## Chapter

# Challenges Faced by Students With HEARING IMPAIRMENT THAT USE Portuguese Sign Language in Mathematics Classes. 

Joana Tinoco, Maria Helena Martinho* and Anabela Cruz-Santos

CIEd, Institute of Education, University of Minho, Braga, Portugal


#### Abstract

This study aims to understand which communication challenges are faced by students with hearing impairment in a Mathematics class. Hearing loss can hinder the communication that is established in the context of Mathematics classroom, since teacher and students are not fluent in the same language: teacher's first language is, mainly, Portuguese, while for students with hearing impairment it is the Portuguese Sign Language (PSL). Given the nature of the research this study follows a qualitative interpretative paradigm, based in a design of a case study with two students with hearing impairment


[^0]Author name
that attended the $6^{\text {th }}$ grade of an inclusive school that is a reference for bilingual education. The results show the communication difficulties experienced by the students which inhibit them from constructing a solid mathematical knowledge, whether in the interpretation of statements or in the interactions they establish, due to the low fluency in Portuguese Language in the written form and in PSL. These students demonstrated poor lexical knowledge in PSL when using the language to work mathematically, specifically with respect to the identification of mathematical terms for which there is no defined gesture.

Keywords: Communication; Hearing Impairment; Mathematics; Portuguese Sign Language.

## Introduction

Communication can be seen as a social process characterized by the interaction through the exchange of information, mutual influence in the negotiation and construction of meanings (Menezes et al. 2014). There has been an increasing interest within research in mathematics education on issues evolving communication, namely when the latter is established within the pedagogical relationship (Guerreiro et al. 2015). Such an interest is further witnessed by the number of recent research studies that have been conducted on this topic (Radford and Barwell 2016; Raymond 2018; Tinoco 2019; Zeegers and Elliot 2019). It is believed that if students are encouraged to become proficient in communication, with the teacher, their peers and even with their parents, they will have more opportunities to explore, organize and connect their thoughts, as well as to acquire new knowledge and different points of view (Cândido 2001; Smole 2001). In particular, in what concerns the mathematics class, such skils will entail the development of logic reasoning, and smooth the connection between mathematical concepts and the symbolic language in which they are expressed (Lestari, Saragih and Hasratuddin 2018; National Council of Teachers os Mathematics 2007; Raymond 2018). Futhermore, they contribute for students to feel included in, and commited to the class dynamics and the construction of knowledge (Douek 2005; Kaya and Aydin 2016; Zeegers and Elliot 2019).

Students with hearing impairment (HI) access the word differently. They are unique learners and distinctiveness must be taken into consideration in regard to their mathematical development (Pagliaro 2006). Although there is evidence that students with HI learn mathematics in a way that does not distinguish them from their peers, and there is no correlation between deafness level and performance in mathematics (Nunes 2012), in practice the learning process needs to be analysed
carefuly. An average gap of about two to three years and a half is observed when comparing school age and chronological age between both groups of students with and without hearing impairment (Baptista 2012; Swanwick, Oddy and Roper 2005; Traxler 2000).

Additionally, students with HI tend to leave school earlier, and therefore are less represented in higher school levels (Nunes 2012). This may be due to their limitations in interacting with their physical and social environments, which limits learning and development opportunities, as well as to the negative impacts resulting from discrimination and exclusion to which they are subjected over time (Melro and César 2010).

Despite the efforts that both the teacher and students with HI may make, it is important to be aware that often teacher and students do not share the Portuguese Sign Language (PSL) as their first language, nor are the teachers bilingual, i.e. fluent in both PSL and the Portuguese language (PL). Furthermore, there is a lack of teachers' training programmes to respond to the language needs of students with HI (Melro and César 2010). Most regular teachers being no fluent in PSL, inclusive schools provide a PSL interpreter in class. However, it has been found that the mere presence of a PSL interpreter in the mathematics class does not ensure, by itself, that the message effectively passes through the various actors in a teaching and learning situation. In part because, PSL is not suitable enough to represent mathematical ideas with the required clarity or depth (Rowley 2001), as many terminological gestures are not yet stabilized or standardized, and much of the specific mathematics vocabulary is not yet integrated in PSL (Almendra 2014; Carvalho 2013; Cruz-Santos and Martinho 2019; Tinoco, Martinho and CruzSantos 2018). Consequently, this contributes to hinder communication processes established in the context of the mathematics classroom. These language issues play a major role in the learning mismatch, but other complex factors cannot be overlooked, for example, related to cognitive development, social and emotional learning and educational experiences (Knoors and Marschark 2015), and their consequences in the learning of mathematics.

