

Particle Filtering for Nonlinear/Non-Gaussian Systems with Energy Harvesting Sensors Subject to Randomly Occurring Sensor Saturations

Weihao Song, Zidong Wang, Jianan Wang, Fuad E. Alsaadi and Jiayuan Shan

Abstract—In this paper, the particle filtering problem is investigated for a class of nonlinear/non-Gaussian systems with energy harvesting sensors subject to randomly occurring sensor saturations (ROSSs). The random occurrences of the sensor saturations are characterized by a series of Bernoulli distributed stochastic variables with known probability distributions. The energy harvesting sensor transmits its measurement output to the remote filter only when the current energy level is sufficient, where the transmission probability of the measurement is recursively calculated by using the probability distribution of the sensor energy level. The effects of the ROSSs and the possible measurement losses induced by insufficient energies are fully considered in the design of filtering scheme, and an explicit expression of the likelihood function is derived. Finally, the numerical simulation examples (including a benchmark example for nonlinear filtering and the applications in moving target tracking problem) are provided to demonstrate the feasibility and effectiveness of the proposed particle filtering algorithm.

Index Terms—Particle filtering, nonlinear/non-Gaussian systems, randomly occurring sensor saturations, energy harvesting sensor, multi-sensor systems

I. INTRODUCTION

As one of the fundamental issues in signal processing and control communities, the filtering problem has attracted considerable research interest during the past few decades [6], [17], [25], [32], [51]. As early as in [15], a general filtering framework has been constructed from a Bayesian point of view, where the probability density function (PDF) of the state of interest conditioned on the measurements has been computed in a recursive manner. For linear systems with Gaussian noises, it is well known that the renowned Kalman filter [33] offers the analytically optimal solution. For general nonlinear and/or non-Gaussian systems, the corresponding optimal solution exists *only* in a conceptual sense because

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the recursive propagation demands the calculation of multi-dimensional integrals that is numerically difficult. To solve this problem, various filtering algorithms have been developed by utilizing different numerical approximation methods with examples including, but are not limited to, extended Kalman filtering with first-order linearization [1], [31], unscented Kalman filtering with unscented transform [20], [21], cubature Kalman filtering with cubature rule [2], and particle filtering with Monte Carlo approximation [3], [12], [28], [61].

Owing to the applicability to general systems with any type of noises, the particle filtering algorithm has captured much attention from both academy and industry, and a rich body of results has been reported from various perspectives, such as reduced communication burden [27], model uncertainty [55] and packet dropouts [62]. Among others, the group importance sampling algorithm has been proposed in [40], which aims to compress the statistical information included in a set of weighted samples into a single summary weighted particle. In [42], a particle selection method has been proposed for cost-reference particle filtering algorithm, which is capable of overcoming the difficulty (incurred by resampling) of parallel computation in conventional particle filters. In addition, the particle filtering problem has been investigated in [45] considering the presence of out-of-sequence measurements.

In many real-world applications of networked systems, it is ubiquitous that the physical sensor cannot produce measurement signals with unlimited amplitude due mainly to the hardware restrictions, and this is referred to as the sensor saturation phenomenon. Such a phenomenon, if not properly considered in the stage of filter design, is likely to result in performance degradation or even algorithm divergence. Accordingly, a large amount of research attention has been devoted to the filtering problem with sensor saturations [11], [16], [37], [38], [47]. For example, the distributed set-membership filters have been designed for systems with sensor saturations, unknown but bounded noises and sector-bounded nonlinearities under the event-triggered communication scheme in [37]. The joint state and fault estimation problem has been investigated with variance constraints in [16] for the time-varying nonlinear systems subject to sensor saturations, parameter uncertainties and randomly occurring faults.

Recently, the phenomenon of randomly occurring sensor saturations (ROSSs) has started to draw some research attention, see e.g. [8], [58]. In a networked environment, due to limited bandwidth and/or unpredictable disturbances, the network itself might undergo data collisions leading to net-

work congestions and subsequent network-induced behaviors (e.g. communication delays and packet dropouts [24], [52], [53]) which, in turn, give rise to the so-called ROSSs [63]. Note that the ROSSs can also result from abrupt environment changes and random/intermittent sensor failures [58]. Up to now, the filtering problem for systems suffering from ROSSs has aroused an initial yet quickly increasing research interest, see [26], [34], [56] and the references therein. Nevertheless, it is worth mentioning that the corresponding filter design issue for general non-Gaussian systems subject to ROSSs has not been investigated adequately.

It is often the case in practice that the energy supply is rather scarce and this inevitably imposes certain constraints on the efficient/smooth usage of the wireless sensor networks since more sensors are battery-operated. To prolong the sensors' lifetime, two kinds of approaches have been put forward, one is to reduce the energy consumption and the other is to increase the energy supply. For the former approach, considering that the wireless transmission of data streams constitutes the main cause for energy consuming, a great deal of work (e.g. [27], [35], [57], [59]) has been concerned with the event-triggered schemes where the data transmission is activated only when certain predefined event occurs. For the latter approach, there has been a notably growing research interest towards the so-called energy harvesting technology [14], [44], [54]. For sensors equipped with energy harvesters, the energy can be harnessed (and then replenished) from the surrounding environment such as solar and mechanical vibrations [23].

Given the practical importance of the energy harvesting sensors, the corresponding filter design problem has recently been gaining some initial attention. For example, in [22], the optimal energy allocation problem has been studied for multi-sensor estimation with energy harvesting and energy sharing technologies. In [18], both the state and the sensor energy level have been estimated for a linear Gaussian system by incorporating the available set-valued and point-valued event-triggered measurements. In [50], a recursion expression for the probability distribution of the energy level has been derived and an effective recursive filtering algorithm has been proposed to address the phenomenon of sensor-energy-induced missing measurements for a class of nonlinear time-delayed systems. It should be noted that, so far, most reported results have been focused on the *linear Gaussian* systems [18], [23] or the nonlinear systems with *special nonlinearities* [50]. The filter design issue for general nonlinear/non-Gaussian systems with energy harvesting sensors subject to ROSSs has not received adequate attention despite its practical significance.

Summarizing the above discussions, in this paper, we aim to deal with the particle filtering problem for a class of nonlinear/non-Gaussian systems involving both ROSSs and probabilistic missing measurements caused by the energy harvesting sensors. Through recursively calculating the probability distribution of the sensor energy level, the missing probability of the measurement is derived. Accordingly, a particle filtering algorithm is developed to restrain the effect from both the ROSSs and the limited sensor energy on the filtering performance by virtue of a modified likelihood function. The main contributions of this paper are highlighted

as follows: 1) a unified framework for sequential Bayesian estimation is proposed for a general class of nonlinear/non-Gaussian systems under the energy-dependent transmission protocol; 2) a modified particle filtering algorithm is developed to address the multi-sensor fusion problem subject to partial measurement-information-loss induced by different energy levels of each individual sensor; and 3) the recursion for importance weight is derived to take into account the joint effects from the ROSSs and the sensor energy constraints.

The remainder of this paper is organized as follows. In Section II, the concerned filtering problem is formulated. In Section III, the particle filtering algorithm is designed by considering the phenomenon of ROSSs and the energy harvesting sensors. In Section IV, two numerical examples are presented to show the usefulness and effectiveness of the proposed particle filtering scheme. Concluding remarks are finally provided in Section V.

Notation. Throughout this paper, the notation used is fairly standard. \mathbb{R}^n represents the n -dimensional Euclidean vector space. $\|\cdot\|$ stands for the Euclidean norm of a vector. The superscript T denotes the operation of transpose. $\text{diag}\{a_1, a_2, \dots, a_n\}$ denotes a diagonal matrix with a_1, a_2, \dots, a_n being the diagonal elements. $p_x(\cdot)$ stands for the PDF of a stochastic variable x , i.e., $x \sim p_x(\cdot)$, and $p_{x|y}(\cdot)$ denotes the conditional PDF of a stochastic variable x given y . $\Pr\{A\}$ denotes the occurrence probability of the discrete event A . $N(a, b)$ represents the Gaussian distribution with mean a and covariance b . $G(a, b)$ denotes the Gamma distribution with shape parameter a and scale parameter b . $\text{Exp}(a)$ stands for the exponential distribution with mean a . $x_{k:l}$ represents the trajectory of x from time instant k to time instant l . Other notations will be provided as the need arises.

