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Back to the actual future

Jacek Wawer[1] · **Alex Malpass**[2]

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Abstract The purpose of the paper is to rethink the role of actuality in the branching model of possibilities. We investigate the idea that the model should be enriched with an additional factor—the so-called Thin Red Line—which is supposed to represent the single possible course of events that gets actualized in time. We believe that this idea was often misconceived which prompted some unfortunate reactions. On the one hand, it suggested problematic semantic models of future tense and and on the other, it provoked questionable lines of criticism. We reassess the debate and point to potential pitfalls, focusing on the semantic dimension of the Thin Red Line theory. Our agenda transcends the semantics, however. We conclude that semantic considerations do not threaten the Thin Red Line theory and that the proper debate should be carried in the domain of metaphysics.

Keywords Branching time · Thin Red Line · Future contingents · Actualism

The paper is structured as follows: In the first section, we motivate introduction of the Thin Red Line (TRL) and explain how it may be applied for semantic purposes. In the second, we outline the semantic objection raised against TRL theory by Belnap

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✉ Jacek Wawer
jacek.wawer@uj.edu.pl

Alex Malpass
a.p.malpass@gmail.com

[1] Department of Philosophy, Jagiellonian University, ul. Grodzka 52, 31-044 Kraków, Poland

[2] Department of Philosophy, University of Bristol, Cotham House, Bristol, UK

and Green (1994) according to which TRL theory treats temporal and modal operators inadequately. We explain how the objection may be overturned by a simple postsemantic maneuver—to interpret a sentence used at the time and the history of the context. The postsemantic move, which we ultimately recommend, appeals to the notion of the history of the context (which we identify with the actual history). In the third section we focus on a particular line of criticism of the simple postsemantics—the postsemantic problem of Belnap et al. (2001). In the next three sections, we discuss two postsemantic responses to the problem—by MacFarlane (2003) and by Malpass and Wawer (2012)—and outline some of their weak points. In the last section, we reassess and undermine the postsemantic problem. We argue that it is underlined by metaphysical assumptions alien to the TRL theory. When TRL is stated in its proper setting of modal actualism, the postsemantic problem has no teeth and the simple theory introduced in the second section turns out to be free of both semantic and postsemantic difficulties.

1 Why do we need the Thin Red Line?

The reasons supporting the TRL are twofold—metaphysical and semantic. As we shall see, these two domains are closely connected and the theoretical choices made in one domain usually affect the decisions made in another.

Let us explore metaphysics first. The branching model is meant to represent the *possible* ways in which the world may develop (for detailed discussion of the model and its significance see e.g., Belnap et al. 2001; Rumberg 2016; Wawer 2016). As Aristotle noticed, the notion of possibility is intelligible only if contrasted with the notion of actuality. Thus, the branching representation of possibilities is incomplete, unless paired with a representation of actuality. This idea was concisely phrased by Belnap et al. (2001):

> "Sure, there are many things that *might* happen, but only one of them is what really *will* happen. (*Qué será, será.*)" If one has objective leanings at all, it is easy to feel a need to treat both conjuncts with *equal* objectivity. (Belnap et al. 2001, pp. 160–1)

Interestingly enough, Belnap et al. (2001) do not treat the second conjunct with equal objectivity (we will return, and question, one of their reasons in due course). Nonetheless, they describe a robust and solid intuition which we call the "actualistic insight." The historical roots of this idea can be found in medieval treatments of future contingents by St. Anselm, William of Ockham, and Richard of Lavenham. The intuition was incorporated into the branching setting with the idea that one of the branches of the tree is the actual branch—the branch that represents what did, does, and will happen. Prior opposed the view, but it was endorsed by, among the others, Rescher and Urquhart (1971), McKim and Davis (1976) and Øhrstrøm (1981).

The second reason to evoke the Thin Red Line (TRL) is of semantic nature. The TRL is meant to provide non-relative truth values to sentences regarding the future. To explain the details, we need to outline the basics of Ockhamist semantics first.

Ockhamism was proposed by Arthur Prior (1966, 1967) and formally clarified by Richmond Thomason (1970). The key feature of the semantics is that sentences are

evaluated at pairs consisting of a possible moment m (a point on the tree of possibilities) and a possible history h (a line through the tree containing the moment). For example,

- The sentence "It will be the case that ϕ" is true at m/h iff "ϕ" is true at some later moment m' in h;
  $m/h \models F\phi$ iff $\exists_{m'}(m' \in h \,\&\, m' > m \,\&\, m'/h \models \phi)$.
- The sentence "It is inevitable that ϕ" is true at a pair m/h if and only if "ϕ" is true at every h' passing through m;
  $m/h \models \Box\phi$ iff $\forall_{h'}(m \in h' \Rightarrow m/h' \models \phi)$.

We should mention that the term "Ockhamism" and its variants are highly ambiguous in the contemporary literature. In the more technically oriented pieces, it is usually used to refer to the formal semantics initially proposed by Prior. Quite often, however, it is used to indicate the general idea that sentences regarding the contingent future may be true (see, e.g. Rosenkranz 2012; Iacona 2014). These two need not go hand in hand. For example, Belnap et al. (2001) endorse Ockhamism in the first sense, while they reject it in the second. Adding to the confusion, Peter Øhrstrøm argues that Prior's Ockhamism does not embody the spirit of William of Ockham's philosophy and he distinguishes between "Ockhamistic" and "Ockham" semantics (Øhrstrøm 2016, sec. 5).

The term TRL-semantics might also be confusing. It might refer to a specific formal semantic project, competing with Prior's Ockhamism, that incorporates a representation of the Thin Red Line to define future tense in the branching models (e.g., Øhrstrøm 2009, defends TRL-semantics in this sense). It might also refer to the general idea that we should somehow appeal to the metaphysical notion of the Thin Red Line when assessing future contingents. The second sense of "TRL-semantics" is similar to the second sense of "Ockhamism" mentioned above. In this paper, we will use the term "Ockhamism", and "Ockhamist semantics", in its original sense—to refer to the formal semantics defined on the branching models by Arthur Prior, and outlined above. The term "TRL-semantics" (also "trl_{fcn}-semantics) will be used to designate the formal projects akin to Øhrstrøm's. To avoid further confusion, we decided to coin a completely new term—"futurism"—to name our own proposal. It might be seen (spoiler alert!) as a version of TRL-semantics, in the second sense, based on Ockhamism, in the first sense.

