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**Citation for published version:**

He, H, Biswas, S, Aquilina, P, Ratnarajah, T & Yang, J 2020, 'Performance Analysis of Multi-Cell Full-duplex Cellular Networks', *IEEE Access*, vol. 8, pp. 206914 - 206930.  
<https://doi.org/10.1109/ACCESS.2020.3037919>

**Digital Object Identifier (DOI):**

[10.1109/ACCESS.2020.3037919](https://doi.org/10.1109/ACCESS.2020.3037919)

**Link:**

[Link to publication record in Edinburgh Research Explorer](#)

**Document Version:**

Peer reviewed version

**Published In:**

IEEE Access

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# Performance Analysis of Multi-Cell Full-duplex Cellular Networks

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This work was supported in part by Anhui Provincial Natural Science Foundation under Grant 1908085QF266, in part by the Strategic Priority Research Program of the Chinese Academy of Sciences under Grant ZDRW-KT-2016-02, in part by Youth Innovation Promotion Association of the Chinese Academy of Sciences under Grant CX2100107001, in part by the China Electronics Technology Group Corporation (CETC) Joint Advanced Research Foundation under Grant 6141B08080101, and in part by the UK Engineering and Physical Sciences Research Council (EPSRC) under Grant EP/P009549/1.

**ABSTRACT** We analyze the performance of a cellular network, where Poisson point process distributed half-duplex (HD) downlink (DL) and uplink (UL) users are served by multiple full-duplex (FD) base stations (BSs). To address the surge in interference in the network due to the simultaneous operation in time and frequency of the FD BSs, we (a) adopt a self-interference cancellation scheme at each BS, and (b) apply linear interference alignment in each cell to cancel the intra-cell interference. Further, to better capture the distribution of the FD BSs, we model the BSs as a Matérn hard-core point process, in which a minimum distance is imposed between points. The performance of both UL and DL users is analyzed by deriving general expressions and closed-form approximations for the outage probability and throughput. Next, simulations are carried out for both macro and micro cell environments under both FD and HD operations with respect to various network parameters. Our results reveal several fundamental characteristics and the necessary conditions required for the successful deployment of such networks.

**INDEX TERMS** Full-duplex Communication, Interference Alignment, Matérn Hard-core Point Process, Stochastic Geometry.

## I. INTRODUCTION

**F**ULL-DUPLEX (FD) wireless communication, which caters for simultaneous transmission and reception at the same time-frequency band, has attracted a great deal of attention for its potential to double the spectrum efficiency and reduce end-to-end latency when compared to half-duplex (HD) systems. However, the primary predicament of FD systems is the strong self-interference (SI), whereby the transmitted power interferes with the uplink (UL) received signals at the FD nodes [1]–[3]. This issue can be tackled primarily in the propagation domain, where the aim is to limit the amount of SI as much as possible. This can be achieved via the use of directional antennas and antenna separation methods (both in terms of distance and via the use of RF absorbers) [4]. However, such techniques on their own do not suppress the SI to a level that is suitable for communication.

Thanks to some recent breakthroughs in analog and digital domain SI cancellation techniques [5], [6], that have allowed to suppress the SI below the noise floor level and make FD communication a true possibility.

However, SI is not the only challenge in realizing a FD wireless network. In particular, in a multi-cell FD wireless network, all transceivers operate at the same time and frequency, implying that receivers are susceptible to interference from every transmitter. The resulting intra-cell and inter-cell interference can cause significant performance degradation in such networks. Therefore, to implement FD operation in practice, it is fundamental to find appropriate techniques to manage the intra-cell and inter-cell interferences.

Some noteworthy developments in this direction can be found in recent literature with the objective of reducing interference and analyzing the performance of FD enabled

networks. In the following subsection, we mention a few references, which although by no means is exhaustive, fairly indicate the scope and trend of research on FD networks.

#### 1) Related literature

The study on FD communication spans over various system configurations and network topologies in literature. For example, the use of FD communication in cognitive radio networks makes it possible for the secondary users to sense spectrum and transmit at the same time. The spectrum sensing approaches and security requirements for FD cognitive radio networks were summarized in [7], while the motivations and challenges of FD cognitive radio networks were also provided. The outage performance and diversity gain of multirelay systems were analyzed in [8], which proved the high spectrum efficiency of FD multirelay channels. For both two-node and three-node FD architectures, authors in [9] shown that employing directional antennas can significantly reduce the loopback interference. The potential FD techniques including SI cancellation, MAC layer protocol design and relay selection were reviewed and summarized in [10], [11], while [11] provided a survey of in-band FD transmission, which reviewed the existing works on the physical layer performance of in-band FD systems. According to [11], FD network topologies can be generally categorized into three types including bi-directional, relay and cellular topology networks. The bi-directional topology consists of pair FD nodes which transmit and receive signals at the same time and frequency. It has been shown that the bi-directional topology networks have the potential to double the spectrum efficiency over that of HD networks [12], [13]. In FD relay networks, the end-to-end latency can be reduced as a relay node can receive data from a source node and transmit data to a destination node simultaneously within all the same frequency band [11], [14]. As for cellular topology, which is the focus of this paper, BSs or users are capable of FD transmission and BSs can serve DL and UL users in the same frequency band. The spectrum efficiency and power efficiency of single-cell and multi-cell FD networks were studied in [15], where power control and user scheduling were adopted. The up- and downlink ergodic achievable rates of a multi-cell multi-user MIMO full-duplex network were studied in [16], where both FD and HD users were considered and linear precoders and receivers were adopted. To mitigate inter-cell interference in FD ultra-dense small cell networks, a resource split approach was proposed in [17], which classified BS into two groups and allocated half of the resources to each group. The system-level performance of FD MIMO cellular networks was analyzed in [18] while [19] optimized the throughput of a FD wireless network with various transmission strategies. For heterogeneous networks, the interference from cells operating in FD mode was studied in [20], where the throughput of a hybrid FD/HD multi-tier heterogeneous network was derived.

In the studies reported above (i.e. [15]–[20]), it was assumed that BSs are randomly located which means that BSs

can be very close to each other. Applying this assumption to the location of BSs may be inaccurate and inconsistent with the actual scenario, where in practice the locations of BSs are carefully planned. Moreover, inter-cell interference from BSs caused significant performance degradation in UL transmission and it was reported in [18] that sufficient spacing between FD BSs in microcells is necessary to make UL transmission feasible. Therefore, in this paper, we adopt a more general and realistic scenario where BSs are distributed according to a Martérn Hard-core point process (HCPP) [21]. In HCPP, a minimum distance, also referred to as the hard-core distance, is imposed between points. It was shown in [22] that HCPP is more accurate than Poisson point process (PPP) when modeling the locations of BSs in real cellular networks obtained from a public database. By setting a hard-core distance and modeling the locations of BSs as a HCPP, it is possible to suppress the inter-cell interference from BSs.

For networks equipped with FD BSs, DL users suffer from additional co-channel interference (CCI) from UL users within the same cell. Since these interferers are from the same cell, distances are short and the power at which they are received is quite significant. Therefore, particular attention must be paid to the management of intra-cell interference for the practical implementation of FD networks. A highly promising technique that can be used to manage intra-cell interference is interference alignment (IA), initially proposed in [23]. Due to its potential to approach the Shannon capacity of interference networks in the high Signal-to-Noise Ratio (SNR) regime, IA has been widely studied in literature. In [24], a distributed numerical IA approach was proposed for HD interference channels, where only local channel state information (CSI) at each node was required to perform minimum weighted leakage interference (Min-WLI) and maximum signal-to-interference-plus-noise ratio (Max-SINR) based IA. Further, in [25] a minimum mean square error (MMSE) based IA algorithm was also proposed for the HD IC. Similarly, for FD systems, while the degrees of freedom (DoF) region was derived in [26], the feasibility of linear IA was studied in [27]. Both the Max-SINR and MMSE based IA algorithms were used to develop beamformers for FD multi-cell networks in [28]. Further, consider a M-cell network, the scalability of FD's multiplexing gain was characterized and analyzed in [29], where IA was adopted to mitigate the uplink-downlink interference. However, the work in [24]–[29] either considered fixed network settings or ignored the impacts of path loss. Accordingly, in this paper we leverage the analysis of [28] and use linear IA to manage interference in FD networks.

#### 2) Contribution of this paper

There have been few works (e.g. [30]–[33]) investigating HCPP in FD networks. In [31], [32], HCPP was used in simulations to model the locations of BSs, and its statistic characteristics were not leveraged in performance analysis of FD networks. In [30], BSs serving one user each were assumed to distributed according to a PPP while only a

subset of BSs which transmit in the same frequency band was modeled as a HCPP. In [33], a fixed number of DL users was considered, and only the capacity of users was analyzed. To the best of the author's knowledge, this paper is the first to study the performance of a general and realistic FD cellular network where FD BSs are modeled as a HCPP while DL and UL HD users are distributed according to PPPs. Both UL and DL performances are analyzed in terms of outage probability, throughput and ergodic capacity, while closed-form approximations for outage probability and throughput are also provided in this work. Monte Carlo simulations are used to verify the analytical results. The main distinctions and findings of this work are summarized below.

- In contrast to previous works that model the spatial distribution of BSs as a PPP, we adopt a more realistic HCPP model by imposing a hard-core distance between FD BSs, which helps to reduce inter-cell interference. Additionally, we use linear IA to cancel out the intra-cell interference from UL users towards DL users. The performance of both UL and DL transmissions are studied and closed-form approximations for outage probability and throughput are derived.
- The analytical and simulations results show that FD communication is always feasible in micro-cellular networks, while for the case of macro-cellular networks, large hard-core distances between the BSs are required to make the UL transmission possible.
- For micro-cellular networks, our results show that interference from neighboring BSs has the most significant impact on system performance, while residual SI can be ignored due to the low transmit power. By applying linear IA and modeling the BS locations as an HCPP, FD networks can provide better performance than HD networks in terms of user capacity and the ratio of UL user capacity in FD networks to that in HD networks increases with the hard-core distance. Moreover, we show that when the hard-core distance is greater than the critical distance, the UL transmission in FD networks has the potential to double the spectrum efficiency.
- For scenarios where BSs have a limited number of transmit antennas, only a limited number of users can be served. Our results show that it can be assumed that all users can be served when the maximum numbers of DL and UL users (which satisfy the IA feasibility conditions) are equal to or greater than twice the mean numbers of PPP distributed DL and UL users in each cell. Moreover, we show that UL user performance can be improved by increasing the maximum number of UL users which satisfies the IA feasibility conditions, while in low and high targeted SINR regime, reducing the maximum number of DL users served by each BS has limited impact on UL performance.
- Finally, compared to PPP-based model, the inter-cell interference can be suppressed by increasing the hard-core distance between BSs. In particular, DL transmis-

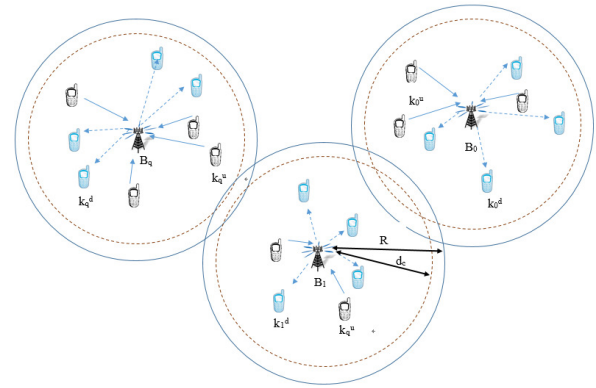


FIGURE 1: System model of a multiple cell network with FD BSs and HD users.

sion can be improved when utilizing FD BSs, while UL transmission can outperform conventional HD networks when the hard-core distance is greater than a specific threshold.

### 3) Notations

Throughout the paper, matrices and vectors are denoted by bold uppercase and bold lowercase letters, respectively. We will use  $B_q$  to represent the  $q$ th BS and  $k_q^d$  ( $k_q^u$ ) to denote the  $k$ th DL(UL) user served by the  $q$ th BS. Circularly symmetric complex Gaussian distribution with mean  $\mu$  and variance  $\tau^2$  is denoted as  $\mathcal{CN}(\mu, \tau^2)$ , while  $\mathcal{U}[n, m]$  represents uniform distribution between  $n$  and  $m$ . Transpose operator of a matrix is denoted as  $(\cdot)^T$  and  $\det(\cdot)$  is the determinant of a matrix. Expectation operation and natural logarithm are represented by  $\mathbb{E}[\cdot]$  and  $\ln(\cdot)$ . Finally,  $\mathbb{C}^{n \times m}$  is used to represent the set of  $n \times m$  complex matrices.

## II. SYSTEM MODEL

We consider a similar setting in [33], where the locations of multiple FD BSs are modeled as a HCPP  $\Phi_M$  with intensity  $\lambda_M$  and hard-core distance  $d_c$ . As shown in Fig.1, each BS is equipped with  $M_d$  transmit antennas and  $M_u$  receive antennas to serve multiple DL and UL HD users in circular regions with radius  $R$ , centered around the BSs. Defer from [33], for each BS the associated DL and UL users are randomly distributed in the circular region according to PPP with intensities  $\lambda_P^d$  and  $\lambda_P^u$  respectively [34]. Furthermore, DL users, requiring  $b_d$  streams each, are equipped with  $N_d$  antennas; while UL users, requiring  $b_u$  streams each, are equipped with  $N_u$  antennas.

