Measuring the Total Radiated Power of Wideband Signals in a Reverberation Chamber

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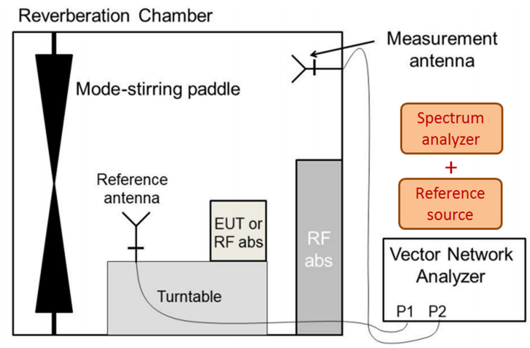
***Abstract*—For a wideband radiated signal, the probability density function (PDF) of the received total power in a reverberation chamber (RC) is no longer an exponential distribution. It is shown in this study that the PDF of the received total power approaches to a normal distribution when the bandwidth of the signal increases. Measurements are performed and the results are validated.**

Keywords—reverberation chamber, total radiated power, statistical distributions

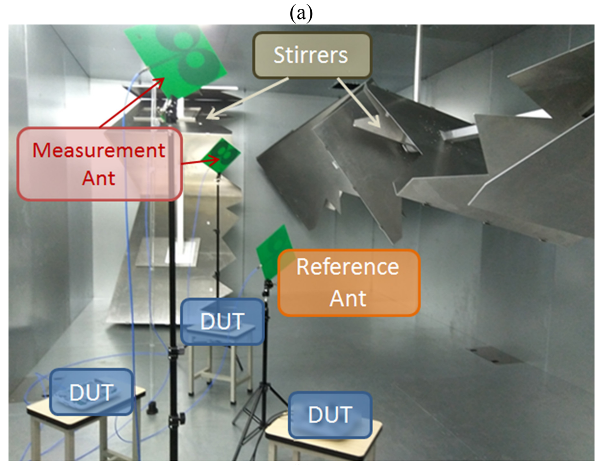
I. Introduction

With the development of electromagnetic compatibility (EMC) and over-the-air (OTA) testing [1]-[4], reverberation chambers (RCs) have been widely used in the radiated emission measurements. An RC has no absorbing materials inside a shielding cavity and offers a statistically uniform environment by reflecting and scattering waves randomly many times. An RC can be understood as a complementary facility compared with an anechoic chamber which is nearly reflectionless. It is well known that the total radiated power (TRP) of a device under test (DUT) can be best measured in a reverberation chamber (RC) accurately [5]-[19].

Fig. 1 shows the schematic plot [2] and a typical measurement scenario in an RC. Although we placed multiple DUTs and measurement antennas in an RC, it is not always necessary. The vector network analyzer (VNA) is also optional and can be replaced by a reference source and a spectrum analyzer (SA). The measurement procedure is quite direct: when the RC is well stirred, once the chamber transfer function (T_{RC}) is known the TRP of the DUT can be obtained by comparing the measured average received power of the DUT ($\langle P_{\mathrm{RxDUT}}\rangle$) and the measured average received power of the reference source ($\langle P_{\mathrm{RxRef}}\rangle$) with known radiated power ($\mathrm{TRP}_{\mathrm{Ref}}$). The TRP of the DUT ($\mathrm{TRP}_{\mathrm{DUT}}$) can be obtained from [1]-[3]



(a)



(b)

Fig. 1. TRP measurement setup: (a) schematic plot [2], (b) measurement scenario in an RC with dimensions of 3.9 m × 6 m × 2.8 m.

$$\frac{\langle P_{\mathrm{RxDUT}}\rangle}{\mathrm{TRP}_{\mathrm{DUT}}} = \frac{\langle P_{\mathrm{RxRef}}\rangle}{\mathrm{TRP}_{\mathrm{Ref}}} = T_{\mathrm{RC}} \tag{1}$$

where $\langle\cdot\rangle$ means the average value over different stirrer positions, and $\mathrm{TRP}_{\mathrm{Ref}}$ represent the net input power from the reference source, which can be obtained from the output power of the reference source, insertion loss of the cable and the total efficiency of the reference antenna. By solving $\mathrm{TRP}_{\mathrm{DUT}}$ from (1), the TRP of the DUT can be obtained as [1]-[3]

$$\mathrm{TRP}_{\mathrm{DUT}} = \frac{\langle P_{\mathrm{RxDUT}}\rangle}{\langle P_{\mathrm{RxRef}}\rangle}\mathrm{TRP}_{\mathrm{Ref}} \tag{2}$$

When the bandwidth of the radiated spectrum of the DUT is narrow (much smaller than the average mode bandwidth of the RC), the probability density function (PDF) of the received power is exponential distribution. However, when the radiated signal is wideband, the received power from different

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frequencies is independent and the PDF of the total received power will no longer be exponential. Thus we should be careful on the use of PDFs as an indicator to assess the performance of an RC (or measurement results).