The Ockhamist semantics has stood the test of time and it is still endorsed by many theorists using the branching model. It owes a lot of its success to the composite nature of the semantic index. Firstly, since it consists of a moment/history pair, it admits natural and independent definitions of temporal and modal operators (the first shifts the moment of the index and the second the history of the index). Secondly, since histories are composed of moments, the dual index allows for a smooth interaction of modal and temporal operators. The source of success of Ockhamism, however, has also become its major curse.

It was clear early on that Ockhamism generates certain controversy (the problem is described as early as in Thomason 1970). If we try to interpret a sentence used at a particular moment, the semantic machinery requires that we pick a particular history, but there seems to be no reason to privilege one history passing through a moment

over another. Therefore, the extension of an expression regarding the future is relative to a completely arbitrary choice of the history parameter of the index.

The standard answer to this problem in the framework of intensional semantics (see e.g. Kaplan 1989) is to postulate that the context of the utterance provides (or, as we shall say, *initializes*) the relevant semantic index (the moment and the history in our case).[1] The typical procedure has been questioned, however, in the branching setting (see in particular Belnap et al. 2001; MacFarlane 2003, 2014). It has been argued that in face of indeterminism, the context of utterance does not initialize a single possible history as *the history of the context*. If it does not, however, we encounter a problem of *initialization failure*: if we want to use Ockhamism, we need to begin the process of interpretation at a particular semantic index, but no such index is initialized by the context of utterance.

To address the problem, John MacFarlane (2003) introduced an intermediate level of linguistic analysis which he calls *postsemantics*. Its role is to dictate how to use Ockhamism to assess a sentence used at a context.[2] More generally, the role of postsemantics is to systematically relate the everyday notion of truth of a sentence used at a context (which we symbolically encode as $\Vdash$) to a more technical notion of Ockhamist truth of a sentence at a semantic index (which we symbolize by $\models$).

This brings us to the second, more technical source of motivation for the TRL. Given that only one of the histories is the actual history, it is evident that it is the history that should be used for semantic purposes. Actuality breaks the tie among the possible histories. Thus, thanks to the actualistic insight encoded by TRL, we can respond to the initialization failure and return to the "flat" postsemantics advocated by David Kaplan and simply evaluate an expression used at a context at *the* (present) moment and *the* (actual) history of the context.

Although this idea might seem as the most natural way to tackle the initialization failure, it has been treated with considerable distrust. It was argued that introduction of the TRL generates considerable formal obstacles that disqualify this semantic theory. We do not believe that future looks so dim for the TRL, but let us first briefly recapitulate those worries.

2 Semantic objection to TRL

The most straightforward application of the actualistic insight into the branching model binds the semantic interpretation of the future tense operator F to the TRL.

[1] The context need not be identified with one of the semantic indexes, like a moment/history pair. It also need not be equated with a moment at a tree, as is sometimes done in the branching setting (see, e.g. Belnap et al. 2001, ch. 8). We carefully separate contexts from indexes to avoid any unnecessary confusion. We tend to think, in line with Kaplan, that a context is a representation of a concrete situation at which an utterance is made, while histories and moments serve as auxiliary formal parameters needed for interpretation of modal and temporal operators. They represent possibilities and temporal elements thereof.

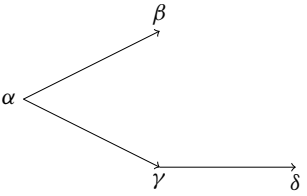
[2] MacFarlane (2014 ch. 9) describes several postsemantic strategies available in the branching setting, including TRL, Peirceanism, three-valued semantics, supervaluationism, and his own relativism (the list is extended in chapter 4 of Wawer 2016).

Definition 1 The sentence "It will be the case that ϕ" is true at moment m iff the sentence "ϕ" is true at later moment m', on the TRL;
$m \models^{TRL} F\phi$ iff $\exists_{m'}(m' > m \ \& \ m' \in TRL \ \& \ m' \models^{TRL} \phi)$.

At first sight, it is exactly what we wanted to achieve. The truth value of a sentence in the future tense is not relative to the modal parameter (the history parameter); rather, it depends solely on what will actually happen. Thanks to existence of the TRL, we do not need to semantically relativize the truth value of a sentence to a history. So, we do not need to initialize the history parameter and the initialization failure does not arise.

This theory is faulty, however, on the semantic level. The difficulty was realized already by Richmond Thomason (1970, pp. 270–271) who, having briefly considered a version of TRL semantics, made the following comment:

> Suppose that β is in the real future of α; then what of the point γ? It isn't in real time, and yet in order to evaluate tensed formulas at γ we must provide it a real future. (Thomason 1970, p. 271)



The worry is that since "real time" is missing at some moments of the tree, it is hard to analyze operator F at such moments. At first, the objection might seem artificial. After all, why should we ever need to evaluate tensed formulas at points which are not in the real time? The answer is that in a compositional semantics, the future operator might be embedded in scope of modal operators which shift the point of evaluation outside of the TRL. This process is well illustrated by the example studied by Belnap and Green:

> The coin will come up heads. It is possible, though, that it will come up tails, and then later (*) it will come up tails again (though at that moment it could come up heads), and then, inevitably, still later it will come up tails yet again.
> The trouble is that at (*) the example says that tails will happen, not merely that it might, whereas the explanation of the future tense given above presupposed that the moment of evaluation was in the TRL. (Belnap and Green 1994, p. 379)

To avoid this difficulty, most proponents of the TRL followed Thomason's imperative that "we must provide γ a real future" and postulated "real futures" of the moments outside of the actual course of events (see e.g. McKim and Davis 1976; Barcellan and Zanardo 1999; Øhrstrøm 2009). We think of this as a turn away from Ockhamism and towards 'Molinism'. This decision marks an important shift in the approach to the TRL. In face of the semantic difficulties, the TRLers abandon the natural idea of the *Absolute Thin Red Line* according to which exactly one of the possibilities is being actualized in the course of time and move to a metaphysically more contentious notion of a *Functional Thin Red Line* which implies that even non-actual moments have (quasi)actual futures. Formally, the technique is very simple, we just replace single TRL with a TRL-function $trl_{fcn}: M \mapsto Hist$ such that $\forall_m m \in trl_{fcn}(m)$.

