TWMS J. App. and Eng. Math. V.11, Special Issue, 2021, pp. 42-50

FUZZY ZERO DIVISOR GRAPH IN A COMMUTATIVE RING

A. KUPPAN[1], J. RAVI SANKAR[1], §

Abstract. Let R be a commutative ring and let $\Gamma(Z_n)$ be the zero divisor graph of a commutative ring R, whose vertices are non-zero zero divisors of Z_n, and such that the two vertices u, v are adjacent if n divides uv. In this paper, we introduce the concept of fuzzy zero zivisor graph in a commutative ring and also discuss the some special cases of $\Gamma_f(Z_{2p})$, $\Gamma_f(Z_{3p})$, $\Gamma_f(Z_{5p})$, $\Gamma_f(Z_{7p})$ and $\Gamma_f(Z_{pq})$. Throughout this paper we denote the Fuzzy Zero Divisor Graph(FZDG) by $\Gamma_f(Z_n)$.

Keywords: Fuzzy graph, Zero divisor graph, Fuzzy zero divisor graph(FZDG).
AMS Subject Classification: 05C25, 05C69.

1. Introduction

The zero divisor graph is very useful to find the algebraic structures and properties of rings. The idea of a zero divisor graph of a commutative ring was introduced by I. Beck's in [2]. Given a ring R, let G(R) denote the graph whose vertex set is R, such that distinct vertices r and s are adjacent provided that rs = 0. I.Beck's main interest was the chromatic number $\chi(G(R))$ of the graph G(R).

Rosenfeld [7] defined a fuzzy graph as a graph that consists of vertices and edges with membership value in the interval [0,1]. More specifically, he defined a fuzzy graph as a pair $G = (\sigma, \mu)$ of functions $\sigma : S \to [0, 1]$ and $\mu : S \times S \to [0, 1]$ where for all $x, y \in S$ we have $\mu(x, y) = \mu(y, x)$ and $\mu(x, y) = \sigma(x) \wedge \sigma(y)$ with $\wedge$ denoting the minimum. The general terminology, notation everything based on the papers [[1], [3] $-$ [6]].

In this paper, an attempt to combine the two concepts: Fuzzy graph theory and zero divisor graph of a commutative ring together by introducing a new concept called fuzzy zero divisor graph of commutative ring. Finally, we discuss some specal cases of $\Gamma_f(Z_{2p})$, $\Gamma_f(Z_{3p})$, $\Gamma_f(Z_{5p})$, $\Gamma_f(Z_{7p})$ and $\Gamma_f(Z_{pq})$.

2. Preliminaries

Definition 2.1. *[8] A fuzzy graph as a pair $G = (\sigma, \mu)$ of functions $\sigma : S \to [0, 1]$ and $\mu : S \times S \to [0, 1]$ where for all $x, y \in S$ we have $\mu(x, y) = \mu(y, x)$ and $\mu(x, y) = \sigma(x) \wedge \sigma(y)$ with $\wedge$ denoting the minimum.*

[1] Department of Mathematics, Vellore Institute of Technology, Vellore, Tamil Nadu, 632014, India.
e-mail: kuppanmyname@gmail.com; ORCID: https://orcid.org/0000-0001-7874-0971.
e-mail: ravisankar.j@vit.ac.in; ORCID: https://orcid.org/0000-0001-9094-503X.
§ Manuscript received: October 19, 2019; accepted: April 8, 2020.

Definition 2.2. *[1] Let R be a commutative ring (with 1) and let $Z(R)$ be its set of zero-divisors. We associate a (simple) graph $\Gamma(R)$ to R with vertices $Z(R)^* = Z(R) - 0$, the set of nonzero zero-divisor of R, and for distinct $x, y \in Z(R)^*$ the vertices x and y are adjacent if and only if $xy = 0$. Thus $\Gamma(R)$ is the empty graph if and only if R is an integral domain.*

3. Fuzzy Zero Divisor Graph

Definition 3.1. *An fuzzy zero divisor graph(FZDG) is of the form $G =< V, V_f, E_f >$ then the vertex set of non-zero zero divisor graph is*

$$\Gamma(Z_n) = V = \{(u, v) : uv = 0 \ (multiplication\ modulo\ n), \forall u, v \in V\} \tag{1}$$

