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This report focuses on a research study the aim of which is to investigate the activity of proving as constituted in a Cypriot classroom for 12-year-old students. By drawing on Cultural Historical Activity Theory, this study explores the way the teacher is working with the students to foreground mathematical argumentation. Analyses of video-recorded whole class discussions show how explaining and exploring provide a key pathway for the development of proving. We refer to these developments as pre-proving. However, inherent contradictions within explaining and exploring hinder the constitution of proving in the classroom.

Keywords: Proof, exploration, explanation, play, CHAT

Introduction

It is now acknowledged that proof and proving should become part of students' experiences throughout their schooling (Hanna, 2000, Yackel and Hanna, 2003, Stylianides, 2007). However, secondary school students as well as undergraduate students face difficulties when giving formal mathematical arguments. At the same time, research that shows how upper primary school students approach and construct proofs is still limited (Stylianou et al, 2009). It is also argued that argumentation, explanation and justification provide a foundation for further work on developing deductive reasoning and the transition to a more formal mathematical study in which proof and proving are central (Yackel and Hanna, 2003). But what is meant by proof and proving? Mathematical argumentation is a discursive activity based on reasoning that supports or disproves an assertion and includes the exploration process, the formulation of hypotheses and conjectures, explaining and justifying the steps towards the outcome and the proof of the statement. Thus, proof is at the core of mathematical argumentation, as a justification, an explanation and a valid argument.

Research has responded to the need to conceptualize proof and proving in such a way that it can be applied not only to older students but also to those in elementary school (Stylianides, 2007). The challenge remains however to understand how proof and proving is shaped by the practices in the mathematics classroom. This is in accordance with Herbst and Balacheff (2009), who argue that the focus should not only be on proof as the culminating stage of mathematical activity, but also on the proving process and how this is shaped by the classroom environment. Thus, in understanding how proving is constituted in the classroom, a wider network of ideas is required as these ideas no doubt have an impact on how proof in the narrow sense is constituted.

To address this issue, we refer to pre-proving, that aspect of mathematical reasoning that might nurture proving. What are the roots of proving? Given that proof is both a justification and an explanation, it can be argued that emphasis should be placed in these two aspects of mathematical reasoning. In considering those functions of proof that are considered important for school mathematics (Hanna, 2000), evidence has been reported that the establishment of sociomathematical norms (Yackel and Cobb, 1996) for explanation and justification (Yackel and Hanna, 2003) might foster deductive reasoning in the classroom. That is, describing, conveying and exchanging ideas through the act of communication, explaining and justifying statements influences

the appearance of proof and the transition from unsophisticated empirical arguments to the level of sophistication that might be expected at the tertiary level. It is through exploration and investigation that all these elements surface and develop in the process of proving. Thus, when discussing the roots of proving, exploration, which activates intuition and encourages thinking, constitutes another notion that should be taken into consideration. Thus, pre-proving refers to those elements that direct mathematical reasoning towards the ultimate goal of formal proving; that is exploration, explanation, justification and communication. In the social environment of the classroom, where hypothesizing, explaining and justifying conjectures is encouraged, the tools and tasks used, the rules of the classroom, the way the students work together, the way the teacher negotiates meanings and other external factors all interact, interrelate and influence each other in forming classroom activity. The purpose of this study is to explore pre-proving and proving in the elementary mathematics classroom and the way the structuring resources of the classroom's setting shape this process.

CHAT based theoretical constructs

As this study is exploring the various forces that impact on the activity of proving, Cultural Historical Activity Theory (CHAT) is being employed as a descriptive and analytical tool alongside collaborative task design (a means of gaining access to the teacher's objectives), to capture the interaction of different levels, such as the actions of teachers, students and the wider field as evidenced in curricula and research documentation. The analysis and discussion in this paper draws upon the following CHAT perspectives: (i) the object of the activity and (ii) the notion of contradictions. Initially, the unit of analysis in CHAT is an activity, a "coherent, stable, relatively long term endeavor directed to an articulated or identifiable goal or object" (Rochelle, 1998, pp.84). The object of a collective activity is something that is constantly in transition and under construction, has both a material entity and is socially constructed and its formation and transformation depends on the motivation and actions of the subject indicating that it proves challenging to define it. Among the basic principles of CHAT is the notion of contradictions. Contradictions are imbalances, ruptures and problems that occur within and between components of the activity system, between different developmental phases of a single activity, or between different activities. These systemic tensions lead to four levels of contradictions (Engeström, 1987). This conceptualization, should be differentiated from mere problems or disorienting dilemmas from the subject-only perspective as they are more deeply rooted in a sociohistorical context (Engeström, 2001). Contradictions are important because they may lead to transformations and expansions of the system and thus become tools for supporting motivation and learning. This paper focuses on a primary contradiction on the teacher's object. The primary contradiction can be identified by focusing on any of the elements of the activity system (subject, tools, object, rules, community, division of labor). For instance, within the mathematics classroom, the clash between the teacher's goal of teaching a specific content of the mathematics curriculum and her need to continually manage student behavior and maintain focus, leads to a primary contradiction within the system's subject (the teacher).

