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An Artificial Neural Network -Based Mathematical Model for a Stochastic Health Care Facility Location Problem

Abstract

This research is conducted to investigate the problem of locating the trauma centers and helicopters' station in order to optimize the trauma care system. The stochastic characteristics of the system, such as stochastic transferring time of the patients, stochastic demand and stochastic servicing time of the patients in trauma centers are taken into account. The problem is first modeled as a stochastic mixed-integer linear mathematical model. In the proposed model, minimizing the total cost, minimizing the transferring time, and minimizing the waiting time inside the trauma center are considered as the three separate objectives. The third objective cannot be expressed by an analytical expression because of the complexity inside a trauma center. Therefore, an artificial neural network (ANN) is first trained by a simulation model and then is utilized to estimate the third objective function. A hybrid multi-objective algorithm is developed based on a non-dominated sorting water flow algorithm in order to search the solution space. Different numerical examples are applied to study the performance of the proposed method. The computational results show that the combination of simulation, ANN, and optimization technique provides an effective means for the highly complex optimization problems.

Keywords: Artificial Neural Network; Mathematical Modeling; Health Care; Facility Location Problem, Operations Research

Highlights

- An artificial neural network is developed for the health care facility location problem.
- A mixed integer non-linear programming model is proposed.
- A simulation model is applied to consider the stochastic parameters.
- Numerical experiments demonstrate the effectiveness of the proposed algorithm.

1. Introduction

With the increase in the need for establishing better healthcare systems, finding the best location of the healthcare resources has become a vital issue in providing better and reliable healthcare services (Shikder et al., 2017). Trauma encompassing the accidents or unintentional injuries is among the most important issues in the healthcare. According to the world health organization (WHO), traumatic injury is a public health problem in the high and low-income countries. Kehoe et al., (2015) stated that the trauma is the leading cause of death in the people between 25– 50 years of age in the United States and the second leading cause for those aged over 75 years old. As a result, the need for trauma centers is essential for each geographical area throughout the country.

A trauma center is a hospital equipped and staffed to immediately provide the surgical services and care to the patients with traumatic injuries. Accredited trauma centers must always be prepared to treat the most disabling and life-threatening injuries. Each trauma center is designated with a certain level corresponding to the degree of care offered by the center (Moritz, 2012). When a trauma case occurs, critical care paramedics are quickly dispatched to the scene either by the ground ambulance or helicopter. They provide the first aid to stabilize the patient at the scene and then, transport him/her to a trauma center. Because any delay in transporting a patient to a trauma center can severely influence his/her survival rate, an appropriate clinical intervention should be provided very soon. Therefore, the optimal location of trauma centers is an integral part of the success of all the trauma systems so as to minimize the distance between the trauma centers and possible incident point. The time needed to transfer the injured to the trauma centers can be reduced by optimal locating the trauma centers. But, establishing the trauma centers is costly and they are generally located in the main hospitals. Therefore, locating a trauma center is equal to selecting a hospital to establish a trauma center in it.

When there are a limited number of trauma centers, transferring will be much more important. Helicopters can quickly take the patient from the incident point to the trauma center. Also, they play an important role by expanding the geographical coverage through transporting the patients from rural areas to the trauma centers. Thus, it is important to simultaneously locate the trauma centers and helicopter stations in designing a trauma care system. In addition to the location problem, the way of allocating the resources in a trauma care system is vital. Helicopters are the most important resources, which should be optimally allocated to the stations. Allocation of the required resources is usually discussed further when the optimal location configuration is addressed. However, the locations of trauma centers influence the demand for the helicopters and vice versa. This dependency is particularly problematic if the problem is not considered integrally.

Thus, in this paper, both problems of locating the trauma centers and helicopter stations are considered along with allocating the rescores in order to optimize the trauma care system. To be more practical, the stochastic characteristics of the system are also taken into account. This stochastic comes from the stochastic transferring time of the patients, stochastic demand, stochastic servicing time of the patients in trauma centers, etc. First, a stochastic mixed-integer linear mathematical model is developed. In the proposed model, minimizing the total cost, minimizing the transferring time, and minimizing the waiting time inside the trauma center are considered as the separate objectives. The first two objectives are expressed by the analytical formula. But, the third objective cannot be expressed by an analytical expression because of the complexity inside a trauma center. Therefore, an artificial neural network (ANN) is utilized to estimate the third objective function. The ANN works as an objective function estimator during the search process.

The main contribution lies in calculating the time of transferring the injured patients to trauma centers by queueing theory, and calculating the waiting time spent inside the trauma center by ANN. To build an efficient ANN, a simulation model is made and through this simulation model the ANN is rained and utilized to estimate the waiting time spent inside the trauma center. Both queueing theory and simulation can handle the stochastic characteristics of the system, such as stochastic transferring time of the patients, stochastic demand and stochastic servicing time of the patients in trauma centers. Therefore, calculating the tome metric is done in a realistic manner. On the other hand, as another contribution, a novel metaheuristic algorithm, named Non-dominated Sorting Water Flow Algorithm (NSWFA), is combined with ANN and is applied to search the solution space.

The remainder of this paper is organized as follows. A brief review of the literature is provided in Section 2. In Section 3, a description of the problem is given along with the mathematical model. Section 4 describes the methodology of the proposed solution. Computational experiments are given in Section 5. Finally, Section 6 is devoted to the conclusions and recommendations for future research.

2. Literature Review

Toregas et al., (1971) and Church and ReVelle (1974) are among the first researchers studied the vehicle location problems of emergency medical services. Toregas et al., (1971) investigated a location set covering problem. They minimized the number of ambulances and find their locations covering all the demand points within a certain distance. Church and ReVelle (1974) proposed the maximal covering location problem, which deals with locating a fixed number of ambulances so as to maximize the amount of demand covered by at least one facility. Branas et al., (2000) presented an integrated model. The model could determine the optimum positions of trauma centers and helicopter stations. A heuristic algorithm was applied to optimize the model. Their computational results showed that the trauma resource allocation model for ambulances and hospitals allows the planners of the trauma systems to better locate their resources with respect to the spatial needs and response times. Branas and ReVelle (2001) considered a joint location problem of trauma centers and helicopters. They modeled this as a deterministic location problem, and formulated the problem as a mixed-integer linear program. They could not obtain the solutions within a reasonable amount of time using the CPLEX directly, so they developed an iterative heuristic to identify the best locations of helicopters, holding the locations of trauma centers fixed, followed by finding the best locations of trauma centers, holding the locations of helicopters fixed, and so on.

Erdemir et al., (2008) developed a mathematical model in order to find the optimum location of helicopter stations. Their model maximized the covered demands considering the service facilities with demands on both nodes and paths. Schuurman et al., (2009) presented a model to maximize the demand coverage of the regions without adding a new helicopter station. A real case with two hospitals and helicopter medical services was used to show the model efficiency. Erdemir et al., (2010) presented two models to optimize the locations of the ambulance, helicopter, and transfer points. The first model minimized the cost of establishing the facilities. The second model maximized the coverage of demands within the given total cost. Fulton et al., (2010) presented an optimization model considering the uncertainty in demand. The model sought to minimize the expected travel time over all the potential scenarios. Their model optimized the redesign and relocation of the emergency service facilities and the location of ground ambulances by finding the optimum location of helicopter stations, hospitals, and helicopter paths. Caunhye et al., (2012) categorized the optimization model for emergency logistics into three groups: (1) facility location, (2) relief distribution and casualty transportation, and (3) other operations. Factors, such as objective function, constraints, data modeling types, and decisions were used to analyze each group. Zanjirani Farahani et al., (2014) provided an overview of the models and algorithms used for health care problems.

Bozorgi-Amiri et al., (2016) considered uncertain demand points and proposed an integer nonlinear programming model. The model was able to find the optimum location of the helicopter stations and helipads. They showed that the demands occur in a square-shaped area. Each side of this square-shaped area follows a uniform distribution. Lee and Jang (2018) addressed the problem of simultaneously locating the trauma centers and helicopters during the planning horizon. They extended the location model into a multi-period location model by introducing an additional decision point namely, the location time. An efficient solution approach, that iteratively updated the helicopters' availability using the previous step of optimization results, was proposed to estimate the availability of the helicopters. They proved that the proposed method is valid and competitive compared to the alternative methods. Sathler et al., (2019) presented a new integrated approach to the public health care planning. They proposed an integrated mathematical model to locate the health care centers and allocate the equipment in order to maximize the demand satisfaction with the minimum resource allocation. The computational results revealed that their approach is generic and suitable to set the location and allocate the resources in the health care if the objective is maximizing the use of specialists' assistance and medical exams.