Without loss of generality, we consider that a typical BS denoted as  $B_0$  and its associated DL and UL users are in  $\varphi_0$  and  $\phi_0$ , respectively. All UL and DL users are assumed to be randomly ordered. BS antennas have a gain of  $G_b$ , while user antennas are unit-gain. The signal received at the  $k$ th DL user

and the UL signal seen at  $B_0$  are respectively given as

$$\begin{aligned} \mathbf{s}_{k_0^d} = & \sum_{i_0^d \in \varphi_0} \sqrt{\frac{G_b \beta}{l_{k_0^d, B_0}^{\alpha_u}}} \mathbf{H}_{k_0^d, B_0} \mathbf{V}_{i_0^d} \mathbf{s}_{i_0^d} + \sum_{j_0^u \in \phi_0} \sqrt{\frac{\beta_u}{l_{k_0^d, j_0^u}^{\alpha_u}}} \mathbf{H}_{k_0^d, j_0^u} \mathbf{V}_{j_0^u} \mathbf{s}_{j_0^u} \\ & + \sum_{B_q \in \Phi_{Mq} \neq 0} \left( \sum_{i_q^d \in \varphi_q} \sqrt{G_b \beta l_{k_0^d, B_q}^{-\alpha_u}} \mathbf{H}_{k_0^d, B_q} \mathbf{V}_{i_q^d} \mathbf{s}_{i_q^d} \right. \\ & \left. + \sum_{j_q^u \in \phi_q} \sqrt{\beta_u l_{k_0^d, j_q^u}^{-\alpha_u}} \mathbf{H}_{k_0^d, j_q^u} \mathbf{V}_{j_q^u} \mathbf{s}_{j_q^u} \right) + \mathbf{n}_{k_0^d}, \end{aligned} \quad (1)$$

$$\begin{aligned} \mathbf{s}_{k_0^u} = & \sum_{j_0^u \in \phi_0} \sqrt{\frac{G_b \beta}{l_{B_0, j_0^u}^{\alpha_b}}} \mathbf{H}_{B_0, j_0^u} \mathbf{V}_{j_0^u} \mathbf{s}_{j_0^u} + \sum_{i_0^d \in \varphi_0} \sqrt{G_b^2 \beta_b} \mathbf{H}_{B_0, B_0} \mathbf{V}_{i_0^d} \mathbf{s}_{i_0^d} \\ & + \sum_{B_q \in \Phi_{Mq} \neq 0} \left( \sum_{i_q^d \in \varphi_q} \sqrt{G_b^2 \beta_b l_{B_0, B_q}^{-\alpha_b}} \mathbf{H}_{B_0, B_q} \mathbf{V}_{i_q^d} \mathbf{s}_{i_q^d} \right. \\ & \left. + \sum_{j_q^u \in \phi_q} \sqrt{G_b \beta l_{B_0, j_q^u}^{-\alpha_b}} \mathbf{H}_{B_0, j_q^u} \mathbf{V}_{j_q^u} \mathbf{s}_{j_q^u} \right) + \mathbf{n}_{B_0}. \end{aligned} \quad (2)$$

Here,  $l_{k_0^d, B_0}$  and  $l_{B_0, j_0^u}$ , denote the distance from  $B_0$  to the  $k$ th DL user and the  $j$ th UL user associated with BS  $B_q$ , respectively. Due to the random locations of UL and DL users, the probability density functions (PDFs) of  $l_{k_0^d, B_0}$  and  $l_{B_0, j_0^u}$  can be written as

$$f_{l_{k_0^d, B_0}}(r) = f_{l_{B_0, j_0^u}}(r) = \frac{2r}{R^2}. \quad (3)$$

We use  $l_{k_0^d, j_q^u}$  to represent the distance from DL user  $k_0^d$  to UL user  $j_q^u$ , and the distance from BS  $B_q$  to  $B_0$  is expressed as  $l_{B_0, B_q}$ . The path loss intercepts of BS-user, user-user and BS-BS are denoted as  $\beta$ ,  $\beta_u$  and  $\beta_b$  respectively, while  $\alpha_u$  and  $\alpha_b$  are the corresponding path loss exponents. Since a multi-slope model can better capture the pathloss among BSs when compared with a single slope model [18], [35], we adopt a two-slope pathloss model for the BS-BS link. For a given distance  $l_{B_0, B_q}$ , we have [33], [35]

$$\begin{cases} \beta_b = \beta_{b_1}, \alpha_b = \alpha_{b_1} & l_{B_0, B_q} \leq R_c \\ \beta_b = \beta_{b_2}, \alpha_b = \alpha_{b_2} & l_{B_0, B_q} > R_c \end{cases} \quad (4)$$

where  $R_c = 4h_{BS}^2/\xi$  is the critical distance,  $h_{BS}$  is the BS antenna height and  $\xi$  is the wavelength.  $\mathbf{H}_{i_0^d, B_q} \in \mathbb{C}^{N_d \times M_d}$  represents the channel from BS  $B_q$  to DL user  $i_0^d$ , and the channel between the  $j$ th UL user communicated with BS  $B_q$  and user  $i_0^d$  is denoted as  $\mathbf{H}_{i_0^d, j_q^u} \in \mathbb{C}^{N_d \times N_u}$ . The channel between BSs  $B_0$  and  $B_q$  is represented as  $\mathbf{H}_{B_0, B_q} \in \mathbb{C}^{M_u \times M_d}$ , while SI channel at  $B_0$  is denoted as  $\mathbf{H}_{B_0, B_0} \in \mathbb{C}^{M_u \times M_d}$ . According to [18], small scale fading between BSs can be modelled as Rayleigh, Rician or disregarded. To keep consistent with BS-user, user-BS and user-user links, we adopt Rayleigh fading for BS-BS links [36]. It is assumed that all channel matrix elements are independent and identically distributed (i.i.d.) complex normal random variables with zero mean and variance one. Moreover,  $\mathbf{s}_{i_0^d} \in \mathbb{C}^{b_d \times 1}$  is the data intend for the  $i$ th DL user associated with BS  $B_q$ , with  $\mathbb{E}\{\mathbf{s}_{i_0^d} \mathbf{s}_{i_0^d}^H\} = P_d \mathbf{I}$ ; while  $\mathbf{s}_{j_q^u}$  represents the transmitted data intended for UL user  $j_q^u$ ,

with  $\mathbb{E}\{\mathbf{s}_{j_q^u} \mathbf{s}_{j_q^u}^H\} = P_u \mathbf{I}$ . The precoders for  $\mathbf{s}_{i_0^d}$  and  $\mathbf{s}_{j_q^u}$  are  $\mathbf{V}_{i_0^d} \in \mathbb{C}^{M_d \times b_d}$  and  $\mathbf{V}_{j_q^u} \in \mathbb{C}^{N_u \times b_u}$ , respectively. Finally,  $\mathbf{n}_{i_0^d}$  and  $\mathbf{n}_{B_0}$  represent additive white Gaussian noise (AWGN) seen at the DL users and the BSs respectively, and both have zero mean and variance  $\sigma^2 \mathbf{I}$ .

Since each BS only serves DL and UL users in its own cell, it is reasonable to assume that perfect CSI is shared between a BS and its associated users. The CSI for the link from the UL users to the DL users can be obtained by measuring the pilot signals from UL users, and then the DL users transmit the CSI to BSs. Unlike other IA related works (e.g. [23], [25]–[28]) where global CSI was assumed, in this work we use only CSI within the cell to keep CSI overheads within acceptable limits. Therefore, intra-cell interference can be eliminated by adopting linear IA techniques. Assuming  $\mathbf{U}_{k_0^d} \in \mathbb{C}^{N_d \times b_d}$  is the unitary receive beamformer applied at DL user  $k_0^d$  and  $\mathbf{U}_{k_0^u} \in \mathbb{C}^{N_u \times b_u}$  is the unitary receiver applied at  $B_0$  to extract the data from UL user  $k_0^u$ , the feasibility conditions for IA to eliminate intra-cell interference are given by [27]

$$|\mathbf{u}_{k_0^d, t}^H \mathbf{H}_{k_0^d, B_0} \mathbf{v}_{k_0^d, t}| > 0 \quad \forall k, t \quad (5a)$$

$$|\mathbf{u}_{k_0^u, t}^H \mathbf{H}_{B_0, k_0^u} \mathbf{v}_{k_0^u, t}| > 0 \quad \forall k, t \quad (5b)$$

$$\mathbf{u}_{k_0^d, t}^H \mathbf{H}_{k_0^d, B_0} \mathbf{v}_{i_0^d, j} = 0 \quad \forall k, t, i, j, (k, t \neq i, j) \quad (5c)$$

$$\mathbf{u}_{k_0^u, t}^H \mathbf{H}_{B_0, k_0^u} \mathbf{v}_{i_0^u, j} = 0 \quad \forall k, t, i, j, (k, t \neq i, j) \quad (5d)$$

$$\mathbf{u}_{k_0^d, t}^H \mathbf{H}_{k_0^d, i_0^u} \mathbf{v}_{i_0^u, j} = 0 \quad \forall k, t, i, j. \quad (5e)$$

Here,  $\mathbf{u}_{k_0^d, t}$ ,  $\mathbf{u}_{k_0^u, t}$ ,  $\mathbf{v}_{k_0^d, t}$  and  $\mathbf{v}_{k_0^u, t}$  are the  $t$ th column of the beamformers  $\mathbf{U}_{k_0^d}$ ,  $\mathbf{U}_{k_0^u}$ ,  $\mathbf{V}_{k_0^d}$  and  $\mathbf{V}_{k_0^u}$  respectively. IA techniques designed for a single-cell with a FD BS serving multiple HD UL and DL users are already available in the literature, with both the MMSE based design (Algorithm 1 in [28]) and Max-SINR based technique (Algorithm 2 in [28]) being suited to our scenario. Using any of the two iterative techniques from [28] to design both precoders and decoders would achieve the IA conditions in (5) for the scenario outlined in this paper. For illustration, Fig. 2 shows the performance comparison between the FD-Max-SINR and FD-MMSE algorithms for a system with 2 FD BSs, 2 DL and 2 UL users associated with each BS and  $b_d = b_u = 2$  with respect to varying antenna number pairs<sup>1</sup>. From the figure it can be seen that while both algorithms show similar sum rate performance, the processing time required for max-SINR algorithm is more than that of the MMSE based one due to its lower computational complexity. Hence, without loss of generality, in this paper we consider the MMSE based design from [28] for our analysis.

Further, Fig. 2.a also illustrates the IA feasibility conditions with respect to the number of antennas at the BSs and

<sup>1</sup>Fig. 2 has been generated by simulating the algorithms proposed in [28] in a centralized manner over 500 Monte Carlo iterations in MATLAB R2017a on a Windows 10 Pro PC (Intel i5-8300 processor (4 cores, each clocked at 2.30 GHz), 8 GB of memory and 8 GB of AMD Radeon RX 570 graphics memory).

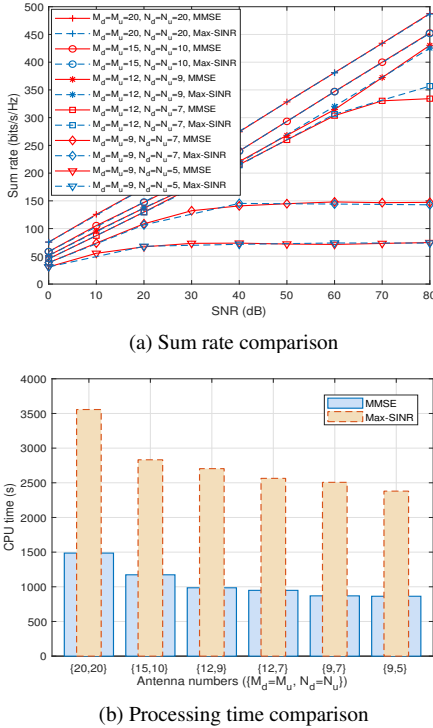


FIGURE 2: Comparison between FD MMSE and FD Max-SINR IA algorithms.

users. In particular, for the antenna pairs  $\{M_d = M_u, N_d = N_u\} = \{20, 20\}, \{15, 10\}, \{12, 9\}$ , the curves have the same slope and the network achieves full DoF. However for  $\{M_d = M_u, N_d = N_u\} = \{12, 7\}, \{9, 7\}, \{9, 5\}$ , the sum rates attenuate with increasing SNR, which indicate that IA is infeasible. At this point, it is worth noting that since the UL users in this paper are modelled as a PPP, the number of UL users in a cell is a Poisson random variable (RV). Therefore, to meet the feasibility conditions of IA, we consider two different scenarios as mentioned below.

- **Scenario I:** The number of antennas at each BS is assumed to be large enough so that the feasibility conditions of IA can always be met, regardless of the number of DL and UL users, required streams and the number of antennas at the users.
- **Scenario II:** In this case, we assume only a limited number of antennas is available at each BS. Given the number of available antennas (at both BS and users) and the amount of data streams required at each DL and UL user, for IA to be feasible within each cell, only  $K_d^{max}$  out of a total  $K_d$  DL users and  $K_u^{max}$  out of a total  $K_u$  UL users can be served by each BS. Note that  $K_d^{max}$  and  $K_u^{max}$  are the maximum numbers of DL and UL users that can be served by each BS in order for IA to be feasible within the cell. Its value can be obtained from the IA feasibility study in [27]. Since the transmit power of BS is higher than that of user, we assume the  $K_d^{max}$  served DL users are randomly selected. The  $K_u^{max}$  users

selected to be served are those nearest to the BS, and signals from the rest of the  $K_u - K_u^{max}$  UL users are treated as residual intra-cell interference.