II. Theory

We first review the PDFs for the narrow band TRP measurement and then generalize them to the wideband cases. When the RC is well stirred with low *K*-factors, the PDFs can be derived analytically using the multiple random variable analysis.

A. Narrow Band DUT

When the radiated spectrum of the DUT is smaller than the average mode bandwidth of the RC, we can consider it as a single frequency source. Suppose X_1, X_2, …, X_M are the measured power samples for the DUT, they are independent random variables with exponential distribution, where *M* is the independent sample number collected in the DUT measurement. The PDF is $e^{-x/\mu_x}/\mu_x$, the expected value and the standard deviation are both μ_x. The average value $\langle P_{\mathrm{RxDUT}}\rangle = \frac{1}{M}\sum_{i=1}^{M} X_i$ has a gamma distribution, and the PDF is [18], [20]-[22]

$$p_{\mathrm{RxDUT}}(x) = \frac{(M/\mu_x)^M}{\Gamma(M)} x^{M-1} e^{-xM/\mu_x},\ \ x = \langle P_{RxDUT}\rangle_M \quad (3)$$

where the expected value is μ_x and the standard deviation is $\mu_x/\sqrt{M}$, $\Gamma(\cdot)$ is the gamma function $\Gamma(x) = \int_0^\infty e^{-t} t^{x-1} dt$ and $\langle\cdot\rangle_M$ represents the average value over *M* independent samples.

The PDF of the measured power from the reference source can also be considered if the sample number is not large in the calibration process. We have the same expression as (3) but with $y = \langle \mathrm{P_{RxRef}}\rangle_N$ (instead of x) where *N* is the independent sample number collected in the reference power measurement (calibration process). It has been found that the ratio of $\langle P_{RxDUT}\rangle_M$ and $\langle P_{\mathrm{RxRef}}\rangle_N$ has a PDF of [18], [20]-[22]

$$p_{\mathrm{TRP}}(z) = \frac{\Gamma(M+N)\mu_x^N \mu_y^M}{\Gamma(M)\Gamma(N)N/M}\left(\frac{zM}{N}\right)^{M-1}\left(\mu_x + \mu_y\frac{zM}{N}\right)^{M+N},$$

$$z = \frac{\langle P_{RxDUT}\rangle_M}{\langle P_{\mathrm{RxRef}}\rangle_N} \quad (4)$$

where the expected value is $\mu_x N/[(N-1)\mu_y]$ and the standard deviation is

$$\frac{N}{N-1}\frac{\mu_x}{\mu_y}\sqrt{\frac{M+N-1}{M(N-2)}} \quad (5)$$

$\Gamma(M)\Gamma(N)/\Gamma(M+N) = B(M,N)$ is the beta function [23]. Note that mean value is biased and the unbiased estimator has a factor of $(N-1)/N$ for μ_x/μ_y. When $M = N$, the relative standard deviation is

$$\sigma_{\mathrm{rTRP}} = \left.\sqrt{\frac{M+N-1}{M(N-2)}}\right|_{M=N} = \sqrt{\frac{2N-1}{N(N-2)}} \quad (6)$$