Belanger et al., (2019) performed a comprehensive review on the static ambulance location problems. They focused on the recent approaches to address the tactical and operational decisions, and the interaction between these two types of decisions. De Freitas et al., (2019) developed a two-step optimization system and a web-based interface to provide an optimum solution for locating and sizing the medical centers and allocating the equipment to meet the secondary care needs of the community. They suggested a location-allocation mathematical model to address the health-care facility location and capacity allocation for secondary care on the state level, adopt the available equipment idle capacity on a hierarchical healthcare system, and recognize the multiple preferences.

Clearly, different types of objectives have been used to solve the health-care facility location problem. Some studies have used more than one objective. Dokmeci (1979) used two objectives of minimizing cost and maximizing utilization. Current et al., (1990) suggested four objectives including (1) cost minimization, (2) demand -oriented, (3) profit maximization, and (4) environmental concern to decide about the facility location. Doerner et al., (2007) studied the problem of locating the mobile healthcare facilities (HCFs). They developed a mathematical model to optimize the number and location of the facilities. Finally, three criteria of efficiency, average distances, and coverage were used to evaluate the model.

Gu et al., (2010) set two objectives including (1) people should have more flexibility to select the service location, and (2) each preventive HCF needs to have a minimum number of clients in order to retain the accreditation to optimize the preventive HCF locations. De Vries et al., (2014) presented an approach to select the locations for new HCFs and to choose for each of these facilities whether to add the healthcare services for the human immunodeficiency virus (HIV). They proposed a two -objective mathematical model to maximize the number of the covered truck -driver patients at these facilities and the maximization of the extent such that, the truck drivers have the continuous access to the needed health service packages. Bonnet et al., (2015) also presented a multi-objective optimization along with simulation to test the optimization output for the performance of the automated external defibrillators in terms of time-to-retrieve. Furthermore, they displayed the results in an interactive decision-making web-based user interface to visualize the potential deployment configurations.

However, in the above-mentioned researches, candidate solutions were obtained by summing the weighted objective functions. This method has several limitations: (1) the knowledge is required about the importance of the objectives, (2) it may lead to only one solution, and (3) trade-offs between the objectives cannot be evaluated (Ngatchou et al., 2005; Yoo & Harman, 2007). Some studies have focused to find the Pareto solutions in the multi -objective (MO) problems in order to cope with these weaknesses. Pareto solutions are the solutions that are greater with respect to the rest of the solutions in the search space when all the objectives are considered but are inferior to the other solutions in the space in case of one or more objectives (Srinivas & Deb, 1994). Pareto plans maintain a range of key index values and reflect the trade-offs between the objectives thus, the planners or decision -makers can select from the Pareto plans. Given the feature of the Pareto solutions, many scholars have attempted to search for the Pareto solutions rather than one best solution for MO problems.

Zhang et al., (2016) proposed a Pareto-based multi-objective genetic algorithm to optimize the location of new HCFs in Hong Kong in 2020. Four objective functions of minimizing the inequity of accessibility, maximizing the accessibility for the whole population, minimizing the number of people who are present outside an acceptable travel distance, and minimizing the cost of constructing the new HCFs were selected. Decerle et al., (2018) presented a new Pareto-based multi-objective algorithm named as the multi-directional local search. They solved the problem so that, the total traveling time of the caregivers, the sum of the penalties due to soft patients' time window and the maximal workload difference among the caregivers were minimized. Finally, they analyzed the effect of balancing each activity of the caregivers on the various objectives. Decerle et al., (2019) developed a multi-objective model to consider the caregivers and qualifications of the visits, temporal dependencies, patient's availability time window, multiple home health care offices, and balanced working time. A Pareto-based multi-objective memetic algorithm was used to simultaneously minimize the total working time of the caregivers, maximize the quality of service, and minimize the maximal working time of the nurses.

Though, the concept of the Pareto solutions for MO problems has been widely used in different areas of operations research, there are a few researches addressed the Pareto solutions for the MO problem of locating the HCFs. Thus, in the current research, the multi-objective optimization (MOO) approach is employed to search for the Pareto solutions of HCF locations. For this purpose, first, a new multi-objective mathematical model is developed. Then, the water flow algorithm is adapted to solve the problem and search for the Pareto solutions. The proposed mathematical model is able to minimize the total cost, the transferring time, and the waiting time inside the trauma center. In this model, the time needed to transfer the injured is considered as a stochastic parameter. This makes the model to be more realistic and applicable.

3. Mathematical Model

The total geographical area that must be covered is partitioned into different areas. The demand in each area i is stochastic with mean λ_i. There are U candidate sites for trauma centers. In fact, there is a hospital in each candidate site where a trauma center can be constructed in it. There are K candidate sites for helicopter stations. Two different modes exist to transfer the patients from each area to a trauma center. In the first mode, the injured is transferred directly to the trauma center by an ambulance sent from the trauma center. In the second mode, the helicopter is used to transfer the injured to the trauma center. In the first mode, there are enough ambulances so that, as soon as a request is arrived, an ambulance is sent to the demand point. But, in the second mode, since the number of available helicopters is limited, the injured have to wait for the helicopters to be available. The exact point of demand in each area is unknown. Therefore, the time needed to transfer the injured is considered as a stochastic parameter. The emergency treatment process will begin after transfer of the injured to the trauma center. The main process of treatment occurs inside the trauma center. Other assumptions are considered as follows:

- A sufficient number of ambulances exist in each trauma center.

- All the demand points must be covered.
- The process of treatment in the trauma centers is predetermined.

The problem is where the trauma centers and helicopter stations are established and how the resources are allocated so that the total cost of building the trauma centers and helicopter stations, total transferring time of the injured patients to trauma centers, and total waiting time of the injured patients inside the trauma centers are minimized. The problem is modeled as a mixed-integer mathematical programming model. The notations used in the developed mathematical model are as follows:

Indices

i	Index for demand area $i = \{1, 2, \ldots, I\}$
k	Index for helicopter station $k = \{1, 2, \ldots, K\}$
u	Index for trauma center $u = \{1, 2, \ldots, U\}$

parameters

$t_{iu\pi}$	The time distance of ambulance between demand area *i* and trauma center *u* in scenario π
$t'_{iuk\pi}$	The time distance of helicopter between helicopter station *k* and demand area *i* plus demand area *i* and trauma center *u* in scenario π
λ_i	Mean arrival rate of demand in the demand area *i*
ch_k	Establishment cost of each helicopter station *k*
ct_u	Establishment cost of each trauma center *u*
H	Total number of available helicopters

The proposed mathematical model has five principal decision variables. Four of them are binary variables and one is an integer variable.

Decision variables

X_{iu}	$\begin{cases} 1 & \text{if demand area } i \text{ is directly serviced by the ambulance from trauma center } u \\ 0 & \text{otherwise} \end{cases}$
Y_{iuk}	$\begin{cases} 1 & \text{if demand area } i \text{ is serviced by trauma center } u \text{ via helicopter station } k \\ 0 & \text{otherwise} \end{cases}$

Z_k	$\begin{cases} 1 & \text{if a helicopter station is established at node } k \\ 0 & \text{otherwise} \end{cases}$
O_u	$\begin{cases} 1 & \text{if a trauma center is established at node } u \\ 0 & \text{otherwise} \end{cases}$
C_k	Number of the helicopters allocated to the helicopter station k
ρ_k	Utilization factor in the helicopter station k
$E(W_k)$	The average waiting time in the helicopter station k

The first objective function minimizes the total costs of establishing the helicopter station and trauma center.

$$OF_1 = Min \sum_{\forall k} Z_k ch_k + \sum_{\forall u} O_u ct_u \tag{1}$$

The second objective function minimizes the total time of transferring the injured patients to the trauma centers. The first part is related to the time of direct transferring from demand point to trauma centers through the ambulance. The second part is related to the time of transferring from demand point to trauma centers through a helicopter and the third part is related to the waiting time in the demand point for a helicopter.

