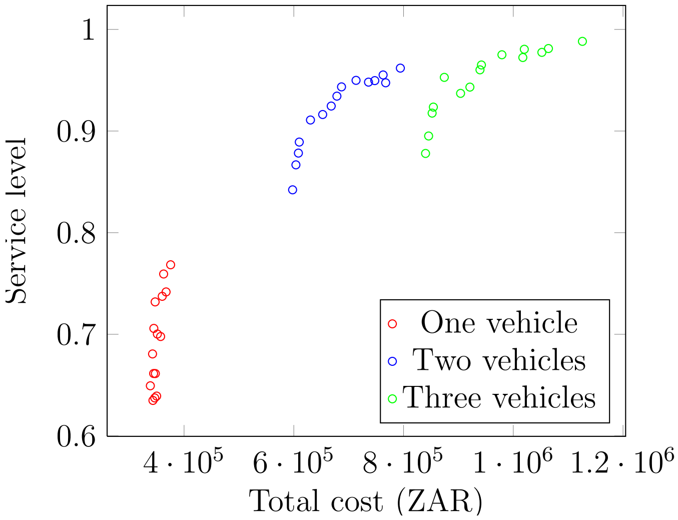

Figure 7.2: Effect of number of vehicles available: Status quo cost structure.

is rarely any cash left in the ATM at time of replenishment (low rebanking cost) and ATMs are often empty (low opportunity cost). Note that there is a very slight increase in cost as service level goes up.

The range of service levels achieved for the one vehicle case is wide. This is primarily due to differences in the replenishment efficiencies of the various vehicle routing methods (discussed in Section 7.3.1). Routing methods with higher efficiencies are able to reach ATMs faster, resulting in higher service levels. ATMs are still not reached fast enough and the service level achievable for one vehicle is significantly lower than for two or three vehicles.

Because service levels are higher for the latter two cases, variable cost is larger (due to higher rebanking and opportunity costs) and there is more variation in the total cost.

7.2.2 Varying the number of vehicles: Adjusted cost structure

The leaps in total cost values observed when vehicles were added using the status quo cost structure lead to the addition of the adjusted cost structure. Results for this structure are shown in Figure 7.3. Cost no longer shows the drastic stepping visible for the status quo cost structure.

The fixed cost component (vehicle cost) of the adjusted structure is smaller than

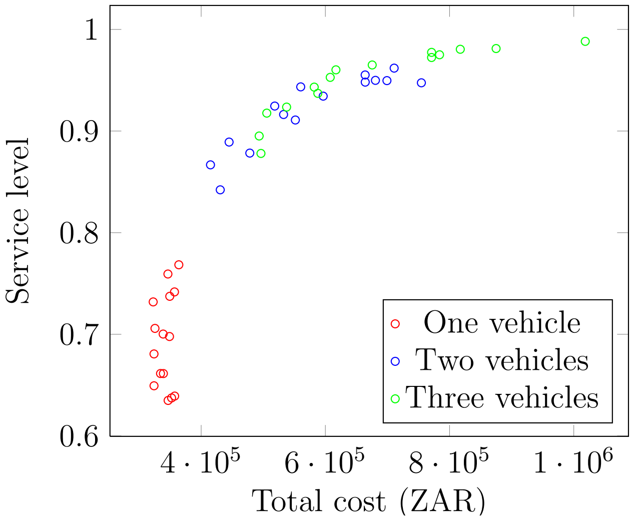

Figure 7.3: Effect of number of vehicles available: Adjusted cost structure.

that of the status quo. The opposite is true of distance cost which is a variable cost component. Distance cost is larger for the adjusted structure than for the status quo. Due to higher variation in total cost, the steps in total cost are less significant and there is more of an overlap visible between the cases where two and three vehicles are available.

Similar to the status quo scenario, when only one vehicle is available, the total cost varies little due to low rebanking and opportunity costs. These low costs result from low service levels. Note that, compared to the current cost structure, there is more variance in total cost for one vehicle when the adjusted cost structure is used. This variance is due to the fact that distance cost becomes significant in the latter case.

7.2.3 Varying the number of vehicles: General observations

In general, distance cost increases from one vehicle to two, but drops from two to three vehicles. This can be explained by the fact that service level improves significantly when two vehicles are available instead of only one. Achieving the higher service level requires that a greater overall distance be covered (also note the increased number of routes completed). Two vehicles are able to cover this distance, where one vehicle was incapable. The increase in service level from two to three vehicles is not as great and subsequently the difference in distance covered is not as large either. Distance cost

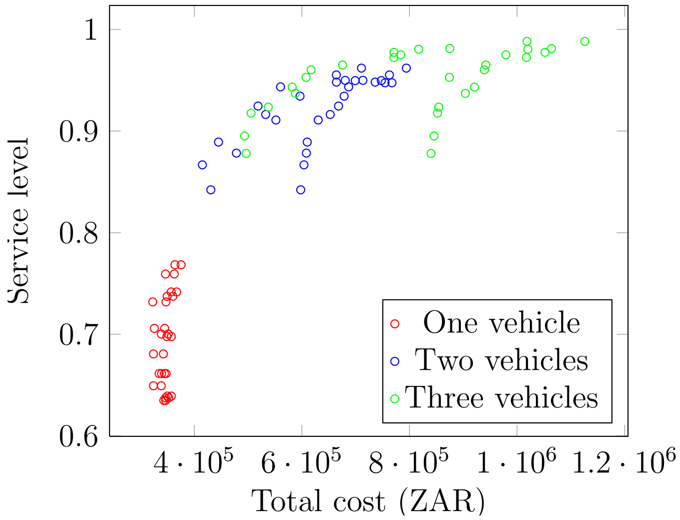

Figure 7.4: Overall effect of the number of vehicles available.

drops because of the cost structuring: for every vehicle used, there are free kilometers available in a month. Adding a vehicle thus increases the amount of free kilometers which leads to a reduction in distance cost.

Average vehicle utilisation decreases as the number of vehicles increase. Vehicle utilisation is calculated in Equation 7.1. Workload per vehicle decreases as vehicles increase. It should be noted that the way average vehicle utilisation is calculated can be misleading. This is because vehicles can sometimes complete only one route in a day. There might be vehicle time left in the day, but not enough to complete another route. In this case vehicles will wait until the following day to perform a route. Vehicle utilisation is never near 100 %. The low utilisation does not mean that a vehicle did not complete all the routes it possibly could. For this reason vehicle utilisation should not be interpreted as absolute values, but rather for the trends that can be identified from these utilisation values.

$$\textit{Average vehicle utilisation} = \frac{\textit{Total vehicle hours available} - \textit{Total vehicle hours not used}}{\textit{Number of vehicles} \times \textit{Vehicle hours available}} \tag{7.1}$$

Together with trends in vehicle utilisation, trends in the number of routes completed for each vehicle scenario are also telling. The number of routes completed increases drastically from one vehicle to two. From two vehicles to three, the increase in number of routes completed is less. This is an indication that for one vehicle, a lot of work exists which cannot be completed. For two vehicles, uncompleted work is less.

Figure 7.4 shows that, generally, using three vehicles achieves the highest service levels at very high cost. Having two vehicles available results in a slight decrease in service level at significantly reduced cost. Routing only one vehicle yields poor service levels at low cost.

7.3 Varying the routing method

An important objective of this study was to evaluate the effect of the VRP for use in the retail banking environment. Chapter 6 briefly discussed the three routing methods that are experimented with: near shortest path (NSP) routing (the implementation of the VRP), first-in-first-out (FIFO) routing and direct replenishment (DR). The respective effects of these routing methods on the system are discussed in this section.

7.3.1 Varying the routing method: Status quo cost structure

FIFO and NSP routes are primarily distance constrained: finding the shortest path between, for example, four ATMs might not exceed this constraint, whereas FIFO routing for the same four ATMs would. The NSP route determined would service four ATMs, whilst one FIFO route services only three (even two) ATMs. Direct replenishment routes contain only one ATM. On average, a single NSP route serves more ATMs than FIFO or DR routes.

Especially for the cases where one or two vehicles are available, NSP routing results in higher service levels than the other two methods. Thus, if vehicle availability is constrained, NSP routing achieves higher service levels due to its replenishment efficiency (an informal measurement of the number of replenishments done on a single route). For the case where three vehicles are available, vehicle availability is not limited and direct replenishment outperforms NSP on service level. For this case, there is a difference in the performance of FIFO and NSP. The one does not, however, perform better than the other.

Due to the fact that variable cost makes out a small part of the total cost for the status quo structure, the effect of the routing methods on cost is not clear. If three vehicles are available, the cost of direct replenishment routing is the highest: if

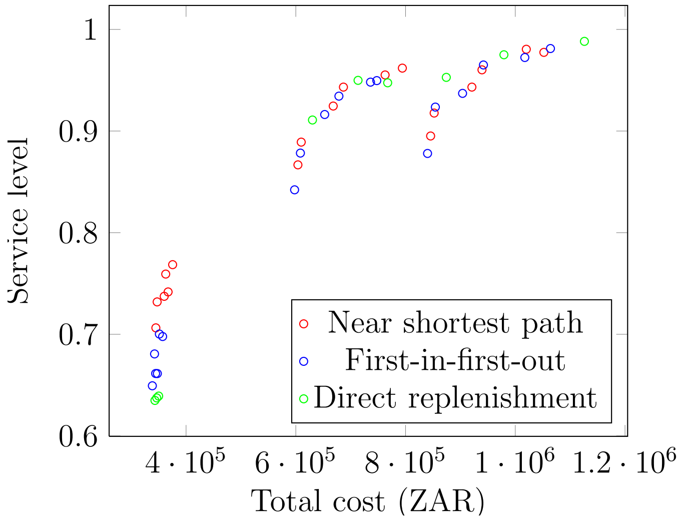

Figure 7.5: Effect of routing method: Status quo cost structure.

vehicle availability is not constrained, the distance covered by DR routing is significantly greater than the distance covered by the other two methods. Covering more kilometers results in an increased distance cost. The service levels achieved using this method are high when three vehicles are available. As previously discussed, achievement of high service levels leads to an increase in rebanking and opportunity cost.

7.3.2 Varying the routing method: Adjusted cost structure

Due to the fact that fixed vehicle cost makes out a smaller and variable distance a larger part of total cost for this the adjusted cost structure, differences due to routing methods are more pronounced. Figure 7.6 shows this.

For the cases where two or three vehicles are available, NSP routing consistently yields high service levels at comparatively low costs. Although the cost associated with DR is generally the highest, service levels are average. Direct replenishment effectively yields only one outstanding result: the experiment returning the highest service level was a DR experiment. The cost associated with this experiment is also the highest. FIFO routing yields results between those of NSP and direct replenishment.

As discussed in Sections 7.2.1 and 7.2.2, there is little variation in the costs associated with the scenario where only one vehicle is available. Direct replenishment yields the lowest service levels at the highest cost. On a cost basis FIFO slightly outperforms

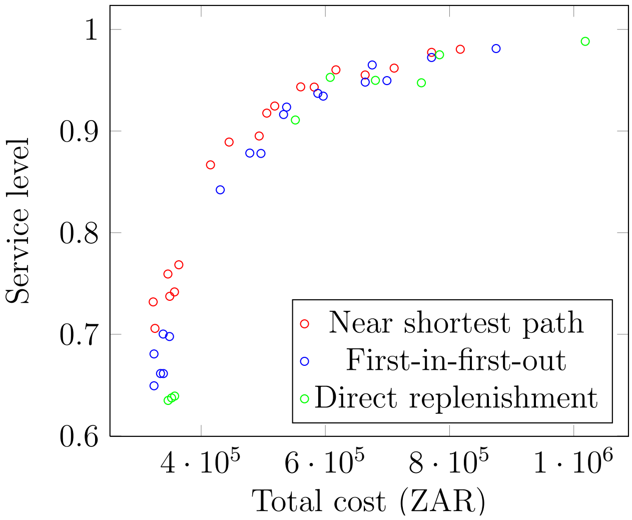

Figure 7.6: Effect of routing method: Adjusted cost structure.

NSP, whilst NSP significantly outperforms FIFO when looking at service level.

7.3.3 Varying the routing method: General observations

Vehicle utilisation is highest when direct replenishment is used and lowest for near shortest path routing. Likewise, the number of routes completed using DR routing is the highest, whereas NSP requires the least number of routes.

Looking at Figure 7.7, it can generally be observed that near shortest path routing yields most of the high-service-level-at-low-cost results. The very best service level is achieved using DR. This very high service level (98.824%) is achieved at a very high cost (R 1 126 000). The worst service level also results from DR. FIFO routing yields average results throughout.

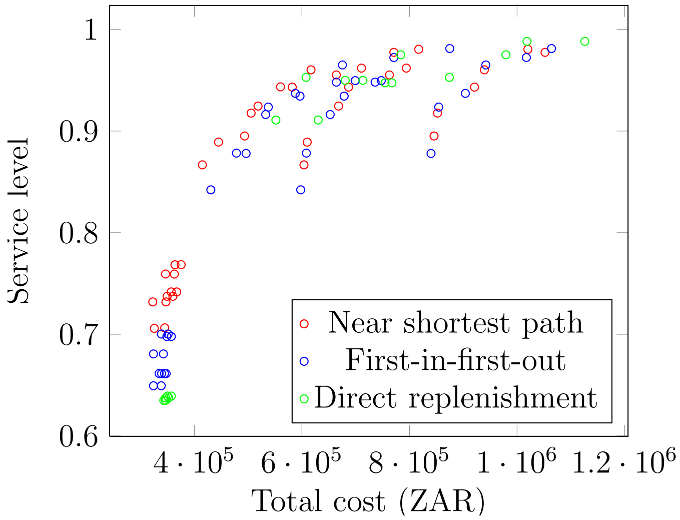

Figure 7.7: Overall effect of routing method.

7.4 Varying the routing point

As discussed in Section 6.3.3, the routing point is dependent on the routing method. Results obtained for a routing point equal to one are the same as results obtained for DR routing. These results should therefore be interpreted for what they are: an indication of the effectiveness of direct replenishment. The scenarios where the routing point is greater than one are applicable only to the NSP and FIFO routing methods. Although results for all experiments are shown, only the NSP and FIFO results will be discussed.

7.4.1 Varying the routing point: Status quo cost structure

For the status quo cost structure, the difference between results achieved for routing points equal to two and three is small. This is shown in Figure 7.8.

For the scenarios were one vehicle is used, routing vehicles when at least three ATMs require replenishment yields higher service levels than a routing point equal to two. This can be explained by the fact that fewer routes are created for the former case and route efficiency (the number of replenishments done per route) tends to be higher. As mentioned in Section 7.3.1, when vehicle availability is constrained, route efficiency is critical. Section 7.3.1 pointed out that NSP is the most efficient routing method. A routing point equal to three is the most route effective routing point.

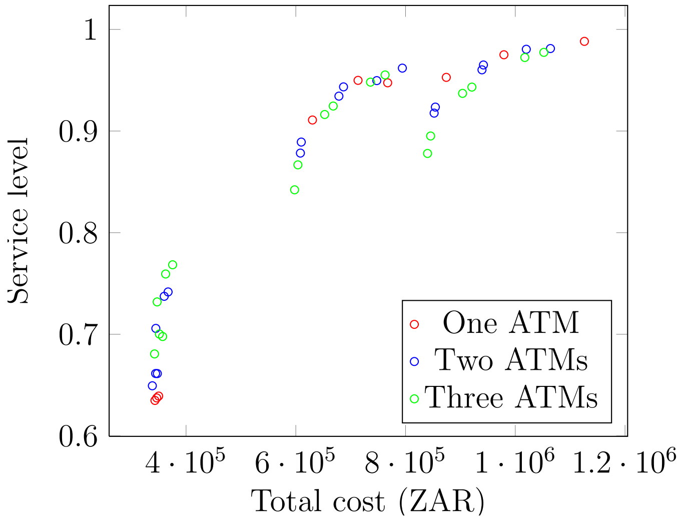

Figure 7.8: Effect of the routing point: Status quo cost structure.

For two and three vehicles, vehicle availability is less constrained and route efficiency is less important. For these cases a routing point equal to two yields better service levels. This is because (more) routes are constructed more often. As a result ATMs are replenished more often. The cost associated with a routing point equal to three is slightly less than that of a routing point equal to two. This can be attributed to the increased number of routes constructed for the latter case and the associated increase in covered distance (and distance cost). The fact that a routing point equal to three leads to ATMs not being serviced as soon as with a routing point equal to two, means that opportunity and rebanking costs are also slightly lower for the former case.

7.4.2 Varying the routing point: Adjusted cost structure

Except for a slight increase in cost variation (which is visible in Figure 7.9), the results obtained for the adjusted cost structure case are similar to those obtained for the status quo. There is no difference in the justification of the results. For the one vehicle scenario, a routing point equal to three yields better service level than a routing equal to two. The difference in cost between these two routing points are negligible.

For the cases where more than one vehicle is available, routing vehicles when at least two ATMs require replenishment yields better service levels. Routing vehicles only when at least three ATMs require replenishment, results in lower costs for these

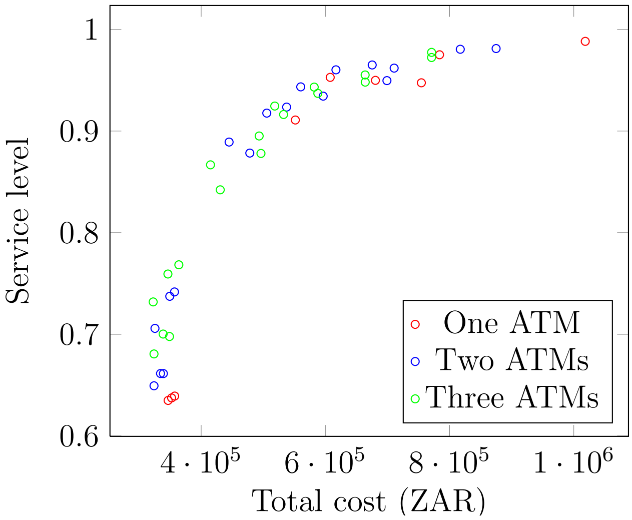

Figure 7.9: Effect of the routing point: Adjusted cost structure.

vehicle scenarios.

7.4.3 Varying the routing point: General observations

Vehicle utilisation is higher for a routing point equal to two than it is for three. As mentioned in Section 6.3.3, the number of routes completed is higher for the former case.

Although the effect of the routing point is noticeable in Figure 7.10, it is small. The routing point clearly affects service level, but its effect on cost is minimal. In short, to obtain as high a service level as possible, a higher routing point should be used if vehicle availability is heavily constrained, whilst a low routing point yields better results if vehicles are available. If cost is a big concern, high routing points result in lower costs as distance cost decreases as the routing point is incremented.

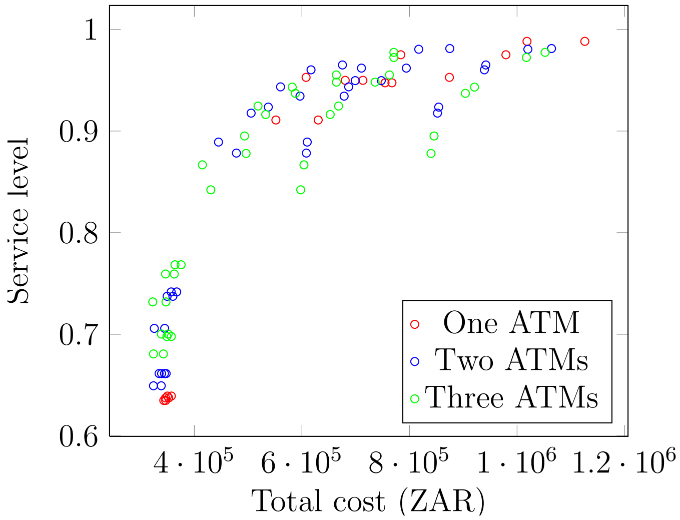

Figure 7.10: Overall effect of the routing point.

7.5 Varying the reorder point

The reorder point was varied from R 150 000, R 350 000 to R 550 000. The results of these different scenarios are discussed below.

7.5.1 Varying the reorder point: Status quo cost structure

The higher the reorder point, the earlier an order is placed. High reorder points therefore result in high service levels. High reorder points also result in high total costs. This can be seen in Figure 7.11 and is due to the fact that opportunity and rebanking costs increase drastically as the reorder point increases. As the reorder point goes up, routes are determined more frequently and the number of routes that are covered rise. The distance covered increases and the associated cost as well.