Let p and q are any prime numbers with $p < q$. Then

$$V_f = \left\{V_1^f \cup V_2^f \mid \forall\ V_1^f, V_2^f \in (0,1)\right\} \ such\ that\ V_f : V \to (0,1) \tag{2}$$

where $V_1^f = \left\{\frac{1}{p}, \frac{2}{p}, \frac{3}{p}, ..., \frac{p-1}{p}\right\}$ and $V_2^f = \left\{\frac{1}{q}, \frac{2}{q}, \frac{3}{q}, ..., \frac{q-1}{q}\right\}$, $0 < V_f < 1$.

$$E_{pq}^f = \left\{E_q^f, E_{2q}^f, E_{3q}^f, ..., E_{pq}^f \mid \forall\ E_{jq}^f \in (0,1]\right\} \ such\ that\ E_f : V \to (0,1] \tag{3}$$

where $j = 1, 2, 3, ..., p$, $0 < E_f \le 1$.
If any one of the condition is not satisfied, then the graph G is not an FZDG.

Definition 3.2. *A fuzzy graph $G : (V_f, E_f)$ is said to be a fuzzy star graph if $\Gamma(Z_n)$ is a zero divisor graph where n=2p and $p > 2$.*

Theorem 3.1. *[4] For (Z_{2p}), where p is any prime number then $\gamma_c(\Gamma(Z_{2p})) = 1$. Also, if n=8,9 then $\gamma_c(\Gamma(Z_n)) = 1$.*

Theorem 3.2. *[4] In (Z_{3p}) where p is any prime with $p > 3$, then $\gamma_c(\Gamma(Z_{3p})) = 2$.*

Theorem 3.3. *[4] If $p > 5$ is any prime, then $\gamma_c(\Gamma(Z_{5p})) = 2$.*

Theorem 3.4. *[4] For any graph (Z_{7p}) where p is any prime with $p > 7$, then $\gamma_c(\Gamma(Z_{7p})) = 2$.*

Theorem 3.5. *[4] If p and q are distinct primes and $q > p$, then $\gamma_c(\Gamma(Z_{pq})) = 2$.*

Theorem 3.6. *If $n = 2p$ where p is any prime and $p > 2$ then $\Gamma_f(Z_{2p})$ be the non-zero fuzzy zero divisor graph is $K_{1,p-1}^f$ fuzzy star graph.*

Proof. Let $\Gamma(Z_{2p})$ be a non-zero zero divisor graph. Then the vertex set of non-zero zero divisor graph $V = \{(u, v) : uv = 0 \ (multiplication\ modulo\ n), \forall u, v \in V\}$. Take two distinct vertex sets V_1 and V_2 in $\Gamma(Z_{2p})$ where $V_1 = \{2\}$ and $V_2 = \{2, 4, 6, ..., 2(p-1)\}$ then clearly we know that a vertex $2 \in V_1$ adjacent to all the vertices in V_2.

Clearly, $\Gamma(Z_{2p})$ is isomorphic with $K_{1,p-1}$.

Case(i): If V_f is fuzzy vertex set. The vertex set of fuzzy zero divisor graph V_f is partition in to two vertex subsets namely, V_{f_1} and V_{f_2}. Let

$$V_{f_1} = \left\{\frac{1}{p}, \frac{2}{p}, \frac{3}{p}, ..., \frac{p-1}{p}\right\}$$

where p is any prime number with $p > 2$.
That is,

$$V_{f_1} = \left\{ \frac{N}{p} \mid N = 1, 2, 3, ..., p-1 \right\}$$

with $V_{f_1} : V \to (0,1)$. Let $V_{f_2} = \left\{\frac{1}{2}\right\}$, with $V_{f_2} : V \to (0,1)$
Since,

$$\begin{aligned} V_f &= V_{f_1} \cup V_{f_2} \\ V_f &= \left\{ \frac{1}{p}, \frac{2}{p}, \frac{3}{p}, ..., \frac{p-1}{p} \right\} \cup \left\{ \frac{1}{2} \right\} \\ &= \left\{ \frac{1}{p}, \frac{2}{p}, \frac{3}{p}, ..., \frac{p-1}{p}, \frac{1}{2} \right\} \\ &= \{V_{f_1} \cup V_{f_2}\} : V \to (0,1) \\ V_f &=: V \to (0,1) \end{aligned}$$

Hence V_f be the fuzzy vertex set.