$$OF_2 = Min \sum_{\forall \pi} \sum_{\forall i} \sum_{\forall u} X_{iu} t_{iu\pi} \lambda_i + \sum_{\forall \pi} \sum_{\forall i} \sum_{\forall u} \sum_{\forall k} Y_{iuk} t'_{iuk\pi} \lambda_i + \sum_{\forall k} E(W_k) \tag{2}$$

The third objective function minimizes the total waiting time spent inside the trauma center for a patient to complete the emergency activities. Calculating this time by the analytical formulation is impossible because of high complexity and high interaction between the elements inside the trauma centers. Therefore, its value is estimated by an ANN.

$$OF_3 = Min\, f(X_{iu}, Y_{iuk}, Z_k, O_u) = ANN \tag{3}$$

Constraint set (4) guarantees that each demand area is serviced by only one of two modes. If one of decision variables of X_{iu} takes value one, all the Y_{iuk} values will be equal to zero showing that the patients in the demand area are transferred to the trauma center by the ambulance. If one of decision variables of Y_{iuk} takes value one, all the X_{iu} values will be equal to zero implying that the patients in the demand area are transferred to the trauma center by the helicopter.

$$\sum_{\forall u} X_{iu} + \sum_{\forall u} \sum_{\forall k} Y_{iuk} = 1 \qquad \forall i \tag{4}$$

Constraint set (5) implies that the demand area *i* can be assigned to the trauma center *u* if a trauma center is already established in point *u*. Constraint set (6) implies that the demand area *i* can be assigned to the trauma center *u* through the helicopter station *k* if a trauma center is already established in point *u* and a helicopter station is already established in point *k*.

$$X_{iu} \leq O_u \qquad \forall i, u \qquad (5)$$

$$Y_{iuk} \leq Z_k O_u \qquad \forall i, u, k \qquad (6)$$

Each helicopter station *k* is modeled as a *M/M/k* queueing system to calculate the waiting time of the injured to reach the helicopter. Readers are referred to the study by Gross and Harris (1985) for justification of the formulas of *M/M/k* queueing systems. According to the formula of *M/M/k* queueing system, the average length of a queue is calculated by Equation (7)

$$L_{qk} = \frac{P0_k \rho^{C_k+1}}{(C_k - 1)!\,(C_k - \rho_k)^2} \qquad (7)$$

Where, ρ_k is utilization factor in the helicopter station *k* and $P0_k$ denotes the probability that there are zero injured in the helicopter station *k* calculated by the following equations.

$$P0_k = \left(\sum_{i=0}^{C_k - 1} \frac{\rho^i}{i!} + \frac{\rho^{C_k}}{C_k!} \left(\frac{C_k \mu_k}{C_k \mu_k - \sum_{\forall i} \sum_{\forall u} Y_{iuk}\, \lambda_i} \right) \right)^{-1} \qquad (8)$$

$$\rho_k = \frac{\sum_{\forall i} \sum_{\forall u} Y_{iuk}\, \lambda_i}{C_k \mu_k} \qquad (9)$$

$$0 \leq \rho_k \leq 1 \qquad (10)$$

In Equations (8) and (9), μ_k is the average number of the injured served by the helicopter station k, and is calculated by Equation (11).

$$\mu_k{}^{-1} = \operatorname*{mean}_{\pi} \frac{\sum_{\forall i} \sum_{\forall u} t'_{iuk\pi} Y_{iuk}}{\sum_{\forall i} \sum_{\forall u} Y_{iuk}} \qquad \forall k \qquad (11)$$

Hence, the average waiting time in the helicopter station k, $E(W_k)$, can be obtained as follows:

$$E(W_k) = \frac{L_{qk}}{\sum_{\forall i} \sum_{\forall u} Y_{iuk}\, \lambda_i} \qquad (12)$$

Constraint set (13) controls the total number of the helicopters.

$$\sum_{\forall k} C_k \leq H \tag{13}$$

The decision variables are kept to be binary or integer by the constraint set (14) and (15).

$$X_{iu}, Y_{iuk}, Z_k, O_u \in \{0, 1\} \qquad \forall i, u, k \tag{14}$$

$$C_k \in N \qquad \forall\, k \tag{15}$$

4. Proposed Approach

This section describes the proposed approach to solve the model. There are two difficulties in solving the model. Firstly, the model is stochastic, multi-objective, and non-deterministic polynomial-time (NP)-hard leading to the fact that the mathematical programming approaches are not so efficient to solve the problem. Another difficulty comes from calculating the third objective function. Because of the high complexity and high interaction between the elements inside the trauma centers, the third objective function cannot be expressed as a formula. Thus, combining a multi-objective metaheuristic optimization algorithm, named Non-dominated Sorting Water Flow Algorithm (NSWFA), along with simulation modeling and ANN is proposed to cope with these difficulties. The NSWFA is used to solve the model. NSWFA can calculate the value of the two first objective functions through Equations (1) and (2). However, there is no equation to calculate the third objective function. First an ANN is trained to calculate the third objective function. To this aim, a series of random solutions are generated. Each solution, called a scenario, is an especial type of decision variables. Then, a discrete-event simulation is used to evaluate the third objective function (the waiting time spent in each trauma center). The obtained values from the simulation model are applied to train an ANN. The trained ANN performs just like an estimator capable of estimating the third objective function. By this way, the value of the third objective function is calculated and NSWFA can search the solution space. In fact, in our NSWFA, an ANN is considered as the evaluator of the third objective function. The combination of simulation, ANN, and NSWFA technique provides an effective means for the highly complex optimization problems.

4.1. Non -Dominated Sorting Water Flow Algorithm

The design of the Non-dominated Sorting Water Flow Algorithm (NSWFA) is a mimic of normal manner of water flowing from higher to lower points. On a surface, one side of a flow will be splitted into multiple sub-flows and on the other hand, sub-flows will merge. Directed by the gravity and based on the fluid momentum, flows can run to higher or lower levels. Water flow will cease and stagnate at the locally or globally lowest depression; when the momentum left cannot expel the water out of the depression, it will stagnate at its current location. No movement is allowed until other flows merge with it or until the water evaporates into the atmosphere. When the evaporated water is accumulated, it will come back to the ground as new flows. This is called the rainfall that occurs occasionally. In order to apply the NSWFA for an optimization problem, the geographical terrain is considered as the solution space, each flow of water is considered as a solution, and the altitude of each flow is the objective value. Movement of a water flow to a lower place is considered as the searching for the optima. Thus, the solution search process is modeled as the water flow. The NSWFA algorithm consists of four primary operations: (1) flow splitting and moving, (2) flow merging, (3) water evaporation, and (4) precipitation. Fig. 1 shows the pseudo-code for the general procedure to implement the NSWFA. In the first step, the initial water flow algorithm parameters including the initial mass of original low, $M0$, initial velocity of original flow, $V0$, upper limit on the number of subflows splitted from a flow, n and number of the iterations, $Iteration$ are set (line 1). Based on the method described in Subsection 4.4, a set of initial flows is randomly generated (line 2). All the flows (solutions) are sorted based on non-dominated sorting approach (line 3), which will be described in Subsection 4.2. Then, the solutions in the same front are sorted based on the crowding distance metric (lines 4-5), which will be described in Subsection 4.3. For each flow, a set of subflows is created based on the method described in Subsection 4.5 and then, each flow moves to its best subflow (lines 7-10). Subflows with the same objective values are merged and the resulting mass and velocity are updated based on Equations (21) and (22) (line 11). Based on the method, which will be described in Subsection 4.7, evaporation operation is performed and the mass is updated for each water flow (line 12). Then, we check whether the precipitation condition is met; if yes, the precipitation operation (Subsection 4.8) is done to generate a set of new solutions (lines 13-15). Finally, the flows are sorted based on non-dominated sorting and crowding distance and the extra flows are the removed solutions (lines 16-19).