Thus, inclusive education cannot fully match the needs of students with HI in mainstream classrooms without adapting the teaching-learning process to their specific strengths and needs (Knoors and Marschark 2015).

The aim of this research was to understand which communication challenges are faced by students with HI in a mathematics class. Therefore, this study was refined into the following research questions: To what extend is PSL suitable to correctly express mathematical concepts and enable communication in mathematics? In which way does communication influence mathematical learning in students with Hearing Impairment?

Author name

As explained, these questions were assessed within mathematics classes in an educational context in which three forms of expression were analyzed, namely, the Portuguese Language (PL), Portuguese Sign Language (PSL), and the mathematics language. We analyzed the way students worked on the tasks presented, either in PL written form or translated into PSL; the way they interpreted the proposed tasks and undertook their realization, individually or in groups; how they justified their reasoning and argued mathematically within the classroom, and, finally, how they presented and discussed the results. From the introduction of the task to its completion, students worked on different mathematical representations. It was also our interest to understand which mathematical representations these students preferred to work and express themselves mathematically.

## Theoretical Context

According to the literature, most children with HI have hearing parents who are very little fluent, or not fluent at all, in sign language and, as such, cannot interact with them and help by providing an early exposure to sign language (Carvalho 2013; Sim-Sim 1999). Therefore, their linguistic and cultural involvement is essentially one of a listening character (Afonso 2007). Non, or reduced early exposition to PSL may bring additional difficulties to the development of student's communication systems (Freire and César 2007; Garcia 2013). This clearly puts such children in disadvantage with respect to language acquisition because the latter does not occur naturally, intuitively or spontaneously.

In these conditions, children are unable to timely perform the necessary appropriation of vocabulary, grammar, expressions, meanings and many other aspects of verbal expressions that are spontaneously acquired by their peers through family interactions or by listening to people around them, television or radio programs (Heward 2000; Ruiz and Ortega 1995; Sousa 2011; Vitova and Balcatova 2012). As a result, interactions between parents and child, and between the child and the school, are limited to a form of nonverbal communication through mime, some often unsuccessful oral communication, and a greater or lesser use of sign language. Actually, it seems consensual that the development of children with HI communication depends on the knowledge and motivation of the family and the school (Almeida et al. 2009; Amaral 1999). Typically, these students arrive at school without mastering the official language of their country (Ruela 2000) or with an underdeveloped language system, which constraints their
appropriation of different curriculum contents, as well as the development of meaningful and challenging interactions that would promote their intelectual development (Freire and César 2007). This also means that students' self-learning opportunities, as well as the use they made of written material may be compromised (Spencer and Marschark 2010).

For many students with HI, the school is the first (and often the unique) possibility to develop a complex language system (Freire and César 2007), both in terms of a common language and of knowledge of the school vocabulary, that enables them to access to national curricula in the various areas (Carvalho 2013). However, research shows that whenever the initiation to PSL takes place at school, a symbolic break with respect to the familiar context is most likely to occur. This can only be minimized with increased interaction with peers and adults with HI (Afonso 2008), which may constitute good linguistic models, stimulating students to learn and resort to this form of communication (Carvalho 2013).

In the Portuguese context, it was found that students with HI reveal difficulties not only in the use of spoken language, but also in the written form of the Portuguese language (Baptista 2008). Actually, basic language structures are often incorrectly used in what concerns morphology, syntax, coordination and subordination of sentences, terms and abstract operations and the understanding words with multiple meanings. These factors lead to a low capacity of understanding the structure of PL, as well as to difficulties in grasping the contents of simple and short written texts, or in using correct written expression. Limitations are also indicated for the reading process.

