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Perceptions, Difficulties and Working Memory Capacity Related to Mathematics Performance

by

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Dedication

I dedicate this thesis to my mother, Maqsooda. Despite experiencing so many tragedies in her life, she gave of herself unstintingly to her family. She encouraged me, supported me and gave me the strength and determination to pursue my studies and my career. I owe her a huge debt of gratitude. Without her, this period of study would not have happened and this thesis would never have been written.

Abstract

There is a general view that students of not have a positive attitude towards mathematics. In general, mathematics is considered a ‘difficult’ subject and sometimes there is a lack of enjoyment. Mathematics is often portrayed as being abstract and unrelated to life.

In the light of the key role mathematics has in the curriculum, the aim of this study is to explore the difficulties and self-perceptions of students aged about 10-12 in Pakistan as they undertake their studies in mathematics. The study uses a survey of student perceptions, working with samples of students drawn from both Urdu and English medium schools (N = 813). In addition, working memory capacity of those in grade 5 (age about 10) was measured and information was gained about their performance in mathematics examinations. The data is analysed to consider how their self-perceptions related to their experiences in learning mathematics which varies with age, language background and gender. Any relationships between these self perceptions, mathematics marks and measured working memory capacity are explored as well. The observed outcomes can be used to inform the agenda for action or further study.

It was found that the vast majority (English medium and Urdu medium) appreciate the role and the importance of studies in mathematics although topics like geometry, fractions, topics with life applications, statistics are causing problems. It is almost certain that these topics place demands on working memory which make understanding very difficult.

In the Urdu medium schools, the curriculum in grade 6 is clearly causing major problems while, in both systems, pressures for success based on examination performance have generated a complete industry of private tutors. Many of the gender differences can be interpreted in terms of the social roles in Pakistani society. However, girls do seem more positive and more committed in relation to their studies in mathematics.

The study has revealed two major issues which need careful consideration. One is the whole issue of memorisation and understanding. The goal of meaningful learning must be stressed more if positive attitudes are to be retained. The whole issue of making the mathematics studied become related in some way to the lifestyle of the learner seems very important but this is not easy without overloading working memory. In considering both of these issues, the critical role of assessment has to be addressed: if assessment offers rewards almost entirely for the recall and correct execution of mathematical procedures, then this will be reflected in textbooks and teaching approaches. Along with curriculum design and teaching approaches which are consistent with the known limitations of working memory, assessment is perhaps the single most important issue to be considered. Very significant correlations were found for grade 5 students when their measured working

memory capacity was related to their mathematics examination performance. Indeed, the correlation value for Urdu medium students is the highest such correlation which has been found in any discipline. This suggests major curriculum design problems in the national syllabus for Urdu medium schools as well as assessment problems.

The study has pinpointed many areas of success along with specific areas where there are serious problems. In this way, an agenda for future research and action has been described.

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Chapter 1

Mathematics and Education in Pakistan

1.1 Introduction

There is a general view that students often do not have a positive attitude towards mathematics. In general, mathematics is considered a “difficult” subject. The majority of students seem to believe that there are externally imposed fixed structural boundaries e.g. assessment procedure, curricula, and grouping practices for each individual learner in mathematics, beyond which learning becomes very difficult and frustrating (Xin Ma, 1997). The lack of opportunity for creativity and enjoyment is a common perception and, sometimes, they find mathematics boring. Another major problem is its perceived irrelevance to the real world. Mathematics is often portrayed as being abstract and unrelated to life but, nonetheless, school pupils seem to realise the importance and place of mathematics for their future careers and are prepared to put up with studies in mathematics. These findings are fairly widespread in the research literature as well as in talking to mathematics teacher, a typical set of observations being made in the review of Xin Ma (1997). Attitudes tend to become more negative as pupils move from elementary (primary) to secondary schools (McLeod, 1994).

In almost all countries, mathematics holds a key position in the curriculum at all levels. At the upper levels of secondary school education, studies in mathematics can be optional. In many countries, the uptake rates are disappointing (e.g. Al-Enezi, 2008) while in others, mathematics is highly popular (Scottish Qualifications Authority, 2007). In Pakistan, mathematics is often regarded as very demanding.

1.2 Study Aims

In the light of the key role mathematics has in the curriculum, the aim of this study is to explore the difficulties and self-perceptions of pupils aged about 10-12 in Pakistan as they undertake their studies in mathematics. Why is mathematics perceived in this way? What are the hidden factors which lead students to this point that they decide not to continue mathematics or continue it “unwillingly”. This age range is important in that it precedes the years of adolescence and, at this age, experiences in mathematics may well lay foundations for attitudes towards the study of mathematics at later stages. Before considering this further, it is useful to outline the way the education system works in Pakistan and where mathematics fits into the overall picture.

1.3 The Pakistan Context

There are two main systems which have evolved in Pakistan: Urdu medium schools and English medium schools. Each is now described briefly, up to age 16. Both systems have similar structures:

Name	Grade	Age
Primary	1-5	5-10
Middle	6-8	11-13
Matriculation	9-10	14-15

Table 1.1 School Structures

Although the structures are outwardly similar, the way the schools operate and the way the curriculum is planned and taught can vary considerably. School mathematics can be presented in numerous ways. These reflect the nature of mathematics itself. The subject has its own terminology and has developed a very large range of symbolic representations. It is is highly conceptual in many areas but depends on the correct following of accepted procedures in order to again 'right' answers. School courses may concentrate on one or more of these features. The traditional tendency is to emphasise the correct conduct of taught procedures in order to gain 'right' answers and, of course, the importance of being able to 'do' mathematics correctly cannot be underestimated. The danger is that conceptual understanding is neglected and this may lead to an inability to be able to apply mathematical skills in novel situations.

This is very much the pattern of Pakistan, especially in Urdu medium schools. In Urdu medium schools, the students are typically from the middle class or the lower class of society, with lower incomes. English is taught as a compulsory subject and, in most schools, it is introduced at grade 6. Students do not have many advanced facilities and have to flourish with limited resources. Here, there is a prescribed textbook in mathematics for every class and the examination is solely based on this book. The text books decided by the same authorities are available for every subject. Concepts are very briefly described in the beginning of every chapter of these books along with a few solved examples and numerous exercises to complete. In the present methodology, stress is laid on solving exercises rather than giving the students a clear idea of the fundamental concepts. Thus, the present teaching methodology of mathematics leads students to rote learning of the textbooks.

In English medium schools there is greater freedom but it tends to end up in a similar position, with pupils working through prescribed worksheets or texts to gain right answers, with minimal grasp of the concepts involved. Thus, "meaningful" learning element is lacking in both mediums. The curriculum is to be followed strictly by the students and the teachers. Mathematics is considered important for future professions but at the same time a very boring and difficult subject for the students. Most of the topics are perceived difficult by the students and they usually have to seek help from tutors after school to cope with mathematics. Indeed, the industry of after school tutoring is extensive.

For Urdu medium schools, the curriculum is constructed by the representatives appointed by the government authorities. Teachers are supposed to follow the curriculum with a very limited freedom to make any changes in the curriculum and assessment process. Policies and rules are governed by the government. From grade 1-8, all subjects are compulsory but at matriculation level two main fields are offered to students: Science or Arts. They have to opt for only one field and their future profession and career is highly dependent on these two options.

It is not easy to develop conceptual understanding related to mathematics. Pupils find it difficult enough to gain any kind of mastery of procedures. Adding on one more layer of complexity may make learning even more difficult. This almost certainly relates to the limitations imposed by working memory capacity and that will be discussed in this thesis.

However, there are some steps that may be useful. If any kind of conceptual understanding is to be developed, then it is vitally important that the curriculum in Pakistan should be thoroughly revised in such a way that concepts are interlinked and it should help to develop the subject matter step by step. Difficulty level of the topics should be according to the age and the mental level of the students at that particular age. Concepts taught at a senior class should be an extension of the concepts taught in the previous class. In this way, the chain of the concepts taught would be interlinked throughout. It is possible that some topics are introduced too early and perhaps in the wrong order. These issues will be explored later and possible ways to encourage more conceptual understanding will be discussed.

There are several keys to making progress in mathematics education in Pakistan, especially at younger ages. There needs to be agreement on what topics are taught when and this must apply to all schools. It is vital that such curriculum decisions are based on the best evidence available. Indeed, the mathematics curriculum needs to be designed for all pupils. Because those qualified in mathematics play a great role in curriculum design, there is a real danger that the curriculum is based on their experiences which tend to be highly successful. The curriculum then is based on the logic of mathematics and the needs and aspirations of those who will use mathematics more academically in future may be met, at the expense of the majority.

Assessment procedures are based on the aim of getting good grades in mathematics. Mixed ability challenges are not much in focus and usually the trend is to make pupils pass the examination. Exam papers are usually not able to assess everyone fairly. The greatest hindrance, perhaps, to curriculum advance lies in the assessment system. The current examination system in Pakistan is based on memory recall, guess papers (a booklet giving an idea to the students of which important questions are likely to be included in the coming examination) and selected questions from past papers. Inevitably, teachers will teach to gain maximum results for the pupils. This will emphasise the correct recall and use of memorised procedures, perhaps with minimal conceptual understanding. While the demand level of examinations must be such that all pupils have opportunity for some level of success, there must be opportunities for those who have gained more in conceptual understanding to receive the rewards for their progress. This will not be easy but, unless the examinations genuinely reflect the desired aims of the courses, teachers and pupils will aim solely at recall and use of memorised procedures as their dominant goal.

In general, the students from the upper class or upper middle class of society attend English medium schools. Most of the schools are private and are free to decide their own curriculum. The medium of instruction is English throughout. Teachers have much freedom to make suggestions or sometimes changes in the curriculum. Text books are decided by the school. Students have great facilities and opportunities to develop at intellectual, personal and academic levels. Unlike Urdu-medium schools, the English medium schools follow curricula from the overseas Cambridge Board, leading to O Levels and A levels at the final stages. The pupils in Urdu medium schools are presented for the FSc Examinations.

Overall, the education system is very divided and there is very limited parity of opportunity. The use of overseas examinations is designed to give high credibility but it makes the system favour the elite at the expense of the majority. It also hinders the development of a truly Pakistani education system.

1.4 Thesis Plan

The aim is to explore aspects of the mathematics learning experiences in grades 5 to 7 (ages approximately 10-12) in both Urdu medium and English medium schools. This involves looking at pupil perceptions of their experiences, the nature of the difficulties they have with mathematics and possible reasons for these difficulties.

In order to understand why difficulties may be arising, there is a need to look at what is known about how children learn, the powerful insights from information processing and the kinds of problems which can lead to the development of more negative attitudes. The nature of attitudes, how they develop and change and how they can be measured are all reviewed. At this point, the focus moves to mathematics itself, what is known about how young pupils think of mathematics, as well as the difficulties they experience.

Several studies have tried to address the basic issues in mathematics education and these findings are summarised to give an overall picture of the difficulties faced by the students in this subject. Reasons for these difficulties will be discussed, this being related strongly to what is known about the processes of learning in highly conceptual areas of the curriculum.

The study uses a survey of pupil perceptions, working with samples of pupils drawn from both Urdu and English medium schools. In addition, working memory capacity of those in grade 5 (age about 10) was measured and information was gained about their performance in mathematics examinations. The data is analysed to consider how their self-perceptions related to their experiences in learning mathematics which varies with age, language background and gender. Any relationships between these self perceptions, mathematics marks and measured working memory capacity are explored as well.

Chapter 2

Learning Theories

Various models of learning and teaching development provide a useful framework for research in education.

2.1 What is Learning?

Learning starts the day we are born and occurs in all areas of life. School learning is only a small part of the learning process. Most of psychologists and educators tend to describe learning as a process by which behaviour is either modified or changed through experience or training. According to Hamachek (1995), it refers not only to an outcome that is manifestly observable, but also to attitudes, feelings and intellectual processes that may not be obvious. Gagné (1964) stated that: *"Learning is a change in human disposition or capability, which can be retained, and which is not simply ascribable to the process of growth."*

According to Reid (2008), learning is a process that leads to any change in behaviour not explainable simply by development. We tend to limit this more to the cognitive but it could be also the psychomotor (e.g. learning a sport or a musical instrument or other skills like swimming, driving etc.). The issues of the conscious effort in learning and the storage of information are under discussion in the field of educational research.

2.2 Background Approaches

Learning theories have been classified into the following two major groups:

Behaviourists theories are also known as stimulus-response theories. Based on the ways animals learn, human learning was first considered as response acquisition. According to behaviourists, learning is a change in observable behaviour, which occurs through stimuli and responses. They interpreted learning in terms of changes in strength of stimulus-response connections, associations, habits or behavioural tendencies. Learners were perceived as passive beings whose learning was influenced by the rewards and punishments from the teachers. Behaviourists theories help us to define the conditions under which particular types of learning must be broken into smaller sub units. Skinner (1904-1990) started to study the human learning process and believed that the learner's mind was a black box and it was impossible to give any explanation of mental processes (Smith *et al*, 1998). Later, a new era of psychological research started to investigate what happens in the human mind ("the black box") and this led to *Cognitive Theories.*

Cognitive Theories involve the research of mental processes or events rather than actual behaviour. Cognitive psychologists developed different models to describe cognitive activities. For cognitive theorists, learning is likely to be holistic. They described learning in terms of reorganisation of perceptual or cognitive fields to gain understanding (Bigge & Shermis, 1999). They tried to investigate how learners obtain, process, store and use information. In contrast to behaviourists who believed that learners are passive beings, cognitive psychologists perceived learners as active processors of information - a metaphor borrowed from the computer world. Among the cognitive theories, constructivism theories and information processing models are important approaches.

The *Information Processing* approach provided models of operation which were applied to the process of human learning. It considers the human being as a processor of information and tracks the way new information flows through the brain. This approach has led to excellent models of learning which have been found to be predictive (Miller 1993). The *constructivism approach* is basically learner-centered and presents the idea that meaningful understanding is constructed by the learners rather than just given to them. They do not only "receive" information, rather they construct the knowledge (Miller, 1993). Children develop their beliefs in early years of life and previously acquired notions influence their new experiences in different ways. They "construct" different sets of ideas in order to make sense of their surroundings. The person who tried to discover the hidden corners of children's minds and whose ideas had a widespread influence in this century was Jean Piaget (1896-1980).

2.3 Piaget's Theory of Cognitive Development

Piaget was born in Neuchatel, Switzerland. He is considered one of the most well-known developmental psychologists but he was also a philosopher, logician and educator. He was the first European to be given the American Psychological Association's award for his distinguished scientific work. Over the course of his career, Piaget wrote more than sixty books and several hundred articles.

He was a keen and natural observer who developed an interest in biology and the natural world from his childhood. He received a PhD in natural sciences from the University of Neuchatel. He then moved to Grange-aux-Belles, France, where he taught in a boys' school run by Alfred Binet, the developer of the Binet Intelligence Test. This experience and his natural interest in physical development processes led him to philosophy and eventually to psychology. These moves are seen as having directly influenced his beliefs in his theory of intellectual development.

Piaget described cognitive development as a process of adaptation to the environment and an extension of biological development (Wadsworth,1979, pg 2). Piaget described cognition as an active and interactive process (Boudourides, 1998). Piaget's concept of cognitive development was greatly influenced by his early work as a biologist where he became impressed by the interaction of molluscs with their environment. From this observation, he came to believe that "*biological acts are acts of adaptation to the physical environment and organisations of environment*" (Wadsworth, 1979, pg 3). He also discovered that mind and body do not operate independently of one another, leading him to see the concept of intellectual development in much same way as biological development. He believed that intellectual and biological activities are both parts of an overall process by which an organism adapts to the environment and organises experience (Wadsworth, 1979, pg 3).

Piaget perceived a child as an organism who grows in an environment that affects its development and adaptation to the surroundings. He asserted that a child tries to make sense of the objects around him and constructs knowledge through experiences provided by the environment. Through his experiments, he discovered that children learn differently from adults. As a biologist, he observed and studied the process of a child's thinking and learning. He tried to explore the influence of a child's environment and experiences on his cognitive development (Atkinson, 1983).

Piaget discovered through his careful observations that a child learns when he goes through different experiences in his environment and then he builds up a hypothesis (Goswami, 1998). Actions on different objects around them help them to develop physical knowledge (Wadsworth, 1979). He defined the following four basic concepts to explain that how and why mental development occurs:

Schema: Schemata are intellectual structures that organise events into groups according to common characteristics. The thought process that brings about adaptation as described by Piaget is called schema. For Piaget, the development of human intellect proceeds through adaptations to the environment by the organisation of actions, or patterns of behaviour called schemata. Schemata are the cognitive or mental structures by which individuals intellectually adapt to and organise the environment.

Schemata are structures that adapt and change with mental development. Schemata are used to process and identify incoming stimuli. Schemata never stop changing or becoming more refined. Schemata reflect the child's current level of understanding and knowledge of the world. When confronted with a stimulus, a child tries to fit the stimulus into an available schema. A child is born with schemata that are reflexive in nature. With the growth of the child, schemata help the child to differentiate several things around him.

With time, schemata become more numerous (as the child is involved in his environment more actively). Also schemata become less sensory and the child is able to be more perceptive. The schemata of the adult evolve from the schemata of the child through adaptation and organisation.

Assimilation: *A cognitive process, by which a person integrates new perpetual, motor or conceptual matter into existing schemata or patterns of behaviour*. (Wadsworth, 1979, pg 14) Assimilation is a constant process and encompasses the child's interaction with the environment in terms of his intellectual structures. The child adapts to his environment and absorbs what is needed for cognitive growth. Assimilation theoretically does not result in a change of schemata, but it does affect the growth of schemata and is thus a part of development.The process of assimilation provides a quantitative change in intellectual structures as it only adds to the existing schemata (Wadsworth, 1979).

Accommodation: When a child deals with a new stimulus, he tries to create a new schema to place the new stimulus or he modifies an existing schema to fit in this new stimulus. Both these possibilities are forms of accommodation. Accommodation is the creation of new schemata or the modification of old schemata and assimilation theoretically does not result in a change of schemata, but it does affect the growth of schemata and is thus a part of development. The process of accommodation provides a qualitative change in intellectual structures (Wadsworth, 1979).

Equilibrium: Piaget described equilibration as a constant adjustment of the balance between assimilation and accommodation. He states that cognitive development, basically, is the logical series of equilibrations. In other words, equilibration is the process of moving from disequilibrium to equilibrium (Flavell, 1963). Disequilibrium is a state of imbalance between assimilation and accommodation. Disequilibrium is a “cognitive conflict”. When a child goes through an experience provided by his environment, he expects a certain thing to happen and if it does not then this discrepancy results in disequilibrium. In other words, disequilibrium activates the process of equilibration and also motivates a child to assimilate or accommodate further (Wadsworth, 1979).

Equilibrium is a state of cognitive balance between assimilation and accommodation. If a child is successful in assimilating a new stimulus into his existing schema, then equilibrium sets in for the moment. If he is not successful, then he tries to accommodate by modifying a schema or creating a new one. After successfully doing it, assimilation of the stimulus proceeds, and equilibrium sets in for the moment (Wadsworth, 1979). As the learner confronts a great variety of experiences, so, every time new information is received new equilibrium has to be formed and hence cognitive development keeps going.

2.4 Piaget's Developmental Stages

Piaget summarised his observations into four stages of development (table 2.4).

Stages of Intellectual Development	Description
Sensorimotor *(birth to 2 years)*	Differentiates self from objects. Recognises self as agent of action and begins to act intentionally. Achieves object permanence, realising that things exist even when no longer present to the senses.
Pre-operational *(2 to 7 years)*	Learns to represent objects by images and words. Thinking is still egocentric with difficulty in seeing the viewpoints of others. Classifies objects according to several feature e.g. colour or height.
Concrete operational *(7-11 years)*	Can think logically about objects and events. Achieves conservation of number (age 6), mass (age 7) and weight (age 9). Can classify objects according to several features; can order them in series along a single dimension.
Formal Operational *(11 years onwards)*	Can think logically about abstract propositions Can test hypotheses systematically Becomes concerned with the hypothetical, the future, and ideological problems.

Figure 2.1 Piaget's Developmental Stages

The Sensorimotor stage (age 0-2): Infants using grasp, suck or look at objects to develop their internal representation. During this stage, behaviour is primarily motor. They do not yet represent events and think conceptually. During this time, there is a transition from sensory-motor cognition, which is action based and dependent on physical interaction with the world, to internalisation of action and the organisation of symbolic knowledge that is associated with concrete operations. Towards the end of this period the child, by exploring the world through sensory experiences and movement, begins to represent the world in terms of mental images and symbols through the acquisition of basic language.

They understand their world via interacting with the environment, which surrounds them. This fundamental response allows infants to obtain knowledge of the world and to attain the idea about it. At the end of this stage, infants can form mental images of reality. During the first two years of life, infants look, touch, grasp and suck the objects around them to develop their internal representation. At this stage, they understand the world by interacting with their environment and there is a transition from sensory motor cognition, which is based on action and depends on the physical interaction with the world. Cognitive development is in progress as they can begin to construct schemata. At the end of this stage children can form mental images.

***The Pre operational Stage* (age 2-7):** At this age, children can remember, imagine and pretend. They start being able to make symbolic mental representations of their actions: They acquire schema of the world around them. This stage is characterised by the development of language and other forms of representation and rapid conceptual development. Reasoning during this stage is pre-logical or semi-logical.

At this stage cognitive development occurs through the use of symbols while language is more developed. Symbols in place of objects and language facility and grammatical rules expand quickly. Memory and imagination are more developed. Children in the pre operational stage tend to be egocentric. Children try to explain things by drawing and their efforts become more factual. The gap between the years of two and seven is the period of the preparation for concrete thought. During this time there is a transition from sensory motor cognition, which is action based and dependent on physical interaction with the world, to internalisation of action and the organisation of symbolic knowledge, which is associated with concrete operations.

***The Concrete Operational Stage* (7-11):** During this stage, children begin to think logically. They can operate in a concrete system of objects and can make relations like ordering, classifying and arranging. The child develops the ability to apply logical thinking to concrete problems. This stage was characterised by the ability to think logically. Nonetheless, the child's reasoning is still imperfect during this period. It was called the concrete stage because starting point of an operation was always some real system of objects and relations. At this point the child has achieved the characteristics of reversibility (inversions and reciprocity) that were associated with conservation. Later on they will be able to generalise from one set of circumstances to another. This stage commences when the formation of classes and series takes place mentally, that is to say, when physical actions become internalised as mental actions or operations. The egocentricity of child decreases substantially and reliable co-operation with others replaces it. The child is able to solve problems with which he is faced, but his solutions are characteristically in terms of direct experiences.

With development the child is able to generalise groups of situations**.** At this stage, children are characterised by logical understanding, the demonstration of logical thought and systematic manipulation of symbols related to concrete objects. The child's logical thinking improves the concrete operational structures such as classification, serialisation and conservation. Children at this stage are able to understand another person's point of view and consider more than one perspective simultaneously and are becoming more logical, flexible, and regular than early childhood, especially how to identify and differentiate between seven types of conservation: number, length, liquid, mass, weight, area, volume. In addition, they are able to distinguish more than one feature of an object at the same time. At this stage, the results of internal actions become reversible thus marking the beginning of mental thought. The development of logic structures continues to require concrete experience to which the logic can be applied.

***The Formal Operational Stage* (11 years onward):** During this period the child starts to use abstract reasoning. Abstract hypotheses can be built along with the capability to hold some variables constant while manipulating other variables in order to determine their influence. Analytical and logical thought no longer requires reference to concrete examples. At this stage, concrete operations become linked together indicating the beginning of the scientific thought. At this stage, the formal operation stage develops: here basic abstract logic is handled and the ideas of hypotheses are attained. This stage coincides approximately with the onset of adolescence. The person becomes able to solve all classes of problems as he develops reasoning and logic. The person manipulates ideas not directly related to the real world and starts to integrate his new intellectual capacities for explanatory purposes.

As logical processes develop abstract ideas can be handled with increasing confidence and there is the beginning of the formation of hypotheses. Steadily, the child becomes able to apply logical reasoning to all classes of problems. In general, this period is characterised by a new mode of reasoning that is no longer limited to dealing with directly representable facts. **P**upils can solve abstract problems, forming conclusions from hypotheses and exploring many possibilities. Thought has become truly logical, abstract, and hypothetical. Formal operational thought resembles the kind of thinking we often call scientific thinking.

The young person steadily develops abilities to conceptualise and hypothesise beyond concrete data. There is increased hypothetical thinking and use of logic to solve problems, leading to all kinds of possibilities. He can identify the present, past and future. He becomes capable of introspection and is able to think about his thoughts and feelings and is able to reason in a manner more fully independent of past and current experience.

According to Piaget, children construct their own knowledge through these stages in the same order but not at the same rate.

2.5 Criticisms of Piaget's work

Researchers have raised many questions on the validity of Piaget's development theory. There are four areas where criticisms can be made and these are now discussed briefly. However, it has to be noted that Piaget just described his empirical observations. He described what he saw. He made little attempt at interpretation. Like any good scientist, he simplified situations to concentrate on a general description of cognitive development. In this way, he laid a foundation on which others have been able to build.

Language

Piaget developed his observations by talking to children about various practical situations and exploring how they saw them. Of course, there might be a possibility of having some language problems while communicating with children during his experiments. For example, while introducing the concept of volume, he displayed two containers on the table for them. One container was 'more' in height than the other but both were of the same capacity. Then he poured the same quantity into both containers. He asked them which container contains 'more' liquid. The children pointed out the one with 'more' height. However, there may be a language problem related to the meaning attached by the child to the word 'more'. Thus, a child might well understand the word 'more' as meaning taller (Margaret Donaldson, 1987)

Sample size

Piaget's research methodology was highly criticised by many researchers. The sample size used by Piaget in his experiments was not large enough, with very little possible statistical analysis, and there was a lack of normative data on age levels (Ausubel, 1978). However, it has to be noted that he never tried to claim statistical significance. He merely described what he observed. Of course, his sample was not a typical cross section of children in Paris at the time. While this might affect the generalisability of his data, it does not affect the significance of what he described. He was a careful observer who presented a model by means of carefully chosen illustrations but the model was based on observations.

Rigidity and inflexibility of cognitive development stages

The boundaries of stages of cognitive development presented by Piaget are sometimes regarded as too rigid and inflexible according to many critics. Ausubel (1978) disagreed with the idea that development is abrupt or occurs in jumps. He asserts that it is more gradual and takes place smoothly. He also explained that intellectual functioning involves more variation at any of these stages than the concept of a stage would suggest to experience. Piaget defined the stages too precisely. The individual does move more smoothly from stage to stage but not in an abrupt fashion. In fact, the individual undergoes periods of transitions from one level to the next. Nonetheless, the stages described by Piaget are still essentially correct.

Cultural context

Different researchers have objected that Piaget neglected the importance of social, environmental and cultural factors in the cognitive development of a child. He mainly described the biological basis of cognitive development and ignored the fact that environment and social interaction play a great role in a child's intellectual growth and development (Bruner, 1966). Piaget ascribed an insufficient role for parents, teachers, peers and other important figures in a child's surroundings and he emphasised more the role of the individual in the process of knowledge construction (Bliss, 1995). The Russian psychologist, Vygotsky (1974), asserted that social and cultural interaction are the most important factors for success in the learning process. It was suggested that when adults and children are involved in the same type of activities in different context e.g. home, school or other social places, the processes involved are not similar. The strategies used by children to solve similar problems always depend on the environment where they face these problems (Wood, 1991). This suggested that problem solving skills are not generic in nature but are essentially context dependent.

Although there has been criticism of the work of Piaget, he is still considered the most outstanding and exceptional cognitive and developmental psychologist of all time. Piaget laid a strong foundation for modern education, having a great impact on educational practice and research (Miller, 1993; Donaldson, 1978). Some of the observations he made may be explained in part by ambiguities of language. His boundaries were perhaps too rigid and his sampling was open to criticism. Nonetheless, his findings are generalisable. The key area where more work was needed was in taking into account the effect of the environment and, particularly, the effect on those around who might be able to draw the child forward cognitively. The work of Vygotsky (1978) filled this gap and is now described.

2.6 Vygotsky

Lev Semyonovich Vygotsky was a Russian philosopher and developmental psychologist. Vygotsky made a great contribution in articulating his distinct views in psychology, especially in learning processes, but his work was suppressed by communist leaders and did not become well recognised outside the former Soviet Union until the reprinting of his famous book "Thought and Language" in 1962, after his death.

Vygotsky strongly rejected the common idea of his time that intelligence was fixed. His widely known social-cognitive theory (1962) contains the following three main themes:

- The importance of social and cultural context
- The role of language
- The idea of zone of proximal development (ZPD)

He, unlike Piaget, considered social and cultural interaction as the key elements to success in the learning process. Piaget considered the role of peer-interaction more important in a child's environment whereas Vygotsky asserted that interaction with a more experienced and skilled person could play a great role in a child's cognitive development. He claimed that every child has a great potential to develop his mental abilities in collaboration with other people in his environment. When children interact with more capable and experienced people, under their assistance they can learn ways to solve problems more effectively than on their own. Culture plays a definite and obvious role. He also pointed out that interaction with peers only could lead to an unnecessary delay in cognitive development and even lead to abnormal development and can create regression according to standards (Tudge & Rogoff, 1989). Vygotsky also advocated that language is a tool for a child, not merely to communicate with others in his environment, but also to plan, understand and develop his own activities.

This theory dealt with the child's capacity to recognise ways to go beyond the limitations of his own stage of development. In this, he referred to the Zone of Proximal Development (ZPD). Vygotsky introduced the idea of the Zone of Proximal Development and defined it as "*the discrepancy between a child's actual mental age and the level which he reaches in solving a problem with assistanc*e". He used a specific term for this assistance: scaffolding. He observed that scaffolding supports the child cognitively and helps him to understand the task fully. The Zone of Proximal Development varies from person to person as, with assistance, some children can reach a higher level of thought and can reflect in a more abstract way on a specific subject area. The children with larger ZPD perform in a better way than others with small ZPD (Bigge & Shermis, 1990).

Vygotsky's main contribution was to define the importance of a more skilled and experienced individual who could move forward a learner in understanding and increasing the levels of abstractness. This does not imply that Piaget's model of stages of development is completely wrong but that social interaction has a part to play in cognitive development.

2.7 Ausubel's Meaningful Learning Model

David Ausubel, an American educational psychologist, presented a model of learning by studying and describing the conditions and factors that lead to meaningful learning. He has made a major contribution to learning by introducing the key principle of his theory, the role of prior knowledge in learning. He states that, *" If I had to reduce all of educational psychology to just one principle, I would say this: the most important single factor influencing learning is what the learner already knows. Ascertain this and teach him accordingly."* (Ausubel, 1968). He also separated the ideas of meaningful and rote learning from reception and discovery learning (figure 2.2).

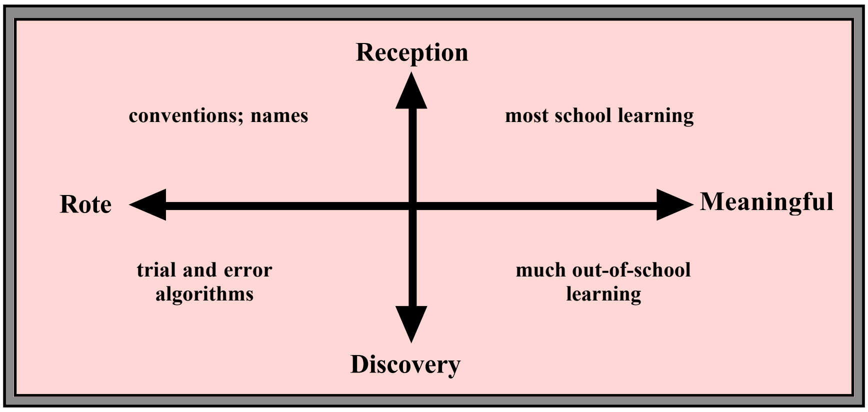

Figure 2.2 Reception Learning & Discovery Learning (Ausubel, 1968)

His model deals with the following two main aspects of human conceptual functions during instructional presentation (Novak, 1978):

- The way a learner learns different kinds of information meaningfully from the verbal/textual presentation of the material to be learned in the classroom.
- The importance of the prior knowledge already learned by the learner during the learning process.

Ausubel understands that prior knowledge plays the key role in providing a bank of frameworks in the learner's cognitive structure. This bank of frameworks gradually develops and grows towards formal reasoning. The scale or level to which the meaningful understanding can occur essentially depends on the quality and organisation of this store or bank of frameworks. Meaningful learning occurs when the new knowledge is linked with the existing conceptual frameworks of knowledge (Ausubel, 1968). He then went on illustrate various types of learning (figure 2.3).

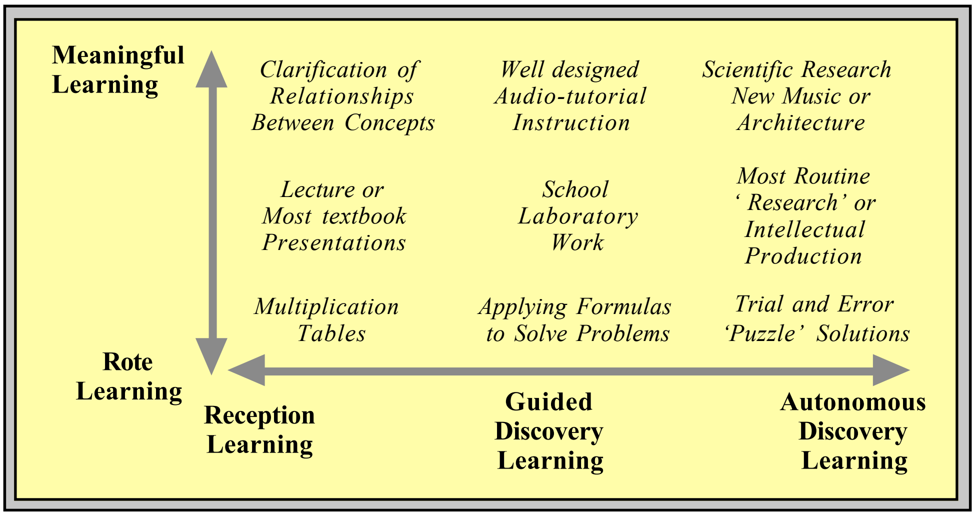

Figure 2.3 Reception Learning & Discovery Learning (detailed) (Ausubel, 1968)

2.7.1 Discovery Learning

Discovery learning is a learner-oriented process in which the main content to be learned is not provided to the learner and he has to discover the main content or concept before incorporating it meaningfully into his cognitive structure. To discover the main content, he is required to arrange, organise and construct the links between the new information and his existing knowledge with the help of procedural instructions provided by the teacher (Ausubel, 1969).

Discovery learning motivates the learner to construct the information into a meaningful pattern rather than depending on his teacher entirely and understanding it without being involved much in any mental activity. Bruner asserts that discovery learning can increase the learner's curiosity that can lead them to explore something new and useful, and in this way the most meaningful learning can take place (Good and Brophy, 1990). Langford (1989) argues that, through discovery learning, real knowledge can be acquired and such knowledge can be preserved much longer in the memory.

2.7.2 Reception Learning

Reception learning is a teacher-centred learning process in which the teacher plays the key role by providing the material to be learned, either directly by means of face to face teaching or indirectly by means of worksheets or textbooks. Thus, Larochelle *et al.* (1998) argue that the teacher is the primary source to arrange suitable conditions under which learning occurs.

In reception learning, the material provided is close to its final form and the learner does not need to discover the main content independently. The learner only has to understand it meaningfully, incorporate it into his cognitive structure, to learn and memorise it and to utilise it in future (Ausubel and Robinson, 1969).

However, Ausubel asserts that most learners learn primarily through reception learning rather than discovery learning but he does not refute the usefulness of discovery learning in certain situations. He argues that the meaningfulness of the material learned does not depend on the method of learning. He believes that reception learning can be made meaningful by presenting the material to be learned in a relevant way. He insists that both reception and discovery learning can be categorised to be either meaningful or rote learning depending on the result after the material to be learned is presented to the learner (Danili, 2001).

2.7.3 Rote Learning

In rote learning the learner does not possess the relevant prior knowledge in his cognitive structure to link with the new information. The new knowledge is mainly verbatim and sequential. Johnstone (1997) described rote learning as, *"…at best, isolated and boxed learning that relates to nothing else in the mind of the learner"*. According to Ausubel and Robinson (1969), the following three conditions tend to encourage rote learning:

"• *The material to be learned lacks logical meaningfulness.*
• *The learner lacks the relevant ideas in his own cognitive structure to construct new knowledge.*
• *The learner lacks the skills to enable him to learn meaningfully."*

Ausubel and Robinson (1969)

Any of the above conditions alone can lead the learner to rote learning.

2.7.4 Meaningful Learning

According to Ausubel, meaningful learning occurs when the learner can relate new ideas to the already existing knowledge in his cognitive structure. The new concept links logically with the established ideas in cognitive structure. Ausubel believes that the quality and organisation of the pre-existing knowledge is the basis of meaningful learning. Ausubel suggested the following three conditions for meaningful learning to occur (Ausubel and Robinson, 1969):

"• *The material to be learned must be relatable to some hypothetical, cognitive structure in a consistent and substantive manner.*
• *The learner must possess appropriate schemata or cognitive structure, which relate to the new information.*
• *The learner must possess the intent to relate the relevant ideas to the new material in a non- arbitrary and substantive manner."*

Johnstone (1997) described meaningful learning as, *"…good , well-integrated, branched, retrievable and usable learning"*.

In summary, *"meaningful learning occurs when the learner's appropriate existing knowledge interacts with the new learning. Rote learning of the new knowle dge occurs when no such interaction takes place*" (West and Fenshman, 1974).

An important point to be noted is that everything cannot be learned meaningfully; rather, rote learning might be useful in learning a foreign language or calligraphy writings. In other words rote learning is closely related to the surface learning approach while meaningful learning links with the deep learning approach (Ghani, 2004).

2.7.5 Subsumption

Subsumption is a key idea presented by Ausubel as an essential process for meaningful learning based on the concept that meaningful learning does not occur by adding new ideas in the learner's mind; rather the new ideas assimilate into and interact with the anchoring concepts. According to Ausubel (1968), to *subsume* is to incorporate new knowledge into a learner's cognitive structure. In other words, the transformed knowledge acquired after the emergence of anchoring ideas and new knowledge is called meaningful learning. According to Ausubel, these anchoring ideas are called *subsumers*.

A subsumer is a pre-existing concept or principle-generalising idea in the learner's cognitive structure providing interaction for the diverse components of the new knowledge (West and Fenshan, 1974). Ausubel explains that subsumption is a continuous process and that its usefulness depends on the growing differentiation and incorporation of the subsumers in the learner's cognitive structures. Thus, a learner whose subsumption process is well developed is capable of solving more complex problems than a learner whose subsumption process is not so elaborate (Ghani, 2004).

2.7.6 Advance Organiser

Advance organiser is another salient idea introduced by Ausubel. He describes it as a kind of conceptual bridge between the new material to be learned and the learner's pre-existing knowledge in his cognitive structure. It is assumed that the key principle for a learner in the learning process is to comprehend what he is learning and it is possible only when the material to be learned can be linked to pre-existing knowledge in his cognitive structure. The advance organiser offers a mechanism to make sure that the correct linkage can take place.

The learner holds many ideas in his long-term memory. As some of the ideas may have been learned using different words, symbols or other linked ideas, some concepts may therefore have been partially forgotten or even confused. The advanced organiser enables the learner to organise or sort out these learned ideas, to bring them to the surface, to remove the ambiguities and make useful links ready so that, on the whole, the new ideas can be associated correctly and meaningfully to pre-existing ideas.

This is more than simply revision although it involves revision of past learning. The key lies in explicitly preparing what is already known so that the new material can be linked on correctly and meaningfully. It assumes that meaningful learning can be described in terms of a correctly linked matrix of ideas, held in long-term memory.

Ausubel (1978) suggested the advance organiser to be used in the following two cases:

(a) When the material is new and the learner does not possess the appropriate subsumers or relevant information to relate to it.
(b) When the relevant subsuming information does exist in the learner's cognitive structure but that information is not developed adequately and is not likely to be recognised effectively and associated with the new information.

The function of an advance organiser is to facilitate meaningful learning, and the advance organiser itself has to be meaningful to the learner for this purpose. Any type of advance organiser cannot possibly function if the learner does not possess the relevant concepts to associate with the new material to be learned (Novak, 1978).

According to Ausubel (1969), the cognitive structure is hierarchically organised and logically structured, which means that the less inclusive or less comprehensive sub-concepts and details of specific records are organised *under* the most inclusive concepts. Therefore, instructions should be given to the students from the most general and inclusive ideas towards the specific ones. Thus, the advance organiser constitutes preliminary information and a complex set of ideas given to the learners before the new lesson has to be learned.

2.7.7 Application of Ausubel's Theory

Ausubel's model explains the importance of anchoring ideas or prior knowledge for future learning. He also clarifies the difference between meaningful learning and rote learning and explains that meaningful learning occurs through interaction of prior knowledge and new information. The 'meaning' cannot be taught to the learners but it has to be constructed by them by organising their prior conceptual frameworks.

His theory reminds teachers to value and assess what the learners already know, how that information has been learned and stored in their memory and how to present the new information effectively to make them link this new information with the prior knowledge. The idea of advance organiser facilitates meaningful learning.

2.8 Gagné's Model of Learning

Robert Gagné (1916-2002) was an American psychologist who was mainly interested in the development of human learning and behaviour in the context of practical problems of training air force personnel. His Air Force training played an important part in shaping his instructional theory.

He describes learning as "*a change in human disposition or capability that persists over time that is not simply assigned to the process of growth*" (Gagné, 1970, pg 3). There are changes which occur in humans from birth onwards. Some of these are simply related to biological development (what Gagné called 'process of growth'). Learning involves changes in the way a person thinks, understands or in what the person is capable of doing (which could be intellectual, attitudinal or physical).

He asserted that learning results in a change in the learner's cognitive structure and learning keeps on progressing constantly. He also pointed out that skills should be learned singly and each new skill is based on the prior learned skills. His theory is based on the following three major factors:

- He introduced a taxonomy or classification of learning outcomes.
- He introduced the internal and external conditions that are essential to attain these learning outcomes.
- His theory introduced nine events of instruction that help develop a unit of instruction.

Planned learning helps every learner to come closer to the aspiration of optimal use of his natural abilities, life experiences and integration with his physical and social environment. Gagné (1974) explained the basic assumptions about instructional design and insisted that instructional planning must be for the individual and must be based upon the cognitive requirements of the learner.

> *"Instructional design has phases that are both immediate (what a teacher does in preparing a lesson plan some hours before the instruction is given) and long- range (concerned with a set of topics to constitute a course sequence or perhaps with an entire instructional system, undertaken by a team of teachers, school committees, organisation of curriculum planners, text books writers and by group of scholars representing the academic disciplines.)"*
>
> (Gagné ,1974)

Systematically designed instruction plays an effective role in individual human development. Basically no one should be educationally disadvantaged and everyone must have an equal opportunity to use his individual talents to the extreme level.

2.8.1 Learning Processes

Gagné introduced two categories of factors involved in a learning process:

(a) External Factors (b) Internal Factors

A learner requires a few internal states during an act of learning such as factual information, intellectual skills and self-management strategies (to manage his behaviour in storing and retrieving information and in sorting out problem solutions). All these internal states depend upon prior learning. The influences of these factors take place through recalling the prior knowledge from learner's memory. In other words, prior learning capabilities facilitate the new learning - the same emphasis of Ausubel, 1968.

He believes that contiguity, repetition and reinforcement are basic and valid learning principles and he puts them in the category of *"external factors"* in the learning process but they may need some new interpretation in the light of modern theory: *"one of their outstanding characteristics is that they refer to controllable instructional events."* (Gagné, 1974).

While presenting a curriculum to the learner, different internal and external learning conditions required for each learning type should be kept in mind (See Figure 2.4)

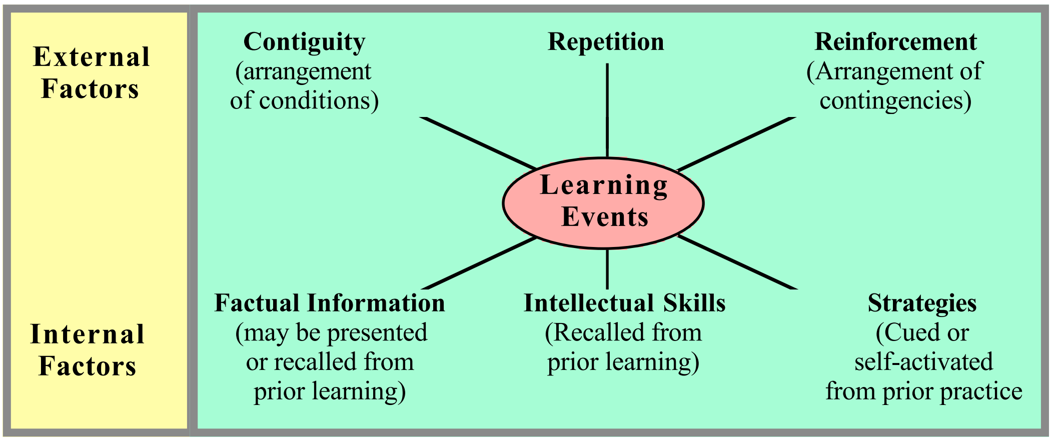

Figure 2.4 External and Internal factors affecting the Learning event (Gagne, 1977, pg 11)

Individuals learn many things in life. A child in early life learns to interact with his environment and also learn to speak and use language as a powerful set of skills that have a great effect on all his subsequent learning in future life. Every individual needs to learn the appropriate use of symbols represented by the environment such as pictures, diagrams, printed letters, words and numerals. By learning these basic and simple symbol-using and language skills, the individual engages himself in highly complex intellectual skills.