7.5.2 Varying the reorder point: Adjusted cost structure

Similar to the status quo cost scenario, higher reorder points yield better service levels at greater cost. The adjusted cost structure shown in Figure 7.12 accentuates this.

7.5.3 Varying the reorder point: General observations

The reorder point has a significant effect on both service level and total cost. It is directly related to both.

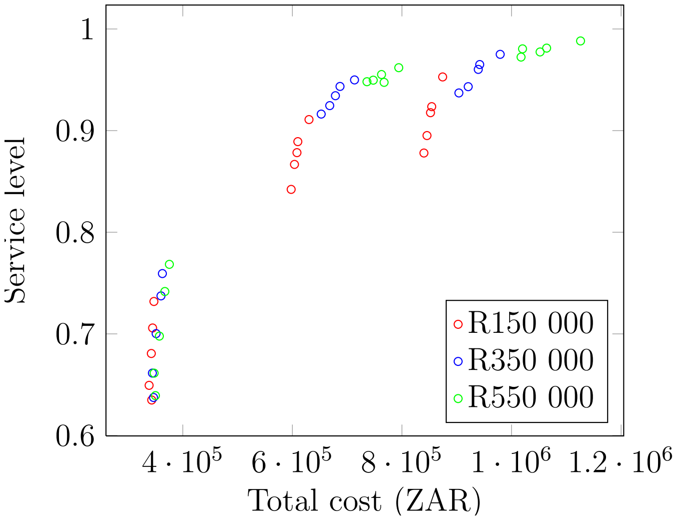

Figure 7.11: Effect of the reorder point: Status quo.

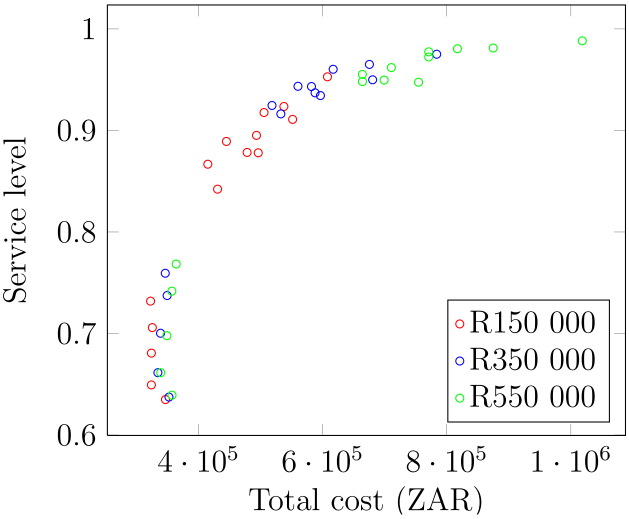

Figure 7.12: Effect of the reorder point: Adjusted cost structure.

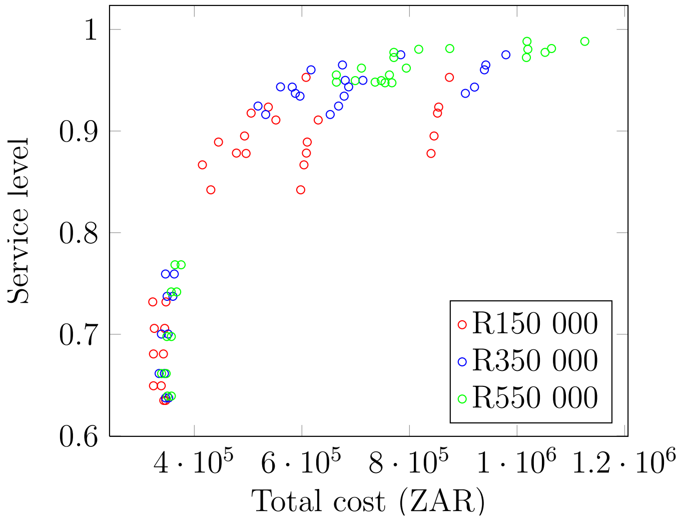

Figure 7.13: Overall effect of the reorder point.

As the the reorder point rises, the number of routes determined and covered escalates. Due to the fact that more routes need to be executed, vehicle utilisation goes up as the reorder point gets bigger. As the number of routes that are executed goes up, the total distance covered increases. Increased distance leads to higher distance cost.

An increase in service level leads to higher rebanking and opportunity costs.

7.6 Summary of global observations

To the student's mind the scenarios discussed above provide a thorough indication of the nature of the ATM network modelled and the effect of certain variables thereon.

An interesting *total cost* → *service level* → *total cost* cycle was observed: adjusting the routing variables (increasing the number of vehicles or lowering the routing point) increases total cost due to higher transportation costs but leads to a service level improved. The improved service level, in turn, results in an escalation of opportunity and rebanking costs.

Other than this observation, main points that deserve to be highlighted are:

- Total cost and service level are both directly related to the number of vehicles available. For one available vehicle, vehicle availability is heavily constrained and results exhibit characteristics that differ from scenarios where more than one vehicle is available.

- Near shortest path routing consistently provides high service levels at low costs (when compared to cost resulting from the other two routing methods). If vehicle availability is not heavily constrained, direct replenishment is able to yield very high service levels at high total costs.
- Although the routing point has an effect on results, there is no definite relation between it and service level or total cost.
- The reorder point is directly related to both service level and total cost.

The observation that the system reacts differently when vehicle availability is heavily constrained and when it is not is important. For the heavily constrained case, routing efficiency is of utmost importance. For the less constrained scenarios, high speed of delivery (a measure of the time between order placement and order fulfilment) yields good results.

7.7 Conclusion: Analysis of results

This chapter provided an analysis of the simulation study results. The effect of the cost scenario used was discussed first. Next, the effect of the number of vehicles on service level and total cost was considered. Having determined that the number of vehicles available has a significant effect, the influence of the routing method was evaluated. After this, the deduction that the routing point has little effect was made. The reorder point, however, has a significant direct relation to both service level and cost.

The next chapter will provide a summary of the project and serve as conclusion the project report.

Chapter 8

Summary and conclusions

Previous chapters contain, amongst others, literature studies, suggested routing and inventory management models, details on the simulation model and, finally, an analysis of the results. This chapter will summarise the final year project and recommend several cash management strategies. It will also show that the work done for this final year project can be of great value of society and that the final year project enriched the student on several levels.

8.1 Project summary

The primary objective of this final year project was to refine work done by du Toit (2011) for his masters. Secondary objectives included evaluating the effect of applying the VRP and continuous review inventory management to the retail banking industry. The project had to deliver a multi-objective optimisation decision support system with which a Pareto optimal set of solutions can be determined.

The student studied MOO problems and discrete-event simulation as documented in Chapter 2. She also researched inventory management (summarised in Chapter 3), the VRP (discussed in Chapter 4), and integer programming (laid out in Chapter 5). The knowledge obtained from this research was used to develop a simulation model of the ATM network. The simulation model is discussed throughout the document but Chapter 6 focused specifically on the model and the simulation experiments that were drawn up. Finally, Chapter 7 provides an analysis of the results of the simulation experiments. Results in Chapter 7 are presented graphically. A Pareto optimal set of solutions can be identified from these plots.

Analysis of experimental results indicate that the application of the VRP to the retail banking industry results in a low cost, high service level vehicle routing strategy. It is also clear that the continuous review reorder point s is directly related to both cost and service level. The adjusted cost scenario provides the majority of the Pareto optimal solutions.

8.2 Recommended cash management strategy

Looking at the simulation results, the adjusted cost structure yields the lowest cost results. Renegotiating the cost structure to look exactly like the one experimented with, is not necessarily feasible. The concept illustrated is important, though: renegotiating the cost structure so that the variable component of the transportation cost is larger and the fixed component smaller, would provide the bank with more control over transportation cost. This could result in significant cost savings, depending on how this additional control is managed.

The nature of the MOO problem is such that there is no one optimal cash management strategy: improving service level leads to escalated total cost. Decision makers therefore need to decide on a service level required and shove their hands deep in their pockets or decide on a total cost level and live with the associated service level. The Pareto plot in Figure 8.1 shows the Pareto optimal set of solutions from which decision makers need to choose a scenario best suited to their requirements. Note that all members of the Pareto optimal set result from the adjusted cost scenario. This reinforces the recommendation made above that greater control over transportation cost can lead to a significant reduction in total cost.

Recognising the fact that the status quo cost structure is the reality, Table 8.1 shows the experiments resulting in the lowest cost or highest service level for both cost scenarios. A comparison of these values further illustrates the difference between results of the two cost scenarios. Table 8.1 also shows the conditions of the scenario **the student recommends: a scenario where service level is a little more than 2.5% less than the maximum, at 30% lower cost.** The adjusted cost scenario would, of course, be preferred. This is not necessarily possible, but the retail bank would do very well implementing the recommended status quo scenario.

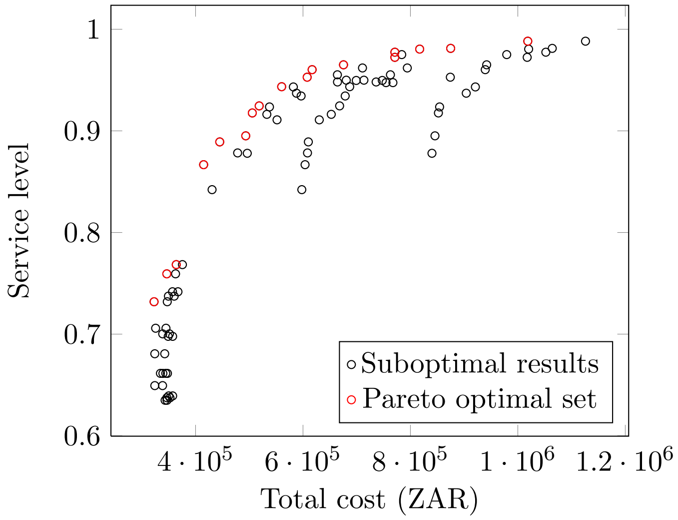

Figure 8.1: The set of Pareto optimal solutions for the MOOP.

		STATUS QUO	ADJUSTED COST
Minimum cost		*Experiment 19*	*Experiment 49*
	Service level	**0.6493**	**0.7319**
	Total cost	**R 338 540**	**R 322 730**
	Number of vehicles	1	1
	Routing method	FIFO	NSP
	Reorder point	R 150 000	R 150 000
	Routing point	2	3
Maximum service level		*Experiment 45*	*Experiment 90*
	Service level	**0.9882**	**0.9882**
	Total cost	**R 1 126 000**	**R 1 018 500**
	Number of vehicles	3	3
	Routing method	Direct replenishment	Direct replenishment
	Reorder point	R 550 000	R 550 000
	Routing point	1	1
Suggested compromise		*Experiment 9*	*Experiment 54*
	Service level	**0.9619**	**0.9619**
	Total cost	**R 794 190**	**R 710 760**
	Number of vehicles	2	2
	Routing method	NSP	NSP
	Reorder point	R 550 000	R 550 000
	Routing point	2	2

Table 8.1: Scenarios resulting in lowest cost, highest service level and a good compromise.

8.3 How this final year project benefits society

In spite of the drive of retail banks toward a cashless society, cash is still the preferred transaction medium of South Africa's poor. Retail banks now offer services affordable to them and as a result many more wage earners make use of ATMs for receiving their wages. The larger the number of wage earners using bank accounts, the smaller the risk of either their employers or themselves getting robbed, becomes.

The nature of business, however, is such that economic feasibility is always a key consideration. The service level provided by the bank will therefore always be limited by the economic feasibility of the associated cost. If ATM service levels in rural areas are too low, wage earners cannot access money in their accounts and thus face a decision: continue to bank their wages and run the risk of being unable to access their money, or stop banking and run the risk of being a victim of crime.

The decision support model developed for this final year project and the findings that resulted would enable retail banks to provide higher service levels at economically justifiable costs. These higher service levels would improve the lives of the millions of South Africans whose livelihoods depend on being able to draw their wages.

8.4 What the student learned

Executing the final year project was an educational experience on both personal and academic levels. On an academic level the student learned more than can be written down in a paragraph or two. Some of the lessons that stand out are listed below.

- Given time and resources, most problems encountered are solvable.
- Often, time and resources are limited and finding a perfect solution is impossible. Under these circumstances it is necessary to find as good a solution as possible and live with it.
- Finding the solution to a problem can be easy; implementing the solution is much more difficult.
- First and second year programming subjects do not make a programmer.
- The answer is seldom found in the first place searched.

- Teamwork also has advantages: the final year project is an individual assignment and, at times, the student wished she had a team mate who would have pointed out things she missed and had to rework.
- Everything takes longer than estimated.
- Donald Knuth is a programming genius.

The student also acquired valuable new skills:

- The simulation package Arena was used for the project. The student had never worked with any simulation software before.
- The study leader required the use of LaTeX instead of MS Word for typesetting the final year document. This is another new software package which the student learned from scratch.

In essence, the student learned that she still has a lot to learn but that she has been well equipped to find the information she requires and make educated assumptions where no information exists.

The student also learned that she likes learning and as a result will start with a masters degree in multi-objective optimisation at the beginning of 2012.

8.5 Conclusion: Summary and conclusions

This chapter provided a summary of the objectives, methodology and results of the final year project. Cash management strategies for different service level and cost outcomes were suggested. The possible enriching effect the work done for the final year project can have on South African society was pointed out. Finally, the lessons and skills acquired by the student as a result of the project were briefly discussed.

Appendix A

Model conceptualisation and translation

This appendix introduces and contains the functional specification used for model conceptualisation and translation.

A.1 Introduction: Functional specification

Model conceptualisation and translation form part of the simulation process discussed by Banks *et al.* (1998) in Chapter 2. Model conceptualisation refers to the abstraction of a real-world system investigated by a series of mathematical and logical relationships. Model translation involves coding the conceptual model developed in Chapters 3, 4 and 5 into computer recognizable form.

Kelton *et al.* (2010) recommend creating a functional specification early in the simulation modelling process. The functional specification assists the modeller in the conceptualisation and translation phases by asking and answering questions regarding the system description, input data needed and output data to be produced.

This appendix contains the functional specification developed for the simulation model, assumptions made (and not included in the rest of the report) whilst developing the conceptual model and during the translation of the conceptual model to an operational model.

A.2 The functional specification

The functional specification details the system under investigation by looking at equipment, product types, operations and transportation involved. It also describes the input data available in the system the output data to be produced by the simulation model.

A.2.1 Equipment

The system in question comprises of a network of ATMs, a count house (equivalent to a distribution center) and delivery vehicles distributing cash from the count house to the ATMs.

ATMs

Automated teller machines serve as service points for customers wanting do draw money. A customer selects the amount of cash required, and the ATM dispenses the required amount as a combination of denominations. The ATMs in question carry four denominations (R20, R50, R100, and R200), even though there are enough cash holding canisters to carry five denominations. The exact algorithm according to which real-world ATMs dispense denominations is unknown at present. A theoretical algorithm is suggested in Chapter 5, whilst its implementation is detailed in Appendix B. This algorithm will be used for the simulated system.

There are 18 ATMs in the network.

Count house

The count house serves as a cash distribution center. Cash is delivered to the count house from a bulk cash supplier situated in East London. From the count house, delivery vehicles distribute cash to the ATM network. This model will include only the transportation of cash from the count house to ATMs.

There is only one count house.

Delivery vehicles

Cash-in-transit (CIT) vehicles perform deliveries to ATMs. Vehicle details are shown in Table A.1. For the simulation study it is assumed that delivery vehicles

STRUCTURE OF VEHICLE COST	
Cost of a dedicated CIT vehicle	R 78 710 per month
Number of free kilometers	4 500
Cost of additional kilometers	R 3.18 per km
OPERATIONAL DETAIL	
Vehicle speed	50 km per hour
CIT delivery time	Monday to Friday from 8:00 - 17:00 Saturdays from 8:00 to 13:00
Branch hours	Monday to Friday from 8:00 - 17:00 Saturdays from 8:00 to 13:00

Table A.1: ATM cash management: Operational and cost details.

can carry all the cash they can deliver in one day. In other words,

$$\textit{Vehicle capacity} >> \textit{ATM replenishment load} \times \textit{Maximum \# of ATMs replenished in a day.} \tag{A.1}$$

The simulation model should allow for one, two or three vehicles to be used.

A.2.2 Product types

Product types that should be considered include the denominations kept in each ATM and the load delivered at time of replenishment.

Denominations

As discussed above, each ATM can hold five canisters of notes. Theoretically, the maximum number of notes per canister is 2500. Due to the fact that dispensing problems occur when canisters are topped up completely, canisters are not filled to the brim. In practice each canister contains 2000 notes.

For the simulation model, the canisters are filled as follows:

- Canister A: R 20-notes
- Canister B: R 50-notes
- Canister C: R 100-notes
- Canister D: R 200-notes
- Canister E: R 200-notes

ATM replenishment load

A replenishment load contains 2000 R 20-, R 50 - and R 100 notes and 4000 R 200 notes.

A.2.3 Operations

Three main types of operations stand out: a customer demands money, an ATM is replenished and the count house is restocked.

A customer draws money

Customers drawing money lowers inventory level in ATM i according to the algorithm developed in Chapter 5.

The retail bank in question installs all its ATMs inside branches. Customers can therefore only make withdrawals during branch hours. The branch hours are shown in Table A.1.

ATM replenishment

ATM replenishment increases inventory level in ATM i by the ATM replenishment load.

Count house replenishment

For the purpose of the simulation study, infinite inventory in the count house is assumed and the effect of count house replenishment can be ignored.

A.2.4 Transport

Cash needs to be transported. Due to the scope of the final year project, only transportation from the count house to ATMs need be considered. This type of transport is handled by CIT vehicles. Distances between ATMs are assumed to be symmetrical and are shown in the distance matrix in Table A.2.

A.2.5 Input data

The two main inputs into the simulation model are customer demand and logistics costs.

Customer demand

ATM	*CH*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*
CH	0	96	12	10	8	173	82	80	116	85	96	118	50	15	144	12	80	116	96
1		0	96	96	96	227	85	174	209	156	106	81	144	96	216	96	174	209	106
2			0	14	15	173	82	80	116	85	96	118	50	9	144	1	80	116	96
3				0	8	173	82	80	116	85	96	118	50	7	144	14	80	116	96
4					0	173	82	80	116	85	96	118	50	11	144	15	80	116	96
5						0	148	131	125	88	268	290	166	173	53	173	131	125	268
6							0	132	163	88	167	162	123	15	144	82	132	163	167
7								0	35	65	170	194	45	80	80	80	1	35	170
8									0	85	201	223	75	116	75	116	35	1	201
9										0	180	204	86	85	59	85	65	85	180
10											0	37	125	96	240	96	170	201	1
11												0	147	118	262	118	194	223	37
12													0	50	117	50	45	75	125
13														0	144	9	80	116	96
14															0	144	80	75	240
15																0	80	116	96
16																	0	35	170
17																		0	201
18																			0

Table A.2: ATM locations: Symmetrical distance matrix.

The student had access to very limited amounts of input data. From du Toit (2011)'s work, the daily amounts withdrawn from each ATM is known for 36 months. All ATMs were not operational for this entire period. Data for all 18 ATMs are only available for six weeks. It was decided to double the data available for these six weeks to obtain a three month simulation period. This should provide a clear enough indication of the effect of vehicle routing and inventory management on fluctuating demand.

The simulation model is designed in such a way that individual customers arrive and demand cash. Information on the total daily amount withdrawn is therefore not sufficient. The student requires the customer arrival rate and a customer demand profile. To obtain the required input data, 18 fictitious customer demand profiles were manufactured. 18 arrival rates were made up as well. These arrival rates and customer demand profiles were adjusted until the amounts demanded μ at the interarrival rate β for one day approximates the total daily amount withdrawn.

Costs

Vehicle costs are shown in Table A.1. Additionally, rebanking cost must be taken into account at R 0.21 per R 100 rebanked. Opportunity cost will be estimated at 6% annual nominal interest, compounded continuously.

