Wetenschappelijke Raad voor het Regeringsbeleid

W 27
A formal presentation of the model used in "Scope for Growth"
TEB-4, Technical-economic bookkeeping version 4

H. A. Berends
F. R. Veeneklaas

The Hague, february 1988

Exemplaren van deze uitgave zijn te bestellen bij het Distributiecentrum Overheidspublikaties, Postbus 20014, 2500 EA 's-Gravenhage, door overmaking van f 10,-- op giro 751 dan wel schriftelijk of telefonisch (070-789880) onder vermelding van titel en ISBN-nummer en het aantal gewenste exemplaren.

ISBN: 90 346 1493 X
Publikatie van de Wetenschappelijke Raad voor het Regeringsbeleid

CONTENTS

1. GENERAL OUTLINE OF THE MODEL

Although the core of the model used in "Scope for Growth" is of the classical input-output type, some adaptations have been introduced to enable it to be used as a tool for prospecting the future. We will begin by describing the model in broad outline, using the familiar terminology of input-output models. Next we will focus on the adaptations. In Section 2 the complete model is presented in detail.

In traditional input-output models, the following equations can be defined for each sector i of the economy:

$$X_i = \sum_{j=1}^{n} x_{i,j} + C_i + I_i + E_i$$

Where gross output of sector i ($=X_i$) equals:

- the sum of intermediate outputs of sector i to be used in the same sector or in other sectors (sum over j of $x_{i,j}$),
- plus the output of sector i that is consumed as end products (C_i),
- plus the output of sector i to be exported (E_i),
- plus the output of sector i to be used as capital goods in other sectors (I_i)

These equations are known as the Leontief equations.

Introducing the technical coefficients $a_{i,j}$:

$$a_{i,j} = \frac{x_{i,j}}{X_j}$$

representing the amount of input from sector i needed to produce one unit of output in sector j and substituting it in the equations above, we get (in matrix notation):

(1) $$X = A X + C + E + I$$

where X is a vector of the form $\begin{bmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{bmatrix}$

The vectors C,E and I have the same form. A is a matrix with characteristic elements $a_{i,j}$.

Usually, assumptions are made for changes in one of the demand components and the corresponding (new) levels of sectoral production needed to satisfy the demand, are then calculated. This is done by rewriting equation (1) as follows:

$$X = (ID - A)^{-1} \cdot (C + E + I)$$

where ID = identity matrix.

The model can be made dynamic by linking the levels of investments in one period of time to the production capacity in the following periods. The needs for imports are usually defined in an extra set of equations, and are related to levels of production, consumption and investment. Also, labour equations are usually introduced, linking production levels to employment.

<u>Special features</u>

Our model, TEB-4 (Technical-Economic Bookkeeping, version 4) differs in three main respects from classical input-output analysis.

1. Instead of varying any of the demand components in order to see what happens to the production level and its sectoral composition, in TEB-4 upper and lower limits are chosen and set for consumption, exports and indirectly for investment of goods and services in the different sectors. These limits reflect expectations, wishes or technical necessities, as explained in

"Scope for Growth". Consequently, there will not be only one result for the level and composition of production given by the model, but a large number of production levels and production compositions will satisfy the demands. Thus room is created for optimization.

2. In TEB-4 the Leontief and other equations have an additional index t for each of the prospected years in the future. As we prospect the next ten years, this increases the number of equations in the model by a factor of 10.

3. The third new feature of our model concerns the environmental equations. These equations are not a traditional part of input-output analysis but are in keeping with it, and relate the emission of pollutants on the one hand to production and consumption levels on the other. Besides the 18 traditional sectors that are defined in the model (see Annex 1), 5 more sectors are defined where pollution abatement activities are undertaken. These sectors do not produce output like the other sectors (the X variables in the model), but produce abated pollution value.

In matrix notation, the following equations form the main framework of the model used:

(1) Leontief

$$X = A \,.\, X + C + E + I$$

(2) Investment

$$I = D \,.\, X + K \,.\, \Delta W$$

(3) Imports

$$M = M_x \,.\, X + M_c \,.\, C + M_i \,.\, I$$

(4) Labour

$$L = f\,(X\,,\,\Delta X\,,\,\lambda^E\,,\,\lambda^D)$$

(5) Pollution

$$V = V_x \cdot X + V_c \cdot C$$

where X, C, E, I, ΔW, M, L, V are vectors of model-variables, A, D, K, M_x, M_c, M_I, V_x, V_c are matrices with the technical, depreciation, capital, import and emission coefficients, and λ^E and λ^D are parameters for the embodied and the disembodied technical progress.

These equations will be presented in detail below.

2 FORMAL PRESENTATION OF THE MODEL

2.1 Introduction

A dynamic input-output model has been formulated, capable of describing the changes in eighteen traditional economic sectors and five (additional) pollution abatement sectors over a future period of ten years. In essence the input-output framework ensures that two kinds of requirements are met. First there is the requirement that everything produced in a sector is also sold. The sales possibilities include supplies to other sectors or within the sector itself, deliveries of capital goods, deliveries to consumers, or exports. No account has been taken of changes in stocks. Secondly there is the requirement that, according to the technical relationships (the production functions) laid down in advance, the factors required to produce that output are available. These consist of imports of raw materials and semi-manufactures, inter-industry sales and labour and capital. The inter-industry sales are both input and output. In a dynamic input-output model of this kind, the same applies to domestically produced investment goods. Capital goods supplied in a given year are assumed to provide capital services for the recipient sector in the following year. Thus investment per sector has been related to the required level of productive capacity.