Moreover, it can be reasonably assumed that an interference cancellation scheme (i.e. analog/digital cancellation) is used at each FD BSs to mitigate the effects of SI on system performance. Since an interference cancellation scheme is adopted at the BSs to mitigate the effects of SI, the strong line-of-sight component in SI can be estimated and removed similar to [4]. Additionally, a specific power value can be used to model the effect of the implementation of analog/digital SI cancellation, and represent the amount of residual SI left [37]. Thus, we model the entries of the residual SI channel as i.i.d. complex normal RVs with zero mean and variance  $\sigma_s^2$ . Using IA to cancel out intra-cell interference and applying the SI cancellation schemes to mitigate the effect of SI, the estimated signals can be written as

$$\begin{aligned} \hat{\mathbf{s}}_{k_0^d} = & \sqrt{G_b \beta l_{k_0^d, B_0}^{-\alpha}} \mathbf{U}_{k_0^d}^H \mathbf{H}_{k_0^d, B_0} \mathbf{V}_{k_0^d} \mathbf{s}_{k_0^d} \\ & + \delta \sum_{j_0^u \in \phi_0'} \sqrt{\beta_u l_{k_0^d, j_0^u}^{-\alpha_u}} \mathbf{U}_{k_0^d}^H \mathbf{H}_{k_0^d, j_0^u} \mathbf{V}_{j_0^u} \mathbf{s}_{j_0^u} \\ & + \sum_{B_q \in \Phi_M, q \neq 0} \mathbf{U}_{k_0^d}^H \left( \sum_{i_q^d \in \tilde{\phi}_q} \sqrt{G_b \beta l_{k_0^d, B_q}^{-\alpha}} \mathbf{H}_{k_0^d, B_q} \mathbf{V}_{i_q^d} \mathbf{s}_{i_q^d} \right. \\ & \left. + \sum_{j_q^u \in \phi_q} \sqrt{\beta_u l_{k_0^d, j_q^u}^{-\alpha_u}} \mathbf{H}_{k_0^d, j_q^u} \mathbf{V}_{j_q^u} \mathbf{s}_{j_q^u} \right) + \mathbf{U}_{k_0^d}^H \mathbf{n}_{k_0^d}, \quad (6) \end{aligned}$$

$$\begin{aligned} \hat{\mathbf{s}}_{k_0^u} = & \sqrt{\frac{G_b \beta}{l_{B_0, k_0^u}^{-\alpha}}} \mathbf{U}_{k_0^u}^H \mathbf{H}_{B_0, k_0^u} \mathbf{V}_{k_0^u} \mathbf{s}_{k_0^u} + \sum_{i_0^d \in \tilde{\phi}_0} \sqrt{G_b^2 \beta_b} \mathbf{U}_{k_0^u}^H \hat{\mathbf{H}}_{B_0, B_0} \mathbf{V}_{i_0^d} \mathbf{s}_{i_0^d} \\ & + \delta \sum_{j_0^u \in \phi_0'} \sqrt{G_b \beta l_{B_0, j_0^u}^{-\alpha}} \mathbf{U}_{k_0^u}^H \mathbf{H}_{B_0, j_0^u} \mathbf{V}_{j_0^u} \mathbf{s}_{j_0^u} \\ & + \sum_{B_q \in \Phi_M, q \neq 0} \mathbf{U}_{k_0^u}^H \left( \sum_{i_q^d \in \tilde{\phi}_q} \sqrt{G_b^2 \beta_b l_{B_0, B_q}^{-\alpha_b}} \mathbf{H}_{B_0, B_q} \mathbf{V}_{i_q^d} \mathbf{s}_{i_q^d} \right. \\ & \left. + \sum_{j_q^u \in \phi_q} \sqrt{G_b \beta l_{B_0, j_q^u}^{-\alpha}} \mathbf{U}_{k_0^u}^H \mathbf{H}_{B_0, j_q^u} \mathbf{V}_{j_q^u} \mathbf{s}_{j_q^u} \right) + \mathbf{U}_{k_0^u}^H \mathbf{n}_{B_0}, \quad (7) \end{aligned}$$

where  $\tilde{\phi}_q = (1 - \delta)\phi_q + \delta\phi_q'$  and  $\phi_q'$  is the subset of served DL users.  $\phi_0'$  is the subset of  $\phi_0$  except for the  $K_u^{max}$  nearest UL users and  $\hat{\mathbf{H}}_{B_0, B_0}$  represents the residual SI channel, with the elements of  $\hat{\mathbf{H}}_{B_0, B_0}$  being i.i.d.  $\mathcal{CN}(0, \sigma_s^2)$ . For case I  $\delta = 0$ , while  $\delta = 1$  in case II.

### III. PRELIMINARIES

In this section, we provide some mathematical preliminaries which will be used in this paper.

#### A. DISTRIBUTIONS OF CHANNEL SQUARED NORMS

Assuming single-stream decoding is performed at DL users and BSs, the distributions of channel squared norms are provided in following lemma.

**Lemma 1.** For the channel between a ( $a \in \{B_0, k_0^d\}$ ) and  $b$  ( $b \in \{B_q, i_q^u\}$  and  $q \neq 0$ ), we have  $2 \times \|\mathbf{u}_{k_0^d, t}^H \mathbf{H}_{a, b} \mathbf{V}_{i_q^y}\|^2$  ( $y \in \{d, u\}$ ) is a Chi-squared distributed RV with  $2b_y$  degrees of freedom (DoF). The PDF of  $\|\mathbf{u}_{k_0^d, t}^H \mathbf{H}_{a, b} \mathbf{V}_{i_q^y}\|^2$  can be expressed as

$$f_{\chi^2}(2x; 2b_y) = \frac{x^{b_y-1}}{2\Gamma(b_y)} e^{-x} \quad y \in \{d, u\}. \quad (8)$$

Especially, when  $q = 0$  and  $k = i$ , we have  $\|\mathbf{u}_{k_0^d, t}^H \mathbf{H}_{k_0^d, B_0} \mathbf{V}_{k_0^d}\|^2$  and  $\|\mathbf{u}_{i_0^u, t}^H \mathbf{H}_{B_0, i_0^u} \mathbf{V}_{i_0^u}\|^2$  are exponentially distributed RVs with unit mean.

*Proof.* Since all columns of  $\mathbf{U}_{k_0^y}$  and  $\mathbf{V}_{i_q^y}$  have unit norm,  $\mathbf{u}_{k_0^d, t}$  and  $\mathbf{V}_{i_q^y}$  have no impact on the signal distribution. Moreover, all entries of  $\mathbf{H}_{a, b}$  are complex Gaussian distributed RVs and  $\text{rank}\{\mathbf{U}_{k_0^y} \mathbf{H}_{a, b} \mathbf{V}_{i_q^y}\} = b_y$  when  $q \neq 0$ . Thus,  $2 \times \|\mathbf{u}_{k_0^d, t}^H \mathbf{H}_{a, b} \mathbf{V}_{i_q^y}\|^2$  is a Chi-squared distributed RV with  $2b_y$  DoF. When  $q = 0$  and  $k = i$ , applying the IA conditions we have  $\|\mathbf{u}_{k_0^d, t}^H \mathbf{H}_{a, b} \mathbf{V}_{k_0^y}\|^2 = \|\mathbf{u}_{k_0^d, t}^H \mathbf{H}_{a, b} \mathbf{v}_{k_0^y}\|^2$ . Thus,  $\|\mathbf{u}_{k_0^d, t}^H \mathbf{H}_{a, b} \mathbf{V}_{k_0^y}\|^2$  is Chi-squared distributed with 2 DoF (i.e. exponentially distributed with unit mean).  $\square$

### B. A USEFUL LEMMA FOR CAPACITY ANALYSIS

To analyse the capacity of fading interference channel, a useful lemma is introduced in this subsection.

**Lemma 2.** For arbitrary non-negative random variables  $\{x_i | i = 1, 2, \dots, N\}$  and  $\{y_j | j = 1, 2, \dots, M\}$ , the following equation can be obtained [38]

$$\mathbb{E} \left[ \ln \left( 1 + \frac{\sum_{i=1}^N x_i}{\sum_{j=1}^M y_j + 1} \right) \right] = \int_0^\infty \frac{\mathcal{M}_y(z) - \mathcal{M}_{x, y}(z)}{z} e^{-z} dz \quad (9)$$

where  $\mathcal{M}_y(z) = \mathbb{E} \left[ e^{-z \sum_{j=1}^M y_j} \right]$  and  $\mathcal{M}_{x, y}(z) = \mathbb{E} \left[ e^{-z(\sum_{i=1}^N x_i + \sum_{j=1}^M y_j)} \right]$ .

## IV. PERFORMANCE ANALYSIS

### A. PERFORMANCE OF A GENERIC DL USER SERVED BY FD BSS

1) Outage probability for each data stream of a generic DL user

Assuming all DL users perform single-stream decoding as in [39], the  $t$ th data stream of DL user  $k_0^d$  is given by

$$\begin{aligned} s_{k_0^d, t} &= \sqrt{G_b \beta l_{k_0^d, B_0}^{-\alpha}} \mathbf{u}_{k_0^d, t}^H \mathbf{H}_{k_0^d, B_0} \mathbf{V}_{k_0^d} \mathbf{s}_{k_0^d} \\ &+ \delta \sum_{j_0^u \in \phi'_0} \sqrt{\beta_u l_{k_0^d, j_0^u}^{-\alpha_u}} \mathbf{u}_{k_0^d, t}^H \mathbf{H}_{k_0^d, j_0^u} \mathbf{V}_{j_0^u} \mathbf{s}_{j_0^u} \\ &+ \sum_{B_q \in \Phi_M, q \neq 0} \left( \sum_{i_q^d \in \tilde{\phi}_q} \sqrt{G_b \beta l_{k_0^d, B_q}^{-\alpha}} \mathbf{u}_{k_0^d, t}^H \mathbf{H}_{k_0^d, B_q} \mathbf{V}_{i_q^d} \mathbf{s}_{i_q^d} \right. \\ &\left. + \sum_{j_q^u \in \phi_q} \sqrt{\beta_u l_{k_0^d, j_q^u}^{-\alpha_u}} \mathbf{u}_{k_0^d, t}^H \mathbf{H}_{k_0^d, j_q^u} \mathbf{V}_{j_q^u} \mathbf{s}_{j_q^u} \right) + \mathbf{u}_{k_0^d, t}^H \mathbf{n}_{k_0^d} \quad (10) \end{aligned}$$

where  $\mathbf{u}_{k_0^d, t}$  is the  $t$ th column of  $\mathbf{U}_{k_0^d}$ . The SINR of the  $t$ th data stream at user  $k_0^d$  is given by

$$\text{SINR}_{k_0^d, t} = \frac{G_b \beta l_{k_0^d, B_0}^{-\alpha} P_d \|\mathbf{u}_{k_0^d, t}^H \mathbf{H}_{k_0^d, B_0} \mathbf{V}_{k_0^d}\|^2}{I_{inter}^d + I_{RUL}^d + \sigma^2}. \quad (11)$$

Here,  $I_{inter}^d = I_{BS}^d + I_{UL}^d$  is the inter-cell interference power, while  $I_{BS}^d$  and  $I_{UL}^d$  denote the interference power from inter-cell BSs and UL users respectively. Thus, we have

$$I_{BS}^d = \sum_{B_q \in \Phi_M, q \neq 0} G_b \beta l_{k_0^d, B_q}^{-\alpha} P_d \sum_{i_q^d \in \tilde{\phi}_q} \|\mathbf{u}_{k_0^d, t}^H \mathbf{H}_{k_0^d, B_q} \mathbf{V}_{i_q^d}\|^2 \quad (12)$$

$$I_{UL}^d = \sum_{B_q \in \Phi_M, q \neq 0} \beta_u P_u \sum_{j_q^u \in \phi_q} l_{k_0^d, j_q^u}^{-\alpha_u} \|\mathbf{u}_{k_0^d, t}^H \mathbf{H}_{k_0^d, j_q^u} \mathbf{V}_{j_q^u}\|^2 \quad (13)$$

and the power of residual intra-cell interference from UL users is given by

$$I_{RUL}^d = \delta \beta_u P_u \sum_{j_0^u \in \phi'_0} l_{k_0^d, j_0^u}^{-\alpha_u} \|\mathbf{u}_{k_0^d, t}^H \mathbf{H}_{k_0^d, j_0^u} \mathbf{V}_{j_0^u}\|^2. \quad (14)$$

Applying the IA conditions, we have  $\|\mathbf{u}_{k_0^d, t}^H \mathbf{H}_{k_0^d, B_0} \mathbf{V}_{k_0^d}\|^2 = \|\mathbf{u}_{k_0^d, t}^H \mathbf{H}_{k_0^d, B_0} \mathbf{v}_{k_0^d}\|^2$ . Since all elements of channel matrices are i.i.d. complex normal random variables with zero mean and variance one,  $\|\mathbf{u}_{k_0^d, t}^H \mathbf{H}_{k_0^d, B_0} \mathbf{V}_{k_0^d}\|^2$  is Chi-squared distributed with 2 degrees of freedom (DoF) (i.e. exponentially distributed with unit mean) and the conditional outage probability (also referred to as cumulative density function (CDF) of  $\text{SINR}_{k_0^d, t}$ ) of the  $t$ th data stream at user  $k_0^d$  is given by

$$\begin{aligned} \mathcal{P} \left( \text{SINR}_{k_0^d, t} < z | l_{k_0^d, B_0} \right) \\ = 1 - \mathbb{E} \left[ \exp \left( -\frac{z l_{k_0^d, B_0}^{-\alpha}}{G_b \beta P_d} (I_{inter}^d + I_{RUL}^d + \sigma^2) \right) \right]. \quad (15) \end{aligned}$$

Using the fact that  $I_{inter}^d$  and  $I_{RUL}^d$  are independent, we have

$$\begin{aligned} \mathcal{P} \left( \text{SINR}_{k_0^d, t} < z | l_{k_0^d, B_0} \right) \\ = 1 - \mathcal{L}_{I_{inter}^d | l_{k_0^d, B_0}}(\mathcal{S}_d) \mathcal{L}_{I_{RUL}^d | l_{k_0^d, B_0}}(\mathcal{S}_d) e^{-\mathcal{S}_d \sigma^2} \quad (16) \end{aligned}$$

where  $\mathcal{S}_d = \frac{z l_{k_0^d, B_0}^{-\alpha}}{G_b \beta P_d}$ ,  $\mathcal{L}_{I_{inter}^d | l_{k_0^d, B_0}}(\mathcal{S}_d) = \mathbb{E}_{I_{inter}^d} \left[ e^{-\mathcal{S}_d I_{inter}^d} \right]$  and  $\mathcal{L}_{I_{RUL}^d | l_{k_0^d, B_0}}(\mathcal{S}_d) = \mathbb{E}_{I_{RUL}^d} \left[ e^{-\mathcal{S}_d I_{RUL}^d} \right]$  are the Laplace transform of  $I_{inter}^d$  and  $I_{RUL}^d$ , respectively, conditioned on given  $l_{k_0^d, B_0}$ .