Case(ii): If E_f is fuzzy edge set. Let as take any two vertices $u, v \in V(\Gamma(Z_{2p}))$ and

$$\Gamma(Z_{2p}) = \{(u,v) : uv = 0 \ multiplication\ modulo\ n, \forall u, v \in V\}.$$

Let $u = 2(p-1)$ and $v = p$ then $uv = 2(p-1).p = 2p(p-1)$. Clearly, 2p must divides $2p(p-1)$, then there exist a edge connect between u and v.
Let E_f be a collection of edges.

$$E_f = \left\{ 1, \frac{1}{2}, \frac{1}{3}, ..., \frac{1}{p-1} \right\},$$

where p is any prime number with $p > 2$.

$$E_f = \left\{ \frac{1}{N} \mid N = 1, 2, 3, ..., p-1 \right\}.$$

Thus, clearly $E_f : V \to (0,1]$.
Clearly we know that every vertex in V_{f_1} is adjacent to all the vertices in V_{f_2}. Hence the graph $K^f_{1,p-1}$ is fuzzy star graph.

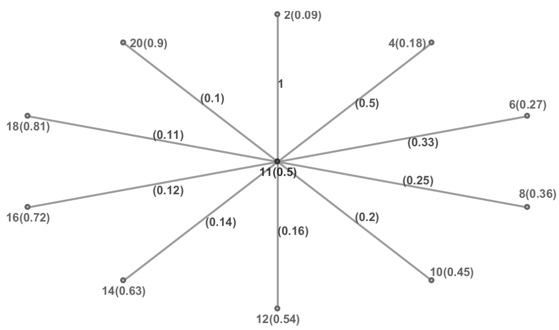

FIGURE 1. $\Gamma_f(Z_{22})$

□

Theorem 3.7. *If $n = 3p$ where p is any prime and $p > 3$ then $\Gamma_f(Z_{3p})$ be the non-zero fuzzy zero divisor graph is $K^f_{2,p-1}$ fuzzy complete bipartite graph.*

Proof. Let p is any prime number with greater than 3. Then the vertex set of $\Gamma(Z_{3p})$ is $V = \{(u, v) : uv = 0 \ \textit{multiplication modulo} \ n, \forall u, v \in V\}$. Take two distinct vertex sets V_1 and V_2 in $\Gamma(Z_{3p})$ where $V_1 = \{p, 2p\}$ and $V_2 = \{3, 6, 9, ..., 3(p-1)\}$. Every vertex in V_1 is adjacent to all the vertices in V_2. Clearly, $\Gamma(Z_{3p})$ is a complete bipartite graph, namely $K_{2,p-1}$.
Let V_f be a fuzzy vertex set. Let us show that $V_f : V \to (0, 1)$.
Fuzzy vertex set V_f is partition in to two vertex subsets, namely V_{f_1} and V_{f_2}.
Let

$$V_{f_1} = \left\{\frac{1}{p}, \frac{2}{p}, \frac{3}{p}, ..., \frac{p-1}{p}\right\},$$

where p is any prime with $p > 3$.
That is,

$$V_{f_1} = \left\{\frac{N}{p} \mid N = 1, 2, 3, ..., p-1\right\},$$

with $V_{f_1} : V \to (0, 1)$.

Let $V_{f_2} = \left\{\frac{1}{3}, \frac{2}{3}\right\}$ with $V_{f_2} \to (0, 1)$
Clearly,

$$\begin{aligned} V_f &= V_{f_1} \cup V_{f_2} \\ &= \left\{\left(\frac{1}{p}, \frac{2}{p}, \frac{3}{p}, ..., \frac{p-1}{p}\right) \cup \left(\frac{1}{3}, \frac{2}{3}\right)\right\} \\ &= \left\{\frac{1}{p}, \frac{2}{p}, \frac{3}{p}, ..., \frac{p-1}{p}, \frac{1}{3}, \frac{2}{3}\right\} \end{aligned}$$

where p is any prime with $p > 3$.