2.8.2 Types of Learning

Gagné determined the following five categories of learning:

1. *Information Learning*: Learning of different facts, principles and generalisations by absorbing and holding them in memory.

2. *Cognitive Strategies*: The internally organised strategies and skills governing the learner's own behaviour, such as focussing, remembering, attending, and self-management of learning and thinking. These skills are developed gradually with the passage of time as individuals are involved in more studying, learning and thinking. The individual selects a specific mental strategy or is involved in a specific mental process to solve a problem.

3. *Attitudes*: A set of beliefs and evaluations. Every individual possesses attitudes of various kinds towards different things, people and situations. Attitude is choosing to behave in a specific way that shows a newly attained belief.

4. *Motor Skills*: The individual learns various physical skills in life: the skills of knowing how to to do different things in life. Examples include how to drive a car, to play football, to know computer skills or the skills learned as part of school instruction like printing letters, drawing, sketching, sculpture and so on.

5. *Intellectual Skills*

> *"An individual interacts with his environment by using symbols. He uses language to deal with his environment symbolically. As the learning of school subjects continues, symbols are used in more complex way: distinguishing, combining, tabulating, classifying, analysing, and quantifying objects, events, and even other symbols. This kind of learned capability is given the name 'Intellectual Skills'."*
>
> (Gagné, 1974).

The human learners learn different kinds of simple and complex intellectual skills. The subject matter in school mathematics is virtually an example of intellectual skills. A relatively simple skill of using symbols is to learn the symbols 5, 20 or the addition symbol '+'. A more complex skill is to add two mixed unlike common fractions. Language and language symbols are used to record and communicate the relationship between concept and rules that exist in every subject.

2.8.3 Types of Intellectual Skills

Gagné described the following major types of intellectual skills:

Discrimination: *"A capability of making different responses to stimuli that differ from each other along one or more physical dimensions"* (Gagné, 1974, pg 39). For example, to distinguish two pictures, one having a square and the other a rectangle.

Concrete Concepts: A capability of putting things into a class and respond to any instance of the class as a member of that class or identifying things by their physical features. The basic meaning of concrete concept is to identify a property or attribute of an object e.g. colour, shape etc.

Defined Concepts: "A defined concept is a rule that classifies objects or events" (Gagné, 1970,pg 189-191). A learner is considered to have a defined concept if he is capable of demonstrating the meaning of some particular class of objects, events or relations. For example, a straight line is a straight locus having no end points; an aunt is a sister of a father or mother; a noun is the name of a place, person or thing.

Rules: *"A rule has been learned when a learner's performance has a kind of regularity over a variety of specific situations*" (Gagne,1974, pg 43). *"A rule is an inferred capability that enables the individual to respond to a class of stimulus situations with a class of performances, such performances being predictably related to the stimuli by a specific class of relations"* (Gagne, 1970, pg 192). In other words, the learner is able to respond with a class of relationship among classes of events and objects. He is able to apply a simple procedure to solve a problem e.g. how to add 56 and 19. At the beginning, rules are simple but gradually they grow in complexity.

Higher Order Rules-Problem Solving: Sometimes rules to be learned by a learner are complex combinations of simpler rules. These more complex rules are also called higher order rules. Higher order rules are invented to solve practical problems or class of problems. It is important to teach students to think clearly and logically while solving a problem. The learner also acquires a new rule or a set of new rules while discovering a workable solution of a specific problem. The learner also recalls relevant rules and also already learned information, concepts and entities, and combines them to form a higher order rule (Gagné, 1977). According to Gagne a higher order rule is to recall the relevant subordinate rules, present a novel problem and demonstrate a new rule in achieving problem solution. An example for mathematics illustrates the idea:

> To add two unlike common fractions, the learner should know the concept of addition, fractions and like fractions. He also needs to recall the subordinate rule to add two like fractions. Therefore, to devise a complex rule (to add two unlike common fractions), we have to make them like by multiplying each fraction by the denominator of the other fraction and then to add them in the same way by using the

subordinate rule of adding like fractions. In this example, the learner combines simpler rules, which he recalls, into a more complex rule to acquire the solution of the problem.

2.8.4 Designing Instructional Sequences

According to Gagné we need to plan sequences of learning events to accomplish a specific instructional objective, because the desired learning cannot take place all at once; rather it occurs in a series of steps or in a succession of individual occasions.

The arrangement of sequence of a topic for each of the types of learned capability are shown in the following table:

Types of Learning Outcomes	Major Principles of Sequencing	Related Sequence Factors
Motor Skills	Provide intensive practice on part-skills of critical importance and practice on total skill.	First of all, learn the executive routine(rule).
Verbal Information	For major sub-topics, order of presentation not important. Individual facts should be preceded or accompanied by meaningful context.	Prior learning of necessary intellectual skills involved in reading,listening, etc. is usually assumed.
Intellectual Skills	Presentation of learning situation for each new skill should be preceded by prior mastery of subordinate skills.	Information relevant to the learning of each new skill should be previously learned or presented in instructions.
Attitudes	Establishment of respect for source as an initial step. Choice situations should be preceded by mastery of any skills involved in these choices,.	Information relevant to choice behaviour should be previously learned or presented in instructions.
Cognitive Strategies	Problem situations should contain previously acquired intellectual skills.	Information relevant to solution of problems should be previously learned or presented in instructions.

Table 2.1 Five Types of Learning (Gagné, 1974, pg 105)

When the topic or sub-topic involves the learning of intellectual skills, sequence (mastery of prerequisite skills) is considered the most important factor. According to Gagné (1977):

> "*The learning of each skill representing a lesson objective will occur most readily when the learner is able to bring to bear those recalled, previously acquired skills which are relevant to the new task. The learning of such tasks also requires information, which may either be recalled or presented as a part of instructions for the learning task*".

2.8.5 Learning Hierarchies for Intellectual Skills

At primary level, a single rule is learned in isolation and then the further complex rules are built up on the basis of these already learned simple rules. According to Gagné, an adult learns related sets of rules pertaining to a larger topic and, during this process, what is learned is an organised set of intellectual skills. The individual rules that arrange such a set are related to each other in a logical sense and also in a psychological sense. The psychological organisation of intellectual skills is represented as learning hierarchy. Two or more concepts may be prerequisite to the learning of a single rule. Similarly two or more rules may be prerequisite to the learning of a super ordinate rule. Once the latter is learned, it may relate to another rule and so on. The complete set of rules, organised in such a way forms a learning hierarchy that illustrates a well-organised course to attain an organised set of intellectual skills which represents understanding of a topic (Gagné, 1977).

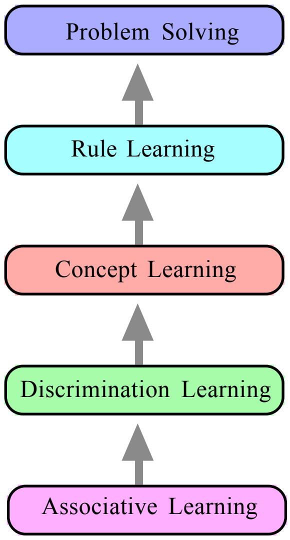

Figure 2.5 Gagne's Intellectual Skill Hierarchy (Gagne, 1977)

2.8.6 Gagné's Model and its Application in Mathematics

In his model, there are important implications for mathematics because logical thinking is involved in it, possibly more than in any other subject. Concepts and ideas in mathematics do build logically on each other and so are interlinked with each other. Also, mathematics is, in part, a skills-based subject, with a product (a right answer).

Gagné's model can be helpful in mathematics-teaching methodologies. The idea of a learning hierarchy is applicable in mathematics because, in mathematics, every single rule (that is the learning objective for every specific problem) is related to the prior set of concepts and rules. Ausubel's meaningful learning concept is important as well. Using learning hierarchy technique, the learning can be made meaningful for the learner as the rules and concepts are interlinked into his cognitive structure meaningfully step by step. Prior concepts play an important role to move ahead logically and finally the learner reaches the level where he fully understands the learning route and achieves the task of solving a specific problem successfully.

Overall, Gagné was a person involved in training and not school or university education but his great contribution lies in his emphasis on:

(a) The need for logical, sequential planning by the teacher;
(b) The usefulness of analysing sequentially what is needed to learn a skill;
(c) Knowledge and ideas building one on to the other;
(d) The aim or a final 'product' (in his case, a practical task brought to a successful conclusion).

However, it does not necessarily follow that his ideas can be translated neatly into school education. School students do not necessarily work in the kind of logical ways which are required in military training. Indeed, looking at chemistry, Howe (1975) found that the learning of 13-14 year old students was not nearly as logical or sequential as implied by Gagné.

2.9 Some Conclusions

This chapter has reviewed some of the learning theories which have been developed from observations. Piaget has shown that there are clear developmental aspects to learning while Ausubel has demonstrated the vital importance of linking new learning to what has been learned before. This linking of new learning to previous knowledge and skills in a logical way is emphasised by Gagné, thus leading to meaningful learning. Ausubel has shown that meaningful learning involves the correct linking of new ideas to previously learned ideas to make a more enriched matrix of ideas. Seeking to gain understanding is the natural process: Piaget demonstrated that the learner looks at the world around and seeks to make sense of it. This ‘making sense’ is the natural process of meaningful learning.

One dimension is still missing and that is how all of this links to the way in which the brain works. Here, the findings from information processing can offer new insights into the mechanisms of learning and that is the theme of the next chapter.

Chapter 3

Information Processing Models

3.1 Background Understandings

From his extensive observations, Piaget (1963) appreciated that a child tries to make sense of the world around him and constructs knowledge through experiences provided by the environment. Piaget, as a biologist, described the four cognitive stages as a developmental process from birth to age 16. He explained the process of a child's thinking and learning by introducing the concepts of schema, assimilation, accommodation and equilibrium. Piaget's description was more focused on maturational factors and needed to be explained further. Later, many researchers tried hard to put Piaget's theory of cognitive development into a framework that was compatible with their own conception of handling information on the human mind.

Pascual-Leone (1970) tried to develop a more comprehensive psychological theory capable of explaining the facts discovered by Piaget and his followers. According to Pascual-Leone, an individual's performance on a cognitive task was considered to be a function of three parameters (which he called Repertoire H, M-Demand and M-Space) that illustrate the psychological system:

(1) ***Repertoire H:*** Repertoire is the mental strategy applied to the task. The basic construct used in Pascual-Leone's theory was ‘schema’, the main concept introduced in Piaget's theory. A considerable number of schemata generating responses are available to the learner in a designated ‘repertoire’. According to Pascual-Leone (Case, 1974), these response-generating schemata can be divided into three main categories:

   (a) *Figurative Schemata:* Figurative schemata are the internal representations of pieces of information that are familiar to the individual and are capable of releasing responses from other super ordinate schemata.

   (b) *Operative Schemata:* Operative schemata are internal representations of orders that can be applied to a specific set of figurative schemata to generate another set of figurative schemata.

   (c) *Executive Schemata:* Executive schemata are internal representations of procedures that were applied in specific problem solving situations to achieve a particular objective. Repertoire was thought to increase in size after continuous use with new schemata.

(2) ***M-Demand:*** The demand placed by the mental strategy on the mental capacity.

(3) ***M-Space*****:** The individual's own mental capacity (central computing space, M-space or M operator). The M capacity increases with age, relating to genetic and maturational influences.

According to Pascual- Leone (Case, 1974), children can be taught to learn different useful strategies for problem solving but the mental capacity cannot be increased through instruction. He argued that both repertoire and computing space (M-space) transform and co-ordinate the information held within the cognitive structure. He also defined structural capacity (M_s) as the individual's maximum mental capacity and the functional capacity (M_f) as the amount of M-space actually utilised in solving a problem.

He suggested that, when an individual confronts a new problem, he collects information and uses it to form a new schema. The already existing information and the information representing the new problem are put into one channel of the central processor M. As a result, the solution of this new problem is established within the individual's repertoire. As the individual interacts with numerous problems in daily life, they continually apply and modify their repertoire of schema. The M capacity increases by one schema for every two years from childhood until maturity.

In the 1950s, it was observed and described that memory appeared to have two separate parts in brain (Brown, 1958; Peterson and Peterson, (1959). These were described as short-term memory and long-term memory. Miller (1956), after many memory experiments found that the average adult capacity of short-term memory is about seven plus minus two (7± 2) separates chunks. The capacity of short-term memory cannot be increased as it is genetic but the process of chunking can increase what can be held.

Miller (1956) introduced the basic theoretical ideas of chunking and the capacity of short-term memory related to information processing framework. He discovered that the short-term memory capacity depends on the age of an individual and it grows on average by 1 unit each two years until about 16. He defined this unit (chunk) as the single item of information as perceived by the individual. In other words, a chunk is the unit of information which can be handled by an individual. A chunk is a piece of information and the learner controls its size. It might be a single number or a single letter or many pieces of information grouped together and seen as one. Chunking is a process by which information can be grouped and this grouping helps to handle more information in the short-term memory space. The more information students can make into a recognisable group by means of chunking, the more complicated ideas they are able to handle at one time.

A major emphasis of cognitive research is to understand how individuals learn. It concerns how information is received, processed, stored and retrieved. All this process is concerned with human memory. Human minds receive information from their environment through five senses: sight, hearing, smell, touch and taste. As we have so much around us to receive, some information is remembered for a short period of time and then forgotten. A small portion of the large amount of information received may stay in our memory for a very long time. However, it is believed that most information received by human minds is almost immediately discarded without even realising it or noticing it (Slavin, 2000).

A key question is why and how an individual's mind holds a piece of information for a short or long time and completely rejects or forgets some other information. Cognitive psychology research has found that information processing (a metaphor borrowed from the field of computer sciences) offers useful insights.

Until the late 1950s, the cognitive psychologists and researchers considered memory as a single unitary faculty. Brown, (1958) and Peterson and Peterson, (1959) suggested a short-term memory (STM) system that operated using different rules from long-term memory (LTM). By the late 1960s, researchers gained clear evidence supporting the idea that STM and LTM are two separate systems (Pickering, 2006).

The work of Atkinson and Shiffrin (1968) on information - processing, is considered the key foundation in cognitive research. Many cognitive thinkers were greatly influenced by the model of Atkinson and Shiffrin who proposed several information processing models (for example: Johnstone and Kellett, 1980; Johnstone, 1984; Ashcraft, 1994; Brunning et al., 1995; Sweller, 1998).

3.2 Memory Models

Numerous models of memory have been developed based on empirical evidence. Many are very similar but this section seeks to trace some of the key developments.

3.2.1 The Atkinson and Shiffrin Memory Model: The well-known memory model presented by Atkinson and Shiffrin (1968) focusses on how human minds store information in memory (figure 3.1).

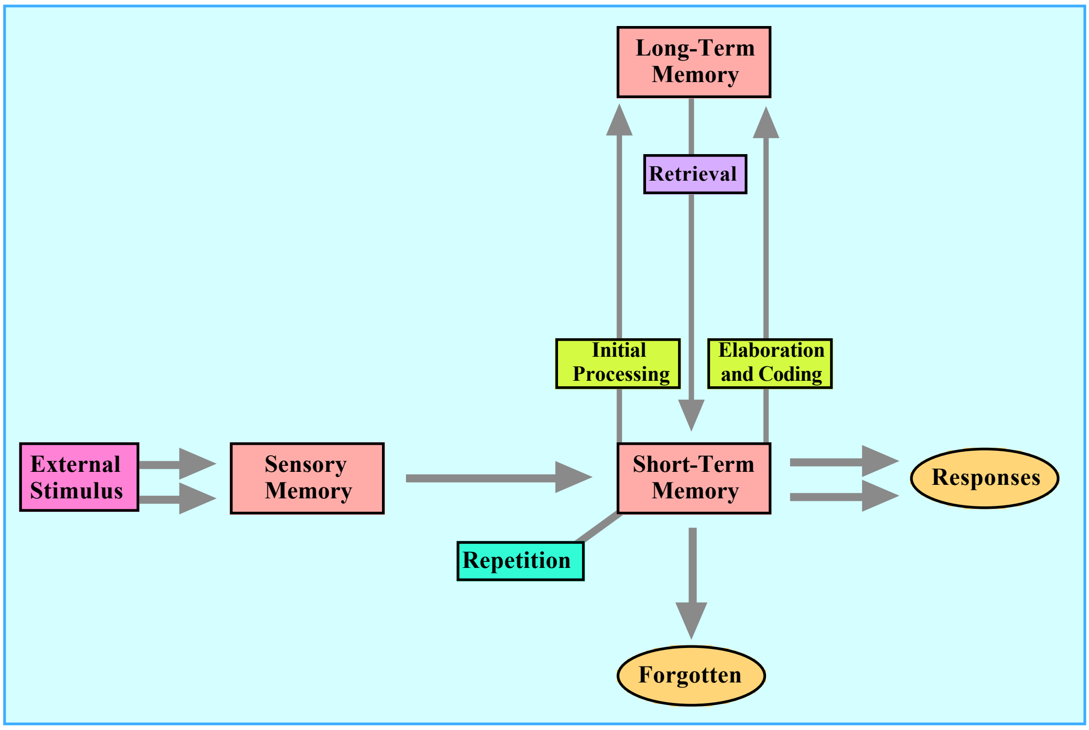

Figure 3.1 The Memory Model of Atkinson and Schiffrin

According to this model, individuals receive information from their environment by a series of temporary sensory memory systems and, if attended to, this information passes into a limited capacity store (short-term memory). If any information is not attended to, then it is forgotten or lost. Once information passes into short-term memory, it can be rehearsed before transferring it into long-term memory. Rehearsal is the repetition, usually subvocal, of what has to be memorised.

3.2.2 The Baddeley and Hitch Working Memory Model: Baddeley and Hitch worked together for three years on a research project concerned with the relationship between short-term memory and long-term memory. They published their model in 1974, providing a simple but robust foundation for the extensive research study of working memory. The phrase 'short-term memory' was replaced by the 'working memory', emphasising that this part of the brain is involved in thinking and not just memorising. Their work focussed very much on the structure of working memory.

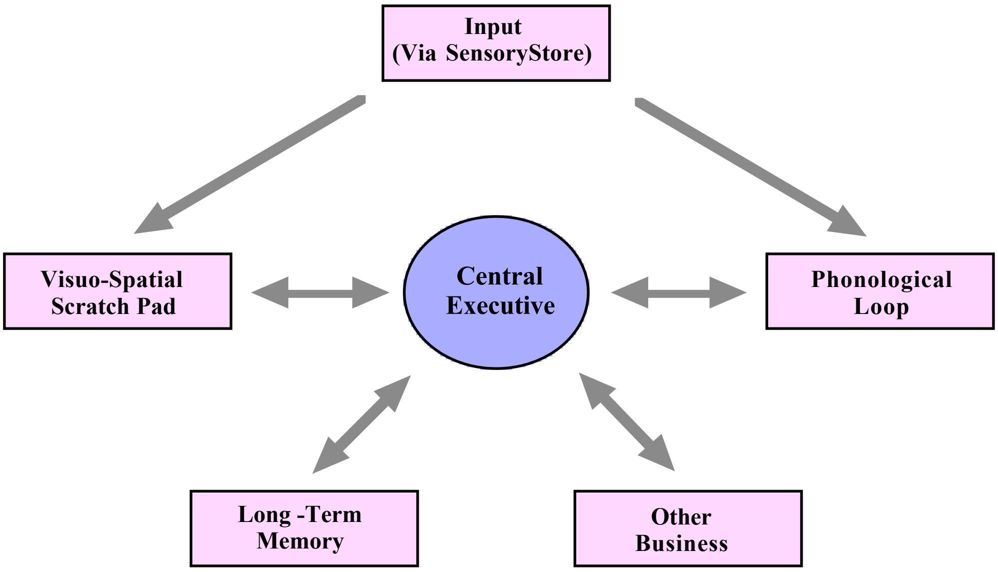

Figure 3.2 Baddeley and Hitch's working memory model (1974)

Their model of working memory consists of the following three major components:

(a) *Central Executive*
(b) Phonological Loop
(c) Visuo- Spatial Sketchpad

The central executive (a central processor for information) was presented as the central system of this model. The capacity of this attentional control system was found to be limited. This master system was supported by two slave systems: the phonological loop and the visuo-spatial sketchpad. The central executive was assumed to deal with the most demanding cognitive tasks. Working memory is a flexible system and its function is to process information in different ways. The phonological loop is used for holding and rehearsing the sound and speech based information. The visuo-spatial sketchpad component performs in the same way for the visual information.

3.2.3 The Brunning Modal Model: Brunning (1995) proposed a model known as the 'modal model', by extracting the common ideas proposed in all other information processing models of his time.

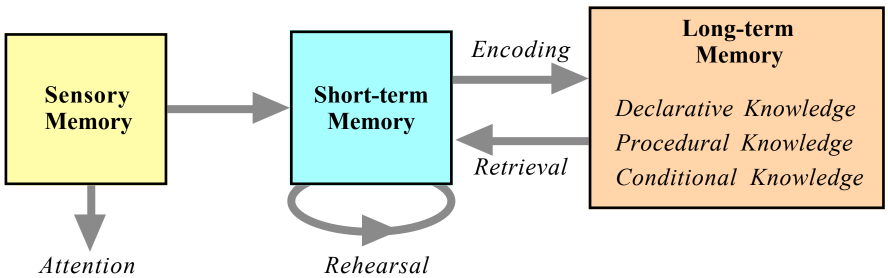

Figure 3.3 The Modal Model of Brunning

Information is received by the sensory memory. If the new information is organised or attended in sensory memory then it is transferred into short-term memory. If the new information is linked with the already existing or stored information in long-term memory, then it is assimilated and accommodated into long-term memory. In this way the new information is stored as cognitive structures.

3.3 Learning Difficulties and Information Processing

A sustained programme conducted by Johnstone discovered the areas of difficulty faced by the learners during the process of learning in mathematics and sciences areas of the curriculum at both school and university levels (e.g.. Johnstone *et al*., 1971; Johnstone and Mughol, 1976; Johnstone and Kellett, 1980; Johnstone and Mahmoud, 1980; Johnstone, 1991). Early in the 1980s, one of his students Kellett had noticed that the possible reason causing these difficulties was information overload. This was developed into a formal hypothesis, which would explain all known areas of difficulty in conceptual subjects.

Kellett (1980) described a relationship between Information Content, Conceptual Understanding and Difficulty and she referred it as the I.C.C.U.D hypothesis. This proposes that:

> *"Where there is a lack of conceptual understanding, pupils may perform reasonably (while not necessarily showing mastery) and not complain of difficulty in low information situations, but in high information situations - i.e. situations in which, at some stage, the expected number of chunks exceeds chunk capacity - performance will drop dramatically, and pupils will complain of difficulty".*

In more detail, she suggested that information content of tasks, related conceptual understanding and adjudged difficulty are connected in the following way:

"(1) The number of chunks represented by a given body of information will depend upon the level of relevant conceptual understanding.

(2) The larger the number of chunks (from given information, together with any additional chunks) required at some stage in the task, the greater its adjudged difficulty, and poorer the observed performance.

(3) In the limit, if chunk capacity is exceeded, no useful information may be extracted if the pupil attempts to handle the given information ' in-a-one'.

(4) When chunk capacity is exceeded, new information may be obtained by using a memory conserving strategy, which allows a sequential consideration of the information.

(5) Conceptual understanding leads to an inefficient (large number of steps) poorly organised strategy, or even an arbitrary or diverging strategy."

Soon after, Johnstone and El-Banna (1986) tested the hypothesis proposed by Kellett and they found to be strongly supported. Johnstone and El-Banna renamed the short-term memory as the working memory. This was the place in the brain where thinking, understanding and problem solving takes place. Their research work will be discussed in more detail later.

3.4 Information Processing Model of Learning

Johnstone (1993) developed a useful information-processing model which is not theoretical but is based on empirical data and observation. This model absorbs the major components of previously proposed models and the key ideas of other research including that of Aschcraft (1994), Piaget (1963), Ausubel (1968), Gagne (1969), Pascual-Leone (1970), and Baddeley (1986).

The model (figure 3.4) seeks to offer description of what is happening when learning is taking place and to account for the limitations of learning. In other words, this model suggests illustration of why learning is sometimes difficult or impossible.

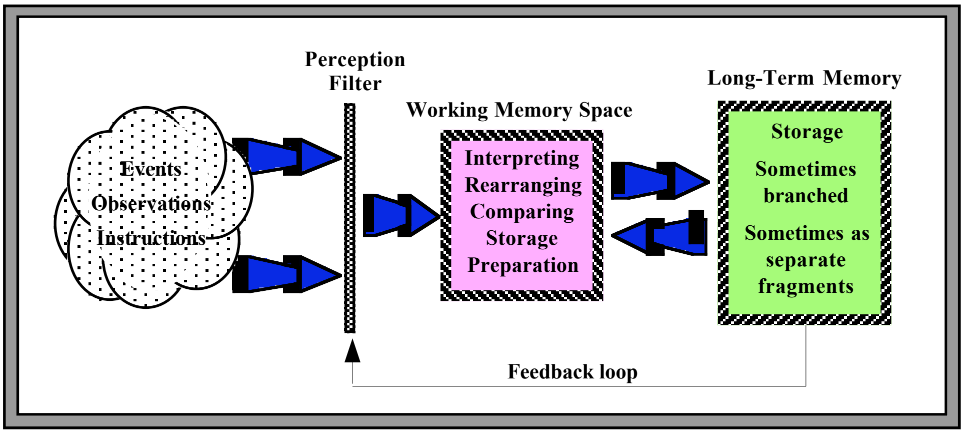

Figure 3.4 Information Processing Model (after Johnstone, 1997)

The model not only offers a picture of what is happening during all learning but it is also predictive, offering clear insights to show how learning can be hindered or enhanced. Many of these insights have been tested and, in every case, the model has been found to predict successfully (Reid, 2008).

There are three major components of this model:

(1) Perception Filter
(2) Working Memory Space
(3) Long-Term Memory

These components will be discussed in detail now.

3.4.1 Perception Filter

According to Johnstone (1997), the perception filter is a mechanism that receives information through events, observations and instructions from the environment. The function of the perception filter is to '*reduce the torrent of sensory stimuli*' to manageable proportions. As it is not possible to respond to everything received from the environment, the filtration system ignores a large part of sensory information and attends to what is important, interesting or, perhaps, sensational.

The perception filter is driven by the long-term memory. Previous knowledge, experiences, beliefs, and attitudes stored in the long-term memory help in the mechanism of selecting and encoding the filtered information. Johnstone, 1997, observes that,

> *"The perception filter must be driven by what we already know and understand. Our previous knowledge, biases, prejudices, preferences, likes and dislikes and beliefs must all play a part".*

The control of the perception filter varies from person to person as every individual has a unique set of held knowledge and beliefs. This observation emphasises that learning does not simply involve the transfer of information from the mind of the teacher to the mind of the learner. The learner, on the basis of past experience, selects and what is learned may be very different of what is taught. The selected and filtered information is then transmitted to the working memory where the subsequent stage of the processing of this information occurs.

3.4.2 Working Memory

Baddeley (1986) noted that the working memory space is a place where we consciously think and perceive. It has a limited capacity and its function is to hold and process (there is also a time factor in processing) the information. These two functions have trade-offs: if there is too much information to hold then less processing can take place and, if too much processing is involved, then it cannot hold much. He found that the working memory has various structures which seem to be part of it. He called these loops: the phonological loop and the visuo- spatial sketchpad

According to Johnstone (1997), the stimuli and information admitted by the perception filter is transmitted to the working memory where it is held and manipulated before being rejected or passed on for storage. The main characteristics of working memory space described by Johnstone (1997) are:

> *"The working memory space has two main functions. It is the conscious part of the mind that is holding ideas and facts while it thinks about them. It is shared holding and thinking space where new information coming through the filter consciously interacts with itself and with information drawn from long-term memory store in order to make sense. However, there is a drawback. The working space is of limited capacity… It is a limited shared space in which there is a trade-off between what has to be held in conscious memory and the processing activities required to handle it, transform it, and get it ready for storage in long-term memory store. If there is too much to hold, there is not enough space for processing; if a lot processing is required, we cannot store too much*".

The capacity of working memory space is limited, 7 ± 2 in adults generally (Miller, 1956) although less is available if processing is involved. The working memory capacity is known to grow on average by 1 unit for each two years until about age 16. There are many ways of measuring the capacity of working memory but the following two are most commonly used world-wide:

(1) Figural Intersection Test (developed by Pascual-Leone, 1970)
(2) Digit Span Task (developed by Miller, 1956)

The capacity of working memory cannot be expanded but we can learn ways to use it more efficiently. Chunking enables us to use this limited space more efficiently. Johnstone (1997) argues that chunking usually depends upon some recognisable conceptual framework that helps us to draw on old material to systematise the new information. According to Johnstone and El-Banna (1986), an individual on the basis of his experience, knowledge and acquired skills controls the process of chunking. The process thus relies on previous experience to bring together several items into one group which then only occupies one space in the working memory.

The terms short-term memory and working memory are both used to describe a part of the brain. Johnstone (1984) clarified the distinction between the short-term memory and working memory. When the space is used to hold information for a short time of a period without any processing, it can be described as short-term memory: for example, if a person memorises a sequence of numbers, he can recall it in the same order within seconds without any processing taking place. Thus, the memory space is being used completely as a short-term memory. If a person applies some arithmetical operations on a set of numbers, a working process takes place and the memory space is now being used as a working memory space.

3.4.2 Working Memory Overload

Working memory capacity is limited and has to be shared for holding and operating processes. A learner is able to handle a learning task easily and confidently if the number of chunks that are needed to be held simultaneously is equal to or less than his working memory capacity. Working memory gets overloaded if the learning task is beyond the working memory capacity of the learner. Barber (1988) has argued that, if the information to be used is beyond the limit of the learner's working memory capacity, then an overload may occur and a loss in productivity or efficiency may arise.

Johnstone (1997) sees working memory overload in a similar way. He observes that, if the learner has too much information to hold, there is not enough space to process that information; if too much processing is required, it cannot hold too much information. Thus, learning difficulties can be seen as arising from working memory overload.

Working Memory Overload ==> Learning Difficulties

Learning in a laboratory is very likely to produce this kind of overload. The information processing model predicts that such an overload will generate learning problems. In an early study, Johnstone and Wham (1982) demonstrated that students' working memory space tends to overload during practical work in science laboratories because too much information is required to be handled at the same time (see figure 3.5).

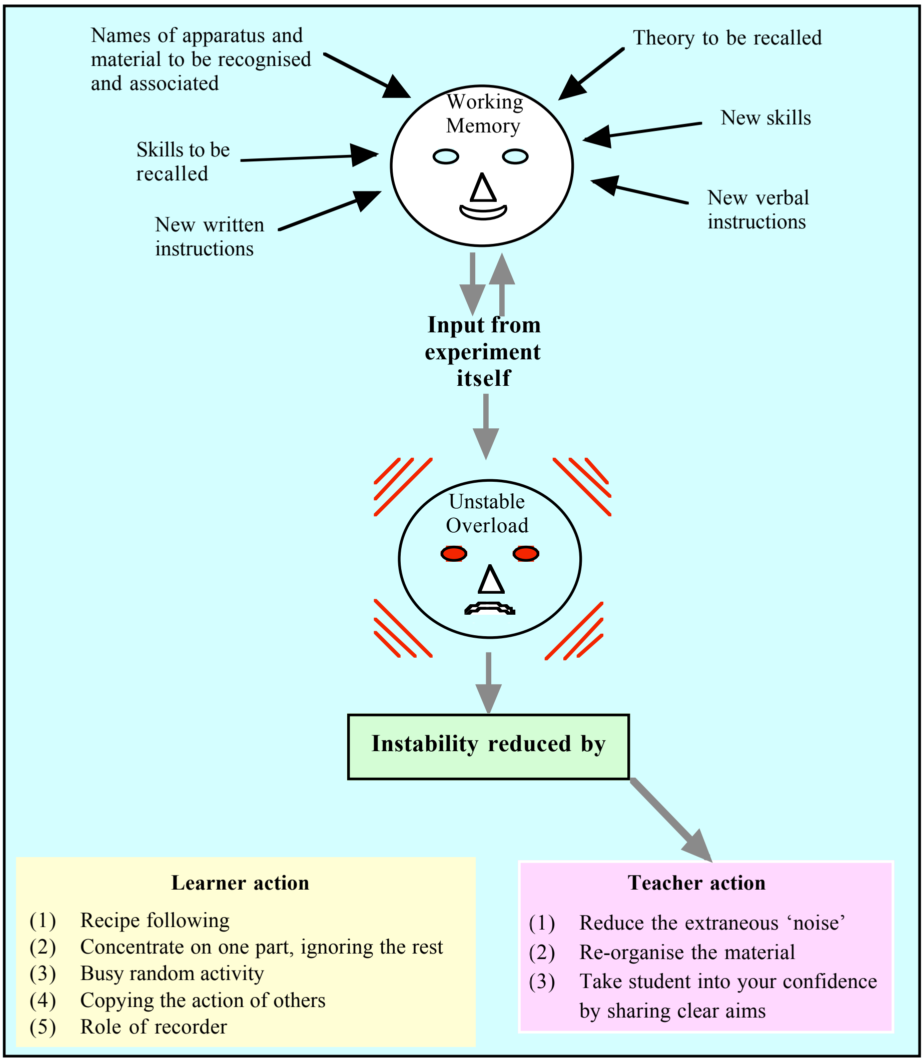

Figure 3.5 Working Memory Overload (Johnstone and Wham, 1982)

In figure 3.5, it is obvious that during an experiment, the student has to recall the theory, names of apparatus and then to recognise the material needed. At the same time he is required to recall the previous skills and to receive the new skills. He also needs to receive the new verbal and written instructions. He eventually loses concentration and reaches a "state of unstable overload'. This state was easily observable when video recording was made of students working in an undergraduate laboratory.

Johnstone and Wham (1982) explained that overloading of the working memory space occurs when the learner is unable to distinguish the signals (relevant and essential information) from the noise (irrelevant information). This idea of distinguishing the signals (relevant and essential information) from the noise (irrelevant information) turns out to be very important. The experienced person is able to select efficiently what is required for the task in hand and, therefore, does not overload the limited working memory with unnecessary information. The novice learner is less able to do this. In addition, there is a skill of being able to select efficiently and this has been explored extensively and will be discussed later.

Johnstone and El-Banna (1986) discovered an important relationship between working memory capacity and the learner's performance. As a tentative indicator of complexity (to examine how well students handled chemistry test problems of varying information complexity) they considered the number of thought steps necessarily required to solve the problem for the least sophisticated students. Their idea of complexity was very similar to Pascual-Leone's memory demand. They plotted the fraction of sample solving a problem correctly against question complexity. Question complexity can be described as the number of thought steps required to obtain the correct answer. They found a significant correlation between the two variables. However, they drew a graph of complexity (information load level) against success for a large range of questions. They expected to obtain a graph of the following shape:

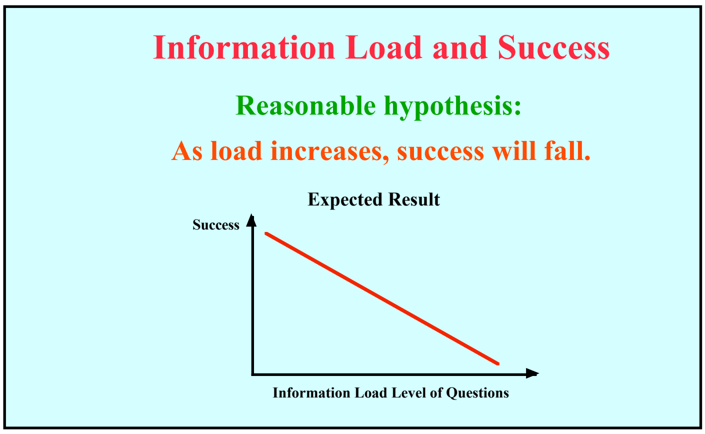

Figure 3.6 Predicted Graph (Johnstone and El-Banna, 1986, 1989)

In fact, the graph they obtained was:

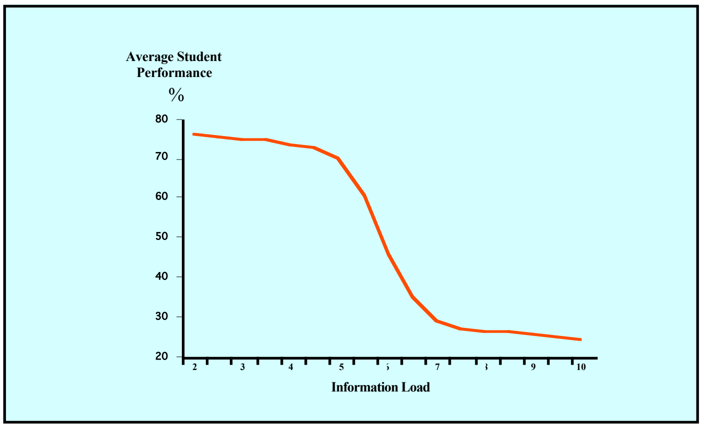

Figure 3.7 Graph Obtained (Johnstone and El-Banna, 1986, 1989)

The results showed that if question complexity exceeds the limit of the student's working memory capacity, the performance catastrophically deteriorates.

They separated students into the following three groups:

(1) Students with below average working memory capacity

(2) Students with average working memory capacity

(3) Students with above average working memory capacity

They then obtained the following graph:

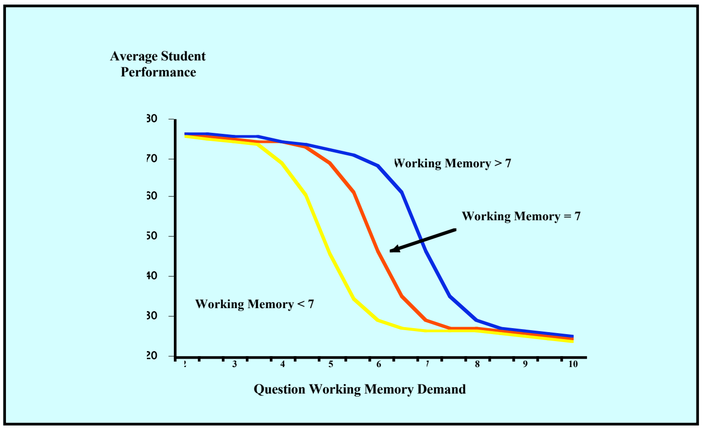

Figure 3.8 Graph with three groups (Johnstone and El-Banna, 1986, 1989)

Other similar studies by Opdenacker *et al.* (1990) with undergraduate medical students solving chemistry problems, Johnstone *et al.* (1993), and Chen (2004), with students solving physics problems, Christou (2001) with algebra story problems.

Johnstone (1980) presented a model (which he called the *Concorde Model*) to illustrate the relationships between demand, working memory capacity and success.

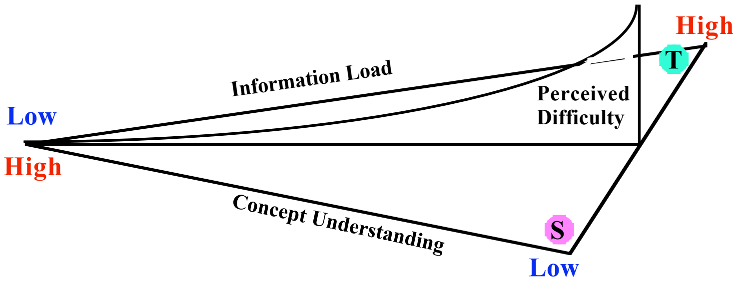

Figure 3.9 Concorde Model (Johnstone, 1980)

He pointed out the possibility of three reasons for learning difficulties:

"(1) The problem lies with the teaching methods;
(2) It is a consequence of the nature of science itself;
(3) It is a function of the teacher's distance from the learning situation dulling his sensitivity."

(Johnstone, 1980)

In this model, he describes that where high concept development and low information content meet, perceived difficulty will be at a minimum. Perceived difficulty will rise to a maximum as concept development decreases and information content increases.

A student learning a new topic as a beginner will be at position S in the diagram. If the teacher presents the material in such a way as to be at position T with high information load, the pupil at S will be unable to see what is going on and will have difficulty in understanding the topic. He stated:

"If the teacher persists at position T then the tail barrier may rapidly become an attitudinal one converting the student view from 'I cannot understand' into 'I shall never understand', laying the basis for the hangover effect we observed so often."

If the teacher presents the new topic at the low end of the information-content axis, the student at S will see what is going on and he will step by step move along the concept development axis to its higher end. The teacher can increase the complexity of information by realising the student's comfort level but the pupil, from his new position, will be able to keep it in sight. As the pupil's conceptual development increases, the information load can be broken into small units and in this way, the perceived load maybe reduced.

Johnstone argued that it was critical for the teacher to be very perceptive and careful in teaching methodologies. The teacher should realise that he or she is familiar with the work but the student, as a beginner, is learning it for the first time. Working memory overload is highly likely.

One of the ways by which overload can occur is the complexity of language (Johnstone and Cassels, 1982). Unfamiliar vocabulary, familiar words giving different meanings when being used in science subjects, using difficult words where simple words would do easily all make learning difficult for students. Johnstone (1980) argued that an unfamiliar or misunderstood word represents at least one chunk in the information load. This was later confirmed by a study on the measurement of working memory capacity where the school pupils were measured twice by the digit span backwards test, once in their mother tongue and once in their second language (in which they were normally taught) (Johnstone and Selepeng, 2001).

Herron *et al.* (1977) considered that scientific concepts are totally different from most other concepts in the everyday world. This is consistent with Johnstone's view (Johnstone, 1984) that the very nature of the sciences poses part of the problem. To grasp a concept, it is almost inevitable that the learner has to hold many ideas at the same time. Working memory overload is thus a strong possibility. Johnstone went on to argue that the methodologies in teaching also caused problems in that the way the new material was being presented generated potential overload.

Although Johnstone was arguing in the context of teaching and learning in the sciences (especially chemistry), the same issues are likely to arise with mathematics. Mathematics is based on conceptual thinking and logic. The novice learner is faced with topics and themes where working memory overload is highly likely. This assumes that the aim is understanding and not just rote recall and application of mathematical routines. Success in these can be achieved by practice until the procedures are essentially automated. If this route is followed, the learner is left with considerable dissatisfaction for understanding has not been achieved and, without understanding, it is extremely difficult to apply the ideas in new situations (Haghanikar, 2003). In mathematics, while solving a problem, new skills are to be related to the already learned skills and students are required to link the new concepts, methods involving different operations and information to the previous knowledge. If too much processing is involved, then the working memories of the students are unable to continue the process efficiently.

The overloading of working memory can easily take place in tests and examinations, especially in conceptual subject like mathematics, which may lead students to give short and incomplete answers (Johnstone,1988). He pointed out that working memory overloading can make further demands on a student during examinations by requiring him to break down a question into sub-goals, chunk information and then use the relevant information to be used and processed by working memory. Irrelevant information can drop down the performance level of a student with limited working memory.

In thinking of chemistry and physics initially, Johnstone (1993) suggested that there were three levels of learning involved (figure 3.10):

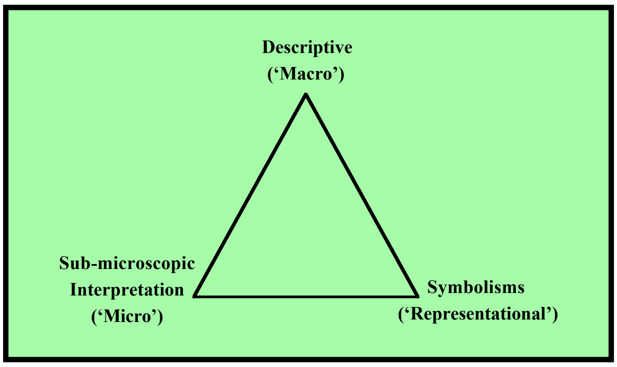

Figure 3.10 Levels of Learning in Physics and Chemistry (Johnstone, 1993)

Macro: The macro level is of tangible, edible and visible.

Sub-Macro: The sub-macro level is of molecular, atomic and kinetic.