II. PROBLEM FORMULATION

Consider the following discrete-time nonlinear system

$$x_{k+1} = f_k(x_k) + \omega_k \quad (1)$$

and N energy harvesting sensors with randomly occurring saturations

$$y_k^i = \gamma_k^i \text{Sat}(h_k^i(x_k)) + (1 - \gamma_k^i) h_k^i(x_k) + \nu_k^i \quad (2)$$

where, for $i = 1, 2, \dots, N$, $x_k \in \mathbb{R}^n$ and $y_k^i \in \mathbb{R}$ are the state of the target plant and the measurement output of the i th sensor at time instant k , respectively. $f_k(\cdot) : \mathbb{R}^n \mapsto \mathbb{R}^n$ and $h_k^i(\cdot) : \mathbb{R}^n \mapsto \mathbb{R}$ denote the state transition function and the measurement function of the i th sensor, respectively. $\omega_k \in \mathbb{R}^n$ represents the process noise satisfying $p_{\omega_k}(\cdot)$ and $\nu_k^i \in \mathbb{R}$ is the measurement noise on sensor i satisfying $p_{\nu_k^i}(\cdot)$. The saturation function $\text{Sat}(\cdot) : \mathbb{R} \mapsto \mathbb{R}$ is modelled by

$$\text{Sat}(\kappa) = \text{sign}(\kappa) \min\{\kappa_{\max}, |\kappa|\} \quad (3)$$

where $\text{sign}(\cdot)$ is the signum function and κ_{\max} represents the saturation level. Moreover, a Bernoulli distributed stochastic variable γ_k^i is defined to characterize the phenomenon of the ROSSs on the i th sensor [58], which takes a value of 0 or 1 with

$$\begin{cases} \Pr\{\gamma_k^i = 1\} = \bar{\gamma}^i \\ \Pr\{\gamma_k^i = 0\} = 1 - \bar{\gamma}^i \end{cases} \quad (4)$$

where $\bar{\gamma}^i \in [0, 1]$ is a known constant.

Next, the following assumptions are made to further clarify the considered system.

Assumption 1: The initial state x_0 satisfies the prior density $p_{x_0}(\cdot)$, i.e., $x_0 \sim p_{x_0}(\cdot)$.

Assumption 2: The process noise ω_k , the N measurement noises $\{\nu_k^i\}_{i=1}^N$ and the stochastic variables $\{\gamma_k^i\}_{i=1}^N$ are all mutually independent and also independent of the initial state x_0 .

Assumption 3: The nonlinear functions $f_k(\cdot)$ and $h_k^i(\cdot)$ as well as the PDFs $p_{\omega_k}(\cdot)$ and $p_{\nu_k^i}(\cdot)$ are all known.

Let the i th energy harvesting sensor be powered by a chargeable battery whose maximum energy storage capacity is denoted by \bar{E}^i and the energy level of the i th sensor at time instant k is defined by $E_k^i \in \{0, 1, \dots, \bar{E}^i\}$. Denote H_k^i as the number of the units of energy harvested by the i th sensor at time instant k , which is modelled as a first-order Markov model. Assume that H_k^i takes a value in the finite non-negative integer set $\mathbb{H} = \{0, 1, \dots, \bar{H}\}$ with transition probability matrix $\Pi = [\pi_{uv}]_{\bar{H} \times \bar{H}}$, where \bar{H} denotes the maximum energy that can be harvested by the sensors. $\pi_{uv} = \Pr\{H_{k+1}^i = v | H_k^i = u\}$ denotes the transition probability for all $u, v \in \mathbb{H}$ and $\sum_{v=0}^{\bar{H}} \pi_{uv} = 1$.

Remark 1: As stated in [14], [23], the energy harvesting process may be correlated among different time instants, e.g. the amount of the harvested solar energy is contingent on the weather and the period of a day. The rationality of the first-order Markov energy harvesting model is justified by the empirical measurements when the solar energy is the energy harvesting source [13].

Assumption 4: The energy harvesting sensor can transmit its measurement to the remote filter only when it stores nonzero units of energy and each measurement transmission consumes one unit of energy.

Define an indicator variable $1_{\{E_k^i > 0\}}$ for the energy consumption as

$$1_{\{E_k^i > 0\}} = \begin{cases} 1, & \text{if } E_k^i > 0 \\ 0, & \text{otherwise} \end{cases}, \quad (5)$$

then the energy level of the i th sensor at each time instant is recursively calculated as [50]

$$E_{k+1}^i = \min \left\{ E_k^i - 1_{\{E_k^i > 0\}} + H_{k+1}^i, \bar{E}^i \right\} \quad (6)$$

with the initial energy level $0 < E_0^i \leq \bar{E}^i$. Note that the energy level is unknown at each time instant $k \geq 1$ due to the unpredictable characteristic of the amount of harvested energy, and the probability distribution of the energy level will be discussed in the subsequent section.

As illustrated in Fig. 1, the i th energy harvesting sensor transmits the new measurement to the remote filter only when the sensor's current energy is sufficient, i.e., $E_k^i > 0$. Due to the above-mentioned property of the energy harvesting sensors, at the remote filter side, the available measurement contributed by the i th sensor at time instant k is described as

$$z_k^i = 1_{\{E_k^i > 0\}} y_k^i + n_k^i, \quad (7)$$

where $n_k^i \in \mathbb{R}$ denotes the channel noise in the reception of the i th sensor's information, which satisfies $p_{n_k^i}(\cdot)$.

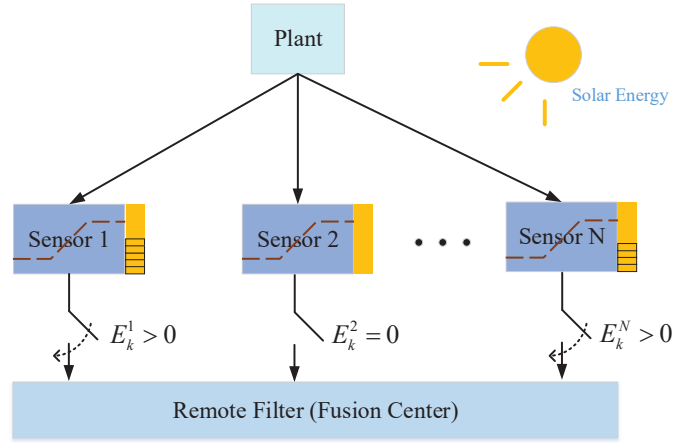


Fig. 1: Block diagram of the networked system with energy harvesting sensors and ROSSs.

Remark 2: Compared with the existing results concerning particle filtering with packet dropout [62] and H_∞ filtering with missing measurements [49], two distinguishing features of the considered model are identified as follows. 1) A novel indicator variable $1_{\{E_k^i > 0\}}$, whose probability distribution is dependent on the evolution dynamics of the energy level (6), is introduced to characterize whether or not the measurement y_k^i is successfully transmitted to the remote filter. Note that $1_{\{E_k^i > 0\}}$ is a time-correlated variable (rather than a Bernoulli distributed stochastic variable with known probability distribution). 2) Both the measurement noise and the channel noise are taken into consideration to better cater for the real-world scenario. In addition, there is no specific requirement on the type of the noise, and therefore the application potential is further increased. In summary, it is these distinguishing features that render some additional difficulties in the subsequent design of filtering scheme.

At time instant k , the available measurements at the remote filter are denoted as $z_k^{1:N} = [z_k^1 \ z_k^2 \ \dots \ z_k^N]^T$. In addition, let us denote $Z_{1:k}^{1:N} = [(z_1^{1:N})^T \ (z_2^{1:N})^T \ \dots \ (z_k^{1:N})^T]^T$ as the vector of all available measurements up to time instant k .

The purpose of this paper is to develop a particle filtering algorithm for the general nonlinear/non-Gaussian systems subject to the sensor energy constraints and the ROSSs such that the estimate of state x_k can be obtained at the remote filter in the sense of minimum mean-square error (MMSE) based on the available actual measurement information $Z_{1:k}^{1:N}$.