If the final demand (i.e. consumption, exports and deliveries of capital goods) per sector is given, the required level of labour, imports, investment and inter-industry sales can then be deduced. The calculation of these requirements is a traditional application of input-output analysis. No fixed values for consumption and exports have, however, been selected in this model; instead intervals have been specified within which they can move in time, thus providing room to examine potential future developments by means of optimizations. With respect to the domestic and foreign demand for end products, margins have been introduced by setting upper and lower limits to the sectoral growth of consumption and exports. In this way, uncertainties with respect to the behaviour of consumers and exporters are reflected in the spread of these margins.

The model is supplemented by equations describing the emission of selected environmental pollutants as a function of sectoral production and consumption levels. The requirements in terms of inputs (labour, capital, imports, raw materials) to control pollution are specified. Whether or not these additional abatement activities will be executed, however, depends on the weights given to the various goals in the optimization process. Finally, a number of goal variables have been included, ranging from economic, socio-economic to environmental objectives.

Though the structure of the model is relatively simple, its size is considerable because 23 economic sectors have been distinguished and we want to prospect the next 10 years. It comprises 2213 equations ("rows" in linear programming terminology), approx. 1400 variables ("columns") and approx. 12,500 non-zero coefficients. All equations are linear, all variables non-negative. 792 of the equations are in the form "equal to", the others being inequalities. In the following sections a formal presentation of the model is given. Eight blocks will be distinguished: (i) gross output (ii) investment and production capacity, (iii) exports, (iv) imports and the balance of trade, (v) consumption, (vi) labour, (vii) the emission and possible abatement of pollutants and (viii) the goal variables. Each block includes definitions and, in most cases, limits on variables or their changeability.

With respect to the notation, the following conventions are observed:

- Capitals: goal variables
- Lower case Latin letters (except a, k, d and q): other model variables
- Lower case Greek letters and a, k, d and q: coefficients
- Index i: sector of origin (row in the input-output table); the terms "(economic) sector" and "industry" are used interchangeably

- Index j: sector of destination (column in the input-output table)
- Index t: year; t=o represents the base year
- Index k: type of pollutant
- Upper index*: exogeneously determined value
- Δ : annual change

All variables are in million guilders and constant prices, unless otherwise stated. All coefficients are in unitary dimensions, unless otherwise stated. Under the formulas presented in this section a short description of the variables and coefficients is given. For the extended definitions see Annexes 4 and 5.

2.2 Gross output

According to the Leontief definition equations, the gross output ("production") of any industry must add up to the sales of intermediate deliveries to itself or to other industries, plus domestic consumption by households and government, plus exports, plus sales of investment goods. This holds true for each of the 18 conventional economic sectors for any year, bringing the total number of equations to 180 (for the sectoral breakdown see Annex 1).

$$(1) \quad x_{i,t} = \sum_{j=1}^{23} a_{i,j,t}\, x_{j,t} + c_{i,t} + e_{i,t} + i_{i,t} \qquad i = 1,\ldots 18 \quad t = 1,\ldots 10$$

With as variables:

x	=	gross output, domestically produced
c	=	consumption by households and government, domestically produced
e	=	exports
r^*	=	interest margins of banks (not equal to zero only for the Business Services sector)
i	=	gross investment, domestically produced

and coefficients:

$a_{i,j}$	=	technical coefficient: sales of industry i needed to produce one unit of output of industry j

2.3 Investment and production capacity

Gross domestically produced investment is divided into replacement investment (its allocation being determined by the depreciation coefficient matrix) and expanding investment (its allocation being determined by the capital coefficient matrix). Furthermore, for some recipient industries investment programmes are predetermined, resulting in exogenous variables for the investment goods producing industries and, as will be shown later, in exogenous variables in the import equations. Predetermined investment programmes are formulated for the sectors Mining & gas distribution, Electricity & water, Housing, and Public administration & defence. For these recipient industries the description of the allocation mechanism by capital and depreciation coefficents is dispensed with: in other words, for j = 2,10,15,18, $k_{ijt} = d_{ijt} = 0$.

$$\text{(1a)} \quad i_{i,t} = \sum_{j=1}^{23} d_{i,j,t}\, x_{j,t} + \sum_{j=1}^{23} k_{i,j,t}\, \Delta w_{j,t} + io^{*}_{i,t} \qquad i = 1,\ldots 18 \quad t = 1,\ldots 10$$

with as variables:

w	=	production capacity
io^*	=	exogenous investments

and coefficients:

$d_{i,j}$	=	depreciation coefficient: sales of investment goods produced by industry i needed to maintain one unit of production capacity in industry j
$k_{i,j}$	=	capital coefficient: sales of investment goods produced in industry i needed to expand the production capacity in industry j by one unit.

Although time indices are attached to the technical, depreciation and capital coefficients - implying that changes in time can be allowed for if such information is available - in the actual computations the coefficients are generally assumed to remain constant over the next ten years.

Investments appear in the Leontief equations as a demand category; they are also relevant in relation to the production capacity as equations (1a) show. By rewriting equations (1a) with only w_t on the left-hand side it is easy to see that capacity in year t is a function of the capacity in the preceding year, the expansion investment and the replacement investment. In the computer model equations (1a) are substituted in equations (1). For the domestically produced part (for the required imports see section 2.5) the production capacity of each industry is in this way implicitly defined.

