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Abstract. Most attempts to justify the use of idealized models to explain appeal to the accuracy of the model with respect to difference-making causes. In this paper, I argue for an alternative way to justify using idealized models to explain that appeals to universality classes. In support of this view, I show that scientific modelers seeking to explain stable limiting behaviors often explicitly appeal to universality classes in order to justify their use of idealized models to explain.

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Universality and Modeling Limiting Behaviors

1. Introduction

Most attempts to justify the use of idealized models to explain appeal to the accuracy of the model with respect to difference-making (or contextually-salient) causes and the irrelevance (or insignificance) of the features distorted by the idealizations (Craver 2006; Kaplan and Craver 2011; Potochnik 2017; Strevens 2008; Weisberg 2013). In this paper, I argue for an alternative way to justify the use of idealized models to explain that appeals to *universality classes*. According to this ‘universality account’, idealized models can be justifiably used to explain when they are within the same universality class as their real-world target system(s) (Batterman and Rice 2014; Rice 2018). Instead of defending any particular account of explanation, my goal here will be to focus directly on the question of how scientific modelers *ought to justify* the use of highly idealized scientific models for purposes of explanation. I will argue that universality classes can link idealized models to real-world systems in ways that can justify their use for purposes of explanation—even when the models drastically distort (or completely ignore) contextually-salient difference-making causes of the explanandum. In support of this view, I then show how the universality account better accommodates cases of modeling stable limiting behaviors across causally diverse systems. In these cases, scientific modelers often *explicitly appeal to universality classes* in order to justify using their highly idealized models to explain.

The following section motivates the need for the universality account by briefly critiquing the standard approach to justifying the use of idealized models to explain. Then, Section 3 lays out the details of how to justify the use of idealized models to explain by appealing to universality classes. Next, Section 4 argues for the adoption of the universality account by presenting an example (that is representative of a larger class of cases) in which scientific modelers explicitly appeal to universality classes to justify their use of idealized models to explain stable limiting behaviors. The final section concludes.

2. The Accurate Representation of Difference Makers

It is widely accepted that there are many idealized models that are used to explain in science (Batterman 2002; Bokulich 2011; Cartwright 1983; Morrison 2015; Potochnik 2017; Rice 2015, 2018; Strevens 2008; Weisberg 2013). This has led philosophers of science to provide various accounts of how models that include idealizations can be justifiably used to explain. The vast majority of these accounts have appealed to one, or typically both, of the following features: (1) the accuracy of the model with respect to the difference-making (or contextually-salient) causes of the explanandum and/or (2) the irrelevance (or non-salience) of the features distorted by the idealizations.

For example, according to most mechanistic accounts, an idealized model will only explain if it provides an accurate representation of the relevant features of the causal mechanism(s) that produced the explanandum (Craver 2006; Kaplan and Craver 2011;

Glennan 2017). Kaplan and Craver (2011) refer to this criterion explicitly as the model-to-mechanism-mapping (3M) requirement.

In addition, most causal accounts require models that explain to provide an accurate representation of (at least some of) the difference-making causal factors that produced the explanandum (Elgin and Sober 2002; Potochnik 2017; Strevens 2008; Weisberg 2013; Woodward 2003). For instance, on Michael Strevens's view, "the overlap between the idealized model and reality...is a standalone set of difference-makers for the target" (Strevens 2008, 318). Moreover, the role of the idealized parts of the model is to, "point to parts of the actual world that do not make a difference to the explanatory target" (Strevens 2008, 318). In addition, Michael Weisberg describes several accounts of what he calls 'minimalist idealization' in which the model "contains only those factors that make a difference" (Weisberg 2013, 100). Indeed, several accounts agree that, "the key to explanation is a special set of explanatorily privileged causal factors. Minimalist idealization is what isolates these causes and thus plays a crucial role for explanation" (Weisberg 2013, 103).

In contrast with Strevens and Weisberg, Angela Potochnik's account allows difference-making causal factors that are not central to the research program to be left out or idealized (Potochnik 2017). However, Potochnik's view still requires that "posits central to representing a focal causal pattern in some phenomenon must accurately represent the causal factors contributing to this pattern..." (Potochnik 2017, 157). Therefore, while the set of important causal factors is delimited somewhat differently, there is still a particular set of

causal factors that need to be accurately represented in order for an idealized model to explain.