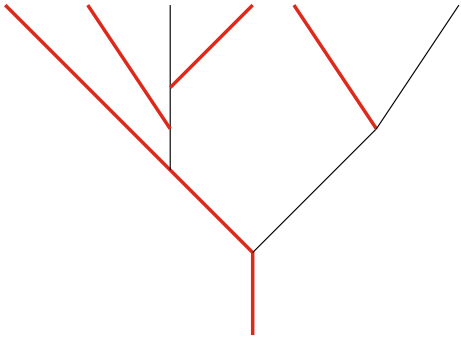


Fig. 1 The thick lines indicate the actual branches of the tree

Intuitively, $trl_{fcn}(m)$ is meant to designate the history that is actual at moment m. With trl_{fcn} at hand, we can interpret the future operator at any moment of the tree:

Definition 2 The sentence "It will be the case that ϕ" is true at moment m iff the sentence "ϕ" is true at later moment m', in the $trl_{fcn}(m)$;
$m \models^{f\text{-}trl} F\phi$ iff $\exists_{m'}(m' > m \ \& \ m' \in trl_{fcn}(m) \ \& \ m' \models^{f\text{-}trl} \phi)$.

Introduction of trl_{fcn} allows for a more appealing formal system, but it forces the TRLers to sacrifice part of their conceptual chastity. When they initially incorporated the notion of the actual course of events into the branching setting, they explicitly argued that there is just *one* possible course of events which gets actualized as time goes by. The actualist intuition supports this verdict. However, in face of the formal problem explicated by Belnap and Green, they gave up this assumption and consented to the central idea of Molinism; i.e. that each moment in the branching world has its own actual future. This idea can be pictorially represented by the tree depicted in Fig. 1.

In such a model, we use the real future to interpret the future tense, but if it is missing we can always use the "second grade" real futures. The future that is not "really" real, but somehow more real than the alternatives. We sympathize with Thomason's comments on this idea:

> At this point we begin to lose track of what a "real future" is, and plainly it would be better to just return to a linear conception of time. (Thomason 1970, p. 271)

In fact, it is exactly what we recommend as a promising path for the TRL theory (see Sect. 7), especially that trl_{fcn} has proven to be problematic not only on the conceptual, but also on the formal level. It turned out that although the functional semantics of the future tense does provide an interpretation of the future tense at an arbitrary moment of the tree, the interaction of the future operator with other tense and modal connectives is highly problematic. Numerous fixes were proposed, but none seem entirely satisfactory (see Wawer 2014, section 2, for a detailed exposition and criticism of trl_{fcn} semantics).

We suggest, therefore, that TRL *should not* be incorporated on the semantic level (e.g. to interpret operator F), but that it should be used *postsemantically*, as a tool linking the truth at context with the truth at index. It means that TRL should be exploited

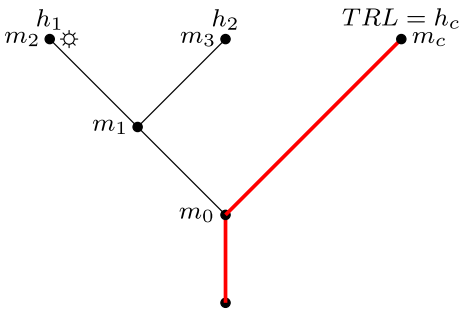


Fig. 2 Moment m_c is the moment of the context, it is a part of the history of the context (the TRL). History h_1 branches off the TRL at moment m_0, before moment m_c. At moment m_2 following moments m_0 and m_1 in history h_1, it is sunny

to provide *the history of the context*, and thus to address the initialization failure, but having accomplished this goal, it should leave the stage. While the relevant history is initialized, the semantic computation should proceed along the standard Ockhamist rules. You can phrase this idea in form of a "flat" postsemantic definition:

Definition 3 *(Futurism)* The sentence "ϕ" is true at context c iff ϕ is true at the moment of the context m_c (the present moment) and the history of the context h_c (the actual history).

$$c \Vdash \phi \textit{ iff } m_c/h_c \vDash \phi$$

Using this strategy, we can stick to the intuitive Ockhamist semantics and also hold on to the simple linear conception of time (there is only one history actualized in time). Therefore, we can avoid both conceptual and technical problems of the trl_{fcn}-semantics. A very similar proposal, though expressed using different vocabulary, was recently entertained by Andrea Iacona (2014). We discuss his ideas in Sect. 6 and 7, and ultimately recommend (our interpretation of) his proposal as a viable version of the TRL theory.

Te get a better grip on the idea, let us test it with a version of the example discussed by Belnap and Green (using the model represented in Fig. 2). Consider the sentence "It might have been the case that it would be sunny" encoded by $P\Diamond FF(sunny)$, used at context c. Under the postsemantics introduced above, $c \Vdash P\Diamond FF(sunny)$ iff $m_c/h_c \vDash P\Diamond FF(sunny)$. Thus, a sentence is simply true at a given context if it is *Ockhamist-true* at the current moment, with respect to the actual history. Then, we proceed with the standard Ockhamist computations. For example, $m_c/h_c \vDash P\Diamond FF(sunny)$ iff there is an earlier moment (like m_0) such that there is a history passing through that moment (like h_1), such that there is a later moment in this history (like m_1) at which $F(sunny)$ is true with respect to that history. Observe that the semantic problem of Belnap and Green does not arise at this point, since we do not need the TRL to interpret the future-tensed sentence $F(sunny)$ at the changed semantic index m_1/h_1. We use the ordinary Ockhamist rule and evaluate sentence $F(sunny)$ with respect to history h_1 which has been introduced by the modal operator $\Diamond$ at moment m_0. Therefore, the problem of interpreting the future operator outside of the history of the context (the actual history) does not arise, and we can easily

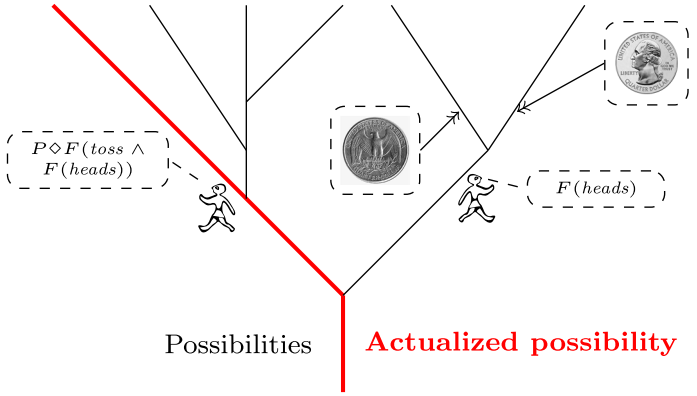


Fig. 3 The sentence $P\Diamond F(toss \wedge F(heads))$ is uttered at a context in TRL. At an earlier moment, a possibility branches off the TRL which later splits into two histories, one where the coin lands heads and the other where it lands tails. The sentence $F(heads)$ is uttered at the non-actual context, just before the tails-history and the heads-history split

compute that the sentence "It might have been the case that it would be sunny" is true at context c.