We now state that production in any year may not exceed the capacity at the end of the previous year.

$$(2) \quad x_{i,t} \leqq w_{i,t-1} \qquad i=1,\ldots 23 \quad t=1,\ldots 10$$

To avoid too high a degree of idle capacity a lower limit of production in relation to capacity is set. This also provides against "ornamental investment" which can appear as an outcome if certain conflicting requirements are imposed (in earlier exercises, for instance, a high level of employment and low use of energy (1)).

$$(3) \quad \sum_{t=1}^{10} x_{i,t} \geqq \phi_i \sum_{t=0}^{9} w_{i,t} \qquad i=1,\ldots 23$$

with the coefficients:

ϕ = minimum average degree of utilization of capital (= o for i = 2, 10, 15, 18)

Expansion of capacity cannot be unlimited, one reason being that there is not an unlimited supply of qualified labour to man the new capacity. Therefore

(4) $w_{i,t} \leq (1 + \omega_{i,t}) \, w_{i,t-1}$ i= 1,...18 i≠2,10,15,18
t= 1,...10

with as coefficients:
ω = maximum permitted annual expansion of capacity.

For those industries where exogenous investment programmes have been specified, the future capacity is also fixed.

(5) $w_{i,t} = w^{*}_{i,t}$ i= 2,10,15,18
t= 1,...10

Sectors 2, 10, 15, 18 are Mining & gas distribution, Electricity & water, Housing, and Public administration & defence. In the case of the pollution abatement sectors (nos. 19-23) an absolute (that is: not related to the capacity in the previous year) limit is set. This represents a financial ceiling to additional environmental investment expenditure.

(6) $\Delta w_{i,t} \leq \omega_{i,t}$ i= 19,...23
t= 1,...10

The contraction of capacity is also subjected to a limit. No deliberate dismantlement of capacity is allowed, implying for each industry a rate of fall not exceeding the rate of depreciation. In other words, negative gross investment is excluded.

$$(7)\quad -\{(1+q^{MK}_{j,t})\sum_{i=1}^{18} k_{i,j,t}\}\,\Delta w_{j,t} \leq \{(1+q^{MD}_{j,t})\sum_{i=1}^{18} d_{i,j,t}\}\,x_{i,t} \qquad j=1,\ldots 23 \quad j\neq 2,10,15,18 \qquad t=1,\ldots 10$$

where:

q^{MK} = import coefficient for expansion investment

q^{MD} = import coefficient for replacement investment.

For those industries with pre-determined investment programmes, restriction (7) is superfluous. For the pollution abatement sectors, where $d_{i,j} = 0$, restriction (7) implies a non-decreasing capacity during the projection period.

A final restriction concerns investment in the last year of the prospected period (t=10). In that year there is no future perspective serving as a guide for adjustment in production capacity. In order to avoid a sudden decrease in production capacity in the last year of the prospected period, an extra restriction is added: the sum of expansion investment, domestically produced and imported, in all sectors should at least equal the sum of these investments in the preceding year.

$$(8)\quad \sum_{j=1}^{18}\{(1+q^{MK}_{j,t})\sum_{i=1}^{18} k_{i,j,t}\}\,\Delta w_{j,10} \geq \sum_{j=1}^{18}\{(1+q^{MK}_{j,t})\sum_{i=1}^{18} k_{i,j,t}\}\,\Delta w_{j,9}$$

2.4 Exports

Exports is one of the demand categories in the Leontief equations (1). As stated above, no traditional explanatory export equations are included in the model. Instead upper and lower limits for the annual change in the volume of exports per sector are formulated. They are expressed as a percentage of the exports in the base year of the survey (indicated by subscript t = o).

(9) $\Delta e_{i,t} \leqq \varepsilon^{U}_{i,t}\, e_{i,0}$ $\qquad$ $i= 1,\ldots 18 \;\; i\neq 12$
$t= 1,\ldots 10$

(10) $\Delta e_{i,t} \geqq \varepsilon^{L}_{i,t}\, e_{i,0}$ $\qquad$ $i=1,\ldots 18 \;\; i\neq 12$
$t= 1,\ldots 10$

with coefficients:
ε^{u} = upper limit for export change
ε^{L} = lower limit for export change

Some industries do not produce export products (Housing, Public administration & defence, and the pollution abatement sectors). As a consequence the upper limit and lower limit are set to zero or the variable is non-existent (in the case of the pollution abatement sector). Other sectors, like Electricity & water, produce in principle exportable products, but are assumed to be non-exporting in the projection period. Still other sectors do export but have a more or less fixed export pattern, e.g. the exports of natural gas which are based on long-term contracts. In this case this information is incorporated in the model by specifying the predicted course (resulting in $\varepsilon^{u} = \varepsilon^{L}$).

The export volume of the Trade sector (meaning here not the value of the goods but the trading margins) is equal to a fixed ratio of the total exports of goods by all the goods producing industries.

(11) $$e_{12,t} = q^{ET} \sum_{i=1}^{9} e_{i,t} \qquad t= 1,...10$$

with the coefficient:

q^{ET} = trade margin for export of goods

As intermediate variable the exports of all sectors each year is defined.