In sum, according to most philosophical accounts, the use of idealized models to explain is justified (or warranted) by showing that they only distort causes, mechanisms, or features that are irrelevant to the explanandum or research program—i.e. their distortions do not get in the way of the accurate representation of the relevant mechanisms, difference-makers, or significant causes. Indeed, the general goal of these accounts is to show that the, “factors distorted by idealized models are details that do not matter to the explanatory target—they are explanatory irrelevancies. The distortions of the idealized model are thus mitigated” (Strevens 2008, 315). Consequently, these accounts depend heavily on a necessary condition for idealized models to explain: the accurate representation relation must hold between the idealized model and some set of difference-making causes that produced the explanandum. Accordingly, I will refer to these as accurate representation of difference makers (henceforth ARDM) accounts.

While this approach has provided one way to justify the use of idealized models to explain, I contend that it cannot be the whole story since there are many scientific models that are used to explain whose idealizations drastically distort difference-making features, many of which are salient to the research program. Examples include models in physics that distort the difference-making components and interactions of fluids, magnets, and quantum dots (Batterman and Rice 2014; Bokulich 2012; Morrison 2015), biological models that distort the difference-making processes of drift and selection (Ariew et al. 2015; Morrison

2015; Rice 2015, 2018), economic models that distort the difference-making features of agents and transactions (Frigg 2010), models used to study human behavior that distort difference-making genetic and environmental factors (Longino 2013), etc. The more general problem is that scientists routinely make use of modeling frameworks that *pervasively* and *holistically* distort both difference-making and non-difference-making causes of the explanandum (Rice 2018). Therefore, in contrast with ARDM views, below I will argue that the story scientific modelers can, should, and often do provide in order to justify their use of these models for purposes of explanation *does not appeal to accurate representation relations between the model and some set of relevant causes*.

In addition, while I am certainly not unique in my attempts to draw on actual scientific practice, when it comes to the task of *justifying* the use of models to explain, most philosophers have attempted to do so by appealing to more general philosophical accounts of explanation (and idealization). In contrast, I suggest a better methodological starting point for justifying the use of idealized models to explain is to *look directly at the justifications actually provided by scientific modelers who produce those explanations*. That is, I will look directly at the justifications scientific modelers give for using their models to explain without defending any particular account of explanation. Taking this approach, we find that very rarely do scientific modelers justify their appeals to idealized models in terms of accurate representation of difference-making causes or distortion of irrelevant causes. Instead, scientific modelers typically justify the introduction of idealizations into their models as a means to applying various modeling techniques they have on hand that they have reason to

believe will lead to the development of an explanation for their explanandum of interest (Batterman 2002; Cartwright 1983; Morrison 2015; Rice 2018). Moreover, as I will argue below, scientific modelers often justify the use of these modeling techniques by appealing to universality classes that identify stable patterns of behavior across classes of real, possible, and model systems that are heterogeneous with respect to their causes.

3. Using Universality to Justify the Use of Idealized Models to Explain

Instead of focusing on the accurate representation of difference makers, an alternative way to establish a connection between idealized models and their target system(s) is to exploit an extremely convenient feature of our universe called *universality*. As physicist Leo Kadanoff (2013) puts it, “Whenever two systems show an unexpected or deeply rooted identity of behavior they are said to be in the same universality class” (178). In short, universality is just a statement of the fact that different physical systems will nonetheless display similar patterns of behavior that are largely independent of their physical features (Batterman 2002). The group of systems that will display similar behaviors despite (perhaps drastic) differences in their physical features are said to be in the same universality class.

What is more, these universality classes often include several *model* systems that involve various idealizing assumptions that enable them to employ various mathematical modeling frameworks used by scientists to provide explanations. Because these model systems are also in the universality class, they will display similar patterns of behavior—although sometimes only in their large-scale limits—despite the fact that the model may

drastically distort the causes and mechanisms responsible for the explanandum in any real-world system. Despite their having drastically different features, discovering universality classes can demonstrate that there is a class of systems, which includes an idealized model system and its target system(s), that will exhibit the same stable patterns of behavior. It is precisely this stability of these universal behaviors across perturbations of the system's physical features that can enable scientists to justifiably use models that drastically distort difference-making causes. I contend that universality is a ubiquitous feature of myriad classes of real, possible, and model systems that can be (and is) exploited by scientific modelers to discover idealized models that drastically distort the difference-making causes of their target system(s) and yet enable them to explain the behaviors of those systems (Batterman and Rice 2104; Rice 2018).