3 Postsemantic objection to TRL

This line of thought must have been intuited by the critics of TRL, since their "formal" complaint is considerably different in (Belnap et al. 2001) than it is in (Belnap and Green 1994):

> The TRL theory sounds all right, but it is not. It has the "logical" defect that it gives no account whatsoever of predictive speech acts occurring at moments of use that lie off the TRL and is by so much useless. (Belnap et al. 2001, p. 162)

The authors are no longer interested, at least not directly, in the analysis of an embedded future tense operator. Instead, they focus on *speech acts* which happen to occur at contexts that lie outside of the actual course of events. This is a subtle, but important difference. They are asking a paradigmatically postsemantic question in the sense of (MacFarlane 2003)—how to use semantic machinery to analyze speech acts that happen not to take place on the TRL?

To illustrate the difference between the semantic and postsemantic objections, let us have a look at the model depicted in Fig. 3 and consider the sentence:

> It was possible that the coin would be tossed and it would land heads. $(P\Diamond F(toss \wedge F(heads)))$

The properly semantic problem regards interpreting the sentence $P\Diamond F(toss \wedge \underline{F}(heads))$ used at an actual context, with attention focused on the underlined occurrence of operator F. The worry is that when the first three operators $P\Diamond F$ shift the semantic index to a non-actual moment and a non-actual history, the embedded sen-

tence $F(heads)$ will end up false or meaningless at the new index. The postsemantic question concerns assessment of the sentence $F(heads)$ *used at the non-actual context* when the coin is being tossed. The picture above clearly illustrates that these are two distinct problems. On the one hand, the sentence $P\Diamond F(toss \wedge F(heads))$ is evidently true, as used at the actual context. On the other hand, it is entirely unclear, if we should call the sentence $F(heads)$ true, while used at the non-actual context.[3]

To put things differently, it is a distinct matter to answer whether *it is true that* ϕ *is possible* than to answer whether ϕ *true, when used in a possible context*. We are by no means the first ones to raise the distinction. The *locus classicus* is (Evans 1985), where a particular semantic theory of temporal logic, which is called T_3, is contrasted with the standard semantics of modal logic:

> The semantic value a complex tensed sentence possesses in a context is, according to T_3, a function of the semantic value which the embedded sentence would possess in another *context*; this is not true of the semantic values of complex statements of a modal logic, or indeed of any other known logic. (Evans 1985, p. 361, emphasis ours)

Evans stresses here that the standard semantics of modal operators *does not* require evaluation of embedded sentences in non-actual contexts, it is enough to shift a relevant parameter of the semantic index. The message for the current discussion is that it is possible to answer Belnap and Green's *semantic* objection without answering Belnap et al.'s (2001) *postsemantic* objection. In the end, it is precisely the strategy we recommend. Nonetheless, there were at least two attempts to address the postsemantic challenge directly. Let us present them briefly to learn from their struggle.

4 TRL-function postsemantics

The postsemantic challenge was first addressed by John MacFarlane (2003). In fact, he was the first to clearly distinguish the semantic and postsemantic dimension of Belnap et al.'s (2001) objections (the first concerns the proper semantic definitions of logical connectives—i.e. the truth at an index—while the second regards the relationship between truth at context and at index). MacFarlane agreed with Belnap et al. (2001) that the absolute TRL is not a promising outset for the accurate postsemantic theory, since the actual history is not available at a non-actual context. He noticed, however, that one can use trl_{fcn} in a non-standard manner: to postsemantically *initialize* the history of evaluation at a context rather than to semantically *interpret* the future tense operator.

[3] There is still another problem in the vicinity: how to interpret an actually used sentence like "Jack could have truly said 'The coin will land heads.'"? We believe that it should be understood more or less as "It could have been that: Jack says 'The coin will land heads' and the coin later lands heads," which is easily interpreted without shifting the context. For discussion of formal details and subtle intricacies of this idea, see (Wawer 2016, sec. 6.3.6)

Definition 4 The sentence "ϕ" is true at context c iff it is true at the moment of the context m_c and history $trl_{fcn}(m_c)$.

$$c\|\overset{f\text{-}trl}{\vdash}\phi \textit{ iff } m_c/trl_{fcn}(m_c) \models \phi$$

The definition kills two birds with one stone. First, it answers the postsemantic challenge (i.e. it accounts for prediction made at arbitrary contexts). And second, it avoids the the typical *semantic* problems of TRL theories:

> This proposal is not touched by Belnap and Green's semantic arguments against the use of a thin red line (*Facing the Future*, pp. 160–170). It uses the very same semantics proper as Belnap and Green endorse, and appeals to the thin red line only in the postsemantics. (MacFarlane 2003, p. 330, n. 10)

The lesson from MacFarlane is that we do not need to modify the standard Ockhamist semantics to subscribe to TRL theory. In particular, the semantics of the future tense operator can remain *history dependent*, but thanks to trl_{fcn} postsemantics, this feature does not generate the initialization failure. When the sentence is used at context c, we begin the process of evaluation at $m_c/trl_{fcn}(m_c)$. Then, we systematically apply the standard Ockhamist rules in semantic analysis. Due to this simple maneuver, we can leave behind, at a single stroke, all the purely semantic problems of TRL-theory.

MacFarlane's postsemantic proposal has been appreciated by Øhrstrøm, the major champions of the semantic version of the Thin Red Line (see Øhrstrøm and Hasle 2015, sec. 5.3). Nonetheless, MacFarlane himself rejected trl_{fcn} theory for *postsemantic* reasons. In his view, it improperly models counterfactual retrospective assessments of future contingent claims (see MacFarlane 2014, pp. 209–211). Instead of discussing the details of MacFarlane's objection let us quickly examine our own earlier theory (Malpass and Wawer 2012), which also addressed the postsemantic challenge.

5 Supervaluational Thin Red Line

Malpass and Wawer, our previous selves, firmly rejected the idea of "alternative actual futures," i.e., actual futures of merely possible moments, that was incorporated into functional TRL theory. We argued that there should be one, and only one, actual course of events in the entire branching model—the unique possible course of events that gets actualized in the course of time.

Aware of the postsemantic objection against the TRL, we nonetheless tried to account for predictions made at non-actual moments. We came up with a kind of patchwork theory of predictions. In simple terms, their idea was that:

- A prediction that is actually made is true iff what it says will actually happen.
- A prediction that could have been made is true iff it would have been true, if it had been made.