Thus, $V_f = \{V_{f_1} \cup V_{f_2}\} : V \to (0, 1)$ which implies that $V_f : V \to (0, 1)$
Hence, V_f is a fuzzy vertex set.

Let E_f be a fuzzy edge set. Let as show that $E_f : V \to (0, 1]$.
Let as take any two vertices $u, v \in V(\Gamma(Z_{3p}))$. Edge set E(G) defined by
$E(G) = \{(u, v) : uv = 0 \ (\textit{multiplication modulo} \ n), \forall u, v \in V\}$. Let u = p, v = 2p or u = 2p, v = p. Then $uv = 2p \times p = 2p^2$ which does not divide by 3p.

Therefore u and v are non-adjacent vertices in $\Gamma(Z_{3p})$. Let x be any other vertex in $\Gamma(Z_{3p})$ such that $ux = vx = 0$. That is the remaining vertices in $\Gamma(Z_{3p})$ are adjacent to both u and v.

Let E_p^f, $E_{2p}^f \in E_f$, where E_p^f and E_{2p}^f are collection of fuzzy edges from p and 2p respectively. since vertex p and 2p are the adjacent to all the vertices in $\Gamma(Z_{3p})$.

$$E_p^f = \left\{1, \frac{1}{2}, \frac{1}{3}, ..., \frac{1}{p-1}\right\}$$

$$E_{2p}^f = \left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, ..., \frac{1}{2(p-1))}\right\}$$

$$E_f = \left\{E_p^f, E_{2p}^f\right\}$$

$$E_f = \left[\left\{1, \frac{1}{2}, \frac{1}{3}, ..., \frac{1}{p-1}\right\}, \left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, ..., \frac{1}{2(p-1))}\right\}\right]$$

where p is any prime number with $p > 3$.
which implies $E_f : V \to (0, 1)$. Hence E_f is a fuzzy edge set. Clearly, $\Gamma_f(Z_{3p}) = (V, V_f, E_f)$ be a fuzzy zero divisor graph in $\Gamma_f(Z_{3p})$ and we know that every vertex in V_{f_1} is adjacent to all the vertices in V_{f_2}. Hence that the graph $K_{2,p-1}^f$ is fuzzy complete bipartite graph. □

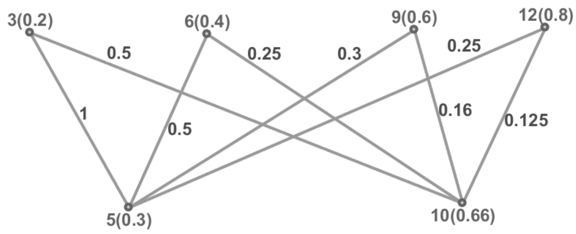

FIGURE 2. $\Gamma_f(Z_{15})$

Theorem 3.8. *If $n = 5p$ where p is any prime and $p > 5$ then $\Gamma_f(Z_{5p})$ be the non-zero fuzzy zero divisor graph is $K_{4,p-1}^f$ fuzzy complete bipartite graph.*

Proof. Let p is any prime number, greater than 5. Then the vertex set of $\Gamma(Z_{5p})$ is $V = \{(u, v) : uv = 0 \text{ multiplication modulo } n, \forall u, v \in V\}$ where n=3p. Take two distinct vertex sets V_1 and V_2 in $\Gamma(Z_{5p})$ where $V_1 = \{p, 2p, 3p, 4p\}$ and $V_2 = \{5, 10, 15, ..., 5(p-1)\}$. Every vertex in V_1 is adjacent to all the vertices in V_2. Clearly, $\Gamma(Z_{5p})$ complete bipartite graph, namely $K_{4,p-1}$.