It is important to note, however, that appealing to the existence of a universality class in order to justify the use of an idealized model to explain is *crucially different from providing an explanation of universality*. There is a large and important literature that analyzes how universality itself can be explained; e.g. by using renormalization techniques (Batterman 2002; Morrison 2015). The account I develop here is explicitly *not* an account of how to explain universality. Instead, I aim to use the fact that various universality classes exist in the world—whether we have explanations for why those behaviors are universal or not—in order to justify the use of various idealized models to provide explanations in science (Rice 2018).

In addition, most philosophical accounts seem to conflate having an accurate representation relation between the model and difference makers with the model's being an explanation; i.e. satisfying this model-world relation is all that is required for the model to explain. In contrast, I argue that being in the same universality class is only one way (of a plurality of ways) to establish a connection between a model and a real-world system that can *justify (or license) various explanatory inferences from the behavior of the idealized model(s) to the behavior of the real-world system(s)*. Consequently, being in the same universality class is not necessary because there are other ways to justify the use of an idealized model for purposes of explanation. It is also not sufficient for explanation since justifying such inferences is not equivalent to providing a complete explanation of the explanandum.

In light of this, then, I should say something about why I take these cases to be genuine instances of scientific explanation. Although I will not be arguing for any particular account of explanation here, almost all accounts (e.g. both causal and noncausal accounts) agree that, “[an] explanation must enable us to see what sort of difference it would have made for the explanandum if the factors cited in the explanans had been different in various possible ways” (Woodward 2003, 11).¹ In line with this approach, I suggest that these idealized models explain because they enable scientists to identify a set of features on which the explanandum counterfactually depends; i.e. they show how the explanandum would have been different if various features of the system had been different in various ways. As we will see below, identifying universality classes, finding idealized models within those classes,

¹ See Bokulich (2011, 2012) and Rice (2015) for noncausal applications of this counterfactual dependence approach.

and applying various mathematical modeling techniques, can reveal a plethora of information about the counterfactual dependencies (and independencies) that hold between various features of the systems within the universality class and the explanandum. This can be accomplished, however, without constructing a model that accurately represents the difference-making features or how those features (causally or mechanistically) produce the explanandum.

One mark in favor of this universality account is that it can easily accommodate the cases used to motivate ARDM accounts. After all, in many cases, the idealized models that are within the same universality class as their target systems will just be those that accurately represent the difference-making causes of their target system(s). When this occurs, the idealized model will undoubtedly produce many of the same patterns of counterfactual dependence for the same reasons those patterns occur in the target system(s). However, in many other cases the same patterns of counterfactual dependence will be produced by *extremely different* sets of difference-making components, causes, and mechanisms across a range of real, possible, and model systems. In these cases, scientists often construct a pervasively distorted minimal model in order to apply the (mathematical) modeling techniques they have on hand and extract universal features that are stable across causally heterogeneous systems. As Goldenfeld and Kadanoff (1999) put it, “We can exploit this kind of ‘universality’ by designing the most convenient ‘minimal model’.” (87). Convenience here refers to the model’s being amenable to ‘mathematical treatment’ in ways that require the distortion of many of the key features of the target system(s) (Cartwright 1983). Despite

these distortions, if the resulting idealized model is in the same universality class as its target system(s), it can be justifiably used to investigate how the explanandum counterfactually depends on various features of the class of systems that display that behavior.

4. Explicit Appeals to Universality in the Explanations of Limiting Behaviors

This section argues that the justifications given by actual scientific modelers attempting to explain stable limiting behaviors are often in line with those suggested by the universality account. Indeed, the justification offered by these modelers *explicitly appeals to universality classes* that contain the model and its target systems. This provides further motivation for adopting the universality account.

4.1. Universality in Bacterial Growth and the Eden Model

Several models from statistical physics that exhibit universal behaviors have recently been applied to problems in biology. For example, many kinds of biological growth can be modeled by an extremely minimal computational model known as The Eden Growth Model (Eden 1961). The simplest version is a lattice model with “occupied” and “unoccupied” sites that follows two simple growth rules:

- (1) Start from a single occupied lattice site—called the “seed” site.
- (2) In each growth step, randomly occupy an empty site that is the nearest neighbor to an occupied site.