(12) (aux.) $$e_t = \sum_{i=1}^{18} e_{i,t} \qquad t= 1,...10$$

2.5 Imports and the balance of trade

Imports per sector and for each year are related to sectoral production, consumption and investment, since each of these have an imported component. Investments are split up into expansion investment and replacement investment, with their own import coefficients. The index j is used to denote the sector for which the imports are destined (just as the index i is used in the export equations, relating to the sectors where exports originated).

(13) $$m_{j,t} = q^{MI}_{j,t} x_{j,t} + (q^{MD}_{j,t} \sum_{i=1}^{18} d_{i,j,t}) x_{j,t} + (q^{MK}_{j,t} \sum_{i=1}^{18} k_{i,j,t}) \Delta w_{j,t} + q^{MC}_{j,t} c_{j,t} + m^{*}_{j,t} \qquad j= 1,...23 \quad t= 1,...10$$

with the variables:

m = imports of goods and services

m* = exogenous imports for predetermined investment in sectors 2, 10, 15 and 18.

and the coefficients:

q^{MX} = import coefficient for raw materials and semi-manufactures

q^{MD} = import coefficient for replacement investment

q^{MK} = import coefficient for expansion investment

q^{MC} = import coefficient for final consumption (excl. non-competing consumer goods and services. These relate to products which cannot be produced in the Netherlands).

In order to arrive at the total volume of imports each year, the imports of non-competing consumer goods and services must be added. A fixed ratio between these goods and services and the total domestically produced consumption is assumed.

(14) (aux.) $m_t = \sum_{j=1}^{23} m_{j,t} + q^{MN} \sum_{i=1}^{18} c_{i,t}$ t= 1,...10

with the variable:

m_t= total imports in year t

and the coefficient:

q^{MN} = import coefficient for non-competing consumer goods and services

The auxiliary equations (12) and (14) combined give us the annual balance of trade. Annual upper and lower limits are placed on this balance, as well as to the average deficit or surplus over time.

(15) $e_t - m_t \leqq \delta_t^U$ t= 1,...10

(16) $e_t - m_t \geqq \delta_t^L$ t= 1,...10

(17) $$\frac{1}{10}\sum_{t=1}^{10} (e_t - m_t) \leq \delta^U$$

(18) $$\frac{1}{10}\sum_{t=1}^{10} (e_t - m_t) \geq \delta^L$$

with the coefficients:

δ_t^u = upper limit for annual balance of trade

δ_t^L = lower limit for annual balance of trade

δ^u = upper limit for average value of the balance of trade

δ^L = lower limit for average value of the balance of trade

2.6 Consumption

Like the sectoral exports, the domestically produced consumption per sector is one of the demand components of gross output as defined in equations (1). And here too no behavioural equations are introduced into the model. Instead margins for the annual change in sectoral consumption are specified. These changes are expressed as a percentage of consumption in the base year ($c_{i,o}$).

(19) $$\Delta c_{i,t} \leq \gamma_{i,t}^U c_{i,0} \qquad i= 1,\ldots 18 \quad i\neq 12 \qquad t= 1,\ldots 10$$

(20) $$\Delta c_{i,t} \geq \gamma_{i,t}^L c_{i,0} \qquad i= 1,\ldots 18 \quad i\neq 12 \qquad t= 1,\ldots 10$$

with the coefficients:

γ^u = upper limit for consumption change

γ^L = lower limit for consumption change

Like exports, the consumption of trade services is assumed to form a fixed ratio of the total domestically produced and imported consumption of goods:

(21) $$c_{12,t} = q^{CT} \sum_{i=1}^{9} (1 + q_{i,t}^{MC}) c_{i,t} \qquad t= 1,...10$$

with the coefficient:

q^{CT} = trade margin for consumer goods

2.7 Labour

The demand for labour is a function of production in the individual sectors. On account of differences in the level and trend of labour productivity, the relation varies per sector. It may, however, be assumed that the productivity of labour does not decline in any sector. In determining the demand for labour, account is taken of the fact that more is invested if production in a given sector increases rapidly thereby resulting in a rejuvenation of the capital goods and hence in higher labour productivity. To model this phenomenon it is necessary to distinguish between two kinds of labour-saving technical progress. The impact of disembodied technical progress, resulting from more efficient organisation, a better educated labour force, learning effects etc., is independent of the rate of growth in production. The labour-saving effect of technical progress embodied in equipment on the other hand, depends on the average age of the capital goods installed and is therefore influenced by the previous rate of expansion of the particular industry. It can be shown (see Annex 2) that by assuming a survival function for equipment and by using a vintage approach the following approximate formula can be deduced.

(22) $$l_{i,t} = \frac{1}{1 + \lambda^D_{i,t}} \, l_{i,t-1} + \frac{\Delta x_{i,t} + \alpha x_{i,t-1} (1 - (1 + \lambda^E_{i,t})^{\theta})}{\bar{\mu}_{i,0} \, \theta (1 + \lambda^D_{i,t})^{t} (1 + \lambda^E_{i,t})^{t} \frac{1 - g_{i,t}}{1 - g^{\theta}_{i,t}}}$$ $i = 1, \ldots 23$, $t = 1, \ldots 10$

where $$g_{i,t} = \frac{1}{1 + \lambda^E_{i,t}}$$

with the variable:

l = demand for labour

and as coefficients:

λ^D = disembodied technical progress

λ^E = embodied technical progress

$\bar{\mu}_0$ = average productivity of labour in the base year

θ = number of vintage years of equipment in operation

α = annual rate of depreciation as a percentage of the stock of equipment

Most important for the working of the model is the fact that (i) both the level of production and the rate of growth in the preceding years determine the demand for labour, (ii) a uniform survival function of equipment for all sectors is assumed and (iii) a linear approximation is applied.