Students with hearing problems are also more vulnerable along the extensive mathematics learning experience (Heward 2000; Kritzer 2009; Traxler 2000; Vitova and Balcarova 2012; Vitova, Zdrazilová and Jezková 2014). Studies show that the understanding of mathematics by students with HI is significantly behind normal standards (Knoors and Marschark 2015; Tinoco, Martinho and CruzSantos 2018; Traxler 2000), eventhough they do not distinguish from their peers in what concerns cognitive skills. This is particularly evident in specific areas such as number (Genovese et al. 2005; Kritzer 2009; Nogueira, Borges and Frizzarini 2013; Nogueira, Zanquetta and Borges 2015; Nunes and Moreno 2002; Pagliaro and Kritzer 2013), geometry (Mason 2000), measurement (Nogueira, Zanquetta and Andrade 2011; Nunes and Moreno 2002; Pagliaro and Kritzer 2013), fractions (Costa and Silveira 2014; Silveira 2006), problem solving (Blatto-Vallee at al. 2007; Lee and Paul 2019; Pagliaro and Kritzer 2013), mental calculation, number comparison (Kritzer 2009; Pagliaro and Kritzer 2013).

However, the potential mismatch mentioned above should not be mistaken

Author name
with cognitive disabilities. Actually, it refers to the fact that when communication processes are compromised, as it is often the case of students with HI, learning and development in general are compromised regardless the potential these individuals may have (Borges and César 2012).

Language difficulties that students with HI have may play a major role in their relatively poor performance, namely getting confused with multiple meanings assigned to words, or multiple labels given to mathematical concepts, or the simple lack of familiarity with mathematical symbols and abreviations (Knoors and Marschark 2015). In problem solving tasks, the use of translations from spoken or written language to sign language may affect the difficulty of the problem itself. Problems could get easier with iconic cues potentially available from sign language (Ansell and Pagliaro 2006), and this may change the nature of the problem so that students with HI interpret it differently, and some times more easily than their peers.

It was emphasized that to understand a problem well enough to be able to start the problem-solving process, the students need first to understand the story underlying the problem, and the language in which it is expressed. In fact, there is a strong correlation between students with HI sign skills and their capacity to resort to-effective problem solving strategies (Pagliaro and Ansell 2012).

We have also to consider the role that communication plays in the mathematics classroom, since the teaching and learning of mathematics is an eminently communicative process (Guerreiro et al. 2015; Menezes et al. 2013). Promoting true communication situations within the mathematics class gives students an opportunity to organize, explore and clarify their thoughts, as mentioned in the introduction to this chapter. The level, or degree, of understanding of a given mathematical concept is closely related to the efficiency of the communication process involved, where students are asked to present their solutions, to guess, to discuss various mathematical representations, to explain their ways of solving, to prove and generalize (Franke, Kazemi and Battey 2007; Yackel and Cobb 1996). In a word, the process of understanding a given concept is enhanced by communication, just as communication is favored by understanding (Alrø and Skovsmose 2006; Cândido 2001).

One of the factors that interfere in the teaching and learning process of mathematics with students with HI is the difficulty of communication between these students and their teachers (Fernandes 2011; Most 2003; Pagliaro 2006). This is due to the mismatch between the language competencies of both groups (typically greater in PSL among students, and in PL among teachers) which offen prevents high quality conversation between them (Spencer and Marschark 2010), and full access to all aspects of the languages used in the class. This possibly
limits the effective learning of the relevant mathematical concepts (Pagliaro 2006).

When the teacher is not fluent in PSL, interpreters take the role of communication mediators, thus increasing the possibility of access for students with HI to formal communication. However, the 2 to 6 seconds that separates teacher's words from the interpreter's gestures contribute to the difficulty of participating in large group discussions within the classroom (Foster 1998; Stinson and Foster 2000). In cases where the student does not master PSL, the situation becomes even more complex.

The existence of an interpreter in the mathematics class does not mean per se that the message will pass through the various actors in the teaching and learning situation effectively (Müller and Gabe 2014). Even with the presence of a PSL interpreter students with HI are in disadvantage and learn less than their peers (Marschark et al. 2006). Moreover, it is very important that the person who translates a message to someone with HI, within a mathematics class, masters PL written form, mathematics and PSL so that communication can proceed as efficiently as possible (Costa and Silveira 2014), taking into account the bilingual education these students are exposed to in inclusive schools.