Running this algorithm N times produces clusters containing N “cells”. By running this algorithm many times, scientists can investigate the topological properties of the ‘typical

cluster' by looking at statistical averages across a range of many possible configurations and taking the limit as $N \rightarrow \infty$. That is, scientists are often interested in the *average properties across a wide range of cases in the limit* rather than having any one of the simulations capture the actual growth process of the physical system(s) (Hermann 1986). Indeed, like many cases in statistical physics, typically one can only get the exact scaling behavior of interest by taking various limits (Hermann 1986, 163). As a result, when growth modelers take these limits, "the situation is analogous to a critical phenomenon at $N \rightarrow \infty$...it is something like a critical *thermodynamic limit*" (Hermann 1986, 157). Moreover, like the critical exponents involved in the universality of phase transitions in physics, the universal limiting behaviors of these growth models depend on various critical exponents of the dynamical equations of the system.

Although the Eden model has been widely used to explain various kinds of biological growth, the model involves a plethora of idealizing assumptions. Here is how Eden discusses his original model:

It has been necessary to make a large number of simplifying and special assumptions so that *the resemblance between the model and the growth of any complicated metazoan or specialized organ or tissue is slight*. We shall assume that each cell is identical with every other cell, that each cell is connected to at least one other cell, that the location of each cell is specified by a node in some regular lattice. For purposes of simplicity the model will be restricted to two dimensions. Any migration of cells, differentiations into specialized cell types, variations in cell size, and simpler properties of organisms will be neglected. Indeed, cells will be assumed immortal and a very special time-to-division distribution function is used. (Eden 1961, 224-25, my emphasis)

In other words, the model distorts most of the features of any real biological growth process (e.g. the cells, their interactions, the structural features of the population, etc.), and instead

focuses on the influence of the overall topological structure of the system on the features of the cluster in the limit. As a result, the model provides little, if any, accurate information about the causes that give rise to these growth patterns in actual biological systems. Indeed, the model explicitly ignores or directly distorts features that the scientific modelers know make a difference to how actual biological growth processes develop.

What is striking is that across a diverse range of biological and non-biological cases, many features of the growing population can be explained by appealing to the universal limiting behaviors of Eden growth models. For example, Eden growth models have been used to model epidemics (Alexandrowicz 1980), the spread of forest fires, (Zhang et al. 1992), and urban growth (Benguigui 1995). This wide range of application is possible because the Eden model is in the same universality class as many actual growth processes.

Specifically, the Eden model is one of a handful of growth models that have been shown to be in the Kardar-Parisi-Zhang (KPZ) universality class (Kardar, Parisi, and Zhang 1986). This class of systems is governed by the Kardar-Parisi-Zhang growth equation:

$$\frac{\partial h}{\partial t} = \nu \nabla^2 h + \frac{\lambda}{2} (\nabla h)^2 + F + \eta(x, t)$$

where ν is the damping coefficient, λ is the growth parameter, F is constant drift, and η is an uncorrelated white noise. As Kardar, Parisi, and Zhang explain, their proposal of this modeling equation was “guided by the idea of universality” and sought to “write down the simplest nonlinear local differential equation governing the growth of the profile to such processes as vapor deposition or the Eden model” (Kardar, Parisi, and Zhang 1986, 889).

While modelers are still working out the details of this universality class, the following features have been identified as important for membership in the KPZ class (Corwin 2016, 233):

Locality: Changes depend only on neighboring sites.

Smoothing: Large valleys are quickly filled in.

Nonlinear slope dependence: Vertical effective growth rate depends nonlinearly on local slope.

Space-time independent noise: Growth is driven by noise which quickly decorrelates in space and time and does not display heavy tails.

Identifying this set of features is key to delimiting the universality class and understanding how the limiting behaviors of interest counterfactually depend on these minimal features.²

Moreover, finding this universality class and the features important for membership also identifies clear links between the minimal features included in the model system(s) and many real-world cases of biological growth. As Ivan Corwin explains, “a variety of physical systems and mathematical models, including randomly growing interfaces, certain stochastic PDEs, traffic models, paths in random environments, and random matrices all demonstrate the same universal statistical behaviors in their long-time/large-scale limit. These systems are

² The Eden model is an extremely minimal model. However, just because it is a minimal model does not mean the explanation provided is a *minimal model explanation*. For details see Batterman and Rice (2014).

said to lie in the *Kardar-Parisi-Zhang (KPZ) universality class*” (Corwin 2016, 230). What this means is that, despite drastic differences in their physical components, causes, mechanisms, and interactions (or in the case of the Eden model, the complete neglect of most of these features), these various real, possible, and model systems will all display similar patterns of limiting behavior.