The total demand for labour each year is calculated as follows:

(23) (aux.) $$l_t = \sum_{i=1}^{23} l_{i,t}$$ $t = 1, \ldots 10$

with the variable:

l_t = total demand for labour, year t

The total demand for labour cannot exceed labour supply minus frictional unemployment.

(24) $l_t \leq ls^*_t - uf^*$

with the variables:
ls^*_t = labour supply in year t
uf^* = level of frictional unemployment

2.8 Pollution

Three types of pollution have been included in the model. The first group of pollutants consists of potentially acidifying substances: sulphur dioxide (SO_2), nitrogen oxides (NO_x) and ammonia (NH_3). The emission of these substances is expressed in the unit of acidification, mol H^+. The second group covers hydrocarbons. Emissions are expressed in thousand kilograms. The third group is formed by pollution for which control measures have been included in the Indicative Multi-Year Environmental Control Programme (IMP-M) 1987-1991, an annually updated action plan issued by the Dutch Ministry of Housing, Physical Planning and the Environment. The content of this category of "other pollution" (in total 17 measures) ranges from noise abatement to replacement of PCBs and from soil decontamination to controlled waste disposal. The emissions are expressed in the model in terms of pollution abatement costs (in guilders). A distinction is made between emission before and after abatement.

With regard to abatement five sectors are distinguished. The first three - sectors 19, 20 and 21 - concern the abatement of potentially acidifying substances in increasing order of cost per unit of acidification abated. Sector 22 relates to the abatement of hydrocarbons and sector 23 to "other pollution".

For the three groups of pollutants, emission coefficients have been included in the model enabling the pre-abatement level of pollution to be related to production and/or consumption of the goods and services produced in the different sectors. In addition emission of pollutants may take place independently of the level of production or consumption. Thus pre-abatement emission can be defined as follows:

(25) $$v_{k,t} = \sum_{i=1}^{23} q^{vx}_{k,i,t}\, x_{i,t} + \sum_{i=1}^{18} q^{vc}_{k,i,t}\, c_{i,t} + v^{*}_{k,t} \qquad k= 1,\ldots 3 \quad t= 1,\ldots 10$$

with the variables:

$v_{k,t}$ = pre-abatement emission of type k in year t

$v^{*}_{k,t}$ = exogenous emission of type k in year t

and the coefficients:

q^{vx} = emission coefficient for production

q^{vc} = emission coefficient for consumption

We will now define the emission level after abatement, i.e. after actions are undertaken in sectors 19-23. No additional abatement, that is on top of the current abatement can be achieved in the first year as the capacity of the specified additional abatement sector at the end of the base year is zero (see equation (3)). So for year 1 holds

(26) $$z_{k,1} = v_{k,1} \qquad k= 1,\ldots 3$$

with the variable:

$z_{k,1}$ = post-abatement emission of pollutant type k

From year 2 onwards the pollution can differ from the pre-abatement emission because of production in the abatement sectors. As regards potentially acidifying substances three such abatement sectors have been distinguished, thereby doing justice to the substantial difference in costs between the various control measures. The costs per mol H^{+} abated are specified for a number of representative methods in each sector, thus enabling the technical, capital, depreciation and labour coefficients as well as the import ratio to be calculated.

Not all annual emissions can be abated by these sectors. The total pre-abatement emission is divided into four segments, indicating the technical ceilings of the three abatement sectors and an unabateble part.

(27) $q^{z}_{i,t}\, x_{i,t} \leq \pi_{i,t}\, v_{1,t}$ $i = 19,\ldots 21$; $t = 2,\ldots 10$

with the coefficients:

q^{z}_{i} = abatement of pollution (for i = 19, ...21 acidifying substances in mol. H^{+}) per guilder of gross output of sector i

π_i = shares of total emission abateble by sector i

Post-abatement emission of acidifying substances can now be defined as

(28) $z_{1,t} = v_{1,t} - \sum_{i=19}^{21} q^{z}_{i,t}\, x_{i,t}$ $t = 2,\ldots 10$

With regard to hydrocarbons and "other pollution" no distinction in the cost-effectiveness of different kinds of abatement techniques has been made. The possibility of an unabateble segment of total pollution however is left open:

(29) $q^{z}_{i,t} x_{i,t} \leq \pi_{i,t} v_{i-20,t}$ $\quad$ i= 22,23
t= 2,...10

with the coefficients:

π_i = share of total emission abateble by sector i

q^{z}_{i} = abatement of pollution (for i = 22 hydrocarbons in grams; for i = 23 "other pollution" in guilders) per guilder of gross output of sector i.

Post-abatement emission of hydrocarbons and "other pollution" is defined as gross emission minus abatement:

(30) $z_{2,t} = v_{2,t} - q^{z}_{22,t} x_{22,t} - q^{ZH}_{19,t} x_{19,t} - q^{ZH}_{20,t} x_{20,t} - q^{ZH}_{21,t} x_{21,t}$ $\quad$ t= 2,...10

(31) $z_{3,t} = v_{3,t} - q^{z}_{23,t} x_{23,t}$ $\quad$ t= 2,...10

with the coefficients:

q^{z}_{i} = abatement of pollution per guilder of gross output of sector i

q^{zH}_{i} = abatement of hydrocarbons (in grams) per guilder of output of the acid abating sectors i = 19, ...21.