## Methodology

The research reported in this chapter followed a case study research design (Gerring 2007), with a qualitative methodology, within the framework of the interpretative research paradigm (Eisner 2017).

Two case studies are presented, identified as the Carla and the Daniel (referred here by fictitious names) cases. Both integrated a 6th grade classroom consisting of four students with HI, within a public shool considered a reference school for bilingual education of children with HI in the North of Portugal.

The first case is focused on Carla, a 11 years old girl, diagnosed with severe to profound deafness at 26 months (thus, at a pre-linguist stage). She wore bilateral prostheses. She is not able to lip reading efficiently, so she depends largely on the simultaneous translation performed by the PSL interpreter. She has been in bilingual classes since preschool, except for the 1st and 2nd grades, where she attended a regular inclusive class. Parents have low to medium levels of education and are not proficient in PSL. Nevertheless, they seem to be very present in her personal and school life, helping her with school tasks and valuing the role of the school in her education.

The second case is focused on Daniel, a 12 years old boy, diagnosed with

Author name
profound deafness, who had recently managed to make some sounds. He depends entirely on sign language or mimes. The hearing loss condition is suspected to date back to birth but was only diagnosed when he was 3 years old. He was clinically analyzed for a cochlear implant, but his parental choice was not to do so. In the $1^{\text {st }}$ school year, he attended a regular inclusive class, but moved to a bilingual class in his 2nd school year. His parents do not know PSL and communication within the family is done through mimes.

The research was mainly based on observation of a total of 58 mathematics lectures within the same class, each one lasting for fourty five minutes, making a total of 43.5 hours of observation. Consent was previously obtained from the school directive board, the teacher, the PSL interpreter, and the students' parents.

Different data collection instruments were used: the students' written productions within the mathematics class, either in the diary, worksheets or on the whiteboard; observation, through the researcher's field notes, as well as audio and video recordings, fully transcribed at a later stage. Three video cameras were used: one directed to the four students, another targeting the whiteboard and the teacher, the last one recording the PSL interpreter. All the recorded episodes concerning PSL dialogues were translated to PL by a different interpreter with no contact with the class, its interpreter and the school itself.

The observation was carried on by a researcher, the first author of this chapter, who made herself familiar with the class along a period of one month before the starting of the observation process. The observer was moderately familiar with PSL. During the classes she was discretely positioned in a corner of the room.

After a preliminary reading all the material produced by the students, as well as of the transcripts of the episodes, the text was marked according to the categories that were previously defined based on the objectives of the study. The episodes were selected in order to answer the research questions. Thus, three groups of categories were considered: interactions in the classroom, linguistic challenges and influence on the communicative process in learning mathematics. In order to characterize each case, all available elements were analyzed, in particular the class episodes, in search of correspondences that might help to understand the relevant behaviors, problems and contexts.

## Results

Carla presented difficulties on vocabulary and the structure of PL. Despite the use of bilateral prostheses, she had severe hearing loss and depends heavily on the

PSL interpreter for translations, constantly shifting her attention between the teacher and the PSL interpreter. To deal with this limitation Carla was always very attentive as the class progressed, trying to follow all the steps that were taken and questioning whenever she has any doubt.

When reading and interpreting written statements, Carla was unaware of some of the vocabulary used in everyday life: for example, production, marked, indicates, works of beneficiation, volume, one by one, proportionality or proportion. It should be noted that the meaning of any of these terms is considered basic vocabulary for students with the cognitive development typical of this age group. Thus, not mastering their meaning could make the interpretation of the tasks that were being solved impossible.

Whenever Carla had difficulties in the structure of PL, she tended to call the PSL interpreter. These situations occurred quite frequently along each class, especially when difficulties appeared when organizing an answer to a problem in written PL, or in the interpretation of Portuguese written statements.

In the situation presented in table 1 students were asked to determine the number of colour pencils and crayons that a boy had in his pencilcase, knowing that he had 12 colour pencils, and pencils and crayons were distributed in a proportion of 3 to 4 .