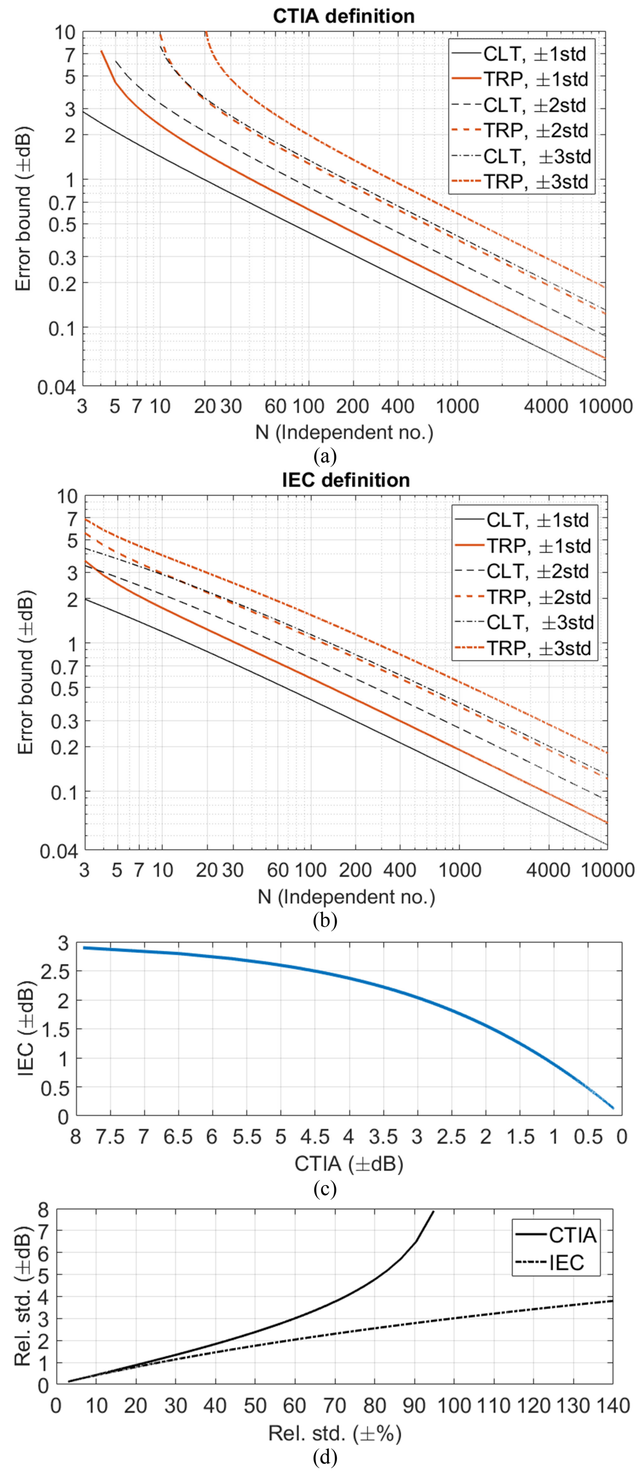

Fig. 2. (a) Error bounds of (6) and values from CLT, note that the standard deviation in dB unit is calculated using $5\log10[(1+\sigma_r)/(1-\sigma_r)]$ (CTIA definition), the probabilities for ±1std, ±2std and ±3std are about 68.2%, 95.4% and 99.7% respectively; (b) the standard deviations in dB unit are calculated using $10\log10(1+\sigma_r)$ (IEC definition); (c) conversion between the relative standard definitions in CITA and IEC standard; (d) conversion between the linear values and dB values.

Not surprisingly when *M* or *N*→ ∞, the relative standard (σ_{rTRP}) deviation approaches $1/\sqrt{N-2}$ or $1/\sqrt{M}$ respectively, which agrees well with the central limit theorem (CLT). Fig. 2(a) and (b) plot the error bounds with different relative standard deviations for $M = N$. The results from CLT (the relative standard deviation is $1/\sqrt{N}$) are also given. The error bounds in Fig. 2(a) and (b) can be considered as the lower limits, as we have not considered other error sources (e.g. from instrument, RC inhomogeneity and reference source). As can be seen, when N is large, the difference between $\sigma_{\rm rTRP}$ and $1/\sqrt{N}$ is small. We have to note that the relative standard deviation definition in dB unit in CTIA [2] and IEC [1] standard are different. In CTIA standard, we have [2]

$$\sigma_{\rm r,dB} = \frac{1}{2}\left(10\log_{10}\frac{1+\sigma_{\rm r,linear}}{1} - 10\log_{10}\frac{1-\sigma_{\rm r,linear}}{1}\right) = 5\log_{10}\frac{1+\sigma_{\rm r,linear}}{1-\sigma_{\rm r,linear}} \quad (7)$$

where $\sigma_{\rm r,linear}$ and $\sigma_{\rm r,dB}$ represent the relative standard deviation in linear unit and dB unit respectively. The average value between the upper bound and the lower bound (relative to the mean value) is used. While in the IEC standard, only the upper bound is considered (the maximum value is more important) [1]

$$\sigma_{\rm r,dB} = 10\log_{10}(1+\sigma_{\rm r,linear}) \quad (8)$$

Fig. 2(c) shows the conversion between the CTIA definition and the IEC definition, note that for small $\sigma_{\rm r,dB}$ the difference is small. While for large $\sigma_{\rm r,dB}$, the error bound from the CTIA definition is bigger than that from the IEC definition. The conversion between dB values and the linear percentage is presented in Fig. 2(d).