We later argued that the question about what will happen is entirely different from the question about what would happen since the first, but not the second, is settled by the flow of time. Since there is no comparable metaphysical factor to settle contingent

counterfactuals, all we can do is to appeal to the modal dimension and consider what would possibly, inevitably, or probably happen in non-actual circumstances. Very similar observation was previously made by Robert Adams:

> The categorical predictions involved in simple foreknowledge may be true by corresponding to future events (…). But in the case of counterfactuals of freedom that are about non-actual creatures or have false consequents, the conditionally predicted actions are not there to be corresponded with because they never actually occur. The truth of counterfactuals is commonly grounded in a logical or causal necessitation of the consequent by the antecedent. (Adams 1991, p. 345)

Malpass and Wawer expressed the difference in postsemantic terms. In their view, actual predictions of contingent events are either true of false, while non-actual predictions of contingent events are neither true nor false. To formally express the difference, they designed a theory which they called Supervaluational Thin Red Line (STRL). It is "Supervaluational," since it incorporates elements of supervaluationism at non-actual contexts and it is "Thin Red Line," since it stresses the special status of the actual predictions. Remember that in supervaluationism:

Definition 5 The sentence "ϕ" is true at context c iff it is true at the moment of the context m_c and at all histories passing through m_c. (Thomason 1970)

$$c\|\!\overset{sup}{\sim}\phi \text{ iff } \forall h(m_c \in h \Rightarrow m_c/h \models \phi).$$

In contrast, in STRL theory the truth value of a prediction at a moment is "sensitive" to the existence of the TRL. If a prediction is made at an actual moment, STRL it behaves as any standard TRL theory and attaches the truth value of the prediction to what will actually happen. If it is made at a non-actual moment, its truth value depends on what is possible and necessary at the non-actual moment, just as it does in supervaluationism. Our idea can be encoded by the following postsemantic definition:

Definition 6 The sentence "ϕ" is true at context c iff it is true at the moment of the context m_c and at TRL, or at all histories passing through m_c.

$$c\|\!\overset{strl}{\sim}\phi \text{ iff } \big(m_c/TRL \models \phi \text{ or } \forall h(m_c \in h \Rightarrow m_c/h \models \phi)\big).$$

This way, we answered the worry that the "TRL theory (…) gives no account whatsoever of predictive speech acts occurring at moments of use that lie off the TRL" (Belnap et al. 2001, p. 162).

6 Iacona's objections

The STRL theory prompted a critical response by Andrea Iacona (2014). According to Iacona, the most problematic feature of STRL theory is that it presupposes a notion of a model where one, and only one, of the histories is distinguished as *the actual history* (TRL). His criticism is encapsulated in the following comment:

> Such a semantics seems incapable of making sense of the counterfactual hypothesis that one of the courses of events that include a given non-actual moment is the actual course of events. (Iacona 2014, p. 2642).

One might wonder why Iacona wants to make a hypothesis that a course of events which contains a non-actual moment is the actual course of events. His rationale is the following:

> If the actual history is fixed once and for all in the model, it turns out that sentences lose some basic semantic properties when they are evaluated at non-actual moments. However, there is a sense in which one may expect that all moments are alike with respect to those properties, namely, the sense in which every moment is actual from its own point of view. (Iacona 2014, p. 2641)

Thus, Iacona rephrases Belnap and Green's *semantic* objection and insists (rightly, we believe) that the semantic properties of linguistic expression should not be sensitive to the TRL, i.e. the meaning of the language should not differ in actual and non-actual circumstances. He observes, though, that according to STRL some moments are distinguished as *absolutely* actual and argues that the distinction later affects the semantic features of various expressions.

We believe that the problem can be averted, if one recognizes semantic and postsemantic dimension of STRL theory.[4] As far as semantics is concerned, STRL satisfies Iacona's requirement that every semantic index has "equal rights" and that interpretation of linguistic expressions does not depend on whether they are evaluated in actual circumstances. Malpass and Wawer, just as MacFarlane, accept ordinary Ockhamism and utilize the Thin Red Line in postsemantics only. Therefore, their theory does not fall pray to the semantic objections (unless those are objections undermining Ockhamist semantics in general).

Let us exemplify the general point and study one specific worry raised by Iacona. He writes that in STRL theory

> $\vee$ is truth-functional at actual moments and non-truth-functional at non-actual moments. (Iacona 2014, p. 2640)

From this, he concludes that that "the meaning of a logical constant varies with the moment of evaluation" (Iacona 2014, p. 2640) which is clearly unacceptable.[5] This conclusion is not warranted, however, since the meaning of a sentential con-

[4] We should be clear that in our previous paper we did not make the distinction between semantics and postsemantics that we stress in this one. We also gave the reader ample reason for alternative interpretations. For example, we referred both to moments and to moment/history pairs as "points of evaluation" which might suggest, and indeed suggested to Andrea Iacona, that we were interested merely in properly semantic considerations. Nonetheless, given that our overarching purpose was to address the postsemantic challenge of Belnap et al. (2001), we charitably assume that STRL should be interpreted as a postsemantic theory.

[5] Interestingly, Malpass and Wawer explicitly agree with Iacona's judgment. In the more technical part of their paper, they reject a theory that require distinct semantic treatment of a truth operator at actual and non-actual moments. With regard to such theory they write: "the meaning of the language seems to change from one point to another. (…) This is an idea which we feel reluctant to accept" (Malpass and Wawer 2012, p. 138).

nective should be characterized by the pattern of its truth values at possible *indexes*, not at possible *contexts* [as Evans advised in the mentioned text; a similar point is made by David Lewis (1970) and David Kaplan (1989)]. Since Malpass and Wawer accept Ockhamist truth, they also subscribe to a thoroughly truth-conditional notion of disjunction—the truth value of a disjunction at an index is a function of the truth values of the disjuncts at the index. The principal goal of any postsemantic theory is *not* to specify the *meaning* of linguistic expressions, but to explain how to assess a sentence used at a particular context. STRL assumes that the procedure is different at actual and non-actual contexts, but it does not affect the semantic properties of the language.