III. ALGORITHM DESIGN AND DISCUSSION

As is well known, the MMSE estimate of the unknown state vector x_k is defined by

$$\hat{x}_k = \int x_k p(x_k | Z_{1:k}^{1:N}) dx_k. \quad (8)$$

Unfortunately, it is difficult to obtain an analytical solution to the problem addressed in this paper due to the fact that the marginal posterior PDF $p(x_k | Z_{1:k}^{1:N})$ is non-Gaussian. As an alternative method, the particle filtering algorithm [3] can be

utilized to obtain an approximate expression of $p(x_{0:k}|Z_{1:k}^{1:N})$ with a set of weighted particle trajectories as

$$p(x_{0:k}|Z_{1:k}^{1:N}) = \sum_{m=1}^M W_k^m \delta(x_{0:k} - x_{0:k}^m) \quad (9)$$

where M denotes the number of the particles and $\delta(\cdot)$ represents the multi-dimensional Dirac delta function. In addition, the particles $\{x_{0:k}^m\}_{m=1}^M$ are drawn from a proposal density function $q(x_{0:k}|Z_{1:k}^{1:N})$ and the corresponding weights $\{W_k^m\}_{m=1}^M$ are calculated via the importance sampling method. To be more specific, the importance weight W_k^m associated with $x_{0:k}^m$ is defined by

$$W_k^m = \frac{p(x_{0:k}^m, Z_{1:k}^{1:N})}{q(x_{0:k}^m|Z_{1:k}^{1:N})}. \quad (10)$$

If the proposal density function is selected such that we can factorize it as

$$q(x_{0:k}^m|Z_{1:k}^{1:N}) = q(x_{0:k}^m|x_{0:k-1}^m, Z_{1:k}^{1:N})q(x_{0:k-1}^m|Z_{1:k-1}^{1:N}), \quad (11)$$

then we have

$$\begin{aligned} W_k^m &= \frac{p(x_{0:k}^m, Z_{1:k}^{1:N})}{q(x_{0:k}^m|Z_{1:k}^{1:N})} \\ &= \frac{p(z_k^{1:N}|x_k^m)p(x_k^m|x_{k-1}^m)p(x_{0:k-1}^m, Z_{1:k-1}^{1:N})}{q(x_k^m|x_{0:k-1}^m, Z_{1:k}^{1:N})q(x_{0:k-1}^m|Z_{1:k-1}^{1:N})}. \end{aligned} \quad (12)$$

According to Assumption 2, it is straightforward to see that

$$p(z_k^{1:N}|x_k^m) = \prod_{i=1}^N p(z_k^i|x_k^m). \quad (13)$$

Substituting (13) into (12) yields

$$W_k^m = W_{k-1}^m \frac{(\prod_{i=1}^N p(z_k^i|x_k^m))p(x_k^m|x_{k-1}^m)}{q(x_k^m|x_{0:k-1}^m, Z_{1:k}^{1:N})}. \quad (14)$$

It is evident from (2) and (7) that the actual measurement z_k^i (contributed by the i th energy harvesting sensor at the remote filter) is dependent on 1) the energy level and 2) whether the sensor saturation occurs or not. Therefore, the update equation of the importance weights, which is related to the indicator variable $1_{\{E_k^i>0\}}$ and the Bernoulli distributed stochastic variable γ_k^i , is distinct from that in the standard sequential importance sampling method. In the sequel, we aim to derive an explicit expression of the likelihood function with the purpose of compensating for the effects resulting from the ROSSs and the probabilistic missing measurements.

Proposition 1: With the measurement model described by (2) and (7), the likelihood function of the system state associated with the i th energy harvesting sensor at time instant k is given by

$$\begin{aligned} p(z_k^i|x_k^m) &= \Pr\{1_{\{E_k^i>0\}} = 1\} [(1 - \bar{\gamma}^i)p_{\nu_k^i+n_k^i}(z_k^i - h_k^i(x_k^m)) \\ &\quad + \bar{\gamma}^i p_{\nu_k^i+n_k^i}(z_k^i - \text{Sat}(h_k^i(x_k^m)))] \\ &\quad + \Pr\{1_{\{E_k^i>0\}} = 0\} p_{n_k^i}(z_k^i) \end{aligned} \quad (15)$$

where $p_{\nu_k^i+n_k^i}(\cdot)$ is the PDF of the sum of the measurement noise ν_k^i and the channel noise n_k^i .

Proof: To prove the result, we will discuss the form of the likelihood function in the following three cases.

Case 1: If the energy of sensor node i is sufficient and the phenomenon of sensor saturation does not occur at time instant k , i.e., $1_{\{E_k^i>0\}} = 1$ and $\gamma_k^i = 0$, it is immediate to see from (2) and (7) that

$$z_k^i = h_k^i(x_k) + \nu_k^i + n_k^i. \quad (16)$$

Then, the likelihood function is evaluated as

$$p(z_k^i|x_k^m, 1_{\{E_k^i>0\}} = 1, \gamma_k^i = 0) = p_{\nu_k^i+n_k^i}(z_k^i - h_k^i(x_k^m)). \quad (17)$$

Case 2: If the energy of sensor node i is sufficient and the phenomenon of sensor saturation occurs at time instant k , i.e., $1_{\{E_k^i>0\}} = 1$ and $\gamma_k^i = 1$, we obtain from (2) and (7) that

$$z_k^i = \text{Sat}(h_k^i(x_k)) + \nu_k^i + n_k^i. \quad (18)$$

Then, we can denote the likelihood function as

$$p(z_k^i|x_k^m, 1_{\{E_k^i>0\}} = 1, \gamma_k^i = 1) = p_{\nu_k^i+n_k^i}(z_k^i - \text{Sat}(h_k^i(x_k^m))). \quad (19)$$

Summarizing the above two cases, it is obtained from (17) and (19) that

$$\begin{aligned} p(z_k^i|x_k^m, 1_{\{E_k^i>0\}} = 1) &= \sum_{j=0}^1 p(z_k^i, \gamma_k^i = j|x_k^m, 1_{\{E_k^i>0\}} = 1) \\ &= \sum_{j=0}^1 p(z_k^i|x_k^m, 1_{\{E_k^i>0\}} = 1, \gamma_k^i = j) \Pr\{\gamma_k^i = j\} \\ &= (1 - \bar{\gamma}^i)p(z_k^i|x_k^m, 1_{\{E_k^i>0\}} = 1, \gamma_k^i = 0) \\ &\quad + \bar{\gamma}^i p(z_k^i|x_k^m, 1_{\{E_k^i>0\}} = 1, \gamma_k^i = 1) \\ &= (1 - \bar{\gamma}^i)p_{\nu_k^i+n_k^i}(z_k^i - h_k^i(x_k^m)) \\ &\quad + \bar{\gamma}^i p_{\nu_k^i+n_k^i}(z_k^i - \text{Sat}(h_k^i(x_k^m))). \end{aligned} \quad (20)$$

Case 3: If the energy of sensor node i is insufficient and the current measurement is not transmitted at time instant k , i.e., $1_{\{E_k^i>0\}} = 0$, then only the channel noise is received at the remote filter and we have

$$z_k^i = n_k^i. \quad (21)$$

Similarly, the likelihood function is rewritten as

$$p(z_k^i|x_k^m, 1_{\{E_k^i>0\}} = 0) = p_{n_k^i}(z_k^i). \quad (22)$$

By noting (20) and (22) together with the law of total probability, the likelihood function $p(z_k^i|x_k^m)$ with regard to the i th sensor at time instant k is expressed by

$$\begin{aligned} p(z_k^i|x_k^m) &= \sum_{j=0}^1 p(z_k^i, 1_{\{E_k^i>0\}} = j|x_k^m) \\ &= \sum_{j=0}^1 p(z_k^i|x_k^m, 1_{\{E_k^i>0\}} = j) \Pr\{1_{\{E_k^i>0\}} = j\} \\ &= \Pr\{1_{\{E_k^i>0\}} = 1\} p(z_k^i|x_k^m, 1_{\{E_k^i>0\}} = 1) \\ &\quad + \Pr\{1_{\{E_k^i>0\}} = 0\} p(z_k^i|x_k^m, 1_{\{E_k^i>0\}} = 0) \end{aligned}$$

$$\begin{aligned}
 &= \Pr\{1_{\{E_k^i > 0\}} = 1\} [(1 - \bar{\gamma}^i) p_{\nu_k^i + n_k^i}(z_k^i - h_k^i(x_k^m)) \\
 &\quad + \bar{\gamma}^i p_{\nu_k^i + n_k^i}(z_k^i - \text{Sat}(h_k^i(x_k^m)))] \\
 &\quad + \Pr\{1_{\{E_k^i > 0\}} = 0\} p_{n_k^i}(z_k^i), \tag{23}
 \end{aligned}$$

which completes the proof. \blacksquare

According to Proposition 1, the explicit expression of the likelihood function corresponding to the i th energy harvesting sensor is now written as (15), which cannot be directly applied in the weight update due to the fact that the transmission probability $\Pr\{1_{\{E_k^i > 0\}} = 1\}$ is by far unknown. Before proceeding further, the following lemma is introduced to deal with such an issue.