Representation: The representational level is of symbols, equations and mathematics

The teacher, as an expert in his or her subject, can easily move from one corner to another or operate and connect any of these three levels but a student as a learner or beginner can not handle these three levels at the same time as his teacher does. Working memory limitations make this impossible until the learner has enough experience to chunk ideas together.

The model was later expanded by Chu (2008) for biology (figure 3.11)

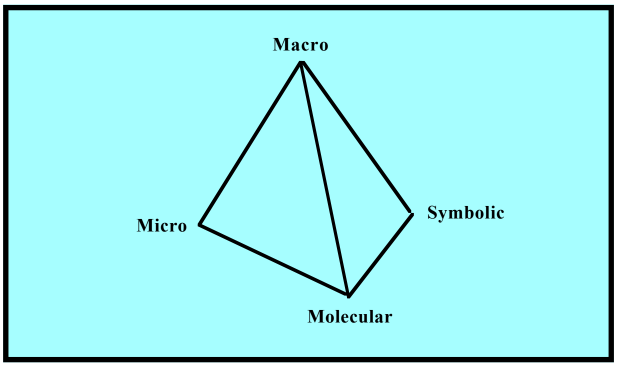

Figure 3.11 Levels of Learning in Biology (Chu, 2008)

It is interesting to speculate on the kind of model which exists for mathematics

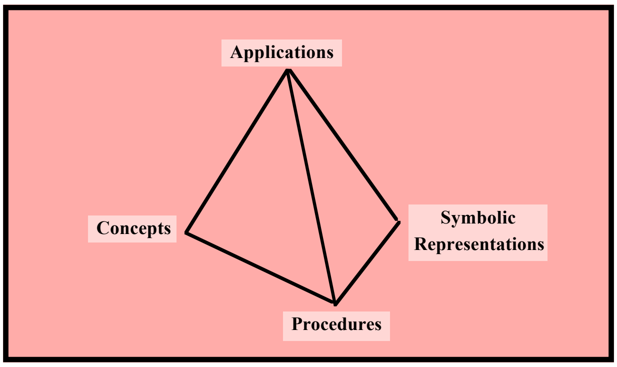

Figure 3.12 Levels of Learning in Mathematics

The way much mathematics is taught is to focus on the procedures, using taught symbolic representations. Pupils can cope with much of this, given enough practice. However, continual repetition of procedures without any attempt to develop any conceptual understandings or any idea of how the mathematical ideas can be applied can leave the learner very dissatisfied. The natural instinct is to seek to gain understanding (Piaget, 1963). However, if the teacher attempts to bring in concepts and applications before the procedures and symbolic representations are more or less automated, working memory is very likely to overload.

Johnstone (1999) argued that, if teachers are teaching students in a traditional style, then the overloading can occur easily because students will try to grasp every point delivered by their teacher, using their limited working memory space. They also try to note down points from the board, try to make sense and, at the same time, try to pay attention to the words spoken by the teacher. All this makes their working memory space overloading almost inevitable.

3.4.4 Long-Term Memory

Processed material in the working memory is transmitted to the long-term memory where it is stored. At the same time, material already stored is being recalled from the long-term memory to be used for processing in working memory. Johnstone (1997) notes the vital importance of being able to store correctly and being able to recall as needed.

Ashcraft, 1994, defined long-term memory as a final destination where information learned and remembered by an individual is stored on a relatively permanent basis while Brunning (1995) saw the long-term memory as a permanent repository of information that one accumulates over periods of days, weeks, months and years. Cooper (1998) described the long-term memory as an immense body of knowledge and skills that a person holds in a relatively permanent and accessible form. Johnstone (1994) notes the importance of the long-term memory as a store where facts are kept, concepts are developed and attitudes are formed.

The long-term memory has unlimited capacity of storing information. It is remarkably stable and long lasting and not easily disrupted. However, the researchers are trying to discover the cause of forgetting: gradual decay in long-term memory because of metabolic changes or inability to retrieve information from it.

Information held in long-term memory can be divided into two main types:

Declarative memory: Declarative memory holds “knowing what”. This memory is the knowledge of things that can be put in words such as our names, recalling the name of the currency of a country, names of the oceans in the world, and also the description of facts or what we have in our consciousness. Declarative memory can be further categorised in two ways:

Declarative memory is called *semantic* when it is abstract and holds general knowledge. It deals with language, rules and concepts. According to Salvin (2000), it contains the facts and vast network of conceptual knowledge underlying an individual's general knowledge which also includes problem solving skills and learning strategies. Declarative memory is *episodic* when it holds personal (autobiographical) knowledge or a record of an individual's past experiences such as good or bad memories, feelings or his conversational record with his friend.

***Procedural memory*:** Procedural memory is our unconscious memory. It deals with “knowing how” to perform certain activities and skills. It is the knowledge that we cannot put into words easily, such as information related to how to run, how to swim or how to drive.

Slavin (2000) has suggested the following three differences between the episodic, semantic and procedural memory to explain the ways by which information is stored and organised:

“• *Information in episodic memory is stored in the form of images that are organised on the basis of when and where events happened.*
• *Information in semantic memory is organised in the form of network of ideas.*
• *Information in procedural memory is stored as a complex of stimulus-response pairing.”*

Human beings are pattern seekers and try to make sense of things around by relating them to an existing system of information and knowledge (Johnstone, 1997). If a person is unable to make sense of anything, it creates a discomfort, which leads to the rejection of the new idea. This raises a real issue for learning in mathematics. At the outset, the emphasis will be on procedures. Any attempt to develop conceptual understanding at the same time almost certainly will bring working memory overload. However, without conceptual understanding, the learner may become dissatisfied.

Johnstone (1997) went on to describe learning as follows:

> *"Learning is the reconstruction of material, provided by the teacher, in the mind of the learner. It is an idiosyncratic reconstruction of what the learner understands, or thinks she understands of the new material provided, tempered by the existing knowledge, beliefs, biases, and misunderstandings in the mind of the learner."*

He suggested the following four ways of storing information in the long-term memory:

"(1) The new knowledge finds a good fit to existing knowledge and is merged to enrich the existing knowledge and understanding (correctly filed).

(2) The new knowledge seems to find a good fit (or at least a reasonable fit) with existing knowledge and is attached and stored, but this may, in fact, be a misfit (a misfiling).

(3) Storage can often have a linear sequence built into it, and that may be the sequence in which things are taught.

(4) The last type of memorisation is that which occurs when the learner can find no connection on which to attach the new knowledge."

The first way of storing information suggested by Johnstone is linked to the meaningful learning described by Ausubel (1968). The information stored through this way is easy to retrieve and almost never lost. The fourth way of storing information can be linked to rote learning, which provides no connection between the new material, and already existing knowledge. The second way is the basis of misconceptions or alternative conceptions. The third way is relevant in mathematics where certain procedures have to be memorised in a set order (like tables).

The information stored and retrieved from the long-term memory plays a key role in selecting what goes through the perception filter and also in helping working memory to process new information (Johnstone, 1993). Our memory system is capable of recognising whether an item of information has been stored or not and can retrieve any particular item by using different helpful strategies such as pattern recognition, rehearsal and effective organisation.

3.5 Message and Noise: Field Dependency

Although the information processing model seems to describe very accurately the processes involved for all learners, the way information is received, collected and organised from our environment varies in many important details from individual to individual. This depends on cognitive structure, previous knowledge and experiences. Therefore, every individual has different cognitive characteristics because of the differences existing in cognitive structures and psychological functioning.

Witkin (1978) was one of the first ones to develop the theory of psychological differentiation and the idea of self-segregation from the world outside and explained that this is where the boundaries are set up between an individual and his immediate environment. Witkin, in his research work (1974, 1977; Witkin and Goodenough, 1981) introduced and developed the idea of field-dependence and field-independence.

He described the ability to distinguish between important and unimportant within a stream of information as the field-dependence/field-independence style. This will be described here under the more general heading of field dependency. Witkin and Goodenough (1981) described the main characteristics of field -dependent and field-independent as:

Field-Independent: An individual who has the ability to break up an organised perceptual field and can easily separate an item from its context is called a field-independent (F.IND). In other words a field-independent person can distinguish between the 'signal' and the 'noise' (Johnstone, 1993). The field-independent person is capable of reconstructing a field by breaking it up into separate items and he can also make a number of changes to the field or to go beyond the information given.

Field-Dependent: An individual who has difficulty to break up an organised perceptual field and cannot easily separate an item from its context is called a field- dependent person (F.D). A Field-dependent person would tend to respond to the dominant properties of the perceptual field presented to him.

To measure the level of field dependency of an individual, Witkin (1977) introduced a 'group embedded figures test' (GEFT). In this test, the individual has to recognise and identify a simple geometric shape within a complex pattern. The more shapes he finds correctly the better he is at the process of separation and is said to be field-independent, and vice versa for field-dependent. The person with middle performance is called field-intermediate (F.INT).

3.6 Field Dependency and Working Memory Space

Many researchers (e.g. Pascual-Leone, 1970; Case and Globerson, 1974) tried to relate field dependency to other cognitive factors such as learning and memory. Their studies showed that in problem solving tasks when the solution depends on using an object in an unfamiliar way, field-independent students are more likely to perform better than field-dependent students.

El-Banna (1987) examined the relationship between performance in a chemistry examination for students of different working memory capacity and varying levels of field-dependence. He discovered that the performance declines when the student is more field-dependent. Al-Naeme (1988) found little difference in performance in a chemistry examination between low working memory capacity field-independent pupils and high working memory capacity field-dependent pupils, illustrated by table 3.1 where the mean marks for each of the nine sub-groups are shown.

		Field Dependency		
	%	*Dependent*	*Intermediate*	*Independent*
Working Memory Capacity	*Low*	47	54	59
	Medium	46	60	72
	High	58	69	78

Table 3.1 Working Memory and Field Dependent (Reid, 2008)

This pattern has been found to occur again and again (e.g.. Christou, 2000; Danili and Reid, 2004). Johnstone *et al*. (1993) presented a possible explanation for above results. They noted that students with high working memory space capacity who are more field-dependent are using part of their working memory to hold unnecessary information: they are occupied with 'noise' as well as 'signal' because of their field-dependency characteristic. They are unable to use their high working memory space capacity efficiently because of the presence of useless information. On the contrary, low capacity and field-independent students will receive the 'signal' and ignore the 'noise' and they will be able to use their low working memory space more efficiently for useful processing.

In order to show relationship between field-dependence/ field-independence and academic performance, significant research (e.g. Witkin *et al*., 1997; El-Banna, 1987; Ziane, 1990; Johnstone & Al-Naeme, 1991; Uz-Zaman, 1996; Tinajero & Paramo, 1997; Danili, 2001) has discovered that field-independents score considerably higher than field-dependents in almost every field of science and mathematics. Indeed, Danili (2004) pointed out that she could find no case where the field dependent students had any advantage in test and examination performance. More recently, Hindal (2007) found strong evidence that this advantage for the field independent was connected to better performance in subjects beyond the maths-science range, including arts and social subjects.

3.7 Conclusions and Implications

The information-processing model offers a comprehensive model to describe what are the essential features of all learning at all levels. The model is also highly predictive and much recent work has shown that simply reorganising the learning situation in line with the insights offered by the model will generate much improved understanding (Danili and Reid, 2006; Hussein, 2007; Chu, 2008) although none of these studies relates to mathematics. One key feature of all these studies is the way the authors redesigned the learning experiences in major sections of school syllabuses to reduce the load on working memory. This did not alter content, the time allocated or involve teacher training. Among other things, the material was re-sequenced, the key 'messages' were highlighted, and there was opportunity to build up ideas step by step. The information model offers some clear pointers to more efficient and effective teaching and learning.

Ausubel found that new material should be linked to the already existing knowledge so that learners attend and respond to the information in such a way that it can be linked meaningfully onto previous knowledge. Johnstone (1997) pointed out that, if essential previous knowledge, concepts or language were missing, the learners would not be able to understand meaningfully what was being conveyed.

As working memory space is fixed and cannot be changed, it might seem useful to teach students to develop strategies to use the space more efficiently. Chunking is one key way to reduce working memory load. However, the process to develop chunking strategies is highly idiosyncratic. It depends enormously on experience and student experiences vary in great measure.

To reduce pressure on limited working memory space, it is important to help pupils to distinguish signal from the noise. During the lessons, teachers should make clear what is important and what is peripheral. For example in mathematics, key formulae used in a problem, basic principles and rules to be used and main points of the concept should be revised and reviewed repeatedly. A few key words can offer the pointer so that the student can focus in on the essential.

As working memory has a limited capacity, it can be overloaded easily if the amount of material to be delivered in a unit of time is not limited. This relates to the issue of speed of processing, an area which is very under-researched. However, Johnstone (1997) makes a key point when he states that, "*Giving more may mean learning less*". The cognitive load may involve an excessive amount of the '*units of sense*' (Johnstone, 1997) such as formula, equation, structure, graphs, definitions, and tables. The more information we deliver in a lesson the less efficiency may be recorded.

This can pose major problems at school level in that school syllabuses are often constructed by those outside the school who have little idea of the learning capacity of the learners and the teaching capacity of the teachers. There need to be fewer topics to be covered in the time. Too much will affect the efficiency of both the teacher and the learner. Of course, topics should be linked to the previous topics in the previous grade and also to the next grade.

One of the most pressing problems is that of forgetting what is taught. The information processing model suggests that, if a theme is taught in such a way that many links are formed between all the items to be remembered, then forgetting is less likely. Ideas come together as a coherent whole and the presence of many links between the ideas means that there are multiple routes to any piece of information meaning that it can be found and recalled more easily at a later stage.

Consolidation is a good way to avoid forgetting. Consolidation is achieved through confirmation, correction, classification, differential practice and review in the course of repeated exposures with feedback, to learning material. Such consolidation means that links between ideas are created and strengthened

If learners do not simply memorise but actually understand the concepts, then the very process of understanding means that ideas are being linked together to make a meaningful whole. Forgetting is less likely later. Similarly, it is possible to use linkages between the symbolic and representation level of the teaching material. Where possible, teachers should provide opportunities for students to use both verbal and imaginable coding information. In mathematics, if we introduce concepts by using concrete material, by using pictures and then by bringing them to abstract level, it will help them to establish powerful links in their long term memory.

It is often observed that, when students get back to school in a new class after the summer holidays, they seem to forget every concept and procedure in mathematics as if they were never taught these topics in the previous class. The problem is that the links between ideas were never made strongly enough and the time gap has allowed the links to become weaker.

By giving them a little practice of these previously done topics, teachers can make them recall the procedures again fairly easily. This is simply re-activating the links between ideas, the basis of pre-learning. Actually this little practice makes them retrieve what is hidden under the pile of new events and information (other than mathematics) that they have encountered during that gap.

It is possible to use child-friendly pictures and also involve them in class activities using concrete material and this can provide a strong base to anchor new concepts. By giving problems in examinations that consist of both pictorial and verbal information, we can provide a chance of comprehending and solution planning to every student as some are good at getting verbal instructions and some are good at imagery level.

Group discussions offer an opportunity to strengthen the links between ideas in long term memory. Small group discussions are rare in mathematics but it is possible to use class discussion to aid learning. There needs to be an atmosphere where pupils can exchange ideas freely and can even reveal areas of difficulty. The actual process of verbally discussing ideas can clarify thinking, correct misconceptions and enable ideas to be connected together more strongly as a meaningful whole. This may lead to better understanding and a greater ability to apply ideas.

Many of the outcomes from the information processing model were later codified in a set of ten principles by Johnstone (1997). These were later called the 'The Commandments' by Gray (1997):

The Ten Educational Commandments

"(1) What you learn is controlled by what you already know and understand.
(2) How you learn is controlled by how you have successfully learned in the past.
(3) If learning is to be meaningful it has to link on to existing knowledge and skills enriching and extending both.
(4) The amount of material to be processed in unit of time is limited.
(5) Feedback and reassurance are necessary for comfortable learning, and assessment should be humane.
(6) Cognisance should be taken of learning styles and motivation.
(7) Students should consolidate their learning by asking themselves about what is going on in their heads.
(8) There should be room for problem solving in its fullest sense to exercise and strengthen linkages.
(9) There should be room to create, defend, try out, and hypothesise.
(10) There should be opportunity given to teach (you don't really learn till you teach)."

Chapter 4

Attitudes

4.1 Definitions

The definition of attitudes has evolved over many decades. Thurstone (1928) offered an early description when he suggested, *"the affect for or against a psychological object*". Likert (1932) described attitudes tersely as, "*a certain range within which responses move"* while Allport (1935) suggested a, *"mental and neural state of readiness to respond, organised through experience, exerting a directive and/or dynamic influence on behaviour"*. The Thurstone description emphasised the affective, the Likert seemed to focus on behaviour while Allport seemed to appreciate that attitudes generate readiness to behave in specific ways.

Krech (1946) described attitudes as aspects of learning and, thinking of problems, he described attitudes as *"attempts at solution"*. According to Katz and Sarnoff (1954), attitudes are, *"stable or fairly stable organisation of cognitive and affective processes"*. Together, these focussed on the cognitive and affective, balancing the behavioural emphasis of some previous work.

Rhine (1958), described an attitude as a *"concept with an evaluation dimension"*. The word 'evaluative' was important in that this stressed that attitudes have to involve some kind of evaluation of someone or something. According to Cook and Sellitz (1964), attitudes do not control behaviour on their own; rather other influences also determine the variety of behaviours. This was an important observation.

Bringing the literature together, Johnstone and Reid (1981) observed that attitudes have three components:

(1)	Knowledge component	(Cognitive)
(2)	Feeling component	(Affective)
(3)	Tendency towards action component	(Conative)

In other words, implicit in any evaluation which leads to attitude formation, there may be knowledge, feeling and experience.

Oppenheim (1992) suggested that attitudes involve,

> *"A state of readiness or predisposition to respond in a certain manner when confronted with certain stimuli…. attitudes are reinforced by beliefs (the cognitive component), often attract strong feelings (the emotional component) which may lead to particular behavioural intents (the action tendency component)".*

This, again, stressed the importance of seeing attitudes in terms of what is known, what is felt and what is experienced or what behaviour might follow. This confirmed that attitudes were not to be seen interms of simple affect or defined in terms of observed behaviour.

After Rhine (1958), Eagly and Chaiken (1993) pointed out that evaluation was a major and important feature of attitudes and described attitudes as:

> *"Attitude is a psychological tendency that is expressed by evaluating a particular entity with some degree of favor or disfavor".*

A person may have some clear knowledge about an attitude object but he will not have an attitude towards it until the evaluative response about this object occurs. In other words, knowing on its own does not imply evaluation while experience and emotion makes evaluation much more likely. This evaluation can be based on the cognitive, affective or behavioural factors or a mixture of them.

An attitude always has an object or is to be directed towards something or someone. According to Chaiken and Eagly (1993),*"Entities that are evaluated are known as attitude objects".* They described that anything that is discriminated can be considered as an attitude object. In other words, anything that becomes an object of thought can be called an attitude object. Attitude objects can be abstract (e.g.. capitalism, learning) or concrete (e.g. TV, rice). They also believed that particular entities, classes of entities, behaviours and the classes of behaviours can function as attitude objects.

According to them an attitude object is always determined from a variety of stimuli. For example, the attitude object '*my mathematics teacher'* is perceived through a variety of stimuli (his/her name, his/her picture, his/her teaching style). Attitudes towards science learning, a major focus in the literature, will be discussed in detail further.

4.2 Attitude as a Latent Construct

Attitudes cannot be measured directly but it is possible to deduce from the overt behaviour. In other words, an attitude can be inferred by considering the observed stimuli and the observed evaluative responses.

Figure 4.1 Attitude as a latent construct (from: Chaiken & Eagly, 1993)

Figure 4.1 illustrates an important point. It is impossible to see an attitude directly. It is latent or hidden in the long term memory. It has to be inferred from some behaviour. However, there is no certainty that observed behaviour will indicate a held attitude perfectly.

4.3 Functions of Attitudes

Individuals develop attitudes for good reasons. After reviewing the literature, Reid (2003) suggested that it was possible to see these reasons under three headings:

> *"Attitudes help us in life to:*
> *(1) Make sense of ourselves;*
> *(2) Make sense of the world around us;*
> *(3) Make sense of relationships."*

Every individual understands the world around in terms of knowledge, experience, feelings and behaviour with some element of logic and determines acceptable patterns of social interaction. All of us are making evaluations of life's experiences and of each other (not necessarily in any condemning sense). This generates attitudes which are held. These in turn influence how we behave towards each other and in various situations.

4.4 Attitude Measurement

Being described as a latent construct, attitudes are not directly observable and so are not directly measurable. Attitudes researchers have been facing problems in measuring attitudes for many decades. Indeed, it is quite obvious that any knowledge about an attitude can only be constructed by inference from the overt responses or indicators. There are real dangers here and Johnstone and Reid (1981) note the serious possibility that, *"attitudes are what attitude measuring devices actually measure"*. This does not totally undermine attempts at measurement. For example, ability at mathematics is often determined by success or otherwise in specific tests of mathematics. In all measurement, there is a need to check that the measurement made actually reflect what is intended. This is part of the whole question of test validity. Thus, attitudes must be inferred or deduced from behaviour with no certainty that the deduction or inference is correct.

There is another important problem. The marks gained in an appropriate mathematics test can be regarded as valid evidence of ability in mathematics. Thus, it is possible to say that a student who scores 80% is better at the mathematics tested than a student who gains 60%. It is not reasonable to suggest that a mark of 80% is better than one of 78%, simply because of the imprecision of the mathematics test as a measuring tool and the imperfections in marking schemes.

With attitudes, there is a much higher imprecision in the measuring tools currently available, and, thus, it is important to note that attitudes cannot be measured in any absolute way. Indeed, attitudes are not open to precise measurement for individuals at all. It is only possible to observe changes in attitudes or differences in attitudes when comparing two or more groups or one group on several occasions.

There are three general approaches to attitude measurement in educational settings:

(a) Observe responses in a written measurement (questionnaires and surveys);
(b) Observe responses to verbal measurement (interviews);
(c) Observe behaviour.

This is no different to other areas of education measurement where written tests, interviews and observation of behaviour (e.g.. lab work, computer programming, cooking or products made in a workshop are all regarded as valid forms of assessment).

The questionnaire is the most commonly used approach but still it is considered as an indirect measurement. To assume that a questionnaire is a true measure of a particular attitude is open to question. However, this question is a problem with typical tests and examinations as well. Thus, Reid (1978) notes that,

> *"Even if a person replies honestly, both consciously and subconsciously, there is no guarantee that his replies will be valid when he is placed in any particular situation relating to his expressed attitudes."*

Rickwood (1984) is more uncertain when he states that,

> *"In the field of attitudes research, there exists a degree of divergence of views and opinions over the nature of attitudes. The range of opinions over the nature of attitude has failed to produce a clear conceptual base on which the measurement of attitude can be based".*

Cook and Selltiz (1964) listed five techniques of attitude measurement and argued that it is dangerous to rely only on one of these techniques, as no one of them is perfect. Only three are used widely in education and it is useful, for example, to see questionnaires and interviews as complementary, each offering a check on the validity of the other.

4.5 The Questionnaire

Questionnaire is an important and popular technique that is widely used to measure attitudes in the field of educational research. Oppenheim (1992, p.100) described the questionnaire as follows:

> "*The questionnaire is an important instrument of research, a tool for data collection…. It can be considered as a set of questions is arranged in a certain order and constructed according specially selected rules. The questionnaire has a job to do; its function is measurement".*

He also explained that a well-constructed questionnaire can lead to *"good results".*

There can be two types of questions in a questionnaire:

(a) Open questions**:** Oppenheim (1992, pg 112) described open questions or free-response questions as those which, *"Are not followed by any kind of choice, and the answers have to be recorded in full".* Open questions are easy to construct and they give a freedom to the respondent to express their opinion freely but such questions may be difficult to answer, involving considerable time, and are not easy to analyse.

(b) Closed questions: In such questions the respondent is offered a set of possible responses and he is supposed to select his choice(s). Closed questions can be more difficult to set as the constructor needs to be extra careful to ensure clarity and lack of ambiguity while offering a fixed range of responses to cover what the respondents think but such questions are easy and quick to answer and analyse. The disadvantage of such questions is to restrict the respondent, as he does not have adequate freedom to express his opinion in his own words.

A mixture of both types of questions in a questionnaire can often give a useful way forward to detect a specific attitude.

There are few important points that are required to keep in mind while constructing a questionnaire (see Reid, 2003):

"• *The attitude object must be specified. The questionnaire should reflect clearly that what a researcher would like to investigate in each question.*
- *The appropriate technique that can reflect the evaluative character of respondent's attitude towards a specific attitude subject should be used.*
- *The constructor should be extra careful about the validity and reliability of the methods used for attitude measurement.*
- *The constructor should focus to deduce the respondent's hidden attitude that cannot be judged by his overt behaviour. For this purpose, the attitude object must be specified and the variety of stimulus, which can help to obtain evaluation about it, should be clearly defined. For example if mathematics lessons are considered as an attitude object then the following stimulus might be defined: mathematics teacher, group activities, outdoor activities, the teaching methodology, class work, duration of the lesson, class room atmosphere etc. etc.*
- *It is useful to get the questions checked critically by other researchers and, if possible, by the teachers who know the pupils to be tested.*
- *Pre-testing is very helpful to check the clarity and the format of the questionnaire. As Solomon (1949) believes that pre-testing is the only way to avoid the possible complications. Pre-testing with a small sample size followed by discussions can be really helpful but pre-testing is not possible always.*
- *Instructions for the teachers involved in the questionnaire project should be clear.*
- *Necessary written and verbal instructions for the pupils involved should be clear and simple.*
- *The timing should be adequate.*
- *The pupils involved must be aware that their answers would not affect their school examination results or their relationship with their teachers. It would encourage them to answer with honesty and without any fear.*
- *The sample size should be large enough to draw some clear results.*
- *The language used in the questions set by the constructor must be simple, clear and according to the level of the pupils.*
- *The font size of the questionnaire should be adequate to help students in reading easily.*

Reid (2003) summarised the procedures in developing questions in diagrammatic form (figure 5.2).

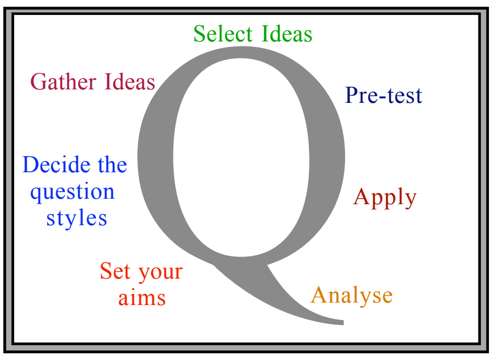

Figure 4.2 Developing a Questionnaire (from Reid, 2003, page 41)

4.6 Validity and Reliability

In educational measurements, the researcher needs to be clear that what is to be measured and whether the measurement is accurate or not.

Reliability: Osgood (1957, p.126) describes reliability: *"The reliability of an instrument is usually said to be the degree to which the same scores can be produced when the same objects are measured repeatedly".* Thus, reliability of any measurement is that the measuring device will produce the same results on more than one equivalent occasion. The time interval between administrations should be long enough for the respondent to forget his previous responses, but short enough to stop long-term factors to influence his attitudes. Based on long experience, Reid (2006) observes that, if the questionnaire is carefully constructed and is applied under the circumstances where respondents are able to be honest in answering and the sample size is large enough to draw some clear conclusion, then the reliability is likely to be good.

This kind of test-retest reliability has to be distinguished from the internal consistency of any measurement tool. This is the extent to which the responses to the questions correlate with each other. This is usually inappropriate in educational measurement where attitudes are highly multifaceted.

Validity: The validity of a measuring instrument can be referred to as the extent to which the instrument measures what the researcher wants it to measure. In other words, the responses to the questions reflect answers to the actual issues the researcher had in mind. The literature has all kinds of validity and Oppenheim (1992) describes these briefly and this is summarised here:

- *Content Validity:* When the items in the measurement or test reflect a balanced coverage of the issues, skills, or knowledge to be measured.
- *Concurrent Validity:* When the outcomes of the measurement can be related to some other valid measures of the same topic, theme or skill.
- *Predictive Validity:* When the outcomes of the measurement can be related to some future criterion such as job performance, recovery from illness or future examination attainment.
- *Construct Validity:* When the outcomes of the measurement can be related to some set of theoretical assumptions about an abstract construct.

According to Reid (2006), there are several ways to check of there is a good level of validity: by having an opinion of a group of experts (Schebeci, 1984), by conducting sample interviews (Reid and Skrybina 2000; Shah, 2004), by developing questions following discussions with the respondents to be involved (Bennet, 2001) and pre-test questions where possible (Reid, 2003). However, as Reid (2006) explained, attitudes cannot be measured in any absolute sense with certainty. Thus, there is no absolute or fixed way to measure validity but careful and sensible checks can provide some encouraging results.

Many years ago, Reid (1978, page 18) discussed some of the problems associated with the use of questionnaires, which are summarised below:

(a) Some attitude targets are difficult to specify and possible ambiguities of language may mislead or misguide the respondents.
(b) Good question selection requires that the selected items must be valid and clear to the respondent, with a sense of reality.
(c) Validity and reliability must be addressed.
(d) The effect of items position on the response pattern is also an important factor that can cause a problem. Kraut (1975) observed the position of particular items in a scale (in his case, Likert scale) and discovered that the later an item was placed, the less was the tendency towards extreme responses, and more the chance of failure to respond.

There is also the problem of unidimensionality. This is discussed fully in Reid (2006) and a summary of his argument is presented here. Attitudes in an educational context are usually highly multidimensional and this can be illustrated with a simple example from an imaginary test of mathematics. Suppose there are 10 questions each of, say, 10 marks. It is only possible add up the marks to get a total out of 100 if there is some assurance that what the 10 questions are testing has something in common. This is straightforward: they will be testing 10 different skills in mathematics, relating to some taught material, assuming that the test is set well. Thus, the mark out of 100 reflects some kind of ability in mathematics in relation to what has been taught.

An imaginary attitude questionnaire can be considered. Suppose there are 10 questions. It is only possible to 'add' up the scores if there is confidence that the 10 questions have something in common. This is often described as a latent construct: some key idea which all the questions are testing.

To show that the 10 questions are relating to the same latent construct (say, attitude to mathematics), correlation is often used. If the questions correlate highly with each other, then it is assumed they are measuring the same thing. This is simply a highly flawed argument. Highly positive correlation cannot be assumed to indicate a common latent construct. Thus, height and weight of humans correlate but it is meaningless to add height to weight to give any meaningful total. Reid (2006) notes the problem with correlation in that a correlation coefficient can be seen as the cosine of the angle between the lines representing the two dimensions of measurement. Because cosines are not linearly related to the angles, the correlation coefficient has to be ***very*** high before the the lines of measurement come near each other (figure 4.3).

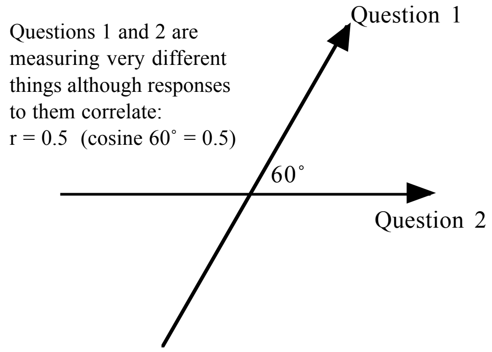

Figure 4.3 Directions of measurement and correlation (from Reid, 2006)

It is interesting to observe that, if the outcomes from the 10 questions in mathematics are correlated with each other, it does not matter if the correlations are high or low because the nature of the topics tested in relation to the syllabus taught is quite straightforward. Experienced teachers can agree, with some measure of confidence, that the test is a fair test of mathematical skills, reflecting the curriculum taught. However, it is simply not possible, in the present state of knowledge, to do this easily for any set of 10 attitude questions.

Thus, while it is possible to describe the mathematics skills which reflect some taught curriculum quite accurately, an attitude like 'an attitude towards mathematics' is highly multi-dimensional and cannot be reduced to a single 'score'. This argument applies equally to Likert and Osgood methods but is not quite so true for Thurstone. These methods are discussed in the next section.

Reid (1978, page 18) also noted some issues related to the actual use of questionnaires. There is the problem of adequate sample size: the sample size should be large enough to draw some clear conclusions and also the random nature of the sample of the population should be considered. There is the problem of social desirability and respondents' dishonesty can cause a problem. To avoid these problems, the constructor should make it difficult for the respondent to spot what is desirable and built-in checks (e.g.. by using repeat questions) can help to see the actual response. Random interviews can also provide some useful information.

4.7 Methods for Attitude Investigation

There are many traditionally accepted methods to measure attitudes. The common and basic character reflected in all these methods is evaluation. The respondents can express their evaluation of an attitude object on a certain scale, which allows them to classify the stimuli between extremely favourable and extremely unfavourable.

Thurstone Method: It was generally believed that it was impossible and unworkable to measure attitudes until Thurstone (1929) presented the key breakthrough in the field of attitude measurement by introducing the use of attitude questionnaires. His work has an eminent historical significance and opened up a new area of further exploration in this field. Through his book "The Measurement Of Attitudes", he laid a mathematical basis for scaling and developed a method summarised by a series of steps (Reid, 1978):

"(1) Specification of attitude variables
(2) Collection of wide range (200) statements and opinions related to attitude.
(3) Editing these to give about 100 statements judged to be relevant, valid and covering a wide range of opinion, including neutral positions, to the attitude variable.
(4) About 300 judges are asked to sort the statements into all 11 piles from "anti" position to attitude to "pro" position, pile of 6 being neutral.
(5) Analysis of these judgements.
(6) Elimination of statements because they have wide ranges in the judgement, or are irrelevant.
(7) Selection of 20 statements along the scale 1-11
(8) Each person tested is asked to mark those items with which he agrees,
(9) The score for each person is the average scale value of all the statements he has endorsed."

Clearly, the Thurstone method was time-consuming and involved many people. The method is rarely used nowadays but he definitely opened a new horizon for researchers interested in attitude measurement.

A few years later, Likert (1932), developed a simpler method when compared to Thurstone's approach.

Likert Method: Likert (1932) mulled over Thurstone's approach and suggested a less laborious method of attitude measurement. His method is widely used these days and provides similar results to the Thurstone method but is less protracted as it avoids the cumbersome collating of statements. No judges are required in this method as the respondent is supposed to place himself on the evaluative scale according to the degree of his preference towards a specific attitude object. In the absence of any judge, the validity and reliability of the method solely depends on the constructor.

The evaluative scale for each statements today normally consists of five positions, running from "strongly agree" to "strongly disagree" including a neutral one. The respondent is supposed to tick one of the five positions provided and in this way can express the level of his agreement or disagreement with a specific statement. Originally, Likert's method was introduced as a scale that helps to estimate the respondent's attitude by the value of the total score obtained. This total score is the sum of scores from evaluation of different items selected and included for the scale (e.g.. 'strongly agree' = 5 down to 'strongly disagree' = 1). The responses for all the questions will be correlated with the total score and the questions correlating best with each other are selected and used to measure the attitude towards a specific attitude object.

An example to illustrate this approach is given below:

Here are some descriptions of the way students **approach** mathematics.

*Tick **one** box on each line.*

	strongly agree	*agree*	*not sure*	*disagree*	*strongly disagree*
Revision sheets help me to understand mathematics well.	☐	☐	☐	☐	☐
Diagram and pictures help me to understand mathematics well.	☐	☐	☐	☐	☐
I can understand the main points easily.	☐	☐	☐	☐	☐
I think mathematics help me in daily life a lot.	☐	☐	☐	☐	☐
I do not want to learn mathematics but it is compulsory.	☐	☐	☐	☐	☐
I enjoy studying mathematics classes.	☐	☐	☐	☐	☐
I work hard in mathematics but cannot get good marks in exam.	☐	☐	☐	☐	☐

Assuming all the questions are positive, he introduced the following scoring system:

Strongly Agree = 5; Agree = 4; Neutral = 3; Disagree = 2; Strongly Disagree = 1

A high score obtained is considered to show an extremely positive attitude and a low score will show a extremely negative attitude. He also argued that the distribution of responses would form a normal distribution.

There are numerous criticisms of the scoring method used by Likert although the style of questions and their use in attitude measurements is of great value. These criticisms are discussed in Reid (2006) and are summarised here:

(a) The method of scoring assumes that every question reflects a common construct, and correlation is used to check this. However, correlation does not demonstrate that this is true.
(b) The 'scores' (5,4,3,2 and 1) are ordinal numbers and cannot be added in this way.
(c) The final score is obtained by adding up scores from evaluation of different items, which may have different meanings. Gardner (1975) illustrates the weakness by stating that, *"to add up the weight, the number of doors, the number*

of cylinders in a motor car to produce a single number would have little meaning".

(d) It is assumed that the gaps between the 'scores (5,4,3,2 and 1) are equal and this cannot be checked.

(e) The method implies that, for example, an 'agree' (=4) is worth twice a 'disagree' (=2). This is meaningless.

(f) The responses to individual questions frequently give distributions very far from normal, making Pearson correlation inappropriate as this is based on the assumptions of approximate normality and the use of ratio numbers.

(g) Responses to specific individual questions often offer valuable insights which scoring and adding obscure.

The same scoring criticisms apply to the Osgood method (below) but both approaches are extremely valuable if inappropriate scoring is avoided. One way forward is to compare the response patterns, question by question, for two groups (e.g. boys and girls) using the chi-square statistics as a contingency test. Chi-square does not rely on any assumption of normality and works well with frequency data.

Osgood's Method (Semantic-Differential Method): Osgood *et al.*(1967) developed a simple technique which they named the semantic-differential which is very popular among attitude researchers these days. Heise (1970), states that, "*Osgood's method is eminently suitable in terms of type sample, administration, easy design, high reliability and validity when compared to other methods."*

This method, at the start, had a seven-point rating scale providing bipolar word-pairs placed at the opposite ends of a scale. Later, Heise (1970) revised and changed from seven-point scale to four or five-point. The respondent's task is to tick only one box on each line to rate the attitude object

An example to illustrate this technique is given below:

What is your opinion about the **subject mathematics?**

*Tick **one** box on each line*

I like mathematics	☐☐☐☐☐☐	I do not like mathematics
Useful in daily life	☐☐☐☐☐☐	Useless in daily life
Easy to understand	☐☐☐☐☐☐	Difficult to understand
Boring subject	☐☐☐☐☐☐	Interesting subject
I do not want to learn it but it is a compulsory subject	☐☐☐☐☐☐	I want to learn it because I enjoy it.

The main advantage of this method is that it takes a shorter time to answer the questions as compared to the Likert method and even children can understand and handle it easily (Reid, 1978). Originally, scores were allocated from the positive end: 6, 5, 4, 3, 2, 1 for the six boxes. The respondent's attitude was acquired by adding up their scores for all questions. This scoring procedure is open to the same criticisms applied to Likert scoring. Used as a scaling method, Heise (1970) notes that Osgood's method has a coefficient of reliability around 0.91 and has a high correlation around 0.90 to more the cumbersome Thurstone method. Overall, the semantic-differential method is considered a reliable and valid method for attitude investigation.

4.8 Interviews

The interview is another useful and powerful technique to investigate attitudes with, numerous possible approaches e.g. open-ended, highly structured, and with fixed questions. It gives more freedom and choice to both the interviewer and the interviewee to express their opinion freely and fully. It provides rich data and sensitive personal information and this technique can be applied to almost everything.

Interview can be considered more helpful and powerful than questionnaire in collecting valid data but they are much more time-consuming and it is extremely difficult to summarise the data to give precise conclusions. It can also be used in checking the validity of the data obtained from questionnaires.

There are two main types of interview.

***Exploratory Interviews*:** According to Oppenheim (1992), the main focal point of these interviews is to develop working ideas or a research hypothesis rather than to collect statistical facts. He further adds that the spontaneous conversation with the interviewee that allows the researcher to obtain deep data about a specific attitude object should be recorded on tape. These interviews are very useful if held before conducting further research to help to clarify the situation, as well as after data collection to check the validity of a questionnaire.

***Standardised Interviews*:** These interviews can also be called verbal questionnaire and are used to collect data. A prepared set of questions is asked to collect the data, and the same questions in the same order are asked of every respondent. Public opinion, research and government surveys are common examples of such interviews.

It is possible to see interviews on a continuum of the extent of open-endedness (figure 4.4).

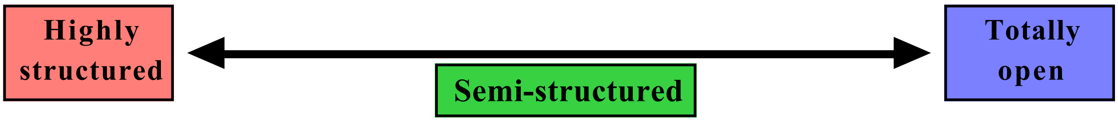

Figure 4.4 Interview Styles

In highly structured interviews, the questions are fixed and decided before hand and such interviews are used to collect the data or to check any misinterpretation with a questionnaire. In a totally open-ended interview, an open question is set to investigate the attitude. For example, what is your opinion about the usefulness of mathematics in daily life? The interviewer can ask some introductory questions to encourage the interviewee and to enable him to talk or give his opinion freely. This kind of interview can be long but provides a rich and deep insight (Reid, 2003).

Overall, interviews offer a useful technique to investigate attitudes but are difficult to plan, open to personal bias and sometimes the data obtained are not easy to interpret.

4.9 Other Questionnaire Approaches

Many questionnaires today are based on the Likert approach, the semantic differential, or both. However, some other approaches have been developed and these are now discussed briefly.

Situational Questions: Students are placed in a supposed or imagined situation to make them think and express their views and ideas on a certain issue. The main purpose of this technique is to investigate their attitude in a simulated real-life situation. An example is given below to illustrate this technique:

> Suppose that the Junior School Headmistress decides not to give mathematics revision homework sheets at weekends but the mathematics teachers suggest to her to continue these sheets because they think that revision sheets make you practice difficult topics at home and also help you to find weak areas. What do you think about mathematics revision sheets? Do they help you to find out your weak areas? Write down three short sentences to express your opinion on the issue.

Rating Questions: The respondent is given a set of responses and is asked to put them in some order or to choose a small number of greatest significance against some criterion (Reid, 2003). The choices made or the order chosen often can give very revealing and clear information. The following example illustrates this approach:

I like mathematics thanks to:

A my parents
B my school teacher
C my tutor
D computer
E my friends
F mathematics lessons
G easy / I am good at it
H mathematics TV programmes

Place these responses in order of their importance by letters A, B ,.. etc. in the boxes below. The letter which comes first is the *most important* and the letter which comes last is the *least important* for you.

☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐

Most important *Least Importan*t

4.10 The Theory of Reasoned Action

In the early days of attitude research, the relationship between held attitudes and behaviour was often not too clear (e.g.. the definitions suggested by Allport, 1935; Krech, 1946). The work of Ajzen and Fishbein clarified this considerably (Ajzen, 1985; Ajzen and Fishbein, 1980). Their first model was *the theory of reasoned action* and this considered behaviour over which humans have control: the world of rational decisions.

According to this model, an individual's overt behaviour (B) depends on his behavioural intentions (BI). In other words, the weakness or strength of the intention will affect the performance of the behaviour accordingly. The stronger the individual's intention, the more he is expected to try and hence, the greater the possibility to perform that behaviour and vice versa. An individual's intention to behave can be predicted by two factors:

(1) ***The person's attitude towards the behaviour (AB)****:* When a person has some information about an attitude object this information results some kind of feelings (positive or negative) about that specific object.

(2) ***The person's subjective norm (SN):*** The second predictor of a behavioural intention, introduced by Fishbein and Ajzen (1975), is the subjective norm. In general, a person tends to perform behaviours that they highly value and also the behaviours that they think are acceptable to other (significant) people.

Their studies suggested that the model did not explain behaviour sufficiently and the model was extended into the Theory of Planned Behaviour.

4.11 Theory Of Planned Behaviour

Ajzen (1985) added one more factor called *perceived behaviour control* (PBC). According to him previous two factors were not sufficient enough to explain behaviour intention. An example illustrates this. Consider a pupil intending to study mathematics in higher classes. While attitudes might be positive and the person thinks that others are supportive, there may be many factors which undermine any intention to continue to study mathematics. For example, these might include previous poor grades and timetable constraints.