$//NSWFA\ procedure$
1: Initializing the parameters of the NSWFA ($M0, V0, n, Iteration$)
2: Creating the initial flows randomly (POP)
3: Sorting the solutions based on the non-dominated sorting
4: Calculating the crowding distance
5: Ranking the flows in the same front by the crowding distance
6: **For** $j = 1$ **to** $Iteration$ **Do**
7:     **For** each flow $i \in \{1, 2, \dots, N\}$ **Do**
8:         Flow splitting and moving
9:         Moving to a neighborhood solution
10:     **End For**
11:         Flow merging.
12:         Water evaporation.
13:     **IF** rainfall is required **DO**
14:         Precipitation.
15:     **End IF**

16: Sorting the solutions based on the non-dominated sorting
17: Calculating the crowding distance
18: Ranking the flows in the same front by the crowding distance
19: Selecting the flows for the next iteration by eliminating the extra flows
20: **End For**

Fig 1. A generic framework for the NSWFA.

4.2. A Fast Non-Dominated Sorting Approach

First a concept, called non-domination is applied to sort the flows. Based on the non-domination definition, a solution dominates the other solutions if its objective function is not worse than the others and at least, one of its objective functions is better than the other ones. Fig. 2 shows a graphical representation of the non-dominated sorting for a minimization problem. Individuals, such as x_1, x_4, x_6, and x_7 are assigned the ranks as rank 1 since there is no individual superiority to them with respect to $f_1(x)$ and $f_2(x)$. The individuals of rank 1 are removed and the individuals with rank 2 are selected in the same way. This procedure is repeated until all the individuals are categorized. The Pareto-optimal front includes the solutions existing in the front 1.

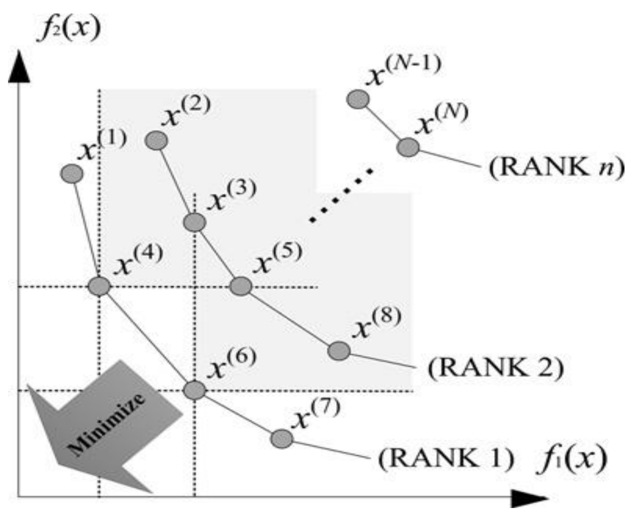

Fig 2. Schematic representation of the non-dominated sorting for two objectives.

4.3. Crowding Distance (CD)

In a situation where two solutions are in the same front, the crowding distance (*CD*) metric is used to rank the solutions. The crowding distance determines the amount of closeness of a solution to its neighbors. The basic idea behind the crowding distance is finding the Euclidean distance between each individual in a front based on their *m* objectives in m-dimensional hyperspace. The formula to calculate the *CD* metric is given in Equation (27), where r is the number of the objectives, f_{i+1}^k is the *kth* objective of the *i+1*st individual ,and f_{i-1}^k is the *k*th objective of the *i-1*st individual after sorting the population according to the *CD*. The individuals with lower *CDs* are put forward in the removal process.

$$CD_i = \frac{1}{r}\sum_{k=1}^{r}\left|f_{i+1}^{k} - f_{i-1}^{k}\right| \tag{27}$$

Afterwards, the normal operators of the water flow algorithm are applied on the flows to improve the current flows and generate the new flows as the next generation. The details of the other operations of the NSWFA to solve the proposed model are described in the following subsections.

4.4. Flow Structure

In the water flow algorithm, each flow is representative of a solution. Therefore, each flow should show all the decision variables. The proposed flow structure is a matrix where each of its cells represents a special area of decision -making. First, the *U+K* number is considered where *U* is the number of the potential trauma centers and *K* is the number of the potential helicopter stations. Therefore, 1 to U presents the trauma center and *U+1* to *K* presents the helicopter station. Then, a random subset is selected. These numbers show where the trauma center and helicopter station should be established. Then, the numbers are randomly sorted; from left to right, each number shows a point for trauma center or helicopter station and its position shows the allocated demand area. Also, some random numbers are generated according to the number of the established helicopter stations so the sum of them is equal to the total available helicopters.

Fig. 3 shows an example for a flow with 5 potential points for the trauma center and 3 potential points for the helicopter stations, respectively. The total number of the available helicopters is equal to 5. As shown in Fig. 3, the points 1, 4, and 5 are selected to establish the trauma center and points 6 and 7 are selected to establish the helicopter station. The demand area 1 is allocated to trauma center 5, demand area 2 is allocated to helicopter station 7, demand area 3 is allocated to helicopter station 5, demand area 4 is allocated to trauma center 1, and demand area 5 is allocated trauma center 4. Also, 2 and 3 helicopters are allocated to stations 6 and 7. This type of solution representation can guarantee that each demand area is serviced by only one of two modes and each demand area is assigned to an open trauma center. The other constraint related to the availability of the facilities inside the trauma center is satisfied by the simulation model.

Potential points	trauma center					helicopter station		
	1	2	3	4	5	6	7	8

Selected points	1	4	5	6	7

Final flow structure	5	6	7	1	4
		2	3		

Fig 3. An example of the solution structure (flow)

4.5. Flow Splitting and Moving Operation

When the NSWFA starts to search, there is only one water flow. The location of this flow is generated randomly. The flow moves to the new locations according to the mass and velocity of flow. Fig. 4 demonstrates the splitting and moving operation for searching the neighborhood solutions. As flow *i* splits into the sub-flows, the number of subflows n_i is obtained based on Equation (16):

$$n_i = max\left\{1, int(\frac{W_i V_i}{T})\right\} \tag{16}$$

Where, *Wi* is the mass of flow *i*, *Vi* is the velocity of flow *i*, and *T* is the base momentum.

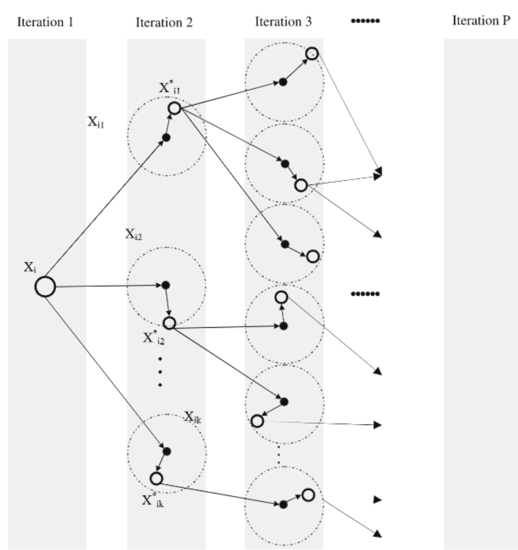

Fig 4. Flow splitting and moving

The design of the flow-moving operation is problem-dependent. In the proposed NSWFA, two neighborhood structures are defined. Both structures are able to preserve the feasibility of the solutions. Neighborhood strategy type 1 is related to the location-allocation decisions. In the neighborhood strategy type 1, two different cells of the first row are randomly selected and swapped (Fig.5). The number of the allocated helicopter is changed by the neighborhood strategy type 2 where two different cells related to the helicopter stations are randomly selected and swapped (Fig. 5).

In the neighborhood strategy type 2, two periods are randomly selected and all the data related to the machine layout and transporter allocation of these periods are exchanged (Fig. 6).