Table 1: dialogue on the number of colored pencils and crayons

| Oral dialogue in PL: | Parallel dialogue in PSL between the <br> interpreter and the student: |
| :--- | :--- |
| Theacher: (...) It is asking for the total <br> number os pencils in the <br> pencilcase. | Interpreter: colour pencils plus crayons, all <br> together. |
| Carla: $9+12$. |  |

In this case, Carla was not able to interpret the story underlying the problem, consequentely, she did not know what to do. Understanding this, the interpreter translated the story problem to PSL, but in the process of translating she gave the answer to the problem by mentioning "colour pencils plus crayons, all together".

In another occasion, students had to solve the problem whose statement contained the expression one by one. This expression caused her a great confusion, since it meant nothing to her. This would be enough to make the proposed task too difficult to interpreter and lead to poor performance in the
process.
Carla was also unaware of the meaning of the word proportionality, but tried to make sense of it by saying "proportion and nationality," using words that were familiar to her. When the teacher changed her question to proportion, she stopped guessing because she could no longer associate this term with any known words.

Over the course of several classes, we found that the translation of distinct mathematical terms was done using the same gesture. Such was the case for the concepts of a fraction, reason and proportion, which are formally distinct but were translated into PSL by the same sequence of gestures. This was previously accorded with the students, as they are omitted in the PSL.

Sometimes the questions posed by Carla were misinterpreted by the teacher, as one may identify in the dialogue given in table 2.

Table 2: example of misinterpretation of questions

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Oral dialogue in PL:
Carla: What's \(d m\) ?
Teacher: Is it your weight? What's being measured? The weight? What is the unit for the weight?
Carla: The weight...
(Another student, Beatriz, answered)
Beatriz: Kilogram.
Teacher: Kilogram. But could it [dm] be?
(Carla turns to the interpreter)
Carla: What's the mame for \(d m\) ?
Interpreter: Decimeter.
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In this dialogue, the student seems to be unaware of the mathematical representation for decimeter and the teacher, realizing the existence of gaps in knowledge triggers a dialogue that was not understood by the student. Facing this lack of understanding, Carla turned frequently to the PSL interpreter seeking her support.

The relationship between PSL and PL is not linear, since both have their own structure, which may conflit and lead to some sort of confusion. To illustrate this statement consider the following situation.

Carla had greater mastery of PSL than PL, so she developed all her reasoning using PSL. When asked to verbalize, Carla translated PSL directly into PL, which resulted in unstructured and non-conventionalized phrases (table 3).

Table 3: example of direct translation of PSL into PL

## Oral dialogue in PL:

Carla: Eight ... its half... eight is half...
Teacher: Eight is half of four? Is half more or less than the starting number?
Carla: More.
Teacher: And the double?
Carla: Eight times two.
Teacher: How much is the double of four?
(...)

Teacher: If it's the double it can't be half, isn't it?
Carla: Eight times two.
Teacher: It is not eight. It is four. Carla, you told me that eight was half of four. Isn't that what you said?
Carla: And four half two.
Teacher: And four is half of two? No! Two is half of four. Four is half of eight. It's the oposite. (...)

To say four is half than eight, in PSL one uses the sequence of gestures "eight - half - four", which means that half of eight is four. Carla, having less fluency in PL than in PSL, used in words this same sequence when she tried to explain to the teacher that half of eight was four. But the teacher did not realize that she was using verbaly the sequence used in the sign language, and wrongly signaled an error.

The non linear relationship between PSL and PL also raised a number of obstacles in issues related to reading numbers, namely integers composed of various digits, decimal numbers or numbers written in the form of a fraction. For example, a student may be unable to read the number 19683 because, in PSL, this number is read digit by digit, for example: "one-nine-six-eight-three", or by combining sequences of numbers as typically done for phone numbers, for example, "nineteen-six-eight-three".

In addition, difficulties also occurred when working with words with double significance, but similar in reading or writing. In this case, the problem arose with the term volume. The teacher was introducing this topic and chose to make a review to see if the students remembered what they had learned in previous years. When the teacher asked what the term volume meant, Carla suggested that it was related to the sound.

