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Title: Small complete caps and saturating sets in Galois spaces I-II

Summary:

Galois spaces are well known to be rich of nice geometric, combinatorial and group theoretic properties that have also found wide and relevant applications in more practical areas, notably Coding Theory and Cryptography. Typical objects linked to linear codes are plane arcs and their generalizations, especially caps, saturating sets and arcs in higher dimensions, whose code theoretic counterparts are distinguished types of error-correcting and covering linear codes, such as MDS codes. Their investigation has received a great stimulus from Coding Theory, especially in the last decades.

An important issue in this context is to ask for explicit constructions of small complete caps and saturating sets. A *cap* in a Galois space is a set of points no three of which are collinear. A *saturating set* is a set of points whose secants (lines through at least two points of the set) cover the whole space. A cap is *complete* if it is also a saturating set. From these geometric objects there arise linear codes which turn out to have very good covering properties, provided that the size of the set is small with respect to the dimension N and the order q of the ambient space.

The aim of these talks is to provide a survey on the state of the art of the research on small complete caps and saturating sets, with particular emphasis on recent developments. In the last decade a number of new results have appeared, and new notions have emerged as powerful tools in dealing with the covering problem, including bicovering arcs, translation caps, and (m) -saturating sets. Also, although caps and saturating sets are rather combinatorial objects, constructions and proofs sometimes heavily rely on concepts and results from Algebraic Geometry in positive characteristic.

In the first talk we explain the close relationship between linear codes with covering radius 2 and caps and saturating sets in Galois spaces. We also deal with caps and saturating sets in the plane. A cap in a Galois plane is often called a *plane arc*. The theory of plane arcs is well developed and quite rich of constructions; however, we decided to focus on plane arcs arising from cubic plane curves, which will be relevant for some recursive constructions of small complete caps. For arcs contained in elliptic curves, a new description relying on the properties of the Tate-Lichtenbaum pairing is given. Also, a construction by Szőnyi of complete arcs contained in cuspidal cubics is generalized.

In the second talk we deal with the 3-dimensional case. The even order case

was substantially settled by Segre in 1959, whereas the problem of constructing complete caps of size close to the trivial lower bound is still wide open for odd q 's. Here we describe how a construction by Pellegrino can give rise to very small complete caps in the odd order case. We also present in more detail the notion of a multiple covering of the farthest-off points, and relate it to that of an (m) -saturating set.

In both talks we deal with inductive methods. In particular, we discuss under what conditions the product construction and the blowing-up construction preserve the completeness of a cap. Then the notions of a translation cap in the even order case, and that of a bicovering arc in the odd order case, come into play as powerful tools to construct small complete caps in higher dimensions. Davydov's recursive construction of saturating sets is also described.