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A truncated Fourier-transform based approach for time-domain diffuse optical tomography

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Abstract: We present a truncated Fourier series approximation for time-domain diffuse optical tomography. The approach estimated optical parameters with accuracies comparable to using the whole time-resolved data, using very low computational time and resources.

1. Introduction

Time-domain diffuse optical tomography (TD-DOT) uses pulsed near-infrared light for imaging spatially varying optical parameters in biological tissues. The image reconstruction problem of TD-DOT involves estimating spatially varying optical parameters, using time-resolved boundary measurements. Several approaches to solve the image reconstruction problem have been proposed. These include using the whole time-resolved measurement data and using integral-transform based moments of the time-resolved measurement data (e.g, Mellin and Laplace transforms) [1]. Use of such moments has led to a significant reduction of computation time, due to compression of the time-resolved measurement data. A comparison of estimation accuracies obtained using different choices of moments, using one *in silico* target, was presented in [1]. It was shown that using only one moment was inadequate to reconstruct both absorption and scattering parameters simultaneously. Also, there were inter-parameter cross-talks, such that the absorption and scattering estimates significantly affected each other. Alternatively, Gibson *et al.* [2] used difference imaging to reconstruct changes of optical parameters from one frequency of Fourier-transformed data. Here, we propose an approach to reconstruct optical parameters in TD-DOT using a truncated Fourier-series approximation [3].

2. Theory

2.1. Truncated Fourier series approximation

Let us consider the discretized TD-DOT observation model

$$y_t = \Gamma_t(\mu_a, \mu_s') + e. \tag{1}$$

Here, y_t is the noisy time-resolved DOT data, e is the additive noise, Γ_t is a forward model of the time-domain light diffusion equation, and μ_a and μ_s' are the spatially distributed absorption and reduced scattering coefficients. The truncated Fourier series approximation of the boundary measurement due to a (delta-function) source Γ_t^δ is given by,

$$\Gamma_t^\delta = \sum_{k=1}^{\mathrm{N}_\omega} \Gamma(\omega_k)\exp(\mathrm{i}\omega_k t), \text{ where } \Gamma(\omega_k) = \frac{1}{\mathrm{T}}\int_0^{\mathrm{T}} \Gamma_t^\delta \exp(-\mathrm{i}\omega_k t) \tag{2}$$

Here the temporal range of the function Γ_t^δ is [0, T]. For light sources with a finite temporal length, the output Γ_t can be expressed as a convolution of the source pulse Q_t and exitance due to a delta source Γ_t^δ. As such, taking a Fourier transform of measurements Γ_t and using Convolution theorem, we can construct the normalized measurement data [3]

$$\tilde{y} = \frac{y_{\mathrm{FT}}}{\mathrm{Q}_{\mathrm{FT}}} = \begin{pmatrix} \mathrm{Re}(\Gamma(\omega_k)) \\ \mathrm{Im}(\Gamma(\omega_k)) \end{pmatrix} + \frac{e_{\mathrm{FT}}}{\mathrm{Q}_{\mathrm{FT}}} = \Gamma_\omega(\mu_a, \mu_s') + \tilde{e} \tag{3}$$

where y_{FT} corresponds to Fourier-transformed data at frequencies ω_k, Q_{FT} is the Fourier-transform of the source pulse and $\tilde{e} = e_{\mathrm{FT}}/\mathrm{Q}_{\mathrm{FT}}$ is the normalized Fourier-transformed noise. The forward model Γ_ω can be modeled using the frequency-domain light diffusion equation.