Lemma 1: Given the dynamics of the energy level $\{E_k^i\}$ described by (6) with initial energy level E_0^i and the first-order Markov energy harvesting process $\{H_k^i\}$ with the initial distribution of H_0^i , the recursion of the probability distribution τ_k^i of the energy level for the i th energy harvesting sensor can be calculated by

$$\begin{cases} p_{k,v}^i = \sum_{u=0}^{\bar{H}} \pi_{uv} p_{k-1,u}^i \\ \tau_k^i = \chi + S_k^i \tau_{k-1}^i \\ \tau_0^i = \underbrace{[0 \ \cdots \ 0]}_{E_0^i} \ 1 \ \underbrace{[0 \ \cdots \ 0]}_{\bar{E}^i - E_0^i} \end{cases} \tag{24}$$

where

$$\begin{aligned}
 \tau_k^i &= [\Pr\{E_k^i = 0\} \ \cdots \ \Pr\{E_k^i = \bar{E}^i\}]^T, \\
 \chi &= \underbrace{[0 \ \cdots \ 0]}_{\bar{E}^i} \ 1]^T, \\
 S_k^i &= \begin{bmatrix} p_{k,0}^i & p_{k,0}^i & 0 & \cdots & 0 \\ p_{k,1}^i & p_{k,1}^i & p_{k,0}^i & \cdots & 0 \\ p_{k,2}^i & p_{k,2}^i & p_{k,1}^i & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ p_{k,\bar{E}^i-1}^i & p_{k,\bar{E}^i-1}^i & p_{k,\bar{E}^i-2}^i & \cdots & p_{k,0}^i \\ -\sum_{s=0}^{\bar{E}^i-1} p_{k,s}^i & -\sum_{s=0}^{\bar{E}^i-1} p_{k,s}^i & -\sum_{s=0}^{\bar{E}^i-2} p_{k,s}^i & \cdots & -p_{k,0}^i \end{bmatrix}
 \end{aligned}$$

and

$$p_{k,v}^i = \Pr\{H_k^i = v\}.$$

Proof: For $v \in \mathbb{H}$, according to the law of total probability, we have

$$\begin{aligned}
 p_{k,v}^i &= \Pr\{H_k^i = v\} \\
 &= \sum_{u=0}^{\bar{H}} \Pr\{H_k^i = v, H_{k-1}^i = u\} \\
 &= \sum_{u=0}^{\bar{H}} \Pr\{H_k^i = v | H_{k-1}^i = u\} \Pr\{H_{k-1}^i = u\} \\
 &= \sum_{u=0}^{\bar{H}} \pi_{uv} p_{k-1,u}^i, \tag{25}
 \end{aligned}$$

by which the probability distribution of the harvested energy quantity can be calculated at each time instant. The remaining part of this proof is similar to that of Lemma 1 in [50] and is omitted here. \blacksquare

From Lemma 1, we immediately obtain the transmission probability of the i th sensor's measurement at time instant k , that is,

$$\lambda_k^i = \Pr\{1_{\{E_k^i > 0\}} = 1\} = [0 \ \underbrace{1 \ \cdots \ 1}_{\bar{E}^i}] \tau_k^i. \tag{26}$$

Remark 3: It should be noted that the structure of Lemma 1 is similar to that in [50], but the distinction lies in that the probability distribution of the harvested energy in Lemma 1 is time-varying and dependent on the previous time instant, which can be calculated recursively with a time-varying matrix S_k^i . In fact, Lemma 1 can be reduced to that in [50] if we simply set the probability transition matrix as an identity matrix. In addition, due to the limited hardware level of the energy harvesting modules, the maximum amount of energy that can be harvested by the i th sensor might be less than the maximum amount that it can store, i.e., $\bar{H} \leq \bar{E}^i - 1$. In this case, some elements in the matrix S_k^i will be always equal to zeros, i.e., $p_{k,v}^i = 0$ for $v > \bar{H}$. Meanwhile, the first $\bar{E}^i - \bar{H} + 1$ elements of the last row in the matrix S_k^i are equal to -1 by noting $\sum_{u=0}^{\bar{H}} p_{k,u}^i = 1$. Consequently, the computational cost of the matrix S_k^i will be reduced.

Now, substituting (15) and (26) into (13), we can evaluate the likelihood function $p(z_k^{1:N} | x_k^m)$ as

$$\begin{aligned}
 &p(z_k^{1:N} | x_k^m) \\
 &= \prod_{i=1}^N \left\{ \lambda_k^i [(1 - \bar{\gamma}^i) p_{\nu_k^i + n_k^i}(z_k^i - h_k^i(x_k^m)) \right. \\
 &\quad \left. + \bar{\gamma}^i p_{\nu_k^i + n_k^i}(z_k^i - \text{Sat}(h_k^i(x_k^m)))] + (1 - \lambda_k^i) p_{n_k^i}(z_k^i) \right\}. \tag{27}
 \end{aligned}$$

Remark 4: Due to the simultaneous presence of the measurement noise and the channel noise, when the energy harvesting sensor i has sufficient energy to send the current measurement to the remote filter, the calculation of the likelihood function depends on the PDF of random variable $(\nu_k^i + n_k^i)$. Noting that ν_k^i and n_k^i are two independent random variables, the PDF of random variable $(\nu_k^i + n_k^i)$ is the convolution of $p_{\nu_k^i}(\cdot)$ and $p_{n_k^i}(\cdot)$. To be more specific, if the sensor saturation phenomenon does not occur at time instant k , then the likelihood function for the m th particle x_k^m is calculated by

$$p_{\nu_k^i + n_k^i}(z_k^i - h_k^i(x_k^m)) = \int p_{\nu_k^i}(\bar{\nu}) p_{n_k^i}(z_k^i - h_k^i(x_k^m) - \bar{\nu}) d\bar{\nu}. \tag{28}$$

In fact, some distributions (e.g. normal distribution and exponential distribution) possess simple convolutions. When the accurate convolution is difficult to obtain in some cases, an alternative approximation of (28) is given by a simulation approach. That is,

$$p_{\nu_k^i + n_k^i}(z_k^i - h_k^i(x_k^m)) \approx \frac{1}{C} \sum_{c=1}^C p_{n_k^i}(z_k^i - h_k^i(x_k^m) - \bar{\nu}^c)$$

where $p_{\nu_k^i}(\bar{\nu})$ is approximated by its particle representation $\frac{1}{C} \sum_{c=1}^C \delta(\bar{\nu} - \bar{\nu}^c)$ and C is the number of the particles.

The implementation of the modified particle filtering algorithm for systems with energy harvesting sensors subject to ROSSs is provided in Algorithm 1.

Algorithm 1 Modified particle filtering with energy harvesting sensors subject to ROSSs

Step 1. Particle initialization

Sample M particles from the prior density, i.e., $x_0^m \sim p_{x_0}(x_0)$, $m = 1, 2, \dots, M$ and the corresponding importance weights W_0^m are all set to be $\frac{1}{M}$. In addition, set the maximum recursive time instant K .

Step 2. Importance sampling

For each $m = 1, \dots, M$, sample particle x_k^m from the transition PDF $p(x_k^m | x_{k-1}^m)$.