Let V_f be a fuzzy vertex set. Let as show that $V_f : V \to (0, 1)$. Fuzzy vertex set V_f is partition in to two vertex subsets, namely V_{f_1} and V_{f_2}.
Let

$$V_{f_1} = \left\{\frac{1}{p}, \frac{2}{p}, \frac{3}{p}, ..., \frac{p-1}{p}\right\},$$

where p is any prime number with $p > 5$.
That is,

$$V_{f_1} = \left\{\frac{N}{p} \mid N = 1, 2, 3, ..., p-1\right\},$$

with $V_{f_1} : V \to (0, 1)$.

Let $V_{f_2} = \left\{\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}\right\}$ with $V_{f_2} \to (0,1)$
Clearly,

$$\begin{aligned} V_f &= V_{f_1} \cup V_{f_2} \\ &= \left\{ \left(\frac{1}{p}, \frac{2}{p}, \frac{3}{p}, ..., \frac{p-1}{p} \right) \cup \left(\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5} \right) \right\} \\ &= \left\{ \frac{1}{p}, \frac{2}{p}, \frac{3}{p}, ..., \frac{p-1}{p}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5} \right\} \end{aligned}$$

where p is any prime number with $p > 3$.

Thus, $V_f = \{V_{f_1} \cup V_{f_2}\} : V \to (0,1)$ which implies that $V_f : V \to (0,1)$
Hence, V_f is a fuzzy vertex set.

Let E_f is fuzzy edge set. Let as show that $E_f : V \to (0,1]$. Let as take any two vertices $u, v \in V(\Gamma(Z_5p))$. Let $u = 2p$ and $v = 3p$ in $\Gamma(Z_{5p})$ then 5p does not divides $uv = 6p^2$, which implies that no two vertices of V_1 and V_2 are adjacent.

Let E_p^f, E_{2p}^f, E_{3p}^f, $E_{4p}^f \in E_f$, where E_p^f, E_{2p}^f, E_{3p}^f and E_{4p}^f are collection of fuzzy edges from p, 2p, 3p and 4p respectively. Since vertex p, 2p, 3p and 4p are the adjacent to all the vertices in $V_2(\Gamma(Z_{5p}))$.

$$\begin{aligned} E_p^f &= \left\{ 1, \frac{1}{2}, \frac{1}{3}, ..., \frac{1}{p-1} \right\} \\ E_{2p}^f &= \left\{ \frac{1}{2}, \frac{1}{4}, \frac{1}{6}, ..., \frac{1}{2(p-1)} \right\} \\ E_{3p} &= \left\{ \frac{1}{3}, \frac{1}{6}, \frac{1}{9}, ..., \frac{1}{3(p-1)} \right\} \\ E_{4p}^f &= \left\{ \frac{1}{4}, \frac{1}{8}, \frac{1}{12}, ..., \frac{1}{4(p-1)} \right\} \\ E_f &= [E_p^f, E_{2p}^f, E_{3p}^f, E_{4p}^f] \\ E_f &= \left[\left\{ 1, \frac{1}{2}, \frac{1}{3}, ..., \frac{1}{p-1} \right\}, \left\{ \frac{1}{2}, \frac{1}{4}, \frac{1}{6}, ..., \frac{1}{2(p-1)} \right\}, \left\{ \frac{1}{3}, \frac{1}{6}, \frac{1}{9}, ..., \frac{1}{3(p-1)} \right\}, \right. \\ &\qquad \left. \left\{ \frac{1}{4}, \frac{1}{8}, \frac{1}{12}, ..., \frac{1}{4(p-1)} \right\} \right] \end{aligned}$$

where p is any prime with $p > 5$, which implies $E_f : V \to (0,1]$. Hence, E_f is a fuzzy edge set. Clearly, $\Gamma_f(Z_{5p}) = (V, V_f, E_f)$ be a fuzzy zero divisor graph in $\Gamma_f(Z_{5p})$. Every vertex in V_{f_1} is adjacent to all the vertices in V_{f_2}. Hence that the graph $K_{4,p-1}^f$ is fuzzy complete bipartite graph.