A.2.6 Output data

The output statistics required are:

- Service level
- Total cost
- Number of cash outs
- The total distance covered
- The number of routes completed
- The number of routes determined
- Average vehicle utilisation
- Vehicle cost
- Distance cost
- Rebanking cost
- Opportunity cost

This concludes the functional specification which was used for implementation of the simulation model. Next, algorithms used for simulation model implementation will be discussed in Appendix B.

Appendix B

Simulation model implementation

This appendix contains the important algorithms used to implement the simulation model. The work described below should be useful to interested parties.

B.1 An algorithm for dispensing notes

Implementation of cash dispensing was done using building blocks in Arena 9.0. Algorithm 1 was developed entirely by the student and is based on Winston (2004)'s branch-and-bound adaption for solving the knapsack problem. As discussed in Chapter 5, notes are dispensed in such a manner that the number of notes dispensed are kept to a minimum and the algorithm below can therefore be seen as a "least-note picking" algorithm.

Algorithm 1 An algorithm for combining notes to meet customer demand

```
1:  [Input.] Customer demand $D$, inventory level for ATM $i$ $CashOnHand(4, i)$ and number of unhelped
    customers $CashOuts(i)$ .
2:  [Initialise.]
3:  Set $Retraced \leftarrow 0$;
4:  $NoteWeight(1) \leftarrow 20$;
5:  $NoteWeight(2) \leftarrow 50$;
6:  $NoteWeight(3) \leftarrow 100$;
7:  $NoteWeight(4) \leftarrow 200$;
8:  $Note \leftarrow 4$;
9:  $NotesRemoved(j) \leftarrow 0$;
10: $NotesAdded(j) \leftarrow 0$;
11: $NoteCombination(j) \leftarrow 0$ for $1 \leq j \leq 4$.
12: [Equal to demand?]
13: If $\sum_{j=1}^{4}$ $NoteCombination(j) \times NoteWeight(j) = D$ then
14:   [Done.] Remove notes:
15:   for $1 \leq j \leq 4$
16:     set $CashOnHand(j,i) \leftarrow CashOnHand(j,i)$ - $NoteCombination(j)$
17:   next $j$
18: else
19:   [Current denomination not available?]
20:   If $CashOnHand(Note, i)$ - $NoteCombination(Note) < 0$ then
21:     [Remove note.]
22:     Set $NotesRemoved(Note) \leftarrow NotesRemoved(Note) + 1$;
23:     $NoteCombination(Note) \leftarrow NotesAdded(Note)$ - $NotesRemoved(Note)$;
24:     go to 12
25:   else
26:     [Too many notes?]
27:     If $\sum_{j=1}^{4}$ $NoteCombination(j) \times$ NoteWeight(j) $> D$ then
28:       go to 21
29:     else
30:        [Note previously removed?]
31:       If $NotesRemoved(Note) \geq 1$ then
32:         [Remove one more?]
33:         If $(NotesRemoved(Note) \geq 1$ AND
34:         $(\sum_{j=1}^{4}$ $NoteCombination(j) \times NoteWeight(j) > D))$
35:         OR $(NotesRemoved(Note) \geq 1$ AND
36:         $(\sum_{j=1}^{4}$ $NoteCombination(j) \times NoteWeight(j) + NoteWeight(1) > D)$ AND
37:         $(\sum_{j=1}^{4}$ $NoteCombination(j) \times NoteWeight(j)) + 2 \times NoteWeight(1) > D)$ then
38:           go to 21
39:         else
40:           [Does smaller note exist?]
41:           If $Note > 1$ then
42:             set $Note \leftarrow Note - 1$;
43:             go to 59
44:           else
45:             [Adjusted combination already?]
46:             If $Retraced = 1$ then
47:               $CashOuts(i) \leftarrow CashOuts(i) + 1$
48:             else
49:               [Adjust combination.]
50:               Set $NotesAdded(Note) \leftarrow 0$;
51:               $NotesRemoved(Note) \leftarrow 0$;
52:               $Note \leftarrow Note + 1$;
53:               $Retraced \leftarrow 1$;
54:               go to 12
55:             end if
56:           end if
57:         end if
58:       else
59:         [Add note of current combination.]
60:         Set $NotesAdded(Note) \leftarrow NotesAdded(Note) + 1$;
61:         $NoteCombination(Note) \leftarrow NotesAdded(Note)$ - $NotesRemoved(Note)$;
62:         go to 12
63:       end if
64:     end if
65:   end if
66: end if.
```

B.2 Vehicle routing

Implementation of vehicle routing was done using a combination of Arena 9.0 and its VBA interface. Blocks in Arena trigger one of three different sets of code depending on the routing method selected.

The algorithms included for vehicle routing are not as detailed as the note picking algorithm.

B.2.1 Near shortest path routing

The implementation of the near shortest path routing method has to be described as the most difficult part of the final year project. Pseudocode for the algorithms developed, adapted and abused are shown below.

Algorithm 2 Main function for NSP routing

```
[Initialise.]
Set necessary variables to 0;
CountROFs ← Number of ATMs in need of replenishment.
[Continue?]
If CountROFs ≥ RoutingPoint then
    continue
else
    break
end if.
[Find initial lower bound.]
Call DETERMINELOWERBOUND.
[Solve the VRP.]
Call BRANCHANDBOUND.
[Write VBA solution to Arena.]
If solutionfound = true then
    call COMPLETEROUTEARRAYS
else
    break
end if.
```

The algorithm used to determine the lower bound is a version of the Greedy algorithm.

The BACKTRACK algorithm was expanded from an algorithm provided by Naverniouk & Chu (2008).

The COMBINATION algorithm was adapted from an algorithm for lexicographic combinations suggested by Knuth (2005).

Algorithm 3 Subfunction: DETERMINELOWERBOUND

```
 1: [Initialise.]
 2: Set necessary variables to 0;
 3: $CheapestEdge \leftarrow 1000000$;
 4: $CurrentPosition \leftarrow CountHouse$;
 5: $SequencePosition \leftarrow 1$.
 6: [All ATMs to be replenished?]
 7: If $SequencePosition = NumberOfATMsToVisit$ then
 8:     return to main
 9: else
10:     continue
11: end if.
12: [Consider all options for next station.]
13: For $Option = 1$ to $NumberOfATMsToVisit$ - $OptionsLost$
14:     [Evaluate current option.]
15:     If $EdgeCost < CheapestEdge$ then
16:         $CheapestEdge \leftarrow EdgeCost$;
17:         $BestOption \leftarrow ThisOption$;
18:         remove $ThisOption$ from $FutureOptions$;
19: next $Option$.
20: [Move to next station.]
21: Set $CurrentPosition \leftarrow BestOption$;
22: $LowerBound \leftarrow LowerBound + CheapestEdge$;
23: $OptionsLost \leftarrow OptionsLost + 1$;
24: $SequencePosition \leftarrow SequencePosition + 1$;
25: go to 6.
```

Algorithm 4 Subfunction: BRANCHANDBOUND

```
 1: [Initialise.]
 2: Set necessary variables to 0.
 3: [Using lower bound, is one route enough?]
 4: If $RoutesRequired = 1$ then
 5:     $BestComboCost \leftarrow LowerBound$;
 6:     $Subset \leftarrow AllATMstoVisit$;
 7:     $NumberOfATMsToVisit \leftarrow CountAllATMstoVisit$;
 8:     Call INITIALISEBACKTRACK(Subset, NumberOfATMsToVisit)
 9: else if ATM 5 requires replenishment then
10:     determine route that services ATM 5
11: else
12:     Call VARYSIZE
13: end if.
14: [Return.]
15: Return to main.
```

Algorithm 5 Subfunction: INITIALISEBACKTRACK

```
1:  [Arguments required.]
2:  Subset, NumberOfATMsToVisit.
3:  [Initialise.]
4:  Set necessary variables to 0.
5:  [Assign values to arguments for BACKTRACK.]
6:  Set Path ← Subset ;
7:  draw up SpecificDistanceMatrix using Subset;
8:  set array seenATM ← False;
9:  CheapestRouteCost ← BestComboCost (Global variable);
10: $i \leftarrow 0$; $t \leftarrow$ CountHouse;
11: FeasibleRouteFound ← False.
12: [Find initial lower bound.]
13: Call BACKTRACK(Path, SpecificDistanceMatrix, SeenATM, CheapestRouteCost, i, t, FeasibleRouteFound).
14: [Return.]
15: Return to calling function.
```

Algorithm 6 Subfunction: BACKTRACK

```
1:  [Arguments required.]
2:  Path, SpecificDistanceMatrix, SeenATM, CheapestRouteCost, i, t, FeasibleRouteFound.
3:  [Initialise.]
4:  Set necessary variables to 0; set $u \leftarrow$ path(i).
5:  [Full route?]
6:  If $u = t$ then
7:      [A feasible route was found.]
8:      set FeasibleRouteFound ← True
9:      if CurrentRouteCost < CheapestRouteCost then
10:         set BestRoute ← CurrentRoute
11:     end if
12: end if.
13: [Start with station 0.]
14: Set $V \leftarrow 0$.
15: [No route between u and V?]
16: If SpecificDistanceMatrix$(u, V)$= 0 then
17:     go to 37
18: end if.
19: [Visited V before?]
20: If SeenATM(V) True then
21:     go to 37
22: end if.
23: [Make V next station.]
24: Set Path(i+1) = $V$.
25: [Determine time required for path thus far.]
26: Set RouteTime = f(CheapestRouteCost).
27: [Continue with route?]
28: If RouteTime > AvailableTime then
29:     [Branch will not yield an optimal solution.]
30:     go to 37
31: end if.
32: [Recurse.]
33: Set SeenATM(V) ← True;
34: call BACKTRACK(Path, SpecificDistanceMatrix, SeenATM, CheapestRouteCost, i+1, t, FeasibleRouteFound).
35: [Recursion done.]
36: Set SeenATM(V) ← False.
37: [Next station.]
38: Next $V$.
39: [Return.]
40: Return to INITIALISEBACKTRACK.
```

Algorithm 7 Subfunction: VARYSIZE

```
 1: [Adjust BestComboCost.]
 2: Set BestComboCost $\leftarrow$ GreedyCost.
 3: [Assign values to arguments required for COMBINATION.]
 4: Set CombinationArray $\leftarrow (0, 1, 2, 3, 4, \ldots)$;
 5: if CountAllATMstoVisit $< 4$ then
 6:     set CombinationSize $\leftarrow$ CountAllATMstoVisit
 7: else
 8:     set CombinationSize $\leftarrow 4$;
 9:     MinATMsOnRoute $\leftarrow 2$;
10:     GoodCombinations $\leftarrow 0$
11: end if.
12: [Find initial lower bound.]
13: While GoodCombinations $= 0$ AND CombinationSize $\geq$ MinATMsOnRoute
14:     Call COMBINATION(CombinationArray, CombinationSize, CountAllATMstoVisit, MinATMsOnRoute);
15:     set CombinationSize $\leftarrow$ CombinationSize - 1.
16: [Return.]
17: Return to BRANCHANDBOUND.
```

Algorithm 8 Subfunction: COMBINATION

```
 1: [Initialise.]
 2: For $1 \leq j \leq t$
 3:     set $c_j \leftarrow j - 1$
 4: next $j$;
 5: set $c_{t+1} \leftarrow n$;
 6: $c_{t+2} \leftarrow 0$.
 7: [Visit.]
 8: Set inSubset $\leftarrow$ the combination $c_t \ldots c_2 c_1$;
 9: notInSubset $\leftarrow$ the remaining ATMs.
10: Call BESTCOMBINATION(inSubset, notInSubset).
11: [Find $j$.]
12: Set $j \leftarrow 1$.
13: While $c_j + 1 = c_{j+1}$
14:     set $c_j \leftarrow j - 1$;
15:     $j \leftarrow j + 1$;
16: repeat until $c_j + 1 \neq c_{j+1}$.
17: [Done?]
18: Terminate the algorithm if $j > t$.
19: Return to VARYSIZE.
20: [Increase $c_j$.]
21: Set $c_j \leftarrow c_j + 1$;
22: return to 7.
```

Algorithm 9 Subfunction: BESTCOMBINATION

```
[Arguments required.]
inSubset, notInSubset.
[Initialise.]
Set necessary variables to 0.
[Assign values to arguments required by INITIALISEBACKTRACK.]
Set Subset ← inSubset;
NumberOfATMstoVisit ← CombinationSize.
[Pass arguments.]
CallINITIALISEBACKTRACK(Subset, NumberOfATMstoVisit).
[Keep best route.]
If CostOfRouteReturned < BestComboCost then
    set BestComboCost ← CostOfRouteReturned;
BestRoute ← RouteReturned
end if.
[Return.]
Return to COMBINATION.
```

Algorithm 10 Subfunction: COMPLETEROUTEARRAYS

```
[Convert VBA route to Arena.]
For $i = 0$ to RouteLength
    write BestRoute(i) to ArenaRoute(i+1)
next $i$.
[Count ATMs not in route.]
Set UnservicedATMs ← 0.
For $j = 1$ to CountAllATMstoVisit
    if ATMIncluded False then
        set UnservicedATMs ← UnservicedATMs + 1
    end if
next $j$.
[Continue?]
Set Arena variable BuildSequenceNow ← 1.
[Return.]
Return to main.
```

B.2.2 First-in-first-out routing

The programming required for FIFO routing was much simpler than the implementation of the NSP.

Algorithm 11 Main function for FIFO routing

```
[Initialise.]
Set necessary variables to 0.
[Continue?]
If $ATMs\ in\ need\ of\ replenishment \geq RoutingPoint$ then
    continue
else
    break
end if.
[Sort ATM orders in order of registration.] Call QUICKSORT.
[Determine a FIFO route.] Call FIFOROUTE.
```

B.2.3 Direct replenishment

The implementation of direct replenishment routing, is a trivial case of FIFO routing as only one ATM is included in every route and the array used for route determination.

Algorithm 12 Subfunction: QUICKSORT

```
1: [Arguments required.]
2: $sortArray,\ startSort,\ endSort.$
3: [Initialise.]
4: Set $i \leftarrow startSort$;
5: $k \leftarrow endSort$.
6: [Sort not complete yet?]
7: If $endSort$ - $startSort \geq 1$ then
8:     set $pivot \leftarrow sortArray(startSort)$;
9:     continue
10: else
11:     break
12: end if.
13: [Terminate search?]
14: If $k \leq i$ then
15:     go to 28
16: end if.
17: [Find next value to swap.]
18: While $sortArray(i) \leq pivot$ AND $i \leq endSort$ AND $k > i$
19:     set $i \leftarrow i + 1$
20: loop.
21: While $sortArray(k) > pivot$ AND $k \geq startSort$ AND $k \geq i$
22:     set $k \leftarrow k - 1$
23: loop.
24: If $k > i$ then
25:     swap $sortArray$, $i$, $k$.
26: Go to 13
27: end if.
28: [Alternative swap.]
29: Swap $sortArray$, $startSort$, $k$.
30: [Recurse.]
31: Call $QUICKSORT(sortArray,\ startSort,\ k\text{-}1)$.
32: Call $QUICKSORT(sortArray,\ k{+}1,\ endSort)$.
33: [Return.]
34: Return to main.
```

Algorithm 13 Subfunction: FIFOROUTE

```
1: [Arguments required.]
2: $DemandArray$.
3: [Initialise.]
4: Set necessary variables to 0.
5: [All ATMs considered?]
6: If $ATMConsidered = NumberOfATMstoVisit$ then
7:     break, returning to main
8: else
9:     continue
10: end if.
11: [Can ATM be added to route?]
12: If $RouteTime \leq TimeAvailable$ then
13:     add ATM
14: end if.
15: [One ATM too many?]
16: If $RouteTime \geq TimeAvailable$ then
17:     remove ATM
18: end if.
19: [Try another ATM]
20: Set $ATMConsidered \leftarrow ATMConsidered + 1$.
21: Go to 5.
```

Algorithm 14 Main function for direct replenishment

```
1: [Initialise.]
2: Set necessary variables to 0.
3: [Determine a FIFO route.]
4: Call DIRECTREPLENISHMENTROUTE.
```

Algorithm 15 Subfunction: DIRECTREPLENISHMENTROUTE

```
1: [Arguments required.]
2: DemandArray.
3: [Initialise.]
4: Set necessary variables to 0.
5: [All ATMs considered?]
6: If $ATMConsidered = NumberOfATMstoVisit$ then
7:     break, returning to main
8: else
9:     continue
10: end if.
11: [Can ATM be added to route?]
12: If $RouteTime \leq TimeAvailable$ then
13:     add ATM
14: end if.
15: [One ATM too many?] If $RouteTime \geq TimeAvailable$ then
16:     remove ATM
17: end if.
18: [Try another ATM]
19: Set $ATMConsidered \leftarrow ATMConsidered + 1$.
20: Go to 5.
```

Appendix C

Simulation model verification and validation

Bekker (2011b) states that “verification allows us to confirm that we have built the model right, whereas validation allows us to confirm that we have built the right model”.

Model verification consists in large part of debugging and is therefore done throughout model development.

The student validated the majority of the simulation model only once it was “completed”. Apostrophes because the validation lead to a significant amount of rework and even after this was done, the model can still not be described as “complete” – there will always be something that can be added or improved.

The student specifically searched for answers to certain questions during model validation. These questions are:

1. Does the actual number of customers created match the expected number of customers created?
2. Are all created customers disposed?
3. Does the actual amount of cash dispensed match the expected amount?
4. Is vehicle cost what it is expected to be? For all three cases?
5. Does the total distance travelled make sense?
6. Is the associated distance cost what it is expected to be?
7. Does the value returned for rebanking cost make sense?

8. Does the value returned for opportunity cost add up?
9. Are the routes determined, the routes used? Or are vehicles doing something else?
10. Do the created routes obey constraints?
11. Are CIT vehicles back at the count house at the end of a day?
12. Do CIT vehicles get "stuck" waiting for work or on route?
13. Does adding or subtracting a truck make a difference to results?
14. Does the algorithm for dispensing notes behave as expected?

These questions were answered satisfactorily for various scenarios spanning the different routing methods, number of vehicles available, routing points and reorder points.

Once the student was satisfied that the model imitated the network to an acceptable extent, the initial experiments used to determine the number of replications required were started. After determining the number of replications required to yield satisfactory confidence interval half-widths, the 90 experiments of which the results are shown in Appendix D were run.

Appendix D

Results of the simulation study

Summaries of the results of all 90 simulation experiments are shown below.