Suppose if he does not want to study mathematics in higher classes but if he says that he would intend to continue mathematics in future, there might be many factors behind this so-called intention. The positive opinion of the important figures, mathematics being a compulsory subject, his fear of not being successful in career without studying mathematics can play an important role to build-up this unwanted intention. The career requirement lies in the area of perceived behavioural control. The views of other people come under the subjective norm. Of course, the decision to take mathematics may not be related to an intention, which is freely considered. Factors may 'force' the decision or it may be that taking the mathematics is done without any serious decision being taken at all. The person, knowing its importance, just flows on and takes the course. All of this led Ajzen (1985) to describe the third component of the theory. The Theory of Planned Behaviour only refers to behaviour which is undertaken after due thought and where the person is taking the choice in freedom: planned behaviour. Perceived behavioural control can be describe as the perception about whether the behaviour is possible. The theory can be shown graphically (figure 4.5):

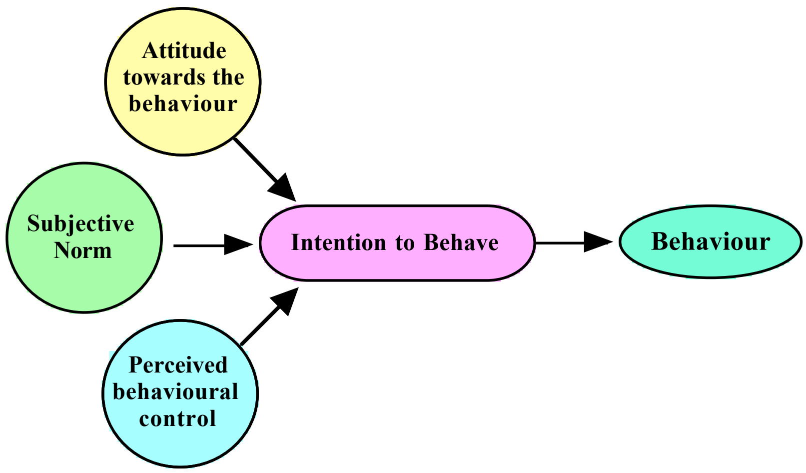

Figure 4.5 The Theory of Planned Behaviour

The results of many studies (Crawley and Black, 1992; Crawley, 1990; Ajzen & Madden, 1985) have shown that the addition of this third factor has improved the predictions of behavioural intentions. Nonetheless, among these three factors, the attitude towards the behaviour is the most powerful factor.

4.12 Attitudes in Science Education

The literature of science education contains a very high number of studies relating to attitudes. These will be discussed very briefly in that they have some parallels with mathematics education.

Attitudes towards science are important to investigate as King (1989) described:

> *"As the details of scientific formulae fall away in the months and years after school, it seems likely that the crucial deposits of science and technology education are to do with attitudes, approaches and even values".*

Long ago, Ramsay and Howe (1969) suggested that a student's attitude towards science may be more important than his understanding of science, since his attitudes determine how he will use his knowledge. Many children join school with great interest in science but learning experiences at school make them feel that science is difficult and inaccessible (Hadden and Johnstone, 1982, 1983).

As a science educator, the main aim of the science teacher at school (especially at secondary level) cannot be to train practising scientists, for only a tiny minority will pursue such careers. The main role of a science teacher is to educate and develop science skills in:

- Pupils who can make sense of life and themselves;
- Pupils who can appreciate the place and role of science in modern society;
- Pupils who can relate their studies to culture, lifestyle and social importance

Many social and moral problems appear with scientific advances and people need to make decisions and judgements about several scientific matters e.g.. pollution, nuclear waste disposal, birth control and cloning etc. For these reasons, attitude development is very important in science education.

There are parallels with mathematics. The aim of school courses cannot be to make academic mathematicians for few will go that way. The aim is to educate in mathematics, its ways of thought and enquiry, its applications in many areas of life and its huge contributions to human understandings of the world. If attitudes are negative, this whole world may be closed off for students.

Gardner (1975) described the attitude towards science as a "*person's attitude to science as a learned disposition to evaluate in certain ways objects, people, actions, situations or propositions involved in the learning of science*". The importance of looking at attitudes in relation to learning has grown over the past 50 years or so (e.g.. Krathwohl, Bloom and Massia, 1964; Choppin and Frankel, 1976; Ramsden, 1998).

Long ago, Khan and Weiss (1973) suggested the areas where attitude development was important (figure 4.6)

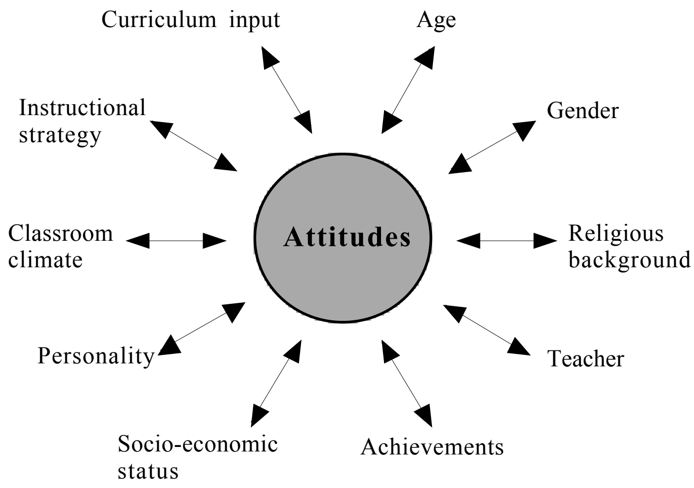

Figure 4.6 Factors Relating to Attitude Development
(derived from Khan and Weiss, 1973)

Many variables like teacher, classroom environment, subject instructions, content and context of the lessons, pupils' socio-economical status, their religious backgrounds, their age and gender, their achievement and personality can influence the formation of attitudes towards science directly or indirectly. However, most of the variables are not open to change by a researcher or teacher. The classroom climate and the way lessons are taught (instructional strategy) are exceptions but most of the rest are pre-determined or are controlled by those well beyond the school. This means that changing attitudes towards mathematics or the sciences depends heavily on the way the curriculum is presented, the way it is assessed (which may be out of the control of the teacher) and the classroom climate.

In the context of the sciences, Reid (2003) lists three areas where attitudes are important and these are summarised here:

(a) ***Attitudes towards the science subject being studied:*** The main concern of educational researchers is to find out how to develop pupils' positive attitude towards the subject being studied.

(b) Attitudes towards study itself: It is also important for the researchers to look at attitudes towards learning; skills for effective learning and using these skills successfully in life. It is necessary to develop a critical understanding and thinking among students about the nature of knowledge i.e. how this knowledge is acquired; what are the approaches towards study; what is the nature and importance of learning as a life-long process..

(c) ***Attitudes towards themes/topics/issues arising in the study of a science subject:*** Study topics and themes which involve important social issues like birth control, pollution, and nuclear industry would make students develop attitudes towards these and related themes.

Clearly, there are parallels with mathematics. The area of attitudes towards mathematics itself and towards study in general apply. However, mathematics does not, at least at school stages, have many implications for society in the way the sciences have.

4.13 The Impact of Teachers on Attitudes

Science teachers can be considered the most important figures who can encourage student's development in science. However, science teachers sometimes just focus on delivering knowledge of science subjects and pay little attention to the application or significance of these subjects in the daily life of their pupils. Allport (1961) suggested that if the school does not teach any value of science, the students would not acknowledge its importance. This suggests that it is important to consider how models of attitude change might apply in education. It is also more important to create a learning atmosphere, which can generate pupils' positive attitudes towards science subjects. The parallels with mathematics are obvious: the teacher has a key role in the development of positive attitudes. Indeed, many researchers (e.g.. Germann, 1988; Reid and Skryabina, 2002) have investigated and found that teachers and their instructional methods play a key role in forming and developing students' attitudes towards science. This is likely to be true also for mathematics.

While some have argued that perceived difficulty is a major issue in developing negative attitudes (e.g.. Cheng, Payne and Witherspoon. 1995), the work of Reid and Skryabina (2002) in relation to physics shows that perceived difficulty and attitude development are not so neatly related as might be thought. This confirmed work done by Reid (1978) where he found that the most popular teaching unit (of a set of 12) was the one which was also regarded as the most difficult.

Reid and Skryabina (2002) found that there were three factors which made physics attractive and these can loosely be thought of as the quality of their experience in the learning situation, the teacher and the perceived value of their studies in career or qualification terms. This confirms the key role of the teacher and the importance of a curriculum where the learner sees the work being studied as related to themselves in their social context and lifestyle.

One of the most influential factors that enable attitudes to grow and develop is contact or interaction with other people (Reid, 2003). The role of an important figure around a pupil may establish a positive or negative attitude towards science. At early years, the role of parents may be critical and at primary level of school, the teacher has a powerful influence. At secondary stages, the parent's influence tends to decrease and the teachers' role is still powerful (Reid & Skryabina, 2002) although this may not be true in all cultures (Alhmali, 2007).

4.14 Attitude Development

Attitude development within education can be seen in terms of (Reid and Jung, 2008):

(a) The communicator (teacher);
(b) The communication (the teaching);
(c) The response (the way the brain operates).

Each of these is considered briefly in turn.

Many researchers (i.e. Hovland, 1953; Reich & Adcock, 1976) stressed that the communicator has to have high credibility in the eyes of students. It is also important to note that the material which has high credibility, but seems unimportant to the learners, can also change their attitudes (Himmelfarb and Eagly, 1974).

Looking at communication, Jung (2008) described several important points which are summarised here:

- The communication must be understood but even if the communication is understood, it is not certain that the learners will change their attitudes (Reich & Adcock, 1976).
- The communicator must take things step by steps. In other words, they should focus on meaningful learning.
- According to McGuire (1968) comprehension is positively related to ability and as students try to make sense about how to relate the information given by the teacher to their daily life so it is important to focus on effective communication procedures. The learners cannot handle too much information because of limited working memory space and as a result, they would not comprehend the teaching properly. Eventually, it may lead to lack of understanding and attitude development would not be possible. In working memory, new learning material needs time to be connecting with the previous knowledge in long-term memory. Then the learner needs time to rethink new ideas and their connection with old ideas. In this way they construct knowledge. Finally, working memory has to clear itself for further new knowledge.

It is difficult to explore the important internal mechanisms in the brain of a learner which might affect attitude development. However, considerable research has offered some important insights. Humans need to be reasonably consistent in thinking. According to Heider (1944) attitude development was more likely when mental inconsistencies are generated, To explain it further, a favourite and respectable teacher's negative view about a subject can generate inconsistency. One possible way to reduce the inconsistency is to separate the view of the teacher from the view of the subject. Another way is to steadily become more positive about the subject.

4.15 The Key Idea of Dissonance

Janis and King (1954) observed when people are forced to do what they do not want to, opinions could be changed but the change did not last. However, Festinger (1957) developed it further and, in an amazing experiment, he offered various rewards to students to do what they did not want to do: they had to report to another student that an excessively boring task was, in fact, interesting. In his findings he showed that, when rewards were offered for doing what they did not want to do, the *smaller* the reward, the *greater* the opinion change in a forcing situation.

Through this study, he developed the idea of ***dissonance.*** He described carefully what he meant by ***dissonance*** and by ***consonance.*** He described the situation where behaviour and attitude were not consistent as ***dissonance*** and the situation where behaviour and attitude were consistent as ***consonance.*** Thus, in his experiment the behaviour was in contradiction with the students' attitude, as they had to tell the next student that the task was interesting, whereas actually it was boring to them. He saw that any attitude change which arose depended on what he called ***total dissonance,*** which took into account the actual ***dissonance*** and the actual ***consonance.*** He described it in a simple mathematical relationship:

$$\textbf{Total Dissonance} = \frac{\textbf{Actual Dissonance}}{\textbf{Actual Dissonance + Actual Consonance}}$$

He presented a hypothesis that any possible attitude change would be related to the total dissonance. Another important point to note is that if the amount of consonance increases, then the total dissonance decreases. This idea is very common in daily life. When faced with information which is inconsistent with what we understand, then dissonance is set up. The natural thing to do is to recall or generate as much consonance in order to ‘dilute’ the dissonance. According to him, he saw total dissonance as a psychological drive.

The work of Festinger has been repeated many times and explored from several perspectives. Essentially, his findings are confirmed (Eagly and Chaiken, 1993). Thus, for attitude development, it is essential to connect the new knowledge and experiences to the previously held knowledge and experiences. This may create dissonance necessary for attitude development.

Jung (2008) noted the following important points:

"• *The possibility of attitude change or development is controlled by the total dissonance and this involves taking into account what is consonant as well as what is dissonant.*
- *Dissonance seems to be a natural process throughout life*
- *When placed in dissonant situations, people tend to seek for consonant cognition, affects or experiences in order to reduce the total dissonance and thus avoid attitude change. This preserves attitude stability and avoids disturbance*
- *Dissonance occurs in the working memory as former knowledge, feeling or experience drawn from long-term memory to interact with new knowledge, feeling or experience*
- *If the working memory is overloaded, the dissonance is impossible. If learning is reduced to memorisation or is the passive reception of information, then dissonance is highly unlikely."*

4.16 Cognitive and Attitudinal

Attitudes related to learning interact continuously with the actual learning processes. Thus, attitudes influence the learning process continuously. As attitudes can be affected by the learning experiences of the learner so it is always hard to undo attitudes once they are built up. *"Students' attitudes may control whether they display their ability completely or almost not at all in learning activities. Therefore, the way to make the most of this interaction should be developed in order to achieve both cognitive and affective objectives in science education"* (Jung, 2008).

Reid (2008) notes that attitudes (which are held in long term memory) may influence the perception filter and can also affect what the learner allows to enter their working memory because people perceive only what they want to perceive. If the learner has a negative attitude towards the learning process itself or the topic being learned, he/she may not be able to focus or concentrate on the learning material and, therefore, new and further steps in the learning process can be disturbed from beginning.

If the learner is not confident about the subject being learned, he/she may not be able to function to his/her capacity fully and it may create dissatisfaction or anxiety in the learner. The learner might hesitate to take cognitive risks which are important to activate thinking and investigation.

By contrast, favourable attitudes can help learners to focus or concentrate on new information positively. Positive attitudes towards the subject being learned help learners to develop their interest; they tend to spend more time in studying that subject; they make every possible effort to understand the information and show readiness to take cognitive risks. According to Nasr (1976), favourable attitudes:

"• *Help the individual learner to reinforce previously learned skills*
- *Lead to the achievement of some important skills, such as communication, cooperation etc.*
- *Lead to the interaction among the learners and also between each learner and his/her teacher*
- *Help the individual learner to make his own decisions*
- *Help the individual learner to reduce the inconsistency caused by introducing new information to him*
- *Help the individual learner to organise knowledge in a way which is simpler to him."*

In his study on algebra story problems, Christou (2001) found that attitudes related to performance. He argued that it was not a simple cause and effect. Positive attitudes generated greater success while greater success generated positive attitudes. Thus, success in a mathematics test or examination may well be important in helping to generate the kind of positive attitudes which will be helpful for future learning.

In the sciences, another factor has been shown to be very important. The learners need to see the science they are studying as related to themselves and their lifestyles. It is more difficult to see the exact equivalent factor in learning mathematics but the issue seems important.

Looking at success in examinations, Jung (2008) noted:

"• *Poor information in long-term memory about the theme under consideration/ attitude object*
- *Poor chunking which leads to working memory overload and consequently miscomprehension or no comprehension about attitude object*
- *Negative attitude storage in long-term memory, which may hinder in further learning*
- *Low confidence and unwillingness to learn further*
- *Memorisation with little understanding, as a way of learning."*

In short, unsuccessful learning experiences can cause students to avoid learning. This may develop negative attitudes and the learners would be unable to utilise their ability to study further as there is no or little prior knowledge in long-term memory and they will not be able to make sense of new learning. This may cause another unsuccessful learning experience.

The possible guiding principles for desirable and well-informed attitudes in science education suggested by Jung (2008) are:

"• *The science curriculum must meet the learner's needs in the sense that it is perceived by the learner as relevant in the context of lifestyle and society;*
- *The science curriculum must meet the learner's needs in the sense that it enables to the learner to make sense of his world;*
- *The material presented to the learner must be accessible: it must be capable of being understood rather than memorised; it must not make unrealistic demands on working memory;*
- *The teaching programme must offer the learner appropriate opportunities to become involved actively with the material being taught: to interest actively and make one's own."*

Keeping these principles in mind, Jung argued that science educators should consider the following points for planning their teaching program:

- *Development of an application-led course related to real life situations such as health, transport, communications;*
- *Presentation of career prospects related to science. Particularly, early subject-related career education for girls is necessary to help them to overcome existing gender-related stereotypes about science;*
- *Development of teaching material related to social context of science. Reid (1978) insisted that historical, domestic, industrial, economic and socio-moral aspects of science should be reflected in a science curriculum.*
- *Presentation of contemporary role models and actual working practices of scientists;*
- *Careful consideration of students' stage of growth, their age and working memory capacities;*
- *Adherences to appropriate difficulty level which arouse students' intention to challenge intellectually;*
- *Development of proper communication skills suitable for students and using various audio-visual materials;*
- *Setting up teaching material of high credibility and high quality;*
- *Providing enough time and opportunity for message-related thinking;*
- *Providing variety of experiences. It may encourage students to participate actively and give them opportunities to internalise. Role-play, problem solving and discussion are examples;*
- *Involving students in the teaching strategies as much as possible. Talking with students, individually and in groups, and making every effort to implement their suggestions could achieve it.*

4.17 Attitude Development through Intra-action

Many researchers have been investigating to find out ways to develop attitudes. Several models have been presented in literature but it is not easy to combine them or draw some general conclusion as these models often apply in specific situations presented by these researchers.

Katz (1959) was the key researcher who considered the purposes of attitudes. He presented the idea of "internal satisfaction": as Piaget said (1962), human beings naturally try to make sense of things. Attitudes serve the purpose for each individual to make sense of life. Attitudes will only develop if they bring some kind of benefit to the person. This can be seen as a kind of satisfaction as the person is enabled to makes sense of the world around, themselves and relationships. Naturally, a person would resist attitude change until he is satisfied internally.

The idea of internal satisfaction is probably connected to the idea of "dissonance" presented by Festinger (1957). The idea of internal satisfaction and dissonance are also clearly related to the idea of "internalisation" suggested by Bloom (1964) and Sherif and Sherif (1967). If an attitude serves the purpose of internal satisfaction for each individual, then that attitude must become the individual's own attitude. That individual internalises this attitude before he owns it.

In accordance with Festinger and other cognitive dissonance approaches, Reid (1978) developed the idea suggested by Osgood (1967) that the key to attitude development is bringing together affective and cognitive elements in such a way that dissonance could occur. Two or more things can only be in dissonance if they are brought tightly together. The word juxtaposition describes this precisely. If the things do not come together (very closely so that they kind of interact or entwine) then dissonance cannot happen.

Reid (1978) described "interactivity" as bringing together meaningfully new cognitive, affective and behavioural elements with the attitudes already held in the long-term memory. The attitude development is stable with time.

Figure 4.7 shown below describes the idea of inter-activity presented by Reid (1978):

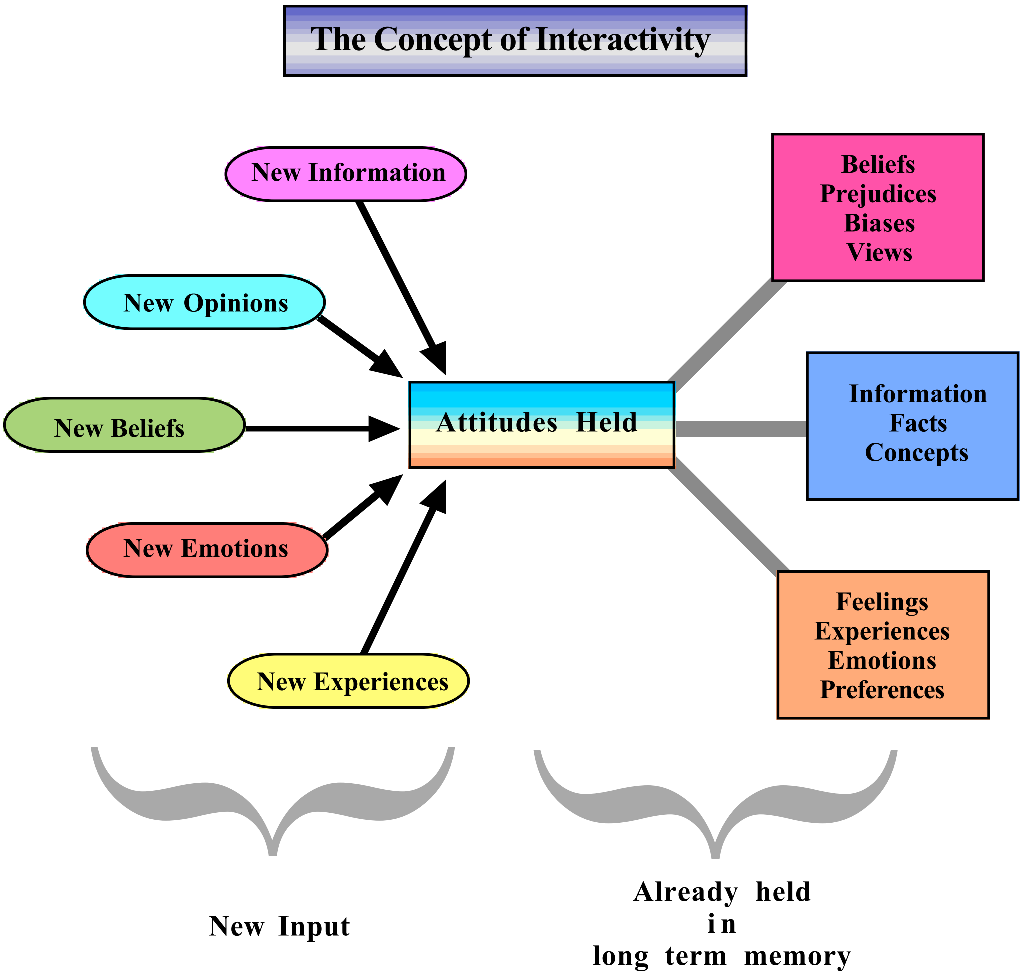

Figure 4.7 Interactivity

The normal English word 'interaction' refers to a two-way relationship. Two people can interact with each other - talking, listening, sharing etc., each changing the other in some way. Reid (1978) took the word "interaction" and used it to describe the way two things interacted *in the brain*. This is very different from normal human interactions. It considers what is going inside the brain, not what happens between people.

For dissonance to occur, two things (e.g. knowledge, attitudes, feelings, experiences of whatever) have to come together, each affecting the other - that is what is involved here. He argued that dissonance can only occur if two or more things interact *inside* the brain in some way. With later knowledge, this must happen in the working memory as some new knowledge, feeling, and/or experience interacts with previously held knowledge, feeling, and/or experience. This may cause a re-evaluation and the development of the attitude concerned.

Chapter 5

Attitudes Towards Mathematics

5.1 Background

Attitudes are often multidimensional and involve three components. Attitudes related to mathematics will follow this general pattern as well and this can be summarised in the following description:

> Attitudes towards mathematics is the psychological tendency of an individual that develops on the basis of evaluation involving knowledge, feelings and experiences about mathematics.

Examples of factors which relate to an individual's attitude towards mathematics include:

(a) **Feelings** - Is he naturally interested in the subject? Does he like mathematics or not? Does he like his teacher? Does he enjoy the subject or find it boring? Does he really want to study the subject or is he studying it because it is compulsory?

(b) **Knowledge** - What does he know about mathematics as a subject? Does he find it easy or difficult? Does he realise its importance in future?

(c) **Experiences** - What is his experience about mathematics. It may involve his teacher's personality and the teaching methodologies used by his teacher. It includes his examination and classroom experiences. The experiences expressed by the important figures around him also play an important role.

All these three components may generate a positive or negative attitude towards mathematics as a subject. However, it has to be noted that attitudes towards mathematics are complex and multi-faceted. Kulm (1980) notes this when he states that, "*It is probably not possible to offer a definition of attitude towards mathematics that would be suitable for all situations, and even if one were agreed on, it would probably be too general to be useful"* Nonetheless, considering the cognitive, affective and experience dimensions may be helpful as Hannula (2002) notes: "*The proposed framework of emotions, associations, expectations and values is useful in describing attitudes and their changes in detail.*"

5.2 Development of Attitudes towards Mathematics

Learners have often perceived mathematics as a difficult subject (Brown *et al.* 2008). Mathematics is abstract and logical in nature; also it is not possible for the teacher to explain every concept of mathematics to the learners in terms of physical representation or to relate it to their daily life. However, mathematics may be helpful in developing their logical thinking; it is useful in their lives or is necessary for them to learn for their future careers. These aspects are to easy to share with young learners.

It is possible that mathematics is able to contribute to the all round development of each pupil by offering understanding which will enable them to develop socially in the sense of being able contribute to society more effectively and being able to make senses of society more effectively. However, mathematics is often portrayed as being abstract and unrelated to life.

There are many claims that studying mathematics develops skills of logic and thinking (Karl, 1984) but most of these are unsubstantiated. It is possible that pupils who have these skills will choose to study mathematics. It is equally possible that mathematics will enhance and enrich such skills if already present in the learner. It is certainly true that mathematics does allow school pupils to use and practice these skills to enormous advantage for their intellectual development as well as career development. Studies in mathematics allow all pupils to look at the world from a quantitative perspective, offering a precision of description, which enables decisions to be taken accurately.

The evidence from many countries is that many pupils do not enjoy school mathematics and seek to avoid it later (McLeod, 1994). Various countries have tried to find ways to improve this but with limited success. The aim should be to enable pupils to derive enjoyment, satisfaction and fulfilment from studying mathematics: to make them to be excited with it and see what it can offer. One of the key aims might be to find and develop ways to do this even more effectively and efficiently.

Attitudes tend to become more negative as pupils move from elementary (primary) to secondary schools (McLeod, 1994). Attitudes to mathematics are no exception. Negative attitudes towards mathematics are difficult to undo (Duffin and Simpson, 2000). Attitudes tend to deteriorate with age simply because work becomes more demanding and also because, as the pupils get older, they may start to think that they will not need mathematics in the future. Johnstone & Hadden (1982) found that interest in science is expressed in their expectations that involvement in exciting experiments would result in the discovery of new knowledge. They also pointed out the natural curiosity and wonder as a key factor at this stage. There is no obvious exact parallel in mathematics.

Although there can be experimental work in mathematics parallel to that in the sciences, is it possible to allow pupils to discover new knowledge in mathematics as well? Of course, it is important to involve them to satisfy their natural curiosity and wonder. This might be done by creating some challenging elements in lessons and also by making them feel that they have *done* or *achieved* something. The starting key is to establish pupil confidence and pleasure in getting things right: this is done by practice and challenge, set in an unthreatening atmosphere. Indeed, the atmosphere must be positively supportive. Perhaps, this is where imagination comes in - can activities and challenges be devised where the pupils can see their mathematics at work and can bring the excitement of mathematics linked to real life? This can be the basis of future research and development.

Oraif (2007) concluded from her research that success breeds confidence. She also found that the key feature underpinning confidence lies in simple success and this success is reflected in speed of learning, understanding and examination success. This confidence leads to more positive attitudes. It provides the possibility for students to enjoy their tasks and also the challenges of further learning. This has enormous implications for mathematics. If the pupils sees success as elusive, then it is not likely that attitudes will remain positive. Success will be rapidly eroded if the pupils are asked to complete task which are beyond them. The importance of working memory limitations may be critical.

The key is for teachers to enable students to grasp and understand the procedures first. By means of clear description followed by adequate practice, the procedures can be more or less automated so that the pupils simply know that they can get it right almost every time. They will have a sense of achievement. At that stage, understanding can be gently introduced, the working memory now being able to cope as the mathematical procedures, being automated, take up little space.

The problem often lies with the assessment. It has to be recognised that:

- Examination assessment is never perfect and it might not reflect the learner's way of thinking well or the fact that the pupils might have a 'bad' day.
- Not everyone will do well. A paper which allows the most able to show their abilities may bring about ‘failure’ for others.
- There is no certainty that papers are set which are able to assess everyone fairly. There is a multitude of learner characteristics.
- What to do about those who do not perform well in examination? How to make them feel successful according to their abilities? How to build up confidence in them? How to offer success to them? If confidence is mostly related to examination success only and is not dependent on other factors, then examinations are almost doomed to consign certain pupils to the dustbin of failure and their confidence will be damaged.

The place and nature of testing and examinations needs careful thought: maybe a major and radical re-thinking of examinations.

Reid (1978) found that difficulty does not neatly relate to positive or negative attitudes. His study results showed that perceived difficulty and enjoyment are not inversely related. Later, Reid and Skryabina (2002) found that very excessive difficulty could cause problems. There is a balance point here. While the demand of challenging tasks does not, of itself, necessarily generate a perception of difficulty, when a task is perceived as being so difficult that the effort is not justified by the rewards, attitudes seem to deteriorate.

In his large study with secondary school students age 12-18 in Libya, Al-Himali (2007) found that students thought that mathematics was not seen as an easy subject. The students there found it over-abstract and the least enjoyable and least attractive part of school. In his results, “polarisation” of views was quite remarkable: while some students ‘loved’ it, many students ‘hated’ it.

Piaget (1963) established beyond doubt that learners are continually seeking to make sense of the world around. It is reasonable to suggest that, when this natural process is blocked in some way, then there will be learner dissatisfaction. The working memory is critical in the process of making sense in that it is here that all thinking takes place. If the tasks they have to do place too much strain on working memory, then understanding cannot take place fully.

If understanding becomes almost impossible because of working memory limitations and if the pupils are faced with examinations and tests, they may have to resort to memorisation in order to pass. This may generate dissatisfaction and positive attitudes may start to decline. If the examination system is too much based on memorisation, it may generate a similar discomfort among students. Also, the fear of failure may shake their confidence because if they failed to reach specified standards, it may close the doors of future opportunities for them.

There can be many reasons for not making sense out of things. The following can be included in them:

(i) Working memory limitations;
(ii) Lack of previous knowledge or experience;
(iii) The way it is taught (teacher, textbook or whatever) which does not link new knowledge in a meaningful way onto previous knowledge;
(iv) There may well be more affective reasons as well: don't like the teacher; I enjoy something else better; it is not important for me so why bother... etc.

In a very recent study, Jung (2008) found a relationship between attitudes towards the sciences and measured working memory capacity. Of even greater significance, she found that those with lower working memory capacities tended to rely more on memorisation while those with higher working memories were more reliant on understanding. However, both groups appeared to *want* to understand. Perhaps, this offers the key: negative attitudes relate to failure to understand. The natural process is to seek to make sense of things. When this is impossible or the opportunities are denied them, then attitude problems will arise.

It is possible to suggest that positive attitudes are much more likely to arise if two conditions are fulfilled:

(1) The learners can cope with what is asked of them. In other words, there is no excessive and unreasonable difficulty. By all means, pupils should be stretched intellectually. However, if the difficulty is excessive, they will be turned off. The art is stretching them only as far as is reasonable so that they do not lose heart, with no prospect of success. The art is very much based on the teacher. When the teacher offers challenges and stimulates the pupils to see these as difficult challenges, with no possible criticisms if they do not always make it - then the learner can take cognitive risks and make a real effort. It arises from a basis of trust between learners and teacher.

(2) The learners can make some sense of what is being taught. In other words, there is some measure of understanding and they can see where it fits in terms of their life experiences, at least in part.

The above discussion leads to a hypothesis presented by Jung (2008). According to her, the following three factors may cause attitude *not* to be positive.

(a) If the working memory capacity is exceeded too often, then understanding cannot take place.

(b) If they cannot see how what they are studying has any significance for them, they may well not see the point of it all and attitudes might fall.

(c) If there is excessive difficulty (which might not relate to working memory at all), attitudes might fall. However, many learners like a challenge and it depends on the nature of the difficulty.

5.3 Variables Affecting Attitude Development in Mathematics Education

Many factors like teacher, classroom environment, gender, achievement, curriculum, culture, home background, important figures around, abilities and age can affect an individual's attitude towards mathematics. Some of these relate to the teacher and much research has explored teacher influences on attitudes towards mathematics and achievement in mathematics. The influence of the teacher is ONE key factor in encouraging the development of positive attitudes. Many researchers (e.g.. Reid, 1978; Ponte, 1991; Johnstone and Reid, 1981; Reid and Skrybina, 2002) note that a teacher's attitude may have the most enormous effect on a pupil, especially if the pupil's attitude towards the teacher is strong, either positively or negatively. It may be the single most important factor (specially at primary level). It is rather generally held that teacher attitudes and effectiveness in a particular subject are important determinants of student attitudes and performance in the subject (especially with younger learners) (see Reid and Skryabina, 2002). There is a remarkable relationship between the views and attitudes of the teachers and those of the students (Al-Enezi, 2004). When teachers are negative towards mathematics, so are the students. When they are positive, they definitely influence the views and attitudes of their students (specially younger ones) towards mathematics.

The key role of teachers can be seen as:

(1) To make mathematics accessible in terms of understanding as well as 'getting it right'.

(2) To give the learners some sense of fun and challenge.

(3) To give the learners the feeling they are being supported and not condemned.

(4) In a mixed ability class, every student cannot score "A" grade as many factors (genetic and natural mathematical abilities etc.) cause different performance results in class and examination, but the teacher can motivate every learner by giving a sense of achievement to every learner according to his mathematical abilities. The teacher should try to make the learner believe that what he can do is to try his level best whether he is in the top of a grade list or not.

Some researchers (N.Orhun, 2007) suggest that teachers should focus on the "learning styles" of the learners as it enables them to adapt their teaching methods to those of the students. This is more or less impossible. Imagine that a teacher has a class of 30, each with a different combination of learning styles - how can any teacher adapt their approach to suit all 30?

Hindal, Reid and Badgaish (2008) note that the phrase 'learning style' can be very confusing as different researchers use the phrase in different ways. They suggest that 'learner characteristics' may be a better term to be used. They argue that there are three aspects to learner characteristics:

- Are they genetically determined?
- Are they learned (formally or by means of experience)?
- Are they preferred (a matter of choice)?

Most may involve all three aspects. The key issue is which learner characteristics bring the greatest benefits to learning and are these characteristics capable of being developed or encouraged?

Some key findings are:

(a) Those who are more confident of their ability to learn mathematics are more likely to continue studying mathematics when it becomes optional but the more important question is to know what causes the confidence - is it previous good performance? The work of Oraif (2008) suggest that it is.

(b) The way in which teachers praise and criticise pupils and their work may have a significant impact on pupils' attitude towards mathematics and achievement in mathematics although there is no simple relationship between the use of praise and pupil achievement (Brophy,1981). Teachers can play a vital role in developing a positive attitude towards mathematics if they are able to judge correctly whether a pupil is a consistent and persistent trier, wants to increase competence or lacks confidence and avoids challenges (Askew, 1995).

(c) Teachers should be cautious about questioning techniques and situations in the classroom. Clearly, students who feel that the teacher is embarrassing them will develop a negative attitude towards mathematics (Askew, 1995).

(d) Al-Himali (2008) tried to investigate the attitudes of students in Libya and his results reflect the perception of students about their teachers. According to these students*: "Teachers must know their subject and be confident and enthusiastic in it. They wish more open types of lessons, with less dependence on lecture type knowledge transmission*." Although set in Libya, this observation may be true in other countries.

(e) Aiken (1961) showed in his studies that students who do not do well in a subject may develop a negative attitude towards that subject and blame their teacher. Aiken and Dreger (1961) found both male and female college students who disliked mathematics blaming their school teachers. However, there is always a tendency for students to blame their teachers if they do not like a subject. If a student is not doing well in mathematics or has a negative attitude towards it, then there are many other factors that may be involved and the teacher cannot be solely blamed for these.

(f) Hannula (1983), unsurprisingly, suggested that the general attitude of a class towards mathematics is related to the quality of teaching and to the social-psychological climate (the general way a class is run) of the class.
Classroom environment is important and, ideally, should be pleasant, encouraging and thought provoking. It is important that the teacher is there to support and enhance confidence (Ortan *et al*. 1994).

(g) As every teacher knows,what is learned and how and where it is learned are inextricably linked. Most of the topics in mathematics cannot be easily related to everyday life and do not come in such a neat form, so it is difficult for pupils whose learning depends on the use of such school-based cues and they will often be unable to apply their learning to real world situations. It is ideal that learners are enabled to apply their mathematical skills in new situations. However, this may be extremely difficult: skill in real world situations would then require the learning of a new set of ideas, rather than the application of existing ideas. The assumption that pupils will be able easily to transfer skills and knowledge learned in one context to other may not be true (Askew, 1995). However, the context in the classroom will be linked strongly to their learning (or lack of it). It is possible to make mathematics learning fun and meaningful. The application of it has often to be left until later.

(h) Mathematics is abstract and involves its own precise form of logic. Therefore, it is hard for the teacher to relate it to real life sometimes but maximum effort should be paid to satisfy the pupils that whatever they are learning will be connected and related to future learning and careers. The teacher should help them to apply their knowledge.

(i) Another significant point was revealed in this study that at the 7th grade, students tend to be more optimistic and willing to accept challenges coming from their teachers but, on the contrary, the 10th grade students seemed to be more independent (Ponte *et. al*., 1994). The main reason may be the natural independence among students at this age. At this age they find a conflict among their own developing ideas and the ideas received from the important figures in their lives.

Learning mathematics cannot be justified by arguing that it develops logical thinking outside mathematics. There is no evidence that mathematicians, as a group, are any more logical than others. The evidence from the problem solving work of Yang (2000) and, later Al-Qasmi (2006), would suggest that the kind of logic developed in mathematics is strictly to be applied in mathematical contexts and does not transfer easily at all. This is the same for scientists where there is little evidence that scientists are any more scientific than others when moving outside their own specialiality. These kinds of skills are context-dependent to a very large extent. Perhaps mathematics can be justified by arguing that the study of mathematics gives pupils an insight into an approach to knowledge which is important and unique to mathematics.

5.4 Curriculum and Assessment Related to Attitudes towards Mathematics

Designing a mathematics curriculum is not easy and there is a need to identify the key aims for learning mathematics with all school pupils. Ideally, the students should be expected to understand and apply ideas and not merely memorise mathematical procedures. Nonetheless, carrying out mathematical procedures correctly to gain right answers is an important skill.

The fundamental question is to identify the reasons for mathematics being in the curriculum. Clearly, there are key mathematical skills and understandings which are needed by everyone in being part of society. There are also many more key skills and understandings which are very important in a wide range of jobs and careers. Mathematics offers a way of thinking and describing the world. Indeed, it offers a way of addressing all kinds of problems in the world, for the benefit of mankind. An insight into these seems vitally important as part of the education of all. Another set of aims might relate to the way mathematics, as a discipline, functions. It has its own logic, its own rational process of thought, its own systematic way of describing and solving problems. All school pupils need some experience of this, as part of their cultural heritage and also as a way of thinking which has been found valuable.

While constructing mathematics curriculum, the following factors are important to be considered, in light of the aims above:

- It should be seen as offering mathematics education for all, not just potential mathematicians. Therefore, it should take into account the kind of mathematics knowledge and experience, which will prove valuable as a contribution to life skills.
- It should meet the the need for the mastering of practical skills which are used in day-to-day life as well as the development of those skills which many careers will find essential.
- The length of the curriculum should be appropriate so that it can provide enough time to the teacher and the learner to comprehend the concepts at a proper pace. Time pressure may affect the performance of both the teacher and the learner.

- It is essential that the mathematics curriculum is appropriate for each age and stage. The material presented to the learners must be accessible; it must be capable of being understood rather than memorised; it must not make unrealistic demands on working memory.
- It should meet the needs of the pupils, given their developmental stage. All the topics should be developed step by step by age: the topics taught in lower level grade should be extended in the next grade level. It would provide a gradual development of concepts to the learners according to their age level.
- The mathematics curriculum should also be related to life (as much as possible) and future needs (Al-Himali, 2007). As much as possible, there should be time for fun and challenge, when mathematics can be seen in relation to real life.

In most countries, there is a tendency in designing mathematics curriculum to consider:

- *The logic and content of the mathematics;*
- *The need of minorities (the most able);*
- *What worked in the past.*

The following factors tend to be ignored:

- *The place of mathematics in society;*
- *The needs of the learners;*
- *The psychology of the learner*

It has been argued (Askew, 1995) that, instead of teaching mathematical knowledge as isolated content, more attention should be paid to the:

(1) Concept being taught
(2) Activity through which the concept is being introduced
(3) Culture

However, this kind of suggestion lacks empirical evidence that it will actually work. While it is important to understand the concepts, the limitations of working memory capacity may be critical. The better way forward is to establish procedures so that pupils can carry these through successfully and they are more or less automatic. Focussing on the conceptual understanding can then follow.

Activity may help - but it also may hinder. The activity or lack of it is not the key issue. The teacher needs to find the right learning experience, which enables the learners to have some chance of understanding the concepts. Indeed, the actual activity may impose its own demands on working memory making the whole process of learning and understanding problematic.

The word culture is often thrown about loosely. It is true that different societies have slightly different emphases on the place of mathematics but this is unlikely to make much difference. Mathematics applies all over the place and is not very culturally sensitive. However, in very poorly developed societies, mathematics will find many fewer applications. There is also educational culture. For example, if the assessment of most subjects relies on the correct recall of information, then pupils may well expect the same in mathematics. At primary stages, the pupils should experience some element of fun, the satisfaction of getting most things right, and trusting the teacher that mathematics is valuable and has meaning.

Ponte *et.al.,* (1991) raises the question of the inclusion of textbooks in the curriculum. There are some advantages in using textbooks:

- Defines the curriculum for pupils, teachers and parents;
- Can give security to learner;
- Makes knowledge seem fixed and 'learnable';
- Can compensate for a poor teacher;
- Can assist pupil who learns best from a written format (more often girls)

However, there are some disadvantages:

- Can make the curriculum too rigid;
- A bad textbook can do immense damage.

In using worksheets, the teacher is more or less writing their own textbook. This can get round the disadvantages of the textbook while retaining the advantages. However, it takes much work and there is no guarantee that the material will be 'good'. In the end, our aim is to generate learners who are able to manage their own learning in increasingly independent ways. All depends to some extent on teachers and texts. However, over-dependence on one or other, or even both is not good.

Of even greater importance is assessment for this determines the priorities for both teachers and learners. If assessment tests the recall and correct application of certain skills, then that is exactly what teachers will seek to develop. If it also takes into account understanding, application and interpretation, then teachers would encourage these elements. Assessment is the key to what happens for teachers are duty bound to get their pupils the best performance they can.

Reid (2008) notes that curricula are often planned without considering the implications of assessment. They may have aims and objectives but no idea how to assess these. They may never have thought through the stifling effect of national assessments. It is essential that a curriculum has clear aims and objectives and that there is a clear link between these and any assessment planned. If this is not done, no matter how good the curriculum, it will be reduced to the way the assessment is carried out.

Overall, while assessment must reflect the aims and emphases of the course, it is also important that assessment itself does not impose unnecessary burdens on limited working memory space. If questions place demands on the pupils where their limited working memory capacity is overloaded, then assessment results may simply reflect working memory capacity (Johnstone and El-Banna, 1986, 1989).

5.5 Parental Influence on Attitude towards Mathematics

No one would deny that parents might play an important role on a child's attitude towards mathematics and Poffenberger and Norton (1959) suggest that parents can affect the child's attitude and performance in three ways:

(a) By parental expectations of child's achievement
(b) By parental encouragement
(c) By parents' own attitudes.

They showed the results of their studies of 390 University of California freshmen. The questionnaire was designed to detect students' own attitudes and the attitude and expectations of their parents towards mathematics. The main findings of this study were that the students' attitudes towards mathematics were positively related to how they rated their father's attitude towards mathematics and their attitude was also related to their reports of the level of achievement in mathematics that their parents expected of them

Xin Ma (2001), in a meta-analysis of research findings, showed that:

- Fathers tend to have more influence on their children's' attitudes towards mathematics than mothers.
- A father’s education has a positive effect on his child's achievement in mathematics
- The American data presented a positive affect of the mother’s education on mathematics attitudes (Ethington & Wolfle, 1984).

According to Xin Ma, “*the relationship between parental education and mathematics attitude may be more culturally diverse than the relationship between parental education and mathematics achievement* “ (2001, pg. 227).

Parental influence can play an important role in developing their child's attitude towards mathematics. Career aspiration, social and cultural trends may influence parents' expectations and it can indirectly affect their child's attitudes towards mathematics. Parents' regular and timely attention can help children to develop a positive attitude towards mathematics. Several studies (Mathews and Pepper, 2005; and Kyriacou and Goulding, 2006) suggest the same point that if any messages about difficulty in relation to future struggle come from parents then it may lead the child to develop a negative attitude towards mathematics. A common feature reflected by most of the studies mentioned above is that students, since early secondary grades, are able to appreciate the importance of mathematics for future careers and it is important for them to fulfil the expectations of their parents, but whether they like or enjoy mathematics or not (and why?) - is another issue.

5.6 Gender Issues

Different researchers have tried to investigate the relationship of attitudes towards mathematics and achievement in mathematics with respect to gender.

Foxman (*et al.*1982), unsurprisingly, found that,

> "*As a group, girls tend to believe, more than do boys, that mathematics is difficult: attitudes to individual problems suggest that boys tend to over-rate the ease of mathematical items, in comparison with girls*" .