Flow structure	5	6	7	1	4
		2	3		

Neighbourhood flow structure	4	6	7	1	5
		2	3		

Fig 5. An example of the neighborhood strategy type 1

Flow structure	5	6	7	1	4
		2	3		

Neighbourhood flow structure	5	6	7	1	4
		3	2		

Fig 6. An example of the neighborhood strategy type 2

As mentioned previously, the mass of flow i must be reasonably distributed to its subflows. To this aim, the non-dominated sorting and crowding distance metric are used to rank the subflows of flow i. The allocated mass to subflow j is distributed in the flow i, U_{ij}, is then calculated by Equation (17).

$$U_{ij} = \left(\frac{n_i + 1 - rank_j}{\sum_{r=1}^{n_i} r} \right) \times W_i \tag{17}$$

Where, $rank_j$ is the rank of subflow j with respect to the other subflows distributed in the flow i. For instance, if flow i splits to 5 subflows, the mass of rank1 subflow is obtained from Equation (18).

$$U_{ij} = \left(\frac{5+1-1}{15} \right) \times W_i = \frac{5}{15} \times W_i \tag{18}$$

Also, velocity of subflow j splitting from flow i is calculated by Equation (19).

$$\mu_{i,j} = \begin{cases} \sqrt{V_i^2 + 2g\overline{\delta_{i,j}}} & if\ V_i^2 + 2g\delta_{i,j} > 0 \\ 0 & otherwise \end{cases} \tag{19}$$

$$\delta_{i,j} = \operatorname*{mean}_k \delta_{i,j}^k$$

Where, 'g' is the gravitational acceleration and $\delta_{i,j}^{k}$ is the difference between flow i from its subflow j in the objective function k. Equation 19 is used to compute the value of $\delta_{i,j}^{k}$ for minimization and maximization and then, is averaged.

$$\delta_{i,j}^{k} = \begin{cases} f_i^k - f_{i,j}^k, \text{ for minimization} \\ f_{i,j}^k - f_i^k, \text{ for maximization} \end{cases} \tag{20}$$

Where, f_i^k is the objective function value k of flow i and $f_{i,j}^k$ represents the kth objective function values of subflow j distributed in the flow i.

4.6. Flow-Merging Operation

Sometimes, two flows move to the same location. In this situation, they are merged into one. The similarity of the flows is considered to determine if two flows are in the same location or not. If all the decision variables are the same, then it can be said that the flows are in the same position. If the flows *i* and *j* share the same location, then flow *j* will be deleted and the mass and velocity of flow *i* will be updated as follows:

$$W_{new} = W_i + W_j \tag{21}$$

$$V_{new} = \frac{V_i W_i + V_j W_j}{W_i + W_j} \tag{22}$$

The NSWFA reduces the number of duplicate solutions to avoid the redundant searches using the flow-merging operation.

4.7. Water Evaporation Operation

As a normal behavior of water, it evaporates and returns to the ground through the precipitation. Water evaporation and precipitation show the mechanism to avoid being trapped in the local optima. In the NSWFA, the velocity-based evaporation is defined such that, the flows with smaller velocities should evaporate faster than those with larger velocities. The formulation of the evaporation proposed in the literature (Wu et al., 2010) is presented in Equations (4.9)–(4.10).

$$W_i = (1 - \rho_i) W_i \tag{23}$$

$$\rho_i = \begin{cases} 1, & if\ \mu_{i,j} = 0 \\ 0, & if\ \frac{\mu_{i,j}}{V_i} \geq 1 \\ 1 - \frac{\mu_{i,j}}{V_i} & if\ 0 \leq \frac{\mu_{i,j}}{V_i} \leq 1 \end{cases} \tag{24}$$

4.8. Water Precipitation Operation

The evaporated water will return to the ground by the precipitation operator after a number of iterations. In this paper, a seasonal rainfall is applied periodically where the precipitation takes place when the number of the evaporated flows reaches to a predefined level. The location of the returned flow is deviated far away from that of flow before evaporation in order to enhance the algorithm's diversity. In this way, the bits of flow are arranged inversely.

4.9. Stopping Criterion

Some convergence experiments are performed and the best criterion is selected to limit the number of replications of the NSWFA. The NSWFA will be stopped when the total number of iterations reaches to a predefined number, which is set according to the result of the experimental design.

5. Computational Experiments

This section describes the applocation of the proposed approach. First, a numerical example is applied to demonstrate how the third objective function is estimated by the ANN. Then, a set of random test problems are generated to evaluate the performance of the NSWFA. The performance of the NSWFA is evaluated against two well-known metaheuristic algorithms named the non-dominated sorting genetic algorithm (NSGA-II) and non-dominated ranked genetic algorithm (NRGA). It should be noted that, the MATLAB software (Version 7.10.0.499, R2010a) was used to code the proposed metaheuristic optimization algorithms, and the programs were executed on a 2 640 GHz laptop with 4 GB RAM.

5.1. Dataset

Due to the lack of a benchmark in the literature for the presented model, the test problems are randomly generated. Table 1 presents the specifications of these problems. The data are generated so that it can be applied for city districts in capital province of Iran. The generated problems are divided into four sets of problem. In the first set, there are 5 demand areas, 2 trauma centers, and 2 helicopter stations, in the second set, there are 10 demand areas, 3 trauma centers, and 3 helicopter stations, in the third set, there are 15 demand areas, 5 trauma centers, and 5 helicopter stations, and in the fourth set, there are 30 demand areas, 8 trauma centers, and 7 helicopter stations. Each demand area is considered with an area of 600 km^2 and a population size uniformly distributed between the following ranges (5×10^5, 7×10^5). The total number of the available helicopters is equal to 15. For each problem size, 8 random problems are generated. The test problems are generated by changing the input parameters of arrival rate of demand, time distance of the ambulance and helicopter, and establishment cost of the trauma center and helicopter station.

Table 1. Random problem generation template

Number of the demand areas	[5, 10, 15, 30]

Number of the trauma centers	[2, 3, 5, 8]
Number of the helicopter stations	[2, 3, 5, 7]
Population of each demand area	Uniform [5×10^5, 7×10^5]
Time distance of the ambulance	Uniform [1.5×10^2 to 3×10^2]
Time distance of the helicopter	Uniform [1.5×10^2 to 3×10^2]
Establishment cost of each helicopter station	Uniform [2×10^4 , 3×10^4]
Establishment cost of each trauma center	Uniform [8×10^4 , 9×10^4]

5.2. Numerical Example

Consider a health care system where there are 10 demand areas, 3 potential trauma centers, 4 potential helicopter stations, and 5 helicopters. Five test problems are generated based on the method described in Subsection 5.1 to train an ANN. For each test problem, a set of solutions is randomly generated. Each solution shows the value of all the decision variables. The simulation model is set according to these decision variables. Then, the simulation model is run and the waiting time spent in each trauma center is determined as the third objective function of the model. According to Laurence and Peterson (1998), the number of experiments must be equal to $(i+h+o) \leq N \leq 10(i+h+o)$ to develop a capable ANN where, i is the number of the decision variables, h is the number of the hidden neurons, and o is number of the objective functions. In our case, i and o are equal to 10 and 1, respectively. The number of nodes in the hidden layer can be estimated by Equation (25) proposed by Chen and Yang (2002).

$$h = \frac{i+o}{2} \tag{25}$$

Consequently, $h = \frac{10+1}{2} \approx 6$ and so the number of the required experiments must be $17 \leq N \leq 170$. Totally, 50 random experiments are generated, each of which are simulated and the total cost and total population coverage are considered as the results of each experiment. MATLAB 2010 toolbox is used to generate an ANN. In this study, the ANN is a multi-layer perceptron (MLP). The learning of such networks is mainly accomplished through the error emission method. Fig. 7 shows the sample of a MLP network where the *Xi* is the decision variable, *Sj* represents the hidden neurons, and *Yi* is the objective function.