Using the density function in (3), the unconditioned outage probability of  $k_0^d$  is obtained as

$$\mathcal{P} \left( \text{SINR}_{k_0^d, t} < z \right) = \int_0^R f_{l_{k_0^d, B_0}}(r) \mathcal{P} \left( \text{SINR}_{k_0^d, t} < z | r \right) dr. \quad (17)$$

Considering the inter-cell interference first, the expression for  $\mathcal{L}_{I_{inter}^d | l_{k_0^d, B_0}}(\mathcal{S}_d)$  can be found in the following proposition.

**Proposition 1.** For a given  $l_{k_0^d, B_0}$ , the coverage probability of the  $t$ th data stream at DL user  $k_0^d$  affected by inter-cell interference is given by (18), where  $G_\varphi^d(x)$  can be written as

$$G_\varphi^d(x) = (1 - \delta)e^{-\mu_d(1-x)} + \delta \left( x^{K_d^{max}} + \sum_{K_d=0}^{K_d^{max}-1} P_d(K_d) \left( x^{K_d} - x^{K_d^{max}} \right) \right). \quad (19)$$

and  $\mu_d = \pi R^2 \lambda_P^d$ ,  $P_d(K_d) = e^{-\mu_d} \frac{\mu_d^{K_d}}{K_d!}$  is the probability that there are  $K_d$  DL users associated with the BS.  $G_\phi^d(l, \theta)$  can be expressed as (20),  $l_{B_0, j_q^u} = \sqrt{l^2 + r^2 - 2lr \cos(\vartheta)}$ ,  $\theta_{j_q^u} = \theta - \arcsin\left(\frac{r}{l_{B_0, j_q^u}} \sin(\vartheta)\right)$  and  $\lambda_{MP} = -\frac{1}{\pi d_c^2} \ln(1 - \pi d_c^2 \lambda_M)$  [21] is the density of parent PPP.  $\rho_M(l)$  is the conditional thinning probability of the MHCPP  $\Phi_M$  given as

$$\rho_M(l) = \begin{cases} \rho_1(l) = \frac{\kappa_1(l, d_c)}{\kappa_2} & \text{if } d_c < l < 2d_c \\ \rho_2(l) = \frac{\lambda_M}{\lambda_{MP}} & \text{if } l \geq 2d_c \\ 0 & \text{otherwise.} \end{cases} \quad (21)$$

Here,  $\kappa_2 = \frac{1 - \exp(-\lambda_{MP} \pi d_c^2)}{\lambda_{MP} \pi d_c^2}$  is the MHCPP retention probability [40] and  $\kappa_1(l, d_c)$  can be written as [41]

$$\kappa_1(l, d_c) = \frac{1}{\lambda_{MP}^2} \left[ \frac{1 - e^{-S_0 \lambda_{MP}}}{S_0^2 - S_0 S_2(l, d_c)} + \frac{e^{\lambda_{MP}(S_2(l, d_c) - 2S_0)} - 1}{-3S_0 S_2(l, d_c) + S_2(l, d_c)^2 + 2S_0^2} + \frac{1 - e^{-\lambda_{MP}(S_1(l, d_c) - S_2(l, d_c) + S_0)}}{S_1(l, d_c) - S_2(l, d_c) + S_0} + \frac{e^{\lambda_{MP}(S_2(l, d_c) - 2S_0)} - 1}{2S_0 - S_2(l, d_c)} \right] \quad (22)$$

where  $S_0 = \pi d_c^2$ ,  $S_1(l, d_c)$  and  $S_2(l, d_c)$  are given by

$$S_1(l, d_c) = \begin{cases} 2d_c^2 \cos^{-1}\left(\frac{l}{2d_c}\right) - \frac{1}{2}l\sqrt{4d_c^2 - l^2} & : 0 < l \leq 2d_c \\ 0 & : l > 2d_c \end{cases} \quad (23)$$

$$S_2(l, d_c) = \begin{cases} \pi l^2 & : 0 < l \leq d_c/2 \\ l^2 \cos^{-1}\left(1 - \frac{d_c^2}{2l^2}\right) + d_c^2 \cos^{-1}\left(\frac{d_c}{2l}\right) & : d_c/2 < l \leq d_c \\ -\frac{1}{2}d_c \sqrt{4l^2 - d_c^2} & : l > \frac{d_c}{2} \end{cases} \quad (24)$$

*Proof.* See Appendix A.  $\square$

Given the expression of  $\mathcal{L}_{I_{inter}^d|k_0^d, B_0}(S_d)$ ,  $\mathcal{L}_{I_{UL}^d|k_0^d, B_0}(S_d)$  (the coverage probability of the  $t$ th data stream at  $k_0^d$  affected by  $I_{UL}^d$ ) can be obtained easily by setting

$$G_\varphi^d \left( \left( 1 + \frac{z l_{k_0^d, B_0}^\alpha}{(l_{k_0^d, B_0}^2 + l^2 - 2l_{k_0^d, B_0} l \cos(\theta))^{\frac{\alpha}{2}}} \right)^{-bd} \right) = 1, \text{ i.e.}$$

$I_{BS}^d = 0$ . Similarly, the coverage probability of the  $t$ th data stream at  $k_0^d$  affected by  $I_{BS}^d$  and denoted as  $\mathcal{L}_{I_{BS}^d|k_0^d, B_0}(S_d)$ , can be obtained by substituting  $G_\phi^d(l, \theta) = 1$  into (18). Moreover, when  $d_c \gg R$  and  $\delta = 1$ , we provide a closed-form approximation of  $\mathcal{L}_{I_{BS}^d|k_0^d, B_0}(S_d)$  in following corollary.

**Corollary 1.** For some given  $l_{k_0^d, B_0}$ , when  $d_c \gg R$ ,  $\delta = 1$  and the intensity of DL users is large enough,  $\mathcal{L}_{I_{BS}^d|k_0^d, B_0}(S_d)$  can be approximated as

$$\hat{\mathcal{L}}_{I_{BS}^d|k_0^d, B_0}(S_d) = F(K_d^{max} b_d, \lambda_M, A_{bu}, d_c, \alpha) \quad (25)$$

where  $A_{bu} = S_d G_b \beta P_d = z l_{k_0^d, B_0}^\alpha$ , function  $F(b, \lambda, A, x, \alpha)$  is given by

$$F(b, \lambda, A, x, \alpha) = \exp \left[ - \sum_{p=0}^{b-1} \binom{p}{b} \frac{2\pi \lambda x^2}{\alpha(b-p) - 2} \left( \frac{A}{x^\alpha} \right)^{b-p} \right] \times {}_2F_1 \left( b, b - (p + \frac{2}{\alpha}); b - (p + \frac{2}{\alpha}) + 1; -\frac{A}{x^\alpha} \right) \quad (26)$$

and  ${}_2F_1(a, b; c; d)$  is hypergeometric function [42, 9.14.1].

*Proof.* See Appendix B.  $\square$

For the possible intra-cell interference, the coverage probability of the  $t$ th data stream at DL user  $k_0^d$  affected by intra-cell UL users is given in Proposition 2.

**Proposition 2.** For some given  $l_{k_0^d, B_0}$ , the coverage probability of the  $t$ th data stream at DL user  $k_0^d$  affected by intra-cell UL users can be written as (27), where  $P(K_u) = \exp(-\lambda_P^u \pi R^2) \frac{(\lambda_P^u \pi R^2)^{K_u}}{K_u!}$  is the probability that there are  $K_u$  UL users in the cell of  $B_0$ ,  $A_{uu} = S_d \beta_u P_u$ ,  $l_{k_0^d, j_0^u} = (l_{k_0^d, B_0}^2 + l_{B_0, j_0^u}^2 - 2l_{k_0^d, B_0} l_{B_0, j_0^u} \cos \theta)^{1/2}$ , and  $f_{K_u^{max}}(r)$  is the PDF of distance from  $B_0$  to the  $K_u^{max}$ th nearest UL user and is given by [43]

$$f_{K_u^{max}}(r) = e^{-\lambda_P^u \pi r^2} \frac{2 (\lambda_P^u \pi r^2)^{K_u^{max}}}{r \Gamma(K_u^{max})} \quad (28)$$

where  $\Gamma(\cdot)$  is Gamma function.

*Proof.* See Appendix C.  $\square$

Up to this point, we have obtained expressions for  $\mathcal{L}_{I_{inter}^d|k_0^d, B_0}(S_d)$  and  $\mathcal{L}_{I_{RU}^d|k_0^d, B_0}(S_d)$ . The outage probability of the  $t$ th data stream of user  $k_0^d$  can be obtained by substituting for  $\mathcal{L}_{I_{inter}^d|k_0^d, B_0}(S_d)$  and  $\mathcal{L}_{I_{RU}^d|k_0^d, B_0}(S_d)$  into (16).

When  $P_d \gg P_u$ , interference from inter-cell BSs becomes dominant and  $\mathcal{P}(SINR_{k_0^d, t} < z | l_{k_0^d, B_0})$  can be approximated as  $1 - \mathcal{L}_{I_{inter}^d|k_0^d, B_0}(S_d) e^{-S_d \sigma^2}$ . Moreover, assuming  $d_c \gg R$ , we have  $\mathcal{L}_{I_{inter}^d|k_0^d, B_0}(S_d) \approx \hat{\mathcal{L}}_{I_{BS}^d|k_0^d, B_0}(S_d)$ , which has been given in (25). Therefore, when  $P_d \gg P_u$



$$\mathcal{L}_{I_{inter}^d|l_{k_0^d},B_0}(S_d) = \exp \left\{ - \int_0^{2\pi} \int_{d_c}^{\infty} \left( 1 - G_{\varphi}^d \left( \left( 1 + \frac{S_d G_b \beta P_d}{(l_{k_0^d}^2 + l^2 - 2l_{k_0^d} l \cos(\theta))^{\frac{\alpha}{2}}} \right)^{-b_d} \right) G_{\phi}^d(l, \theta) \right) \lambda_{MP} \rho_M(l) dl d\theta \right\} \quad (18)$$

$$G_{\phi}^d(l, \theta) = \exp \left[ -\lambda_P^u \int_0^{2\pi} \int_0^R \left( 1 - \left( 1 + \frac{S_d \beta_u P_u}{(l_{B_0, j_q^u}^2 + l_{k_0^d, B_0}^2 - 2l_{B_0, j_q^u} l_{k_0^d, B_0} \cos(\theta_{j_q^u}))^{\alpha_u/2}} \right)^{-b_u} \right) r dr d\theta \right] \quad (20)$$

$$\mathcal{L}_{I_{RUL}^d|l_{k_0^d},B_0}(S_d) = 1 - \delta + \delta \left\{ \sum_{K_u=0}^{K_u^{max}-1} P(K_u) + \int_{r=0}^R f_{K_u^{max}}(r) \exp \left[ -\lambda_P^u \int_0^{2\pi} \int_r^R \left( 1 - \left( 1 + \frac{A_{uu}}{l_{k_0^d, j_0^u}^{\alpha_u}} \right)^{-b_u} \right) l dl d\theta \right] dr \right\} \quad (27)$$

and  $d_c \gg R$ , we have

$$\begin{aligned} & \mathcal{P} \left( SINR_{k_0^d, t} < z | l_{k_0^d, B_0} \right) \\ & \approx 1 - F(K_d^{max} b_d, \lambda_M, A_{bu}, d_c, \alpha) \exp(-S_d \sigma^2). \end{aligned} \quad (29)$$

### 2) Throughput of DL user $k_0^d$

Since each DL user receives  $b_d$  data streams from the corresponding BS, the throughput of DL user  $k_0^d$  can be expressed as

$$\begin{aligned} T_{k_0^d} &= b_d \mathcal{R} \left( 1 - \mathcal{P} \left( SINR_{k_0^d, t} < z | l_{k_0^d, B_0} \right) \right) \\ &= b_d \mathcal{R} \mathcal{L}_{I_{inter}^d|l_{k_0^d},B_0}(S_d) \mathcal{L}_{I_{RUL}^d|l_{k_0^d},B_0}(S_d) \exp(-S_d \sigma^2) \end{aligned} \quad (30)$$

where  $\mathcal{R}$  is the required data rate and  $z = 2^{\mathcal{R}} - 1$ .