□

Theorem 3.9. *If $n = 7p$ where p is any prime and $p > 7$ then $\Gamma_f(Z_{7p})$ be the non-zero fuzzy zero divisor graph is $K_{6,p-1}^f$ fuzzy complete bipartite graph*

Proof. From the theorem 3.6, theorem 3.7 and theorem 3.8, $\Gamma_f(Z_{7p})$ is a fuzzy complete bipartite graph namely $K_{6,p-1}^f$.

□

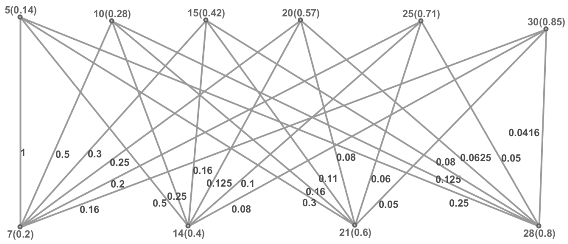

FIGURE 3. $\Gamma_f(Z_{35})$

Theorem 3.10. *If $n = pq$ where p and q are distinct prime numbers and $q > p$ then $\Gamma_f(Z_{pq})$ be the non-zero fuzzy zero divisor graph is $K^f_{p-1,q-1}$ fuzzy complete bipartite graph.*

Proof. Let p and q are any prime numbers and $p < q$. Then the vertex set of $\Gamma(Z_{pq})$ is $V = \{(u, v) : uv = 0 \ multiplication \ modulo \ n, \forall u, v \in V\}$ where n=pq. Take two distinct vertex sets V_1 and V_2 in $\Gamma(Z_{pq})$ where $V_1 = \{p, 2p, 3p, ..., p(q-1)\}$ and $V_2 = \{q, 2q, 3q, ..., (p-1)q\}$.

Every vertex in V_1 adjacent to all the vertices in V_2. Clearly, $\Gamma(Z_{pq})$ complete bipartite graph, namely $K_{p-1,q-1}$.

Let V_f is fuzzy vertex set. Let as show that $V_f : V \to (0, 1)$. Fuzzy vertex set V_f partition in to two sets, namely V_{f_1} and V_{f_2}.

Let

$$V_{f_1} = \left\{\frac{1}{p}, \frac{2}{p}, \frac{3}{p}, ..., \frac{(p-1)}{p}\right\}$$

where p and q are any prime numbers with $p < q$. That is,

$$V_{f_1} = \left\{\frac{N}{p} \mid N = 1, 2, 3, ..., p-1\right\}$$

with $V_{f_1} : V \to (0, 1)$. Let

$$V_{f_2} = \left\{\frac{1}{q}, \frac{2}{q}, \frac{3}{q}, ..., \frac{(q-1)}{q}\right\}$$

with $V_{f_2} : V \to (0, 1)$. where p and q are any prime numbers with $p < q$.

$$\begin{aligned} V_f &= V_{f_1} \cup V_{f_2} \\ &= \left\{\frac{1}{p}, \frac{2}{p}, \frac{3}{p}, ..., \frac{(p-1)}{p}\right\} \cup \left\{\frac{1}{q}, \frac{2}{q}, \frac{3}{q}, ..., \frac{(q-1)}{q}\right\} \\ &= \left\{\frac{1}{p}, \frac{2}{p}, \frac{3}{p}, ..., \frac{(p-1)}{p}, \frac{1}{q}, \frac{2}{q}, \frac{3}{q}, ..., \frac{(q-1)}{q}\right\} \end{aligned}$$

where p and q are any prime numbers with $p < q$. Thus, $V_f = \{V_{f_1} \cup V_{f_2}\} : V \to (0, 1)$ which implies that $V_f : V \to (0, 1)$. Hence V_f is a fuzzy vertex set.

Let E_f is fuzzy edge set. Let as show that Let

$$E_p^f = \left\{1, \frac{1}{2}, \frac{1}{3}, ..., \frac{1}{q-1}\right\}$$

$$E_{2p}^f = \left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, ..., \frac{1}{2(q-1)}\right\}$$

$$E_{3p} = \left\{\frac{1}{3}, \frac{1}{6}, \frac{1}{9}, ..., \frac{1}{p(q-1)}\right\}$$

.