Step 3. Weight update

Calculate the unnormalized weights $\{\tilde{W}_k^m\}_{m=1}^M$ based on (27) as

$$\begin{aligned} & \tilde{W}_k^m \\ &= W_{k-1}^m \prod_{i=1}^N \left\{ \lambda_k^i \left[(1 - \bar{\gamma}^i) p_{\nu_k^i + n_k^i}(z_k^i - h_k^i(x_k^m)) \right. \right. \\ & \quad \left. \left. + \bar{\gamma}^i p_{\nu_k^i + n_k^i}(z_k^i - \text{Sat}(h_k^i(x_k^m))) \right] \right. \\ & \quad \left. + (1 - \lambda_k^i) p_{n_k^i}(z_k^i) \right\}. \end{aligned}$$

Step 4. Normalization

Normalize the importance weights as

$$W_k^m = \frac{\tilde{W}_k^m}{\sum_{m=1}^M \tilde{W}_k^m}.$$

Step 5. State estimate update

Calculate the state estimate \hat{x}_k and estimation error covariance P_k as

$$\begin{aligned} \hat{x}_k &= \sum_{m=1}^M W_k^m x_k^m, \\ P_k &= \sum_{m=1}^M W_k^m (x_k^m - \hat{x}_k)(x_k^m - \hat{x}_k)^T. \end{aligned}$$

Step 6. Resampling

Resample the particles based on the normalized importance weights $\{W_k^m\}_{m=1}^M$.

Step 7. If $k < K$, then go to Step 2; otherwise go to Step 8.

Step 8. Stop.

As can be seen from Algorithm 1, the proposed recursive algorithm is mainly constructed by: 1) sampling new particles from the proposal density function $q(x_k | x_{0:k-1}, Z_{1:k}^{1:N})$ by using previous particles (*Steps 1* and *2*); and 2) collecting the measurements from all the energy harvesting sensors and updating the importance weights according to the available measurements $z_k^{1:N}$, which are dependent on the energy level of each individual sensor and the ROSSs (*Steps 3* and *4*). For the convenience of implementation, the state transition PDF $p(x_k | x_{k-1})$ determined by (1) is chosen as the proposal density function in Algorithm 1. Nevertheless, after a few iterations, the particle degeneracy phenomenon may occur, which means that all but one particle will have insignificant

weights. Therefore, a resampling procedure [3] is introduced to reduce the effect of the degeneracy phenomenon (*Step 6*). Note that the proposed algorithm can be straightforwardly extended to the adaptive resampling case, i.e., the resampling procedure is executed only when the effective sample size [41] is less than a predefined threshold.

Remark 5: Up to now, the filtering problem has been addressed for a class of nonlinear/non-Gaussian systems subject to the ROSSs and the sensor energy constraints in the framework of sequential Bayesian estimation. By recursively calculating the probability distribution of each sensor's energy level, the probability of measurement transmission is obtained at each time instant. Accordingly, we have derived a modified likelihood function to update the importance weights. It is worth figuring out that our proposed algorithm is also applicable to the case without sensor energy constraints, i.e., $\lambda_k^i = 1$, and the case where the missing measurement is governed by a Bernoulli distributed stochastic variable with known probability distribution [58]. Meanwhile, if we set $\bar{\gamma}^i = 0$, then our proposed algorithm degenerates to the case with only sensor energy constraints.

Remark 6: It should be pointed out that, in recent years, many particle filtering methods have been proposed to address the model uncertainty by adopting multiple candidate state-space models. A common approach available in the literature is to utilize a batch of particle filters, each of which corresponds to one of the candidate models (see e.g. [5], [9], [29], [30], [42], [55]), and another kind of strategy is to jointly make inferences on the states and model index [10]. Actually, due to the random nature of the sensor saturations and measurement information loss, the measurement process of each sensor can also be described by a system with three candidate models (i.e., $1_{\{E_k^i > 0\}} = 1$ and $\gamma_k^i = 0$, $1_{\{E_k^i > 0\}} = 1$ and $\gamma_k^i = 1$, and $1_{\{E_k^i > 0\}} = 0$) by following the line of [29], [30]. However, it is worth noting that, in this paper, the missing probability of the measurement has been derived in a recursive manner and we are able to directly derive the likelihood function based on the statistical characteristics of the ROSSs and measurement information loss, avoiding the use of multiple particle filters and the inference on the model index.

Remark 7: The particle filtering problem for nonlinear/non-Gaussian systems has become a hot topic for a few decades with successful applications in many areas. Comparing to the rich body of existing results in the literature, the main results established in this paper own the following specific merits: 1) the problem addressed is new in the sense that both the energy-dependent transmission protocol and the phenomena of ROSS are taken into account; 2) the particle filtering algorithm proposed is new that tackles the multi-sensor fusion issue subject to partial measurement-information-loss induced by different energy levels of each individual sensor; and 3) the recursion derived for importance weight is new as the joint effects from the ROSSs and the sensor energy constraints are explicitly reflected.

IV. SIMULATION RESULTS

In this section, some illustrative examples are presented to verify the effectiveness of our modified particle filtering

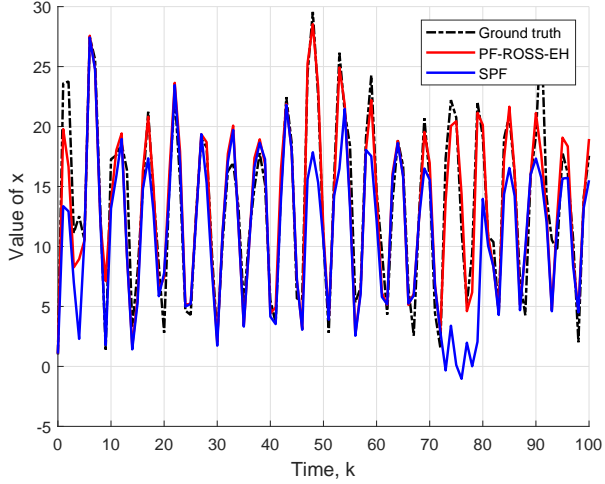


Fig. 2: Estimation results of PF-ROSS-EH and SPF.

algorithm.

A. Numerical example

In this subsection, a simple non-stationary model (modified from [64], [65]) is employed to show the feasibility of the proposed algorithm (abbreviated as PF-ROSS-EH).

Consider a nonlinear and non-Gaussian system described by (1)-(2) with the following nonlinear functions

$$\begin{cases} f_k(x_k) = 0.5x_k + 25\frac{x_k}{1+x_k^2} + 8\cos(1.2k) \\ h_k^i(x_k) = \frac{(x_k - s_i)^2}{20}, i = 1, 2 \end{cases} \quad (29)$$

where $s_1 = 0.1$ and $s_2 = -0.1$. The process noise ω_k satisfies the Gamma distribution, i.e., $\omega_k \sim G(2, 2)$. The measurement noises ν_k^i and channel noises n_k^i obey the exponential distributions, i.e., $\nu_k^i \sim \text{Exp}(2)$ and $n_k^i \sim \text{Exp}(2)$ for $i = 1, 2$. The occurring probability of the sensor saturation is set to be $\bar{\gamma}^i = 0.5$ and the saturation level is $\kappa_{\max} = 10$. The maximum number of the storable energy for both sensors is $\bar{E}^i = 3$ and the initial energy is $E_0^i = 1$. The state space of the energy harvesting process is defined as $\mathbb{H} = \{0, 1, 2, 3\}$, and the transition probability matrix is given by

$$\begin{bmatrix} \pi_{00} & \pi_{01} & \pi_{02} & \pi_{03} \\ \pi_{10} & \pi_{11} & \pi_{12} & \pi_{13} \\ \pi_{20} & \pi_{21} & \pi_{22} & \pi_{23} \\ \pi_{30} & \pi_{31} & \pi_{32} & \pi_{33} \end{bmatrix} = \begin{bmatrix} 0.5 & 0.2 & 0.2 & 0.1 \\ 0.4 & 0.4 & 0.1 & 0.1 \\ 0.3 & 0.3 & 0.2 & 0.2 \\ 0.3 & 0.1 & 0.2 & 0.4 \end{bmatrix}.$$

In addition, the initial state is $x_0 = 1$ and the initial 200 particles are drawn from a Gaussian prior distribution $N(1, 4)$.

For comparison, the standard particle filtering algorithm (abbreviated as SPF) is utilized without considering the ROSSs and the possibly failed measurement transmission. The estimation results are shown in Fig. 2. It can be seen that our PF-ROSS-EH provides a more accurate estimate than the SPF, which demonstrates the feasibility and effectiveness of our proposed filtering algorithm.