Carla's low level of hearing adds some barriers to her mathematical work. Actually, it was found that she was unaware of some of the common lexicon

Author name
expected to be known by her peers with typical development. She has also difficulties with phrase construction, either oral or written, actually proceeding in quite different way with respect to the usual construction in PL. This fact stems from the different structure of PL and PSL, which also explians difficulties in reading numbers and associating gestures to mathematical terms. The existence of words similar in spelling or phonetic values, but different in meaning, also underly several situations in which the student showed additional difficulties compared to what was expected from her peers.

Daniel had serious problems in reading PL and managing the meaning of words associated to everyday situations, such as produce, left over, check, market, reality, promotion or odd. For example, he was unaware of the meaning of checking, a word for which there is no gesture in PSL. If it had not been detected and explained by the PSL interpreter and the teacher, Daniel would not have been able to solve the proposed task, which asked to sort fractions using the major and minor symbols, because he simply didn't know the meaning of what was being asked.

In one class, the teacher was trying to differentiate percentage from the real value, stressing that, being different quantities, one could not operate with each other, and in particular, calculate the difference between them. The teacher mentioned a basket of oranges and asked the students what kind of fruit they could get from that basket. Daniel thought the question was about the number of oranges in the basket, rather than the kind of fruit in it, failing to understand what was being requested despite the simultaneous translation to PSL.

In Daniel's case, the sentence construction was only analyzed in its written component, due to the intrinsic characteristics of this student. Actually, the difficulties in writing in PL were evident from the written answers that concluded the resolution of a problem. Daniel seems to give answers detached from the questions, disregarding details such as units. It was found that his poor fluency in written PL led to almost imperceptible answers for someone out of context because of their atypical grammatical structure.

In other episodes the student only mastered the gesture corresponding to a given word but did not know its written form in PL, for example the words sticks or odd (table 4).

Table 4: example of mastery of the gesture corresponding to a given word without knowing its written form in PL


Note that, although in the end Daniel gives the right answer, qualifying 47 as an odd number, it is not clear whether he answered correctly because he understood the question and knew the answer, or by chance.

Similarly, the reading of numbers, or in the case of this student the writing of numbers in extension, was a linguistic challenge. For example, in one task students were supposed to work with the number 19683, and the teacher asked them to read it.

Given Daniel's difficulties in reading the number 19683, the teacher suggested alternatively that he reads the number 2013, corresponding to the current calendar year. Even in this case the student replied by reading the number digit by digit, as he does in PSL. Not accepting this answer, the teacher asked Daniel to go to the board and write down how to read the number. Daniel, however, was unable to do so, guessing that it would start with a thousand. Then, the PSL interpreter suggested that the student should start by writing two, without the teacher having asked for it. Understanding his difficulties, the teacher went to the board and helped Daniel to write the requested number.

Daniel showed significant limitations in what concerns acquisition and development of vocabulary for everyday use in PL. Further limitations were detected at the level of mathematical vocabulary as a result of the limited interactions he was subjected in his daily life due to his profound HI. These

Author name
limitations also explain his difficulties in sentence construction according to the linguistic aspects of the Portuguese language and the meaning of words with similar spellings. Finally, the reading of numbers composed of several digits resulted in a barrier for this student.

## DISCUSSION

This study showed that students experienced communication difficulties due to the low fluency in PL and in PSL. These difficulties prevent them from constructing a solid mathematical knowledge, both in what concerns the interpretation of statements and the interactions that they establish in the classroom. Both Carla and Daniel, whose cases were presented in this chapter, have parents that do not communicate fluently in PSL. Hence both students may have been exposed to limited social and familiar interactions, which might have lead to a poor development and learning of the language system. Actually, it is well-known that these limitations can lead to constraints regarding the appropriation of different curricular contents, and the development of meaningful interactions which promote continuous learning development (Genovese et al. 2005; Lee and Paul 2019; Vitova and Balcarova 2012; Vitova, Zdrazilová and Jezková 2014).