Thus, STRL may be defended against the Iacona's objection. It does not mean, however, that the theory is free of charges. The aspect of STRL which we find most peculiar today is the idea of assessment of a sentence used at a non-actual context. It seems most natural to extend the democracy advocated by Iacona from the semantic to the postsemantic level and agree that whenever one considers a sentence used in a given context, one should assume that the context is actual. Theoretical parsimony recommends a uniform postsemantic treatment to be applied to all contexts, while Malpass and Wawer insist to treat some contexts as actual and others as as non-actual.[6]

To be fair to our former sleves, we should note that their persistent struggle to interpret sentences used at non-actual contexts is not accidental. They explained that the major purpose of their project was to answer the postsemantic challenge of Belnap et al. (2001) and to give some account of "predictive speech acts occurring at moments of use that *lie off the TRL*" (Belnap et al. 2001, p. 162, emphasis ours). Whoever aims to address the challenge *cannot* treat an arbitrary context as if it was actual. For this reason Malpass and Wawer and MacFarlane introduce postsemantic theories that explicitly dictate how to assess sentences used at non-actual contexts. We believe that it is also why they ultimately fail.

7 Postsemantic challenge reassessed

Therefore, rather than offering yet another postsemantic TRL theory, we decided to challenge the postsemantic challenge of Belnap et al. (2001). We believe that it is based on a tacit metaphysical assumption which should be rejected by any proponent of TRL theory. To use Wittgenstein's metaphor, an attempt to interpret sentences used at non-actual contexts

> commits us to a particular way of looking at the matter. (…) The decisive movement in the conjuring trick has been made, and it was the very one that we thought quite innocent. (Wittgenstein, *Philosophical Investigations*, §308.)

[6] We seem to have packed two independent insights into a single project: (i) that every future contingent, like "The coin will land heads," is either true or false at a context and (ii) that none actually used counterfactual with a contingent consequent, like "Had I tossed the coin, it would have landed heads," can be assigned a truth value at a context. An attempt to formally grasp the second insight is sketched in (Wawer and Wroński 2015).

The original sin of Malpass and Wawer (and MacFarlane) was to accept Belnap's postsemantic challenge at all. What should have immediately raised our eyebrows, but did not, is the presupposition of the challenge. Namely, Belnap et al. (2001) assume that there are people, who live outside of the actual world and who make their own predictions. It might be reasonable within the modal metaphysics accepted by Belnap et al. (2001), but TRL theory is motivated by the actualist insight which rejects such a picture. It should be clear to any TRLer that as soon as one considers a person making a prediction, one needs to assume that the person occupies the actual world. Otherwise, one gives up the actualist credentials and implicitly accept the modal-realistic perspective on the tree of possibilities. One lets into the world plenty of people who live in non-actual possibilities and who make predictions of their own that need to be taken care off.

The postsemantic objection is symptomatic of the entire line of criticism of the TRL raised by Belnap et al. (2001, see in particular sec. 6D). It reveals the metaphysical underpinning of their account of the branching model (the authors, however, would probably distance themselves from accusation of being involved in any sort of metaphysics). Almost all branching theorists agree that the model is meant to represent the possible courses of events, but Belnap et al. seem to buy into a particular brand of modal metaphysics—the brand associated with the modal realism of David Lewis.[7] In particular, they accept what might be called "modal neutrality". According to this view, no point in a modal space can be privileged as objectively and absolutely actual. They consider actuality as indexical and perspectival. Every possible world (possible moment) is actual from its own perspective and none such perspective is distinguished as *the correct one*. Their devotion to the idea of modal neutrality is quite well captured by the claim that their "theory of agents and choices in branching time *pictures the causal structure of our world as made up of alternative courses of events branching tree-like toward the future*" (Belnap et al. 2001, p. v, emphasis ours).

The principle of modal neutrality is hard to reconcile with the notion of the absolutely actual branch—the TRL. As soon as we accept that every modal perspective is on a par with our own perspective, it is hard to simultaneously admit that only one of the possibilities is absolutely different from all the others (i.e. not merely relatively or indexically actual). The tension is exploited in Belnap et al.'s arguments against the TRL. The postsemantic objection is a good example. Modal neutrality requires that all the speakers in the branching worlds are actual to themselves, but absolute actuality demands that only some of them are objectively actual. Therefore, some unlucky speakers, who consider themselves actual, are not *really* actual and do not have the TRL at their disposal and hence, they cannot use it for (post)semantic purposes. This, and other of their objections, leads to a conclusion which can be put shortly: if *Our World* is branching, then we cannot distinguish just one branch as absolutely actual.

[7] There certainly are some significant differences: see Lewis (1986), sec. 4.2, for his criticism of overlapping worlds; see also Belnap et al. (2001), sec. 7A.6, for their criticism of Lewisian worlds. Nonetheless, they seem to share certain core principles. Belnap has probably inherited a realist account of branching from Richmond Thomason who sympathizes with Lewis' account of modality (see for example Thomason 1984, p. 215, n. 14); Belnap et al. themselves also draw some parallels between their, and Lewis' vision of modality (see Belnap et al. 2001, p. 163).

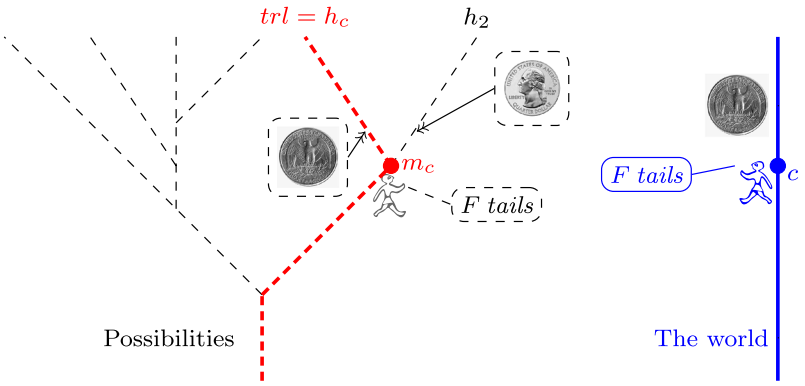


Fig. 4 On the left-hand side, the tree of possibilities is depicted. The unique possibility successively actualized by the world is represented by the thick, densely dashed history. On the right-hand side, the linearly developing world itself is depicted. The coin happens to land tails in the world, so the actualized possibility needs to represent the coin as landing tails

As they say, one philosopher's *modus ponens* is another's *modus tollens*. We accept the implication mentioned above, but we derive the opposite conclusion. Since we accept an absolute distinction between the actual and the possible, we abandon the idea that our world is branching. We deny the principle of modal neutrality and accept that the perspective of just one possibility—our possibility—offers the accurate account of reality. The pictorial representation of the TRL theory presented above, in Sect. 2 and 3, is therefore slightly misleading. A more appropriate diagram clearly distinguishes the (linearly developing) world where utterances take place from the sphere of (branching) possibilities. The latter are necessary to analyze *actually used* sentences regarding what might have happened, but they should not be confused with the world itself.[8] Look at Fig. 4 for a more accurate representation.