In equation (30) the activities of the acid-abating sectors appear because some of the measures against acidifying substances also reduce the emission of hydrocarbons (e.g. catalysts in motor vehicles).

2.9 Goal variables

An unlimited number of goal variables can be formulated: all possible linear combinations of variables in the model. In practice we specified nine of them of which seven were normally used in the optimization process (modelling the emission of hydrocarbons ran into data problems due to the difficulty of finding an adequate summation measure for the different types of hydrocarbons).

1. Consumption (C)

Average total consumption, domestically produced and imported, is labelled goal variable C.

$$(32)\quad C = \frac{1}{10}(1 + q^{MN}) \sum_{t=1}^{10} \sum_{i=1}^{18} c_{i,t} + \frac{1}{10} \sum_{t=1}^{10} \sum_{i=1}^{18} q^{MC}_{i,t}\, c_{i,t}$$

with as variables:
c = consumption by households and government, domestically produced

and as coefficients:
q^{MN} = import coefficient for non-competing consumer goods and services
q^{MC} = import coefficient for competing consumer goods and services

2. Consumption pattern (F)

Not only can the total volume of consumption be an objective, but a balanced development pattern of the consumption per sector and between sectors can also be desired. A "balanced development pattern" is here made operational by defining an additional variable as the "lowest" growth rate of consumption in any year in any sector. "Lowest" is defined as the deviation from the lower limit set, taking into account the distance of this lower limit to the upper limit. This is done by expressing the deviation as a percentage of the distance. This percentage deviation can be calculated for each sector and each year. The minimum value of this set of deviations is the variable to be maximized.

For example: assume that the consumption from sector 1 in a certain year grows by 2%; the lower limit being -2% and the upper +6%. The deviation from the lower limit is thus (2-(-2))/(6-(-2)) = 50%. Call this percentage

F_1. The consumption from sector 2 growths by 3%, its lower limit being 0% and upper limit 9%. The deviation, expressed as a percentage of the distance between lower and upper limit, is for sector 2 (3-0)/(9-0) = 33% = F_2. Define F as:

$$F = \min (F_1, F_2) = 33\%.$$

In maximizing F the consumption from sector 2, being the "lowest", is the relevant growth rate. Extending this principle to the 18 sectors and ten years prospected, equation (20) is reformulated as

$$(20)^* \quad \Delta c_{i,t} \geqq \{ \gamma^L_{i,t} + F(\gamma^U_{i,t} - \gamma^L_{i,t}) \} c_{i,0} \qquad i = 1,\ldots 18 \quad i \neq 12, \quad t = 1,\ldots 10$$

with the variable:
c = consumption by households and government, domestically procuced

and as coefficients:
γ^L = lower limit for consumption growth
γ^u = upper limit for consumption growth

F is the goal variable to be maximized.

3. Demand for labour (L)

Average total demand for labour, an indicator for employment, is the third goal variable specified. Its definition is straightforward.

$$(34) \quad L = \frac{1}{10} \sum_{t=1}^{10} l_t$$

with as variable
l_t = demand for labour in year t

4. "Highest" unemployment (W)

As with consumption, not only total employment but also the course of unemployment in time can be an objective. Therefore a variable, representing the largest deviation from a pre-determined trajectory, is defined as a goal variable to be minimized.

(35) $W \geq l_t^* - l_t$ $\qquad$ $t = 1, \ldots 10$

with the variables:

l_t = demand for labour in year t

l^*_t = target value for employment in year t

5. Exports (E)

Average total exports E is the fifth goal variable.

(36) $E = \frac{1}{10} \sum_{t=1}^{10} e_t$

with the variable:

e_t = exports in year t

6. Labour productivity

The labour productivity of a country is an indicator for international competitiveness and wealth. Increasing this productivity can form an objective of economic policy. To avoid non-linearity this is operationalized by formulating a goal variable equal to the difference between the index of total value added and that of employment at the end of the projection period (average over the years 8-10). (Both indices are set at 100 in the base year). Value added is defined as gross output minus inter-industry sales and imports.

(37) $$P = \{ \frac{1}{3} \sum_{t=8}^{10} (\sum_{j=1}^{23} (1 - q^{MI}_{j,t} - \sum_{i=1}^{18} a_{i,j,t}) x_{j,t}) \} / y_0 - \{ \frac{1}{3} \sum_{t=8}^{10} l_t \} / l_0$$

with the (exogenous) variables:

y_0 = total value added in the base year

l_0 = total employment in the base year (thousand man-years)

7-9. Acidification (Z), Hydrocarbons (H) and "Other pollution" (G)

The average emission of unabated potentially acidifying substances, of unabated hydrocarbons and of unabated "other pollution" are the three environmental goal variables.

(38) $$Z = \frac{1}{10} \sum_{t=1}^{10} z_{1,t}$$

(39) $$H = \frac{1}{10} \sum_{t=1}^{10} z_{2,t}$$

(40) $$G = \frac{1}{10} \sum_{t=1}^{10} z_{3,t}$$

with the variable:

$z_{k,t}$ = post-abatement emission of pollutant k in year t

The model is completed by setting upper and lower limits on all goal variables.