The low fluency in PL, particularly in its written form, clearly emerged from the observation of the classes. It was reflected in the difficulties of interpretation of written problem and task statements, as well as the construction of the corresponding responses. It was noticed that students, after reading an assignment individually, waited either for the teacher or the interpreter to instruct on how to proceed (Daniel), or to confirm if their understanding was adequate (Carla). Then the teacher provided the explanation required, and the PSL interpreter translated it in detail, listing and giving meanings to each word that may require such an explanation. This often led to the resolution of the task by the teacher or the PSL interpreter with the student's collaboration, influencing both the student's performance and the development of his/her resolution strategies (Rahmi et al. 2017; Yackel and Cobb 1996).

Daniel was the one who had the greatest difficulty in interpreting written utterances, always needing their translation to PSL. However, it was also evident that neither with the translation to PSL or any further explanation, the utterances were fully understood, which leads to consider Daniel's low fluency in writing both in PL and PSL. In the same sense, Vitova, Zdrazilová and Jezková (2014) pointed out reading difficulties by students with HI.

There are several words or expressions which have been identified as part of the common lexicon of students with typical cognitive development but unknown to these students. Examples include the notions of volume, works of beneficiation, owner, production (of apples), (price) marked, indicates (the terms), one by one, proportionality, proportion, checking, municipal market, reality, among others. As discussed elsewhere (Genovese et al. 2005; Lee and Paul 2019), this may potentially interfere with the quality of students' mathematical work.

Another challenge concerns words which exhibited the same spelling in PL but have different meanings depending on the context. For example, volume (sound or as occupied space), reason (mathematical or philosophical) or concentrated (concentrated juice or attention).

The word volume was associated, by Carla, with the sound level, rather than thought as a denotation of the "space occupied by an object", as intended. Carla also questioned the meaning of the word reason, which for her was not a mathematical, but a philosophical term associated with being right about a cause or motive. Daniel could not figure out what the correct meaning to use for the written word one, whether the number 1 or the definite article. This leads us to conclude that the need to know and distinguish the various meanings of a word in order to decide which one makes more sense in a given context added difficulty for these students.

The production of written texts was another challenge. The need to write a text in PL, for example, to give a complete answer to a problem or justify a piece of reasoning, raised an additional difficulty for these students, leading them to seek help, particularly from the PSL interpreter.

When asked for some written explanation or justification, the students showed little fluency, producing poorly written texts in terms of syntactic construction. They had difficulties in phrasing and ordering, and used a very unusual structure, far from the defined standards for PL in the written form, and producing answers with phrases that seemed to be detached from the question. Similar problems have been identified by Lee and Paul (2019).

The teacher's misinterpretation of the student's speech when it follows a literal translation from the gestual language, led to embarrassment on both sides. For example, the sequence of words used to refer to half a number is different in both languages. In PL one says that 50 is half of 100 , while in PSL the same sentence is expressed through the sequence of gestures 100 _half_50.

The same problem appears in reading and writing numbers composed of several digits, or of decimal numbers, since in PL the reading respects the positional value of each digit, unlike what happens in PSL where reading is done digit by digit, making it difficult to memorize a non trivial word-number
sequence. Difficulties in reading decimal numbers were obstacles to the comparison of these numbers. Actually, disregarding the positional value of each digit leads to the loss of the decimal system regularities. Kritzer (2009) also points out difficulties associated with understanding the concept of number, namely in what concerns reading and writing.

Along the class observations, we noticed that the use of the gesture which designates the word equal was also used, following an interpreter's suggestion, for equivalent. This led to unclear dialogues whenever the teacher tried to distinguish between these two concepts. Also, when working with equivalent fractions, the translation was equal fractions, which is of course problematic in terms of mathematical correctness. Similarly, the combined gesture for fraction and reason was the same which led to equally strange episodes when the teacher tried to distinguishing between them. Costa and Silveira (2014) also identified difficulties associated with understanding the concept of fraction.

In Daniel's case, we found some confusion between the use of the multiplication sign, which was often mistaken for addition as the gestures corresponding to these concepts are very close. In other cases, Daniel knew both the word and its meaning in PSL, but was unable to recognise it in written form of PL. This was not due to the fact that he didn't recognize the operation or the words themselves, but because he could not make an association with the word written in the PL.

In what concerns discussion or written justification of results, we found an equally poor scenario. Students rarely went to the board to solve tasks. When the teacher asked someone to do it and to explain to the other classmates what he or she had done, they just described, as simply as possible, the steps they had written, translating the mathematical language used in the resolution to PL and PSL, without adding any justification.