Thus, to properly counter the postsemantic challenge, one should insist that there are no people who happen to live in merely possible circumstances and who perform speech acts that "lie off the TRL." Therefore, there is nothing to give an account of and the challenge never gets off the ground. It means that when Belnap et al. (2001, p. 162) complain that "the TRL (…) gives no account whatsoever of predictive speech acts occurring at moments of use that lie off the TRL" one should not be much concerned. It is not a "logical defect," as they describe it, but it is the most reasonable consequence of the actualist worldview. The best way to address Belnap et al.'s (2001) postsemantic worry, is not to worry about it at all.

Our cautious attitude towards the branching model is faithful to Arthur Prior's legacy. The proponent of the model was rather scarce on its metaphysical interpretation, but there is good reason to believe that he also would not treat the branching tree as a depiction of the structure of the world itself. Remember that Prior contributed enormously to invention and development of relational models of modal and tense

[8] The metaphysical status of branching possibilities is a whole new subject which we do not intend to tackle here. Importantly though, even if an actualist accepts that possibilities are real, they insist that they are *differently* real than the concrete world we occupy.

logic, but he was very skeptical throughout his career about their philosophical gravity. In 1958, when he comments on an interpretation of a system of tense logic (PF-calculus) in terms of first-order logic (1-calculus), he notes:

> The interpretation of the PF-calculus within the 1-calculus is clearly a device of considerable metalogical utility. (…)
> There are strong reasons, however, for refusing to attach this metaphysical significance to the interpretability of the PF-calculus in the 1-calculus. (Prior 1958, pp. 115–116)

He reaffirms the same attitude with respect to modal logic in his posthumously published book.

> So, the original, normal or standard interpretation of the calculus sketched in 1.1 [a version of the modal system S5], i.e., the interpretation of it as a logic of necessity and possibility, can be presented as just a special case of the interpretation of it as a mildly odd formulation of the uniform monadic lower predicate calculus. It *can* be so presented. But do we illuminate the subject of modal logic by so presenting it? To this I want to say, No; or at all events, Not much. (…) [P]ossible worlds, in the sense of possible states of affairs, are not *really* individuals (just as numbers are not *really* individuals). To say that a state of affairs obtains is just to say that something is the case; to say that something is a possible state of affairs is just to say that something could be the case. (Prior and Fine 1977, pp. 53–54)

Hence, even though Prior was sometimes interpreted as an advocate of branching of time (see e.g., Thomason 1970; Rescher and Urquhart 1971), he would likely agree with us that a branching model is just a useful set theoretical and diagrammatic representation of more primitive modal facts regarding what might have been the case. Drawing trees is a convenient way of representing temporal possibilities on a piece of paper, but we should keep in mind that suggestive drawings are potentially dangerous in the philosophical practice. The tree might help us to conceptualize relations between time and modality, but it should not mislead us to thinking that there is a multitude of branches "out there" occupied by the "tree-dwellers" similar to us.[9]

Our project is particularly keen to Prior's demand not to reify modality and to accept the fundamental distinction between the possible and the actual. With respect to the temporal domain, we try not to prejudge whether the notions of past, present, and future deserve a similar, fundamental status. Thus, we are less committed than Prior to presentism and A-theory. In fact, we believe that TRL theory as such is compatible with a variety of views in metaphysics of time, ranging from presentism, through growing block and moving-spotlight, to eternalism. For us, the rejection of modal realism is the distinctive move here, and this can be combined with any of the standard temporal ontologies to produce a version of TRL theory. Prior was also rather skeptical with regard to the notion of "true future" among all the possible ones, as he believed that

[9] A similar conjecture regarding Prior's skeptical attitude to reification of the branching model is made in (Hasle and Øhrstrøm 2016, p. 3411).

the notion has deterministic overtones (see Prior 2003). In this respect, we also part ways with the great New Zealander.

7.1 Elitist and egalitarian TRL

We believe that instead of fighting the postsemantic objection an actualist should ignore it and insist that every context is a part of the TRL. There are two ways to look at the thesis—"elitist" and "egalitarian". The elitist assumes the perspective of the actual history and stubbornly insist that no moment outside of the TRL can be reasonably conceived as the moment of the context. Such an approach was once chosen by one of us (Wawer 2014). The author argues that since there are no other worlds, "parallel" or "branched," in which people make (true or false) assertions, no moment outside of TRL can be reasonably treated as the moment of the context and thus, the postsemantic problem does not arise.

Such an actualist obviously admits that some assertions which have never been made *might have been made*. For example, they readily admit that Elizabeth Warren might have said "I fully endorse Bernie Sanders" during the 2016 Democratic primaries in the US. They might even admit that there is a possible moment and a possible history in which the assertion is being made. They keep in mind, however, that such a claim is not to be taken completely literally. It is a useful paraphrase that capture a more fundamental truth about the modal reality. The sentence paraphrased depends on the particular version of actualism, but the claim that there is a possible scenario in which Elizabeth Warren utters "A" might be understood as:

- Elizabeth Warren could have said "A".
- E.W. had a disposition/power/tendency to say "A".
- There is an (abstract) maximally compatible set of propositions representing E.W. as saying "A".
- There is an (abstract) maximal state of affairs in which E.W. says "A".
- There is a way things could have been containing E.W. saying "A".
- Etc.

As far as the TRL theory advocated in this paper goes, any of these suggested metaphysical pictures would be acceptable. However, we currently find the second option to be quite promising, and the sort of thing we naturally have in mind when we think of the TRL. The idea is that the possible futures in the branching tree are reducible to more fundamental facts about the powers or dispositions of actually existing entities. So imagine that the apple in this bowl will actually get eaten. On a possible, but non-actual, future the apple does not get eaten, but instead remains in the bowl until it goes rotten. On the view advocated here, the presence of this possible future is grounded in the "potentiality" that the apple has to rot. The "ground floor" level of reality has actual objects with "modal profiles," or sets of potential states that they could occupy in the appropriate circumstances. It is out of this "furniture" that we "build" the tree of possibilities. While this is only a sketch, hopefully it does enough to flesh out somewhat the sort of picture we have in mind. Elaborated theories along these lines include the works by Barbara Vetter (2015) and Antje Rumberg (2016, the

second is particularly relevant since it is developed in the framework of branching model).