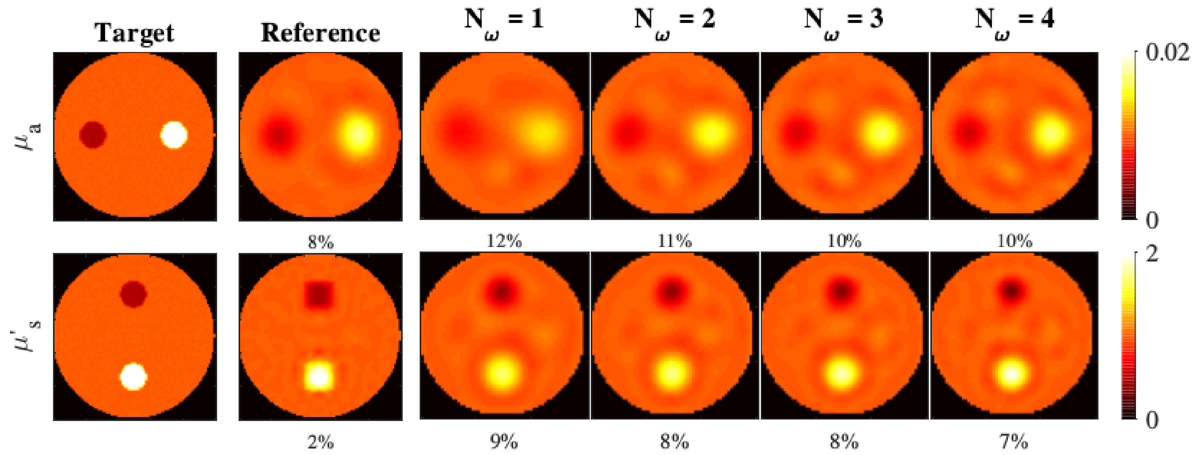

Fig. 1. Absorption (first row) and scattering (second row) distributions. First column: Simulated target. Second column: Reference estimates using whole time-domain data. Columns 3-6: Estimates using one to four Fourier frequencies (N_ω). Below each image is the relative estimation error.

2.2. *Bayesian approach to the inverse problem of TD-DOT*

The parameters (μ_a, μ'_s) were estimated using a Bayesian approach by minimizing

$$(\hat{\mu}_\mathrm{a}, \hat{\mu}'_\mathrm{s}) = \arg\min_{\mu_a, \mu'_s} \{\|L_{\tilde{e}}(\tilde{y} - \Gamma_\omega(\mu_a, \mu'_s) - \tilde{e}_*)\|^2 + \|L_{\mu_a}(\mu_a - \mu_{a,*})\|^2 + \|L_{\mu'_s}(\mu_s - \mu'_{s,*})\|^2\}, \tag{4}$$

using data at different number of frequencies, where $\tilde{e}_*$, is the noise mean and $L_{\tilde{e}}$ is the Cholesky of the inverse of noise covariance. Prior means $(\mu_{a,*}, \mu'_{s,*})$ and Cholesky matrices $(L_{\mu_a}, L_{\mu'_s})$ were specified using Gaussian Ornstein-Uhlenbeck prior. The reference estimates were calculated using the full time-resolved data y_t as

$$(\hat{\mu}_\mathrm{a}, \hat{\mu}'_\mathrm{s}) = \arg\min_{\mu_a, \mu'_s} \{\|L_e(y_t - \Gamma_t(\mu_a, \mu'_s) - e_*)\|^2 + \|L_{\mu_a}(\mu_a - \mu_{a,*})\|^2 + \|L_{\mu'_s}(\mu_s - \mu'_{s,*})\|^2\}, \tag{5}$$

where e_*, is the noise mean and $L_{\tilde{e}}$ is the Cholesky of the inverse of noise covariance.

3. Results

In the numerical studies, the domain was a circle with radius 25mm, shown in the first column of Fig. 1. The measurement setup consisted of 16 sources and 16 detectors, modeled as Gaussian surface patches with 2mm width, located at equi-spaced angular intervals on the boundary. The reference estimate and estimates using four Fourier frequencies ($\mathrm{N}_\omega = 1, \ldots, 4$) is also displayed. The computation time was 18624s for reference estimate; 21s, 18s, 45s and 70s for estimates with N_ω = 1,..,4, i.e, the Fourier transformed estimates required very low computation time compared to the time-resolved data. The relative errors of these estimates are displayed below each image. The estimation errors did not change considerably after four frequencies and hence are not shown.

The results show that the proposed approach can recover reliable estimates of optical parameters, using only a few Fourier frequencies ($\sim$ 4), using significantly low computational resources. Based on the results, we suggest that the truncated Fourier series could provide an efficient modeling protocol for TD-DOT.

References

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