.

.

$$E_{pq}^f = \left\{\frac{1}{p}, \frac{1}{2p}, \frac{1}{3p}, ..., \frac{1}{p(q-1)}\right\}$$

$$E_f = [E_p^f, E_{2p}^f, E_{3p}^f, ..., E_{pq}^f]$$

$$E_f = \left[\left\{1, \frac{1}{2}, \frac{1}{3}, ..., \frac{1}{q-1}\right\}, \left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, ..., \frac{1}{2(q-1)}\right\}, \left\{\frac{1}{3}, \frac{1}{6}, \frac{1}{9}, ..., \frac{1}{p(q-1)}\right\}, ..., \left\{\frac{1}{p}, \frac{1}{2p}, \frac{1}{3p}, ..., \frac{1}{p(q-1)}\right\}\right]$$

where p and q are any prime numbers with $p < q$. which implies that $E_f : V \to (0, 1]$. Hence E_f is a fuzzy edge set.
Clearly, $\Gamma_f(Z_{pq}) = (V, V_f, E_f)$ is called fuzzy zero divisor graph. Therefore every vertex in V_{f_1} is adjacent to all the vertices in V_{f_2}. Hence that the graph $K_{p-1,q-1}^f$ is fuzzy complete bipartite graph. □

4. Conclusion

In this paper, we have defined the Fuzzy Zero Divisor Graph of a commutative ring. Also established some special cases of $\Gamma_f(Z_{2p})$, $\Gamma_f(Z_{3p})$, $\Gamma_f(Z_{5p})$, $\Gamma_f(Z_{7p})$ and $\Gamma_f(Z_{pq})$. In future we will study some more properties and applications of FZDG.

References

[1] Anderson, D. F. and Livingston, P. S., (1999), The Zero-Divisor Graph of a Commutative Ring, J. Algebra., 217, pp. 434-447
[2] Beck, I., (1998), Coloring of Commutative Ring, J. Algebra., 116, pp. 208-226
[3] Ravi Sankar, J and Meena, S., (2012), Changing and unchanging the Domination Number of a commutative ring, International Journal of Algebra, 6, No-27, pp. 1343 - 1352.
[4] Ravi Sankar, J and Meena, S., (2012), Connected Domination Number of a commutative ring, International Journal of Mathematical Research, 5, No-1, pp. 5 - 11.
[5] Ravi Sankar, J and Meena, S., (2013), On Weak Domination in a Zero Divisor Graph, International Journal of Applied Mathematics, 26, No-1, pp. 83 - 91.
[6] Ravi Sankar, J and Meena, S., (2014), Notes on the Strong Domination in a Zero Divisor Graph, Advances in Algebra, 7, No-1, pp. 1-7.
[7] Rosenfeld, A.: Fuzzy graphs. In: Zadeh, L.A., Fu, K.S., Shimura, M. (eds.) Fuzzy Sets and Their Applications, pp. 77–95. Academic Press, New York (1975)
[8] Zadeh, L. A., (1965), Fuzzy Sets, Information Science., 8, pp. 338-353.

A.Kuppan was born in Tamil Nadu, India. He obtained his M.Sc.(Mathematics) degree in 2017 from Islamiah college, Vaniyambadi, Tamil Nadu and M.Phil.(Mathematics) degree from Department of Mathematics, Islamiah college, Vaniyambadi in 2018. Currently, he is a research scholar (Ph.D) in the Department of Mathematics, Vellore Institute of Technology, Vellore, Tamil Nadu, India. His research interest focus mainly on Algebra, Graph theory and Fuzzy graph.

J.Ravi Sankar is an Assistant Professor (Senior) of Mathematics in Vellore Institute of Technology, Vellore, Tamil Nadu, India. He is Completed Ph.D. from Manonmanium Sundaranar University. He has published over 45 research papers in various international and national journals and currently guiding four students for Ph. D. an area of interest of research includes various topics in Algebra, Fuzzy Mathematics, Discrete Mathematics, Applied Mathematics, Graph Theory like Domination, Coloring.