In general, girls tend to have more negative attitudes towards mathematics than boys (Frost *et al.*1994; Leder, 1995). By contrast, Ma & Kishor (1997) reported that the relationship between attitude towards mathematics and achievement in mathematics is the same for males and females. In addition, they showed that gender does not interact with grade and ethnicity to affect the attitude towards mathematics and achievement in mathematics relationship.

Xin Ma (1997) in his study "Reciprocal Relationship between Attitude toward Mathematics and Achievement in Mathematics" showed that the gender differences in this revised model were insignificant for both attitude towards mathematics and the achievement in mathematics. He also added in his findings, the results of American Studies (Friedman, 1989, 1994; Frost, Hyde &Fennema, 1994; Hyde, J.S. Fennema, E & Lamon, S.J., 1990) that concluded the similar finding that: "*Sex differences are not pronounced in both mathematics attitude and mathematics achievement*".

Xin Ma (1999) reported the same findings that there is no significant gender difference on the relationship between attitude towards mathematics and achievement in mathematics. According to Ma, these recent studies are reflecting an opposite finding of Aiken's results that *"measure of attitudes and anxiety may be better predictors of the achievement of females than males"*. (pg. 567).

Orhun (2007) investigated the attitudes of university students towards mathematics and achievement in mathematics with respect to learning style according to gender. He found that differences among learning modes preferred by female and male students may exist but their attitudes towards mathematics and achievement in mathematics were not, themselves, dependent on gender.

Hargreaves *et al.* (2008) tried to investigate the gender differential performance in "gifted and talented" 9 and 13 year olds in mathematics assessment in England. The result of this study showed that there was no significant gender difference in performance for the 9 or 13 year olds. *"…attitudinal differences were found, including a seemingly commonly held stereotypical view of mathematics as boys' subject. Further findings reveal that where "gifted" girls perform as well as "gifted" boys, their confidence in the subject is lower than their performance might suggest."* According to them, their results are important since the uptake of higher-level mathematically based courses by girls in England is poor.

All the research results shown above come from different countries with different sample sizes but do not offer any generalised conclusion. The relationship of attitude towards mathematics and achievement in mathematics with respect to gender involves many factors. Firstly, the social norms and cultural traditions and trends play an important role in this regard. If a country basically has a male dominant culture and if traditionally mathematics is viewed as more as a man's interest or occupation, then girls may perceive mathematics to be not very useful for them. Many of the above researchers in their studies also have pointed out that the attitude of many girls towards mathematics appeared to decline steadily through the years of secondary schooling.

In Pakistan, society is male dominant and girls usually do not get very strong support from their parents and society to opt for careers which are based on mathematics. Things are changing slowly and steadily and girls in these countries are coming forward to science professions but still the ratio is not satisfactorily high. Parents' support, encouragement from schools at early years, social acceptance, equal career opportunities, mathematics curriculum designed for keeping everybody in mind regardless of social, age and gender differences, all can help girls to be more confident and comfortable with mathematics.

The overall research findings suggest that there are few significant gender differences in attitudes or performance in relation to mathematics. Where there are gender differences in society, these may be reflected in relation to mathematics. However, there appear to be no cognitive differences which cause gender differences.

5.7 The Relationship between Attitude and Achievement

Several researchers have found the following results through various studies to investigate the relationship of attitudes towards mathematics and achievement in mathematics:

Hannula (2002) noted that, "*The negative attitude towards mathematics can be a successful defence strategy of a positive self-concept*". In other words, people may say, "I am good but I do not like mathematics" or "I do not like mathematics for so and so reason but it does not mean that I could not do it". They are confident about their abilities but they give some other reason for their negative attitudes towards mathematics.
McLeod (1994) observed that attitudes tend to become more negative as pupils move from primary to secondary schools. However, this may be common for many subjects as the work of Hadden and Johnstone (1983) suggested.

In their meta-analysis, Ma and Kishore (1997) suggest that gender differences in the relationship between attitude towards mathematics and achievement in mathematics are too weak to have any practical implication. They observe that, "*From the narrative reviews about the effect of grade level on the relationship between attitude towards mathematics and achievement in mathematics, we may expect that the relationship either remains weak throughout the elementary or secondary school levels (Aiken, 1976) or shows a slow increase over grade levels (Aiken 1970a)".*

The results of this meta-analysis show that the relationship between attitude towards mathematics and achievement in mathematics may not be strong at the elementary school level but may be strong enough for practical consideration at the secondary school level. Perhaps the key lies in the power of assessment. At secondary stages, assessment will determine grades and future careers and future study. Assessment at primary stages is less critical in this regard. Their results also show that the relationship between attitude towards mathematics and achievement in mathematics is significantly stronger for Asian students than for White or Black students.

They also considered ability one of the important factors that affects the relationship between attitude towards mathematics and achievement in mathematics and insisted on the inclusion of mathematics ability as a key variable in further research on the relationship between attitude towards mathematics and achievement in mathematics. However, this raises many questions: how can mathematics ability be described? Can any standard be set? There is no objective way to define mathematics ability. It tends to be defined in terms of those who do well in mathematics exams. Then mathematics examinations are seen testing mathematics ability, giving a circular argument. There can be no absolute standard at all. In simple terms, bad performance in mathematics examinations (for whatever reason) will tend to undermine pupil confidence and their belief that they can 'do' mathematics. This may generate negative attitudes. This is self-evidently obvious!

Xin Ma (1997) noted that a highly rated variable is curriculum tracking in which students are grouped for instructions mainly on the basis of their academic ability. In other words, learning of mathematics is best when the students learn in groups where everyone is of approximately the same ability. This procedure is adopted widely in various ways in much education. However, it has to be clearly distinguished from 'streaming'. In streaming, pupils are placed in classes according to their overall abilities and are taught all subjects in these classes. This approach is open to much justified criticism. In 'setting', the pupils are grouped for their mathematics learning by their ability in mathematics. This can be done by forming groups (by ability) within the class or taking, say, 100 pupils and reorganising them into 3 classes simply for their mathematics lessons. They may be organised in different groups for other subjects.

Ma (1997) also recommended a separate analysis of the relationship between attitude towards mathematics and achievement in mathematics based on gender groups, grade levels and ethnic backgrounds. Researchers are encouraged to be aware that the relationship between attitude towards mathematics and achievement in mathematics can be culturally shaped and reinforced.

Brown *et al.* (2008), working with a sample of over 1500 students in 17 schools in England, investigated students' attitude towards mathematics. These students were close to the moment of choice. The results showed that: "*The most prevalent reason that students wrote for not continuing with mathematics was the* ***perceived difficulty*** *of the subject. Thus the rejection on grounds of difficulty is common, even amongst those predicted to do well enough to achieve a grade A."* Perceived difficulty may reflect a real difficulty or it may simply reflect that the learner thinks there is a difficulty (perhaps based on previous experiences). Perceived difficulty may be a major influence on the development of negative attitudes. Difficulty may arise for several reason: working memory overload is one, lack of appropriate knowledge, inability to relate what they know to what they are being taught, etc.

Results shown by Mathews and Pepper (2005) and Kyriacou and Goulding (2006) suggest the same point that external messages about difficulty in relation to future struggle can accelerate the feeling of being unable to understand mathematics. The experiences and comments shared by their friends, siblings or teachers can play an important role. It is possible that these reports of difficulty provide an external and hence acceptable rational for students not to continue. Direct messages from their teachers and indirect messages from school or examination practices can have a serious implication. "*We derive our identities in part from messages we receive about ourselves; in this way low expectations create low attainment as much as they respond to it. The messages received from the important figures around, the structure of the curriculum, the examination system and the grouping practices in schools, all this make students to maintain low self efficiency in mathematics which has been highlighted by other researcher*s" (Xin Ma, 1997, discussing: Hannula 2002; Pietch *et al*. 2003; Matthews and Pepper 2005; Kyriacou and Goulding 2006).

The study results reviewed by Xin Ma (1997) also pointed out that externally imposed structural boundaries, such as examination systems, curricula, and grouping practices may encourage perceived cognitive boundaries. Some students appeared to believe that there were fixed boundaries for each individual person in mathematics, beyond which learning becomes extremely difficult and frustrating, and several students pointed towards this personal fixed boundary effect within their reasons for not continuing with mathematics.

According to these studies, the second and third most prevalent reasons for not continuing with mathematics are lack of enjoyment and a belief that the subject is boring. Matthew and Papper (2005) also reported the same point that the same perception is expressed by high attaining as well as low attaining students. They also considered the possibility that these negative perceptions have the potential to be intimately bound up with the complex ideas associated with difficulty. Students may wish to distance themselves from emotional involvement in a subject in which they do not feel very successful. However it may involve other feelings e.g. the lack of opportunity for creativity. The study also found that, "*towards the end of primary school, expressions of boredom were indicative not only the lack of stimulation (fun) but also of a lack of challenge, a loss of control over tasks and direction, and a felt inability to be seen and acknowledged as successful in a subject."* (Xin Ma, 1997, pg.9)

Nardi and Steward (2003) also identify *a, "mystification through reduction effect, in which teachers, in an attempt to make mathematics simpler, reduce it to a list of rules, and thereby fail either enhance a proper understanding of the underlying concepts or to provide intellectual challenge."* This idea shows that if a teacher reduces mathematics to a set of instructions to be memorised, following these instructions will enable the pupils to get 'right' answers and pass exams. The pupils may *understand* virtually nothing!!

Students need to get more effective messages from society (especially mathematics education community) about the place of mathematics in life. Quilter and Harper (1988) found that one of the two most important reasons for students failing to continue with mathematics was its perceived irrelevance to the real world. Matthew and Pepper (2005) reported the same view with senior secondary school students while students at primary levels also raise the same point of the relevance of mathematics to daily life. Teachers, parents and government need to convey the usefulness of mathematics generally through the curriculum, but they also need to make sure it is experienced as useful in the classroom. Thus, perceived difficulty (coming from experience or external messages) was commonly cited as a major reason for avoiding mathematics in further studies. Lack of interest and boredom were also found as major reasons. According to the findings, at age 11-14, fun is the key aspect for students of effective mathematics teaching (Steward & Nardi 2002).

5.8 Anxiety and Attitude towards Mathematics

According to Richardson & Suinn (1972) mathematical anxiety is a feeling of tension, helplessness and mental disorganisation a person has when required to manipulate numbers and shapes. Cemen (1987) described mathematics anxiety as, "*an anxious state in response to mathematics-related situations that are perceived as threatening to self-esteem" while* Wood (1988) described anxiety as, *"the general lack of comfort that someone might experience when required to perform mathematically*".

Mathematics anxiety is probably multidimensional (Hart, 1989; Wigfield and Meece, 1988):

- *Dislike (an attitudinal element)*
- *Worry (a cognitive element)*
- *Fear (an emotional element)*

As an attitude has emotional as well as cognitive features, a relationship between anxiety towards mathematics and achievement in mathematics should be expected. A number of studies (e.g. McGowan, 1961; Reese, 1960) have found small but statistically significant correlation between anxiety towards mathematics and achievement in mathematics (Aiken, 1970).

Xin Ma (1999), in a meta-analysis, concluded the following main points to explain the relationship between anxiety towards mathematics and achievement in mathematics:

- It was found that the relationship between mathematics anxiety and mathematics achievement is consistent across gender groups, grade levels (Grade 4 through 6, Grade 7 through 9, and grade 10 through 12), ethnic groups (mixed and unmixed), instruments used to measure anxiety, and years of publication.
- In this meta-analysis, he showed that the typical correlation for the relationship between anxiety towards mathematics and achievement in mathematics was $r = -0.27$.
- He founded the same results as previously reported (e.g. Armstrong, 1985; Eccles, 1985; Hackett, 1985; Wigfield & Meec, 1998), regarding the significance of the relationship between anxiety towards mathematics and achievement in mathematics for school pupils. According to him, the most important finding in this analysis was an immense improvement in mathematics achievement when mathematics anxiety is reduced.
- Handler (1990), recommended that a cognitive process approach reduces mathematics anxiety through:
    - (a) Making knowledge workable for the learner
    - (b) Joining skills and content (applying the learned knowledge)
    - (c) Linking motivation to cognition
    - (d) Using social communities (working together).

Handler also found some interesting facts in this meta-analysis:

- Firstly, he found no significant gender difference on the relationship between anxiety towards mathematics and achievement in mathematics. Ma and Kishore (1997) showed the similar findings and reported that the relationship between anxiety towards mathematics and achievement in mathematics is same for males and females.
- Secondly, he studied the most important issue of the development characteristics of the relationship. After studying three grade-levels group (Grade 4 to 6, Grade 7 to 9, and grade 10 to 12), he concluded that the relationship between anxiety towards mathematics and achievement in mathematics was significant from grade 4 on.

According to Lazarus (1974), "*mathematics anxiety can arise any time during the schooling.*" He further explained that once mathematics anxiety occurs, its relationship with mathematics achievement is consistent across grade levels. He combined this finding with previous reports (e.g., Brush, 1985; Hembree, 1990; Meece, 1981; Wigfield & Meece, 1988) showing the fact that uneasiness, worry, and anxiety associated with the learning of mathematics increase during the early adolescent years, and demonstrated that an increasing decline in mathematics achievement during early secondary schooling is possible for adolescent students with mathematics anxiety. This study also pointed out that screening and treatment programmes should be brought in during the upper elementary grades.

- Thirdly, he found that,

  "Researchers studying participants of varied ethnic backgrounds tended to find a relationship similar to that found by researchers who studied participants with homogenous ethnic backgrounds. This result indicates that the ethnic formation of a sample does not bias the relationship." However, he insisted that, *"more studies are needed to examine the relationship from the racial-ethnic perspective before one can conclude that measures of anxiety towards mathematics may predict the level of mathematics achievement equally well across ethnic groups"*.

Xin Ma also concluded that social and academic characteristics of students could play a key role in unfolding the anxiety-achievement dynamics in mathematics. According to Cemen (1987) & Zaslavsky (1994), these characteristics can be classified as:

(1) Personal (e.g. gender, age, ethnicity, and social class)
(2) Environmental (e.g. social stereotypes, mathematics experiences and parental encouragement)
(3) Dispositional (e.g. attitude, confidence and self- esteem)
(4) Situational (e.g. classroom factors, instructional format, and curriculum factors)

Xin Ma (1997) pointed out the following important issues about the *outliers* (data which is abnormally high or low)***:***

- In this meta-analysis, he referred to the results presented by Bush (1991), "*who found a positive, significant relationship between mathematics anxiety and mathematics achievement and argued that mathematics anxiety tends to rise in students whose mathematics performance is improving"* and gave a reason for this contradiction: Bush probably used a sample of students who were extremely interested in mathematics or either gifted student who knew the importance of mathematics in career building. According to Cemen (1987), such students are able to control their anxiety and channel it into the given task. Their strong self-esteem and high levels of task related confidence help them in this regard. Cemen further explained that once this control takes place, their anxiety facilitates their performance. Xin Ma described that certain level of mathematics anxiety can benefit the mathematics performance of this special group of gifted students.

- Ma gave another example to illustrate the dynamic nature of the relationship between anxiety towards mathematics and achievement in mathematics by referring the results presented by Rensick *et al,* 1982 who found that *"a decrease in mathematics anxiety is not associated with improvement in mathematics performance".* According to Ma the reason for such result was the sample size of college students with extensive mathematics background and the level of their mathematics anxiety was limited but their achievement level might be very high. He further described that: *mathematics anxiety can facilitate mathematics performance, can debilitate mathematics performance or cannot be associated with mathematics performance.*

Xin Ma (1997, pg 521) discusses the work of Cemen (1987) who described in her model that negative mathematics experience, lack of parental encouragement, lack of confidence, classroom environment, instructional format-all these factors may produce anxiety towards mathematics.

There is almost no work which has explored factors that may cause anxiety towards mathematics or the affective factors that can help to reduce it. However, many studies (Armstrong; 1985; Betz, 1978; Brush, 1978; Burton, 1979; Donady & Tobias, 1977; Hendel, 1980; Preston, 1986/1987; Richardson & Suinn, 1972) have described the consequences of being anxious towards mathematics such as inability to do mathematics, the decline in mathematics performance, the avoidance of mathematics courses, the limitation in selecting college majors and future careers, and the negative feeling of guilt.

5.9 Mathematics in Scotland

It is useful to look at country like Scotland where many of the problems associated with mathematics education at school level are much less apparent: mathematics is chosen by many at both senior school and university levels (Humes, 1999).

Scotland has had a long history of universal school education (dating from about 1560). Scotland also had four universities by the early 16th century - open to all. Overall, Scottish education is perhaps the earliest universal education system while the number of universities for such a small population was abnormally high by European standards. Thus, education held high status. Mathematics was an integral part of this from the start.

The system has generated a small number of leading intellects who have offered to the world great insights. Many of these arose from the fields of mathematics, the sciences and engineering as well as medicine. For example, Napier (1550-1617), a graduate of St Andrews University, invented logarithms, which transformed computation.

This has left a culture in Scotland where mathematics, the sciences, engineering and medicine are held in high regard. This can be seen in the uptake levels in Higher Grade Mathematics (used for university entry) and Scotland has never suffered from the common pattern where mathematics is unpopular and taken by increasingly few.

For secondary school education, there is a full teaching force in mathematics, all qualified and, indeed, often very well qualified. The university uptakes in mathematics are buoyant. Indeed, subjects dependent on mathematics (like the sciences) are also highly popular at school level and university numbers are high (figure 5.1).

Mathematics syllabuses at school level are of reasonable length and levels of difficulty. Major syllabus changes took place in the mid 1960s but not all of these were effective and were later adjusted to give curricula which are designed to educate in mathematics rather than just produce mathematicians: mathematics for 'everyone'. This has generated a population where mathematics is taken by large numbers to the final stages of schooling, where there is no great population of mathophobes, and where large numbers choose to go on to take degrees very successfully in mathematics.

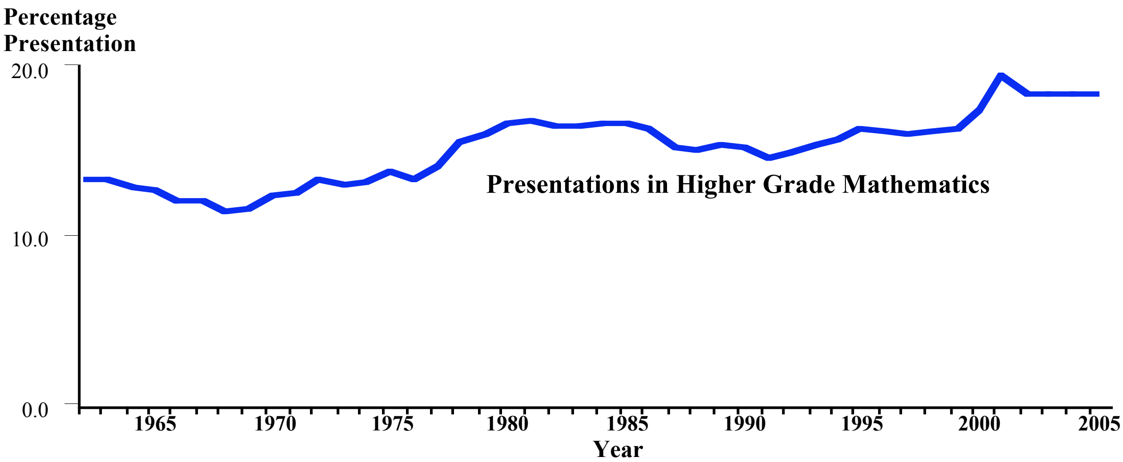

Figure 5.1 Uptake in Mathematics at the Higher Grade in Scotland (Source: Scottish Qualifications Authority, 1962-1997)

By contrast, in England, there seems to be the view at the upper end of secondary that the reason for taking mathematics is to become a mathematician or to choose a career to some degree heavily dependent on mathematics (Stewart *et al,* 2007). In Scotland, the reason for taking mathematics at the upper end of secondary is that mathematics is seen as an integral part of overall education. This over-simplified picture illustrates the problem. Mathematics syllabuses are designed by mathematicians. By definition, they have more than passing commitment to mathematics. This can often lead to the syllabus reflecting the needs of those committed to mathematics and not to the general educational benefit of all.

The university entrance examinations in Scotland (known as Higher Grades) show the consistent popularity of mathematics. In fact, all four science subjects are very popular there and no other subjects, except English which is more or less compulsory, come near them in the numbers uptake (see figure 5.2).

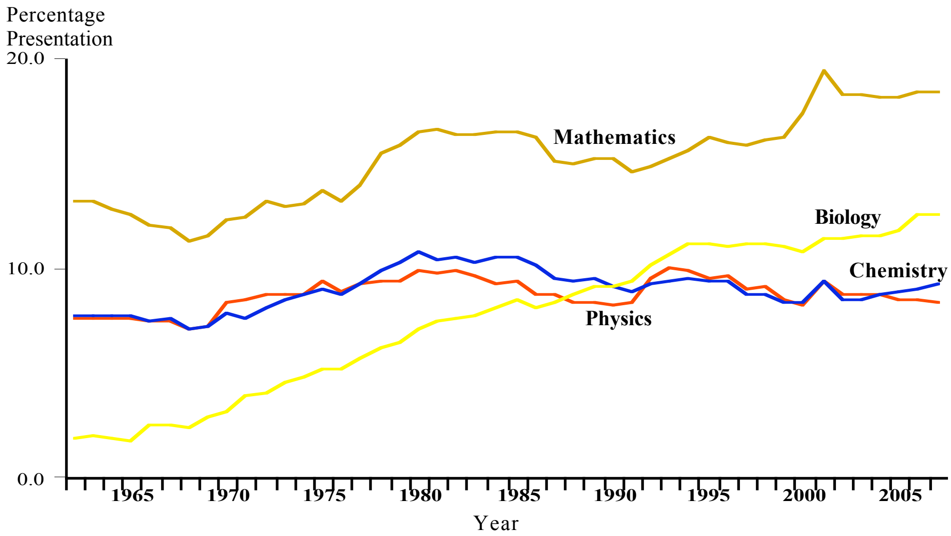

Figure 5.2 Mathematics and the Sciences at the Higher Grade (Source: Scottish Qualifications Authority, 1962-1997)

5.10 Some Conclusions

In thinking of mathematics at school level, it is important to consider the attitudes of the learners in relation to mathematics and their studies. The general pattern is that attitudes in relation to studies in mathematics tend to deteriorate with age . At the same time, the mathematics to be learned becomes more demanding. It is highly likely that increasing difficulties will bring about poorer performance, with concomitant attitude decline.

Many factors can encourage the development of more positive attitudes. Perhaps the most important influence lies with the teacher. In order to be able to cope more easily with increasing difficulty, the role of the teacher in providing support and encouragement may be critical, especially for the girls. However, there are few gender differences which are gender related.

Of course, it is important that the mathematics curriculum is set at an appropriate level and is not excessively demanding, known difficulties in mathematics education form the focus of the next chapter.

Chapter 6

Difficulties in Mathematics

Many researchers have tried to investigate the areas of difficulty in mathematics. Several studies have identified topics and approaches that are causing problems for the learners in learning mathematics. This chapter seeks to summarise some of the main findings related to learning difficulties in mathematics with special emphasis on primary school stages.

6.1 Difficulties in Arithmetic:

Harries and Barmby (2007) in their study investigated the importance of *representations* with respect to the comprehension of multiplication by primary school pupils. They looked at the use of array representation for reasoning (see appendix 3) with and understanding of multiplication. Harrie and Patrick (2006) carried out a preliminary work (examining year 4 and year 6 pupils) on array representation for multiplication calculations, and using a novel methodological approach of recording children's workings on a computer, they observed and suggested a few important points which are summarised below:

- The representation can provide a focus for discussions and reasoning of the calculation being considered;
- The array can provide a representation of multiplication that not only emphasises the replication element of multiplication, but can also be used to aid the calculations of the total;
- Pupils could be imaginative in their use of the array to calculate the total number of counters.
- The distributive property of multiplication can be used to perform the calculation and this could be an important development point.
- They also stressed on further work to gain better understanding of how pupils reason with a representation and how they make sense of the array in the context of multiplication.

Usually textbooks present multiplication as merely a quicker way of doing repeated addition but Clark and Kamii (1996) argued that multiplicative thinking is clearly distinguishable from additive thinking and that multiplicative thinking appears early and develops very slowly. They also offered a pedagogical implication of their finding that children should be allowed to solve multiplication problems in their own ways. Some children will solve them with multiplication while others will use addition to solve the same problem. Children should be presented challenging problems and encouraged to compare different ways in which the same problem can be solved. After interviewing 336 pupils in grade 1-5 individually and using a Piagetian task to study their development from additive to multiplicative thinking, they found that the introduction of multiplication is appropriate in second grade, but the educators must not expect all children to use multiplication, even in fifth grade. They also concluded that, before introducing multiplication, we must understand the nature of multiplicative thinking must be understood. It is also important to determine carefully at what grade level children can be expected to understand multiplication or any other topic in mathematics.

Teaching and learning of the fundamental concepts of fractions usually have been a problem at primary level. Mostly students' understanding is not more than the knowledge of a procedure or formula application to solve problems but it is also not rich in connection among symbols, models, pictures, and context (Kouba *et al.,* 1997). According to Moss and Case (1999), difficulties in understanding fractions are partially related to teaching methodologies that focus on syntactic knowledge (rules or formulas) over semantic knowledge (meaningful concept).

Cramer *et al.* (2002) presented a study that contrasted the achievement of students using either commercial curricula (CC) for initial fraction learning with the achievement of students using the Rational Number Project (RNP) fractions curriculum. The RNP focuses on the use of multiple physical models and translations within and between modes of presentation e.g. pictorial, manipulative, verbal, real-world and symbolic. During a 28-30 days instructional programme 1600 students of grade 4 and grade 5 were involved. The data reflected, "*great differences in the quality of students' thinking as they solved order and estimation tasks conceptually by building on their constructed mental images of fractions, whereas CC students relied more often on standard, often rote, procedures when solving identical fractions tasks".* These results were consistent with earlier RNP work with smaller sample size in several teaching experiment settings.

Division of fractions is considered one of the most mechanical and least understood topics in primary school (Fendal, 1987; Payen, 1976). Pupils are mostly not very successful in various tasks related to this topic (Carpenter,1988; Hart, 1981).

According to Tirosh (2000, pg. 7), the literature shows that children mostly make the following three mistakes when solving the problems related to division of fractions.

(1) Algorithmically based mistake
(2) Intuitively based mistake
(3) Mistakes based on formal knowledge

Tirosh (2000) in his research presented and discussed, *"an attempt to promote development of prospective elementary teachers' own subject-matter knowledge of division of fractions as well as their awareness of the nature and the likely sources of related common misconceptions held by children."* Her data showed that most participants knew how to divide fractions but could not explain the procedure. The prospective teachers were not completely aware of major sources of students' incorrect responses in this domain . She also suggested that the teacher education programs should attempt to familiarise prospective teachers with common, sometimes erroneous, cognitive processes used by students in dividing fractions and the effects of use of such processes.

Arithmetic word problems are an important part of the curriculum at elementary level. The main reason for introducing these word problems is to help pupils to apply, in daily life, the formal mathematical knowledge and skills learned at school. However, it has been argued for several years in research that extensive experiences with traditional arithmetic word problems induces in students a strong tendency to approach word problems in a mindless, superficial, routine-based way in their struggle to identify the correct arithmetic operation needed to get the right solution (Mulligan *et.al.,* 1997)**.** Verschaffel *et al.,* (1999) tried to collect in a systematic way empirical data about the scope and the nature of upper elementary school pupils' difficulties with modelling and solving non routine additive word problems. They emphasised only those problems in which straightforward addition or subtraction of the 2 given numbers yields either 1 more or 1 less than the correct answer. 99 pupils from six grade 5 classes and 100 pupils from six grade 6 classes from five elementary schools were involved and they had great difficulties in solving these problems, with various shortcomings underlying these difficulties. The study showed that many errors resulted from the superficial, stereotyped approach of adding or subtracting the 2 given numbers without considering the appropriateness of that action in relation to the problem context. It also showed that some other errors had different origins, such as misconceptions about numbers and arithmetic operations.

Rudnitsky *et al*., (1995) presented a study in which they tried to design and field-test instruction intended to help students construct knowledge about addition and subtraction word problems and determine if this knowledge would transfer to actually solving problems. The study also tested the following two related hypotheses:

(a) *Structure-plus- writing instruction will result in improved word-problem solving,*
(b) *This improvement will be more enduring than that resulting from a more traditional heuristic and practice-based approach.*"

To verify these hypotheses, 401 third-grade and fourth-grade students from 21 class-rooms in six schools were involved in a study in which the problem solving of children taught by a structure-plus-writing approach was compared to that of,

(a) A control group receiving no explicit instruction in arithmetic word-problem solving
(b) A group receiving instruction based largely on practice and explicit heuristics

Both hypotheses were supported by the results. The structure-plus-writing group performed better than the group receiving practice and explicit heuristics instruction. Moreover, the structure-plus-writing group not only performed better but also, "*actually widened the gap, as shown by retention test results 10 weeks after treatment. The effectiveness of instruction based on a structured approach to authoring arithmetic word problems was strongly supported.*"

Mokros and Russell (1995) tried to understand the characteristics of fourth to eighth graders' constructions of "average" as a representative number summarising a data set. They described that the issue of mathematical representativeness arises when we need to describe a set of data in a succinct way. 21 primary level students were interviewed, using a series of open-ended problems and were asked to construct their own notion of representativeness. They identified and analysed the following five basic constructions of representativeness and these approaches demonstrate the ways in which students are (or are not) developing useful, general definitions for the statistical concept of average. These five basic constructions are shown below [pg. 26]:

Average as the mode (The value or item occurring most frequently in a series of observations or statistical data)

Students with this predominant approach-

* *consistently use mode to construct a distribution or interpret an existing one;*
* *lack flexibility in choosing strategies;*
* *are unable to build a distribution when not allowed to use the given average as a data point;*
* *use the algorithm for finding the mean infrequently or incorrectly;*
* *view the mode only as "the most," not as representative of the data set as a whole;*
* *frequently use egocentric reasoning in their solutions.*

Average as algorithm

Students with this predominant approach-

* *view finding an average as carrying out the school-learned procedure for finding the arithmetic mean;*
* *often exhibit a variety of useless and circular strategies that confuse total, average, and data;*
* *have limited strategies for determining the reasonableness of their solutions.*

Average as reasonable

Students with this predominant approach-

* *view an average as a tool for making sense of the data;*
* *choose an average that is representative of the data, both from a mathematical perspective and from a common-sense perspective;*
* *use their real-life experiences to judge if an average is reasonable;*
* *may use the algorithm for finding the mean; if so, the result of the calculation is scrutinized for reasonableness;*
* *believe that the mean of a particular data set is not one precise mathematical value, but an approximation that can have one of several values.*

Average as midpoint

Students with this predominant approach-

* *view an average as a tool for making sense of the data;*
* *choose an average that is representative of the data, both from a mathematical perspective and from a common-sense perspective;*
* *look for a "middle" to represent a set of data; this middle is alternately defined as the median, the middle of the X axis, or the middle of the range;*
* *use symmetry when constructing a data distribution around the average. They show great fluency in constructing a data set when symmetry is allowed but have significant trouble constructing or interpreting nonsymmetrical distributions;*
* *use the mean fluently as a way to "check" answers. They seem to believe the mean and middle are basically equivalent measures.*

Average as mathematical point of balance

Students with this predominant approach-

* *view an average as a tool for making sense of the data;*
* *look for a point of balance to represent the data;*
* *take into account the values of all the data points;*
* *use the mean with a beginning understanding of the quantitative relationships among data, total, and average; they are able to work from a given average to data, from a given average to total, from a given total to data.*
* *break problems into smaller parts and find "submeans" as a way to solve more difficult averaging problems.*

Irwin (2001) tried to investigate the role of students' everyday knowledge of decimals in supporting the development of their knowledge of decimals. He stressed that decimal fractions need to be anchored in some way to students' existing knowledge. 16 students of age 11 and 12, from a lower economic area, were involved in pair work (one member of each pair a more able student and one less able student) and were asked to solve problems that tapped into common misconceptions about decimal fractions. Half the pairs worked on problems presented in familiar contexts and half worked on problems without the context.

The results revealed that children who worked on contextual problems made a significant progress in their knowledge of decimals as compared to those who worked on non contextuals problems. It was postulated that the students who performed well in solving contextual problems were able to integrate everyday knowledge with understandings of decimals from school mathematics. They did this by reflecting on their routine life knowledge as it pointed to the meaning of decimal numbers and the results of decimal calculations. They believed that integrating Vygotsky's (1987) and Wardekker's (1998) theory on the transition of everyday life to scientific or scholastic knowledge with that of Piaget (1932/1965) on the conditions in which peer collaboration leads to learning may lead to a complete understanding of why this intervention was successful. They also suggested that the teachers of lower income and diverse classrooms should be aware of students' everyday knowledge and any misconceptions developed on the way to achieving scientific knowledge .

Ratio and proportion are important concepts in mathematics curricula at primary level. According to Lo (1997), the concepts of ratio and proportion do not develop in isolation and are part of the individual's multiplicative conceptual field. This field includes other concepts such as multiplication, division, and rational numbers. Vergnaud (1988, pg.1) used the term "*multiplicative conceptual field*" to refer to all the situations that can be analysed as simple or multiple proportion problems. In a single case study, she tried to clarify the beginning of this development process. Through several sessions, two main teaching goals were set. The first goal was to build a deeper understanding of number structures such as multiples and divisors, and this knowledge was essential to develop a strategy for identifying the ratio unit in her ratio-unit/ build up method systematically. The second goal was to help the student construct and integrate his knowledge of division. Also, special attention was paid to the student's mental images that might have affected his choice of strategies (Lo, 1997, pg: 217).

It was found that limited understanding of multiplication, division, fraction and decimals could be considered the roots of students' difficulties in developing ratio and proportion concepts. The findings also supported the view that the development of ratio and proportion is embedded within the development of the multiplicative conceptual fields. The study also showed that the process of extending the meanings of multiplication and division operations from single-digit numbers to multidigit numbers should be taken into account seriously. She suggested that students should be provided more experience with tasks involving geometry and measurement, because both provide rich context for developing concepts of numbers and operations at all grade levels. [Lo, 1997, pg: 234].

Mulligan and Mitchelmore (1997) defined an intuitive model as an internal mental structure corresponding to a class of calculation strategies. Their study was an extension of previous research on young children's intuitive models of whole-number multiplication and division by widening the range of semantic structures included and by including the effect of maturation. They observed the sample of female students (Grades 2 and 3) four times as they solved the same set of 24 word problems. From the correct responses, 12 distinct calculation strategies were identified and grouped into categories from which the children's intuitive models of multiplication and division were inferred. They found that:

> *"The students used 3 main intuitive models: direct counting, repeated addition, and multiplicative operation. A fourth model, repeated subtraction, only occurred in division problems. All the intuitive models were used with all semantic structures, their frequency varying as a complex interaction of age, size of numbers, language, and semantic structure. The results are interpreted as showing that children acquire an expanding repertoire of intuitive models and that the model they employ to solve any particular problem reflects the mathematical structure they impose on it."*

Lucangeli *et al.* 1998, tried to investigate how to validate an economic model of the cognitive and metacognitive abilities involved in mathematical word problems. They chose the following seven abilities and to measure them they devised a series of mathematical word problems for 64 students from grade 3; 78 students from grade 4; 68 students from grade 5; 67 students from grade 6 and 87 students from grade 7:

1. *Text comprehension*
2. *Problem Representation*
3. *Problem Categorization*
4. *Solution Estimate*
5. *Planning the Solution*
6. *Procedure Self-Evaluation*
7. *Calculus Self- Evaluation*

They suggested that five of these are important: The semantic comprehension of the relevant information in the text, the capacity to have a good visual representation of the data, the capacity to recognise the deep structure of the problem, the capacity to order correctly the steps to arrive at the solution and a good capacity to evaluate the procedure utilised in the solution. They also concluded that the model obtained by them may be utilised in order to devise practical instruments for the analysis of mathematical word problem difficulties.

6.2 Difficulties in Geometry

Kato *et al.* (2002) considered the idea that one should build on a child's existing ideas and tried to investigate the relationship between abstraction and representation. They interviewed sixty Japanese children between the ages of 3 years 4 months and 7 years 5 months to investigate the relationship between their levels of abstraction (as assessed by a task involving conservation of number) and their levels of representation (as assessed by a task asking for their graphic representation of small groups of objects). They concluded that "*abstraction and representation are closely related and that children can represent at or below their level of abstraction but not above this level.*"

Clements *et al.* (1999) investigated criteria pre-school children use to recognise members of a class of shapes from other figures. They based their study on three dominant lines of inquiry presented by the theories of Piaget, van Hieles, and cognitive psychologists (Clements & Battista, 1992b). They carried out individual interviews of 97 children ages 3 to 6, emphasizing identification and descriptions of shapes and reasons for these identifications. They concluded that, "*young children initially form schemas on the basis of feature analysis of visual forms. While these schemas are developing, children continue to rely primarily on visual matching to distinguish shapes*." They also found that these children were capable of recognising components and simple properties of familiar shapes. According to them, their results supported previous claims (Clements and Battista, 1992b) that:

> *"A prerecognitive level exists before van Hiele Level 1 ("visual level") and that Level 1 should be reconceptualized as syncretic (i.e., a synthesis of verbal declarative and imagistic knowledge, each interacting with the other) instead of visual."*
>
> (Clements, 1992)

They also stressed that more research is needed to identify the specific, original intuitions and ideas that young children develop about geometric figures. They designed their study to investigate the geometric concepts formed by young children. In their study, they raised the following critical questions and tried to find out the answers (Clements and Battista, 1992b, pg 194):

> *"What criteria do pre-school children use to distinguish members of a class of shapes (e.g., circles or triangles) from other figures?*
> *Do they use criteria in a consistent manner?*
> *Are the content, complexity, and stability of these criteria related to age or gender? What implications do findings have for theoretical descriptions of children's geometric thinking?*"

Friel *et al.* (2001) tried to bring together perspectives concerning the processing and use of statistical graphs to identify critical factors that appear to influence graph comprehension and suggested instructional implications. After providing a synthesis of information about the nature and structure of graphs, they defined graph comprehension. They described the following four critical factors that may affect graph comprehension:

- *The purposes of using graphs*
- *Task characteristics,*
- *Discipline characteristics*
- *Characteristics of graph readers"*

They also defined a construct called '*graph sense*' and suggested a sequence for ordering the introduction of graphs. They concluded with a discussion of issues involved in making sense of quantitative information using graphs and ways instruction may be modified to promote such sense making.

Watson and Moritz (2000) described ways to develop the concepts of sampling. They explained that a key element in developing ideas associated with statistical inference involves developing concepts of sampling. They tried to investigate and understand the characteristics of students' constructions of the concept of sample. By using open-ended questions related to sampling, they interviewed sixty-two students in Grades 3, 6, and 9; written responses to a questionnaire were also analysed. They identified and described the following six categories of construction in relation to the sophistication of developing concepts of sampling These categories present helpful and unhelpful foundations for an appropriate understanding of representativeness and hence will help curriculum developers and teachers plan interventions. Their six categories of sampling suggested by them are listed below:

Small Samplers Without Selection:

* *may provide examples of samples, such as food products*
* *may describe a sample as a small bit or, more rarely, as a try or test*
* *agree to a sample size of fewer than 15*
* *suggest no method of selection or an idiosyncratic method*

Small Samplers With Primitive Random Selection:

* *provide examples of samples, such as food products*
* *describe a sample as either a small bit or a try or test*
* *agree to a sample size of fewer than 15*
* *suggest selection by random without description or with a simple instruction to choose "any," perhaps from different schools*

Small Samplers With Preselection of Results:

* *provide examples of samples, such as food products*
* *describe a sample as both a small bit and a try or test*
* *agree to a sample size of fewer than 15*
* *suggest selection of people by weight, either a spread of fat and skinny or people of normal weight*

Large Samplers With Random or Distributed Selection :

* *provide examples of samples, such as food products*
* *describe a sample as both a small bit and a try or test*
* *may refer to term average*
* *suggest a sample size of at least 20 or a percentage of the population*
* *suggest selection based on a random process or distribution by geography*

Large Samplers Sensitive to Bias:

* *provide examples of samples, sometimes involving surveying*
* *describe a sample as both a small bit and a try or test*
* *may refer to terms average or representative*
* *suggest a sample size of at least 20 or a percentage of the population*
* *suggest selection based on a random process or distribution by geography*
* *express concern for selection of samples to avoid bias*
* *identify biased samples in newspaper articles reporting on results of surveys*

Equivocal Samplers:

* *provide examples and descriptions of samples*
* *may indicate indifference about sample size, sometimes based on irrelevant aspects*
* *may base decisions on small size with appropriate selection methods or with partial sensitivity to bias, or base decisions on large sample size with inappropriate selection methods*

Watson and Moritz (2000, pg. 54)

Geometry can help learners to develop spatial thinking and visualisation skills and plays a key role in developing their ability in deductive reasoning and proving (Battista, 2007; Royal Society, 2001). The notion of definitions is very important in this regard as definitions assign properties to mathematical objects. According to Henri Poincaré (1914, pg. 452), to make learners understand the definition, it is important to show the object defined and also the neighbouring objects as well from which it has to be “distinguished”.

Fujita and Jones (2007) tried to describe the findings concerning learners' knowledge of the definition, and classification relationship between, quadrilaterals and, also to suggest a theoretical framing that is intended to inform further analyses in this research area. Fujita and Jones (2007) noted that defining and classifying quadrilaterals, though an established component of the school mathematics curriculum, appears to be a difficult topic for many learners. They explained that the reasons for such difficulties relate to the complexities in learning to analyse the attributes of different quadrilaterals and to distinguish between critical and non-critical aspects. They further discussed that such learning, if it is to be effective, requires logical deduction, together with suitable interactions between concepts and images. With the sample of 263 learners, the main purpose of their study was to present a theoretical framing that is intended to inform further studies of this important topic within mathematics education research. This theoretical framing relates prototype phenomenon and implicit models to common cognitive paths in the understanding of the relationship between quadrilaterals.

Outhred and Mitchelmore (2000) discussed the strategies young children use to solve rectangular covering tasks before they have been taught area measurement. 115 children from Grades 1 to 4 were given various array-based tasks, and their drawings were analysed. In their study, five developmental levels in the strategies young children use to solve rectangular covering tasks were identified and it was argued that these levels show the successive acquisition of four basic principles: complete covering, spatial structure, size relations, and multiplicative structure. These principles constitute children's intuitive understanding of area measurement.

They also highlighted the importance of understanding the relation between the size of the unit and the dimensions of the rectangle in learning about rectangular covering and a good understanding of linear measurement, without which children are unlikely to learn the relation between unit size and rectangle dimensions. They explained the role of multiplication and identified the significance of a relational understanding of length measurement. Implications for the learning of area measurement are addressed. They also described the significance of the formation of an *iterable* row as the foundation of an understanding of array structure and also identified the significance of the relation between the size of the unit and the dimensions of the rectangle.

6.3 What is the *Origin* of these Difficulties?

The research studies in the previous section have discussed the prevalence of several difficulties in mathematics, focussing especially on the younger learners. The results vary according to the nature of the samples, and criteria used. The main conclusion gained from most studies is that many children have difficulties with mathematics, and a significant number have relatively specific difficulties with mathematics. Such difficulties appear to be equally common in boys and girls. The real question here is what is the ***origin*** of these difficulties? What is wrong and where? Looking at the learning processes can offer some insights.

The difficulties in certain areas of mathematics are so widespread and occur across cultures, curricula and methodologies. Therefore, it is unlikely that it is caused by the teachers. Firstly, the teachers do not decide curriculum. They are supposed to follow the syllabus, textbooks and other resources provided by the authorities and they also do not have much freedom to make any amendments or changes in the provided curriculum. This fact is true for several countries e.g. in Pakistan, where two distinct mediums of instruction are in system: English medium and Urdu medium. In Urdu medium schools, the government authorities in the Education Department (Text Book Board) decide the curriculum and textbooks and teachers have to teach what is prescribed. In English medium schools, conditions are better and teachers have some freedom to present suggestions.

Teaching approaches, teaching strategies and order of topics are usually determined by the logic of mathematics as a discipline. Educationalists usually determine what they see as the logical order for courses in mathematics. Therefore, it is likely that the problem arises because topics are presented at the wrong time, or by the very nature of mathematics itself.
Mathematics curricula are rarely determined by practising teachers. Inevitably, mathematics curricula are planned and designed by those who themselves are mathematicians and who are committed to mathematics. These are people who have been successful in the study of mathematics and yet the curriculum they design has to serve the needs of the whole population, including those for whom mathematics may be neither attractive nor easy. Inevitably, such curricula are designed around the logic of mathematics and the needs of those who will use or depend on mathematics later in life. All of this can result in material being included which poses problems for learners, and the teachers are then faced with developing ways to teach material which is not readily accessible to the pupils at that age (Al-Enezi, 2008). Further problems can occur with assessment. Teachers do not decide national certification and yet teachers may be criticised if their pupils are not successful enough. This can lead to a dependence on the memorisation of procedures, understanding being a casualty.