In a more specific application, Bonachela et al. (2011), found that in the case of bacterial growth, “Similar values for the exponents measured in different bacterial strains indicate that, despite the varying microscopic details and interactions specific to each strain, ...they show the same *universal behavior*” (307). A large part of demonstrating that their theoretical growth model is in a universality class that includes their target system(s) involves matching the systems’ critical exponents. The characteristic critical exponents of the KPZ universality class are a dynamic scaling exponent $z = 3/2$, a growth exponent $\beta = 1/3$, and a fluctuation (or roughness) exponent $\alpha = 1/2$ (Bonachela et al. 2011, 307). In discovering the match between these critical exponents, these modelers determined that their growth model and many actual system(s) of bacterial growth are likely part of the KPZ universality class (Bonachela et al. 2011, 307).

However, while they suggest that some bacterial colonies are likely within the KPZ universality class, they found that one of the critical exponents of their bacterial colonies did not reliably match the exponents of their idealized model within the KPZ class. As a result, they *looked for a model in an alternative universality class* that could capture all the observed critical exponents. This shift to a new universality class was achieved by

constructing an idealized growth model in which the thermal noise was replaced by a quenched noise. This resulted in the discovery of a universality class they refer to as the *quenched* KPZ universality class (qKPZ) (Bonachela et al. 2011, 311). They then used this universality class's inclusion of their idealized model and the real bacterial colonies in order to justify drawing explanatory inferences from their idealized growth model.

In sum, showing that their idealized growth models are in the same universality classes as various real-world bacterial systems is what *justifies* their use of these highly idealized models to explain various limiting patterns of bacterial growth—even if the models and the various real-world systems display those limiting patterns because of very different causes, mechanisms, and interactions. Moreover, by identifying these universality classes, these modelers were able to identify several features that the large-scale limiting behaviors of bacterial growth depend on. For example, they found that the universal behaviors of interest depend on the kind of noise involved in the system. Changing this feature results in the system being in a different universality class that will display different limiting behaviors. Furthermore, their investigation of idealized models within various universality classes demonstrated that the limiting behaviors they are interested in depend on the system's critical exponents and fractal dimension. This dependence information is crucial to explaining why those limiting behaviors occur across the diverse systems included in the universality class.

Discovering these universality classes also helped these modelers determine features of the system(s) that are *irrelevant* to the universal behaviors of interest. For example, they found that “irregular cell shape, long-range cell motility, and extracellular compounds are not

necessary for our idealized cell groups to exhibit the same universal behavior as real bacterial colonies.” (Bonachela et al. 2011, 313). In other words, investigation of the models within this universality class was also able to reveal explanatory information about the irrelevance of many features to the occurrence of the explanandum. As a result, by showing that their idealized models are in the same universality class as real bacterial colonies, these modelers were able to justifiably use their pervasively distorted idealized model to provide explanatory information about why we see the same limiting behaviors in various real systems.

While I have focused on just one case, the use of universality classes to justify the use of idealized models to explain has been employed by many scientific modelers. For example, Thomas Gisiger claims that:

Universality has been described as a physicist’s dream come true. Indeed, what it tells us is that a system, whether it is a sample in a laboratory or a mathematical model, is very insensitive to details of its dynamics or structure near critical points. From a theoretical point of view, to study a given physical system, one only has to consider the simplest mathematical model possibly conceivable in the same universality class. (Gisiger 2001, 173)

Gisiger then suggests that, “This argument can be extended to evolution and ecology... *One just has to consider the simplest model conceivable in the same universality class as the ecosystems.*” (Gisiger 2001, 191). In addition, in defense of agent-based models (ABM) in economics Parunak et al. suggest:

To an unbiased observer, [these models’] success seems almost magical...leading some users to hesitate in trusting the results. Universality helps explain this unreasonable success...The existence and widespread manifestation of universality can help build confidence in ABMs, as well as guide in their refinement as users gain experience in how universality manifests itself in specific configurations. (Parunak et al. 2004, 936)

In short, identifying universality classes and understanding the details of various instances of

universality can justify the use of highly idealized models that drastically distort the difference-making causes of the explanandum.

5. Conclusion

In light of these cases, I suggest that philosophers of science investigate the ways scientists use universality classes to justify their uses of idealized models to explain and move away from relying exclusively on the accurate representation requirements derived from general philosophical accounts of explanation. Doing so will better capture the justifications that should be, and are, offered by scientific modelers and will help develop a more pluralistic account of the model-world relationships that can be used to justify appeals to idealized models to explain.

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