The students in the two cases discussed in this chapter are not proficient in either PSL or PL written form, which triggers linguistic challenges that they must strive to overcome. We found several kinds of difficulties concerning the knowledge of common and frequently used PL vocabulary, lack of gestures in PSL to designate mathematical terms and the influence of combined gestures on mathematical knowledge, difficulties in correctly reading numbers composed of several digits or decimal numbers, and difficulties in understanding the teacher. In general, due to the different structures of PL and PSL, students have shown several difficulties in discussing or justifying their reasoning, either in PL written form, or in PSL.

## Conclusion

The focus of this study was to understand the communication challenges faced by students with HI in the context of the mathematics class in an inclusive school with a bilingual education curriculum suitable for this group of students. The analysis of two case studies provided evidence that most students experienced communication difficulties due to the low fluency in both PL and PSL. These difficulties, detailed in the previous sections, clearly emerge when interpreting problem statements or trying to discuss mathematical topics in the classroom. In a broader sense, they prevent the construction of a solid and operational mathematical knowledge. Similar findings are documented in the literature (Genovese et al. 2005; Lee and Paul 2019; Rahmi et al. 2017; Rowley 2001; Vitova, Zdrazilová and Jezková 2014).

The study reported in this chapter contributes at two levels with respect to the current research in the area.

Firstly, it focuses on the Portuguese speaking context were research results on this topic are scarce. We provide evidence that PSL is not sufficiently expressive to represent mathematical concepts with the necessary clarity and depth. In particular, we found that many gestures for mathematics are not yet established, and much of the specific vocabulary of the mathematics discipline is not yet integrated into the PSL lexicon. To fill these gaps, often the interpreter has to spell a word in PSL, where each letter is represented by a gesture for Portuguese, or even to create new gestures for the teaching content. We identified several gestures used in class (e.g. to capture notions like ratio, fraction, proportion, and equivalent) that are not part of the PSL gestuary, and as such only assume mathematical meaning for a specific group of students, namely those that were exposed at that specific class/teacher/interpreter.

As a second contribution, this study, based on the continued observation of two students in a classroom context over several months, showed that, although they attended an inclusive school that is a reference for hearing loss, with adequate human and material resources (for example, the presence of a interpreter in all classes), several communication difficulties remain. Actually, students with HI have to be considered as individuals that possess their own language and their own ways of apropriating and construct knowledge. Language has a major influence in the interpretation of the tasks proposed in the classroom. To succeed in these classes, and to be included, students must understand what is the meaning of what is being taught. They must be able to discuss the contents in order to interpret correctly. But to do so they need to understand PL in the written form and PSL. Otherwise the translations will often introduce misunderstandings or erode the precise meaning of the problem statement.

Teaching mathematics to students with HI requires the development of a number of skills to bridge the gap between the world of students with HI and those of their peers, which are often in clear pedagogical opposition. Ideally, teachers should master PSL, Mathematics, and the Portuguese Language, as well as specific teaching and learning methodologies that take into account the characteristics of these subjects (Neves and Silva 2011). In order to provide a good mathematical learning experience to students with HI , it is not enough to have a PSL interpreter in the classroom. It is also necessary to include these students within the school community, through the collaborative and a universal design of inclusive activities, according to the unique linguistic and communicative characteristics of these students in order to offer a bilingual education, as established by the Portuguese Law for an Inclusive Education for all students with special needs.

All the those aspects emerged in this study, although this research study was limited to a school grade and carried on in the context of an inclusive school for hearing loss. Further research is deeply needed. First of all, similar studies should be conducted at different school years, and younger ages, to gather a broader body of knowledge. Equally interesting would be to focus specific research efforts on interpreters with a reasonable knowlegde in mathematical contents, or teachers proficient in PSL.

A final, main perspective for future work concerns to use all the knowledge gathered to extend PSL to better capture the details of mathematical expression. This is a most pressing task for Portugal in order to promote a more inclusive education, in the spirit of the United Nations Sustainable Developement Goals 4 and 10 .

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[^0]:    * Corresponding author e-mail: mhm@ie.uminho.pt

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