Importantly, none of the paraphrases above suggests that beside the concrete acts of utterances taking place in our world, there are similar acts occurring in other worlds. For example, *the proposition representing E.W. as saying "A"* or *E.W.'s disposition to say "A"* are radically different entities than *E.W.'s actual utterances*. Therefore, we do not need to worry about utterances occurring in other worlds, since, strictly speaking, no such utterances exist (there exist just appropriate sets of propositions representing such utterances, dispositions to such utterances, etc.). The postsemantic task was to match a concrete act of utterance with a particular semantic index and since all the utterances are made in the single world, all of them will designate the single actualized history as the relevant semantic parameter of truth.

The resulting formal model might be seen as a revival of the idea which accompanied Saul Kripke, when he first introduced the semantics for modal logic (Kripke 1959, 1963). In the models he considered, in contrast to what is usually called Kripke-models today, each structure was equipped with a distinguished world, G, which Kripke describes as "*the real world*". Having done that, he identifies simple truth with truth-in-the-distinguished-world (Kripke 1963, p. 69).[10] By analogy, we define truth at a context as truth-in-the-distinguished history.

We should mention that Kripke does not distinguish between semantics and post-semantics, nor does he mention truth at context in his paper. Nonetheless similarities between his approach and the elitist proposal are remarkable. First of all, Kripke does distinguish between two notions: truth and truth-in-a-world. The latter is relevant for all properly semantic purposes (like definitions of logical connectives), while the former bridges the gap between the more technical and the more mundane notion of truth. These two are analogous to our notions of truth-at-context and truth-at-index. Moreover, Kripke fixes a single possible world as the unique world relevant for (simple) truth. Similarly, in the elitist model, a single possible history is fixed as *the* history relevant for truth at context. For Kripke, just like for an elitist, to consider an alternative history as the distinguished history is to construct an entirely new model of the modal reality. Thus, it is safe to say that Kripke's "real world" is closely related to our "thin red line".

The other way of looking at the assumption that all contexts are actual is more democratic. An egalitarian doesn't require a separate model for every possible context. She accepts that within a single model, an arbitrary moment on the tree can be *treated as* a moment of the context. However, even within the liberal perspective, the actualist presupposition sneaks in. You can consider any possible moment as the moment of the context, nonetheless, whenever you do so, you need to assume that the possible moment is also actual. And if it is actual, it is a part of the actual history. Therefore, one of the possible histories passing through this possible moment needs to be the actualized history, i.e. the TRL. It is unintelligible to assume that the actualized history

[10] A very similar distinction in the context of branching model was entertained by Arthur Prior (1966). For historical precision, let us mention that Kripke (1959) initially used the term "valid" to describe truth-in-the-distinguished world. He later replaces "valid" with "true" and admits that "the present terminology is clearly an improvement." (Kripke 1963, p. 70)

is "missing" at a context, since it is to assume that an utterance takes place "outside" of the world. This egalitarian solution seems to be in line with the view advocated by Iacona in the mentioned paper:

> The crux of the matter is the relation between plain truth, which is a property that belongs to sentences of natural language (relative to times), and truth in a history, which is defined in rigorous way for formulas of a formal language (relative to moments). (…) When a future contingent is uttered at t, the utterance involves reference to one in particular among the many courses of events that are possible at t, the actual course of events. (Iacona 2014, pp. 2646–7)

No matter which perspective one takes—elitist or egalitarian[11] —it is granted that every prediction occurs in the actual world. Hence Belnap et al.'s (2001) problem of predictions made at non-actual contexts does not arise. Importantly, one should never combine both these perspectives into a single theory. That is, one should never simultaneously assume that one particular history represents the actual course of events (elitism), and then consider a possible moment outside of this history as the moment of the context (egalitarianism).

This type of controversial assumption was implicitly presupposed by many objections historically raised against the TRL. This might be exemplified by MacFarlane's criticism of trl_{fcn} postsemantics (see MacFarlane 2014, sec. 9.4). We will skip the details, but let us just note that in course of his argument, MacFarlane assumes—in the *elitist* manner—that one of the histories is absolutely actual. In the next step, he envisages however—in the *egalitarian* fashion—a person who occupies a moment which lies outside of the actual history. Then, he arrives at a peculiar situation where a non-actual person is asked to assess a previous, actual utterance. The thought experiment ends rather poorly for the trl_{fcn} postsemantics (it turns out that the non-actual person needs to admit that what happens to her, was not going to happen). This failure should serve as a warning against any theory that blends "elitist" and "egalitarian" points of view. Many of the objections (semantic, metaphysical, and epistemic) that Belnap et al. (2001) raised against the idea of the Thin Red Line seem to presuppose a similar, confused version of the TRL theory.[12]

It is safer then, to stick to either elitism or egalitarianism and always remind oneself that every moment of the context needs to be treated as a part of the actualized history. In consequence, one can accept futurism—the simple-minded actualist postsemantics outlined in Definition 3—which states that a sentence is true, when used at context c if and only if it is true at the moment of the context (the present moment) and the history of the context (the actual history). When applied to the issue of future contingents, the postsemantics implies that the sentence, "There will be a sea battle," is true at a given context if and only if there actually will be a sea battle at some later moment. Not only we believe that it is a consistent postsemantic principle, but we claim that few theories of future contingents can an offer equally elegant response to the ancient, Aristotelian puzzle.

[11] One could even argue that these two are just different articulations of the same basic idea.

[12] A historical reconstruction conducted in (Wawer 2016, sec. 5.1) reveals that some TRLers might have easily suggested such a metaphysical view to their critics.

8 Summary

Our aim was to assess the role of the Thin Red Line in the branching setting. We argued that it is brought in by the actualistic insight to resolve the initialization failure rendered by Ockhamist semantics. We first considered a semantic version of the Thin Red Line theory and its criticism championed by Belnap and Green (1994). Then, we looked into postsemantic answers to the initialization failure and investigated how they deal with the postsemantic challenge raised by Belnap et al. (2001). We explained that no semantic objection can be used to discredit these theories, but they generate controversies since they treat some contexts as non-actual. To rescue the TRL theory, we reassessed the postsemantic challenge and argued that its force is overstated. We concluded that as long as one sticks to the actualist modal metaphysics, one need not be bothered with such an objection. One can simply evaluate a sentence used at a context at the moment of the context and the history of the context. It means that neither semantics nor postsemantics gives good reasons to give up the Thin Red Line and bivalence of future contingents that it generates. The overarching lesson of the argument is that if there is any reason to disqualify the Thin Red Line, it should be sought for in the domain of metaphysics rather than semantics or postsemantics.

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