EXPERIMENT 1					
Details					
Routing method	Near-shortest path routing		Number of vehicles used	1	
Routing point	2		Re-order point for ATMs	150000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.7058	0.006	Total cost	344900	1297.6
Number of cash-outs	33822	690.08	Vehicle cost	236130	0
Total distance travelled (km)	15868	108.53	Cost of distance travelled	7674.9	327.32
Number of routes determined	67.987	0.57377	Opportunity cost	99537	967.36
Number of routes completed	65.3	0.52356	Rebanking cost	1553.9	160.83
Average vehicle utilisation	0.54378	0.00434			

EXPERIMENT 2					
Details					
Routing method	Near-shortest path routing		Number of vehicles used	1	
Routing point	2		Re-order point for ATMs	350000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.73737	0.0066	Total cost	360190	2067
Number of cash-outs	30204	761.91	Vehicle cost	236130	0
Total distance travelled	16745	137.49	Cost of distance travelled	10321	436.96
Number of routes determined	72.7	0.75207	Opportunity cost	105770	1134.6
Number of routes completed	68.85	0.67664	Rebanking cost	7966.9	738.94
Average vehicle utilisation	0.57788	0.00502			

EXPERIMENT 3					
Details					
Routing method	Near-shortest path routing		Number of vehicles used	1	
Routing point	2		Re-order point for ATMs	550000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.74169	0.00621	Total cost	367250	2479.8
Number of cash-outs	29705	717.92	Vehicle cost	236130	0
Total distance travelled	16807	100.91	Cost of distance travelled	10516	320.9
Number of routes determined	74.425	0.6591	Opportunity cost	108080	1246.5
Number of routes completed	69.562	0.60347	Rebanking cost	12526	1233
Average vehicle utilisation	0.58248	0.00374			

EXPERIMENT 4					
Details					
Routing method	Near-shortest path routing		Number of vehicles used	1	
Routing point	3		Re-order point for ATMs	150000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.73187	0.00482	Total cost	347290	1162.6
Number of cash-outs	30821	552.91	Vehicle cost	236130	0
Total distance travelled	15348	114.98	Cost of distance travelled	6319.7	321.31
Number of routes determined	61.162	0.4327	Opportunity cost	102850	795.6
Number of routes completed	58.65	0.44674	Rebanking cost	1986.4	205.29
Average vehicle utilisation	0.5302	0.00441			

EXPERIMENT 5					
Details					
Routing method	Near-shortest path routing		Number of vehicles used	1	
Routing point	3		Re-order point for ATMs	350000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.75933	0.00389	Total cost	362750	1588.1
Number of cash-outs	27674	448.64	Vehicle cost	236130	0
Total distance travelled	16169	94.161	Cost of distance travelled	8509.2	298.3
Number of routes determined	66	0.41545	Opportunity cost	108750	762.65
Number of routes completed	62.4	0.39913	Rebanking cost	9369.4	755.65
Average vehicle utilisation	0.56346	0.00356			

EXPERIMENT 6					
Details					
Routing method	Near-shortest path routing		Number of vehicles used	1	
Routing point	3		Re-order point for ATMs	550000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.7684	0.00501	Total cost	375510	2332.7
Number of cash-outs	26631	583.06	Vehicle cost	236130	0
Total distance travelled	16671	103.53	Cost of distance travelled	10084	329.24
Number of routes determined	69.075	0.52887	Opportunity cost	112230	1013.2
Number of routes completed	64.812	0.50325	Rebanking cost	17073	1194.8
Average vehicle utilisation	0.58244	0.00388			

EXPERIMENT 7					
Details					
Routing method	Near-shortest path routing		Number of vehicles used	2	
Routing point	2		Re-order point for ATMs	150000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.88915	0.00327	Total cost	609960	666.77
Number of cash-outs	12749	377.09	Vehicle cost	472260	0
Total distance travelled	20573	188.49	Cost of distance travelled	9.2617	13.124
Number of routes determined	85.362	0.69724	Opportunity cost	126560	449.21
Number of routes completed	84.262	0.73169	Rebanking cost	11136	301.4
Average vehicle utilisation	0.33356	0.00309			

EXPERIMENT 8					
Details					
Routing method	Near-shortest path routing		Number of vehicles used	2	
Routing point	2		Re-order point for ATMs	350000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.94346	0.00241	Total cost	686960	1952
Number of cash-outs	6497.5	277.09	Vehicle cost	472260	0
Total distance travelled	23635	197.54	Cost of distance travelled	371.14	123.84
Number of routes determined	98.787	0.90152	Opportunity cost	147550	601.99
Number of routes completed	97.912	0.84327	Rebanking cost	66773	1384
Average vehicle utilisation	0.39424	0.00371			

EXPERIMENT 9					
Details					
Routing method	Near-shortest path routing		Number of vehicles used	2	
Routing point	2		Re-order point for ATMs	550000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.96191	0.00192	Total cost	794190	3764
Number of cash-outs	4380	220.11	Vehicle cost	472260	0
Total distance travelled	27285	212.64	Cost of distance travelled	3568.8	424.89
Number of routes determined	115.9	0.94575	Opportunity cost	165140	669.52
Number of routes completed	114.35	0.81309	Rebanking cost	153220	3007.3
Average vehicle utilisation	0.46303	0.00371			

EXPERIMENT 10					
Details					
Routing method	Near-shortest path routing		Number of vehicles used	2	
Routing point	3		Re-order point for ATMs	150000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.86673	0.00401	Total cost	603850	789.18
Number of cash-outs	15323	461.32	Vehicle cost	472260	0
Total distance travelled	18653	125.13	Cost of distance travelled	0	0
Number of routes determined	73.762	0.59537	Opportunity cost	123080	542.88
Number of routes completed	72.575	0.60446	Rebanking cost	8510	328.8
Average vehicle utilisation	0.30157	0.0024			

EXPERIMENT 11					
Details					
Routing method	Near-shortest path routing		Number of vehicles used	2	
Routing point	3		Re-order point for ATMs	350000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.92459	0.00237	Total cost	668190	1700.9
Number of cash-outs	8673.3	272.78	Vehicle cost	472260	0
Total distance travelled	21794	139.51	Cost of distance travelled	33.549	26.366
Number of routes determined	86.587	0.63832	Opportunity cost	142440	566.23
Number of routes completed	84.8	0.61455	Rebanking cost	53454	1174.5
Average vehicle utilisation	0.36332	0.00267			

EXPERIMENT 12					
Details					
Routing method	Near-shortest path routing		Number of vehicles used	2	
Routing point	3		Re-order point for ATMs	550000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.95519	0.00185	Total cost	762760	3038.1
Number of cash-outs	5154.4	213.3	Vehicle cost	472260	0
Total distance travelled	25914	124.28	Cost of distance travelled	1395.2	204.03
Number of routes determined	102.76	0.66917	Opportunity cost	159960	622.84
Number of routes completed	100.6	0.62813	Rebanking cost	129140	2387.7
Average vehicle utilisation	0.43788	0.0025			

EXPERIMENT 13					
c					
Routing method	Near-shortest path routing		Number of vehicles used	3	
Routing point	2		Re-order point for ATMs	150000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.91763	0.00219	Total cost	852110	626.87
Number of cash-outs	9469.8	250.4	Vehicle cost	708390	0
Total distance travelled	21150	198.02	Cost of distance travelled	0	0
Number of routes determined	88.037	0.77472	Opportunity cost	130040	405.4
Number of routes completed	87.275	0.78581	Rebanking cost	13677	338.62
Average vehicle utilisation	0.21228	0.00238			

EXPERIMENT 14					
Details					
Routing method	Near-shortest path routing		Number of vehicles used	3	
Routing point	2		Re-order point for ATMs	350000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.96023	0.00179	Total cost	939310	1508
Number of cash-outs	4572.9	205.84	Vehicle cost	708390	0
Total distance travelled	24575	220.21	Cost of distance travelled	0	0
Number of routes determined	102.07	0.98038	Opportunity cost	151430	473.44
Number of routes completed	101.36	0.92846	Rebanking cost	79490	1094.7
Average vehicle utilisation	0.25732	0.00284			

EXPERIMENT 15						
Details						
Routing method	Near-shortest path routing		Number of vehicles used	3		
Routing point	2		Re-order point for ATMs	550000		
Minimum number of ATMs in route	2		Number of replications	80		
Results						
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.98046	0.00111	Total cost	1020100	2396.8
Number of cash-outs	2246.5	127.98	Vehicle cost	708390	0
Total distance travelled	29665	300.26	Cost of distance travelled	0	0
Number of routes determined	125.03	1.2623	Opportunity cost	173860	405.66
Number of routes completed	124.18	1.152	Rebanking cost	197820	2030.7
Average vehicle utilisation	0.32007	0.00339			

EXPERIMENT 16					
Details					
Routing method	Near-shortest path routing		Number of vehicles used	3	
Routing point	3		Re-order point for ATMs	150000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.89513	0.00236	Total cost	845710	602.59
Number of cash-outs	12061	273.89	Vehicle cost	708390	0
Total distance travelled	19485	100.66	Cost of distance travelled	0	0
Number of routes determined	76.862	0.38963	Opportunity cost	126490	399.69
Number of routes completed	75.787	0.39928	Rebanking cost	10829	311.95
Average vehicle utilisation	0.19544	0.00189			

EXPERIMENT 17					
Details					
Routing method	Near-shortest path routing		Number of vehicles used	3	
Routing point	3		Re-order point for ATMs	350000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.94323	0.00231	Total cost	920940	1876.2
Number of cash-outs	6528.7	265.73	Vehicle cost	708390	0
Total distance travelled	22634	164.48	Cost of distance travelled	0	0
Number of routes determined	90.15	0.73287	Opportunity cost	147060	624.92
Number of routes completed	88.325	0.74984	Rebanking cost	65491	1310.3
Average vehicle utilisation	0.238	0.00261			

EXPERIMENT 18					
Details					
Routing method	Near-shortest path routing		Number of vehicles used	3	
Routing point	3		Re-order point for ATMs	550000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.97741	0.00128	Total cost	1052100	3339.7
Number of cash-outs	2597	146.82	Vehicle cost	708390	0
Total distance travelled	28238	172.52	Cost of distance travelled	0	0
Number of routes determined	111.75	0.78297	Opportunity cost	169340	604.02
Number of routes completed	109.88	0.79311	Rebanking cost	174360	2763.4
Average vehicle utilisation	0.30318	0.00217			

EXPERIMENT 19					
Details					
Routing method	First-in-first-out routing		Number of vehicles used	1	
Routing point	2		Re-order point for ATMs	150000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.64934	0.00474	Total cost	338540	1256.2
Number of cash-outs	40318	543.44	Vehicle cost	236130	0
Total distance travelled	16362	113.29	Cost of distance travelled	9147.7	354.24
Number of routes determined	77.45	0.69325	Opportunity cost	92731	893.92
Number of routes completed	72.875	0.69461	Rebanking cost	535.14	95.164
Average vehicle utilisation	0.55212	0.00452			

EXPERIMENT 20					
Details					
Routing method	First-in-first-out routing		Number of vehicles used	1	
Routing point	2		Re-order point for ATMs	350000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.66141	0.00488	Total cost	344500	1343.8
Number of cash-outs	38932	564.17	Vehicle cost	236130	0
Total distance travelled	16832	111.36	Cost of distance travelled	10600	353.4
Number of routes determined	81	0.75766	Opportunity cost	95265	856.22
Number of routes completed	74.912	0.71886	Rebanking cost	2503.4	288.84
Average vehicle utilisation	0.56873	0.00419			

EXPERIMENT 21					
Details					
Routing method	First-in-first-out routing		Number of vehicles used	1	
Routing point	2		Re-order point for ATMs	550000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.66136	0.00467	Total cost	347670	1322.5
Number of cash-outs	38942	542.24	Vehicle cost	236130	0
Total distance travelled	16984	108.15	Cost of distance travelled	11079	343.93
Number of routes determined	83.65	0.80121	Opportunity cost	95953	798.91
Number of routes completed	76.487	0.80986	Rebanking cost	4504.4	391.59
Average vehicle utilisation	0.5715	0.00376			

EXPERIMENT 22					
Details					
Routing method	First-in-first-out routing		Number of vehicles used	1	
Routing point	3		Re-order point for ATMs	150000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.68068	0.00475	Total cost	342550	1090.4
Number of cash-outs	36716	549.45	Vehicle cost	236130	0
Total distance travelled	15953	102.85	Cost of distance travelled	7964.7	294.35
Number of routes determined	65.25	0.52975	Opportunity cost	97802	828.47
Number of routes completed	62.025	0.52669	Rebanking cost	656.8	103.03
Average vehicle utilisation	0.54462	0.00373			

EXPERIMENT 23					
Details					
Routing method	First-in-first-out routing		Number of vehicles used	1	
Routing point	3		Re-order point for ATMs	350000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.70025	0.00397	Total cost	351330	1207.2
Number of cash-outs	34459	462.48	Vehicle cost	236130	0
Total distance travelled	16562	88.606	Cost of distance travelled	9747	280
Number of routes determined	69.575	0.52645	Opportunity cost	101630	700.89
Number of routes completed	64.837	0.47898	Rebanking cost	3818.1	460.84
Average vehicle utilisation	0.56732	0.00344			

EXPERIMENT 24					
Details					
Routing method	First-in-first-out routing		Number of vehicles used	1	
Routing point	3		Re-order point for ATMs	550000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.69781	0.00409	Total cost	357260	1331.9
Number of cash-outs	34752	471.17	Vehicle cost	236130	0
Total distance travelled	17010	101.35	Cost of distance travelled	11165	321.58
Number of routes determined	72.437	0.5797	Opportunity cost	102100	717.09
Number of routes completed	67.15	0.54239	Rebanking cost	7865.8	493.89
Average vehicle utilisation	0.58013	0.00379			

EXPERIMENT 25					
Details					
Routing method	First-in-first-out routing		Number of vehicles used	2	
Routing point	2		Re-order point for ATMs	150000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.87831	0.004	Total cost	608380	877.5
Number of cash-outs	13993	461.15	Vehicle cost	472260	0
Total distance travelled	23371	138.69	Cost of distance travelled	432.91	130.36
Number of routes determined	106.53	0.77924	Opportunity cost	127160	564.7
Number of routes completed	103.68	0.72365	Rebanking cost	8534.3	313.08
Average vehicle utilisation	0.37917	0.00264			

EXPERIMENT 26					
Details					
Routing method	First-in-first-out routing		Number of vehicles used	2	
Routing point	2		Re-order point for ATMs	350000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.93434	0.0026	Total cost	678570	2027.7
Number of cash-outs	7550.7	298.9	Vehicle cost	472260	0
Total distance travelled	27422	147.25	Cost of distance travelled	3942.1	325.89
Number of routes determined	126.62	0.90484	Opportunity cost	146530	622.81
Number of routes completed	123.45	0.86476	Rebanking cost	55840	1319.7
Average vehicle utilisation	0.45318	0.00263			

EXPERIMENT 27					
Details					
Routing method	First-in-first-out routing		Number of vehicles used	2	
Routing point	2		Re-order point for ATMs	550000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.94962	0.00223	Total cost	747620	2857.4
Number of cash-outs	5791.8	256.25	Vehicle cost	472260	0
Total distance travelled	30695	156.65	Cost of distance travelled	12001	481.65
Number of routes determined	142.93	1.0109	Opportunity cost	157570	576.92
Number of routes completed	137.97	0.97332	Rebanking cost	105790	2046.7
Average vehicle utilisation	0.51466	0.00275			

EXPERIMENT 28					
Details					
Routing method	First-in-first-out routing		Number of vehicles used	2	
Routing point	3		Re-order point for ATMs	150000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.84216	0.00318	Total cost	597800	590.18
Number of cash-outs	18153	368.02	Vehicle cost	472260	0
Total distance travelled	20409	141.98	Cost of distance travelled	0	0
Number of routes determined	81.9	0.61476	Opportunity cost	121700	504.18
Number of routes completed	79.275	0.64854	Rebanking cost	3842.7	175.79
Average vehicle utilisation	0.33058	0.00289			

EXPERIMENT 29					
Details					
Routing method	First-in-first-out routing		Number of vehicles used	2	
Routing point	3		Re-order point for ATMs	350000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.91626	0.0029	Total cost	652640	1593.3
Number of cash-outs	9626.6	334.58	Vehicle cost	472260	0
Total distance travelled	24174	142.03	Cost of distance travelled	503.91	159.4
Number of routes determined	97.55	0.73531	Opportunity cost	141530	613.41
Number of routes completed	94.862	0.71579	Rebanking cost	38343	1053.8
Average vehicle utilisation	0.39875	0.00294			

EXPERIMENT 30					
Details					
Routing method	First-in-first-out routing		Number of vehicles used	2	
Routing point	3		Re-order point for ATMs	550000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.94812	0.00225	Total cost	736140	3092.4
Number of cash-outs	5969.4	259.6	Vehicle cost	472260	0
Total distance travelled	28380	14782	Cost of distance travelled	6007.4	424.64
Number of routes determined	115.55	0.81607	Opportunity cost	157420	669.46
Number of routes completed	111.86	0.79773	Rebanking cost	100450	2178
Average vehicle utilisation	0.47612	0.0027			

EXPERIMENT 31					
Details					
Routing method	First-in-first-out routing		Number of vehicles used	3	
Routing point	2		Re-order point for ATMs	150000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.92357	0.00217	Total cost	854340	571.19
Number of cash-outs	8786.9	248.44	Vehicle cost	708390	0
Total distance travelled	24811	122.35	Cost of distance travelled	0	0
Number of routes determined	114.03	0.73404	Opportunity cost	133620	355.92
Number of routes completed	111.76	0.73561	Rebanking cost	12328	313.78
Average vehicle utilisation	0.25191	0.00181			

EXPERIMENT 32					
Details					
Routing method	First-in-first-out routing		Number of vehicles used	3	
Routing point	2		Re-order point for ATMs	350000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.96501	0.00142	Total cost	941970	1287.1
Number of cash-outs	4024.4	163.42	Vehicle cost	708390	0
Total distance travelled	29371	140.24	Cost of distance travelled	0	0
Number of routes determined	135.75	0.96971	Opportunity cost	154800	379.25
Number of routes completed	132.78	0.91629	Rebanking cost	78787	980.11
Average vehicle utilisation	0.30846	0.00211			

EXPERIMENT 33					
Details					
Routing method	First-in-first-out routing		Number of vehicles used	3	
Routing point	2		Re-order point for ATMs	550000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.98118	0.00111	Total cost	1064100	2936.4
Number of cash-outs	2164.4	127.82	Vehicle cost	708390	0
Total distance travelled	35502	187.18	Cost of distance travelled	381.75	129.15
Number of routes determined	164.77	0.9779	Opportunity cost	173840	498.84
Number of routes completed	161.13	0.97906	Rebanking cost	181510	2432.8
Average vehicle utilisation	0.38224	0.00224			

EXPERIMENT 34					
Details					
Routing method	First-in-first-out routing		Number of vehicles used	3	
Routing point	3		Re-order point for ATMs	150000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.87795	0.00285	Total cost	840110	569.56
Number of cash-outs	14036	326.53	Vehicle cost	708390	0
Total distance travelled	21353	129.79	Cost of distance travelled	0	0
Number of routes determined	85.1	0.69389	Opportunity cost	126450	450.7
Number of routes completed	83.2	0.68723	Rebanking cost	5271.7	211.05
Average vehicle utilisation	0.21704	0.00226			

EXPERIMENT 35					
Details					
Routing method	First-in-first-out routing		Number of vehicles used	3	
Routing point	3		Re-order point for ATMs	350000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.93706	0.00244	Total cost	903940	1841.1
Number of cash-outs	7236.5	282.02	Vehicle cost	708390	0
Total distance travelled	25197	162.56	Cost of distance travelled	0	0
Number of routes determined	101.61	0.77641	Opportunity cost	145990	670.15
Number of routes completed	98.762	0.81634	Rebanking cost	49554	1239.1
Average vehicle utilisation	0.26445	0.00227			

EXPERIMENT 36					
Details					
Routing method	First-in-first-out routing		Number of vehicles used	3	
Routing point	3		Re-order point for ATMs	550000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.97236	0.00151	Total cost	1017400	2849.9
Number of cash-outs	3179.2	174.35	Vehicle cost	708390	0
Total distance travelled	30973	169.01	Cost of distance travelled	2.385	4.7938
Number of routes determined	124.71	0.93186	Opportunity cost	167930	585.26
Number of routes completed	122	0.92345	Rebanking cost	141050	2311.3
Average vehicle utilisation	0.32956	0.00203			

EXPERIMENT 37					
Details					
Routing method	Direct replenishment		Number of vehicles used	1	
Routing point	1		Re-order point for ATMs	150000	
Minimum number of ATMs in route	1		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.63491	0.00495	Total cost	343250	1411.6
Number of cash-outs	41987	572.59	Vehicle cost	236130	0
Total distance travelled	18245	148.08	Cost of distance travelled	15091	470.9
Number of routes determined	128.67	1.0429	Opportunity cost	91314	933.79
Number of routes completed	120	1.0927	Rebanking cost	712.03	88.05
Average vehicle utilisation	0.61111	0.00574			

EXPERIMENT 38					
Details					
Routing method	Direct replenishment		Number of vehicles used	1	
Routing point	1		Re-order point for ATMs	350000	
Minimum number of ATMs in route	1		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.63739	0.00446	Total cost	346570	1309.3
Number of cash-outs	41709	511.64	Vehicle cost	236130	0
Total distance travelled	18486	142.23	Cost of distance travelled	15856	452.29
Number of routes determined	129.97	1.0125	Opportunity cost	92436	806.8
Number of routes completed	120.3	1.08	Rebanking cost	2142.1	178.47
Average vehicle utilisation	0.61984	0.00531			

EXPERIMENT 39					
Details					
Routing method	Direct replenishment		Number of vehicles used	1	
Routing point	1		Re-order point for ATMs	550000	
Minimum number of ATMs in route	1		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.63922	0.00478	Total cost	350070	1487.8
Number of cash-outs	41486	548.2	Vehicle cost	236130	0
Total distance travelled	18634	136.6	Cost of distance travelled	16326	434.41
Number of routes determined	133.73	0.97841	Opportunity cost	93496	888.78
Number of routes completed	121.56	0.98368	Rebanking cost	4120.2	306
Average vehicle utilisation	0.62579	0.00512			