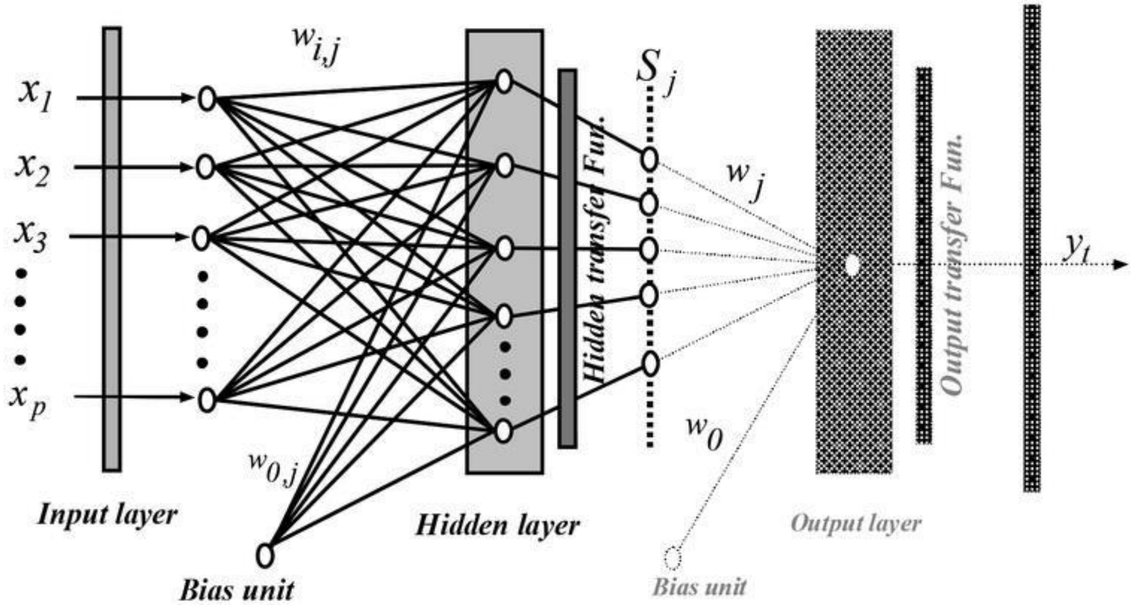

Fig 7. Structure of the MLP

It should be mentioned that the efficiency of neural network is determined by R^2 measure. The R^2 measure shows the ability of the neural network to predict the objective function based on the input factors. Fig. 8 demonstrates the values of R^2. The higher value of R^2 shows that the efficiency of the designed neural network is satisfactory and it could be used for predicting the values of the objective function. According to R^2=0.9771, the ANN model has been properly trained and has good quality predictions. Now, we have a trained ANN, which is capable of better estimation of the objective function values.

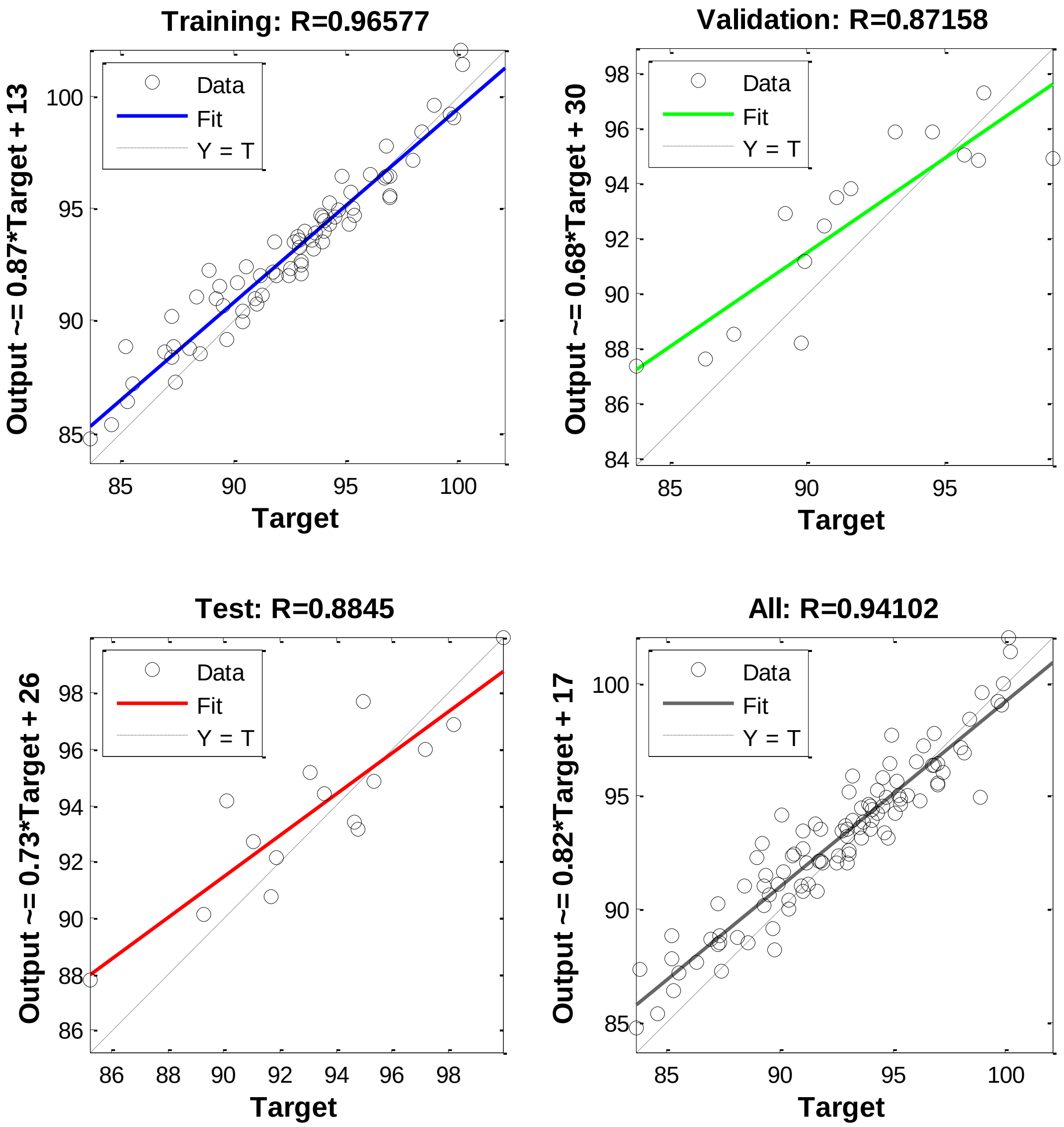

Fig 8. Efficiency of the neural network in the data fitness

5.3. Multi-Objective Metrics

The following metrics were selected to evaluate the performances of the multi-objective meta-heuristic algorithms:

1. Diversity (D): It measures the diversity of the Pareto front.
2. Spacing (S): It measures the standard deviation of the distances among solutions of the Pareto front.
3. Number of the Pareto Fronts (NPF): It shows the number of the solutions in the first Pareto front.

4. Time (CPU time): It is referred to the CPU time of running the algorithms until they are terminated.

5.4. Parameter Tuning

In this research, the parameters are tuned based on the Taguchi approach in order to regulate the algorithms for better performance as much as possible. The levels of the factors are first determined in order to apply the Taguchi method. Then, the appropriate experiments are designed and performed based on the number of factors and their levels. Since, there are 4 factors in 3 levels, therefore, the appropriate array must have at least 13 trials. The array L_{16} is used for the experiment. The multi-objective coefficient of variation (MOCV) is considered as the response for the experiments.

$$MOCV = \frac{S}{Diversity} \tag{26}$$

In the Taguchi design of experiment, the result of each experiment is converted into a Taguchi parameter defined as the signal-to-noise ratio (S/N). These S/N ratios of trials are averaged and for each parameter, the level with the maximum value of the S/N is selected as the optimum level. Table 2 shows all the levels for each parameter where the best values are highlighted.

Table 2. Algorithm parameters and their level

Solving methodology	Parameter		Range	Level 1	Level 2	Level 3	Level 4
NSGA-II	Popsize	Initial pop size	30-200	30	70	**140**	200
	P_c	Percent of crossover	0.7-0.9	0.7	**0.8**	0.85	0.9
	P_m	Percent of mutation	0.1-0.2	0.1	0.15	**0.17**	0.2
	Iteration	Number of generation	200-500	200	300	**400**	500
NRGA	Popsize	Initial pop size	50-200	50	100	**150**	200
	P_c	Percent of crossover	0.7-0.9	0.7	0.8	**0.85**	0.9
	P_m	Percent of mutation	0.1-0.3	0.1	**0.15**	0.2	0.3
	Iteration	Number of generation	200-600	200	**350**	500	600
NSWFA	M_0	Initial mass of the original flow	30-60	30	**40**	50	60
	V_0	Initial velocity of the original flow	10-30	10	**15**	25	30
	n	Upper limit on the number of subflows splitted from a flow	2-8	2	4	**6**	8
	Iteration	Number of Iteration	100-400	100	200	**300**	400

5.5. Computational Results

In this section, the performances of the proposed tuned multi-objective solving methodologies are evaluated and compared using the multi-objective metrics. Appendix A shows the results of applying the algorithms on the 32 test problems. It should be mentioned that while, in diversity and NOS metrics, bigger values are desired, smaller values are considered to be better for spacing and CPU time. Fig. 9 shows the performance of the algorithms in all the metrics.