Assuming  $d_c \gg R$  and in the high  $P_d/P_u$  regime, the throughput of DL user  $k_0^d$  can be approximated as

$$T_{k_0^d} \approx b_d \mathcal{R} F(K_d^{max} b_d, \lambda_M, A_{bu}, d_c, \alpha) \exp(-S_d \sigma^2). \quad (31)$$

### 3) Capacity of the channel between BS and a generic DL user

For DL user  $k_0^d$ , the link capacity can be written as

$$\begin{aligned} C_{k_0^d|l_{k_0^d},B_0} &= \mathbb{E} \left[ b_d \log_2 \left( 1 + SINR_{k_0^d, t} \right) \right] \\ &= \mathbb{E} \left[ b_d \log_2 \left( 1 + \frac{l_{k_0^d, B_0}^{-\alpha} G_b \beta P_d \|\mathbf{u}_{k_0^d, t}^H \mathbf{H}_{k_0^d, B_0} \mathbf{V}_{k_0^d}\|^2}{I_{inter}^d + I_{RUL}^d + \sigma^2} \right) \right]. \end{aligned} \quad (32)$$

Invoking (9),  $C_{k_0^d|l_{k_0^d},B_0}$  can be rewritten as

$$\begin{aligned} C_{k_0^d|l_{k_0^d},B_0} &= \frac{b_d}{\ln 2} \int_0^{\infty} \mathbb{E} \left[ e^{-\frac{z}{\sigma^2} (I_{inter}^d + I_{RUL}^d)} \right] \\ & \times \left( 1 - \mathbb{E} \left[ e^{-\frac{z G_b \beta P_d}{\sigma^2 l_{k_0^d, B_0}^{\alpha}} \|\mathbf{u}_{k_0^d, t}^H \mathbf{H}_{k_0^d, B_0} \mathbf{V}_{k_0^d}\|^2} \right] \right) \frac{e^{-z}}{z} dz \\ &= \frac{b_d}{\ln 2} \int_0^{\infty} \mathcal{L}_{I_{inter}^d|l_{k_0^d},B_0} \left( \frac{z}{\sigma^2} \right) \mathcal{L}_{I_{RUL}^d|l_{k_0^d},B_0} \left( \frac{z}{\sigma^2} \right) \\ & \times \frac{e^{-z} G_b \beta P_d}{\sigma^2 l_{k_0^d, B_0}^{\alpha} + z G_b \beta P_d} dz \end{aligned} \quad (33)$$

where the expressions for  $\mathcal{L}_{I_{inter}^d|l_{k_0^d},B_0}(S_d)$  and  $\mathcal{L}_{I_{RUL}^d|l_{k_0^d},B_0}(S_d)$  are already known from (18) and (27).

In particular, when  $d_c \gg R$ ,  $\delta = 1$  and in the high  $P_d/P_u$  regime, the link capacity can be approximated as

$$\begin{aligned} \hat{C}_{k_0^d|l_{k_0^d},B_0} &= \frac{b_d}{\ln 2} \int_0^{\infty} F \left( K_d^{max} b_d, \lambda_M, \frac{z G_b \beta P_d}{\sigma^2}, d_c, \alpha \right) \\ & \times \frac{e^{-z} G_b \beta P_d}{\sigma^2 l_{k_0^d, B_0}^{\alpha} + z G_b \beta P_d} dz. \end{aligned} \quad (34)$$

For a generic DL user, (33) can be unconditioned to obtain

$$C_{k_0^d} = \int_{l_{k_0^d, B_0}=0}^R f(l_{k_0^d}) C_{k_0^d|l_{k_0^d},B_0} dl_{k_0^d, B_0}. \quad (35)$$

## B. PERFORMANCE OF A GENERIC UL USER SERVED BY FD BS

### 1) Outage probability for each data stream of a generic UL user

Consider UL transmission, it is assumed that all BSs perform single-stream decoding. The  $t$ th data stream from UL user  $k_0^u$  can be written as (36), where  $\mathbf{u}_{k_0^u, t}$  is the  $t$ th column of  $\mathbf{U}_{k_0^u}$ . The SINR of the  $t$ th data stream from UL user  $K_0^u$  can be expressed as

$$SINR_{k_0^u, t} = \frac{l_{B_0, k_0^u}^{-\alpha} G_b \beta P_u \|\mathbf{u}_{k_0^u, t}^H \mathbf{H}_{B_0, k_0^u} \mathbf{V}_{k_0^u}\|^2}{I_{SI} + I_{RUL}^u + I_{inter}^u + \sigma^2} \quad (37)$$

where  $I_{SI} = \sum_{i_0^d \in \varphi_0} G_b^2 \beta P_d \|\mathbf{u}_{k_0^u, t}^H \hat{\mathbf{H}}_{B_0, B_0} \mathbf{V}_{i_0^d}\|^2$  is the power of the residual SI, and  $I_{RUL}^u$  and  $I_{inter}^u = I_{BS}^u + I_{UL}^u$  are the power of intra-cell and inter-cell interference. Here,  $I_{RUL}^u$ ,  $I_{BS}^u$  and  $I_{UL}^u$  are respectively given by

$$\begin{aligned} I_{RUL}^u &= \delta \sum_{j_0^u \in \phi_0'} l_{B_0, j_0^u}^{-\alpha} G_b \beta P_u \|\mathbf{u}_{k_0^u, t}^H \mathbf{H}_{B_0, j_0^u} \mathbf{V}_{j_0^u}\|^2 \quad (38) \\ I_{BS}^u &= \sum_{B_q \in \Phi_M, q \neq 0} \sum_{i_q^d \in \varphi_q} l_{B_0, B_q}^{-\alpha} G_b^2 \beta P_d \|\mathbf{u}_{k_0^u, t}^H \mathbf{H}_{B_0, B_q} \mathbf{V}_{i_q^d}\|^2 \\ I_{UL}^u &= \sum_{B_q \in \Phi_M, q \neq 0} \sum_{j_q^u \in \phi_q} l_{B_0, j_q^u}^{-\alpha} G_b \beta P_u \|\mathbf{u}_{k_0^u, t}^H \mathbf{H}_{B_0, j_q^u} \mathbf{V}_{j_q^u}\|^2. \end{aligned} \quad (39)$$

$$s_{k_0^u,t} = \sqrt{G_b \beta l_{B_0,k_0^u}^{-\alpha}} \mathbf{u}_{k_0^u,t}^H \mathbf{H}_{B_0,k_0^u} \mathbf{V}_{k_0^u} \mathbf{s}_{k_0^u} + \sum_{i_0^d \in \bar{\varphi}_0} \sqrt{G_b^2 \beta} \mathbf{u}_{k_0^u,t}^H \hat{\mathbf{H}}_{B_0,B_0} \mathbf{V}_{i_0^d} \mathbf{s}_{i_0^d} + \delta \sum_{j_0^u \in \phi_0} \sqrt{\frac{G_b \beta}{l_{B_0,j_0^u}^{-\alpha}}} \mathbf{u}_{k_0^u,t}^H \mathbf{H}_{B_0,j_0^u} \mathbf{V}_{j_0^u} \mathbf{s}_{j_0^u} + \sum_{B_q \in \Phi_{Mq} \neq 0} \left( \sum_{i_q^d \in \bar{\varphi}_q} \sqrt{G_b^2 \beta l_{B_0,B_q}^{-\alpha}} \mathbf{u}_{k_0^u,t}^H \mathbf{H}_{B_0,B_q} \mathbf{V}_{i_q^d} \mathbf{s}_{i_q^d} + \sum_{j_q^u \in \phi_q} \sqrt{G_b \beta l_{B_0,j_q^u}^{-\alpha}} \mathbf{u}_{k_0^u,t}^H \mathbf{H}_{B_0,j_q^u} \mathbf{V}_{j_q^u} \mathbf{s}_{j_q^u} \right) + \mathbf{u}_{k_0^u,t}^H \mathbf{n}_{k_0^d} \quad (36)$$

Therefore, given the distance between UL user  $k_0^u$  and BS  $B_0$ , the conditional outage probability of the  $t$ th data stream from UL user  $k_0^u$  can be written as

$$\mathcal{P}(\text{SINR}_{k_0^u,t} < z | l_{B_0,k_0^u}) = 1 - \mathcal{L}_{I_{SI}|l_{B_0,k_0^u}}(\mathcal{S}_u) \times \mathcal{L}_{I_{RUL}^u|l_{B_0,k_0^u}}(\mathcal{S}_u) \mathcal{L}_{I_{inter}^u|l_{B_0,k_0^u}}(\mathcal{S}_u) \exp(-\mathcal{S}_u \sigma^2) \quad (41)$$

where  $\mathcal{S}_u = \frac{z l_{B_0,k_0^u}^{-\alpha}}{G_b \beta P_u}$ ,  $\mathcal{L}_{I_{SI}|l_{B_0,k_0^u}}(\mathcal{S}_u) = \mathbb{E}_{I_{SI}} [e^{-\mathcal{S}_u I_{SI}}]$ ,  $\mathcal{L}_{I_{RUL}^u|l_{B_0,k_0^u}}(\mathcal{S}_u) = \mathbb{E}_{I_{RUL}^u} [e^{-\mathcal{S}_u I_{RUL}^u}]$  and  $\mathcal{L}_{I_{inter}^u|l_{B_0,k_0^u}}(\mathcal{S}_u) = \mathbb{E}_{I_{inter}^u} [e^{-\mathcal{S}_u I_{inter}^u}]$ .

Given the PDF of  $l_{B_0,k_0^u}$  in (3), the unconditional outage probability of UL user  $k_0^u$  can be computed as

$$\mathcal{P}(\text{SINR}_{k_0^u,t} < z) = \int_0^R f_{l_{B_0,k_0^u}}(r) \mathcal{P}(\text{SINR}_{k_0^u,t} < z | r) dr. \quad (42)$$

Considering the residual SI first, the coverage probability is given in following proposition.

**Proposition 3.** For given  $l_{B_0,k_0^u}$ , the coverage probability of the  $t$ th data stream from UL user  $k_0^u$  affected by the residual SI at BS is given by

$$\mathcal{L}_{I_{SI}|l_{B_0,k_0^u}}(\mathcal{S}_u) = G_\varphi^d \left( (1 + \mathcal{S}_u G_b^2 \beta_{b_1} P_d \sigma_s^2)^{-b_d} \right), \quad (43)$$

where  $G_\varphi^d(x)$  has been provided in (19).

*Proof.* See Appendix D.  $\square$

With respect to inter-cell interference, the coverage probability of the  $t$ th data stream from UL user  $k_0^u$  is given in Proposition 4.

**Proposition 4.** Given the distance between UL user  $k_0^u$  and BS  $B_0$ , the coverage probability of the  $t$ th data stream from UL user  $k_0^u$  affected by the inter-cell interference can be written as

$$\begin{aligned} & \mathcal{L}_{I_{inter}^u|l_{B_0,k_0^u}}(\mathcal{S}_u) \\ &= \exp \left[ -2\pi \int_{d_c}^\infty \left( 1 - G_\varphi^d \left( \left( 1 + \frac{A_{ub}}{l_{B_0,B_q}^{\alpha_b}} \right)^{-b_d} \right) \right. \right. \\ & \quad \left. \left. \times G_\varphi^u(l_{B_0,B_q}) \right) \lambda_M \rho_M(l_{B_0,B_q}) dl_{B_0,B_q} \right] \quad (44) \end{aligned}$$

where  $A_{ub} = \mathcal{S}_u G_b^2 \beta_b P_d$  and  $G_\varphi^u(l_{B_0,B_q})$  is given by

$$\begin{aligned} G_\varphi^u(l_{B_0,B_q}) &= \exp \left[ -\lambda_P^u \int_0^{2\pi} \int_0^R \left( 1 - \left( \frac{\mathcal{S}_u G_b \beta P_u}{(l_{B_0,B_q}^2 + r^2 - 2l_{B_0,B_q} r \cos(\vartheta))^{2\alpha/2}} \right)^{-b_u} \right) r dr d\vartheta \right]. \quad (45) \end{aligned}$$

*Proof.* The proof of Proposition 4 follows steps similar to those outlined in the proof of Proposition 1, and is thus omitted.  $\square$

For the  $t$ th data stream from  $k_0^u$ , the expressions of coverage probabilities affected by  $I_{BS}^u$  and  $I_{UL}^u$  can be obtained by substituting  $G_\varphi^u(l_{B_0,B_q}) = 1$  and  $G_\varphi^d \left( \left( 1 + \frac{A_{ub}}{l_{B_0,B_q}^{\alpha_b}} \right)^{-b_d} \right) = 1$  into (44) respectively.

**Corollary 2.** Given the distance between UL user  $k_0^u$  and  $B_0$ , the coverage probability of the  $t$ th data stream from UL user  $k_0^u$  affected by inter-cell interference from DL BSs can be approximated as

$$\begin{aligned} \hat{\mathcal{L}}_{I_{BS}^u}(\mathcal{S}_u) &= \frac{F(K_d^{max} b_d, \lambda_M, A_{ub}, d_c, \alpha_{b_1})}{F(K_d^{max} b_d, \lambda_M, A_{ub}, R_c, \alpha_{b_1})} \\ & \quad \times F(K_d^{max} b_d, \lambda_M, A_{ub}, R_c, \alpha_{b_2}). \quad (46) \end{aligned}$$

*Proof.* The proof of Corollary 2 follows steps similar to those outlined in the proof of Corollary 1, and is thus omitted.  $\square$

As for the interference from intra-cell UL users, the coverage probability can be found in following proposition.