EXPERIMENT 40					
Details					
Routing method	Direct replenishment		Number of vehicles used	2	
Routing point	1		Re-order point for ATMs	150000	
Minimum number of ATMs in route	1		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.91086	0.00304	Total cost	630350	955.57
Number of cash-outs	10253	350	Vehicle cost	472260	0
Total distance travelled	27878	116.67	Cost of distance travelled	6375.4	346.96
Number of routes determined	189.87	1.1069	Opportunity cost	134740	451.68
Number of routes completed	186.06	0.98336	Rebanking cost	16973	423.39
Average vehicle utilisation	0.45314	0.00227			

EXPERIMENT 41					
Details					
Routing method	Direct replenishment		Number of vehicles used	2	
Routing point	1		Re-order point for ATMs	350000	
Minimum number of ATMs in route	1		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.94987	0.0021	Total cost	713440	2294.6
Number of cash-outs	5765.1	242.15	Vehicle cost	472260	0
Total distance travelled	32313	141.36	Cost of distance travelled	16945	440.79
Number of routes determined	218.13	1.1716	Opportunity cost	152560	579.03
Number of routes completed	212.93	1.16	Rebanking cost	71672	1439.2
Average vehicle utilisation	0.53652	0.00254			

Experiment 42					
Details					
Routing method	Direct replenishment		Number of vehicles used	2	
Routing point	1		Re-order point for ATMs	550000	
Minimum number of ATMs in route	1		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.94747	0.00205	Total cost	767430	3207.3
Number of cash-outs	6037.8	236.07	Vehicle cost	472260	0
Total distance travelled	34408	167.19	Cost of distance travelled	23578	529.25
Number of routes determined	235.16	1.3942	Opportunity cost	159490	622.56
Number of routes completed	225.58	1.359	Rebanking cost	112100	2191.1
Average vehicle utilisation	0.57679	0.00292			

EXPERIMENT 43					
Details					
Routing method	Direct replenishment		Number of vehicles used	3	
Routing point	1		Re-order point for ATMs	150000	
Minimum number of ATMs in route	1		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.95281	0.00165	Total cost	874290	705.4
Number of cash-outs	5428	189.64	Vehicle cost	708390	0
Total distance travelled	29381	111.97	Cost of distance travelled	0	0
Number of routes determined	198.28	1.1183	Opportunity cost	140370	366.48
Number of routes completed	196.55	1.134	Rebanking cost	25532	416.81
Average vehicle utilisation	0.30211	0.00134			

EXPERIMENT 44					
Details					
Routing method	Direct replenishment		Number of vehicles used	3	
Routing point	1		Re-order point for ATMs	350000	
Minimum number of ATMs in route	1		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.97505	0.00128	Total cost	979310	1425.5
Number of cash-outs	2868.9	146.95	Vehicle cost	708390	0
Total distance travelled	35004	133.62	Cost of distance travelled	314.66	96.757
Number of routes determined	234.77	1.3554	Opportunity cost	161620	381.91
Number of routes completed	232.15	1.3283	Rebanking cost	108980	1091.9
Average vehicle utilisation	0.36641	0.00145			

EXPERIMENT 45					
Details					
Routing method	Direct replenishment		Number of vehicles used	3	
Routing point	1		Re-order point for ATMs	550000	
Minimum number of ATMs in route	1		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.98824	0.00106	Total cost	1126000	3321.2
Number of cash-outs	1352.3	121.68	Vehicle cost	708390	0
Total distance travelled	42550	194.69	Cost of distance travelled	8397.1	501.12
Number of routes determined	289.62	1.7014	Opportunity cost	181320	442.69
Number of routes completed	286.02	1.6571	Rebanking cost	227940	2519.6
Average vehicle utilisation	0.4611	0.00247			

EXPERIMENT 46					
Details					
Routing method	Near-shortest path routing		Number of vehicles used	1	
Routing point	2		Re-order point for ATMs	150000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.7058	0.006	Total cost	325600	2189.5
Number of cash-outs	33822	690.08	Vehicle cost	118070	0
Total distance travelled (km)	15868	108.53	Cost of distance travelled	106440	327.32
Number of routes determined	67.987	0.57377	Opportunity cost	99537	967.36
Number of routes completed	65.3	0.52356	Rebanking cost	1553.9	160.83
Average vehicle utilisation	0.54378	0.00434			

EXPERIMENT 47					
Details					
Routing method	Near-shortest path routing		Number of vehicles used	1	
Routing point	2		Re-order point for ATMs	350000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.73737	0.0066	Total cost	349410	3147.2
Number of cash-outs	30204	761.91	Vehicle cost	118070	0
Total distance travelled (km)	16745	137.49	Cost of distance travelled	117600	1748.8
Number of routes determined	72.7	0.75207	Opportunity cost	105770	1134.6
Number of routes completed	68.85	0.67664	Rebanking cost	7966.9	738.94
Average vehicle utilisation	0.57788	0.00502			

EXPERIMENT 48					
Details					
Routing method	Near-shortest path routing		Number of vehicles used	1	
Routing point	2		Re-order point for ATMs	550000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.74169	0.00621	Total cost	357050	3025
Number of cash-outs	29705	717.92	Vehicle cost	118070	0
Total distance travelled (km)	16807	100.91	Cost of distance travelled	118390	128306
Number of routes determined	74.425	0.6591	Opportunity cost	108080	1246.5
Number of routes completed	69.562	0.60347	Rebanking cost	12526	1233
Average vehicle utilisation	0.58248	0.00374			

EXPERIMENT 49					
Details					
Routing method	Near-shortest path routing		Number of vehicles used	1	
Routing point	3		Re-order point for ATMs	150000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.73187	0.00482	Total cost	322730	2235.8
Number of cash-outs	30821	552.91	Vehicle cost	118070	0
Total distance travelled	15348	114.98	Cost of distance travelled	99827	1462
Number of routes determined	61.162	0.4327	Opportunity cost	102850	795.6
Number of routes completed	58.65	0.44674	Rebanking cost	1986.4	205.29
Average vehicle utilisation	0.5302	0.00441			

EXPERIMENT 50					
Details					
Routing method	Near-shortest path routing		Number of vehicles used	1	
Routing point	3		Re-order point for ATMs	350000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.75933	0.00389	Total cost	346450	2326
Number of cash-outs	27674	448.64	Vehicle cost	118070	0
Total distance travelled	16169	94.161	Cost of distance travelled	110270	1197.7
Number of routes determined	66	0.41545	Opportunity cost	108750	762.65
Number of routes completed	62.4	0.39913	Rebanking cost	9369.4	755.65
Average vehicle utilisation	0.56346	0.00356			

EXPERIMENT 51					
Details					
Routing method	Near-shortest path routing		Number of vehicles used	1	
Routing point	3		Re-order point for ATMs	550000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.7684	0.00501	Total cost	364020	3128.1
Number of cash-outs	26631	583.06	Vehicle cost	118070	0
Total distance travelled	16671	103.53	Cost of distance travelled	116660	1316.9
Number of routes determined	69.075	0.52887	Opportunity cost	112230	1013.2
Number of routes completed	64.812	0.50325	Rebanking cost	17073	1194.8
Average vehicle utilisation	0.58244	0.00388			

EXPERIMENT 52					
Details					
Routing method	Near-shortest path routing		Number of vehicles used	2	
Routing point	2		Re-order point for ATMs	150000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.88915	0.00327	Total cost	444980	2731.3
Number of cash-outs	12749	377.09	Vehicle cost	236130	0
Total distance travelled	20573	188.49	Cost of distance travelled	71156	2318.7
Number of routes determined	85.362	0.69724	Opportunity cost	126560	449.21
Number of routes completed	84.262	0.73169	Rebanking cost	11136	301.4
Average vehicle utilisation	0.33356	0.00309			

EXPERIMENT 53					
Details					
Routing method	Near-shortest path routing		Number of vehicles used	2	
Routing point	2		Re-order point for ATMs	350000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.94346	0.00241	Total cost	560300	3576.3
Number of cash-outs	6497.5	277.09	Vehicle cost	236130	0
Total distance travelled	23635	197.54	Cost of distance travelled	109840	2512.7
Number of routes determined	98.787	0.90152	Opportunity cost	147550	601.99
Number of routes completed	97.912	0.84327	Rebanking cost	66773	1384
Average vehicle utilisation	0.39424	0.00371			

EXPERIMENT 54					
Details					
Routing method	Near-shortest path routing		Number of vehicles used	2	
Routing point	2		Re-order point for ATMs	550000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.96191	0.00192	Total cost	710760	4945.4
Number of cash-outs	4380	220.11	Vehicle cost	236130	0
Total distance travelled	27285	212.64	Cost of distance travelled	156270	2704.7
Number of routes determined	115.9	0.94575	Opportunity cost	165140	669.52
Number of routes completed	114.35	0.81309	Rebanking cost	153220	3007.3
Average vehicle utilisation	0.46303	0.00371			

EXPERIMENT 55					
Details					
Routing method	Near-shortest path routing		Number of vehicles used	2	
Routing point	3		Re-order point for ATMs	150000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.86673	0.00401	Total cost	415010	2094.8
Number of cash-outs	15323	461.32	Vehicle cost	236130	0
Total distance travelled	18653	125.13	Cost of distance travelled	47285	0
Number of routes determined	73.762	0.59537	Opportunity cost	123080	542.88
Number of routes completed	72.575	0.60446	Rebanking cost	8510	328.8
Average vehicle utilisation	0.30157	0.0024			

EXPERIMENT 56					
Details					
Routing method	Near-shortest path routing		Number of vehicles used	2	
Routing point	3		Re-order point for ATMs	350000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.92459	0.00237	Total cost	518450	2989.2
Number of cash-outs	8673.3	272.78	Vehicle cost	236130	0
Total distance travelled	21794	139.51	Cost of distance travelled	86423	1774.6
Number of routes determined	86.587	0.63832	Opportunity cost	142440	566.23
Number of routes completed	84.8	0.61455	Rebanking cost	53454	1174.5
Average vehicle utilisation	0.36332	0.00267			

EXPERIMENT 57					
Details					
Routing method	Near-shortest path routing		Number of vehicles used	2	
Routing point	3		Re-order point for ATMs	550000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.95519	0.00185	Total cost	664060	4038.6
Number of cash-outs	5154.4	213.3	Vehicle cost	236130	0
Total distance travelled	25914	124.28	Cost of distance travelled	133830	1580.9
Number of routes determined	102.76	0.66917	Opportunity cost	159960	622.84
Number of routes completed	100.6	0.62813	Rebanking cost	129140	2387.7
Average vehicle utilisation	0.43788	0.0025			

EXPERIMENT 58					
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Routing method	Near-shortest path routing		Number of vehicles used	3	
Routing point	2		Re-order point for ATMs	150000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.91763	0.00219	Total cost	505630	1385.5
Number of cash-outs	9469.8	250.4	Vehicle cost	354200	0
Total distance travelled	21150	198.02	Cost of distance travelled	7711.6	1081.4
Number of routes determined	88.037	0.77472	Opportunity cost	130040	405.4
Number of routes completed	87.275	0.78581	Rebanking cost	13677	338.62
Average vehicle utilisation	0.21228	0.00238			

EXPERIMENT 59					
Details					
Routing method	Near-shortest path routing		Number of vehicles used	3	
Routing point	2		Re-order point for ATMs	350000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.96023	0.00179	Total cost	617000	3173.7
Number of cash-outs	4572.9	205.84	Vehicle cost	354200	0
Total distance travelled	24575	220.21	Cost of distance travelled	31883	2250.3
Number of routes determined	102.07	0.98038	Opportunity cost	151430	473.44
Number of routes completed	101.36	0.92846	Rebanking cost	79490	1094.7
Average vehicle utilisation	0.25732	0.00284			

EXPERIMENT 60					
Details					
Routing method	Near-shortest path routing		Number of vehicles used	3	
Routing point	2		Re-order point for ATMs	550000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.98046	0.00111	Total cost	817240	4954.5
Number of cash-outs	2246.5	127.98	Vehicle cost	354200	0
Total distance travelled	29665	300.26	Cost of distance travelled	91360	3763.1
Number of routes determined	125.03	1.2623	Opportunity cost	173860	405.66
Number of routes completed	124.18	1.152	Rebanking cost	197820	2030.7
Average vehicle utilisation	0.32007	0.00339			

EXPERIMENT 61					
Details					
Routing method	Near-shortest path routing		Number of vehicles used	3	
Routing point	3		Re-order point for ATMs	150000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.89513	0.00236	Total cost	493400	745.25
Number of cash-outs	12061	273.89	Vehicle cost	354200	0
Total distance travelled	19485	100.66	Cost of distance travelled	1890.8	421.4
Number of routes determined	76.862	0.38963	Opportunity cost	126490	399.69
Number of routes completed	75.787	0.39928	Rebanking cost	10829	311.95
Average vehicle utilisation	0.19544	0.00189			

EXPERIMENT 62					
Details					
Routing method	Near-shortest path routing		Number of vehicles used	3	
Routing point	3		Re-order point for ATMs	350000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.94323	0.00231	Total cost	582050	3033.9
Number of cash-outs	6528.7	265.73	Vehicle cost	354200	0
Total distance travelled	22634	164.48	Cost of distance travelled	15298	1437
Number of routes determined	90.15	0.73287	Opportunity cost	147060	624.92
Number of routes completed	88.325	0.74984	Rebanking cost	65491	1310.3
Average vehicle utilisation	0.238	0.00261			

EXPERIMENT 63					
Details					
Routing method	Near-shortest path routing		Number of vehicles used	3	
Routing point	3		Re-order point for ATMs	550000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.97741	0.00128	Total cost	771020	5150.8
Number of cash-outs	2597	146.82	Vehicle cost	354200	0
Total distance travelled	28238	172.52	Cost of distance travelled	73127	2168.8
Number of routes determined	111.75	0.78297	Opportunity cost	169340	604.02
Number of routes completed	109.88	0.79311	Rebanking cost	174360	2763.4
Average vehicle utilisation	0.30318	0.00217			

EXPERIMENT 64					
Details					
Routing method	First-in-first-out routing		Number of vehicles used	1	
Routing point	2		Re-order point for ATMs	150000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.64934	0.00474	Total cost	324070	2280.4
Number of cash-outs	40318	543.44	Vehicle cost	118070	0
Total distance travelled	16362	113.29	Cost of distance travelled	112730	1441
Number of routes determined	77.45	0.69325	Opportunity cost	92731	893.92
Number of routes completed	72.875	0.69461	Rebanking cost	535.14	95.164
Average vehicle utilisation	0.55212	0.00452			

EXPERIMENT 65					
Details					
Routing method	First-in-first-out routing		Number of vehicles used	1	
Routing point	2		Re-order point for ATMs	350000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.66141	0.00488	Total cost	334550	2288.3
Number of cash-outs	38932	564.17	Vehicle cost	118070	0
Total distance travelled	16832	111.36	Cost of distance travelled	118710	1416.5
Number of routes determined	81	0.75766	Opportunity cost	95265	856.22
Number of routes completed	74.912	0.71886	Rebanking cost	2503.4	288.84
Average vehicle utilisation	0.56873	0.00419			

EXPERIMENT 66					
Details					
Routing method	First-in-first-out routing		Number of vehicles used	1	
Routing point	2		Re-order point for ATMs	550000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.66136	0.00467	Total cost	339160	2255.5
Number of cash-outs	38942	542.24	Vehicle cost	118070	0
Total distance travelled	16984	108.15	Cost of distance travelled	120640	1375.7
Number of routes determined	83.65	0.80121	Opportunity cost	95953	798.91
Number of routes completed	76.487	0.80986	Rebanking cost	4504.4	391.59
Average vehicle utilisation	0.5715	0.00376			

EXPERIMENT 67					
Details					
Routing method	First-in-first-out routing		Number of vehicles used	1	
Routing point	3		Re-order point for ATMs	150000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.68068	0.00475	Total cost	324050	2001.1
Number of cash-outs	36716	549.45	Vehicle cost	118070	0
Total distance travelled	15953	102.85	Cost of distance travelled	107520	1308.3
Number of routes determined	65.25	0.52975	Opportunity cost	97802	828.47
Number of routes completed	62.025	0.52669	Rebanking cost	656.8	103.03
Average vehicle utilisation	0.54462	0.00373			

EXPERIMENT 68					
Details					
Routing method	First-in-first-out routing		Number of vehicles used	1	
Routing point	3		Re-order point for ATMs	350000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.70025	0.00397	Total cost	338790	1913.4
Number of cash-outs	34459	462.48	Vehicle cost	118070	0
Total distance travelled	16562	88.606	Cost of distance travelled	115270	1127
Number of routes determined	69.575	0.52645	Opportunity cost	101630	700.89
Number of routes completed	64.837	0.47898	Rebanking cost	3818.1	460.84
Average vehicle utilisation	0.56732	0.00344			

EXPERIMENT 69					
Details					
Routing method	First-in-first-out routing		Number of vehicles used	1	
Routing point	3		Re-order point for ATMs	550000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.69781	0.00409	Total cost	349010	2157.4
Number of cash-outs	34752	471.17	Vehicle cost	118070	0
Total distance travelled	17010	101.35	Cost of distance travelled	120980	1289.1
Number of routes determined	72.437	0.5797	Opportunity cost	102100	717.09
Number of routes completed	67.15	0.54239	Rebanking cost	7865.8	493.89
Average vehicle utilisation	0.58013	0.00379			

EXPERIMENT 70					
Details					
Routing method	First-in-first-out routing		Number of vehicles used	2	
Routing point	2		Re-order point for ATMs	150000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.87831	0.004	Total cost	478300	2356.6
Number of cash-outs	13993	461.15	Vehicle cost	236130	0
Total distance travelled	23371	138.69	Cost of distance travelled	106480	1764.1
Number of routes determined	106.53	0.77924	Opportunity cost	127160	564.7
Number of routes completed	103.68	0.72365	Rebanking cost	8534.3	313.08
Average vehicle utilisation	0.37917	0.00264			

EXPERIMENT 71					
Details					
Routing method	First-in-first-out routing		Number of vehicles used	2	
Routing point	2		Re-order point for ATMs	350000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.93434	0.0026	Total cost	596500	3277.1
Number of cash-outs	7550.7	298.9	Vehicle cost	236130	0
Total distance travelled	27422	147.25	Cost of distance travelled	158010	1873
Number of routes determined	126.62	0.90484	Opportunity cost	146530	622.81
Number of routes completed	123.45	0.86476	Rebanking cost	55840	1319.7
Average vehicle utilisation	0.45318	0.00263			

EXPERIMENT 72					
Details					
Routing method	First-in-first-out routing		Number of vehicles used	2	
Routing point	2		Re-order point for ATMs	550000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.94962	0.00223	Total cost	699130	4055.9
Number of cash-outs	5791.8	256.25	Vehicle cost	236130	0
Total distance travelled	30695	156.65	Cost of distance travelled	199650	1992.6
Number of routes determined	142.93	1.0109	Opportunity cost	157570	576.92
Number of routes completed	137.97	0.97332	Rebanking cost	105790	2046.7
Average vehicle utilisation	0.51466	0.00275			

EXPERIMENT 73					
Details					
Routing method	First-in-first-out routing		Number of vehicles used	2	
Routing point	3		Re-order point for ATMs	150000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.84216	0.00318	Total cost	430720	2071.5
Number of cash-outs	18153	368.02	Vehicle cost	236130	0
Total distance travelled	20409	141.98	Cost of distance travelled	69048	1762.7
Number of routes determined	81.9	0.61476	Opportunity cost	121700	504.18
Number of routes completed	79.275	0.64854	Rebanking cost	3842.7	175.79
Average vehicle utilisation	0.33058	0.00289			

EXPERIMENT 74					
Details					
Routing method	First-in-first-out routing		Number of vehicles used	2	
Routing point	3		Re-order point for ATMs	350000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.91626	0.0029	Total cost	532700	2928.3
Number of cash-outs	9626.6	334.58	Vehicle cost	236130	0
Total distance travelled	24174	142.03	Cost of distance travelled	116700	1806.6
Number of routes determined	97.55	0.73531	Opportunity cost	141530	613.41
Number of routes completed	94.862	0.71579	Rebanking cost	38343	1053.8
Average vehicle utilisation	0.39875	0.00294			