Data collection and analysis

This study was conducted in a year 6 classroom in a primary school in Cyprus. Apart from the researcher, the participants were the teacher, a Deputy Principal at the school who endorses the integration of technology in teaching mathematics, and 22 students (11-12 years old) of mixed abilities. Even though using computers was part of the classroom's routine, the students were not familiar with dynamic geometry environments, DGEs. The data collection process as relevant to this paper included video data from the classroom observations and field notes. The content of the curriculum covered during the classroom observations was the area of triangles, and the circumference and the area of circle. The overall process of analysis of the collected data was one of progressive focusing. According to Stake (1981, pp.1), progressive focusing is "accomplished in multiple stages: first observation of the site, then further inquiry, beginning to focus on relevant issues, and then seeking to explain". The systematization of the data led to the evolution of two broad activities: (i) the activity of exploration including the exploration of mathematical situations, exploration for supporting mathematical connections and exploration of the DGE and (ii) the activity of explanation which focuses on clarifying aspects of one's mathematical thinking to others, and sometimes justifying for them the validity of a statement. These activities were then interpreted through the lens of CHAT, by generating the activity systems of both exploration and explanation. Achieving this also made possible the identification of tensions.

There is insufficient scope in this short paper to consider in detail these various levels and so this specific study focuses on illustrative episodes, which were generated during classroom discussion, to show one aspect of how the teacher was working with the students to foreground mathematical argumentation. To elaborate more, while the teacher was endeavoring to provide opportunities for exploration and investigation, it was observed that the teacher would sometimes interrupt this exploration. This interruption was often followed by the teacher either translating students' exploration as playing and/or was providing the step that needed to be followed. This paper focuses on the teacher using the word 'play' as part of the activity of exploration. The relevance of this emphasis of the paper lies in the connection that exists between exploration and play. That is, analyzing 'play' provides important information in portraying the activity of exploration and identifying the way this might influence the activity of explanation, and thus, shed light on how proving is constituted in the classroom.

Results

This section provides a chronological overview of the protocols that illustrate the teacher intervening in the classroom by using the word 'play'.

Protocol 1

On the first lesson related with the area of triangles, the students are expected to say the area of rectangles presented on the interactive whiteboard and make explicit the way they worked towards the answer.

Teacher: 12 again … but why are your playing? We are not doing something on the computers now. Stop.

In this protocol, the teacher is relating exploration of DGE with ‘play’. That is, exploration was interpreted by the teacher as ‘playing’ instead of learning.

Protocol 2

After demonstrating on the interactive whiteboard how to construct a rectangle in which the triangle is inscribed, the students worked in pairs and constructed rectangles on a DGE. When they finished, the teacher asked:

Teacher: Now that you constructed the rectangles, can they help you to find the area of the triangles?

Students: Yes.

Teacher: What is the area of triangle PRS?

Student1: 3.

At this point, the teacher interrupted the classroom discussion as she was concerned with a student ‘playing’ with the computers.

Teacher: Student2 you are still talking. You are playing all the time and I will move you from the computers.

In this protocol, the teacher is relating exploration of DGE with ‘play’. That is, exploration was interpreted by the teacher as ‘playing’ instead of learning.

Protocol 3

On the third lesson, after revising the mathematical formula for the area of triangles, the students would construct triangles with specific areas on the DGE. Before the teacher demonstrated to the class the steps followed so as to construct a triangle on the DGE, she said:

Teacher: Now we will go back to the DGE to play with its features. I will give you instructions and you will construct … to see how it operates and then we will move on to a game where you will play in pairs on the computers online.