**Proposition 5.** For some given  $l_{B_0,k_0^u}$ , the coverage probability of the  $t$ th data stream from UL user  $k_0^u$  affected by the residual intra-cell interference from UL users is given by (47).

*Proof.* The proof of Proposition 5 follows steps similar to those outlined in the proof of Proposition 2, and is thus omitted.  $\square$

To this point, the expressions for  $\mathcal{L}_{I_{SI}|l_{B_0,k_0^u}}(\mathcal{S}_u)$ ,  $\mathcal{L}_{I_{BS}^u|l_{B_0,k_0^u}}(\mathcal{S}_u)$ ,  $\mathcal{L}_{I_{RUL}^u|l_{B_0,k_0^u}}(\mathcal{S}_u)$  and  $\mathcal{L}_{I_{UL}^u|l_{B_0,k_0^u}}(\mathcal{S}_u)$  have been derived, the conditional outage probability of the  $t$ th data stream from UL user  $k_0^u$  can be obtained by substituting these into (41).

Since users are randomly distributed in the circular region centered around their corresponding BS, the locations of the user are dependent on the locations of the BSs. However, it is reasonable to assume that the user locations are independent with the BS location when analysing the inter-cell interference. This assumption has been proved to be tight in [18]. Therefore, when  $P_d \gg P_u$ ,  $\mathcal{P}(\text{SINR}_{k_0^u} < z | l_{B_0,k_0^u})$  can be approximated as

$$\begin{aligned} \hat{\mathcal{P}}(\text{SINR}_{k_0^u} < z | l_{B_0,k_0^u}) &= 1 - \mathcal{L}_{I_{SI}|l_{B_0,k_0^u}}(\mathcal{S}_u) \\ & \quad \times \hat{\mathcal{L}}_{I_{BS}^u|l_{B_0,k_0^u}}(\mathcal{S}_u) \exp(-\mathcal{S}_u \sigma^2). \quad (48) \end{aligned}$$

$$\mathcal{L}_{I_{RUL}^u|l_{B_0,k_0^u}}(\mathcal{S}_u) = 1 - \delta + \delta \left\{ \sum_{K_u=0}^{K_u^{max}-1} P(K_u) + \int_{l_{B_0,k_0^u}}^R f_{K_u^{max}}(r) \left( 1 + \frac{\mathcal{S}_u G_b \beta P_u}{r^\alpha} \right)^{-b_u} \frac{F(bu, \lambda_P^u, \mathcal{S}_u G_b \beta P_u, r)}{F(bu, \lambda_P^u, \mathcal{S}_u G_b \beta P_u, R)} dr \right\} \quad (47)$$

2) Throughput of UL user  $k_0^u$

The throughput of link between UL user  $k_0^u$  and its associated BS can be expressed as

$$\begin{aligned} T_{k_0^u} &= b_u \mathcal{R} (1 - \mathcal{P}(SINR_{k_0^u,t} < z|l_{B_0,k_0^u})) \\ &= b_u \mathcal{R} \mathcal{L}_{I_{SI}^u|l_{B_0,k_0^u}}(\mathcal{S}_u) \mathcal{L}_{I_{inter}^u|l_{B_0,k_0^u}}(\mathcal{S}_u) \\ &\quad \times \mathcal{L}_{I_{RUL}^u|l_{B_0,k_0^u}}(\mathcal{S}_u) e^{-\mathcal{S}_u \sigma^2}. \end{aligned} \quad (49)$$

Assuming  $P_d \gg P_u$ ,  $T_{k_0^u}$  can be approximated as

$$\hat{T}_{k_0^u} = b_u \mathcal{R} \mathcal{L}_{I_{SI}^u|l_{B_0,k_0^u}}(\mathcal{S}_u) \hat{\mathcal{L}}_{I_{BS}^u|l_{B_0,k_0^u}}(\mathcal{S}_u) e^{-\mathcal{S}_u \sigma^2}. \quad (50)$$

3) Capacity of the link between a generic UL user and BS

For UL user  $k_0^u$ , the conditional link capacity is given by

$$\begin{aligned} C_{k_0^u|l_{B_0,k_0^u}} &= \mathbb{E} [b_u \log_2 (1 + SINR_{k_0^u,t})] \\ &= \frac{b_u}{\ln 2} \int_0^\infty \mathcal{L}_{I_{SI}^u|l_{B_0,k_0^u}} \left( \frac{z}{\sigma^2} \right) \mathcal{L}_{I_{inter}^u|l_{B_0,k_0^u}} \left( \frac{z}{\sigma^2} \right) \\ &\quad \times \mathcal{L}_{I_{RUL}^u|l_{B_0,k_0^u}} \left( \frac{z}{\sigma^2} \right) \frac{e^{-z} G_b \beta P_u}{\sigma^2 l_{B_0,k_0^u}^\alpha + z G_b \beta P_u} dz. \end{aligned} \quad (51)$$

When  $P_d \gg P_u$ , the conditional link capacity can be approximated as

$$\begin{aligned} \hat{C}_{k_0^u|l_{B_0,k_0^u}} &= \frac{b_u}{\ln 2} \int_0^\infty \mathcal{L}_{I_{SI}^u|l_{B_0,k_0^u}} \left( \frac{z}{\sigma^2} \right) \hat{\mathcal{L}}_{I_{BS}^u|l_{B_0,k_0^u}} \left( \frac{z}{\sigma^2} \right) \\ &\quad \times \frac{e^{-z} G_b \beta P_u}{\sigma^2 l_{B_0,k_0^u}^\alpha + z G_b \beta P_u} dz. \end{aligned} \quad (52)$$

For a generic UL user, the unconditional link capacity can be expressed as

$$C_{k_0^u} = \int_0^R f_{l_{B_0,k_0^u}}(r) C_{k_0^u|r} dr. \quad (53)$$

C. PERFORMANCE OF HD SYSTEMS

To compare the performance of FD and HD networks, we provide expressions for the capacity in HD networks. Since separate time-frequency signaling channels are utilized for DL and UL transmissions in HD communication, we scale the capacity of a fully DL or UL HD network by 1/2, so as to provide a fair comparison with FD.

By turning off the UL user transmission, we have  $I_{UL}^d = 0$  and  $I_{RUL}^d = 0$ . Thus, the link capacity of DL user  $k_0^d$  (i.e. the counterpart to (33)) can be written as

$$C_{k_0^d|l_{k_0^d,B_0}}^{HD} = \frac{1}{2} \frac{b_d}{\ln 2} \int_0^\infty \mathcal{L}_{I_{BS}^d} \left( \frac{z}{\sigma^2} \right) \frac{e^{-z} G_b \beta P_d}{\sigma^2 l_{k_0^d,B_0}^\alpha + z G_b \beta P_d} dz. \quad (54)$$

Assuming  $d_c \gg R$  and  $\delta = 1$ , we have

$$\begin{aligned} \hat{C}_{k_0^d|l_{k_0^d,B_0}}^{HD} &= \frac{1}{2} \frac{b_d}{\ln 2} \int_0^\infty F \left( K_d^{max} b_d, \lambda_M, \frac{z G_b \beta P_d}{\sigma^2}, d_c, \alpha \right) \\ &\quad \times \frac{e^{-z} G_b \beta P_d}{\sigma^2 l_{k_0^d,B_0}^\alpha + z G_b \beta P_d} dz. \end{aligned} \quad (55)$$

For UL user  $k_0^u$ , the HD counterpart to (51) can be obtained by turning off BS transmission (i.e.  $I_{SI} = 0$  and  $I_{BS}^u = 0$ ) and written as

$$\begin{aligned} C_{k_0^u|l_{B_0,k_0^u}}^{HD} &= \frac{1}{2} \frac{b_u}{\ln 2} \int_0^\infty \mathcal{L}_{I_{UL}^u|l_{B_0,k_0^u}} \left( \frac{z}{\sigma^2} \right) \\ &\quad \times \mathcal{L}_{I_{RUL}^u|l_{B_0,k_0^u}} \left( \frac{z}{\sigma^2} \right) \frac{e^{-z} G_b \beta P_u}{\sigma^2 l_{B_0,k_0^u}^\alpha + z G_b \beta P_u} dz. \end{aligned} \quad (56)$$

Assuming  $\delta = 1$ , we have

$$\hat{C}_{k_0^u|l_{B_0,k_0^u}}^{HD} = \frac{1}{2} \frac{b_u}{\ln 2} \int_0^\infty \mathcal{L}_{I_{UL}^u} \left( \frac{z}{\sigma^2} \right) \frac{e^{-z} G_b \beta P_u}{\sigma^2 l_{B_0,k_0^u}^\alpha + z G_b \beta P_u} dz. \quad (57)$$

V. NUMERICAL RESULTS

In this section, Monte Carlo simulations are used to verify the accuracy of the derived expressions. We also study the effect of varying the hard-core distance between the BSs, and applying IA to each cell. Two different network settings are considered, namely, (a) a micro-cell scenario, and (b) a macro-cell scenario. Throughout all the simulations, unless otherwise specified, we assume  $M_u = M_d = 10$ ,  $N_d = N_u = 6$ ,  $b_d = b_u = 2$  and  $l_{k_0^d,B_0} = l_{B_0,k_0^u} = 100m$ . For micro-cellular networks, we assume  $R = 300m$ ,  $d_c = 290m$ ,  $\lambda_M = 3.54 BSs/km^2$  (i.e. one BS per circular cell of radius 300 m) and  $\lambda_P^d = \lambda_P^u = 7.08 users/km^2$ . As for macro-cellular networks, it is assumed that  $R = 1000m$ ,  $d_c = 950m$ ,  $\lambda_M = 0.32 BSs/km^2$  (i.e. one BS per circular cell of radius 1000 m) and  $\lambda_P^d = \lambda_P^u = 3.84 users/km^2$ . All the other parameters are provided in Table 1.

For a FD micro-cellular network, CDFs of DL and UL SINR are plotted versus targeted SINR in Fig. 3. It can be observed that both DL and UL transmissions are feasible in FD micro-cellular networks and DL transmission outperforms UL transmission. This is because different pathloss parameters are applied to BS-BS and BS-user links, and UL transmission suffers more significant interference from BSs with distances below  $R_c$ .

For randomly distributed DL and UL users in a FD micro-cellular network where  $\lambda_{MP} = 3.54 BSs/km^2$  and targeted SINR=-5 dB, outage probabilities are plotted versus hard-core distance in Fig. 4 where different numbers of data streams required at each user are considered. It can be noticed from Fig. 4, that outage probabilities decrease with hard-

TABLE 1: Parameters for different networks

Parameter	$P_d$	$P_u$	$G_b$	$\alpha$	$\alpha_{b_1}$	$\alpha_{b_2}$	$\alpha_u$
Micro-cell	24 dBm	23 dBm	5 dBi	3.75	3	4	4
Macro-cell	46 dBm	23 dBm	15 dBi	3.75	3	4	4
Parameter	$\beta$	$\beta_{b_1}$	$\beta_{b_2}$	$\beta_u$	$h_{BS}$	$\xi$	$R_c$
Micro-cell	-32.9 dB	-38.45 dB	-49.36 dB	-55.78 dB	4m	0.15m	427m
Macro-cell	-15.3 dB	-38.45 dB	1.0439 dB	-55.78 dB	20m	0.15m	10667m

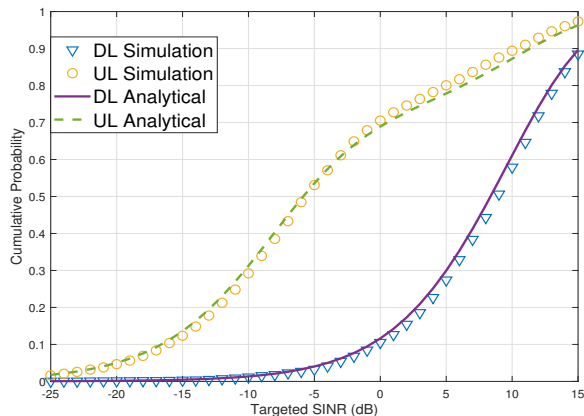


FIGURE 3: CDFs of DL and UL SINR in a full-duplex micro-cellular network.

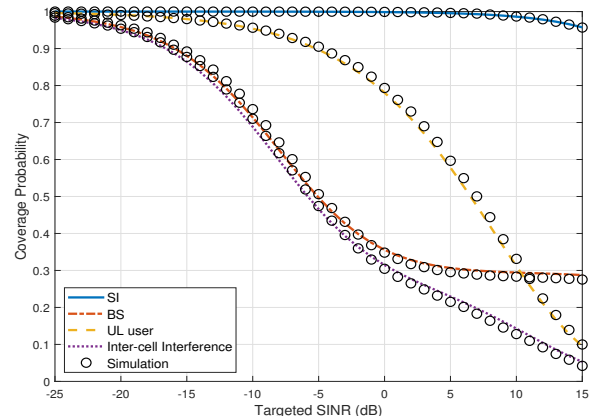


FIGURE 5: Coverage probability of UL user affected by different interferences in a full-duplex micro-cellular network.