B. *Wideband DUT*

For a wideband signal (the radiated spectrum is much wider than the average mode bandwidth of an RC), the measured power samples are actually already superimposed from different frequencies. This is equivalent to the frequency stir process. Suppose we have L independent samples in the measured bandwidth ($L \approx \mathrm{BW}/\Delta f$, where BW is the bandwidth of the radiated signal, $\Delta f = f/Q$ is the average mode bandwidth [3]), each measured power samples from W_1, W_2, … , W_M can be represented using

$$W_m = \sum_{i=1}^{L} T_{{\rm RC}i} X_{m,i}\,, \quad m = 1, 2, \ldots M \quad (9)$$

where $T_{{\rm RC}i}$ is the chamber transfer function for different frequencies, $X_{m,i}$ is the radiated power samples at frequency $f(i)$ which has an exponential PDF. It can be found that although the PDF of the received power is different, it does not affect the use of (2) to measure the TRP. When the measured power spectrum is corrected considering the frequency dependency of $T_{\rm RC}$ in the calibration process, the mean value is unbiased. Because of the frequency stir, we have an even smaller relative standard deviation ($1/\sqrt{ML}$). Empirically, the final relative standard deviation can be calculated approximately from the uncertainty propagation [24], [25] of $z = x/y$ as

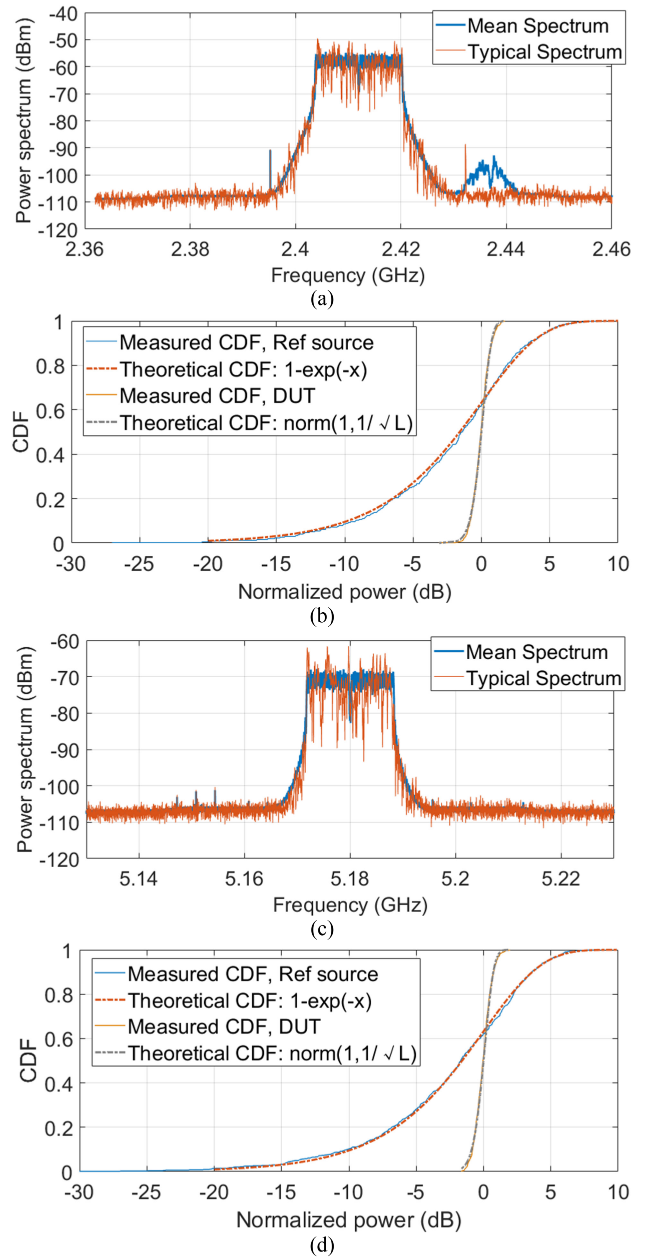

Fig. 3. (a) Measured spectrum at 2.4 GHz; (b) CDFs of the measured normalized power samples at 2.4 GHz, norm(1, $1/\sqrt{L}$) means the CDF of the normalized normal distribution with a standard deviation of $1/\sqrt{L}$, estimated $L \approx 50$; (c) Measured spectrum at 5.18 GHz; (d) CDFs of the measured normalized power samples at 5.18 GHz, estimated $L \approx 49$.