EXPERIMENT 75					
Details					
Routing method	First-in-first-out routing		Number of vehicles used	2	
Routing point	3		Re-order point for ATMs	550000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.94812	0.00225	Total cost	664210	4286.2
Number of cash-outs	5969.4	259.6	Vehicle cost	236130	0
Total distance travelled	28380	14782	Cost of distance travelled	170200	1880.3
Number of routes determined	115.55	0.81607	Opportunity cost	157420	669.46
Number of routes completed	111.86	0.79773	Rebanking cost	100450	2178
Average vehicle utilisation	0.47612	0.0027			

EXPERIMENT 76					
Details					
Routing method	First-in-first-out routing		Number of vehicles used	3	
Routing point	2		Re-order point for ATMs	150000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.92357	0.00217	Total cost	537590	1806.4
Number of cash-outs	8786.9	248.44	Vehicle cost	354200	0
Total distance travelled	24811	122.35	Cost of distance travelled	37449	1590.6
Number of routes determined	114.03	0.73404	Opportunity cost	133620	355.92
Number of routes completed	111.76	0.73561	Rebanking cost	12328	313.78
Average vehicle utilisation	0.25191	0.00181			

EXPERIMENT 77					
Details					
Routing method	First-in-first-out routing		Number of vehicles used	3	
Routing point	2		Re-order point for ATMs	350000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.96501	0.00142	Total cost	675420	2421.5
Number of cash-outs	4024.4	163.42	Vehicle cost	354200	0
Total distance travelled	29371	140.24	Cost of distance travelled	87638	1748.1
Number of routes determined	135.75	0.96971	Opportunity cost	154800	379.25
Number of routes completed	132.78	0.91629	Rebanking cost	78787	980.11
Average vehicle utilisation	0.30846	0.00211			

EXPERIMENT 78					
Details					
Routing method	First-in-first-out routing		Number of vehicles used	3	
Routing point	2		Re-order point for ATMs	550000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.98118	0.00111	Total cost	874940	4716.9
Number of cash-outs	2164.4	127.82	Vehicle cost	354200	0
Total distance travelled	35502	187.18	Cost of distance travelled	165390	2381
Number of routes determined	164.77	0.9779	Opportunity cost	173840	498.84
Number of routes completed	161.13	0.97906	Rebanking cost	181510	2432.8
Average vehicle utilisation	0.38224	0.00224			

EXPERIMENT 79					
Details					
Routing method	First-in-first-out routing		Number of vehicles used	3	
Routing point	3		Re-order point for ATMs	150000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.87795	0.00285	Total cost	496270	1355.2
Number of cash-outs	14036	326.53	Vehicle cost	354200	0
Total distance travelled	21353	129.79	Cost of distance travelled	10350	1028.5
Number of routes determined	85.1	0.69389	Opportunity cost	126450	450.7
Number of routes completed	83.2	0.68723	Rebanking cost	5271.7	211.05
Average vehicle utilisation	0.21704	0.00226			

EXPERIMENT 80					
Details					
Routing method	First-in-first-out routing		Number of vehicles used	3	
Routing point	3		Re-order point for ATMs	350000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.93706	0.00244	Total cost	587850	3241.6
Number of cash-outs	7236.5	282.02	Vehicle cost	354200	0
Total distance travelled	25197	162.56	Cost of distance travelled	38106	1834.5
Number of routes determined	101.61	0.77641	Opportunity cost	145990	670.15
Number of routes completed	98.762	0.81634	Rebanking cost	49554	1239.1
Average vehicle utilisation	0.26445	0.00227			

EXPERIMENT 81					
Details					
Routing method	First-in-first-out routing		Number of vehicles used	3	
Routing point	3		Re-order point for ATMs	550000	
Minimum number of ATMs in route	2		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.97236	0.00151	Total cost	770980	4449.4
Number of cash-outs	3179.2	174.35	Vehicle cost	354200	0
Total distance travelled	30973	169.01	Cost of distance travelled	107800	2148.2
Number of routes determined	124.71	0.93186	Opportunity cost	167930	585.26
Number of routes completed	122	0.92345	Rebanking cost	141050	2311.3
Average vehicle utilisation	0.32956	0.00203			

EXPERIMENT 82					
Details					
Routing method	Direct replenishment		Number of vehicles used	1	
Routing point	1		Re-order point for ATMs	150000	
Minimum number of ATMs in route	1		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.63491	0.00495	Total cost	346780	2791.5
Number of cash-outs	41987	572.59	Vehicle cost	118070	0
Total distance travelled	18245	148.08	Cost of distance travelled	136690	1883.6
Number of routes determined	128.67	1.0429	Opportunity cost	91314	933.79
Number of routes completed	120	1.0927	Rebanking cost	712.03	88.05
Average vehicle utilisation	0.61111	0.00574			

EXPERIMENT 83					
Details					
Routing method	Direct replenishment		Number of vehicles used	1	
Routing point	1		Re-order point for ATMs	350000	
Minimum number of ATMs in route	1		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.63739	0.00446	Total cost	352390	2638.9
Number of cash-outs	41709	511.64	Vehicle cost	118070	0
Total distance travelled	18486	142.23	Cost of distance travelled	139750	1809.1
Number of routes determined	129.97	1.0125	Opportunity cost	92436	806.8
Number of routes completed	120.3	1.08	Rebanking cost	2142.1	178.47
Average vehicle utilisation	0.61984	0.00531			

EXPERIMENT 84					
Details					
Routing method	Direct replenishment		Number of vehicles used	1	
Routing point	1		Re-order point for ATMs	550000	
Minimum number of ATMs in route	1		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.63922	0.00478	Total cost	357310	2739.6
Number of cash-outs	41486	548.2	Vehicle cost	118070	0
Total distance travelled	18634	136.6	Cost of distance travelled	141630	1737.6
Number of routes determined	133.73	0.97841	Opportunity cost	93496	888.78
Number of routes completed	121.56	0.98368	Rebanking cost	4120.2	306
Average vehicle utilisation	0.62579	0.00512			

EXPERIMENT 85					
Details					
Routing method	Direct replenishment		Number of vehicles used	2	
Routing point	1		Re-order point for ATMs	150000	
Minimum number of ATMs in route	1		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.91086	0.00304	Total cost	551660	2007.2
Number of cash-outs	10253	350	Vehicle cost	236130	0
Total distance travelled	27878	116.67	Cost of distance travelled	163810	1484.1
Number of routes determined	189.87	1.1069	Opportunity cost	134740	451.68
Number of routes completed	186.06	0.98336	Rebanking cost	16973	423.39
Average vehicle utilisation	0.45314	0.00227			

EXPERIMENT 86					
Details					
Routing method	Direct replenishment		Number of vehicles used	2	
Routing point	1		Re-order point for ATMs	350000	
Minimum number of ATMs in route	1		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.94987	0.0021	Total cost	680600	3486.1
Number of cash-outs	5765.1	242.15	Vehicle cost	236130	0
Total distance travelled	32313	141.36	Cost of distance travelled	220230	1798.1
Number of routes determined	218.13	1.1716	Opportunity cost	152560	579.03
Number of routes completed	212.93	1.16	Rebanking cost	71672	1439.2
Average vehicle utilisation	0.53652	0.00254			

Experiment 87					
Details					
Routing method	Direct replenishment		Number of vehicles used	2	
Routing point	1		Re-order point for ATMs	550000	
Minimum number of ATMs in route	1		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.94747	0.00205	Total cost	754600	4644.1
Number of cash-outs	6037.8	236.07	Vehicle cost	236130	0
Total distance travelled	34408	167.19	Cost of distance travelled	246880	2126.6
Number of routes determined	235.16	1.3942	Opportunity cost	159490	622.56
Number of routes completed	225.58	1.359	Rebanking cost	112100	2191.1
Average vehicle utilisation	0.57679	0.00292			

EXPERIMENT 88					
Details					
Routing method	Direct replenishment		Number of vehicles used	3	
Routing point	1		Re-order point for ATMs	150000	
Minimum number of ATMs in route	1		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.95281	0.00165	Total cost	607800	1948.6
Number of cash-outs	5428	189.64	Vehicle cost	354200	0
Total distance travelled	29381	111.97	Cost of distance travelled	87699	1399.1
Number of routes determined	198.28	1.1183	Opportunity cost	140370	366.48
Number of routes completed	196.55	1.134	Rebanking cost	25532	416.81
Average vehicle utilisation	0.30211	0.00134			

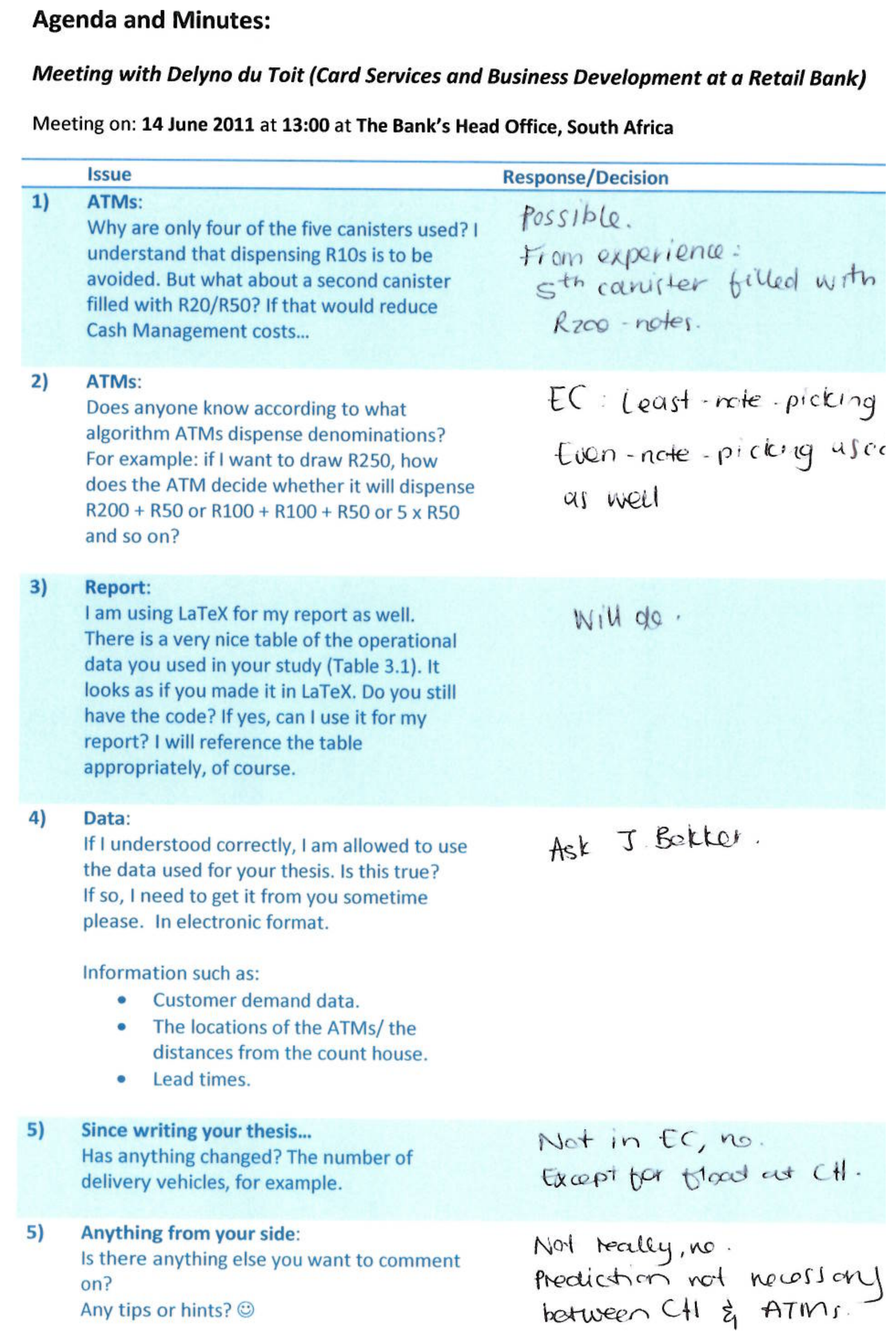
EXPERIMENT 89					
Details					
Routing method	Direct replenishment		Number of vehicles used	3	
Routing point	1		Re-order point for ATMs	350000	
Minimum number of ATMs in route	1		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.97505	0.00128	Total cost	783850	2831
Number of cash-outs	2868.9	146.95	Vehicle cost	354200	0
Total distance travelled	35004	133.62	Cost of distance travelled	159050	1699.7
Number of routes determined	234.77	1.3554	Opportunity cost	161620	381.91
Number of routes completed	232.15	1.3283	Rebanking cost	108980	1091.9
Average vehicle utilisation	0.36641	0.00145			

EXPERIMENT 90					
Details					
Routing method	Direct replenishment		Number of vehicles used	3	
Routing point	1		Re-order point for ATMs	550000	
Minimum number of ATMs in route	1		Number of replications	80	
Results					
Output	*Average*	*95% CI half-width*	*Output*	*Average*	*95% CI half-width*
OPERATIONAL			COST		
Service level	0.98824	0.00106	Total cost	1018500	5147
Number of cash-outs	1352.3	121.68	Vehicle cost	354200	0
Total distance travelled	42550	194.69	Cost of distance travelled	255040	2476.5
Number of routes determined	289.62	1.7014	Opportunity cost	181320	442.69
Number of routes completed	286.02	1.6571	Rebanking cost	227940	2519.6
Average vehicle utilisation	0.4611	0.00247			

Appendix E

Delyno du Toit: Meeting agenda and minutes

The agenda and minutes for the official meeting held with Delyno du Toit follows.



Agenda and Minutes:

Meeting with Delyno du Toit (Card Services and Business Development at a Retail Bank)

Meeting on: **14 June 2011** at **13:00** at **The Bank's Head Office, South Africa**

	Issue	Response/Decision
1)	**ATMs:** Why are only four of the five canisters used? I understand that dispensing R10s is to be avoided. But what about a second canister filled with R20/R50? If that would reduce Cash Management costs…	Possible. From experience: 5th canister filled with R200-notes.
2)	**ATMs:** Does anyone know according to what algorithm ATMs dispense denominations? For example: if I want to draw R250, how does the ATM decide whether it will dispense R200 + R50 or R100 + R100 + R50 or 5 x R50 and so on?	EC: Least-note-picking Even-note-picking usec as well
3)	**Report:** I am using LaTeX for my report as well. There is a very nice table of the operational data you used in your study (Table 3.1). It looks as if you made it in LaTeX. Do you still have the code? If yes, can I use it for my report? I will reference the table appropriately, of course.	Will do.
4)	**Data:** If I understood correctly, I am allowed to use the data used for your thesis. Is this true? If so, I need to get it from you sometime please. In electronic format. Information such as: • Customer demand data. • The locations of the ATMs/ the distances from the count house. • Lead times.	Ask J. Bekker.
5)	**Since writing your thesis…** Has anything changed? The number of delivery vehicles, for example.	Not in EC, no. Except for flood at CH.
5)	**Anything from your side:** Is there anything else you want to comment on? Any tips or hints? ☺	Not really, no. Prediction not necessary between CH & ATMs.

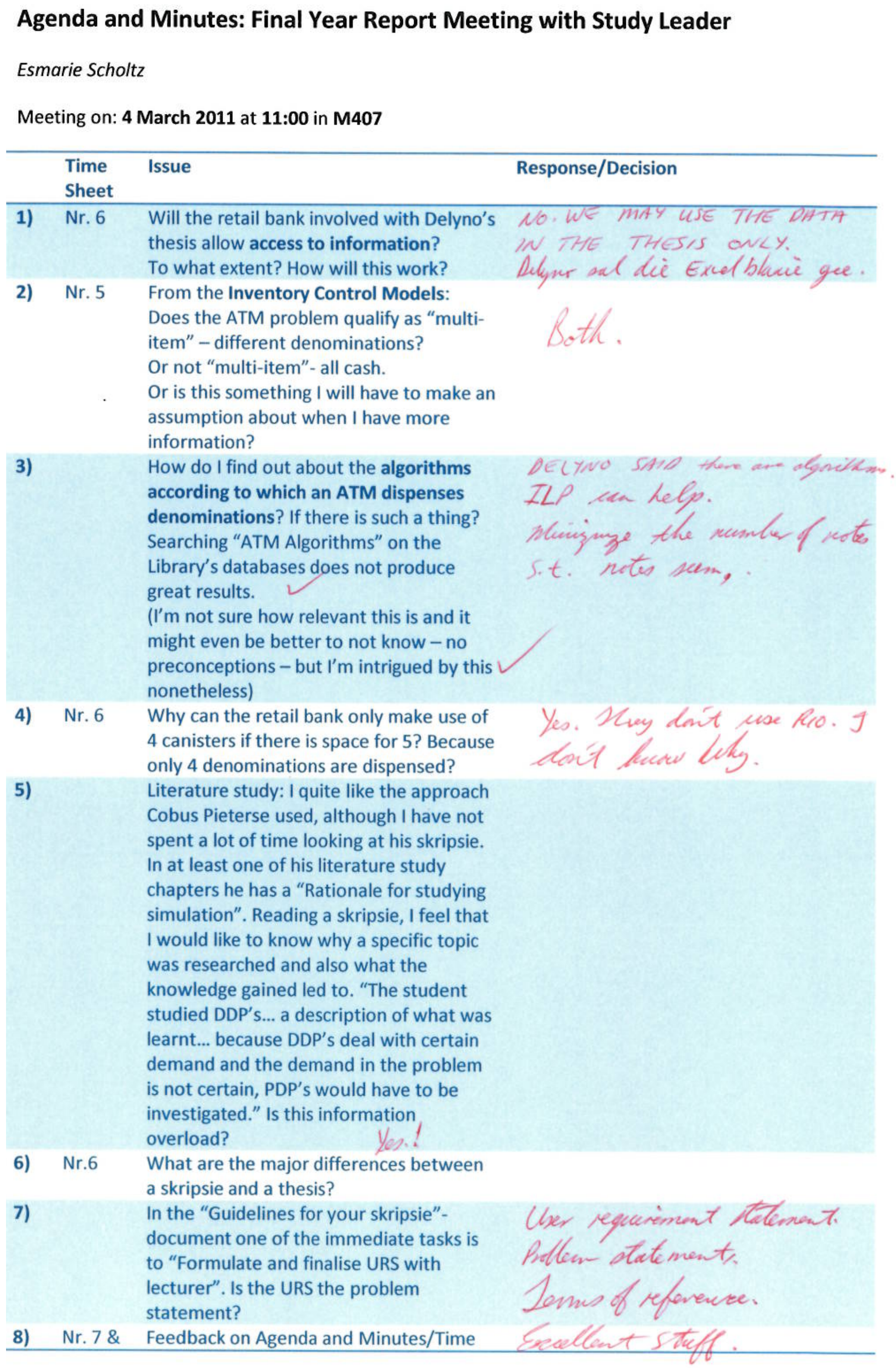
Figure E.1: Agenda and minutes for meeting held with Delyno du Toit.

Appendix F

Documentation of meetings with study leader

The study leader recommended that agendas be drawn up for project meetings to ensure that meetings are effective. The compilation of periodic time sheets was also recommended. These agendas and time sheets could be used to solve disputes that might arise during project execution. The agendas and time sheets also illustrate the student's individual project management capability.

The meeting documents for two project meetings are contained in this appendix. The first set was drawn up for one of the very first meetings, whilst the second set was used for one of the last. Due to space constraints, all agendas, time sheets, and supplements cannot be included. The reader is, however, invited to request these as the student has kept and filed agendas, minutes, and time sheets for all official project meetings.