B. Application to 2-D and 3-D target tracking problems

In this subsection, the proposed algorithm is firstly used to track a target that moves in a two-dimensional plane. The state of the target at time instant k is denoted as

$$x_k = [s_{x,k}^t, v_{x,k}^t, s_{y,k}^t, v_{y,k}^t]^T,$$

where $(s_{x,k}^t, s_{y,k}^t)$ and $(v_{x,k}^t, v_{y,k}^t)$ are the position and velocity of the target's centroid, respectively.

Following [7], the dynamics of the target is represented by the white noise acceleration model described by

$$x_{k+1} = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix} x_k + \omega_k \quad (30)$$

where T denotes the sampling period and ω_k denotes the zero-mean Gaussian white noise sequences with covariance matrix Q_k determined by

$$Q_k = \Delta \begin{bmatrix} \frac{T^3}{3} & \frac{T^2}{2} & 0 & 0 \\ \frac{T^2}{2} & T & 0 & 0 \\ 0 & 0 & \frac{T^3}{3} & \frac{T^2}{2} \\ 0 & 0 & \frac{T^2}{2} & T \end{bmatrix} \quad (31)$$

and Δ denotes the acceleration variance.

Let the moving target emit a radio or acoustic signal. The N passive received-signal-strength (RSS) sensors are deployed in the reconnaissance region to measure the emitted signal energy of the target. To be specific, the measurement model of the i th RSS sensor located at $(s_{x,k}^i, s_{y,k}^i)$ is represented by [7], [46]

$$h_k^i(x_k) = P_0 - 10n_r \log_{10} \left(\frac{\|[s_{x,k}^t, s_{y,k}^t]^T - [s_{x,k}^i, s_{y,k}^i]^T\|}{d_0} \right), \quad (32)$$

$i = 1, 2, \dots, N$, where P_0 is the received signal energy at the reference distance d_0 and n_r is the path loss exponent.

Due to the effect of the ROSSs, the actual measurement output of the i th sensor is given by

$$y_k^i = \gamma_k^i \text{Sat}(h_k^i(x_k)) + (1 - \gamma_k^i) h_k^i(x_k) + \nu_k^i \quad (33)$$

where ν_k^i denotes the zero-mean Gaussian white noise sequence with variance $\sigma_{\nu,i}^2$. Afterwards, if the sensor energy is sufficient, the measurement output is transmitted to the remote filter via a noisy communication channel (see (7)). The communication noise n_k^i is modelled by a zero-mean Gaussian white noise sequence with variance $\sigma_{n,i}^2$.

To evaluate the performance of the proposed algorithm, the root mean-square error (RMSE) on the position and velocity are respectively defined as follows:

$$\text{RMSE}_{\text{Pos},k} = \sqrt{\frac{1}{MC} \sum_{j=1}^{MC} [(s_{x,k}^{t,j} - \hat{s}_{x,k}^{t,j})^2 + (s_{y,k}^{t,j} - \hat{s}_{y,k}^{t,j})^2]},$$

$$\text{RMSE}_{\text{Vel},k} = \sqrt{\frac{1}{MC} \sum_{j=1}^{MC} [(v_{x,k}^{t,j} - \hat{v}_{x,k}^{t,j})^2 + (v_{y,k}^{t,j} - \hat{v}_{y,k}^{t,j})^2]}$$

where MC denotes the total number of the Monte Carlo trials, $[s_{x,k}^{t,j}, s_{y,k}^{t,j}]^T$ and $[v_{x,k}^{t,j}, v_{y,k}^{t,j}]^T$ stand for the realization of $[s_{x,k}^t, s_{y,k}^t]^T$ and $[v_{x,k}^t, v_{y,k}^t]^T$, whose estimates in the j th Monte Carlo trial are denoted by $[\hat{s}_{x,k}^{t,j}, \hat{s}_{y,k}^{t,j}]^T$ and $[\hat{v}_{x,k}^{t,j}, \hat{v}_{y,k}^{t,j}]^T$. For the convenience of readers, a summary of the notation involved in this application is provided in TABLE I.

TABLE I: Summary of the involved notation.

Variables	Descriptions
$s_{x,k}^t/s_{y,k}^t$	the target position in the X/Y coordinate direction
$v_{x,k}^t/v_{y,k}^t$	the target velocity in the X/Y coordinate direction
$s_{x,k}^{t,j}/s_{y,k}^{t,j}$	the realization of $s_{x,k}^t/s_{y,k}^t$ in the j th Monte Carlo trial
$v_{x,k}^{t,j}/v_{y,k}^{t,j}$	the realization of $v_{x,k}^t/v_{y,k}^t$ in the j th Monte Carlo trial
$\hat{s}_{x,k}^{t,j}/\hat{s}_{y,k}^{t,j}$	the estimate of $s_{x,k}^t/s_{y,k}^t$ in the j th Monte Carlo trial
$\hat{v}_{x,k}^{t,j}/\hat{v}_{y,k}^{t,j}$	the estimate of $v_{x,k}^t/v_{y,k}^t$ in the j th Monte Carlo trial
$s_{x,k}^{s,i}/s_{y,k}^{s,i}$	the i th sensor's position in the X/Y coordinate direction
T	the sampling period
Δ	the acceleration variance
N	the number of sensors
κ_{\max}	the saturation level
P_0	the received signal energy at the reference distance d_0
n_r	the path loss exponent
$\sigma_{v,i}^2$	the measurement noise variance
$\sigma_{n,i}^2$	the channel noise variance
$\tilde{\gamma}^i$	the occurrence probability of sensor saturations
E_0^i	the initial energy level of the i th sensor
\bar{E}^i	the maximum energy that the i th sensor can store
\bar{H}	the harvestable maximum energy at each time
Π_1/Π_2	the transition probability matrix
MC	the total number of the Monte Carlo trials
M	the number of the particles

In the simulation, the number of particles is set to be $M = 500$ and 100 times of independent Monte Carlo trials are conducted (i.e., $MC = 100$). In each trial, the true trajectory is generated independently with initial state $x_0 = [25\text{m } 0.3\text{m/s } 10\text{m } 0.4\text{m/s}]^T$. Similar to [7], the position components of the initial particles are drawn from a Gaussian prior distribution with mean $[25, 10]^T$ and covariance matrix $\text{diag}\{25^2, 25^2\}$, while the velocity components are calculated by the resultant velocity and the azimuth which are drawn from a Gaussian prior distribution with mean $[\sqrt{(0.3)^2 + (0.4)^2}, \arctan(0.4/0.3)]^T$ and covariance matrix $\text{diag}\{0.2^2, (\pi/30)^2\}$. In addition, the state space of the energy harvesting process and the corresponding transition probability matrix are the same as that in Section IV-A. For the sake of clarity, the values of other parameters involved in the simulation are displayed in TABLE II.

The simulation results obtained in one Monte Carlo trial are shown in Figs. 3-5. Fig. 3 sketches the true target trajectory and its corresponding estimate, which shows that our proposed PF-ROSS-EH is able to well track the moving target. Fig. 4 depicts the measurement output of Sensor 4 and the phenomenon of ROSSs while Fig. 5 displays the energy level of Sensor 4 and the transmission instants of measurement output.

For the purpose of comparison, five scenarios, including tracking with our PF-ROSS-EH, tracking with SPF, tracking with standard particle filter using the ideal measurements (unaffected by the ROSSs and the sensor energy constraints,

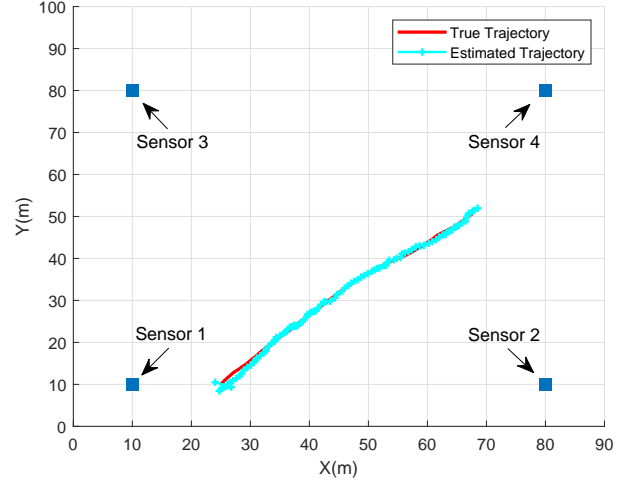


Fig. 3: The true target trajectory and its estimate in one trial.