A major source of the problem lies in the nature of mathematics and learning. Mathematics learning might involve four processes (figure 6.2).

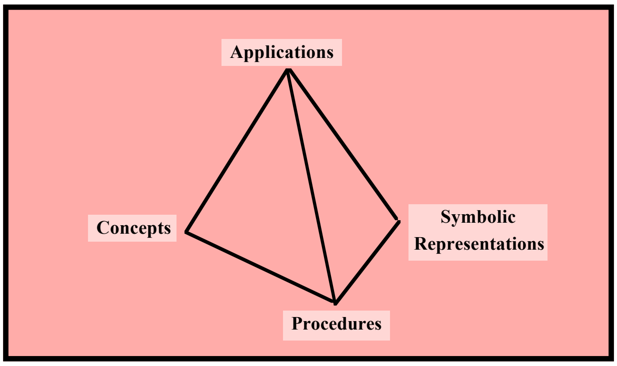

Figure 6.1 The Mathematics Tetrahedron

The work of Piaget (1962) has established the young learner as a person who is trying to make sense of what is experienced. In mathematics, trying to master the processes and symbolism may well create enough pressure on limited working memory capacity. The learner cannot cope with concept (understanding), procedure, symbolism and application all at the same time.

Adding on the 'making sense of' dimension will almost certainly generate overload and yet it is this dimension, which is the natural way of learning. Ausubel (1968) talks of meaningful learning where what is understood is 'internalised'. This is critical but the limitations of working memory may make such understanding and internalisation very difficult. There are good arguments, therefore, for making the mathematics taught meaningful. There are also good arguments that this may prove very difficult.

The key has to be in establishing confidence and competence in the processes and symbolisms at one point in time and, later, adding on the understanding. In this way, the limitations imposed by working memory may be reduced. Thus, for example, some mathematical procedure is taught and then mastered by numerous examples before, perhaps during the next lesson, the teacher says something like, 'Let's think about what you can now do - what does it mean and how can be use it?' With the procedure more or less automated, enough working memory space is now available to think about its meaning and significance.

6.4 Assessment

The powerful influence of assessment on the learners' attitudes cannot be underestimated. The difficulty is that national assessment is determined outside the schools and often loses touch with the realities of the learning situation. Too often, assessment focusses on what is easy to assess. This usually over-emphasises recall of information or procedures. The recent work of Hindal (2007) shows the devastating power of assessment in reducing almost everything to a recall-recognition measurement exercise while the effects in relation to attitudes has been discussed on pages 81-82.

The aims of mathematics education must involve conceptual understanding as well as the ability to be able to use and apply the mathematics learned in new situations. These are very difficult skills to develop and, if the assessment system gives little reward for such skills, the temptation is to concentrate on the correct use of learned procedures in routine situations. It will not be easy to resolve such problems but they must firstly be recognised and then research carried out to find ways to assess conceptual understanding and application.

Chapter 7

The Experiments Conducted

7.1 Introduction

The aim is to explore aspects of mathematics learning experiences in grades 5 to 7 (ages approximately 10-12) in both Urdu medium and English medium schools. This involves looking at pupil perceptions of their experiences, the nature of the difficulties they have with mathematics and possible reasons for these difficulties.

To achieve this, a short survey was used with pupils in grades 5, 6 and 7, drawn for both systems. This survey not only looked at the pupil perceptions related to their experiences in mathematics but also surveyed topic areas to see where they were having difficulties. For grade 5, working memory capacity was measured and their mathematics marks were gained.

7.2 Samples Used

Samples of pupils were drawn from a range of Urdu medium schools in the Lahore area of the country, involving schools which are typical of schools in Pakistan. The schools drew from a diversity of areas and social backgrounds. For English medium schools, two boys schools were selected, girls schools not being readily available. The samples are summarised in table 7.1.

Grade	Urdu Medium		English Medium	Totals
	Girls	*Boys*	*Boys*	
5	74	76	150	**300**
6	33	115	106	**254**
7	107	51	101	**259**
Totals	**214**	**242**	**375**	**831**

Table 7.1 Samples Chosen

7.3 The Survey Employed

A list of all the aspects of mathematics was drawn up and questions were developed to explore these. Lists of topics in the mathematics curriculum for each year group for each language system were drawn up. The draft survey was considered by experienced mathematics teachers and minor amendments incorporated. The survey was translated into Urdu and the translation checked. The survey for grade 5, English medium, is shown below. The other surveys were identical except for the list of topics covered in question 9. All six surveys are shown in full in the appendix.

What do you think about Mathematics ?

(1) What is your opinion about the **subject mathematics?**
*Tick **one** box on each line*

I like mathematics	☐ ☐ ☐ ☐ ☐ ☐	I do not like mathematics
Useful in daily life	☐ ☐ ☐ ☐ ☐ ☐	Useless in daily life
Easy to understand	☐ ☐ ☐ ☐ ☐ ☐	Difficult to understand
Boring subject	☐ ☐ ☐ ☐ ☐ ☐	Interesting subject
I do not want to learn it but it is a compulsory subject	☐ ☐ ☐ ☐ ☐ ☐	I want to learn it because I enjoy it.

(2) What is your opinion about **mathematics lessons?**
*Tick **one** box on each line*

Easy lessons	☐ ☐ ☐ ☐ ☐ ☐	Difficult lessons
I understand my lessons completely	☐ ☐ ☐ ☐ ☐ ☐	I do not understand my lessons completely
I like the way my teacher explains the methods	☐ ☐ ☐ ☐ ☐ ☐	I do not like the way my teacher explains the methods.
I just memorise the procedures in class	☐ ☐ ☐ ☐ ☐ ☐	I actually understand the procedures in class
I do not like doing too much class work daily	☐ ☐ ☐ ☐ ☐ ☐	I enjoy doing my class work daily
I revise my lessons regularly	☐ ☐ ☐ ☐ ☐ ☐	I revise them just before the exam or a test.

(3) How do you feel **yourself** in your **mathematics course** at school?

I feel I am trying hard to do well in mathematics	☐ ☐ ☐ ☐ ☐ ☐	It is my fault I cannot study mathematics well.
I hate homework because I can't do it on my own	☐ ☐ ☐ ☐ ☐ ☐	I enjoy homework because I can do it on my own
I am getting better at the subject	☐ ☐ ☐ ☐ ☐ ☐	I am getting worse at the subject.
Enough revision at school to help me understand well	☐ ☐ ☐ ☐ ☐ ☐	Not enough revision at school ... me understand well
I understand at school with little extra help at home	☐ ☐ ☐ ☐ ☐ ☐	I understand at school only with extra help from home

(4) **Imagine** you have **problem** in understanding a **new** topic or concept. What is your likely reaction?
Tick as many boxes as you wish.

☐ Start to panic	☐ See it as a challenge	☐ Seek help from my teacher
☐ Seek help from my tutor	☐ No worries, I will understand it with time.	☐ Seek help from a family member

(5) Here are some descriptions of the way students **approach** mathematics.
*Tick **one** box on each line.*

	strongly agree	*agree*	*not sure*	*disagree*	*strongly disagree*
Revision sheets help me to understand mathematics well	☐	☐	☐	☐	☐
Diagram and pictures help me to understand mathematics well	☐	☐	☐	☐	☐
I can understand the main points easily.	☐	☐	☐	☐	☐
I think mathematics help me in daily life a lot.	☐	☐	☐	☐	☐
I do not want to learn mathematics but it is compulsory	☐	☐	☐	☐	☐
I enjoy studying mathematics classes	☐	☐	☐	☐	☐
I work hard in mathematics but cannot get good marks in exam	☐	☐	☐	☐	☐

(6) Would you like to learn **more** mathematics in **next classes?**

Tick either 'yes' or 'no' and give a reason.

☐ Yes, because

☐ No, because

(7) I like mathematics thanks to :
*Tick **as many** boxes as you wish*

☐ My parents ☐ My teacher ☐ My tutor ☐ Mathematics lessons
☐ Mathematics TV programs ☐ Computers ☐ My friends ☐ Easy/I am good at it
☐ Other - please show:

(8) Think about **examination/tests in mathematics** at your school.
*Tick **as many** you feel true for you*

☐ I tend to panic near the exam.
☐ I think there is enough revision at school before exams.
☐ I cannot do well in the paper because I study late night and feel sleepy.
☐ I like challenging questions in exam.
☐ I do not like lengthy questions because I can make more mistakes in them.
☐ I find it difficult to revise the whole year syllabus in final exam.
☐ The topics included in half yearly exams should not be included in final exams.
☐ I do not like short questions because it does not give me chance to express that how much I know
☐ I like fill in the blanks and true/false type questions in exam.
☐ I like questions with colourful pictures and diagrams.
☐ Sometimes I leave my questions incomplete because there is not enough space for solutions.
☐ I think that the allowed time limit is very short in mathematics paper.

(9) Think of the following topics in your **mathematics syllabus**.

Easy	I understood it first time
Moderate	I found it difficult but I understand it now
Difficult	I still do not understand it

Tick the **suitable** box for each topic to show if you find that topic, easy, moderate or difficult

	Easy	*Moderate*	*Difficult*
Writing numbers in words and figures	☐	☐	☐
Rounding off Integers	☐	☐	☐
Number Sequences	☐	☐	☐
Addition and Subtraction	☐	☐	☐
Multiplication	☐	☐	☐
Division	☐	☐	☐
Common Fractions	☐	☐	☐
Decimal Fractions	☐	☐	☐
Percentages	☐	☐	☐
Use of Brackets	☐	☐	☐
Even and Odd Numbers	☐	☐	☐
Problem Solving	☐	☐	☐
Organising and Using Data	☐	☐	☐
Shapes and Measures	☐	☐	☐
Rectangles	☐	☐	☐
Triangles	☐	☐	☐
Symmetry	☐	☐	☐
Angles	☐	☐	☐
Recognising Parallel / Perpendicular lines	☐	☐	☐
Length, Mass, Capacity measurements	☐	☐	☐
Area	☐	☐	☐
Perimeters of rectangles and regular polygons	☐	☐	☐
Time	☐	☐	☐

7.4 Measurement of Working Memory Capacity

To determine an individual's working memory capacity, the Figural Intersection Test, designed by Pascual-Leone (1970), was used. There are two sets of simple geometric shapes, the set of shapes on the right (the presentation set) and the other set of overlapping shapes on the left (the test set) of a page (Figure 7.1). The presentation set consists of a number of shapes separated from each other. The test set consists of the same shapes but overlapping, so that there exists a common area which is inside all the shapes of the presentation set. What the pupils have to do is to look for and shade-in the common area of intersection in the test set. In some items, a misleading irrelevant shape (not present in the presentation set) is included in the test set. Altogether, there are 20 items in the test, as shown in the appendix.

Test set **Presentation set**

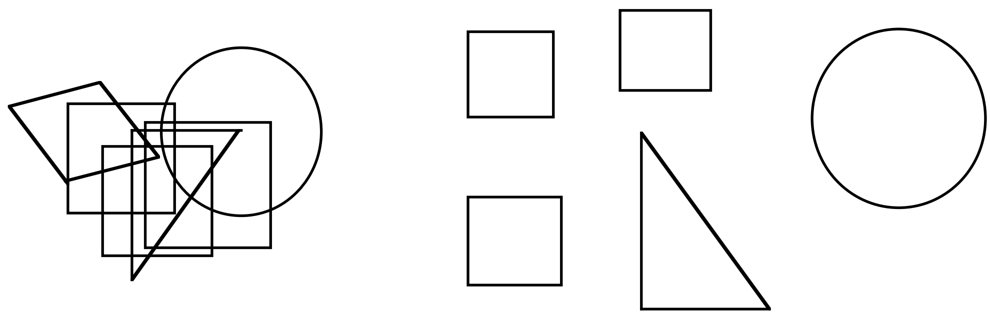

Figure 7.1 Example of Test Item

The number of shapes varies from 2 to 8. The number of shapes in the test set is equal to the score given if the item is marked correctly. The test is timed. In this test, as the number of shapes increased, the task became more complex. Every item has to be completed in about 20-25 seconds.

7.5 Data Analysis

The data from the surveys were entered on to a spreadsheet and the overall frequencies for each year group, for each language group, and for gender were obtained. Chi-Square ($\chi 2$), as a contingency test, was used for comparison of groups. The chi-square test is one of the most widely used non-parametric tests. It compares frequency distributions where the data do not form normal distributions, as in the survey here.

There are two different applications of the chi-square test. In the '*goodness-of-fit*' test, a frequency distribution is compared to a distribution from a control group. This was not used here. In this study, the chi-square as a *contingency test* was used, for example, to compare two or more independent samples: year groups, gender, or languages.

Where any category falls below a certain value, the value of the chi-square tends to be inflated, giving a potentially misleading result. In this study, a minimum of 10 or 5% was imposed (see Reid, 2003). Data were grouped where necessary. The chi-square values obtained were compared to those from chi-square tables to indicate significant differences. This involved the use of degrees of freedom. The degree of freedom (df) is stated for any calculated chi-square value. The value of the degree of freedom for any analysis is obtained from the following calculations:

$$\mathbf{df = (r\text{-}1) \times (c\text{-}1)}$$

*(where **r** is the number of rows and **c** is the number of columns in the contingency table)*

A more detailed description of the use of chi-square is in the appendix.

It frequently happens that two measurements relate to each other: a high value in one is associated with a high value in the other. The extent to which any two measurements are related in this way is shown by calculating the correlation coefficient. There are three ways of calculating a correlation coefficient, depending on the type of measurement:

(a) With integer data (like examination marks), Pearson correlation is used. This assumes an approximately normal distribution.
(b) With ordered data (like examination grades), Spearman correlation is used. This does not assume a normal distribution.
(c) With ordered data where there are only a small number of categories, Kendall's Tau-b correlation used. This does not assume a normal distribution.

In this study, any relationships involving survey questions were analysed using Kendall's Tau-b. Relationships between marks and working memory capacity were explored using Pearson correlation as these variables are integers with an approximately normal distribution.

Patterns of correlations can be analysed using factor analysis and this is described in chapter 12. With a large number of measurements, it is possible to calculate numerous correlation coefficients. It is possible that there is some underlying ‘reason’ for the pattern of correlations obtained. Factor analysis can be used to explore if any underlying structure exists. Factor analysis cannot indicate what the underlying factors (or components) are. This can only be done by value judgements, looking at the specific questions involved. The aim here was to see if there was any pattern which underlay the way attitudes related to mathematics develop and exist.

Chapter 8

Age and Attitudes Related to Mathematics
Urdu Medium Schools

8.1 Introduction

The questionnaire was used with three year groups drawn from typical Urdu medium schools (see table 8.1).

Grade	Sample	Girls	Boys
5	150	74	76
6	148	33	115
7	158	107	51
Total	**456**	**214**	**242**

Table 8.1 Sample Details

In addition, the working memory capacity of the grade 5 students was measured using the figural intersection test.

The aim was to see how attitudes developed with age in Urdu medium schools and to explore any relationships with working memory capacity. This chapter will only discuss the survey data.

All the data obtained were entered into a spreadsheet before analysis**.** The data for each item of each question will be presented as percentages for clarity. However, all statistical analyses were carried out using frequency data. Where there is a statistical significant difference between the age groups, this is shown with colour coding:

$p < 0.05$	*green*
$p < 0.01$	*amber*
$p < 0.001$	*red*

8.2 Data Analysis

(1) What is your opinion about the subject mathematics?										
	Grade	**%**						**χ2**	**df**	**p**
I like mathematics	5	74	19	0	0	1	5			
	6	43	8	8	12	11	20	89.1	4	$p < 0.001$
	7	82	7	3	1	3	4			
Useful in daily life	5	79	17	1	0	1	2			
	6	43	16	10	13	16	3	65.1	2	$p < 0.001$
	7	81	13	4	1	1	1			
Easy to understand	5	47	24	15	0	3	11			
	6	30	18	12	16	10	15	61.6	6	$p < 0.001$
	7	65	10	4	2	4	16			
Interesting subject	5	63	5	12	2	5	13			
	6	32	14	24	10	12	9	73.5	6	$p < 0.001$
	7	70	10	4	3	3	11			
I want to learn it because I enjoy it	5	63	15	1	1	4	17			
	6	40	11	12	14	16	8	24.6	2	$p < 0.001$
	7	63	6	8	4	2	17			

Table 8.2 Data Question 1

Three features stand out in the data in table 8.2. Firstly, grade 6 is responding in a very different way when compared to grades 5 and 7. As this sample is drawn from the same schools and the same teachers as the others, the differences must be caused by the curriculum in grade 6. Secondly, there is some polarisation of views in many of the items: a significant proportion are opting for the two extreme positions. Thirdly, for the majority, views are very positive.

(2) What is your opinion about mathematics lessons?										
	Grade	**%**						**χ2**	**df**	**p**
Easy	5	83	4	2	1	2	9			
	6	38	5	4	10	15	29	100.5	4	$p < 0.001$
	7	67	9	1	2	3	19			
I understand my lessons completely	5	56	28	3	4	1	8			
	6	31	15	11	18	14	12	81.2	6	$p < 0.001$
	7	64	9	3	3	8	13			
I like the way my teacher explains the methods	5	73	7	11	1	3	5			
	6	45	9	9	18	14	6	59.5	6	$p < 0.001$
	7	75	9	3	3	2	9			
I actually understand the procedures in class	5	68	2	13	5	3	9			
	6	30	14	19	13	11	14	26.0	4	$p < 0.001$
	7	67	10	4	2	5	11			
I enjoy doing my class work daily	5	57	19	2	2	9	11			
	6	30	8	11	18	16	17	76.8	8	$p < 0.001$
	7	50	11	2	6	6	25			
I revise my lessons regularly	5	64	7	3	1	5	21			
	6	43	15	9	11	10	13	45.3	6	$p < 0.001$
	7	69	6	4	3	3	16			

Table 8.3 Data Question 2

Table 8.3 shows the same three features: grade 6 is responding in a very different way, there is some polarisation of views in many of the items, views are very positive for the majority.

(3) How do you feel yourself in your mathematics course at school?	Grade	%						χ2	df	p
I feel I am trying hard to do well in mathematics	5	87	5	0	1	0	7			
	6	41	7	7	12	7	26	75.6	4	$p < 0.001$
	7	72	7	3	1	1	15			
I enjoy homework because I can do it on my own	5	37	13	1	3	21	26			
	6	20	16	14	12	14	25	68.6	8	$p < 0.001$
	7	41	10	1	3	11	35			
I am getting better at the subject	5	65	11	15	1	3	4			
	6	39	16	9	14	12	10	48.3	6	$p < 0.001$
	7	72	13	1	5	3	7			
Enough revision at school to help me understand well	5	48	9	11	12	14	5			
	6	34	18	14	17	9	9	49.4	10	$p < 0.001$
	7	63	8	6	4	5	14			
I understand at school with little extra help at home	5	49	9	5	9	10	18			
	6	34	19	7	12	11	18	35.0	6	$p < 0.001$
	7	61	7	3	1	8	21			

Table 8.4 Data Question 3

Table 8.4 shows the same three features: grade 6 is again responding in a very different way while there is some polarisation of opinion in many of the items. However, views are usually positive for the majority.

(4) Imagine you have problem in understanding a new topic or concept. What is your likely reaction?	Grade		χ2	df	p
Start to panic	5	54			
	6	55	0.8	2	n.s.
	7	50			
Seek help from my tutor	5	41			
	6	50	26.2	2	$p < 0.001$
	7	70			
See it as a challenge	5	77			
	6	63	6.8	2	$p < 0.05$
	7	68			
No worries, I will understand it with time.	5	47			
	6	68	28.6	2	$p < 0.001$
	7	76			
Seek help from my teacher	5	85			
	6	89	1.6	2	n.s.
	7	90			
Seek help from a family member	5	72			
	6	59	10.1	2	$p < 0.01$
	7	75			

Table 8.5 Data Question 4

In table 8.5, several patterns are evident. '*Seeking help from the teacher*' stands out as the most popular option but all options are selected by by large numbers. In two options, there is a clear trend with age: the place of the tutor increases in importance with age as is an increasing confidence in being able to cope. In all schools in Pakistan, the tendency is for most pupils to have help from tutors after school hours. This arises from the immense pressure on pupils to gain good passes in examinations.

(5) Here are some descriptions of the way students approach mathematics.	Grade	*SA*	*S*	*N*	*D*	*SD*	χ2	df	p
Revision sheets help me to understand mathematics well	5	81	10	7	1	1			
	6	61	23	11	3	3	32.3	4	$p < 0.001$
	7	84	15	1	0	1			
Diagram and pictures help me to understand mathematics well	5	67	26	3	1	3			
	6	38	29	21	11	1	42.7	6	$p < 0.001$
	7	50	34	8	6	3			
I can understand the main points easily.	5	65	21	11	3	1			
	6	43	23	15	16	3	37.8	6	$p < 0.001$
	7	57	28	11	4	1			
I think mathematics help me in daily life a lot.	5	59	23	5	11	1			
	6	51	18	15	11	4	32.1	6	$p < 0.001$
	7	75	16	5	4	0			
I do not want to learn mathematics but it is compulsory	5	29	15	7	27	22			
	6	24	26	12	28	10	18.4	8	$p < 0.05$
	7	27	22	13	18	22			
I work hard in mathematics but cannot get good marks in exam	5	27	13	5	11	43			
	6	26	24	17	16	16	36.6	6	$p < 0.001$
	7	23	14	10	20	33			

Table 8.6 Data Question 5

Table 8.6 shows the same features seen before: grade 6 is responding in a very different way and there is some polarisation of views in many of the items, Nonetheless, views are very positive for the majority. Looking at the first three items in this question as well as the final item, the different responses of grade 6 all reflect a perceived lack of ability to understand. Given that the pupils are taught in the same schools with the same teachers in the same language, all following the same curriculum, this suggests very strongly that the curriculum for grade 6 is excessively demanding in some way.

(6) Would you like to learn more mathematics in your next class?					
Grade	*Yes*	*No*	χ2	df	p
5	97	3	*valid value not possible***		
6	84	16			
7	96	4			

Table 8.7 Data Question 6

Grade 6 stands out again. Nonetheless, the vast majority would like more mathematics, suggesting that they see the importance and value of the subject. It is quite encouraging that, although grade 6 show much greater unhappiness with many aspects of the difficulty of their course, most still want to continue.

** While a chi square value may be calculated, it is likely to over-inflate any observed differences in that two boxes drop below 5% (see appendix).

(7) I like mathematics thanks to ...					
	Grade		**χ2**	**df**	**p**
My Parents	5	60	1.1	2	n.s.
	6	63			
	7	57			
Mathematics TV programs	5	5	43.0	2	$p < 0.001$
	6	37			
	7	27			
My Teacher	5	85	12.1	2	$p < 0.01$
	6	71			
	7	84			
Computers	5	13	36.0	2	$p < 0.001$
	6	45			
	7	35			
My Tutor	5	68	6.8	2	$p < 0.05$
	6	61			
	7	75			
My friends	5	53	4.3	2	n.s.
	6	62			
	7	51			
Mathematics Lessons	5	75	16.7	2	$p < 0.001$
	6	56			
	7	75			
Easy/ I am good at it	5	55	0.4	2	n.s.
	6	57			
	7	59			
Other	5	2	1.1	2	n.s.
	6	4			
	7	3			

Table 8.8 Data Question 7

The enormous importance of the teacher stands out although again, grade 6 are less convinced, perhaps holding the teacher in some way responsible for the perceived difficulty. This confirms much previous work which shows the vital importance of the teacher in terms of understanding, confidence and positive attitudes in subjects lilke mathematics, chemistry and physics (see Reid and Skryabina, 2002).

(8) Think about examination/tests in mathematics at your school					
	Grade		χ2	df	p
I tend to panic near the exam.	5	58	4.0	2	n.s.
	6	68			
	7	58			
I think there is enough revision at school before exams.	5	83	Chi-square invalid		
	6	74			
	7	96			
I cannot do well in the paper because I study late night and feel sleepy.	5	49	14.6	2	p < 0.001
	6	49			
	7	30			
I like challenging questions in exam.	5	66	0.7	2	n.s.
	6	63			
	7	67			
I do not like lengthy questions because I can make more mista in them.	5	53	1.4	2	n.s.
	6	55			
	7	60			
I find it difficult to revise the whole year syllabus in final exam.	5	56	0.3	2	n.s.
	6	53			
	7	54			
The topics included in half yearly exams should not be included in final exams.	5	67	1.0	2	n.s.
	6	62			
	7	63			
I do not like short questions because it does not give me chance to express that how much I know	5	43	5.0	2	n.s.
	6	48			
	7	35			
I like fill in the blanks and true/false type questions in exam.	5	82	29.8	2	p < 0.001
	6	59			
	7	83			
I like questions with colourful pictures and diagrams	5	78	10.4	2	p < 0.01
	6	61			
	7	69			
Sometimes I leave my questions incomplete because there is not enough space for solutions.	5	48	0.4	2	n.s.
	6	45			
	7	45			
I think that the allowed time limit is very short.	5	59	9.3	2	p < 0.01
	6	75			
	7	62			

Table 8.9 Data Question 8

In question 8, for the majority of the items, the three years groups respond similarly. However, grade 6 responded differently in areas where the asessment might be reflecting their perceived difficulties. It is worth noting that revision is seen as adequate by the vast majority.

(9) Look at the following topics in your mathematics syllabus.			**Grade 5 N = 150**
	Easy	*Moderate*	*Difficult*
Write in words and numerals	89	9	2
Place value	77	22	1
Factors and multiples	58	25	17
G.C.D by prime factorisation	66	24	10
G.C.D by division	64	30	5
L.C.M by prime factorisation	55	35	10
L.C.M by division	63	28	9
Word problems on LC.M and G.C.D	49	38	13
Types of common fractions	48	44	9
Writing C.F in the simplest form	61	33	6
Comparison of C.F	40	48	12
Addition and Subtraction of C.F	53	38	9
Associative & Commutative properties of C.F (addition)	42	46	12
Multiplication of C.F	52	37	10
Associative and commutative properties of C.F (multiplication)	39	44	17
Division of C.F	54	36	10
Word Problems on C.F	46	42	13
Decimal Fractions	56	32	13
Compound Quantities	46	36	17
Measurement of a line segment	48	33	19
To draw a circle when the measurement of the radius is given	44	40	17
To draw an isosceles and a scalene triangle	33	44	24
Kinds of triangles and the perimeter of a triangle	44	30	27
Kinds of angles and their measurement	48	31	21
To draw a perpendicular on a line	54	26	21
To find the perimeter of a square and a triangle	52	32	16
Graphs	53	25	22

Table 8.10 Data Question 9 (grade 5)

In this and the next two tables, the 'difficult' column is considered mainly. Pupils were invited to tick this column if they still did not feel they understood the topic. Colour coding has again been used and the percentages shown in the following way (this is arbitrary and not based on any statistical analysis):

'Difficulty' from 11-15%	*green*
'Difficulty' from 16-20%	*amber*
'Difficulty' > 20%	*red*

Looking at those coded red (>20) and amber (16-20), the majority of areas of perceived difficulty relate to geometry.

(9) Look at the following topics in your mathematics syllabus.			**Grade 6 N = 148**
	Easy	*Moderate*	*Difficult*
Place Value of Natural Numbers	66	24	10
Writing Natural Numbers in Order	51	39	11
Common Fractions	43	40	17
Equivalent Fractions	45	33	22
Writing C.F in the Simplest Form	36	45	19
Reduction of C.F to the Simplest Form	37	41	22
Reducible and Non Reducible C.F	45	37	18
Decimal Fraction	48	35	17
Converting C.F to Decimal Fractions	35	45	20
Converting Decimal Fractions to C.F	45	36	20
Rounding Off Decimal Fractions	47	37	16
Additon & Subtraction of Decimal Fractions	52	34	14
Multiplication Of Decimal Fractions	43	39	18
Divison Of Decimal Fractions	48	39	13
Use of Brackets	35	45	21
Direct and Inverse Proportion	37	48	16
Average	47	36	17
Angles	43	40	17
Parallel Lines and Perpendicular Lines	33	48	19
Construction of Triangles	30	45	26
Classification of the kinds of Squares	39	41	20
To Measure the Area of a Square and a Rectangle	35	44	21
Finding the Volume of a Cube and a Cuboid	35	51	14
Graphs	37	38	26

Table 8.11 Data Question 9 (grade 6)

Using the same colour coding as in the previous table, it is clear that almost every topic is regraded as 'difficult'. Indeed, overall, almost one fifth of the year group (the mean is over 17%) say that they never understood most of the topics at all. This is very different when compared to the other two year groups where the mean of the 'difficulty' column is 13%. This confirms that the curriculum is too demanding for grade 6. Looking at the detailed topics, one major area occurring several times relates to fractions. Either this is being taught too early or what is required of the students is too demanding.

(9) Look at the following topics in your mathematics syllabus.			**Grade 7 N = 158**
	Easy	*Moderate*	*Difficult*
Proper, Improper, Compund and Equivalent Fractions	70	27	3
Writing C.F in the Simplest Form	60	35	4
Commutative and Associative Property in C.F	49	45	6
Distributive property in C.F	53	41	6
Use of Brackets in C.F	48	44	9
Word Problems involving C.F and Decimal Fractions	41	47	12
Ratio and Proportion	61	30	10
Inheritance & Partnership	35	41	25
Direct Proportion and Inverse Proportion	44	35	20
Integers, Directed Numbers and Number Line	63	32	5
Addition and Subtraction of Directed Numbers	67	27	6
Percentage	70	25	4
Zakat (Islamic giving)	43	46	11
Income Tax, Property Tax and Custom	29	39	32
Commission and Discount	29	45	27
Loss and Profit	70	25	5
Weighted Average	32	48	20
Algebra	51	34	15
Finding the Area of a Right Angled Triangle	46	39	16
Finding the Area of a Parallalogram	49	36	15
Finding the Area of a Triangle	53	37	11
Finding the Volume of a Cube and a Cuboid	41	39	20
Bisection of a Line Segment	30	44	26
Bisection of an Angle	41	42	18
Construction of an Angle with the help of a Compass	63	31	6
Construction of a triangle with the help of a Compass	57	35	8
Pie Graph	51	35	14

Table 8.12 Data Question 9 (grade 7)

Again, geometry features quite a bit in areas regarded as 'difficult' but the greatest problems lie in areas where mathematics is being applied to life: tax, inheritance, business. There may be two problems here Firstly, at this age, there will be a great lack of experience of such areas of life. However, secondly, teaching in these areas requires a mastery of the mathematics involved at the same time as its application in life. This may simply reflect overload of limited working memory capacity.

The other observation which stands out is that fractions and related topics do not now seem to be causing the kinds of problems seen in grade 6. Perhaps, this set of topics was introduced one year too early or a simpler introduction in grade 6 is required. Perhaps, the topic might best be left to grade 7.

8.3 Conclusions

By looking at the data shown above, the following features are prominent.

- Views are generally positive for most of the items.
- Grade 6 is responding in a very different way throughout when compared to grades 5 and 7. As this sample is drawn from the same schools and the same teachers as the others, it is likely that the differences are being caused by the curriculum in grade 6. This would need further exploration to confirm it.
- '*Seeking help from the teacher*' can be seen as the most popular option but the place of the tutor increases in importance with age. In all schools in Pakistan, the tendency is for most pupils to have help from tutors after school hours, this growing with age. This arises from the immense pressure on pupils to gain good passes in examinations.
- The different responses of grade 6 reflect a perceived lack of ability to understand, suggesting very strongly that the curriculum for grade 6 is inappropriately demanding in some way.
- The vast majority (even grade 6 students) would like to learn more mathematics which shows that they see the importance and value of the subject.
- The majority of students from all year groups strongly appreciate the importance of the teacher in terms of understanding but grade 6 seems slightly less convinced. Perhaps they relate their perceived difficulty to their teachers.
- Revision is considered important by all the year groups.

In looking at areas of greatest perceived difficulty, the following conclusions can be drawn:

- At grade 5, geometry is the area of greatest difficulty.
- At grade 6, most of the topics (especially fractions and geometry) are considered difficult. This shows that the curriculum at this level is too demanding and must be reconsidered.
- At grade 7, while geometry topics are regarded as 'difficult' the major problems lie in areas where mathematics is being applied to daily life: topics like tax, inheritance, business. The reason for this problem may be the lack of experience in such areas of life. Also, teaching in these areas requires a mastery of the mathematics involved at the same time as its application in life. Fractions are not considered difficult at this level. It may be suggested that at grade 6, a simpler introduction of fractions is required and then this topic should be developed further at grade 7.

Chapter 9

Age and Attitudes Related to Mathematics
English Medium Schools

9.1 Introduction

The questionnaire was used with three years groups drawn from typical English medium schools (see table 9.1).

Grade	Sample
5	150
6	106
7	101
Total	**357**

Table 9.1 Sample Details

The aim was to see how attitudes developed with age in English medium schools and explore any relationships with working memory capacity. This chapter will only discuss the survey data.

All the data obtained were entered into a spreadsheet before analysis**.** The data for each item of each question will be presented as percentages for clarity. However, all statistical analyses were carried out using frequency data. The full questionnaire is shown in the appendix but the essentials of each question are shown here. Items were randomly positive and negative in the questionnaire. Some have been reversed here so that all items in a question are positive, simply for clarity.

Where there is a statistical significance difference between the age groups, this is shown with colour coding:

$p < 0.05$	*green*
$p < 0.01$	*amber*
$p < 0.001$	*red*

9.2 Data and Analysis

(1) What is your opinion about the subject mathematics?										
	Grade	**%**						**χ2**	**df**	**p**
I like mathematics	5	62	23	3	7	1	4			
	6	53	31	9	4	2	2	18.1	6	$p < 0.01$
	7	49	25	13	3	4	6			
Useful in daily life	5	63	23	8	2	1	2			
	6	59	26	13	1	1	1	5.7	6	n.s.
	7	56	26	14	2	1	1			
Easy to understand	5	37	25	14	8	5	12			
	6	34	33	19	7	4	4	16.2	8	$p < 0.05$
	7	33	25	20	9	3	11			
Interesting subject	5	60	18	8	7	1	7			
	6	51	23	8	8	4	5	15.8	8	$p < 0.05$
	7	42	22	13	10	4	9			
I want to learn it because I enjoy it	5	62	16	8	4	1	8			
	6	45	25	14	7	5	4	22.9	6	$p < 0.001$
	7	40	23	14	9	4	10			

Table 9.2 Data Question 1

Views are very positive in all the items although they become less positive with age.

(2) What is your opinion about mathematics lessons?										
	Grade	**%**						**χ2**	**df**	**p**
Easy	5	44	18	12	10	3	12			
	6	32	32	19	9	5	3	26.0	8	$p < 0.01$
	7	28	28	18	12	4	10			
I understand my lessons completely	5	45	24	10	9	6	6			
	6	32	37	20	7	3	1	30.1	8	$p < 0.001$
	7	29	31	22	11	4	4			
I like the way my teacher explains the methods	5	76	12	7	3	1	1			
	6	54	24	12	5	2	4	33.1	6	$p < 0.001$
	7	56	17	10	6	4	8			
I actually understand the procedures in class	5	34	12	16	11	8	19			
	6	34	15	10	11	15	14	12.2	10	n.s.
	7	34	14	15	13	10	15			
I enjoy doing my class work daily	5	39	14	14	9	6	19			
	6	32	20	11	12	12	13	11.8	8	n.s.
	7	28	16	12	11	11	23			
I revise my lessons regularly	5	47	13	15	8	5	12			
	6	44	20	15	4	7	10	16.7	10	n.s.
	7	36	21	12	9	7	15			

Table 9.3 Data Question 2

Views are quite widespread and some polarisation can be observed. However, they are very positive about the way teachers explain how to do mathematics. Understandably, mathematics becomes less easy with age, with a reduction in understanding. Their perception of their teacher's ability to explain does fall with age as well but this probably simply reflects increasingly levels of demand.

(3) How do you feel yourself in your mathematics course at school?										
	Grade	**%**						**χ2**	**df**	**p**
I feel I am trying hard to do well in mathematics	5	63	22	4	3	3	5	5.4	4	n.s.
	6	58	28	9	3	1	1			
	7	54	26	12	3	3	2			
I enjoy homework because I can do it on my own	5	49	13	13	12	2	12	19.6	8	$p < 0.05$
	6	37	27	15	9	8	4			
	7	33	24	18	10	4	11			
I am getting better at the subject	5	59	24	9	4	2	3	8.1	6	n.s.
	6	63	21	12	2	1	1			
	7	56	27	11	3	2	1			
Enough revision at school to help me understand well	5	59	15	10	5	3	8	23.9	8	$p < 0.01$
	6	45	24	16	5	3	7			
	7	41	26	9	6	8	9			
I understand at school with little extra help at home	5	49	20	12	5	2	12	7.4	8	n.s.
	6	45	26	13	3	3	9			
	7	41	27	12	6	5	10			

Table 9.4 Data Question 3

In general, their views are very positive, with slight polarisation showing in some questions. However, as might be expected, homework becomes less attractive with age and more revision is needed as they get older.

(4) Imagine you have problem in understanding a new topic or concept. What is your likely reaction?					
	Grade		**χ2**	**df**	**p**
Start to panic	5	20	5.7	2	n.s.
	6	12			
	7	13			
Seek help from my tutor	5	34	11.8	2	$p < 0.01$
	6	39			
	7	31			
See it as a challenge	5	54	2.5	2	n.s.
	6	46			
	7	48			
No worries, I will understand it with time.	5	47	1.1	2	n.s.
	6	46			
	7	42			
Seek help from my teacher	5	60	0.6	2	n.s.
	6	63			
	7	60			
Seek help from a family member	5	41	3.3	2	n.s.
	6	49			
	7	50			

Table 9.5 Data Question 4

There are few differences with age but it is interesting to note the perceived importance of teachers when faced with understanding problems, stressing the importance of the teacher in a subject like mathematics which is so highly conceptual (rather like physics: Reid and Skryabina, 2002)

(5) Here are some descriptions of the way students approach mathematics.	Grade	*SA*	*S*	*N*	*D*	*SD*	χ2	df	p
Revision sheets help me to understand mathematics well	5	49	37	9	2	3	8.6	6	n.s.
	6	37	46	14	1	2			
	7	40	40	14	3	3			
Diagram and pictures help me to understand mathematics well	5	31	34	30	5	1	1.9	4	n.s.
	6	29	37	25	6	3			
	7	25	38	22	8	7			
I can understand the main points easily.	5	44	33	18	5	1	9.4	6	n.s.
	6	51	33	13	2	1			
	7	40	40	15	3	2			
I think mathematics help me in daily life a lot.	5	62	28	8	1	1	12.6	6	$p < 0.05$
	6	59	28	9	3	1			
	7	52	26	14	5	3			
I do not want to learn mathematics but it is compulsory	5	14	9	10	14	53	19.2	8	$p < 0.05$
	6	6	6	8	26	55			
	7	10	10	7	26	47			
I enjoy studying mathematics classes	5	60	23	7	7	3	28.3	6	$p < 0.001$
	6	48	36	9	3	4			
	7	38	34	17	7	4			
I work hard in mathematics but cannot get good marks in exam	5	16	10	17	20	37	10.6	8	n.s.
	6	11	14	15	26	34			
	7	16	17	12	25	31			

Table 9.6 Data Question 5

Their responses overall are very positive, suggesting that mathematics is regarded well in English speaking schools at these age levels. However, enjoyment falls quite considerably with age and, as they get older, they are less convinced that mathematics can help them in daily life. This may well reflect the topics being studied which they see as less related to their lives with age.

Grade	*Yes*	*No*	χ2	df	p
(6) Would you like to learn more mathematics in Prep School?					
5	90	10	9.3	2	$p < 0.01$
6	92	8			
7	84	16			

Table 9.7 Data Question 6

Grade 7 stands out a bit as less positive. Nonetheless, the vast majority would like to study more mathematics in higher classes, suggesting that they realize the importance and value of the subject.

(7) I like mathematics thanks to ...	Grade		χ2	df	p
My Parents	5	75	15.3	2	$p < 0.001$
	6	73			
	7	60			
Mathematics TV programs	5	15	11.1	2	$p < 0.01$
	6	6			
	7	7			
My Teacher	5	7	2.8	2	n.s.
	6	6			
	7	10			
Computers	5	85	6.9	2	$p < 0.05$
	6	79			
	7	74			
My Tutor	5	11	0.4	2	n.s.
	6	14			
	7	12			
My friends	5	38	7.5	2	$p < 0.05$
	6	45			
	7	34			
Mathematics Lessons	5	24	2.6	2	n.s.
	6	23			
	7	29			
Easy/ I am good at it	5	62	7.1	2	$p < 0.05$
	6	56			
	7	49			
Other	5	42	0.3	2	n.s.
	6	40			
	7	41			

Table 9.8 Data Question 7

There are two dominant influences shown here: parents and the computers. It is strange that teachers and tutors do not figure strongly. It is also surprising that parents and the computers are seen as strong influences in making mathematics attractive. However, these parents have *chosen* English medium schools for their children and have a great concern for education. There may well be greater access to computers as well.

However, the influence of the actual mathematics lessons and how good they see themselves at the subject are both quite marked. There are some trends with age but these largely reflect experience and maturation.

(8) Think about examination/tests in mathematics at your school	Grade		χ2	df	p
I tend to panic near the exam.	5	32	1.2	2	n.s.
	6	36			
	7	33			
I think there is enough revision at school before exams.	5	67	14.3	2	$p < 0.001$
	6	53			
	7	47			
I cannot do well in the paper because I study late night and feel sleepy.	5	10	10.3	2	$p < 0.01$
	6	11			
	7	29			
I like challenging questions in exam.	5	71	20.3	2	$p < 0.001$
	6	62			
	7	49			
I do not like lengthy questions because I can make more mistakes in them.	5	47	3.6	2	n.s.
	6	53			
	7	57			
I find it difficult to revise the whole year syllabus in final exam.	5	38	4.4	2	n.s.
	6	48			
	7	47			
The topics included in half yearly exams should not be included in final exams	5	40	13.8	2	$p < 0.001$
	6	51			
	7	59			
I do not like short questions because it does not give me chance to express that how much I know	5	29	5.0	2	n.s.
	6	26			
	7	20			
I like fill in the blanks and true/false type questions in exam.	5	80	11.5	2	$p < 0.01$
	6	84			
	7	73			
I like questions with colourful pictures and diagrams	5	50	5.1	2	n.s.
	6	51			
	7	42			
Sometimes I leave my questions incomplete because there is not enough space for solutions.	5	16	1.7	2	n.s.
	6	16			
	7	16			
I think that the allowed time limit is very short.	5	44	19.8	2	$p < 0.001$
	6	46			
	7	63			

Table 9.9 Data Question 8

It is interesting to note that the pupils strongly like questions involving the minimum of writing although, as they get older, this declines slightly. Where there are differences with age, these are largly a reflection of more difficult work as they get older as well as signs of maturation.

It is somewhat surprising that large proportions in all year groups said they want challenging question in examinations although it does fall with age. Is this related to a desire for many to show their abilities?

Two English medium schools were surveyed and each has its own curriculum. The curriculum for Urdu medium schools is determined nationally but English medium schools develop their own curricula. The response patterns are shown separately for school 1 and school 2 and reflect the actual specific curriculum in each of the schools.

School 1		**Grade 5**	N= 46
(9) Look at the following topics in your mathematics syllabus.			
	Easy	*Moderate*	*Difficult*
Place Value	94	4	2
Write in words	96	4	0
Write in numerals	91	9	0
Addition	98	2	0
Subtraction	91	9	0
Multiplication	76	22	2
Division	72	20	9
Distributive Property	30	50	20
Angles	76	20	4
Triangles	78	17	4
Common Fractions	70	24	7
Decimal Fractions	74	22	4
Compound Quantities	65	26	9
Perimeter	65	20	15
Area	63	15	22
Volume	35	37	28
Capacity	33	37	30

Table 9.10 Data Question 9 (School 1, Grade 5)

Topics tend to be regarded as ‘difficult’ or ‘easy’. In topics like volume, capacity and even area, many ideas have to be handled at the same time, the concept of volume being one that Piaget (1963) found was not accessible below a given age. This almost cetainly arises because of limited working memory capacity.

School 1		**Grade 6**	N = 197
(9) Look at the following topics in your mathematics syllabus.			
	Easy	*Moderate*	*Difficult*
Place Value	89	10	1
Multiplication	94	4	2
Division	91	7	2
Common Fractions	63	34	4
Decimal Fractions	63	31	6
Use of brackets	60	36	4
Percentage	57	36	7
Time	58	37	5
Quadrilaterals	44	44	12
Bilateral Symmetry	45	43	12
Constructions of angles	76	20	5
Construction of triangles	70	24	6
Perimeter	69	26	5
Area	71	25	4
Information Handling (basic statistics)	35	49	16
G.C.D	88	11	1
L.C.M	88	11	1

Table 9.11 Data Question 9 (School 1, Grade 6)

Unlike the Urdu medium schools, there are few difficulties. However, statistical ideas pose problems again along with some areas of geometry.