In this protocol, the teacher relates both exploration of the DGE, and playing a game on the computer with ‘play’, something encouraging and constructive.

Protocol 4

On the first lesson related with circle, after defining circle, the class moved to the computers. Before engaging in tasks so as to explore mathematical relationships related with circle, the teacher introduced the class to a new DGE. Eventually, the teacher, referring to a circle presented on the DGE made the following comment:

Teacher: I can make it bigger or smaller. Look what we will do next. We will play later. Construct a circle and move it. Click on the center. Did you all do it? Nice. Stop.

This protocol focuses on the teacher relating exploration of DGE to ‘play’, something that has a positive value.

Protocol 5

On the second lesson, after revising the definition of a circle, the teacher asked the students to tell her the mathematical relationships explored the day before. At this point, several students could not give an answer. The following comment comprises the teacher's interpretation of the hesitation these students had in participating in the classroom discussion:

Teacher: You shouldn't only play but concentrate and listen in the classroom.

In this protocol, the teacher is relating exploration of DGE with 'play'. That is, exploration was interpreted by the teacher as 'playing' instead of learning.

Protocol 6

In the following part of the lesson, the class engaged in discussing ways in calculating the area of circle. Among the students' ideas was to count the squares inside a circle. However, it was concluded that this could prove difficult to achieve. Other students hypothesized that the area might be equal to circumference times radius. Others said that the area could be equal to circumference times diameter. The teacher encouraged them to investigate and test these hypotheses while exploring a task on a DGE, by making the following comment:

Teacher: I will leave you for a while to play.

In this protocol, the teacher is relating exploration of DGE with 'play'. That is, exploration is translated as something encouraging and constructive.

Protocol 7

During the third lesson that followed the exploration of the mathematical formulas of the circumference and area of circle, the teacher asked the students to find the radius and area of a circle with a given circumference.

Student1: But how?

Student2: I do not understand.

At this point, the teacher interpreted the queries the students had as a result of 'playing' with the computers.

Teacher: We came up to some conclusions. We have been working on the computers for two days now. We should not only play but also find …

Through classroom discussion, the students were able to use the mathematical formula, separate the variables, use division and find the radius and area of a circle with a given circumference.

In this protocol, the teacher is relating exploration of DGE with 'play'. That is, exploration was interpreted by the teacher as 'playing' instead of learning.

Discussion

Analysis of the above protocols indicates that the word 'play' as used by the teacher has differing connotations. This leads to the emergence of two contrasting values, play/learn.

Initially, one value the word ‘play’ entailed was related with the teacher interpreting exploration as ‘playing’ instead of learning. That is, while the teacher would encourage the students to explore an activity in order to reach some conclusions, she would also make a negative comment about this exploration as something that had no didactical value. In protocols 1 and 2, the teacher is relating exploration of DGE that preceded the classroom discussion with ‘play’. That is, exploration was interpreted by the teacher as ‘playing’ *instead* of learning, though, in protocol 2, the student was still exploring the DGE, so as to construct the triangle. In protocol 5, the teacher translated the fact that some students could not really summarize the work which was done previously as ‘play’ instead of learning. Furthermore, in protocol 7, the teacher is relating the exploration of DGE (as in protocol 6 which is described subsequently) with ‘play’, something that has no didactical value. What is striking is the fact that the teacher is referring to the DGE tasks that were designed in collaboration with the researcher in such a way that could initiate the formation of hypotheses and mathematical argumentation. This intervention was followed by the teacher guiding the classroom discussion.

The word ‘play’ had an opposite value when used by the teacher to refer in a general way to the exploration of the activity. In protocols 3 and 4, exploring the features of the DGE and working in pairs for the construction of triangles, and investigation of mathematical relationships accordingly, is translated as something encouraging and constructive. What should be noted though for protocol 4, is that it appears that exploring the environment by following the teacher’s instructions has more value than the students exploring the environment themselves, which is called ‘play’. In protocol 3, the teacher announces that the students will have the opportunity to play a game on the computer. In this protocol, the word play is used with its authentic meaning, even though, from an educational and didactical perspective, it can be considered as a form of reflection, evaluation and further understanding. In protocol 6, the teacher is encouraging investigation and exploration of a mathematical situation that would lead to explanation and justification.