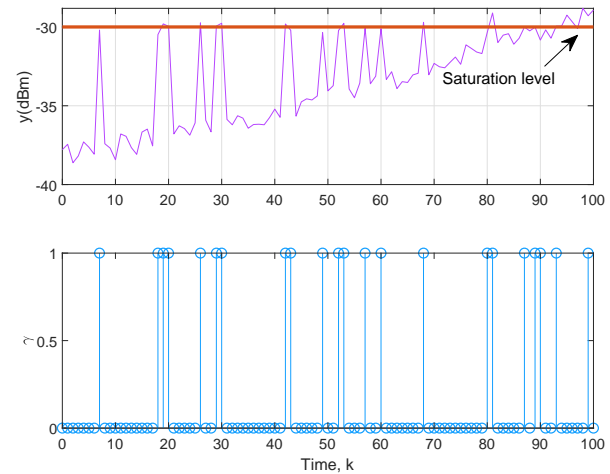


Fig. 4: The measurement output with ROSSs (Sensor 4).

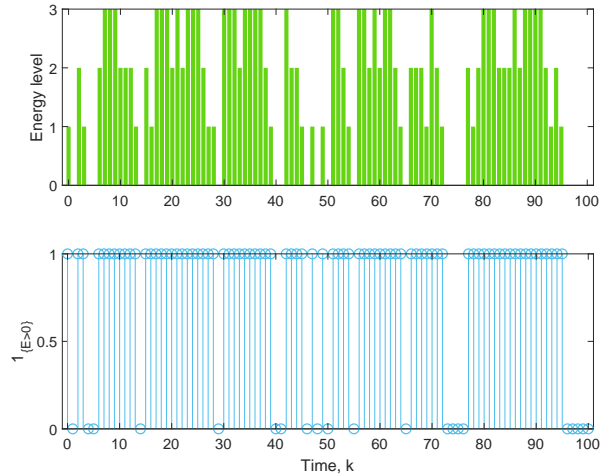


Fig. 5: The energy level and the associated transmission time instants (Sensor 4).

TABLE II: Parameter settings.

Parameters	Values	Parameters	Values
T	1 s	P_0	1 dBm
N	4	κ_{\max}	30 dBm
d_0	1 m	$\sigma_{v,1}^2$	0.1 dBm ²
n_r	2	$\sigma_{v,2}^2$	0.049 dBm ²
$\bar{\gamma}^i$	0.2	$\sigma_{v,3}^2$	0.064 dBm ²
E_0^i	1	$\sigma_{v,4}^2$	0.144 dBm ²
\bar{E}^i	3	$\sigma_{n,i}^2$	0.001 dBm ²
\bar{H}	3	Δ	0.0016 m ² /s ⁴

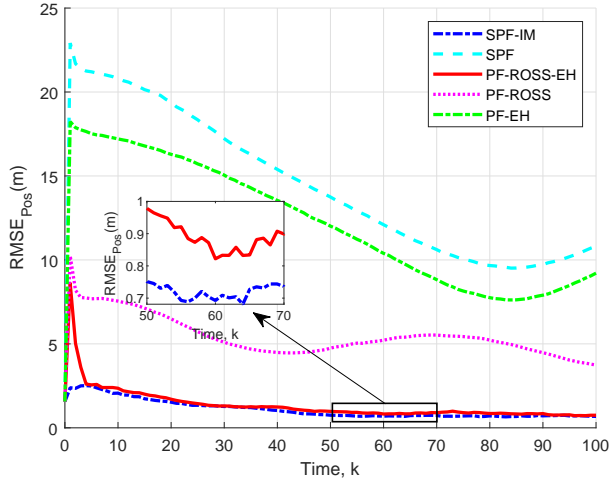


Fig. 6: Position RMSEs of PF-ROSS-EH, PF-ROSS, PF-EH, SPF and SPF-IM.

and abbreviated as SPF-IM), tracking with the particle filter only compensating for the effect of ROSSs (abbreviated as PF-ROSS), and tracking with the particle filter only compensating for the effect of possibly failed measurement transmission (denoted as PF-EH), will be considered to show the tracking performance. Naturally, we expect that using the unaffected measurements will obtain the best tracking performance among the three scenarios, while neglecting the effect of the above-mentioned phenomena will lead to the worst tracking performance.

The evolutions of the RMSEs on the position estimate and velocity estimate are respectively given in Figs. 6-7. It can be observed from Figs. 6-7 that the tracking performance of our proposed PF-ROSS-EH approaches to that of the SPF-IM and is much better than that of the SPF, PF-ROSS, and PF-EH. This is reasonable since we have made much effort to compensate for the effect of the ROSSs and the possible missing measurements.

Next, we will conduct further simulations with different occurrence probabilities of the sensor saturations, different numbers of sensors and different transition probability matrices in the energy harvesting process to analyze their respective effect on the tracking performance. The simulation results with regard to different occurrence probabilities of the sensor saturations (i.e., $\bar{\gamma}^i = 0.2, 0.5, 0.8$) and different numbers

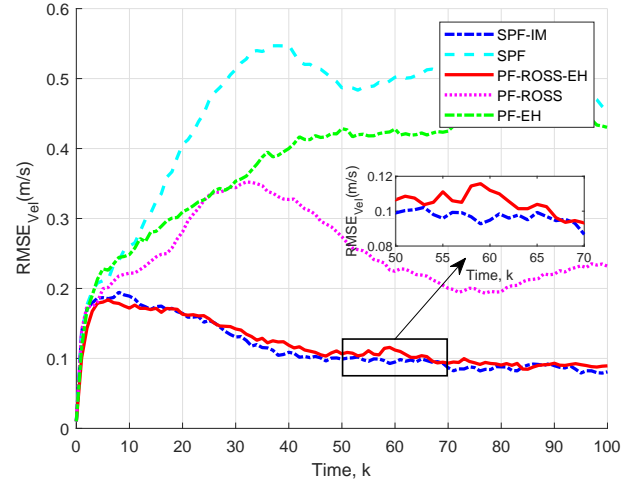


Fig. 7: Velocity RMSEs of PF-ROSS-EH, PF-ROSS, PF-EH, SPF and SPF-IM.

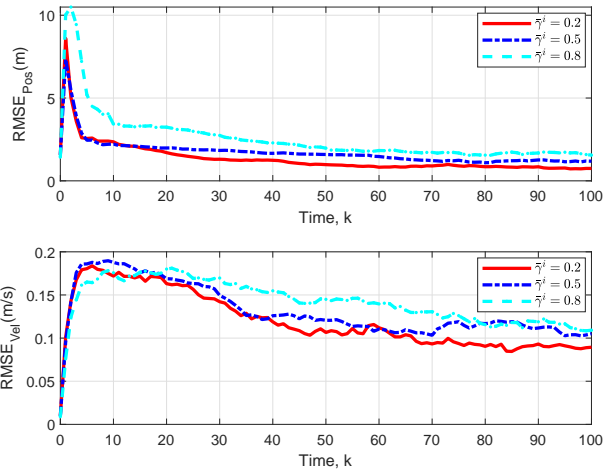


Fig. 8: Position and velocity RMSEs of PF-ROSS-EH with different occurrence probabilities of the sensor saturations.

of sensors (i.e., $N = 2, 4, 6$) are, respectively, displayed in Figs. 8-9, which show that the occurrence probability of the random sensor saturations and the number of sensors both have notable effects on the tracking performance. Specifically, the tracking performance degrades with the increase of the occurrence probability of sensor saturations, and improves as the number of sensors increases. On the other hand, we denote the transition probability matrix used before as Π_1 and choose another transition probability matrix Π_2 as

$$\Pi_2 = \begin{bmatrix} 0.8 & 0.1 & 0.05 & 0.05 \\ 0.7 & 0.1 & 0.1 & 0.1 \\ 0.6 & 0.2 & 0.1 & 0.1 \\ 0.5 & 0.2 & 0.2 & 0.1 \end{bmatrix}.$$

The corresponding simulation results are illustrated in Fig. 10. From Fig. 10, we can see that the tracking performance of the proposed algorithm with transition probability matrix Π_1 is superior to that with Π_2 , which implies that the energy

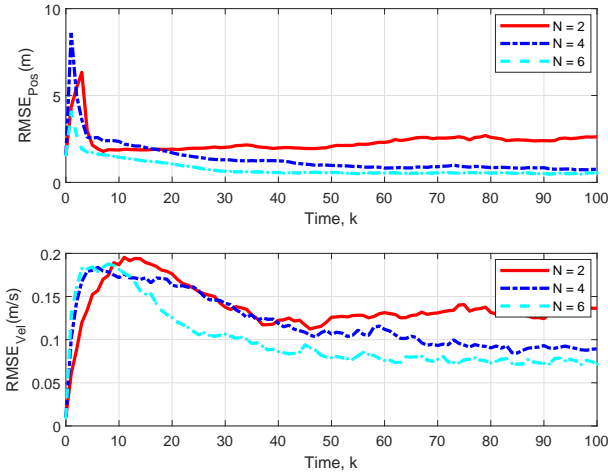


Fig. 9: Position and velocity RMSEs of PF-ROSS-EH with different numbers of sensors.