School 1	**Grade 7**	N =157	
(9) Look at the following topics in your mathematics syllabus.			
	Easy	*Moderate*	*Difficult*
Sets	88	11	1
Integers	84	14	3
BODMAS	67	30	3
Factors	68	28	5
Multiples	79	18	3
G.C.D	81	17	2
L.C.M	88	11	1
Link between G.C.D and L.C.M	53	38	10
Basic Algebra	64	26	10
Polynomials	64	26	10
Multiplication of polynomials	64	23	13
Division of Polynomials	55	31	14
Algebraic Sentences	56	28	16
Solution of Linear Equations	57	31	12
Square Roots	96	4	0
Percentage	75	20	5
Polygons	44	46	10
Line and Angles	66	28	6
Constructions of Angles	75	19	6
Construction of Triangles	74	21	5
Perimeter	68	25	7
Area	63	31	7

Table 9.12 Data Question 9 (School 1, Grade 7)

Some key ideas of algebra are introduced here and some of these clearly cause problems. The way these topics are approached gives considerable detail and the examples can be quite complex. Again, this may well be posing some working memory problems.

School 2	**Grade 5**	N = 104	
(9) Look at the following topics in your mathematics syllabus.			
	Easy	*Moderate*	*Difficult*
Writing numbers in words and figures	81	15	4
Rounding off Integers	75	20	5
Number Sequences	82	14	3
Addition and Subtraction	98	1	1
Multiplication	97	2	1
Division	88	11	2
Common Fraction	56	39	5
Decimal Fractions	53	39	8
Percentages	51	38	12
Use of Brackets	48	39	13
Even and Odd Numbers	90	8	2
Problem Solving	64	29	7
Organising and Using Data (basic statistics)	30	51	19
Shapes and Measures	76	19	5
Rectangles	93	6	1
Triangles	93	6	1
Symmetry	89	8	3
Angles	84	14	3
Recognising Parallel / Perpendicular Lines	64	27	10
Compound Quantities	47	40	14
Area	62	24	14
Perimeter of a rectangle and Regular Polygons	70	20	10
Time	78	19	3

Table 9.13 Data Question 9 (School 2, Grade 5)

Looking at table 9.13, only a very few topics seem to give great difficulty, judging by the 'difficult' column. The topics regarded as 'difficult' tend to be those which require some kind of conceptual understanding in order to gain success. Statistics is notorious as an area of difficulty, often generating very negative attitudes (see Ghani, 2004).

School 2		**Grade 6**	N = 106
(9) Look at the following topics in your mathematics syllabus.			
	Easy	*Moderate*	*Difficult*
Rounding Off Integers	68	30	2
Ordering the Set of Integers	48	43	9
Subtraction and Addition	98	1	1
Multiplication	97	3	0
Division	91	8	2
Number Sequences	81	15	4
Finding Squares of Numbers	78	19	3
Factorising numbers into Prime Factors	31	53	16
Fractions	64	27	9
Decimal Fractions	83	14	3
Percentsges	51	39	10
Use Of Brackets	60	36	4
Problem Solving	66	28	6
Organising and Using Data (basic statistics)	45	41	14
Graphs	70	24	7
Classifying Quadrilaterals	39	41	21
Making Shapes with Increasing Accuracy	44	43	12
Identifying Nets for a closed Shape	45	34	21
Angles Measurement	91	10	0
Angles of a Triangle	84	13	3
Rotation of Shapes	63	28	9
Length, Mass, Capacity Measurement	56	34	10
Perimeter of Simple Compound shapes	75	21	5
Area of a Simple Compound Shape	77	18	5
Different Times Around the World	45	36	19

Table 9.14 Data Question 9 (School 2, Grade 6)

Again, not too many topics are regarded as 'difficult'. Statistics and some topics in geometry again show as difficult. It is highly likely that such topics place great demands on limited working memory space. For example, seeing the geometrical shape in relation to its net does require holding many ideas together at the same time.

School 2		**Grade 7**	N = 101
(9) Look at the following topics in your mathematics syllabus.			
	Easy	*Moderate*	*Difficult*
Understanding Decimal Notations	50	40	11
Place Value	64	33	3
Multiplication\ Division of Whole Numbers	89	10	1
Brackets	80	18	2
Fractions	67	27	6
Percentages	69	25	6
Percentage Increase and Decrease	46	43	12
HCF	62	25	13
LCM	72	19	9
Ordering Fractions, Decimals, Percentages	46	49	6
Use of Negative Numbers	58	31	11
Ratio and Proportion	51	32	18
Rounding Off Decimals\ Whole Numbers	73	19	8
Algebra	62	26	12
Collecting and Organising Data	55	29	17
Finding Mode, Median and Range	79	18	3
Finding Probabilities	62	29	9
12-hour \ 24-hour Clock	70	23	7
Using Formula for the Area of a Triangle	69	18	13
Perimeter of Compound shapes	54	30	17
Measuring\ Drawing Lines and Angles	55	33	13
Construction of a Triangle	59	30	11
Reflection in a given Line	49	35	17
Translation, Rotation about a given point	39	36	26
Enlargement of 2D-Shapes	29	38	34
Deducing the formula for the Area of a Parallelogram	36	34	31
Deducing the formula for the area of a Trapezium	27	36	38

Table 9.15 Data Question 9 (School 2, Grade 7)

Again statistics and geometry give the greatest problems.

9.3 Conclusions

The following general conclusions can be drawn from the data above although the conclusions cannot be generalised to all English-medium schools in that ecah follows its own curriculum.

- The majority of the students in all year groups is very positive about the way teachers explain how to do mathematics. Understandably, the higher the level the less easy the mathematics becomes, with a reduction in understanding.
- As might be expected, homework becomes less attractive and more revision is required with age.
- Seeking help fron the teacher becomes very important when faced with understanding problems.
- Overall, positive responses suggest that mathematics is regarded well in English speaking schools at these age levels. However, enjoyment falls considerably with age and they are less convinced that mathematics is useful in daily life as they grow older.
- The majority of the students would like to study more mathematics in higher classes which shows that they know the importance of the subject, although this declines in grade 7.
- Parents and computers are seen as strong influences in making mathematics attractive. However, the influence of the actual mathematics lessons and how good they see themselves at the subject are both quite marked although teachers and tutors do not figure strongly
- Pupils strongly like questions involving the minimum of writing although, as they get older, this declines slightly. Perhaps, this simply reflects the desire for minimum work.
- Where there are differences with age, these are largely a reflection of more difficult work as they get older as well as signs of maturation.

In looking at areas of greatest perceived difficulty, the following conclusions can be drawn:

- At Grade 5, the topics regarded as ‘difficult’ tend to be those which require some kind of conceptual understanding in order to gain success. Statistics is notorious as an area of difficulty (Ghani, 2004). Concepts like volume, capacity and even area are main problems. In these topics, many ideas have to be handled at the same time and working memory capacity cannot cope easily.
- At Grade 6, unlike the Urdu medium schools, there are few difficulties. However, statistics and some topics in geometry are again regarded as difficult. Limited working memory space may well be the reason for this.
- At grade 7, statistics and geometry are considered the major problems. Some key ideas of algebra are introduced at this level and some of these clearly cause problems. Again, this may well be posing some working memory problems.

Chapter 10

Attitudes Related to Mathematics and Gender

10.1 Introduction

The questionnaire was used with groups drawn from typical Urdu medium schools (see table 10.1). The schools in the English medium sector are for boys only. For this reaons, the gender comparisons only involve Urdu medium schools

Grade	Sample
Girls	214
Boys	242
Total	**456**

Table 10.1 Sample Details

All the data obtained were entered into a spreadsheet before analysis**.** The data for each item of each question will be presented as percentages for clarity. However, all statistical analyses were carried out using frequency data. Where there is a statistical significance difference between the age groups, this is shown with colour coding:

$p < 0.05$	*green*
$p < 0.01$	*amber*
$p < 0.001$	*red*

10.2 Data and Analysis

(1) What is your opinion about the subject mathematics?	Gender	%						χ2	df	p
I like mathematics	Girls	71	12	2	2	4	9			
	Boys	63	11	5	6	5	10	6.3	3	n.s.
Useful in daily life	Girls	78	13	4	1	2	1			
	Boys	59	17	6	7	8	3	22.4	2	$p < 0.001$
Easy to understand	Girls	53	14	5	4	6	19			
	Boys	43	20	15	7	5	10	26.0	4	$p < 0.001$
Interesting subject	Girls	68	10	6	2	4	10			
	Boys	44	9	19	8	9	12	39.1	4	$p < 0.001$
I want to learn it because I enjoy it	Girls	67	10	5	2	2	14			
	Boys	45	11	9	10	11	15	35.1	4	$p < 0.001$

Table 10.2 Data Question 1

Firstly, the views are mostly positive for the majority. A main feature stands out that for every item girls have a distinctively more positive attitude towards mathematics than boys.

(2) What is your opinion about mathematics lessons?	Gender	%						χ2	df	p
Easy	Girls	61	9	1	2	7	20			
	Boys	64	3	3	6	6	18	0.4	3	n.s.
I understand my lessons completely	Girls	59	10	4	5	6	16			
	Boys	43	23	7	11	9	7	35.4	5	$p < 0.001$
I like the way my teacher explains the methods	Girls	79	8	4	1	3	6			
	Boys	52	9	11	12	9	7	40.7	3	$p < 0.001$
I actually understand the procedures in class	Girls	73	9	5	2	5	7			
	Boys	39	9	18	11	8	15	61.9	3	$p < 0.001$
I enjoy doing my class work daily	Girls	54	14	2	3	8	19			
	Boys	39	12	7	14	12	17	28.0	4	$p < 0.001$
I revise my lessons regularly	Girls	73	10	3	2	3	8			
	Boys	46	8	7	7	8	24	46.1	3	$p < 0.001$

Table 10.3 Data Question 2

Girls are again more positive about mathematics lessons, sometimes very markedly so. Looking at the final item, girls say they do much more regular revision of their lessons. This might help them to perform better then boys. Also, girls look more happy with the teaching ways of their teachers than boys. Girls perhaps tend to be more conformist and eager to please but, in Pakistan culture, there are fewer social opportunities for girls and less competition with study demands.

(3) How do you feel yourself in your mathematics course at school?										
	Gender							**χ2**	**df**	**p**
I feel I am trying hard to do well in mathematics	Girls	69	11	1	3	2	14			
	Boys	65	3	5	7	3	17	1.0	2	n.s.
I enjoy homework because I can do it on my own	Girls	52	6	3	4	8	28			
	Boys	15	19	7	7	22	30	84.7	4	$p < 0.001$
I am getting better at the subject	Girls	70	16	3	1	4	6			
	Boys	49	11	13	12	7	8	44.8	3	$p < 0.001$
Enough revision at school to help me understand well	Girls	71	9	6	4	2	8			
	Boys	29	14	15	17	15	10	89.5	4	$p < 0.001$
I understand at school with little extra help at home	Girls	57	8	5	1	7	22			
	Boys	40	14	5	13	12	16	33.8	4	$p < 0.001$

Table 10.4 Question 3

There is an enormous difference of responses among girls and boys in doing homework on their own, reflecting, perhaps, the nature of girls at this age as well as less competing social demands. Girls seem far more confident about their performance in the subject. Also, they seem to be satisfied by having enough revision and support at school than boys.

(4) Imagine you have problem in understanding a new topic or concept. What is your likely reaction?					
	Gender		**χ2**	**df**	**p**
Start to panic	Girls	44			
	Boys	61	1.0	1	n.s.
Seek help from my tutor	Girls	63			
	Boys	46	12.2	1	$p < 0.001$
See it as a challenge	Girls	62			
	Boys	76	9.7	1	$p < 0.01$
No worries, I will understand it with time.	Girls	71			
	Boys	58	7.9	1	$p < 0.05$
Seek help from my teacher	Girls	84			
	Boys	92	7.0	1	$p < 0.05$
Seek help from a family member	Girls	59			
	Boys	77	17.9	1	$p < 0.001$

Table 10.5 Question 4

Again girls seem to be more confident than boys. The place and importance of the tutor is more established for girls than boys and this may be a contribution to their confidence and positive attitudes about the subject. Boys, according to their nature, like to see problems more as a challenge. Seeking help from the teacher again stands out distinctively for both girls and boys. Boys seem to seek more help from their family members which again shows their need of more support. In general, girls at this age seem more willing to seek help.

(5) Here are some descriptions of the way students approach mathematics.	Gender	*SA*	*S*	*N*	*D*	*SD*	χ2	df	p
Revision sheets help me to understand mathematics well	Girls	71	20	8	1	1			
	Boys	80	12	5	1	2	5.9	3	n.s.
Diagram and pictures help me to understand mathematics well	Girls	41	36	9	9	5			
	Boys	61	24	11	3	0	18.3	2	$p < 0.001$
I can understand the main points easily.	Girls	49	33	11	6	1			
	Boys	61	16	13	9	2	19.6	3	$p < 0.001$
I think mathematics help me in daily life a lot.	Girls	72	21	4	2	1			
	Boys	54	17	12	15	3	32.6	2	$p < 0.001$
I do not want to learn mathematics but it is compulsory	Girls	19	23	13	21	25			
	Boys	34	19	9	27	12	24.7	4	$p < 0.001$
I enjoy studying mathematics classes	Girls	18	16	12	22	32			
	Boys	31	18	10	10	31	19.0	4	$p < 0.001$

Table 10.6 Question 5

Boys have more positive veiws about the use of diagrams and pictures, and say they understand mathematics better. Nonetheless, they tend to reject mathematics more than the girls but still say they are enjoying it. These may not appear to be consistent patterns of views but they probably reflect the boys' greater tendency to reject being forced to do things and their general greater confidence. Girls are more aware of the way mathematics can help them in daily life but it is difficult to interpret this view.

(6) Would you like to learn more mathematics in Prep School?					
Gender	***Yes***	***No***	**χ2**	**df**	**p**
Girls	93	7			
Boys	92	8	0.3	1	n.s.

Table 10.7 Question 6

Both girls are boys seem to realize the importance of mathematics as a subject and would like to continue in next classes.

(7) I like mathematics thanks to ...					
	Gender		**$\chi2$**	**df**	**p**
My Parents	Girls	43			
	Boys	75	50.5	1	$p < 0.001$
Mathematics TV programs	Girls	15			
	Boys	29	12.5	1	$p < 0.001$
My Teacher	Girls	3			
	Boys	3	Chi-square invalid		
Computers	Girls	76			
	Boys	84	4.3	1	n.s.
My Tutor	Girls	28			
	Boys	34	1.8	1	n.s.
My friends	Girls	62			
	Boys	74	7.4	1	$p < 0.01$
Mathematics Lessons	Girls	36			
	Boys	72	60.6	1	$p < 0.001$
Easy/ I am good at it	Girls	66			
	Boys	71	1.2	1	n.s.
Other	Girls	47			
	Boys	66	17.4	1	$p < 0.001$

Table 10.8 Question 7

Boys seem more dependent on others in influencing them towards mathematics. They also say they are much more influenced by the actual mathematics lessons. Being good at the subject is also a greater influence. In general, there is tendency for boys to need to be 'pushed' and directed more at these ages; this is mainly a maturity aspect. In addition, for some parents (more marked for those in the Urdu medium sector), they see their sons as future wage-earners while their daughters are seen as wives and home-makers.

(8) Think about examination/tests in mathematics at your school					
	Gender		χ2	df	p
I tend to panic near the exam.	Girls	56			
	Boys	66	4.9	1	$p < 0.05$
I think there is enough revision at school before exams.	Girls	94			
	Boys	76	26.7	1	$p < 0.001$
I cannot do well in the paper because I study late night and feel sleepy.	Girls	23			
	Boys	60	63.7	1	$p < 0.001$
I like challenging questions in exam.	Girls	60			
	Boys	70	4.6	1	$p < 0.05$
I do not like lengthy questions because I can make more mistakes in them.	Girls	54			
	Boys	59	1.1	1	n.s.
I find it difficult to revise the whole year syllabus in final exam.	Girls	46			
	Boys	62	12.0	1	$p < 0.001$
The topics included in half yearly exams should not be included in final exams	Girls	49			
	Boys	77	39.2	1	$p < 0.001$
I do not like short questions because it does not give me chance to express that how much I know	Girls	30			
	Boys	52	22.0	1	$p < 0.001$
I like fill in the blanks and true/false type questions in exam.	Girls	75			
	Boys	74	0.2	1	n.s.
I like questions with colourful pictures and diagrams	Girls	62			
	Boys	76	9.7	1	$p < 0.01$
Sometimes I leave my questions incomplete because there is not enough space for solutions.	Girls	36			
	Boys	55	15.8	1	$p < 0.001$
I think that the allowed time limit is very short.	Girls	56			
	Boys	74	69.9	1	$p < 0.001$

Table 10.9 Question 8

Again girls seem more confident about and look satisfied by the way revision is done at school. The third item shows the stress and panic for boys before exams which might stem from their lack of confidence about the subject. Boys see it as more difficult to revise the whole syllabus near the exam. The next item confirms that boys are strongly wanting less syllabus in the final exam. Items 8-12 confirms again that boys want to express themselves more through legnthy questions, colourful pictures and diagrams. Boys are asking for more time and space to express themselves. The majority of the students is positive about having true/false and fill in the blanks type questions.

10.3 Conclusions

Overall, the following general pattern of gender differences is apparent:

- Many of the the differences reflect the different rates of maturation, the different social expectations for boys and girls in Pakistani society and their somewhat different lifestyles.
- In general, the girls tend to be more positive and more committed. Sometimes, the differences are extremely marked.
- While girls seem to be willing to seek help more, the boys are more dependent on others for influencing them towards mathematics.
- Inevitably, the boys seem less conformist and show greater confidence at times.
- Girls are more aware of the way mathematics can help them in daily life but it is difficult to interpret this view.

Chapter 11

Age and Attitudes Related to Mathematics
Urdu and English Medium Schools

11.1 Introduction

The final analysis of the questionnaire looks at a comparison between Urdu and English medium schools. Only boys are considered as the English medium schools are boys only. The samples involved are shown in table 11.1.

Grade	Urdu Sample	English Sample
5	150	150
6	148	106
7	158	101
Total	**456**	**357**

Table 11.1 Sample Details

All the data obtained were entered into a spreadsheet before analysis**.** The data for each item of each question will be presented as percentages for clarity. However, all statistical analyses were carried out using frequency data. Where there is a statistical significance difference between the age groups, this is shown with colour coding:

$p < 0.05$	*green*
$p < 0.01$	*amber*
$p < 0.001$	*red*

11.2 Data Analysis

(1) What is your opinion about the subject mathematics?										
	Language	**%**						**χ2**	**df**	**p**
I like mathematics	English	58	23	7	5	2	5			
	Urdu	63	11	5	6	5	10	25.1	4	$p < 0.001$
Useful in daily life	English	62	24	10	1	1	1			
	Urdu	59	17	6	7	8	3	11.0	2	$p < 0.01$
Easy to understand	English	37	23	17	8	4	11			
	Urdu	43	20	15	7	5	10	2.8	5	n.s.
Interesting subject	English	53	19	9	9	2	8			
	Urdu	44	9	19	8	9	12	34.1	4	$p < 0.001$
I want to learn it because I enjoy it	English	49	21	11	6	3	10			
	Urdu	45	11	9	10	11	15	25.6	5	$p < 0.001$

Table 11.2 Data Question 1

The majority of the students in both mediums holds positive attitudes about mathematics and seem to realize the importance in daily life. Boys in English medium school have more positive views in four of the five areas considered. This may simply reflect the selective nature of English medium schools.

(2) What is your opinion about mathematics lessons?										
	Language	**%**						**χ2**	**df**	**p**
Easy	English	37	23	15	11	4	10			
	Urdu	64	3	3	6	6	18	75.3	4	$p < 0.001$
I understand my lessons completely	English	37	29	16	9	5	4			
	Urdu	43	23	7	11	9	7	17.9	5	$p < 0.001$
I like the way my teacher explains the methods	English	65	20	9	3	1	3			
	Urdu	52	9	11	12	9	7	55.2	3	$p < 0.001$
I actually understand the procedures in class	English	30	13	14	13	11	19			
	Urdu	39	9	18	11	8	15	11.0	5	n.s.
I enjoy doing my class work daily	English	33	15	12	10	9	22			
	Urdu	39	12	7	14	12	17	10.9	5	n.s.
I revise my lessons regularly	English	43	17	15	8	5	13			
	Urdu	46	8	7	7	8	24	27.7	5	$p < 0.001$

Table 11.3 Data Question 2

Again, in general, the views of the English medium pupils are slightly more positive than those of the Urdu medium pupils. Indeed, there is a slight tendency for the view of Urdu medium pupils to be little more polarised.

(3) How do you feel yourself in your mathematics course at school?										
	Language							**χ2**	**df**	**p**
I feel I am trying hard to do well in mathematics	English	59	26	8	2	2	4			
	Urdu	65	3	5	7	3	17	77.1	3	p < 0.001
I enjoy homework because I can do it on my own	English	39	20	13	10	5	13			
	Urdu	15	19	7	7	22	30	95.2	5	p < 0.001
I am getting better at the subject	English	59	24	11	3	2	2			
	Urdu	49	11	13	12	7	8	60.9	3	p < 0.001
Enough revision at school to help me understand well	English	48	20	13	6	4	9			
	Urdu	29	14	15	17	15	10	43.7	4	p < 0.001
I understand at school with little extra help at home	English	48	24	13	3	3	9			
	Urdu	40	14	5	13	12	16	59.0	4	p < 0.001

Table 11.4 Data Question 3

The comparison here gives very high chi-square values indicating quite marked differences between the self perceptions of the pupils in the two systems. In every case, the English medium boys are more positive and there is a tendency for greater polarisation with the Urdu sample. The most marked result relates to homework where it is clear that Urdu pupils are finding difficulties working on their own. This may reflect the inadequacy of what is done in the classroom and the relative lack of family or tutor support outside the classroom.

(4) Imagine you have problem in understanding a new topic or concept. What is your likely reaction?					
	Language		**χ2**	**df**	**p**
Start to panic	English	18			
	Urdu	61	118	1	p < 0.001
Seek help from my tutor	English	33			
	Urdu	46	10.7	1	p < 0.01
See it as a challenge	English	46			
	Urdu	76	51.2	1	p < 0.001
No worries, I will understand it with time.	English	38			
	Urdu	58	22.7	1	p < 0.001
Seek help from my teacher	English	59			
	Urdu	92	78.7	1	p < 0.001
Seek help from a family member	English	38			
	Urdu	77	89.1	1	p < 0.001

Table 11.5 Data Question 4

Again, the chi-square values show enormous differences. It is quite stark that 61% say they will panic in Urdu schools. However, the Urdu pupils are much more prepared to seek help from tutor, teacher or family member. Surprisingly, they are also more phlegmatic that they will understand it with time. Perhaps, this arises from the more difficult social backgrounds which are more common with Urdu-medium pupils.

(5) Here are some descriptions of the way students approach mathematics.	Language	SA	A	N	D	SD	χ2	df	p
Revision sheets help me to understand mathematics well	English	45	42	10	1	2			
	Urdu	80	12	5	1	2	84.5	2	$p < 0.001$
Diagram and pictures help me to understand mathematics well	English	29	39	23	6	3			
	Urdu	61	24	11	3	0	61.6	2	$p < 0.001$
I can understand the main points easily.	English	43	33	18	4	1			
	Urdu	61	16	13	9	2	34.5	3	$p < 0.001$
I think mathematics help me in daily life a lot.	English	61	26	9	3	1			
	Urdu	54	17	12	15	13	24.4	2	$p < 0.001$
I do not want to learn mathematics but it is compulsory	English	12	9	8	20	53			
	Urdu	34	19	9	27	12	112	4	$p < 0.001$
I enjoy studying mathematics classes	English	52	31	9	6	3			
	Urdu	31	18	10	10	31	92.3	3	$p < 0.001$

Table 11.6 Data Question 5

By looking at the first three items, it is clear that the majority in Urdu medium schools is far more positive about the importance of the revision sheets, diagrams and pictures as compared to English medium schools. Also, Urdu medium school students seem more satisfied when understanding their mathematics lessons. The majority in both mediums seems to learn mathematics willingly and realize the usefulness of the subject in daily life..The last item reflects the lack of enjoyment in lessons in Urdu medium schools.

(6) Would you like to learn more mathematics in Prep School?					
Language	Yes	No	χ2	df	p
English	90	10			
Urdu	92	8	0.5	1	n.s.

Table 11.7 Data Question 6

The majority of students in both English and Urdu medium schools want to continue learning mathematics in next classes.

(7) I like mathematics thanks to ...					
	Gender	**%**	**$\chi 2$**	**df**	**p**
My Parents	English	65	6.7	1	$p < 0.01$
	Urdu	75			
Mathematics TV programs	English	11	32.6	1	$p < 0.001$
	Urdu	29			
My Teacher	English	7	3.3	1	n.s.
	Urdu	3			
Computers	English	85	0.1	1	n.s.
	Urdu	84			
My Tutor	English	11	45.7	1	$p < 0.001$
	Urdu	34			
My friends	English	40	83.3	1	$p < 0.001$
	Urdu	74			
Mathematics Lessons	English	23	143.4	1	$p < 0.001$
	Urdu	72			
Easy/ I am good at it	English	59	9.8	1	$p < 0.01$
	Urdu	71			
Other	English	37	49.0	1	$p < 0.001$
	Urdu	66			

Table 11.8 Data Question 7

There are some surprising differences here. For Urdu pupils, the greater influence of parents, TV, tutor and friends can be seen. However, the enormously greater impact of the actual mathematics lessons stands out. It is unlikely that mathematics lessons in Urdu schools are so dramatically more stimulating than those in English medium schools. It is probably more likely that the Urdu pupils are more dependent on what actually goes on in class. They also say that being good at mathematics is a greater influence but this may reflect the more demanding courses in English medium schools, with an increased school and social pressure to achieve high standards.

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(8) Think about examination/tests in mathematics at your school					
	Language	**%**	**$\chi 2$**	**df**	**p**
I tend to panic near the exam.	English	41			
	Urdu	66	35.5	1	$p < 0.001$
I think there is enough revision at school before exams.	English	52			
	Urdu	76	37.8	1	$p < 0.001$
I cannot do well in the paper because I study late night and feel sleepy.	English	13			
	Urdu	60	####	1	$p < 0.001$
I like challenging questions in exam.	English	60			
	Urdu	70	6.1	1	$p < 0.05$
I do not like lengthy questions because I can make more mistakes in them.	English	53			
	Urdu	59	1.7	1	n.s.
I find it difficult to revise the whole year syllabus in final exam.	English	47			
	Urdu	62	13.4	1	$p < 0.001$
The topics included in half yearly exams should not be included in final exams	English	48			
	Urdu	77	49.3	1	$p < 0.001$
I do not like short questions because it does not give me chance to express that how much I know	English	23			
	Urdu	52	55.0	1	$p < 0.001$
I like fill in the blanks and true/false type questions in exam.	English	79			
	Urdu	74	2.0	1	n.s.
I like questions with colourful pictures and diagrams	English	50			
	Urdu	76	40.8	1	$p < 0.001$
Sometimes I leave my questions incomplete because there is not enough space for solutions.	English	20			
	Urdu	55	74.5	1	$p < 0.001$
I think that the allowed time limit is very short.	English	56			
	Urdu	74	21.2	1	$p < 0.001$

Table 11.9 Data Question 8

On a positive note, Urdu medium pupils show a greater tendency to think there is enough revision, like challenging questions, do not like short questions, and like questions with colourful pictures and diagrams. Negatively, Urdu medium pupils show a greater tendency to panic, lack sleep, be unhappy with the way end of year examinations are constructed, and are more critical of lack of writing space and of time.

In almost every option, the Urdu medium pupils show higher numbers. This indicates that, overall, the Urdu pupils have ticked far more boxes than those from the English medium schools. This is possibly because, in English medium schools, there are regular monthly class tests, while, in Urdu medium schools, they only face two major examinations each year. The English students may be more confident as a result of more experience. This masks some important differences in the order of priority of the 12 options. Thus, for example, in responding to ‘not liking questions which are lengthy’, while the percentages selecting this option are similar, this choice is the fourth highest choice for the English medium pupils and only the tenth choice out of the twelve for the Urdu pupils. The view that the topics from the half year examination should *not* be included in the end of year examination shows a marked difference in percentage terms but this is even more marked in that the Urdu pupils select this as their most ticked option while the English medium pupils have this as their seventh choice.

11.3 Conclusions

It has to be recognised that the boys attending English medium schools have parents who have specifically chosen to send their sons there and this requires the payment of fees. Therefore, the boys attending English medium schools may come from very different social and economic backgrounds and their homes will express somewhat different aspirations in relation to education.

Overall, the following conclusions can be drawn:

- In general, while both groups are positive about their experiences in mathematics, the boys from the Urdu medium schools are somewhat less positive and, in some places, there is a slightly greater extent of polarisation: sizeable numbers are very positive and sizeable numbers are very negative.
- When faced with problems or examinations, the reactions of the two groups are very markedly different. This can be seen in the way the Urdu medium boys say they will 'panic' more. It probably underpins the tendency for the Urdu medium boys to be more dependent: on revision sheets, diagrams and pictures, teachers and the actual lessons.
- In thinking of examinations, it is very strange that the Urdu boys make many more selections. Does this reflect a greater dissatisfaction with the kind of examinations to which they are exposed? Perhaps, behind this is a unease with the recall-procedure approach to all testing. Some very interesting findings were found in Saudi Arabia with much older students where there was clear evidence of their strong unhappiness with the restrictive nature of assessment: greater credit was given for quantity of correctly recalled information or procedures. Almost nothing came from evidence of understanding and critical thought (Oraif, 2007).

Chapter 12

Correlations and Factor Analysis

12.1 Introduction

In this chapter, relationships between various measurements will be considered. Correlation is the usual technique used while factor analysis is often used when seeking to find any patterns from multiple correlations between many variables.

12.2 Factor Analysis

In looking at the survey questions 1, 2, 3 and 5 (24 items in all), quite a number of these show quite high intercorrelations (Kendall's Tau-b) with each other. The items in these four questions were never designed with any underlying structure in mind. Nonetheless, a factor analysis can offer insights to see if there is any underlying structure to explain these correlations (see appendix for these correlations). The data for the entire sample (N = 813) were analysed using principal components analysis. Using varimax rotation, it was found that 9 components accounted for 68% of the variance. The scree plot is shown below.

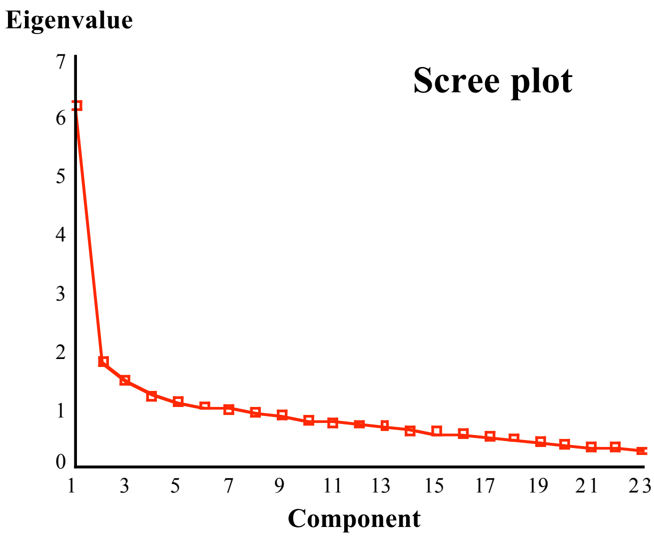

Figure 12.1 Scree Plot

Principal components analysis will identify factors (components) which can explain the large number of inter-item correlations. It does not identify the nature of the factors and this has to be done by looking at the items which correlate most closely with each of the nine components. These correlations are known as loadings and these are shown in table 12.1. The possible nature of these nine components is also shown in this table. Only loadings above 0.3 are shown, simply for clarity.

	Components								
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
	Enjoyment, interest	*Easiness, difficulty*	*Effort*	*Memorising or understanding*	*Useful in daily life*	*Type of revision*	*Quality of explanation*	*Extent of help at home*	*Capacity to understand*
1A	**-0.66**	**0.41**							
1B					**0.85**				
1C		**0.80**							
1D	**0.76**								
1E	**0.80**								
2A		**0.78**							
2B		**0.68**							
2C						**0.58**	**0.34**		
2D				**0.83**					
2E	**0.41**			**0.45**					
2F			**0.72**						
3A			**0.56**						
3B	**0.43**			**0.37**					
3C			**0.69**						
3D						**0.82**			
3E								**0.88**	
5A						**0.40**			**-0.50**
5B							**0.89**		
5C									**0.73**
5D	**-0.41**				**0.67**				
5E	**0.66**								

Table 12.1 Loadings Table

The possible nature of the nine components was deduced by looking in detail at the specific questions where the loadings were high (poisitive or negative). They seem to make sense but they are tentative. Many of these components reveal influences which are not unexpected: enjoyment, difficulty, factors relating to understanding, effort, and perceived usefulness in daily life. In an education system where examinations dominate so strongly, sources of help are important: help at home, quality of teacher explanation, and revision. Examinations very often encourage memorisation and this emerges as one of the components.

The loadings table does not provide a list of the key factors which will solve any problems of attitude deterioration but the nine components do suggest areas which may be important. It reveals that enjoyment and interest are very important in developing positive attitudes related to mathematics. It also suggests that a mathematics curriculum must be of appropriate difficulty: excessive difficulty may have powerful negative effects on attitudes. Two factors are interesting: the memorising-understanding dimension and the perception of usefulness in daily life.

The work of Jung (2008) has shown very clearly that understanding is what learners are seeking but, if the difficulties are too great, the learners have to resort to memorisation with concomitant attitude deterioration. Examinations may have a very powerful effect here.

The other factor of interest is the perception or otherwise of the useful of mathematics in daily life. This has been found to be critical in the sciences (see Reid and Skryabina, 2002) and it seems important here also. It is relatively easy to introduce applications into syllabuses in biology, chemistry and physics at school level. It may be very much more demanding to achieve this in mathematics in that introducing another layer of thought will almost certainly overwhelm the working memory. This issue has been discussed by Al-Enezi (2008) and no easy solution is yet apparent.

12.3 Gender Issues

If the items in questions 1, 2, 3, and 5 are correlated with each other for boys and girls separately, some interesting differences can be seen (table 12.2).

First Item		Second Item		Girls	Boys	Δ
1a	Like mathematics	1e	Want to learn because I enjoy it	0.60	0.24	**0.36**
1a	Like mathematics	2d	Understand procedures	0.38	0.06	**0.32**
1a	Like mathematics	5e	Do not want to learn	-0.26	0.03	**-0.29**
1c	Easy to understand	5b	Diagrams and pictures help understand	0.04	0.31	**-0.27**
1c	Easy to understand	1e	Want to learn because I enjoy it	0.41	0.16	**0.25**
1c	Easy to understand	2e	Enjoy classwork	0.42	0.13	**0.29**
1d	Interesting	5f	Enjoy studying mathematics classes	-0.23	0.17	**-0.40**
1e	Do not want to learn	5f	Enjoy studying mathematics classes	0.29	-0.03	**0.32**
2a	Easy lessons	3b	Enjoy homework .. can do on my own	0.21	-0.12	**0.33**
2a	Easy lessons	5b	Diagrams and pictures help understand	0.04	0.39	**-0.35**
2a	Easy lessons	5e	Do not want to learn but compulsory	-0.23	0.03	**-0.26**
2b	Understand lessons completely	5e	Do not want to learn but compulsory	-0.24	0.05	**-0.29**
2c	Like the way my teacher explains	5f	Enjoy studying mathematics classes	-0.13	0.14	**-0.27**
2e	Enjoy classwork	3a	I feel I am trying hard to do well	0.48	0.18	**0.30**
2e	Enjoy classwork	2f	Revise my lessons regularly	0.29	-0.01	**0.30**
2e	Enjoy classwork	3d	Enough revision to help understand well	0.25	-0.04	**0.29**
2e	Enjoy classwork	5f	Enjoy studying mathematics classes	0.20	-0.06	**0.26**
2f	Revise my lessons regularly	2d	Understand procedures	-0.37	-0.09	**0.28**
2f	Revise my lessons regularly	3a	I feel I am trying hard to do well	0.58	0.25	**0.33**
2f	Revise my lessons regularly	3e	Understand with little help from home	0.09	0.35	**-0.26**
3a	I feel I am trying hard to do well	3b	Enjoy homework .. can do on my own	0.25	-0.13	**0.38**
3a	I feel I am trying hard to do well	3e	Understand with little help from home	-0.01	0.25	**-0.26**
3b	Enjoy homework .. can do on my own	3c	I am getting better at the subject	0.31	0.00	**0.31**
3b	Enjoy homework .. can do on my own	5a	Revision sheets help me understand	0.20	-0.07	**0.27**
3b	Enjoy homework .. can do on my own	5b	Diagrams and pictures help understand	0.08	-0.20	**0.28**
3b	Enjoy homework .. can do on my own	5c	Understand the main points easily	0.24	-0.03	**0.27**
3b	Enjoy homework .. can do on my own	5f	Enjoy studying mathematics classes	-0.23	0.09	**-0.32**
3c	I am getting better at the subject	3e	Understand with little help from home	0.11	0.36	**-0.25**
3c	I am getting better at the subject	5f	Enjoy studying mathematics classes	-0.25	0.09	**-0.34**
5a	Revision sheets help me understand	5e	Do not want to learn but compulsory	-0.15	0.13	**-0.28**
5a	Revision sheets help me understand	5f	Enjoy studying mathematics classes	-0.31	0.05	**-0.36**
5e	Do not want to learn but compulsory	5b	Diagrams and pictures help understand	0.06	-0.26	**-0.32**
5e	Do not want to learn but compulsory	5c	Understand the main points easily	0.20	-0.13	**-0.33**
5f	Enjoy studying mathematics classes	5d	Mathematics helps in daily life	0.20	-0.24	**-0.44**
5f	Enjoy studying mathematics classes	5e	Do not want to learn but compulsory	-0.34	0.02	**0.36**

Table 12.2 Inter-Item Correlations Related to Gender

Table 12.2 summarises the inter-item correlations for boys and girls, with the final column giving the difference between the two correlation values. Only data where the difference between boys and girls exceeds 0.25 are shown. This figure is somewhat arbitrary. However, a correlation of around 0.25 is significant at $p < 0.001$ for each gender. Thus, it is reasonable to consider this value as indicating that the *difference* between the girls and boys correlation values will be highly significant. Green shading indicates the girls with the higher correlation, yellow shading shows that the boys having a higher correlation.

It has been observed already that there are many gender differences in overall perceptions (chapter 10). The gender differences of correlations between items also show many gender differences. Indeed, some of the patterns obtained are to easy to explain. However, the lifestyles of boys and girls in Pakistan are very different, with much more social freedom for boys and more time spent at home on school-related work from the girls. This may explain some of the differences observed.

For girls, understanding seems more important. Thus, their liking of the subject tends to be more related to understanding as well as the feeling they are getting better. The role of the teacher is also more important for the girls, similar to that observed by Reid and Skryabina (2002b) in relation to physics. There is a greater influence of hard work, revision, and general commitment from the girls and this probably reflects their more restricted lifestyle where work at home is more systematic. They see understanding as related to hard work and effort. Interestingly, the boys show stronger relationships relating to the use of pictures and diagrams. This may be genuine or it may simply relate to the way they perceive pictures and diagrams: perhaps a shortcut to success involving less effort. In general, boys find geometry easier than girls.

12.4 Working Memory Correlations

The working memory capacity of the grade 5 students was measured using the figural intersection test. The descriptive data are shown in table 12.3.

Sample	Minimum	Maximum	Mean	Standard Deviation
300	2	8	4.1	1.2

Table 12.3 Working Memory Data

It was not possible to gain access to students at grade 6 and 7. Grade 5 students have an average age of about 10-11 years. Students of age 10 might be expected to have a mean working memory capacity of 4 and the results obtained here are consistent with this.

The working memory capacity measurements were correlated with the marks in mathematics using Pearson correlation. The results are shown in table 12.b.

	Sample	Pearson r	Probability
Grade 5 Urdu Medium	150	0.69	< 0.001
Grade 5 English Medium	150	0.43	< 0.001

Table 12.4 Working Memory Correlations

It was not possible to gain the marks from one of the two English Medium schools and the sample, therefore, is reduced.

Working memory capacity measurements will only correlate with performance measurements if one or both of the following conditions are fulfilled (Reid, 2008):

(a) The teaching and learning process is such that students with higher working memory capacities have an advantage.
(b) The assessment is such that students with higher working memory capacities have an advantage.

Typically, correlation values which lie between 0.2 and 0.65 are obtained when working memory capacities are correlated with examination performance. The value of 0.69 is unusually high indicating that about 50% of the variance in their mathematics marks is controlled by their working memory capacity. (The extent of variance is calculated by squaring the correlation coefficient and converting to a percentage.) Of course, correlation does not of itself imply causation. However, the work of Johnstone and El-banna (1986, 1989) shows clearly that causation is involved.

12.5 Conclusions

The factor analysis suggest two important aspects which need much further study. The whole issue of memorisation and understanding is vitally important in the development of positive attitudes. The natural process for human beings is to seek to understand, the work of Piaget (1963) illustrating this. Jung (2008) has shown recently that where understanding is difficult due to working memory overloading, then the learner has to turn to memorisation and attitudes tend to deteriorate. This is a clear issue for mathematics. The goal of understanding must be stressed more if positive attitudes are to be retained.

Secondly, the whole issue of making the mathematics studied become related in some way to the lifestyle of the learner seems very important although ways to do this are not easy. Perhaps the best way forward is to re-examine the syllabus topics, asking if such topics should be taught at all in the light of the need to develop meaningful applications.

The correlation values obtained between performance in mathematics and working memory capacity reveal yet again the key role of limited working memory capacity in all learning. Reid (2002) showed that, at least in terms of assessment, questions can be developed which do *not* place undue stress on the working memory. The questions are still demanding but the demand lies with the understanding of the mathematics, not the capacity of the individual's working memory. It is very clear that, in Pakistan, at this age, both the curriculum and the assessment appear to be placing the working memory under stress. This is a major issue needing addressed by curriculum planners and by those who set examinations. The situation with the Urdu curriculum and assessment is particularly a matter of concern.

The effect of working memory capacity on mathematics performance can be illustrated in table 12.5. The work of Johnstone and El-banna (1986, 1989) confirms that the correlation is, in fact, cause and effect.

Working Memory Capacity	***Urdu Medium*** N = 150	***English Medium*** N = 150
	Mathematics Marks (%)	
Above average	63.0	55.7
Average	50.0	48.1
Below average	39.7	46.4

Table 12.5 Differences in Mean Marks Related to Working Memory Capacity

Table 12.5 illustrates the considerable difference in mean examination marks caused by limited working memory capacity.

Chapter 13

Conclusions

13.1 Review of Work Done

Frequently, mathematics is considered as a difficult subject. In Pakistan, the subject occupies a core place in the curriculum but there is very little freedom offered to either teachers or students, especially in Urdu medium schools.

The aim in this study has been to explore aspects of the mathematics learning experiences in grades 5 to 7 (ages approximately 10-12) in both Urdu medium and English medium schools. This has involved looking at pupil perceptions of their experiences, the nature of the difficulties they have with mathematics and possible reasons for these difficulties.

The study has surveyed pupil perceptions from both Urdu and English medium schools. In addition, working memory capacity of those in grade 5 (age about 10) were measured and information was gained about their performance in mathematics examinations. The data have been analysed to consider how their self-perceptions related to their experiences in learning mathematics which varies with age, language background and gender. Any relationships between these self perceptions, mathematics marks and measured working memory capacity were explored as well.

13.2 The Main Findings

The study was exploratory and aimed to investigate aspects of the mathematics learning experiences in grades 5 to 7. Any observed outcomes can be used to inform the agenda for action or further study.

In Pakistan, the education system is very divided, with English medium schools tending to draw from the more educated and wealthier part of the population. Therefore, differences are to be expected. At the same time, boys and girls are often treated differently in the social structure of Pakistan and are often taught separately.

In both systems, the vast majority appreciate the role and the importance of studies in mathematics. Quite a number of topics pose problems in both systems: geometry, fractions, topics with life applications, statistics, and some specific problems like volume and capacity. It is almost certain that these topics place demands on working memory which make understanding very difficult. Either such topics should be taught later (when working memory capacity has increased or greater experience offers strategies to chunk) or taught and assessed in ways which place less demand on limited working memory.