The ‘play’ dichotomy relates to the notion of the play paradox (Hoyles and Noss, 1992) and the notion of the planning paradox (Ainley et al, 2006). Hoyles and Noss (1992) introduce the notion of the play paradox to describe the multiplicity of paths that are available to students when using a tool in an exploration related with a mathematical task. That is, the students, through their exploration, might not encounter the mathematical ideas that were perceived as the objectives set by the teacher or the curriculum materials. Thus, the teacher may decide to close down an exploration opportunity as she may interpret students’ exploration as shifting away from her own objectives. In a similar way, Ainley et al. (2006) call the conflict that may occur in the daily mathematical classrooms, due to contextualize tasks as the planning paradox. This tension may also be related to the notion of ownership as perceived by Papert (1993) in his formulation of Constructionism. That is, while the students are provided with the necessary tools to participate and to take ownership of the learning process, the teacher is at the same time attempting to avoid facing these paradoxes.

Considering the dichotomy related with the word ‘play’ through the CHAT constructs, this tension is a manifestation of a primary contradiction. The primary contradiction that emerges is inherent in the component related with the object of the activity system. In the activity of exploring as part of pre-proving, the object for the teacher is related with exploring triangles and circles. At a first glance, this object seems to be clear and distinct. However, this object is multifaceted. To be more precise, the object for the teacher is related with the investigation of situations that lead to

conclusions related with the aforementioned parts of the mathematics curriculum. The teacher on one hand understands the importance of providing enjoyable exploring opportunities that keep students' motivation and interest to engage with the problem. As a result, the teacher provides opportunities that can be approached by the students in their own way. On the other hand, students, through the exploration of these opportunities are expected to reach those conclusions regarding triangles and circles as pre-determined by the teacher. The two poles of the object lead to a constant struggle in the teacher's everyday practice. The teacher, due to this multifaceted object, is faced with the play/learn dichotomy and thus the play and the planning paradoxes. That is, students' free exploration may lead to paths other than those expected by the teacher. This initially shows that the students share the teacher's object. Thus, the object related with exploring is being reached. However, if the exploration moves away from the teacher's motive, the teacher will inevitably close down the exploration opportunity and guide the students towards the exploration that leads to the conclusions that satisfy her. Time management and the pressure of the coverage of the curriculum further highlight this tension. Inevitably, even though closing down the exploration is necessary, the object will not be met because of this contradiction.

Manifestation of this contradiction leads to a clash between the activity of exploration and explanation and, subsequently, with the way pre-proving activity occurs in the classroom. It has been illustrated that pre-proving activity is closely connected with exploration and explanation. That is, those aspects of reasoning that appear to have the qualities of proving, even though they may not be proving in themselves, entail exploration and explanation that provide a point of reference for proof production. Correspondingly, the object of developing proving in the classroom is related with these notions. The object of the central system of pre-proving activity is related with exploration that leads to explaining and justifying for a specific part of the mathematics curriculum. However, closing down the exploration has an impact on how explanation and justification are established in the classroom. Furthermore, closing down an exploration opportunity may have a negative impact on the students' ability to approach the construction of a proof. Referring to exploration as 'play' may also have a negative impact on students' confidence in relying on their intuitions when exploring a situation.

Concluding remarks

The aim of this paper was to shed some light on the area related with the activity of proving as constituted in the naturalistic setting of the mathematics primary school classroom. The elements that drive pre-proving activity and influence the way proving may be established in the classroom have been identified. That is, in mathematical argumentation, pre-proving is coming out of reasoning through exploring, explaining and justifying and can lead to proving. This paper reports on a teacher whose object is related with exploration that leads to explaining and justifying. However, this object is being conflicted as while a play-like exploration can facilitate learning, this can prove quite challenging for the teacher, as she wishes to maintain focus and is worried that exploring detracts from that focus. The contradiction between emphasizing exploring and maintaining focus is one of the tensions which make the constitution of pre-proving in the classroom inherently complex. However, this does not tell the whole story. Exploring opportunities that were closed down were exploited so as to negotiate and establish socio-mathematical norms in the classroom. As these norms are related with the very nature, functions and characteristics of

proof and proving, they can lead to explaining and justifying. Consequently, their establishment strengthens the activity of explanation and thus, the activity of proving.

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