$$\sigma_{\rm rTRP} \approx \frac{1}{z}\sqrt{\left(\frac{\partial z}{\partial x}\sigma_x\right)^2 + \left(\frac{\partial z}{\partial y}\sigma_y\right)^2}\,\Bigg|_{\substack{\sigma_x = \frac{x}{\sqrt{ML}} \\ \sigma_x = \frac{y}{\sqrt{N}}}} = \sqrt{\frac{ML+N}{MLN}} \quad (10)$$

As expected, when $L = 1$, (10) reduces to the narrow band cases and is about the same as (6) when M and N are large. When $T_{\rm RC}$ is approximately constant and the radiated spectrum is relatively flat, the PDF of the measured power samples W_m, $m = 1..M$ can be derived as

$$p_{W_m}(x) = \frac{(1/\mu_w)^L}{\Gamma(L)} x^{L-1} e^{-x/\mu_w}, \quad x = W_m \quad (11)$$

where μ_w is the mean value of W_m. Note that (11) is the same as (3) and the frequency stirring effect is already included in each measured power sample. When M measured samples are averaged, the result has the same PDF as (11) but with M replaced with ML. The following steps are the same as (4)-(6) but with M replaced with ML. Note that (11) is rigorous only when the product of T_{RC} and the radiated spectrum is flat. However, if we are not interested in the analytical expression of W_m, the CLT still holds even when T_{RC} and the radiated spectrum are not flat; i.e. when ML is large, the measured PDF approaches to a normal distribution.

It is interesting to note that when the BW is wide (large L), the standard deviation for each measured sample (at different position or stirrer position) is small. Ideally for large L, the measured power is almost the same for different stirrer positions, and no mechanical stirring is necessary.

III. Measurements

We recorded 720 channel power samples for DUT measurement in Fig. 1(b) (2°/step for both horizontal and vertical stirrers with 4 different initial vertical stirrer positions). The spectrum of the DUT (a WiFi device) is recorded and illustrated in Fig. 3(a) and (c) at around 2.4 GHz and 5.18 GHz respectively. The mean spectrum (averaged over 720 stirrer positions) and the typical spectrum (at one stirrer position) are given. Fig. 3(b) and (d) show the measured cumulative distribution functions (CDFs) of 720 samples.

The measured samples are normalized to the mean value. As can be seen, the measured power of the reference source with a single frequency is exponential distribution while the measured power (channel power) of the DUT is normal distribution. The independent sample number L (which is also the independent sample number in the frequency stir) in Fig. 3(b) and (d) are estimated from the relative standard deviation of the measured samples.

As expected, the measured power for broad band signal has a smaller standard deviation and good agreement is achieved between measurement and theory. Hypothesis testing has been performed to verify the measured empirical CDFs, and the results fail to reject the theoretical CDFs at a significant level of 5%. The final measured TRP of the DUT can be calculated using (2) and the relative standard deviation can be estimated using (6) (with M replaced by ML) or (10). At 2.4 GHz, $M = 720$, $L \approx 50$ and $N = 720$, $\sigma_{\mathrm{rTRP}} \approx 3.76\%$ and the measured TRP of the DUT is 1.10 dBm $\pm$ 0.16 dB (1std); At 5.18 GHz, $M = 720$, $L \approx 49$ and $N = 720$, $\sigma_{\mathrm{rTRP}} \approx 3.76\%$ and the measured TRP of the DUT is 0.04 dBm $\pm$ 0.16 dB (1std). The independency of the mechanical stirrer positions has been verified using the autocorrelation coefficient [1], [2], and (7) is used for the linear-to-dB conversion (CTIA definition). These error bounds are the theoretical lower limits without uncertainties from instruments, RC inhomogeneity and the reference source.

IV. Conclusions

We have reviewed the PDFs for the narrow band signal TRP measurement in an RC, and have generalized to the wideband case. The differences of the standard deviation definitions between the CTIA and the IEC standard are also presented.

The narrow band measurement is a special case with the signal bandwidth smaller than the average mode bandwidth of the RC. When the signal bandwidth is wider than the average mode bandwidth of the RC, an inherent frequency stirring is introduced in power measurement. It is interesting to find that for the wideband signal TRP measurement, the measured result has smaller standard deviations. Ideally, when the bandwidth is wide enough (large L), the measured power is uniform for different stirrer positions and a single sample measurement ($M = 1$) is enough (no need to stir) which can be much more efficient than the 3D scan method in an anechoic chamber.

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