NOTES

(1) Netherlands Scientific Council of Government Policy, A Policy-Oriented Survey of the Future; Report to the Government no. 25, Staatsuitgeverij, The Hague, 1983.
English summary available.

Annex 1 SECTORAL BREAKDOWN

	Row nr. National Accounts	Standard Industrial Classification (SBI) 1974
1. Agriculture (incl. horticulture, forestry and fisheries)	1, 2	01-03
2. Mining and gas distribution (oil and natural gas extraction and exploration, other mining, natural gas distribution companies)	3, 4, 38	12, 19, 402
3. Chemicals (basic and endproducts of the chemical industry, rubber and plastics processing industry)	26-28	11, 29-31
4. Heavy industry (basic metals, construction materials, earthenware, glass and glass products)	29, 30	32, 33
5. Metal-processing industry (metal products and machinery, transport equipment, instruments and optical goods)	31, 32, 34-36	34, 35, 37-39
6. Electrotechnical industry	33	36
7. Foodstuffs industry (incl. beverages and tobacco)	5-14	20-21
8. Petroleum industry (oil refineries and manufacture of petroleum and coal products)	25	28
9. Other industry (textiles, clothing, footwear and leather goods, timber and furniture industry, paper and cardboard, paperware and corrugated cardboard, printing and publishing)	15-24	22-27
10. Electricity generation and water supply	37, 39	401, 403
11. Construction and installation on construction projects	40	5
12. Trade (wholesale and retail trade plus intermediaries)	41	61-66
13. Consumer services (hotels, restaurants, cafés, repair of consumer goods, social services, private households with salaried staff)	42, 43, 54 56-58	67, 68, 91 929, 94-99
14. Transport and communications	44-46	71-77
15. Housing	49	83
16. Business services (incl. banking and insurance)	47, 48, 50	81, 82, 84, 85
17. Health care and education (incl. veterinary services)	53, 55	921-928, 93
18. Public administration and defence (civilian and military)	51, 52	906, 907

Additional sectors:
19. 'Low cost' acidification abatement sector
20. 'Medium cost' acidification abatement sector
21. 'High cost' acidification abatement sector
22. Hydrocarbons abatement sector (not used in this study)
23. Other environmental measures.

Annex 2 THE LABOUR DEMAND FUNCTION

Equation (22) in section 2.7 is derived as follows:

The demand for labour is a function of production in the individual sectors:

$$(1) \quad L_{i,t} = \frac{X_{i,t}}{\eta_{i,t}}$$

Where :

$L_{i,t}$ = labour demand in year t in sector i,

$X_{i,t}$ = value of production in sector i in year t.

$\eta_{i,t}$ = labour productivity of the capital goods used in the production of sector i in year t.

As the capital goods used have different productivities according to their age and as we want to introduce technical progress, we can rewrite (1) as follows:

$$(2) \quad L_{t,i} = \sum_{\tau=1}^{\theta} \frac{X_{t,\tau,i}}{\eta_{t,\tau,i}}$$

Where:

$X_{t,\tau,i}$ = the value of production in year t in sector i, produced with capital goods τ years old.

$\eta_{t,\tau,i}$ = labour productivity of capital goods used in production in sector i, in year t, with capital goods τ years old.

θ = number of vintage years of capital goods used in the production process.

As we want to take into account embodied and disembodied technical progress influencing labour productivity, we specify the following equations that show how labour productivity increases or decreases over time:

(3) $$\eta_t = \eta_0 \cdot (1+\lambda_D)^t$$

(4) $$\eta_t = \eta_0 \cdot (1+\lambda_E)^t$$

(5) $$\eta_{-t} = \eta_0 \cdot (1+\lambda_E)^{-t}$$

Rewriting (5) with τ for the years in the past (and t for the years in the future) and combining (3),(4) and (5), we obtain the following equations, where for reasons of clarity we omit index i:

(6) $$\eta_{t,\tau} = \eta_0 \cdot (1+\lambda_D)^t \cdot (1+\lambda_E)^{t-\tau}$$

Where :

η_0 = labour productivity in the base year preceding the prospected period.

λ_E and λ_D = coefficients representing embodied and disembodied technical progress.

Substituting equation (6) in equation (2) results in:

(7) $$L_t = \sum_{\tau=1}^{\theta} \frac{X_{t,\tau}}{\eta_0 \cdot (1+\lambda_D)^t \cdot (1+\lambda_E)^{t-\tau}}$$

Multiplying the right-hand side by $1/(1+\lambda_D)$, rewriting it for period t-1 and then subtracting this new expression from both the left- and right-hand side of equation (7), we obtain:

(8) $$L_t = \frac{L_{t-1}}{1+\lambda_D} + \frac{X_{t,1}}{\eta_0 \cdot (1+\lambda_D)^t \cdot (1+\lambda_E)^{t-1}} - \frac{X_{t-1,\theta}}{\eta_0 \cdot (1+\lambda_D)^t \cdot (1+\lambda_E)^{t-1-\theta}}$$

because $X_{t,2} = X_{t-1,1}$

and $X_{t,3} = X_{t-1,2}$, etc.

and the expressions between the first and the last ones cancel each other out.

Equation (8) shows that the labour demand in year t is reduced by disembodied technical progress (first term on the right-hand side), but may expand because of (new) production made possible by access to new capital goods (τ =1, second term on the right-hand side), and again is reduced by dismantlement of capital goods (from the oldest year, $\tau = \theta$, the third term on the right-hand side) with lower labour productivity per output.