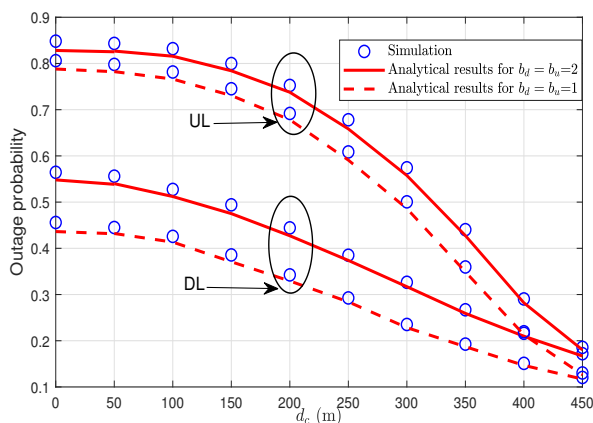


FIGURE 4: Outage probabilities of randomly distributed DL and UL users in a full-duplex micro-cellular network.

core distance. When hard-core distance is greater than critical distance  $R_c$ , UL and DL users experience almost the same outage probability. This is because BSs and users in micro-cellular networks have similar transmit power and BSs within critical distance suffer less path loss than these with distances to  $B_0$  greater than critical distance. Moreover, Fig. 4 shows that outage probabilities of DL and UL users can be improved by reducing the number of data streams required at each user. When  $d_c = 0$ , HCPP converges to PPP and it can be observed that FD transmission in PPP-based model may be infeasible and a suitable distance between BSs should be imposed.

Consider a micro-cellular network, the effect of different interferences on UL transmission are studied in Fig. 5 where

the coverage probabilities of a UL user with link length  $l_{B_0, k_0^u}$  are plotted against SINR. Assuming an analog/digital SI cancellation scheme is adopted at each BS, the covariance of elements in the residual SI channel is specified as  $\sigma_s^2 = -110$  dB [6]. Due to the low BS-BS pathloss for distances below  $R_c$ , it can be noticed that inter-cell interference among BSs has much more significant impact on UL transmission than interference from UL users. Since we adopt a two-slope model for the BS-BS pathloss, interference from BSs with distance to the origin below  $R_c$  is dominant. Therefore, in the high SINR regime, the coverage probability of a UL user affected by inter-cell interference from BSs converges to a fixed value which is the probability that there is no interferer BS with distance to the origin less than  $R_c$ . The average number of BSs distributed within  $R_c$  can be calculated as  $\mu = \int_{d_c}^{R_c} 2\pi r \lambda_M \rho_M(r) dr$ . Then, we have  $e^{-\mu} = 0.2435$  which matches the result in Fig. 5. Moreover, due to the use of SI cancellation scheme and low DL transmission power in micro-cell networks, SI plays an inconsequential role in UL transmission. It can be concluded that UL transmission in micro-cell networks is mainly affected by inter-cell interference from BSs and UL users when an SI cancellation scheme is applied at each BS.

Assuming BSs have limited number of transmit antenna, the CDFs of UL SINR at BSs in a FD micro-cell network are characterized as shown in Fig. 6. In Fig. 6a, we set  $K_d^{max} = 4$  and vary  $K_u^{max}$ , the maximum numbers of UL user which allow IA to still be feasible. Fig. 6a shows that UL performance can be improved by increasing  $K_u^{max}$ , i.e. increasing the number of antenna at each BS. Since UL users

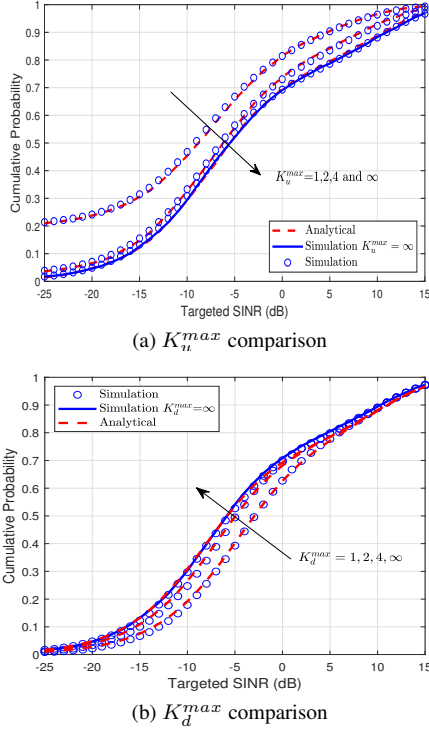


FIGURE 6: CDFs of SINR at a UL user in a FD micro-cellular network with limited number of transmit antennas at BS.

are distributed as a PPP, when  $K_u > K_u^{max}$ , only the nearest  $K_u^{max}$  UL users are served while the rest  $K_u - K_u^{max}$  users are treated as interferers. Therefore, increasing  $K_u^{max}$  will suppress the intra-cell interference. Different from Fig.6a, in Fig.6b we set  $K_u^{max} = 4$  and vary  $K_d^{max}$ . It can be observed that increasing  $K_d^{max}$  has a negative impact on UL performance, since the inter-cell interference from BSs increases with  $K_d^{max}$ . However, it worth to be noticed that in low and high targeted SINR regimes, the UL performance could not be improved by reducing  $K_d^{max}$ . By increasing  $K_u^{max}$  or  $K_d^{max}$ , it can be recognized that the performance gap between  $K_u^{max} \geq 2\pi R^2 \lambda_P^u$  (or  $K_d^{max} \geq 2\pi R^2 \lambda_P^d$ ) (i.e. Case II and in this case  $K_u^{max} = 4$  (or  $K_d^{max} = 4$ )) and  $K_u^{max} = \infty$  (or  $K_d^{max} = \infty$ ) (i.e. Case I) is uniformly marginal. Therefore, by setting  $K_u^{max} \geq 2\pi R^2 \lambda_P^u$  or  $K_d^{max} \geq 2\pi R^2 \lambda_P^d$ , Case II converges to Case I and the intra-cell interference is marginal and can be ignored. In the remainder of this paper, we assume  $K_u^{max} = K_d^{max} = \infty$ .

For a macro-cellular network, the throughput of DL and UL users are plotted against hard-core distance in Fig. 7 when the targeted data rate  $r_t = 2$  bits per channel use. It can be observed that our approximations match simulation results quite closely. In the small hard-core distance regime, it can be seen that UL transmission provides poor performance, while DL has much better performance than UL. This is caused by the fact that DL transmission power from BSs is much larger than UL transmission power from users, and SI at BSs

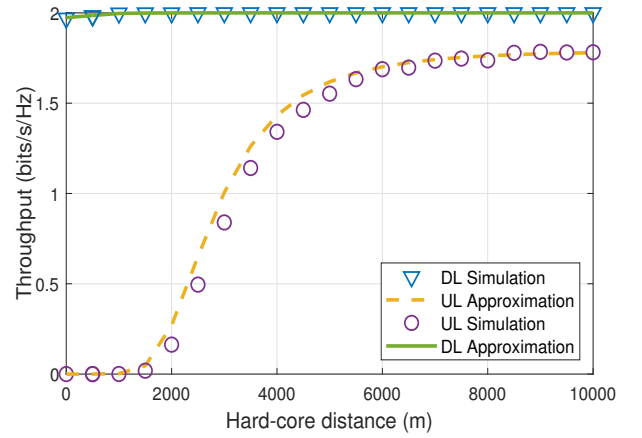


FIGURE 7: Throughput of DL and UL users in a full-duplex macro-cellular network.

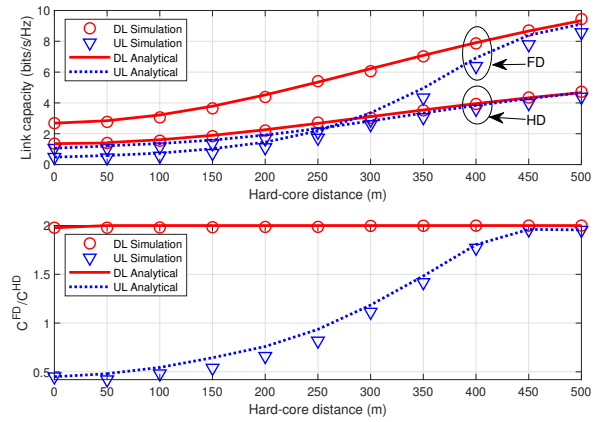


FIGURE 8: Comparison of DL and UL user capacities in FD and HD micro-cellular networks where  $l_{k_0^d, B_0}$  and  $l_{B_0, k_0^u}$  are given.

and inter-cell interference from BSs become overwhelming when  $P_d \gg P_u$ . Due to the low outage probability in DL transmission, throughput of DL users is almost constant throughout, while throughput of UL users is increased with hard-core distance. Therefore, UL transmission in FD macro-cellular networks is feasible when a larger hard-core distance between BSs is adopted.

In Fig. 8, we compare the capacities of DL and UL users in FD and HD micro-cell networks for given  $l_{k_0^d, B_0}$  and  $l_{B_0, k_0^u}$ ,  $\lambda_P^d = \lambda_P^u = 10.19$  users/km<sup>2</sup> and  $\lambda_{MP} = 5.09$  BS/km<sup>2</sup>. It can be observed that the capacity of DL users in FD micro-cell networks is almost twice that in HD micro-cell networks. This is because for DL transmission, interference from inter-cell BS is dominant and the capacity of a DL user  $k_0^d$  can be approximated as  $\frac{b_d}{\ln 2} \int_0^\infty \mathcal{L}_{I_{BS}^d} \left( \frac{z G_b \beta P_d}{\sigma^2 l_{k_0^d, B_0}^\alpha} \right) \frac{e^{-z} G_b \beta P_d}{\sigma^2 l_{k_0^d, B_0}^\alpha + z G_b \beta P_d} dz$ , which equals  $2 \times C_{K_0^d | l_{k_0^d, B_0}}^{HD}$ . In the low hard-core distance regime, due to the additional interference from DL BSs in FD networks, HD networks provide better UL link capacity. However, the interference from DL BSs in FD networks is suppressed by

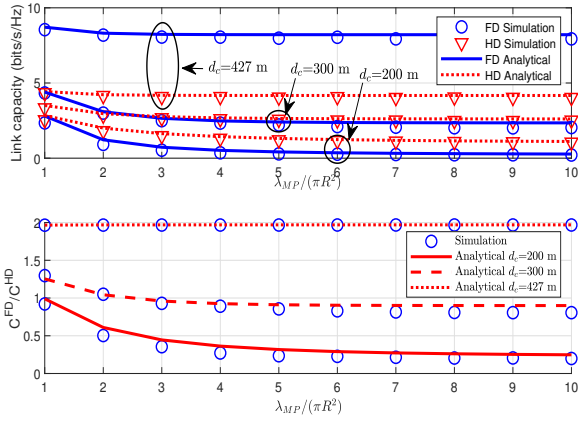


FIGURE 9: Comparison of UL user capacities in FD and HD micro-cellular networks.

increasing hard-core distance, and UL transmission in FD networks outperforms that of HD networks when hard-core distance is large enough.

To further analyze the impacts of hard-core distance and the intensity of candidate BSs on UL transmission, we compare the UL user capacities in FD and HD micro-cellular networks with different hard-core distances in Fig. 9. It can be observed that a suitable hard-core distance between FD BSs should be imposed to make UL transmission feasible in micro-cell networks. Moreover, Fig. 9 shows that the ratio of UL user capacity in FD networks to that in HD networks increases with hard-core distance. And, when hard-core distance is greater than or equal to the critical distance (i.e.,  $d_c \geq R_c$ ), UL transmission in FD networks has the potential to double the spectrum efficiency.

## VI. CONCLUSION

The performance of a FD cellular network, where a HCPP was used to model the BSs and linear IA was applied at each cell to cancel the intra-cell interference was studied. In addition to providing general expressions for the performance metrics, closed-form approximations of the outage probability and throughput were derived. Simulation and analytical results showed that FD communication is feasible in micro-cellular networks, while large hard-core distances are required to make UL transmission in FD macro-cellular networks a reality. Moreover, results also showed that interference from BSs plays a crucial role in micro-cell systems. When the maximum number of UL users for which IA is feasible is equal to or greater than twice the mean number of UL users in each cell, the residual intra-cell interference from UL users can be ignored. It was clearly seen that even IA is utilized for intra-cell interference management in FD networks, suitable spacing between FD BSs should be imposed to make UL transmission feasible. Moreover, we show that FD networks tend to double the UL user capacity over their HD counterparts when the hard-core distance is greater than or equal to the critical distance.

## APPENDIX

### A. PROOF OF PROPOSITION 1

$\mathcal{L}_{inter}^{d,l_{k_0^d,B_0}}(z)$  can be rewritten as (58).