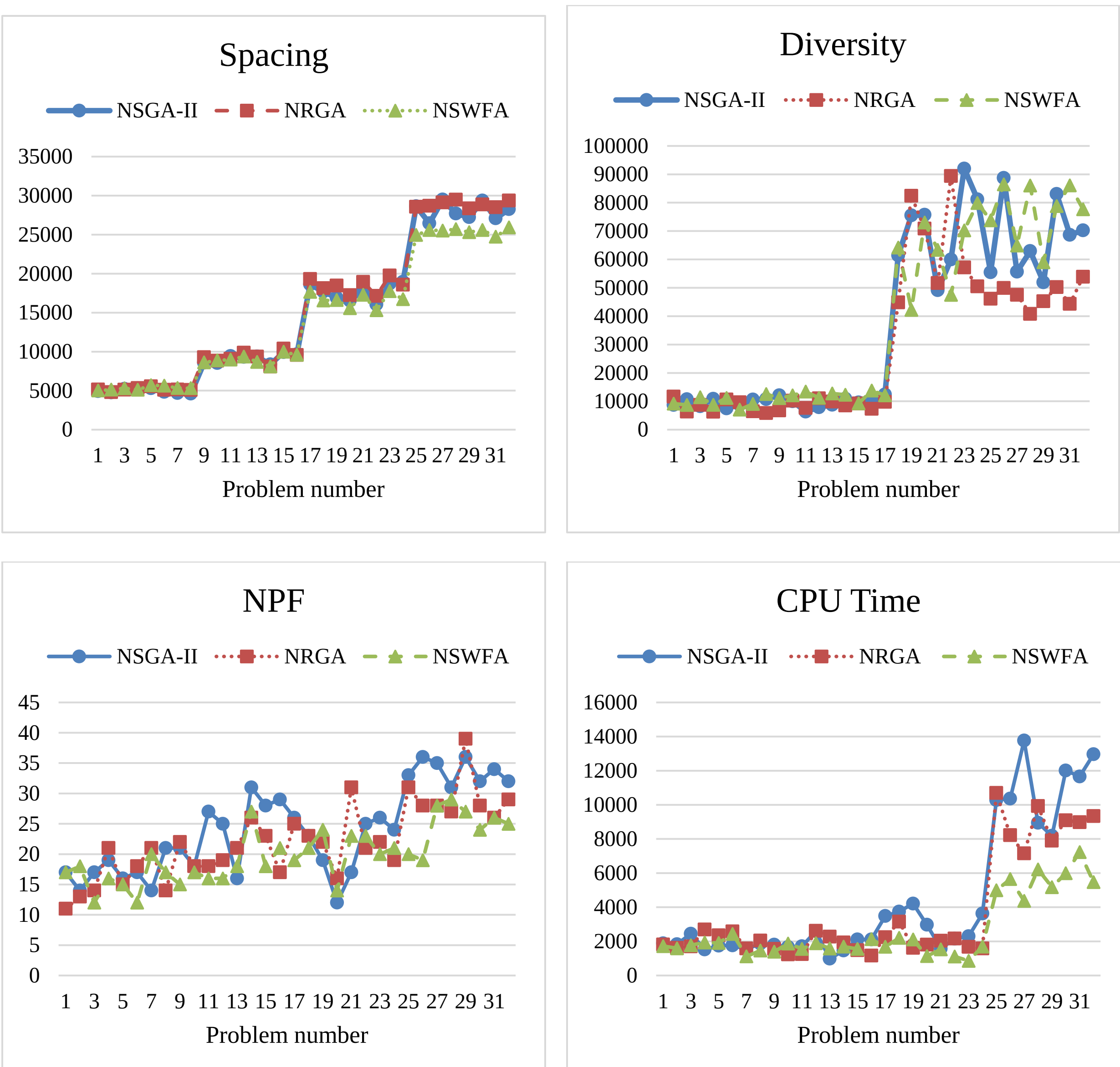

Fig 9. Comparison of the proposed algorithms according to the metrics of spacing, diversity, NOS, and CPU time

Fig. 10 shows a graphical illustration of the obtained Pareto fronts for the problems 12 and 20. As shown in Fig. 10, the solutions of NSWFA have achieved the best values in all the objective functions indicating the effectiveness of the NSWFA in terms of solution quality.

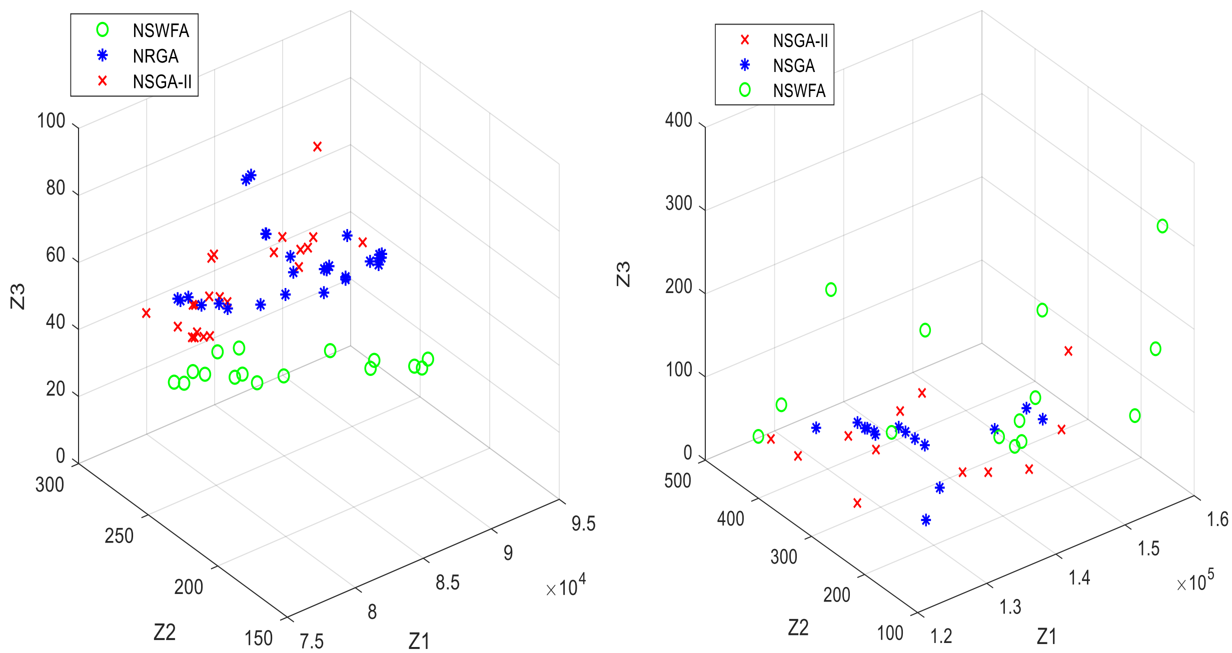

Fig 10. Visual presentation of the obtained Pareto-fronts for the test problems 12 and 20

The analysis of variance (ANOVA) test is used to statistically evaluate the algorithms. To this aim, the results of the algorithms in each metric are normalized based on the formulation of the relative percentage deviation (RPD):

$$RPD(i,j,k) = \frac{Alg_{sol}(i,j,k) - min_{sol}(k)}{min_{sol}(k)} \times 100 \tag{27}$$

In Equation (32), $Alg_{sol}(i,j,k)$ shows the value of algorithm i in the performance metric j in the problem number k and $min_{sol}(j,k)$ is the best value of the performance metric j in the problem number k between all the algorithms. Table 3 presents the ANOVA results so that, where there is a significant difference between the algorithms, the null hypothesis is rejected.

Table 3. ANOVA test results

Metric's name	F-value	P-value	Test results
Spacing	12.18	1.995e-5	Null hypothesis is rejected
Diversity	13.81	5.599e-06	Null hypothesis is rejected
NPF	10.57	7.30e-5	Null hypothesis is rejected
CPU Time	12.80	1.232e-5	Null hypothesis is rejected

Based on the statistical outputs presented in Table 3, there is a significant difference in all the performance metrics. Therefore, Tukeys' test is applied to statistically rank the algorithms for more investigation. Fig. 11 shows the results of the Tukeys' 95% confidence intervals. According to Fig. 11, in terms of the spacing metrics, it can be said that the NSWFA algorithm is considerably better than the other two algorithms. Also, NSGA-II can perform better than the NRGA. NRGA has the lowest diversity performance among all the algorithms. But, the NSWFA has the best efficiency in this metric. The NSGA-II had the better performance in terms of the NPF metric among the other algorithms as shown in Fig. 11. Regarding the CPU time index, the NSWFA was statistically better than the other two algorithms.