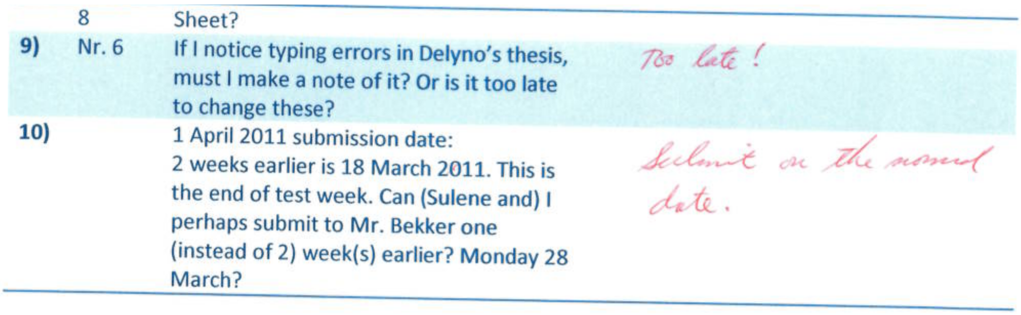
Agenda and Minutes: Final Year Report Meeting with Study Leader

Esmarie Scholtz

Meeting on: **4 March 2011** at **11:00** in **M407**

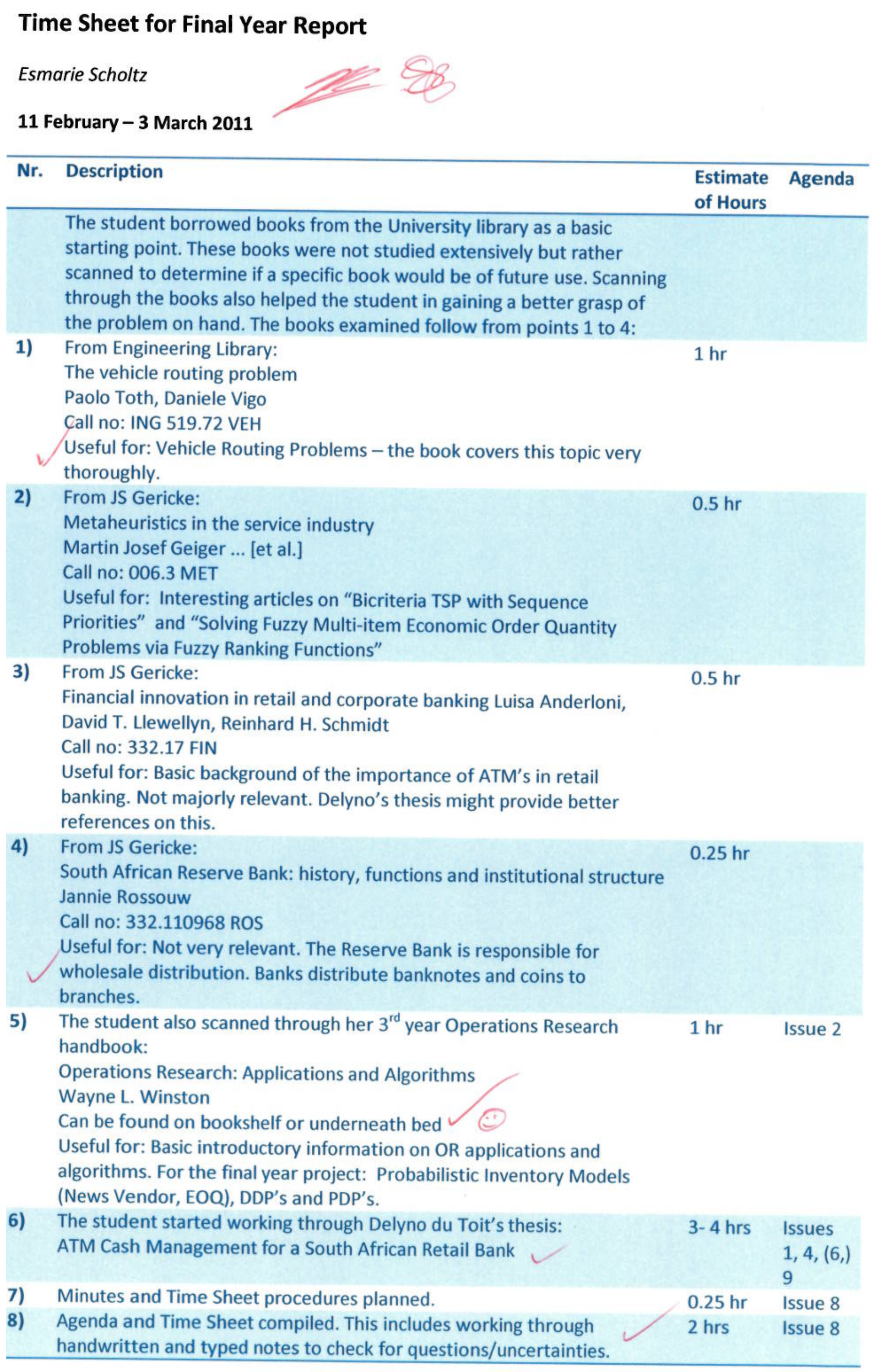
	Time Sheet	Issue	Response/Decision
1)	Nr. 6	Will the retail bank involved with Delyno's thesis allow **access to information**? To what extent? How will this work?	NO. WE MAY USE THE DATA IN THE THESIS ONLY. Delyno sal die Excel blaaie gee.
2)	Nr. 5	From the **Inventory Control Models**: Does the ATM problem qualify as "multi-item" – different denominations? Or not "multi-item"- all cash. Or is this something I will have to make an assumption about when I have more information?	Both.
3)		How do I find out about the **algorithms according to which an ATM dispenses denominations**? If there is such a thing? Searching "ATM Algorithms" on the Library's databases does not produce great results. ✓ (I'm not sure how relevant this is and it might even be better to not know – no preconceptions – but I'm intrigued by this nonetheless) ✓	DELYNO SAID there are algorithms. ILP can help. Minimize the number of notes s.t. notes sum, .
4)	Nr. 6	Why can the retail bank only make use of 4 canisters if there is space for 5? Because only 4 denominations are dispensed?	Yes. They don't use R10. I don't know why.
5)		Literature study: I quite like the approach Cobus Pieterse used, although I have not spent a lot of time looking at his skripsie. In at least one of his literature study chapters he has a "Rationale for studying simulation". Reading a skripsie, I feel that I would like to know why a specific topic was researched and also what the knowledge gained led to. "The student studied DDP's… a description of what was learnt… because DDP's deal with certain demand and the demand in the problem is not certain, PDP's would have to be investigated." Is this information overload? Yes.!	
6)	Nr.6	What are the major differences between a skripsie and a thesis?	
7)		In the "Guidelines for your skripsie"- document one of the immediate tasks is to "Formulate and finalise URS with lecturer". Is the URS the problem statement?	User requirement statement. Problem statement. Terms of reference.
8)	Nr. 7 &	Feedback on Agenda and Minutes/Time	Excellent Stuff.

Figure F.1: Page one of agenda and minutes for meeting on 4 March 2011.



	8	Sheet?	
9)	Nr. 6	If I notice typing errors in Delyno's thesis, must I make a note of it? Or is it too late to change these?	Too late!
10)		1 April 2011 submission date: 2 weeks earlier is 18 March 2011. This is the end of test week. Can (Sulene and) I perhaps submit to Mr. Bekker one (instead of 2) week(s) earlier? Monday 28 March?	Submit on the normal date.

Figure F.2: Page two of agenda and minutes for meeting on 4 March 2011.



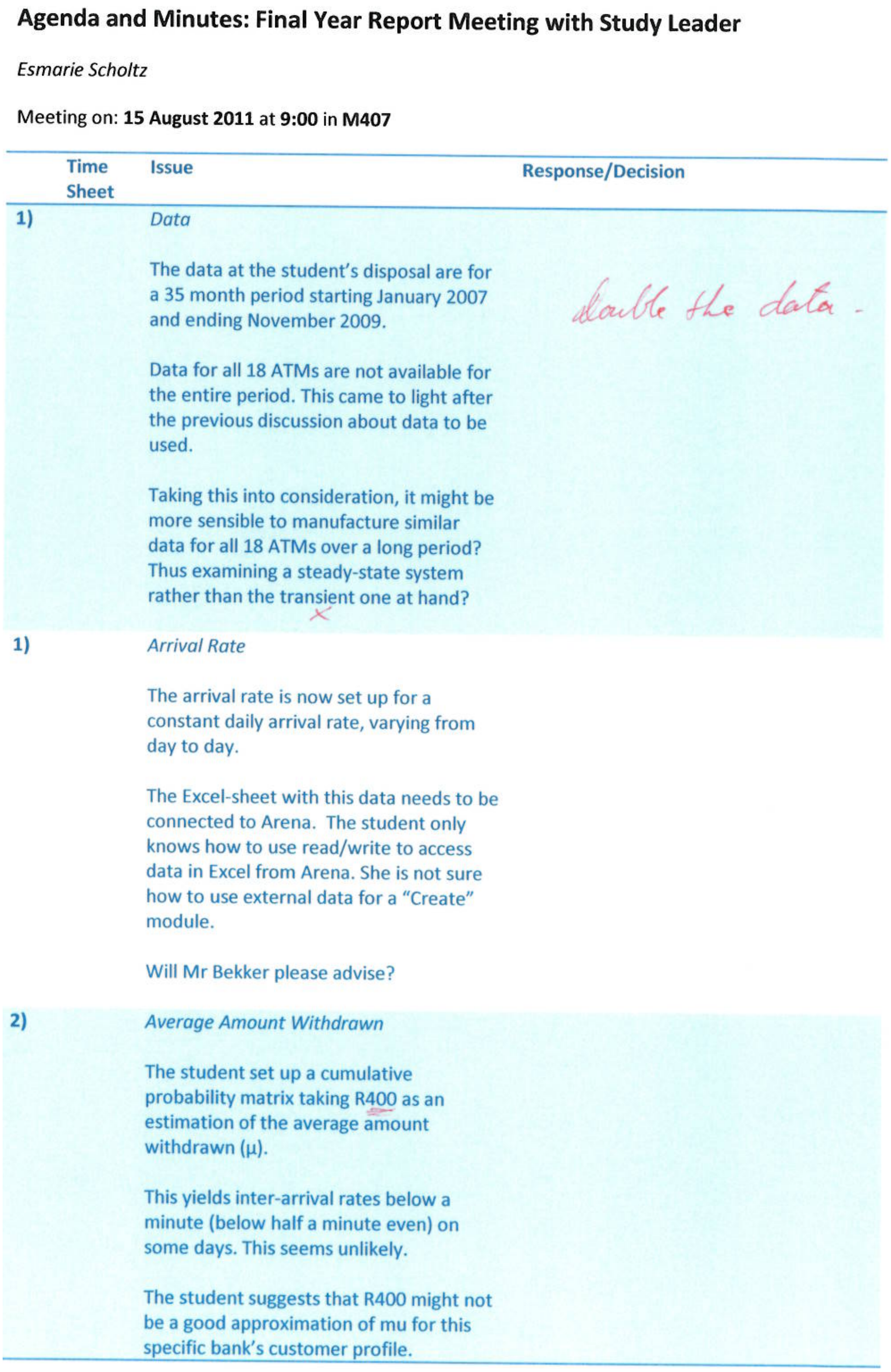
Time Sheet for Final Year Report

Esmarie Scholtz

11 February – 3 March 2011

Nr.	Description	Estimate of Hours	Agenda
	The student borrowed books from the University library as a basic starting point. These books were not studied extensively but rather scanned to determine if a specific book would be of future use. Scanning through the books also helped the student in gaining a better grasp of the problem on hand. The books examined follow from points 1 to 4:		
1)	From Engineering Library: The vehicle routing problem Paolo Toth, Daniele Vigo Call no: ING 519.72 VEH Useful for: Vehicle Routing Problems – the book covers this topic very thoroughly.	1 hr	
2)	From JS Gericke: Metaheuristics in the service industry Martin Josef Geiger ... [et al.] Call no: 006.3 MET Useful for: Interesting articles on “Bicriteria TSP with Sequence Priorities” and “Solving Fuzzy Multi-item Economic Order Quantity Problems via Fuzzy Ranking Functions”	0.5 hr	
3)	From JS Gericke: Financial innovation in retail and corporate banking Luisa Anderloni, David T. Llewellyn, Reinhard H. Schmidt Call no: 332.17 FIN Useful for: Basic background of the importance of ATM’s in retail banking. Not majorly relevant. Delyno’s thesis might provide better references on this.	0.5 hr	
4)	From JS Gericke: South African Reserve Bank: history, functions and institutional structure Jannie Rossouw Call no: 332.110968 ROS Useful for: Not very relevant. The Reserve Bank is responsible for wholesale distribution. Banks distribute banknotes and coins to branches.	0.25 hr	
5)	The student also scanned through her 3rd year Operations Research handbook: Operations Research: Applications and Algorithms Wayne L. Winston Can be found on bookshelf or underneath bed Useful for: Basic introductory information on OR applications and algorithms. For the final year project: Probabilistic Inventory Models (News Vendor, EOQ), DDP’s and PDP’s.	1 hr	Issue 2
6)	The student started working through Delyno du Toit’s thesis: ATM Cash Management for a South African Retail Bank	3- 4 hrs	Issues 1, 4, (6,) 9
7)	Minutes and Time Sheet procedures planned.	0.25 hr	Issue 8
8)	Agenda and Time Sheet compiled. This includes working through handwritten and typed notes to check for questions/uncertainties.	2 hrs	Issue 8

Figure F.3: Time sheet for meeting on 4 March 2011.



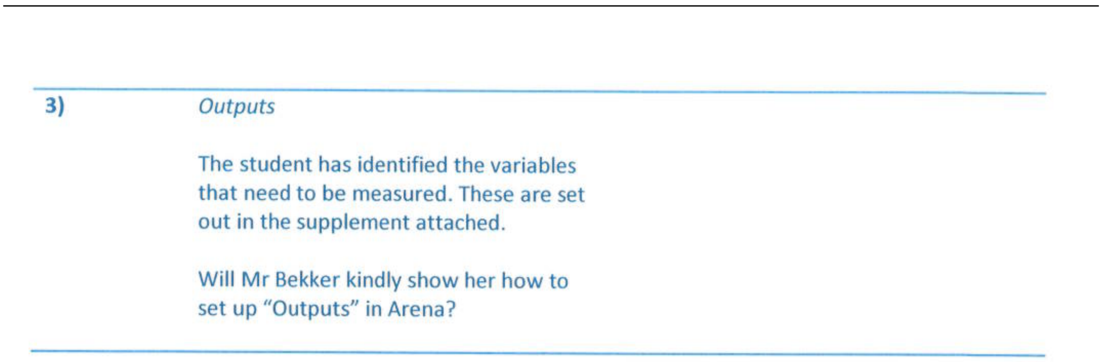
Agenda and Minutes: Final Year Report Meeting with Study Leader

Esmarie Scholtz

Meeting on: **15 August 2011** at **9:00** in **M407**

	Time Sheet	Issue	Response/Decision
1)		*Data* The data at the student's disposal are for a 35 month period starting January 2007 and ending November 2009. Data for all 18 ATMs are not available for the entire period. This came to light after the previous discussion about data to be used. Taking this into consideration, it might be more sensible to manufacture similar data for all 18 ATMs over a long period? Thus examining a steady-state system rather than the transient one at hand?	double the data.
1)		*Arrival Rate* The arrival rate is now set up for a constant daily arrival rate, varying from day to day. The Excel-sheet with this data needs to be connected to Arena. The student only knows how to use read/write to access data in Excel from Arena. She is not sure how to use external data for a "Create" module. Will Mr Bekker please advise?	
2)		*Average Amount Withdrawn* The student set up a cumulative probability matrix taking R400 as an estimation of the average amount withdrawn (μ). This yields inter-arrival rates below a minute (below half a minute even) on some days. This seems unlikely. The student suggests that R400 might not be a good approximation of mu for this specific bank's customer profile.	

Figure F.4: Page one of agenda and minutes for meeting on 15 August 2011.

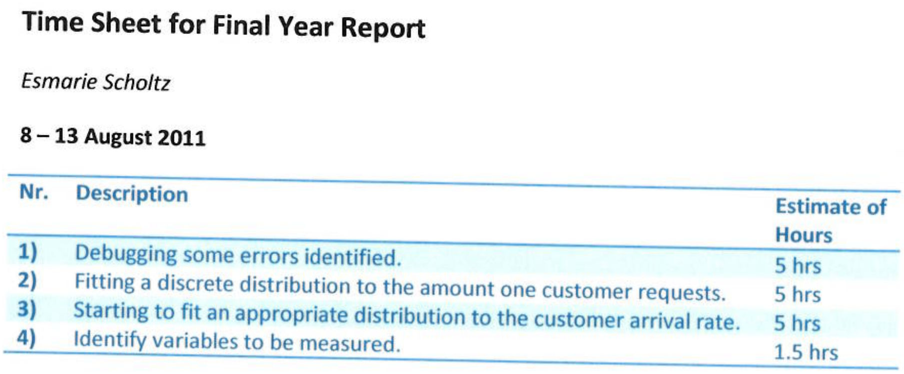


3) *Outputs*

The student has identified the variables that need to be measured. These are set out in the supplement attached.

Will Mr Bekker kindly show her how to set up “Outputs” in Arena?

Figure F.5: Page two of agenda and minutes for meeting on 15 August 2011.



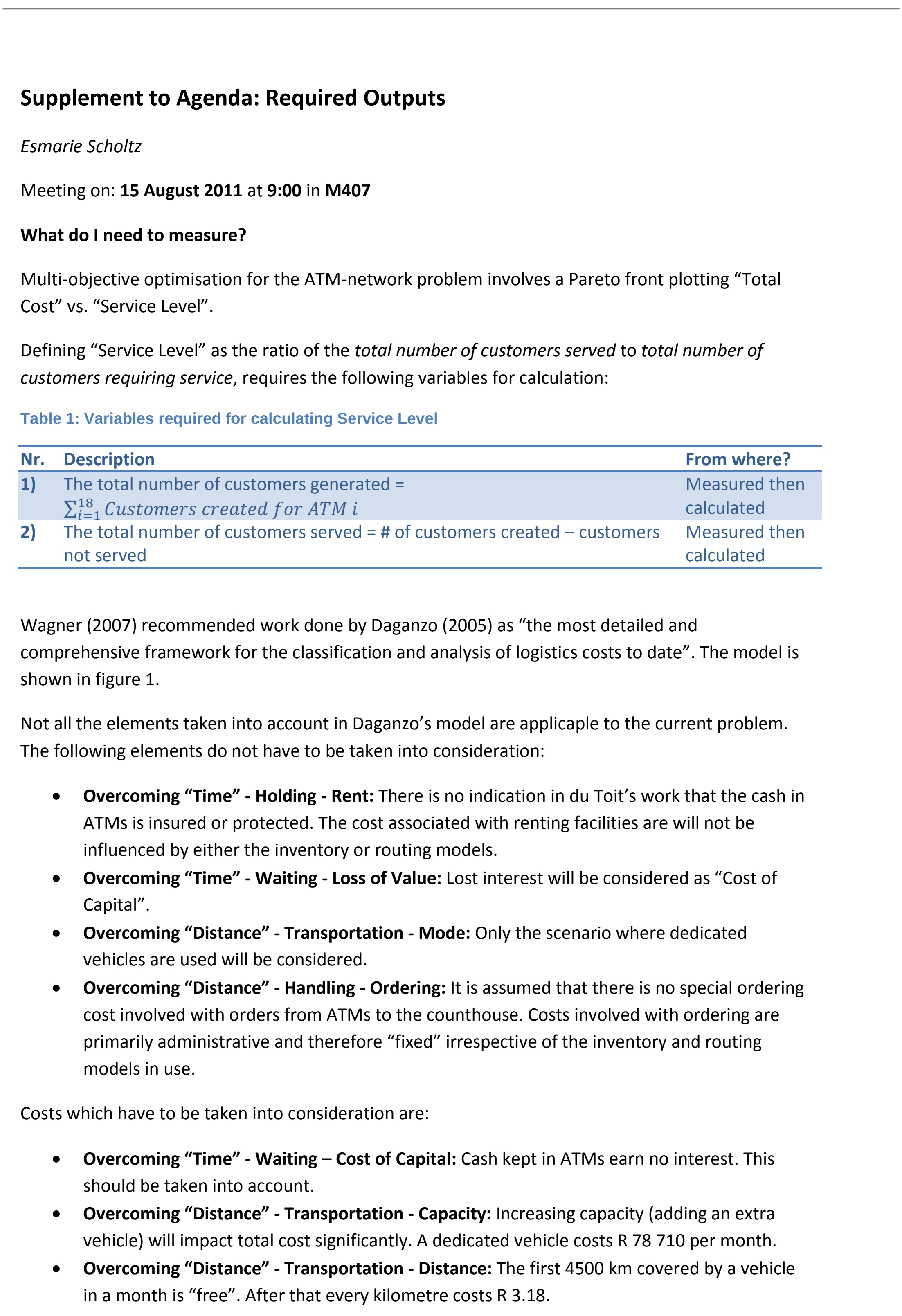
Time Sheet for Final Year Report

Esmarie Scholtz

8 – 13 August 2011

Nr.	Description	Estimate of Hours
1)	Debugging some errors identified.	5 hrs
2)	Fitting a discrete distribution to the amount one customer requests.	5 hrs
3)	Starting to fit an appropriate distribution to the customer arrival rate.	5 hrs
4)	Identify variables to be measured.	1.5 hrs

Figure F.6: Time sheet for meeting on 15 August 2011.