The PDF of  $\|\mathbf{u}_{k_0^d,t}^H \mathbf{H}_{k_0^d,B_q} \mathbf{V}_{i_q^d}\|^2$  has been provided in (8). Therefore, we have

$$\mathbb{E}_{\mathbf{H}_{k_0^d,B_q}} \left[ e^{-\frac{z l_{k_0^d,B_0}^{\alpha_u}}{l_{k_0^d,B_q}^{\alpha_u}} \|\mathbf{u}_{k_0^d,t}^H \mathbf{H}_{k_0^d,B_q} \mathbf{V}_{i_q^d}\|^2} \right] = \left( 1 + \frac{z l_{k_0^d,B_0}^{\alpha_u}}{l_{k_0^d,B_q}^{\alpha_u}} \right)^{-b_d}. \quad (59)$$

Similarly,  $2 \times \|\mathbf{u}_{k_0^d,t}^H \mathbf{H}_{k_0^d,j_q^u} \mathbf{V}_{j_q^u}\|^2$  is a Chi-squared distributed RV with  $2b_u$  DoF and we have

$$\mathbb{E}_{\mathbf{H}_{k_0^d,j_q^u}} \left[ e^{-S_d \frac{\beta_u P_u}{l_{k_0^d,j_q^u}^{\alpha_u}} \|\mathbf{u}_{k_0^d,t}^H \mathbf{H}_{k_0^d,j_q^u} \mathbf{V}_{j_q^u}\|^2} \right] = \left( 1 + S_d \frac{\beta_u P_u}{l_{k_0^d,j_q^u}^{\alpha_u}} \right)^{-b_u} \quad (60)$$

The coverage probability affected by inter-cell interference can be rewritten as

$$\begin{aligned} & \mathcal{L}_{inter}^{d,l_{k_0^d,B_0}}(z) \\ &= \mathbb{E}_{\Phi_M} \left[ \prod_{B_q \in \Phi_M, q \neq 0} G_\varphi^d \left( \left( 1 + \frac{z l_{k_0^d,B_0}^{\alpha_u}}{l_{k_0^d,B_q}^{\alpha_u}} \right)^{-b_d} \right) G_\phi^d(l, \theta) \right] \end{aligned} \quad (61)$$

where the function  $G_\varphi^d(x)$  is given by

$$\begin{aligned} G_\varphi^d(x) &= \mathbb{E}_{\tilde{\varphi}_q} \left[ \prod_{i_q^d \in \tilde{\varphi}_q} x \right] \\ &= (1 - \delta) \mathbb{E}_{\varphi_q} \left[ \prod_{i_q^d \in \varphi_q} x \right] + \delta \mathbb{E}_{\varphi'_q} \left[ \prod_{i_q^d \in \varphi'_q} x \right] \end{aligned} \quad (62)$$

and (19) can be obtained.

$$\begin{aligned} G_\phi^d(l, \theta) &= \mathbb{E}_{\phi_q} \left[ \prod_{j_q^u \in \phi_q} \left( 1 + S_d \frac{\beta_u P_u}{l_{k_0^d,j_q^u}^{\alpha_u}} \right)^{-b_u} \right], \quad l_{k_0^d,j_q^u} = \\ &= \sqrt{l_{B_0,j_q^u}^2 + l_{k_0^d,B_0}^2 - 2l_{B_0,j_q^u} l_{k_0^d,B_0} \cos(\theta_{j_q^u})}, \quad l_{B_0,j_q^u} = \\ &= \sqrt{l^2 + r^2 - 2lr \cos(\vartheta)} \text{ and } \theta_{j_q^u} = \theta - \arcsin \left( \frac{r}{l_{B_0,j_q^u}} \sin(\vartheta) \right). \end{aligned}$$

Let  $v_1(j_q^u) = \left( 1 + S_d \frac{\beta_u P_u}{l_{k_0^d,j_q^u}^{\alpha_u}} \right)^{-b_u}$  and  $\forall j_q^u \in \phi_q$ , we have  $0 \leq v_1(j_q^u) \leq 1$ . For  $\phi_q$ , the probability generating function (PGF) is given by

$$G_\phi^d(l, \theta) = \exp \left[ -\lambda_P^u \int_{R^2} (1 - v_1(j_q^u)) ds \right]. \quad (63)$$

Since the integration area is the circular region centered by  $B_q$  with radius  $R$ , (20) can be obtained.

To evaluate the expectation in (61), PGF of HCPPs is required. However, it has been reported in the literature that exact PGF of HCPPs does not exist. In [44], it has been proved that the mean interferences at nodes of a MHCPP and a PPP with the same intensity are comparable while the

$$\begin{aligned} \mathcal{L}_{I_{inter}^d|l_{k_0^d,B_0}}(z) = & \mathbb{E}_{\Phi_M} \left\{ \prod_{B_q \in \Phi_M, q \neq 0} \mathbb{E}_{\tilde{\varphi}_q} \left[ \prod_{i_q^d \in \tilde{\varphi}_q} \mathbb{E}_{\mathbf{H}_{k_0^d,B_q}} \left[ \exp \left( -\frac{z l_{k_0^d,B_0}^\alpha}{l_{k_0^d,q}^\alpha} \|\mathbf{u}_{k_0^d,t}^H \mathbf{H}_{k_0^d,B_q} \mathbf{V}_{i_q^d}\|^2 \right) \right] \right] \right. \\ & \left. \times \mathbb{E}_{\phi_q} \left[ \prod_{j_q^u \in \phi_q} \mathbb{E}_{\mathbf{H}_{k_0^d,j_q^u}} \left[ \exp \left( -\frac{z l_{k_0^d,B_0}^\alpha \beta_u P_u}{l_{k_0^d,j_q^u}^\alpha G_b \beta P_d} \|\mathbf{u}_{k_0^d,t}^H \mathbf{H}_{k_0^d,j_q^u} \mathbf{V}_{j_q^u}\|^2 \right) \right] \right] \right\} \end{aligned} \quad (58)$$

gap never exceeds 1dB. Therefore, we use PGF of PPP to approximate that of MHCPP (this approximation is shown to be tight in simulation section). Assuming  $v_2(B_q) =$

$$\begin{aligned} G_\varphi^d \left( \left( 1 + \frac{z l_{k_0^d,B_0}^\alpha}{l_{k_0^d,B_q}^\alpha} \right)^{-b_d} \right) G_\phi^d(l, \theta), \text{ we have} \\ \mathbb{E} \left[ \prod_{B_q \in \Phi_M/B_0} v_2(B_q) \right] \approx e^{-\int_0^{2\pi} \int_d^\infty (1-v_2(B_q))^{\lambda_{MPPM}(l) dl d\theta} } \end{aligned} \quad (64)$$

where  $l_{k_0^d,B_q} = (l_{k_0^d,B_0}^2 + l^2 - 2l_{k_0^d,B_0} l \cos(\theta))^{1/2}$ ,  $l$  is the distance from  $B_q$  to  $B_0$ , and  $\theta$  is the angle difference of BS  $B_q$  and user  $k_0^d$ . Due to the exist of hard-core distance between BSs, the value of  $l$  varies from  $d_c$  to  $\infty$ . Substituting the expression for  $\rho_M(l)$  into the above equation, the proof of Proposition 1 is complete.

### B. PROOF OF COROLLARY 1

Since  $l$  is the distance between  $B_0$  and  $B_q$ , we have  $l \geq d_c$ . When  $d_c \gg R$ ,  $l \gg l_{k_0^d,B_0}$  and the distance from  $B_q$  to  $k_0^d$  can be approximated as  $l$ . Moreover, when  $\delta = 1$  and the intensity of DL users is large enough, we have  $G_\varphi^d(x) \approx x^{K_d^{max}}$ . Therefore, setting  $G_\phi^d(l, \eta) = 1$  and  $\rho_1(l) \approx \rho_2(l)$ , (61) can be approximated as

$$\hat{\mathcal{L}}_{BS^d|l_{k_0^d,B_0}}(\mathcal{S}_d) = e^{-2\pi\lambda_M \int_{d_c}^\infty \left( 1 - \left( 1 + \frac{A_{bu}}{l^\alpha} \right)^{-K_d^{max} b_d} \right) l dl} \quad (65)$$

where  $\left( 1 - \left( 1 + \frac{A_{bu}}{l^\alpha} \right)^{-K_d^{max} b_d} \right)$  can be rewritten as

$$\begin{aligned} & \left( 1 - \left( 1 + \frac{A_{bu}}{l^\alpha} \right)^{-K_d^{max} b_d} \right) \\ & = \sum_{p=0}^{K_d^{max} b_d - 1} \binom{p}{K_d^{max} b_d} \frac{A_{bu}^{-p} l^{\alpha p}}{\left( 1 + \frac{l^\alpha}{A_{bu}} \right)^{K_d^{max} b_d}}. \end{aligned} \quad (66)$$

Letting  $t = l^\alpha$ , we have  $l = t^{1/\alpha}$ ,  $dl = \frac{1}{\alpha} t^{1/\alpha - 1} dt$  and

$$\begin{aligned} \hat{\mathcal{L}}_{BS^d|l_{k_0^d,B_0}}(\mathcal{S}_d) = & \exp \left( -2\pi\lambda_M \sum_{p=0}^{K_d^{max} b_d - 1} \binom{p}{K_d^{max} b_d} \right. \\ & \left. \times \int_{(d_c)^\alpha}^\infty \frac{A_{bu}^{-p} t^{p+2/\alpha-1}}{\alpha \left( 1 + \frac{1}{A_{bu}} t \right)^{K_d^{max} b_d}} dt \right). \end{aligned} \quad (67)$$

Finally, applying [42, 3.194.2] (25) is obtained, proving Corollary 1.

### C. PROOF OF PROPOSITION 2

When  $\delta = 0$ , we have  $\mathcal{L}_{I_{RUL}^d}(\mathcal{S}_d) = 1$ . For the second case, distance from the  $K_u^{max}$ th nearest UL user to  $B_0$  is denoted as  $r_{K_u^{max}}$ . All the intra-cell interference from UL users can be cancelled when  $r_{K_u^{max}} \geq R$ . Given  $r_{K_u^{max}} (r_{K_u^{max}} < R)$ , the UL users distributed in the ring with radius  $r_{K_u^{max}} \leq r \leq R$  and centered by  $B_0$  form a subset of PPP  $\phi'_0$ , since they are randomly distributed in the ring and their number is Poisson distributed. Thus,  $\mathcal{L}_{I_{RUL}^d}(z)$  can be written as (68). Following steps similar to the proof of Proposition 1,  $\mathcal{L}_{I_{RUL}^d}(z)$  can be rewritten as

$$\begin{aligned} \mathcal{L}_{I_{RUL}^d|l_{k_0^d,B_0}}(\mathcal{S}_d) = & 1 - \delta + \delta \left\{ \sum_{K_u=0}^{K_u^{max}-1} P(K_u) \right. \\ & \left. + \int_0^R f_{K_u^{max}}(r) \mathbb{E}_{\phi'_0} \left[ \prod_{j_0^u \in \phi'_0} \left( 1 + \frac{A_{uu}}{l_{k_0^d,j_0^u}^\alpha} \right)^{-b_u} \right] dr \right\} \end{aligned} \quad (69)$$

where the expectation can be obtained as

$$\begin{aligned} & \mathbb{E}_{\phi'_0} \left[ \prod_{j_0^u \in \phi'_0} \left( 1 + \frac{A_{uu}}{l_{k_0^d,j_0^u}^\alpha} \right)^{-b_u} \right] \\ & = \exp \left[ -\lambda_P^u \int_0^{2\pi} \int_{r_{K_u^{max}}}^R \left( 1 - \left( 1 + \frac{A_{uu}}{l_{k_0^d,j_0^u}^\alpha} \right)^{-b_u} \right) l dl d\theta \right]. \end{aligned} \quad (70)$$

This completes the proof of Proposition 2.

### D. PROOF OF PROPOSITION 3

Since the elements of  $\hat{\mathbf{H}}_{B_0,B_0}$  are i.i.d. RVs with distribution  $\mathcal{CN}(0, \sigma_s^2)$ . Thus,  $\frac{2}{\sigma_s^2} \|\mathbf{u}_{k_0^d,t}^H \hat{\mathbf{H}}_{B_0,B_0} \mathbf{V}_{i_0^d}\|^2$  is a Chi-squared distributed RV with  $2b_d$  DoF and its PDF can be written as

$$f_{\chi^2} \left( \frac{2}{\sigma_s^2} x; 2b_d \right) = \frac{x^{b_d-1}}{2\sigma_s^{2(b_d-1)} \Gamma(b_d)} e^{-\frac{x}{\sigma_s^2}}. \quad (71)$$

The expression for  $\mathcal{L}_{I_{SI}^d|l_{B_0,k_0^d}}(\mathcal{S}_u)$  can be obtained as

$$\begin{aligned} & \mathcal{L}_{I_{SI}^d|l_{B_0,k_0^d}}(\mathcal{S}_u) \\ & = \mathbb{E}_{\tilde{\varphi}_0} \left\{ \prod_{i_0^d \in \tilde{\varphi}_0} \int_0^\infty f_{\chi^2} \left( \frac{2}{\sigma_s^2} x; 2b_d \right) e^{-\mathcal{S}_u G_0^2 \beta_{b_1} P_d x} d \left( \frac{2}{\sigma_s^2} x \right) \right\} \\ & = \mathbb{E}_{\tilde{\varphi}_0} \left\{ \prod_{i_0^d \in \tilde{\varphi}_0} \left( 1 + \mathcal{S}_u G_0^2 \beta_{b_1} P_d \sigma_s^2 \right)^{-b_d} \right\} \end{aligned} \quad (72)$$

$$\mathcal{L}_{RU}^d |l_{k_0^d, B_0}(\mathcal{S}_d) = 1 - \delta + \delta \left\{ \int_R f_{K_u^{max}}(r) dr + \int_0^R f_{K_u^{max}}(r) \mathbb{E}_{J_{RU}^d} \left[ e^{-\mathcal{S}_d \sum_{j_0^u \in \phi_0^u} \frac{\beta_u P_u}{\Gamma_{k_0^d, j_0^u}^{\alpha_u}} \| \mathbf{u}_{k_0^d, t}^H \mathbf{H}_{k_0^d, j_0^u} \mathbf{V}_{j_0^u} \|^2} \right] dr \right\} \quad (68)$$

Solving the above expectation, the proof of Proposition 3 is obtained.

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