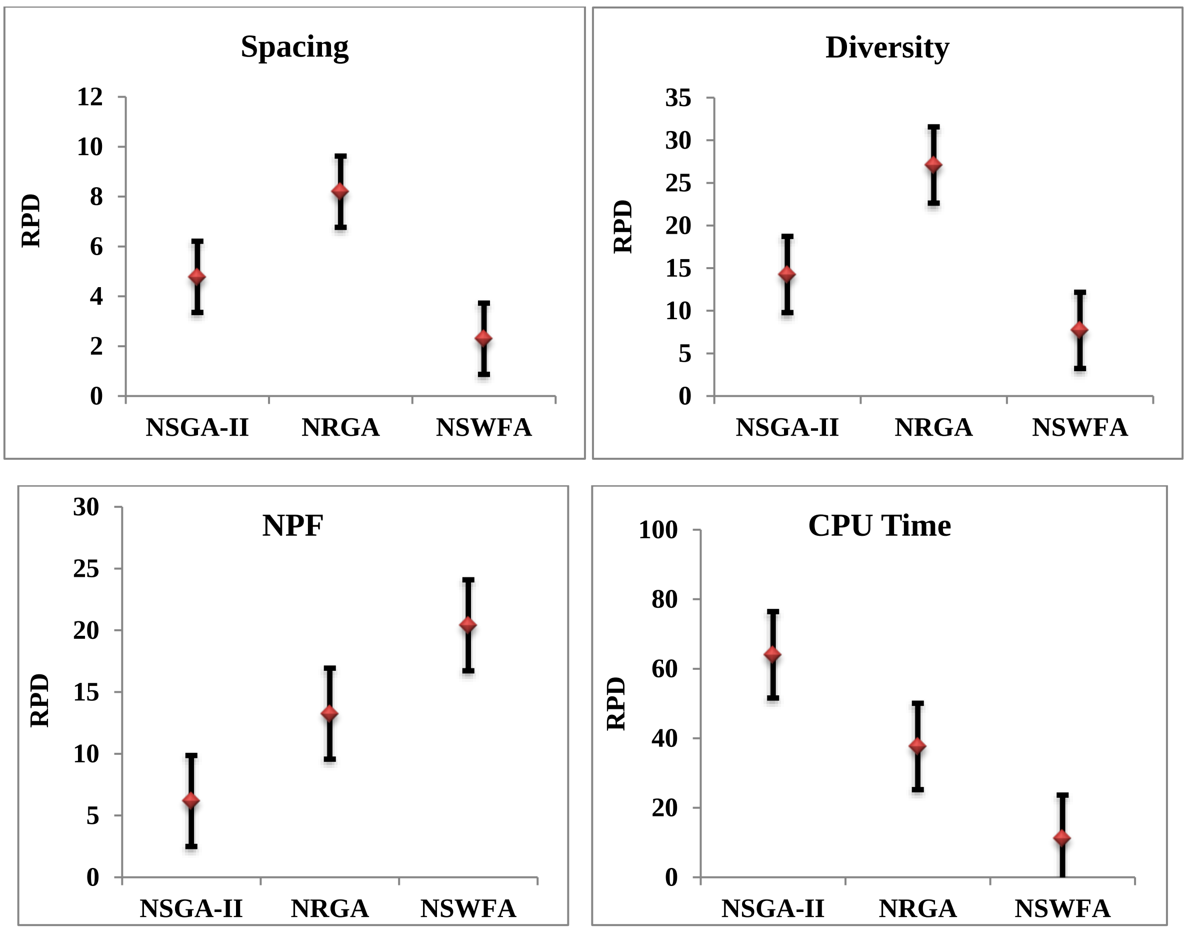

Fig 11. The simultaneous Tukeys' 95% confidence intervals for the metrics

6. Conclusion

In this paper, a new multi-objective mathematical model was developed and a new solution approach was presented for health care facility location problem where transferring time of the patients, demand and servicing time of the patients were considered as the stochastic parameters. In addition to minimizing the costs, the proposed model considered the time-based metrics, which is very important in the health care optimization problems. Considering the high complexity and interaction between the elements of the model, optimizing it by typical mathematical programing approaches was impossible. Consequently, a new approach was presented. The proposed approach was a hybrid method encompassing the simulation, ANN, and multi-objective metaheuristic algorithm. A novel multi-objective metaheuristic algorithm called the NSWFA was applied. Some experimental computations were accomplished to evaluate the effectiveness of the proposed NSWFA. In these experiments, the proposed algorithms were statistically compared with two well-known meta-heuristic algorithms using 32 test problems through the four multi-objective metrics. Results showed that the algorithm generally acts more effectively than the other two algorithms. For future researches, other features, such as transferring point of the patient, allocation of the buffers between the machines, and unequal area of the machines can be incorporated in the presented model. Also, developing a regression model to estimate the third objective function and applying a non-linear solver to optimize the model are among the remarkable directions for the future researches. It is also suggested to evaluate the proposed NSWFA in other fields of operations research like the facility layout problems.

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Appendix A: Multi-objective performance measures obtained for each algorithm

	Spacing			Diversity (D)			NPF			CPU time		
	NSGA-II	NRGA	NSWFA	NSGA-II	NRGA	NSWFA	NSGA-II	NRGA	NSWFA	NSGA-II	NRGA	NSWFA
P1	4933	5125	5019	8702	11625	9151	17	11	17	1877	1814	1721
P2	4752	4785	5124	10709	6336	8733	14	13	18	1834	1620	1595
P3	5231	5112	5304	8237	8694	11419	17	14	12	2447	1704	1773
P4	5294	5313	5118	10873	6282	8774	19	21	16	1521	2695	1936
P5	5288	5540	5696	7471	10628	11186	16	15	15	1743	2350	1906
P6	4827	5089	5632	8825	9641	7052	17	18	12	1761	2577	2438
P7	4673	5121	5341	10590	6458	9049	14	21	20	1534	1585	1121
P8	4599	5079	5307	10788	5817	12561	21	14	17	1871	2046	1462
P9	8583	9273	8611	12107	6762	11146	21	22	15	1801	1561	1391
P10	8527	8824	8879	10011	10223	12095	18	18	17	1668	1229	1875
P11	9425	9070	8974	6347	7635	13434	27	18	16	1713	1248	1557
P12	9314	9858	9375	7892	11011	11224	25	19	16	2595	2615	1891
P13	9365	9339	8680	8753	9871	12756	16	21	18	984	2277	1585
P14	8356	8076	8100	10940	8525	12305	31	26	27	1462	1929	1693
P15	9930	10376	9974	9592	9253	9259	28	23	18	2109	1476	1556
P16	9641	9559	9586	9717	7348	13770	29	17	21	2112	1165	2125
P17	18623	19291	17651	12426	9812	12120	26	25	19	3483	2237	1681
P18	17685	18115	16554	61408	44849	64184	23	23	21	3742	3149	2203
P19	16911	18448	16603	75518	82398	42240	19	22	24	4212	1616	2110
P20	16529	17230	15566	75726	70858	73017	12	16	14	2969	1807	1137
P21	17911	18928	17270	49135	51673	63370	17	31	23	1578	2038	1534
P22	16044	17135	15310	60004	89401	47509	25	21	23	2150	2164	1112
P23	18709	19744	17743	91992	57157	70267	26	22	20	2299	1678	861
P24	18934	18571	16712	81193	50485	79888	24	19	21	3630	1584	1692

P25	28626	28557	24941	55463	46126	73798	33	31	20	10280	10694	4995
P26	26444	28684	25576	88702	49939	86470	36	28	19	10364	8217	5649
P27	29478	29106	25516	55656	47519	64925	35	28	28	13764	7152	4371
P28	27702	29467	25691	62944	40774	86049	31	27	29	8947	9927	6217
P29	27225	28355	25313	51939	45270	59029	36	39	27	8183	7899	5175
P30	29360	28854	25592	83085	50247	78813	32	28	24	12009	9091	5993
P31	27087	28496	24721	68679	44311	86127	34	26	26	11663	8977	7231
P32	28248	29354	25924	70238	53891	77671	32	29	25	12963	9335	5473