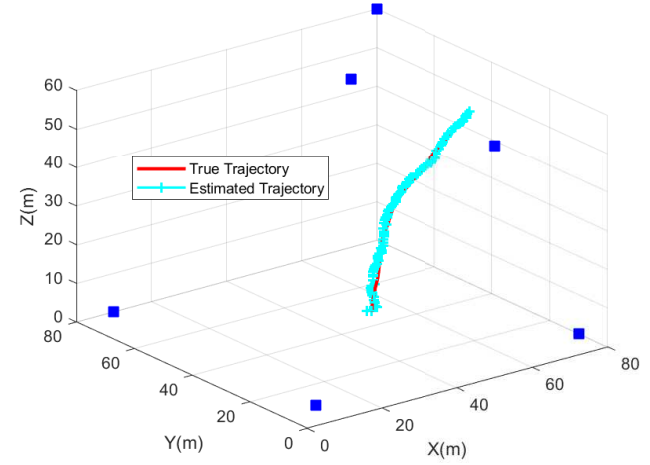


Fig. 11: The true target trajectory and its estimate in the 3-D scenario.

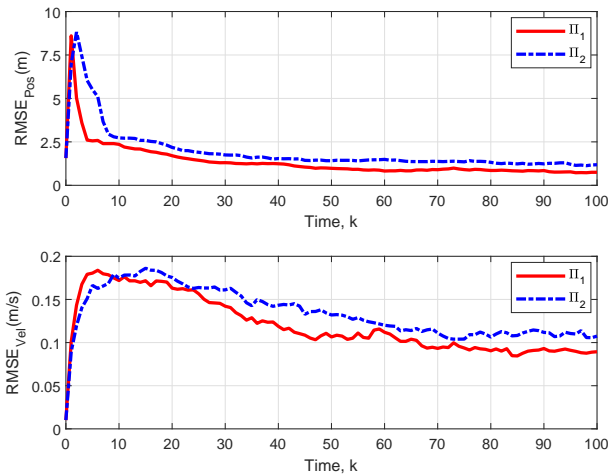


Fig. 10: Position and velocity RMSEs of PF-ROSS-EH with different energy harvesting processes.

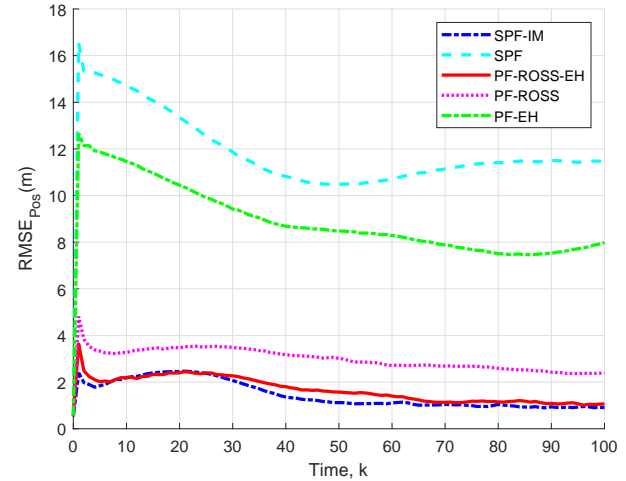


Fig. 12: Position RMSEs of PF-ROSS-EH, PF-ROSS, PF-EH, SPF and SPF-IM in the 3-D scenario.

harvesting process has a non-negligible effect on the filtering accuracy. In fact, the results are intuitively reasonable because the energy harvesting process with transition probability matrix Π_2 is more likely to harvest zero energy and is easier to incur missing measurement.

In what follows, the 3-D moving target tracking scenario is considered to further demonstrate the effectiveness of the proposed algorithm. Similarly, the dynamics of the target is represented by the white noise acceleration model [4] and the state of the target at time instant k is represented by

$$x_k = [s_{x,k}^t, v_{x,k}^t, s_{y,k}^t, v_{y,k}^t, s_{z,k}^t, v_{z,k}^t]^T,$$

where $(s_{x,k}^t, s_{y,k}^t, s_{z,k}^t)$ and $(v_{x,k}^t, v_{y,k}^t, v_{z,k}^t)$ are the position and velocity of the target's centroid, respectively. The measurement model of the i th RSS sensor is based on the distance between the moving target and the i th sensor in the 3-D scenario. The RMSEs on the position and

velocity are similarly defined. The initial state is chosen as $x_0 = [25\text{m } 0.3\text{m/s } 10\text{m } 0.4\text{m/s } 20\text{m } 0.3\text{m/s}]^T$, and the initial particles are sampled from a Gaussian prior distribution with mean x_0 and covariance $\text{diag}\{10^2, 0.3^2, 10^2, 0.3^2, 10^2, 0.1^2\}$. In the simulation, the number of sensors is set as $N = 6$ and the number of particles is $M = 1000$. Other parameters are the same as those in the 2-D scenario.

The simulation results in the 3-D scenario are displayed in Figs. 11-13, which again verify the effectiveness of the proposed particle filtering algorithm in the simultaneous presence of the ROSSs and possibly failed measurement transmission.

V. CONCLUSIONS

In this paper, a particle filtering algorithm has been developed to solve the filtering problem for a class of nonlinear/non-

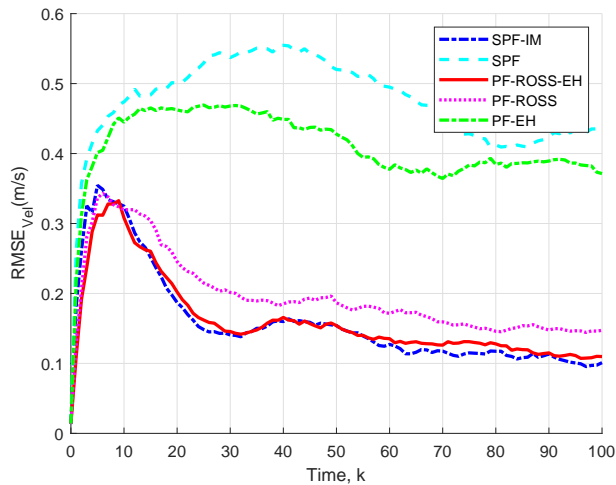


Fig. 13: Velocity RMSEs of PF-ROSS-EH, PF-ROSS, PF-EH, SPF and SPF-IM in the 3-D scenario.

Gaussian systems with energy harvesting sensors subject to the ROSSs. According to the established first-order Markov model of the energy harvesting process, the occurrence probability of the missing measurement has been calculated at each time instant. A modified likelihood function, which takes the occurrence probability of the ROSSs and the missing probability of the measurements induced by insufficient energy into consideration, has been derived to attenuate the effects from both ROSSs and sensor energy constraints on the filtering quality. Finally, the usefulness of the proposed filtering scheme has been demonstrated by the illustrative examples. Future research topics would be the extensions of the present results to more sophisticated energy harvesting models (e.g. non-Markovian model), and to distributed state estimation problem with large datasets [36], [48], [60], in which the herding algorithms [19], the compressed Monte Carlo schemes [39] and the group importance sampling schemes [40] might be considered to reduce the computational burden and the communication cost.

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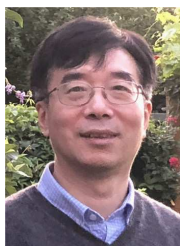
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