Supplement to Agenda: Required Outputs

Esmarie Scholtz

Meeting on: **15 August 2011** at **9:00** in **M407**

What do I need to measure?

Multi-objective optimisation for the ATM-network problem involves a Pareto front plotting "Total Cost" vs. "Service Level".

Defining "Service Level" as the ratio of the *total number of customers served* to *total number of customers requiring service*, requires the following variables for calculation:

Table 1: Variables required for calculating Service Level

Nr.	Description	From where?
1)	The total number of customers generated = $\sum_{i=1}^{18} Customers\ created\ for\ ATM\ i$	Measured then calculated
2)	The total number of customers served = # of customers created – customers not served	Measured then calculated

Wagner (2007) recommended work done by Daganzo (2005) as "the most detailed and comprehensive framework for the classification and analysis of logistics costs to date". The model is shown in figure 1.

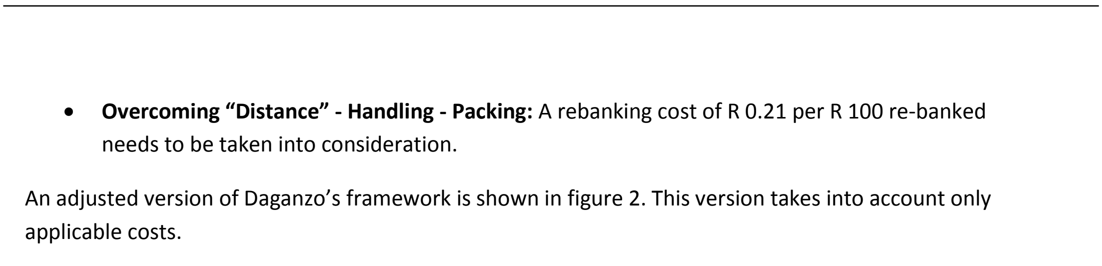
Not all the elements taken into account in Daganzo's model are applicape to the current problem. The following elements do not have to be taken into consideration:

- **Overcoming "Time" - Holding - Rent:** There is no indication in du Toit's work that the cash in ATMs is insured or protected. The cost associated with renting facilities are will not be influenced by either the inventory or routing models.
- **Overcoming "Time" - Waiting - Loss of Value:** Lost interest will be considered as "Cost of Capital".
- **Overcoming "Distance" - Transportation - Mode:** Only the scenario where dedicated vehicles are used will be considered.
- **Overcoming "Distance" - Handling - Ordering:** It is assumed that there is no special ordering cost involved with orders from ATMs to the counthouse. Costs involved with ordering are primarily administrative and therefore "fixed" irrespective of the inventory and routing models in use.

Costs which have to be taken into consideration are:

- **Overcoming "Time" - Waiting – Cost of Capital:** Cash kept in ATMs earn no interest. This should be taken into account.
- **Overcoming "Distance" - Transportation - Capacity:** Increasing capacity (adding an extra vehicle) will impact total cost significantly. A dedicated vehicle costs R 78 710 per month.
- **Overcoming "Distance" - Transportation - Distance:** The first 4500 km covered by a vehicle in a month is "free". After that every kilometre costs R 3.18.

Figure F.7: Page one of the supplement to the agenda for meeting on 15 August 2011.



- **Overcoming "Distance" - Handling - Packing:** A rebanking cost of R 0.21 per R 100 re-banked needs to be taken into consideration.

An adjusted version of Daganzo's framework is shown in figure 2. This version takes into account only applicable costs.

Figure F.8: Page two of the supplement to the agenda for meeting on 15 August 2011.

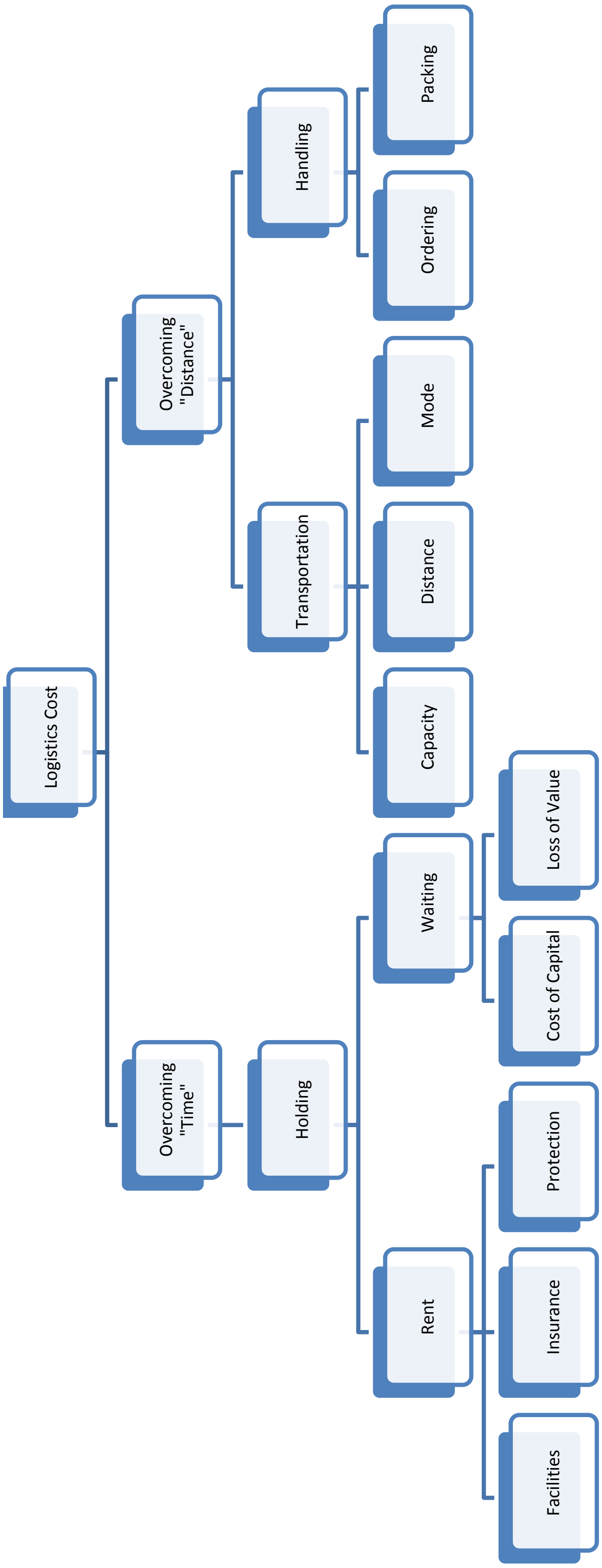


Figure F.9: Page three of the supplement to the agenda for meeting on 15 August 2011.

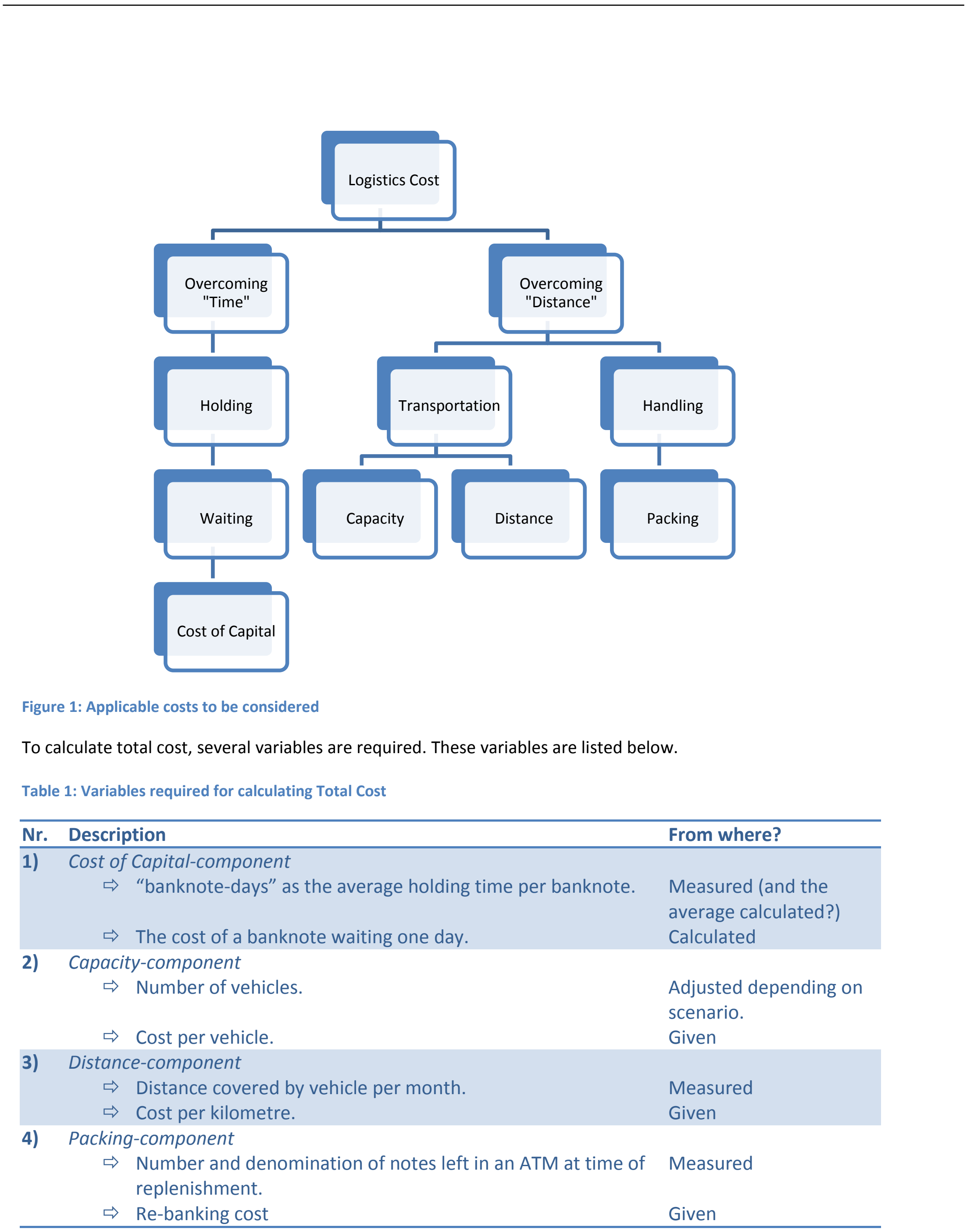


Figure 1: Applicable costs to be considered

To calculate total cost, several variables are required. These variables are listed below.

Table 1: Variables required for calculating Total Cost

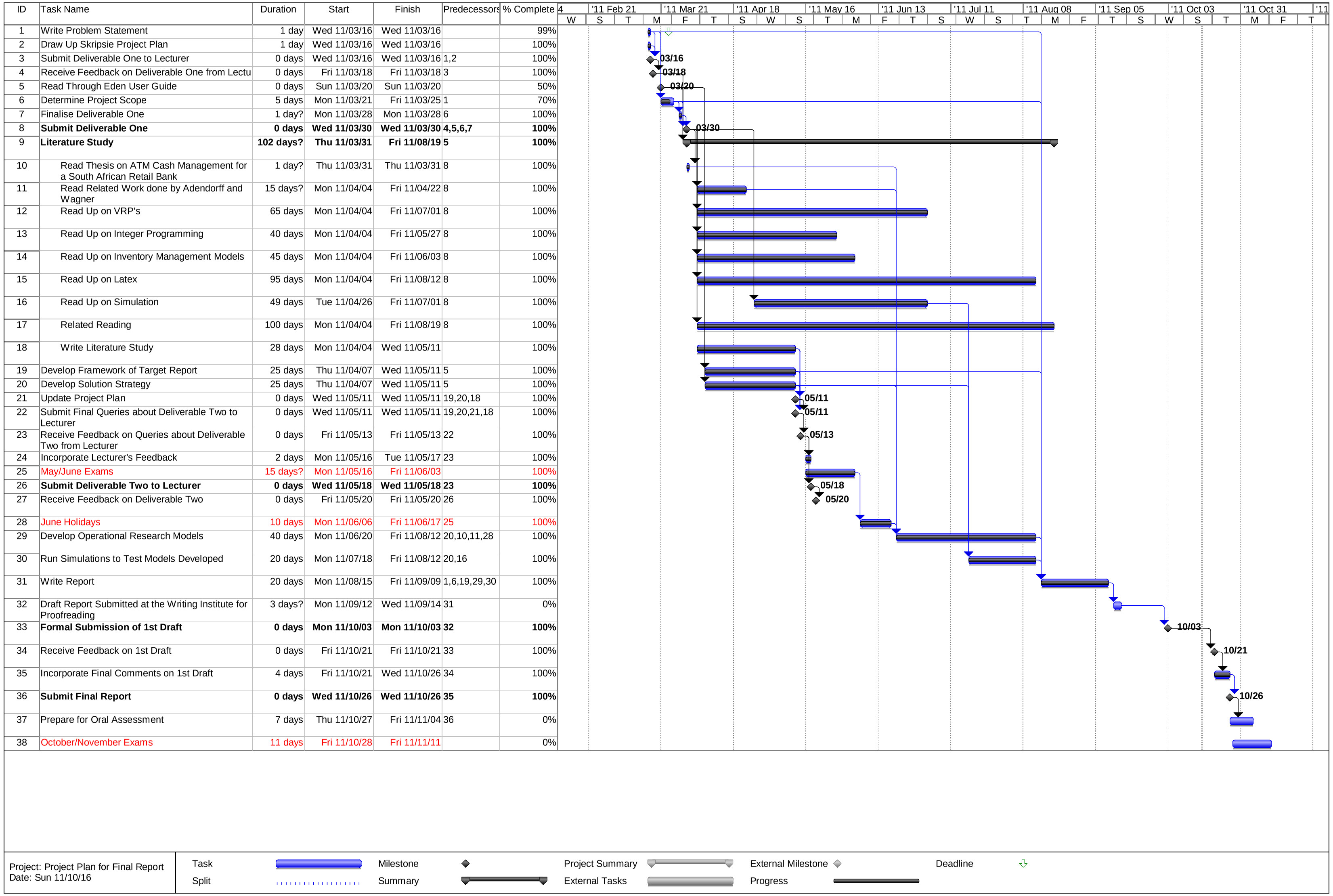
Nr.	Description	From where?
1)	*Cost of Capital-component*	
	⇨ "banknote-days" as the average holding time per banknote.	Measured (and the average calculated?)
	⇨ The cost of a banknote waiting one day.	Calculated
2)	*Capacity-component*	
	⇨ Number of vehicles.	Adjusted depending on scenario.
	⇨ Cost per vehicle.	Given
3)	*Distance-component*	
	⇨ Distance covered by vehicle per month.	Measured
	⇨ Cost per kilometre.	Given
4)	*Packing-component*	
	⇨ Number and denomination of notes left in an ATM at time of replenishment.	Measured
	⇨ Re-banking cost	Given

Figure F.10: Page four of the supplement to the agenda for meeting on 15 August 2011.

Appendix G

Project plan

The final year project plan is shown in Figure G.1. This idea of this Gantt chart plan was to give the student an indication of the progress of the final year project. It was last updated as a final adjustment to the project report.



ID	Task Name	Duration	Start	Finish	Predecessors	% Complete
1	Write Problem Statement	1 day	Wed 11/03/16	Wed 11/03/16		99%
2	Draw Up Skripsie Project Plan	1 day	Wed 11/03/16	Wed 11/03/16		100%
3	Submit Deliverable One to Lecturer	0 days	Wed 11/03/16	Wed 11/03/16	1,2	100%
4	Receive Feedback on Deliverable One from Lectu	0 days	Fri 11/03/18	Fri 11/03/18	3	100%
5	Read Through Eden User Guide	0 days	Sun 11/03/20	Sun 11/03/20		50%
6	Determine Project Scope	5 days	Mon 11/03/21	Fri 11/03/25	1	70%
7	Finalise Deliverable One	1 day?	Mon 11/03/28	Mon 11/03/28	6	100%
8	**Submit Deliverable One**	**0 days**	**Wed 11/03/30**	**Wed 11/03/30**	**4,5,6,7**	**100%**
9	**Literature Study**	**102 days?**	**Thu 11/03/31**	**Fri 11/08/19**	**5**	**100%**
10	Read Thesis on ATM Cash Management for a South African Retail Bank	1 day?	Thu 11/03/31	Thu 11/03/31	8	100%
11	Read Related Work done by Adendorff and Wagner	15 days?	Mon 11/04/04	Fri 11/04/22	8	100%
12	Read Up on VRP's	65 days	Mon 11/04/04	Fri 11/07/01	8	100%
13	Read Up on Integer Programming	40 days	Mon 11/04/04	Fri 11/05/27	8	100%
14	Read Up on Inventory Management Models	45 days	Mon 11/04/04	Fri 11/06/03	8	100%
15	Read Up on Latex	95 days	Mon 11/04/04	Fri 11/08/12	8	100%
16	Read Up on Simulation	49 days	Tue 11/04/26	Fri 11/07/01	8	100%
17	Related Reading	100 days	Mon 11/04/04	Fri 11/08/19	8	100%
18	Write Literature Study	28 days	Mon 11/04/04	Wed 11/05/11		100%
19	Develop Framework of Target Report	25 days	Thu 11/04/07	Wed 11/05/11	5	100%
20	Develop Solution Strategy	25 days	Thu 11/04/07	Wed 11/05/11	5	100%
21	Update Project Plan	0 days	Wed 11/05/11	Wed 11/05/11	19,20,18	100%
22	Submit Final Queries about Deliverable Two to Lecturer	0 days	Wed 11/05/11	Wed 11/05/11	19,20,21,18	100%
23	Receive Feedback on Queries about Deliverable Two from Lecturer	0 days	Fri 11/05/13	Fri 11/05/13	22	100%
24	Incorporate Lecturer's Feedback	2 days	Mon 11/05/16	Tue 11/05/17	23	100%
25	May/June Exams	15 days?	Mon 11/05/16	Fri 11/06/03		100%
26	**Submit Deliverable Two to Lecturer**	**0 days**	**Wed 11/05/18**	**Wed 11/05/18**	**23**	**100%**
27	Receive Feedback on Deliverable Two	0 days	Fri 11/05/20	Fri 11/05/20	26	100%
28	June Holidays	10 days	Mon 11/06/06	Fri 11/06/17	25	100%
29	Develop Operational Research Models	40 days	Mon 11/06/20	Fri 11/08/12	20,10,11,28	100%
30	Run Simulations to Test Models Developed	20 days	Mon 11/07/18	Fri 11/08/12	20,16	100%
31	Write Report	20 days	Mon 11/08/15	Fri 11/09/09	1,6,19,29,30	100%
32	Draft Report Submitted at the Writing Institute for Proofreading	3 days?	Mon 11/09/12	Wed 11/09/14	31	0%
33	**Formal Submission of 1st Draft**	**0 days**	**Mon 11/10/03**	**Mon 11/10/03**	**32**	**100%**
34	Receive Feedback on 1st Draft	0 days	Fri 11/10/21	Fri 11/10/21	33	100%
35	Incorporate Final Comments on 1st Draft	4 days	Fri 11/10/21	Wed 11/10/26	34	100%
36	**Submit Final Report**	**0 days**	**Wed 11/10/26**	**Wed 11/10/26**	**35**	**100%**
37	Prepare for Oral Assessment	7 days	Thu 11/10/27	Fri 11/11/04	36	0%
38	October/November Exams	11 days	Fri 11/10/28	Fri 11/11/11		0%

Project: Project Plan for Final Report
Date: Sun 11/10/16

Figure G.1: Project plan for the final year project.

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