In the Urdu medium schools, it was suggested that the curriculum in grade 6 is likely to be causing major problems. Indeed, in both systems, pressures for success based on examination performance have generated a complete industry of private tutors. Nonetheless, the role of teachers is still seen as very important. It is interesting to note the different reaction towards examinations from the students of the two systems, with the less experienced Urdu students showing much less confidence. However, it has to be recognised that generalising the findings for all English-medium schools is not possible in that each follows its own curriculum.

Many of the gender differences can be interpreted in terms of the social roles in Pakistani society. However, girls do seem more positive and more committed in relation to their studies in mathematics.

The study has revealed two major issues which need careful consideration. One is the whole issue of memorisation and understanding. The goal of meaningful learning must be stressed more if positive attitudes are to be retained. However, if the working memory is overloaded too often, the students may well turn to memorisation of procedures simply to passs examination.

The whole issue of making the mathematics studied become related in some way to the lifestyle of the learner seems very important. This is not easy without overloading working memory. However, mathematics curricula need to be re-examined to see what is possible. In considering both of these issues, the critical role of assessment has to be addressed. Thus, if assessment offers rewards almost entirely for the recall and correct execution of mathematical procedures, then this will be reflected in textbooks and teaching approaches. Along with curriculum design and teaching approaches which are consistent with the known limitations of working memory, assessment is perhaps the single most important issue to be considered.

The correlation of 0.69, which was found for grade 5 Urdu students when their measured working memory capacity was related to their mathematics examination performance, is extraordinarily high. Indeed, it is the highest such correlation which has been found in any discipline. This suggests major curriculum design problems in the national syllabus for Urdu medium schools.

13.3 Strengths and Weakness of the Study

It was found that the pupils were very happy to complete the surveys, perhaps appreciating the opportunity to express their views. The working memory test was also quite acceptable to the school authorities and to the pupils. However, it proved difficult to gain access to schools to obtain mathematics marks. This probably reflects the enormous power and importance of examination grades in Pakistan.

Data from large samples were obtained and this gives confidence that the outcomes are generalisable for Pakistan. The aim was to offer an overview of the situation in mathematics education at these ages in order to pinpoint an agenda for action in Pakistan and this was achieved in large measure. It would have been helpful to gain more information relating to working memory capacity but it proved difficult to gain access to the pupils for enough time.

13.4 Future work

A future study might look at working memory capacity in relation to specific mathematics topics, the way they are taught, and the way they are assessed. Another area urgently needing exploration is why the mathematics in grade 6 in Urdu medium schools is creating problems. Clearly some topics in each year group in each system are causing problems. An analysis of the ways these topics are taught and assessed in the light of limited working memory capacity is urgently required. These topics should be taught and assessed differently or, perhaps, they are better left to a later stage in the curriculum. Indeed, questions need to be asked about why some topics are taught at all: are they of any relevance or importance as a part of general education?

13.5 Implications and Final Comments

This study has had the limited aim of exploring student experiences in grade 5, 6, and 7 in mathematics in Pakistan. It has pinpointed a number of areas of concern, mostly relating to curriculum construction and assessment. Teachers have very limited opportunities to influence either of these areas. It is hoped that this study has defined an agenda for further research and action. The aim is that students in Pakistan can find their studies in mathematics increasingly accessible and satisfying.

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Appendix 1

Surveys Used

What do you think about Mathematics?

Name: .... Roll No:................

This questionnaire seeks to find out what you think about mathematics
Your answers will not be seen by your teachers
Your answers will help future planning in mathematics
Please answer completely honestly!

Here is a way to describe a racing car

quick	☑ ☐ ☐ ☐ ☐ ☐	slow
important	☐ ☐ ☑ ☐ ☐ ☐	unimportant
safe	☐ ☐ ☐ ☐ ☑ ☐	dangerous

The positions of the ticks between the word pairs show that you consider it as <u>very</u> quick, slightly more important than unimportant and <u>quite</u> dangerous.

Use the same method to answer questions 1 to 3
Remember: only tick ONE box on each line!

(1) What is your opinion about the **subject mathematics?**
*Tick **one** box on each line*

I like mathematics	☐ ☐ ☐ ☐ ☐ ☐	I do not like mathematics
Useful in daily life	☐ ☐ ☐ ☐ ☐ ☐	Useless in daily life
Easy to understand	☐ ☐ ☐ ☐ ☐ ☐	Difficult to understand
Boring subject	☐ ☐ ☐ ☐ ☐ ☐	Interesting subject
I do not want to learn it but it is a compulsory subject	☐ ☐ ☐ ☐ ☐ ☐	I want to learn it because I enjoy it.

(2) What is your opinion about **mathematics lessons?**
*Tick **one** box on each line*

Easy lessons	☐ ☐ ☐ ☐ ☐ ☐	Difficult lessons
I understand my lessons completely	☐ ☐ ☐ ☐ ☐ ☐	I do not understand my lessons completely
I like the way my teacher explains the methods	☐ ☐ ☐ ☐ ☐ ☐	I do not like the way my teacher explains the methods.
I just memorise the procedures in class	☐ ☐ ☐ ☐ ☐ ☐	I actually understand the procedures.in class
I do not like doing too much class work daily	☐ ☐ ☐ ☐ ☐ ☐	I enjoy doing my class work daily
I revise my lessons regularly	☐ ☐ ☐ ☐ ☐ ☐	I revise them just before the exam or a test.

(3) How do you feel **yourself** in your **mathematics course** at school?

I feel I am trying hard to do well in mathematics	☐ ☐ ☐ ☐ ☐ ☐	It is my fault I cannot study mathematics well.
I hate homework because I can't do it on my own	☐ ☐ ☐ ☐ ☐ ☐	I enjoy homework because I can do it on my own
I am getting better at the subject	☐ ☐ ☐ ☐ ☐ ☐	I am getting worst at the subject.
Enough revision at school to help me understand well	☐ ☐ ☐ ☐ ☐ ☐	Not enough revision at school to help me understand well
I understand at school with little extra help at home	☐ ☐ ☐ ☐ ☐ ☐	I understand at school only with extra help from home

(4) **Imagine** you have **problem** in understanding a **new** topic or concept. What is your likely reaction?
Tick as many boxes as you wish.

☐ Start to panic ☐ See it as a challenge ☐ Seek help from my teacher
☐ Seek help from my tutor ☐ No worries, I will understand it with time. ☐ Seek help from a family member

(5) Here are some descriptions of the way students **approach** mathematics.
*Tick **one** box on each line.*

	strongly agree	*agree*	*not sure*	*disagree*	*strongly disagree*
Revision sheets help me to understand mathematics well	☐	☐	☐	☐	☐
Diagram and pictures help me to understand mathematics well	☐	☐	☐	☐	☐
I can understand the main points easily.	☐	☐	☐	☐	☐
I think mathematics help me in daily life a lot.	☐	☐	☐	☐	☐
I do not want to learn mathematics but it is compulsory	☐	☐	☐	☐	☐
I enjoy studying mathematics classes	☐	☐	☐	☐	☐
I work hard in mathematics but cannot get good marks in exam	☐	☐	☐	☐	☐

(6) Would you like to learn **more** mathematics in **next classes?**
Tick either 'yes' or 'no' and give a reason.

☐ Yes, because ....

☐ No, because ....

(7) I like mathematics thanks to :
*Tick **as many** boxes as you wish*

☐ My parents	☐ My teacher	☐ My tutor	☐ Mathematics lessons
☐ Mathematics TV programs	☐ Computers	☐ My friends	☐ Easy/I am good at it

☐ Other - please show: ....

(8) Think about **examination/tests in mathematics** at your school.
*Tick **as many** you feel true for you*

☐ I tend to panic near the exam.
☐ I think there is enough revision at school before exams.
☐ I cannot do well in the paper because I study late night and feel sleepy.
☐ I like challenging questions in exam.
☐ I do not like lengthy questions because I can make more mistakes in them.
☐ I find it difficult to revise the whole year syllabus in final exam.
☐ The topics included in half yearly exams should not be included in final exams.
☐ I do not like short questions because it does not give me chance to express that how much I know
☐ I like fill in the blanks and true/false type questions in exam.
☐ I like questions with colourful pictures and diagrams.
☐ Sometimes I leave my questions incomplete because there is not enough space for solutions.
☐ I think that the allowed time limit is very short in mathematics paper.

(9) Think of the following topics in your **mathematics syllabus**.

Easy	I understood it first time
Moderate	I found it difficult but I understand it now
Difficult	I still do not understand it

Tick the **suitable** box for each topic to show if you find that topic, easy, moderate or difficult

	Easy	*Moderate*	*Difficult*
Writing numbers in words and figures	☐	☐	☐
Rounding off Integers	☐	☐	☐
Number Sequences	☐	☐	☐
Addition and Subtraction	☐	☐	☐
Multiplication	☐	☐	☐
Division	☐	☐	☐
Common Fractions	☐	☐	☐
Decimal Fractions	☐	☐	☐
Percentages	☐	☐	☐
Use of Brackets	☐	☐	☐
Even and Odd Numbers	☐	☐	☐
Problem Solving	☐	☐	☐
Orgainising and Using Data	☐	☐	☐
Shapes and Measures	☐	☐	☐
Rectangles	☐	☐	☐
Triangles	☐	☐	☐
Symmetry	☐	☐	☐
Angles	☐	☐	☐
Recognising Parallel / Perpendicular lines	☐	☐	☐
Length, Mass, Capacity measurements	☐	☐	☐
Area	☐	☐	☐
Perimeters of rectangles and regular polygons	☐	☐	☐
Time	☐	☐	☐

What do you think about Mathematics?

Name: ....   Roll No:.................

This questionnaire seeks to find out what you think about mathematics
Your answers will not be seen by your teachers
Your answers will help future planning in mathematics
Please answer completely honestly!

Here is a way to describe a racing car

quick	☑ ☐ ☐ ☐ ☐ ☐	slow
important	☐ ☐ ☑ ☐ ☐ ☐	unimportant
safe	☐ ☐ ☐ ☐ ☑ ☐	dangerous

The positions of the ticks between the word pairs show that you consider it as <u>very</u> quick, slightly more important than unimportant and <u>quite</u> dangerous.

Use the same method to answer questions 1 to 3
Remember: only tick ONE box on each line!

(1) What is your opinion about the **subject mathematics?**
*Tick **one** box on each line*

I like mathematics	☐ ☐ ☐ ☐ ☐ ☐	I do not like mathematics
Useful in daily life	☐ ☐ ☐ ☐ ☐ ☐	Useless in daily life
Easy to understand	☐ ☐ ☐ ☐ ☐ ☐	Difficult to understand
Boring subject	☐ ☐ ☐ ☐ ☐ ☐	Interesting subject
I do not want to learn it but it is a compulsory subject	☐ ☐ ☐ ☐ ☐ ☐	I want to learn it because I enjoy it.

(2) What is your opinion about **mathematics lessons?**
*Tick **one** box on each line*

Easy lessons	☐ ☐ ☐ ☐ ☐ ☐	Difficult lessons
I understand my lessons completely	☐ ☐ ☐ ☐ ☐ ☐	I do not understand my lessons completely
I like the way my teacher explains the methods	☐ ☐ ☐ ☐ ☐ ☐	I do not like the way my teacher explains the methods.
I just memorise the procedures in class	☐ ☐ ☐ ☐ ☐ ☐	I actually understand the procedures.in class
I do not like doing too much class work daily	☐ ☐ ☐ ☐ ☐ ☐	I enjoy doing my class work daily
I revise my lessons regularly	☐ ☐ ☐ ☐ ☐ ☐	I revise them just before the exam or a test.

(3) How do you feel **yourself** in your **mathematics course** at school?

I feel I am trying hard to do well in mathematics	☐ ☐ ☐ ☐ ☐ ☐	It is my fault I cannot study mathematics well.
I hate homework because I can't do it on my own	☐ ☐ ☐ ☐ ☐ ☐	I enjoy homework because I can do it on my own
I am getting better at the subject	☐ ☐ ☐ ☐ ☐ ☐	I am getting worst at the subject.
Enough revision at school to help me understand well	☐ ☐ ☐ ☐ ☐ ☐	Not enough revision at school to help me understand well
I understand at school with little extra help at home	☐ ☐ ☐ ☐ ☐ ☐	I understand at school only with extra help from home

(4) **Imagine** you have **problem** in understanding a **new** topic or concept. What is your likely reaction?
Tick as many boxes as you wish.

☐ Start to panic	☐ See it as a challenge	☐ Seek help from my teacher
☐ Seek help from my tutor	☐ No worries, I will understand it with time.	☐ Seek help from a family member

(5) Here are some descriptions of the way students **approach** mathematics.
*Tick **one** box on each line.*

	strongly agree	*agree*	*not sure*	*disagree*	*strongly disagree*
Revision sheets help me to understand mathematics well	☐	☐	☐	☐	☐
Diagram and pictures help me to understand mathematics well	☐	☐	☐	☐	☐
I can understand the main points easily.	☐	☐	☐	☐	☐
I think mathematics help me in daily life a lot.	☐	☐	☐	☐	☐
I do not want to learn mathematics but it is compulsory	☐	☐	☐	☐	☐
I enjoy studying mathematics classes	☐	☐	☐	☐	☐
I work hard in mathematics but cannot get good marks in exam	☐	☐	☐	☐	☐

(6) Would you like to learn **more** mathematics in **next classes?**
Tick either 'yes' or 'no' and give a reason.

☐ Yes, because ....

☐ No, because ....

(7) I like mathematics thanks to :
*Tick **as many** boxes as you wish*

☐ My parents ☐ My teacher ☐ My tutor ☐ Mathematics lessons
☐ Mathematics TV programs ☐ Computers ☐ My friends ☐ Easy/I am good at it
☐ Other - please show: ....

(8) Think about **examination/tests in mathematics** at your school.
*Tick **as many** you feel true for you*

☐ I tend to panic near the exam.
☐ I think there is enough revision at school before exams.
☐ I cannot do well in the paper because I study late night and feel sleepy.
☐ I like challenging questions in exam.
☐ I do not like lengthy questions because I can make more mistakes in them.
☐ I find it difficult to revise the whole year syllabus in final exam.
☐ The topics included in half yearly exams should not be included in final exams.
☐ I do not like short questions because it does not give me chance to express that how much I know
☐ I like fill in the blanks and true/false type questions in exam.
☐ I like questions with colourful pictures and diagrams.
☐ Sometimes I leave my questions incomplete because there is not enough space for solutions.
☐ I think that the allowed time limit is very short in mathematics paper.

(9) Think of the following topics in your **mathematics syllabus**.

Easy	I understood it first time
Moderate	I found it difficult but I understand it now
Difficult	I still do not understand it

Tick the **suitable** box for each topic to show if you find that topic, easy, moderate or difficult

	Easy	*Moderate*	*Difficult*
Writing numbers in words and figures	☐	☐	☐
Rounding off Integers	☐	☐	☐
Number Sequences	☐	☐	☐
Addition and Subtraction	☐	☐	☐
Multiplication	☐	☐	☐
Division	☐	☐	☐
Common Fractions	☐	☐	☐
Decimal Fractions	☐	☐	☐
Percentages	☐	☐	☐
Use of Brackets	☐	☐	☐
Even and Odd Numbers	☐	☐	☐
Problem Solving	☐	☐	☐
Orgainising and Using Data	☐	☐	☐
Shapes and Measures	☐	☐	☐
Rectangles	☐	☐	☐
Triangles	☐	☐	☐
Symmetry	☐	☐	☐
Angles	☐	☐	☐
Recognising Parallel / Perpendicular lines	☐	☐	☐
Length, Mass, Capacity measurements	☐	☐	☐
Area	☐	☐	☐
Perimeters of rectangles and regular polygons	☐	☐	☐
Time	☐	☐	☐

What do you think about Mathematics?

Name: .... Roll No:.................

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Here is a way to describe a racing car

quick	☑	☐	☐	☐	☐	☐	slow
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The positions of the ticks between the word pairs show that you consider it as <u>very</u> quick, slightly more important than unimportant and <u>quite</u> dangerous.

Use the same method to answer questions 1 to 3
Remember: only tick ONE box on each line!

(1) What is your opinion about the **subject mathematics?**
*Tick **one** box on each line*

I like mathematics ☐ ☐ ☐ ☐ ☐ ☐ I do not like mathematics
Useful in daily life ☐ ☐ ☐ ☐ ☐ ☐ Useless in daily life
Easy to understand ☐ ☐ ☐ ☐ ☐ ☐ Difficult to understand
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I feel I am trying hard to do well in mathematics ☐ ☐ ☐ ☐ ☐ ☐ It is my fault I cannot study mathematics well.
I hate homework because I can't do it on my own ☐ ☐ ☐ ☐ ☐ ☐ I enjoy homework because I can do it on my own
I am getting better at the subject ☐ ☐ ☐ ☐ ☐ ☐ I am getting worst at the subject.
Enough revision at school to help me understand well ☐ ☐ ☐ ☐ ☐ ☐ Not enough revision at school to help me understand well
I understand at school with little extra help at home ☐ ☐ ☐ ☐ ☐ ☐ I understand at school only with extra help from home

(4) **Imagine** you have **problem** in understanding a **new** topic or concept. What is your likely reaction?
Tick as many boxes as you wish.

☐ Start to panic ☐ See it as a challenge ☐ Seek help from my teacher
☐ Seek help from my tutor ☐ No worries, I will understand it with time. ☐ Seek help from a family member

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Diagram and pictures help me to understand mathematics well	☐	☐	☐	☐	☐
I can understand the main points easily.	☐	☐	☐	☐	☐
I think mathematics help me in daily life a lot.	☐	☐	☐	☐	☐
I do not want to learn mathematics but it is compulsory	☐	☐	☐	☐	☐
I enjoy studying mathematics classes	☐	☐	☐	☐	☐
I work hard in mathematics but cannot get good marks in exam	☐	☐	☐	☐	☐

(6) Would you like to learn **more** mathematics in **next classes?**
Tick either 'yes' or 'no' and give a reason.

☐ Yes, because ....

☐ No, because ....

(7) I like mathematics thanks to :
*Tick **as many** boxes as you wish*

☐ My parents	☐ My teacher	☐ My tutor	☐ Mathematics lessons
☐ Mathematics TV programs	☐ Computers	☐ My friends	☐ Easy/I am good at it

☐ Other - please show: ....

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Easy	I understood it first time
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Difficult	I still do not understand it

Tick the **suitable** box for each topic to show if you find that topic, easy, moderate or difficult

	Easy	*Moderate*	*Difficult*
Writing numbers in words and figures	☐	☐	☐
Rounding off Integers	☐	☐	☐
Number Sequences	☐	☐	☐
Addition and Subtraction	☐	☐	☐
Multiplication	☐	☐	☐
Division	☐	☐	☐
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Percentages	☐	☐	☐
Use of Brackets	☐	☐	☐
Even and Odd Numbers	☐	☐	☐
Problem Solving	☐	☐	☐
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Angles	☐	☐	☐
Recognising Parallel / Perpendicular lines	☐	☐	☐
Length, Mass, Capacity measurements	☐	☐	☐
Area	☐	☐	☐
Perimeters of rectangles and regular polygons	☐	☐	☐
Time	☐	☐	☐

Appendix 2

Figural Intersection Test

Figure Intersection Test

Name: .... ☐ Boy ☐ Girl

This is a test of your ability to find the overlap of a number of simple shapes.
The results of this test will not affect your schoolwork in any way.

There are two sets of simple geometric shapes, one on the right and the other on the left.
The set on the left contains the same shapes (as on the right) but overlapping so that there exists a common area which is inside all of the shapes.

Your task is to **look for** and **shade** in the common **area of overlap.**

Note these points:

(1) The shape on the left may differ in size or position from those on the right, but they match in *shape and proportions*.

(2) In some items on the left, some extra shapes appear which are not present in the right hand set, and which do not form a common area of intersection with all of the other shapes. These are presented to mislead. You should ignore them.

(3) The overlap should be shaded ***clearly*** by using a pen or pencil.

Here are three examples to get you started.

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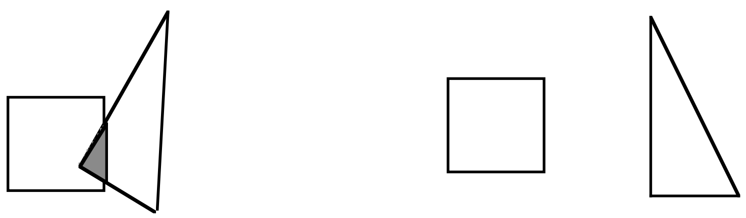

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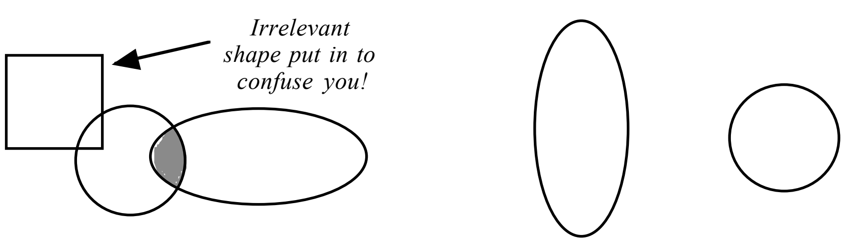

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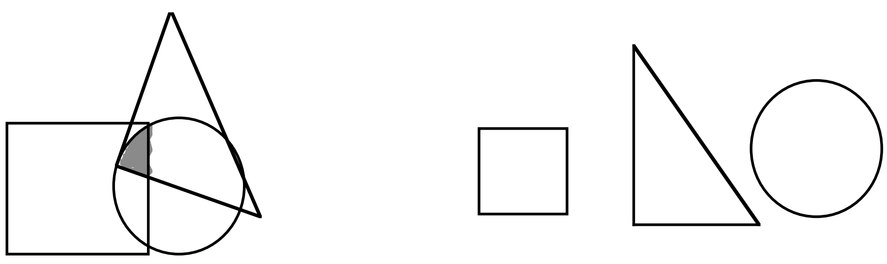

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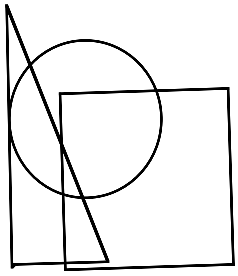
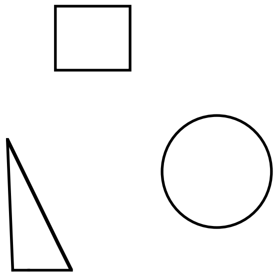
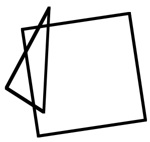
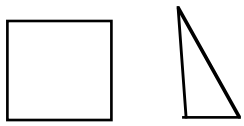
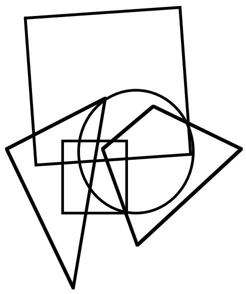
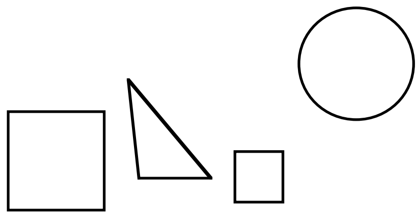
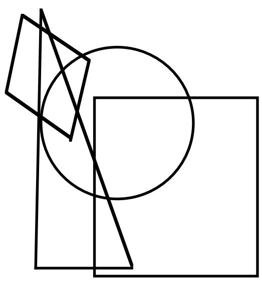
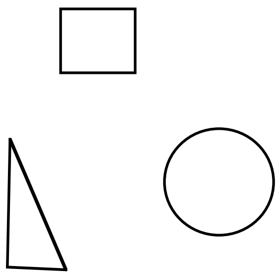
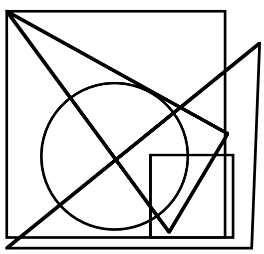
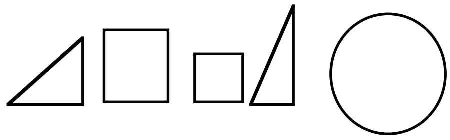
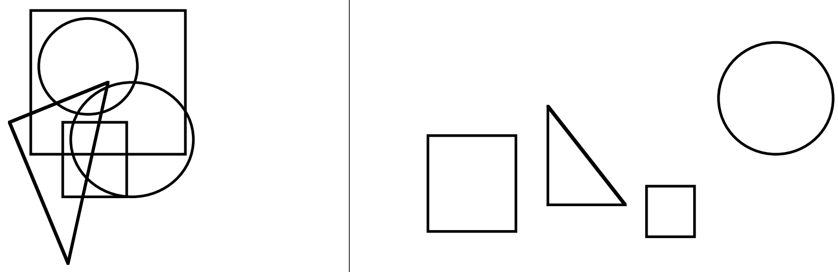
 1	
 2	
 3	
 4	
 5	



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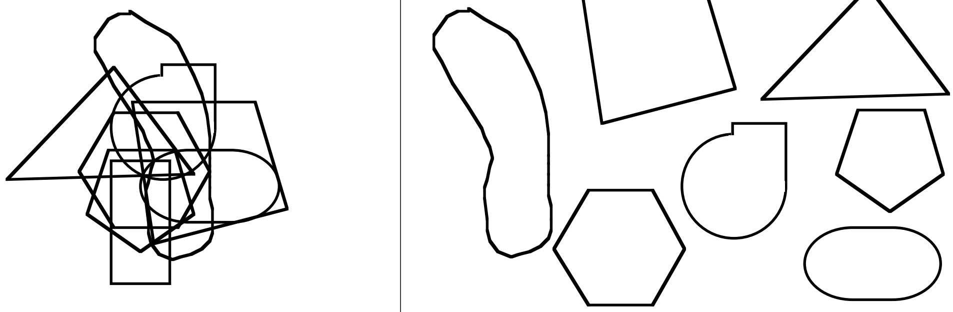

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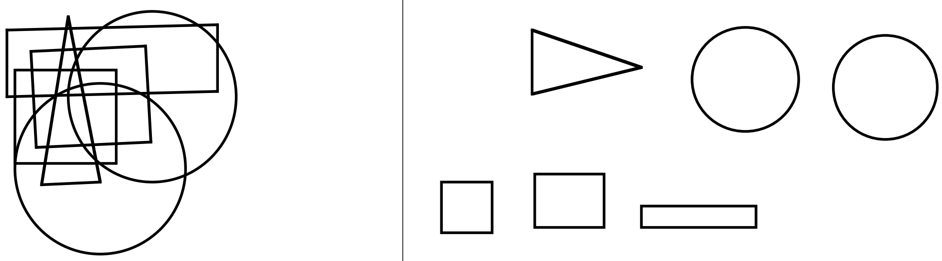

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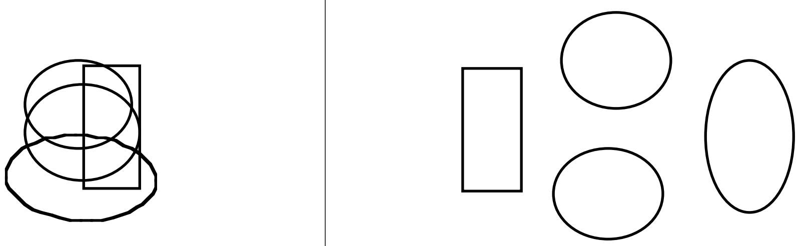

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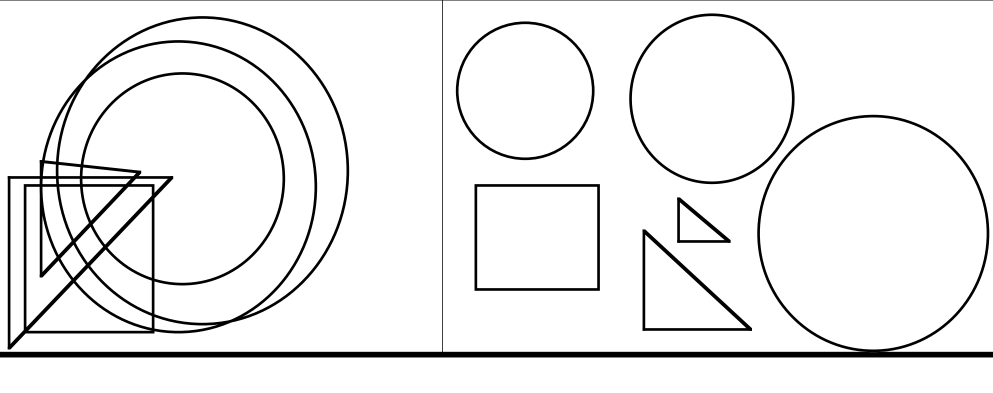

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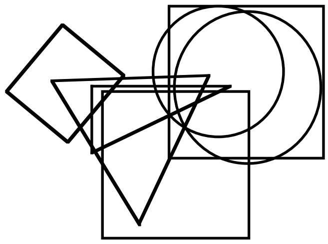
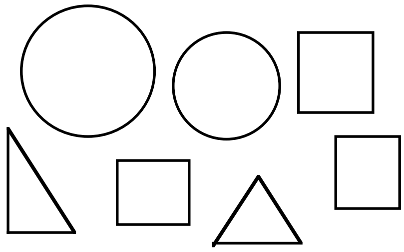
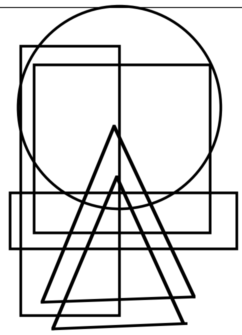
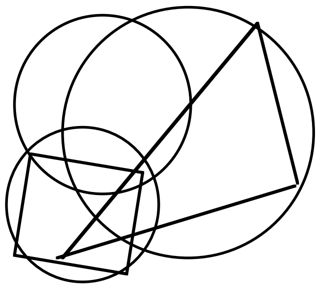
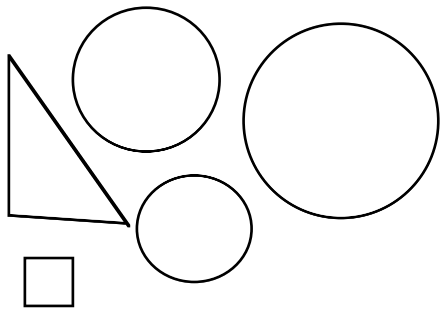
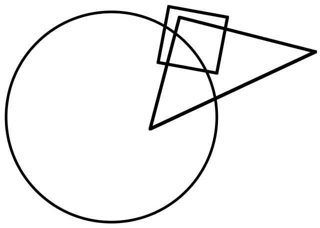
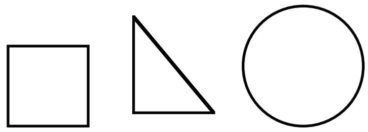
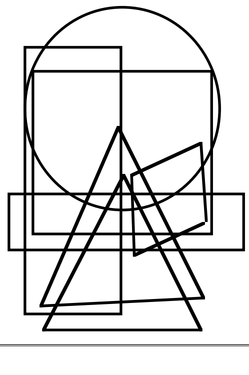
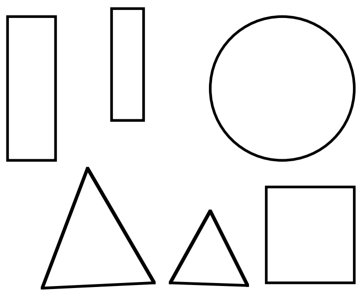
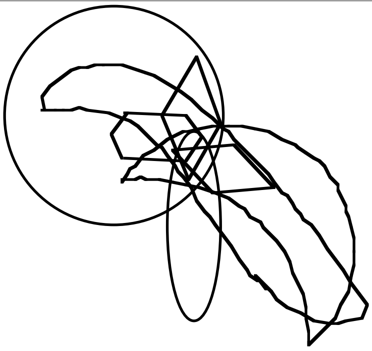
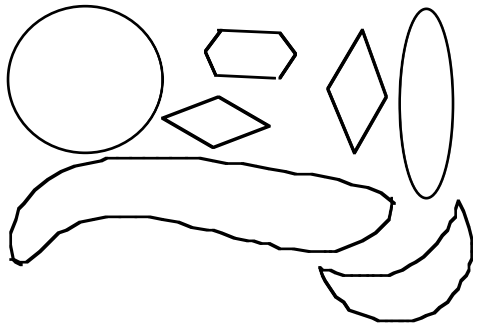
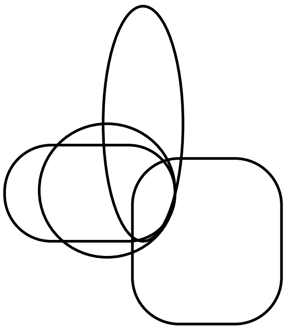
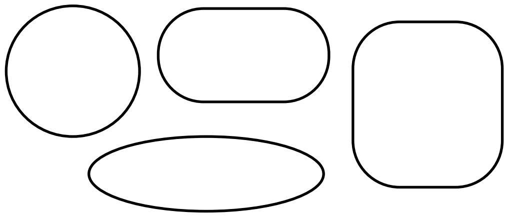
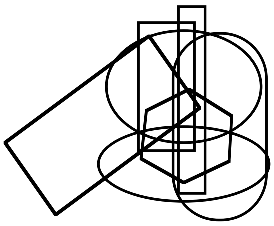
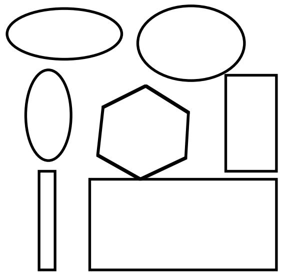
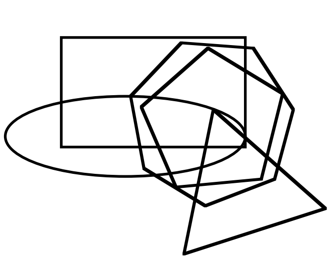
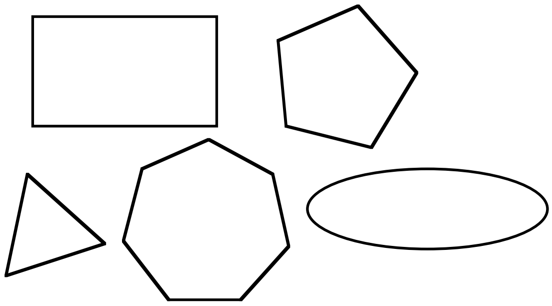
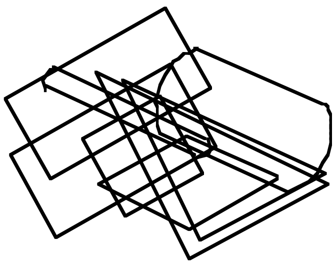
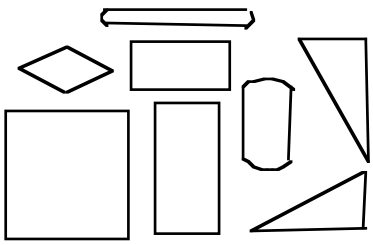
 11	
 12	
 13	
 14	
 15	

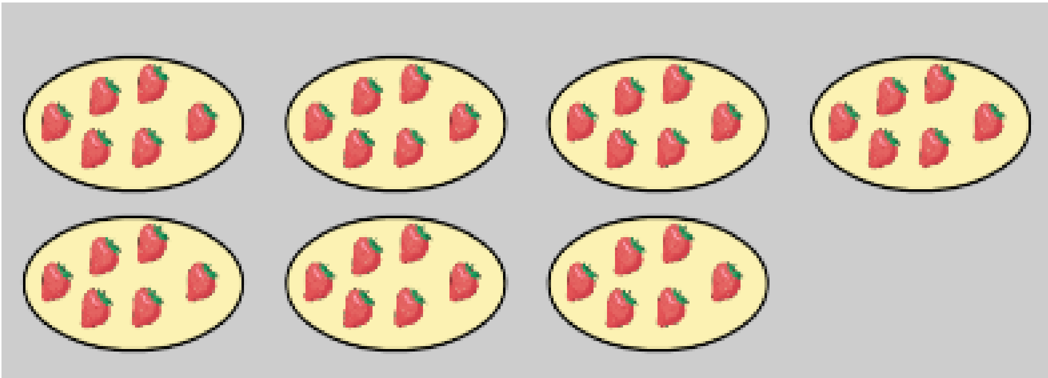
 16	
 17	
 18	
 19	
 20	

Appendix 3

Array Representation Figure

Array Representations

This is based on a paper by Tony Harries & Patrick Barmby (2007), entitled, “Representing and Understanding Multiplication”. For example, they described some common examples of ways of representing multiplication and below is what they called “plates of strawberries” representations: in the example below, 7 is multiplied by 6.



Their description of an ***array representation*** can be illustrated as follows:

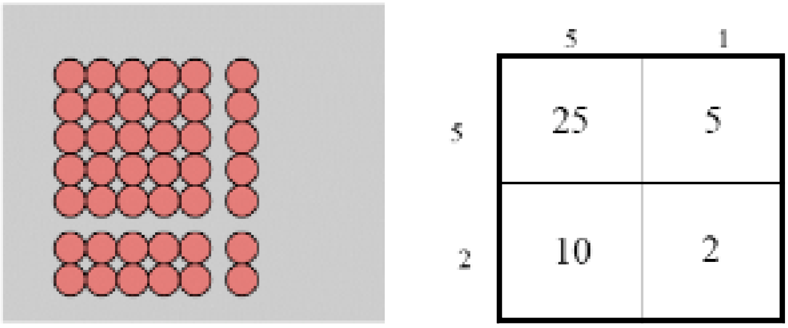


Figure 5: Linking the array representations with the grid method

Appendix 4

Correlation Data

Inter-Item Correlations

(Questions 1,2,3 and 5)

Each item of questions 1, 2, 3 and 5 were correlated with each other, using Kendall's Tau-b correlation. The other questions of the survey could not be correlated in this way because of their different format. These data were further analysed using Principal Components Analysis using Varimax Rotation to explore for the presence of any underlying factors (see chapter 12).

	1a	1b	1c	1d	1e	2a	2b	2c	2d	2e	2f	3a	3b	3c	3d	3e	5a	5b	5c	5d	5e
1b	0.34																				
1c	0.52	0.30																			
1d	-0.34	-0.32	-0.29																		
1e	-0.39	-0.23	-0.26	0.47																	
2a	0.45	0.30	0.50	-0.26	-0.27																
2b	0.44	0.35	0.47	-0.29	-0.29	0.51															
2c	0.28	0.41	0.29	-0.32	-0.23	0.32	0.35														
2d	-0.15	-0.17	-0.17	0.31	0.27	-0.18	-0.17	-0.17													
2e	-0.28	-0.21	-0.22	0.40	0.38	-0.24	-0.22	-0.23	0.36												
2f	0.22	0.29	0.23	-0.27	-0.19	0.24	0.28	0.33	0.21	-0.15											
3a	0.44	0.35	0.31	-0.30	-0.29	0.48	0.44	0.34	0.22	-0.26	0.32										
3b	-0.10	-0.29	-0.10	0.25	0.19	-0.05	-0.08	-0.09	0.18	0.29	-0.18	-0.05									
3c	0.35	0.37	0.35	-0.35	-0.26	0.38	0.42	0.43	-0.21	-0.23	0.38	0.48	-0.15								
3d	0.23	0.24	0.23	-0.22	-0.13	0.16	0.27	0.31	-0.11	-0.19	0.27	0.18	-0.13	0.28							
3e	0.10	0.18	0.08	-0.15	-0.05	0.11	0.16	0.22	-0.05	0.14	0.23	0.16	-0.12	0.27	0.22						
5a	0.34	0.26	0.26	-0.26	-0.25	0.33	0.31	0.17	-0.20	-0.27	0.20	0.25	-0.07	0.25	0.08	0.03					
5b	0.22	0.24	0.18	-0.13	-0.10	0.24	0.20	0.16	-0.11	-0.13	0.12	0.24	0.05	0.23	0.01	0.09	0.28				
5c	0.22	0.23	0.30	-0.24	-0.20	0.28	0.31	0.22	-0.17	-0.22	0.23	0.28	-0.10	0.29	0.15	0.07	0.31	0.27			
5d	0.24	0.43	0.27	-0.36	-0.25	0.26	0.29	0.33	-0.21	-0.22	0.29	0.25	-0.09	0.34	0.23	0.17	0.23	0.18	0.31		
5e	-0.12	0.03	-0.06	0.19	0.21	-0.06	-0.07	-0.02	0.17	0.16	-0.09	-0.06	0.26	-0.09	-0.05	-0.05	0.00	0.12	0.01	-0.05	
5f	0.07	0.01	0.05	-0.07	0.00	-0.03	0.02	0.05	0.12	0.00	-0.05	0.01	-0.04	0.19	0.08	0.00	-0.02	-0.01	0.04	0.12	-0.05

$p < 0.05$

$p < 0.01$

$p < 0.001$

Appendix 5

Statistical Notes

Statistics Used

The Chi-square Test ($\chi 2$)

The chi-square test is said to to be one of the most widely used tests for statistical data generated by non-parametric analysis. There are two different of applications of chi-square test. These are used in this study.

(1) Goodness of Fit Test

This tests how well the experimental (sampling) distribution fits the control (hypothesised) distribution. An example of this could be a comparison between a group of experimentally observed responses to a group of control responses. For example,

	Positive	**Neutral**	**Negative**	
Experimental	55	95	23	N(experimental) = 173
Control	34	100	43	N(control) = 177
				(using raw numbers)

A calculation of observed and expected frequencies leads to:

	Positive	**Neutral**	**Negative**
fo = observed frequency	*55*	95	23
fe = expected frequency	33	97	42

Where *fe* = [N(experimental)/N(control)] X (control data) or (173/177) X (control data)

The degree of freedom (df) for this comparison is 2. This comparison is significant at two degrees of freedom at greater than 1%. (**$\chi 2$** critical at 1% level = 9.21)

(2) Contingency Test

This chi-square test is commonly used in analysing data where two groups or variables are compared. Each of the variable may have two or more categories which are independent from each other. The data for this comparison is generated from the frequencies in the categories. In this study, the chi-square as a contingency test was used, for example, to compare two or more independent samples like, year groups, gender, or ages. The data is generated from one population group. For example,

	Positive	Neutral	Negative
Male (experimental)	55	95	23
Female (experimental)	34	100	43

(Actual data above)

	Positive	Neutral	Negative	**N**
Male (experimental)	55 (44)	95 (96)	23 (33)	**173**
Female (experimental)	34 (45)	100 (97)	43 (33)	**177**
Totals	**89**	**195**	**66**	**350**

(Expected frequencies above in red)

The expected frequencies are shown in red in brackets ( ), and are calculated as follows:

e.g. 44 = (173/350) x 89

$$\chi 2 = 2.75 + 0.01 + 3.03 + 2.69 + 0.09 + 3.03$$

$$= \mathbf{11.60}$$

At two degrees of freedom, this is significant at 1%. (**$\chi 2$** critical at 1% level = 9.21)

The degree of freedom (df) must be stated for any calculated chi-square value. The value of the degree of freedom for any analysis is obtained from the following calculations:

df = (r-1) x (c-1)
*where **r** is the number of rows and **c** is the number of columns in the contingency table.*

Limitations on the Use of $\chi 2$

It is known that when values within a category are small, there is a chance that the calculation of $\chi 2$ may occasionally produce inflated results which may lead to wrong interpretations. It is safe to impose a 10 or 5% limit on all categories. When the category falls below either of these, then categories are grouped and the df falls accordingly.

Correlation

It frequently happens that two measurements relate to each other: a high value in one is associated with a high value in the other. The extent to which any two measurements are related in this way is shown by calculating the correlation coefficient. There are three ways of calculating a correlation coefficient, depending on the type of measurement:

(a) With integer data (like examination marks), Pearson correlation is used. This assumes an approximately normal distribution.
(b) With ordered data (like examination grades), Spearman correlation is used. This does not assume a normal distribution.
(c) With ordered data where there are only a small number of categories, Kendall's Tau-b correlation used. This does not assume a normal distribution.

Sometimes, the two variables to be related use different types of measurement. In this case, none of the methods is perfect and it is better to use more than one and compare outcomes. It is possible to use a Pearson correlation when one variable is integer and other is dichotomous. The coefficient is now called a point biserial coefficient.

Factor Analysis

When a large number of measurements are made with a sample of people, the outcomes of these measurements may correlate with each other. The patterns of correlations obtained can sometimes arise because of some underlying reasons. Factor analysis examines all the measurements in all the variables with all the sample and explores whether there is some simple structure of variables which can offer an explorartion for the data obtained.

Principal Components Analysis is one method to explore the underlying structure. Ideally, any structure should account for around 70% of the variance and the Scree Plot can sometimes suggest the number of variables needed in the structure. The underlying structure is described in terms of components (factors) and the correlation of each of the original measurements are referred to as "loadings". Ideally, each original measurement should load highly onto a very small number of components.

The actual nature of the components can only be detemined by the researcher following exploration of each measurement and the way it loads onto the factors.