Rewriting equation (8) in order to relate the demand for labour directly to the total level of production, and not to various levels of production made with different vintages of capital goods, we must introduce α , representing the percentage of capital goods set aside annually.
Therefore we define:

(9) $$X_{t,1} = \Delta X_t + \alpha \cdot X_{t-1}$$

Equation (8) can be rewritten as:

(10) $$L_t = \frac{L_{t-1}}{1+\lambda_D} + \frac{\Delta X_t + \alpha \cdot X_{t-1}}{\eta_0 \cdot (1+\lambda_D)^t \cdot (1+\lambda_E)^{t-1}} - \frac{X_{t-1,\theta}}{\eta_0 \cdot (1+\lambda_D)^t \cdot (1+\lambda_E)^{t-1-\theta}}$$

We note that :

(11) $$X_{t-1,\theta} = \alpha \cdot X_{t-1}$$

or, in words: the oldest part of the capital goods still in use (from year θ) equals that part that is set aside (thus α). Using (11), equation (10) becomes:

$$(12)\quad L_t = \frac{L_{t-1}}{1+\lambda_D} + \frac{\Delta X + \alpha \cdot X_{t-1}}{\eta_0 \cdot (1+\lambda_D)^t \cdot (1+\lambda_E)^{t-1}} - \frac{\alpha \cdot X_{t-1}}{\eta_0 \cdot (1+\lambda_D)^t \cdot (1+\lambda_E)^{t-1-\theta}}$$

Equation (12) shows the labour demand in period t as a function of the labour demand in period (t-1), and also as a function of the production increase and of production in the previous period. As production increases, so will labour demand. It can be rewritten as follows:

$$(13)\quad L = \frac{L_{t-1}}{1+\lambda_D} + \frac{\Delta X + \alpha \cdot X_{t-1} \cdot (1-(1+\lambda_E)^{\theta})}{\eta_0 \cdot (1+\lambda_D)^t \cdot (1+\lambda_E)^{t-1}}$$

A practical problem is that the labour productivity of capital goods produced in the base year (the term η_0) is not known and we want to write it in terms of the average labour productivity in the base year. This average equals:

$$(14)\quad \bar{\eta}_0 = \frac{\eta(-1) + \eta(-2) + \ldots + \eta(-\theta)}{\theta} =$$

$$= \frac{\eta_0 \cdot (1+\lambda_E)^{-1} + \eta_0 \cdot (1+\lambda_E)^{-2} + \ldots + \eta_0 \cdot (1+\lambda_E)^{-\theta}}{\theta} =$$

$$= \frac{\eta_0}{\theta} \cdot \sum_{\tau=1}^{\theta} \left(\frac{1}{1+\lambda_E} \right)^{\tau} =$$

$$= \frac{\eta_0}{\theta} \cdot \frac{(1+\lambda_E)^{\theta} - 1}{\lambda_E \cdot (1+\lambda_E)^{\theta}}$$

Substituting the η_0 as presented in (14) in equation (13), we obtain:

$$(15) \quad L_t = \frac{L_{t-1}}{(1+\lambda_D)} + \frac{\Delta X.((1+\lambda_E)^{\theta}-1) - \alpha . X_{t-1}((1+\lambda_E)^{\theta}-1)^2}{\bar{\eta}_0 . \theta . \lambda_E . (1+\lambda_D)^t . (1+\lambda_E)^{t-1+\theta}}$$

This expression equals:

$$(16) \quad L_t = \frac{L_{t-1}}{1+\lambda_D} + \frac{\Delta X_t + \alpha . X_{t-1} . (1-(1+\lambda_E)^{\theta})}{\bar{\eta}_0 . \theta . (1+\lambda_D)^t . (1+\lambda_E)^t . \frac{1-g}{1-g^{\theta}}}$$

Where $g = \frac{1}{1+\lambda_E}$

Equation (16) is the same as equation (22) presented in the main description of the model.

Annex 3 A COMPARISON BETWEEN LINEAR PROGRAMMING IN GENERAL AND REVISED SIMPLEX

In general

Linear programming consists of solving the following problem:
maximize a variable Z which is a linear function of n variables x(1), x(2),...,x(n).

(1) $Z = c(1).x(1) + c(2).x(2).+ \ldots +c(n).x(n)$

(Z is the objective)

while at the same time fulfilling a set of conditions:

(2)
$$
\begin{aligned}
&a(1,1).x(1) + a(1,2).x(2) + \ldots + a(1,n).x(n) \leq b(1)\\
&a(2,1).x(1) + a(2,2).x(2) + \ldots + a(2,n).x(n) \leq b(2)\\
&\ldots\\
&a(m,1).x(1) + a(m,2).x(2) + \ldots + a(m,n).x(n) \leq b(m)
\end{aligned}
$$

These are the m restrictions.

In matrix notation:
(1) $Z = C . X$
(2) $A . X \leq B$

Where Z is a variable (scalar),
C is a row vector of coefficients c with length n;
X is a vector of variables with length n;
A is an mxn matrix of coefficients;
B is a vector of fixed values with length m.

First we introduce the m slack variables x(n+1),...,x(n+m) in order to transform the inequalities (2) into equations:

(3) $A . X + I . X_S = B$

Where I is an m x m identity matrix,

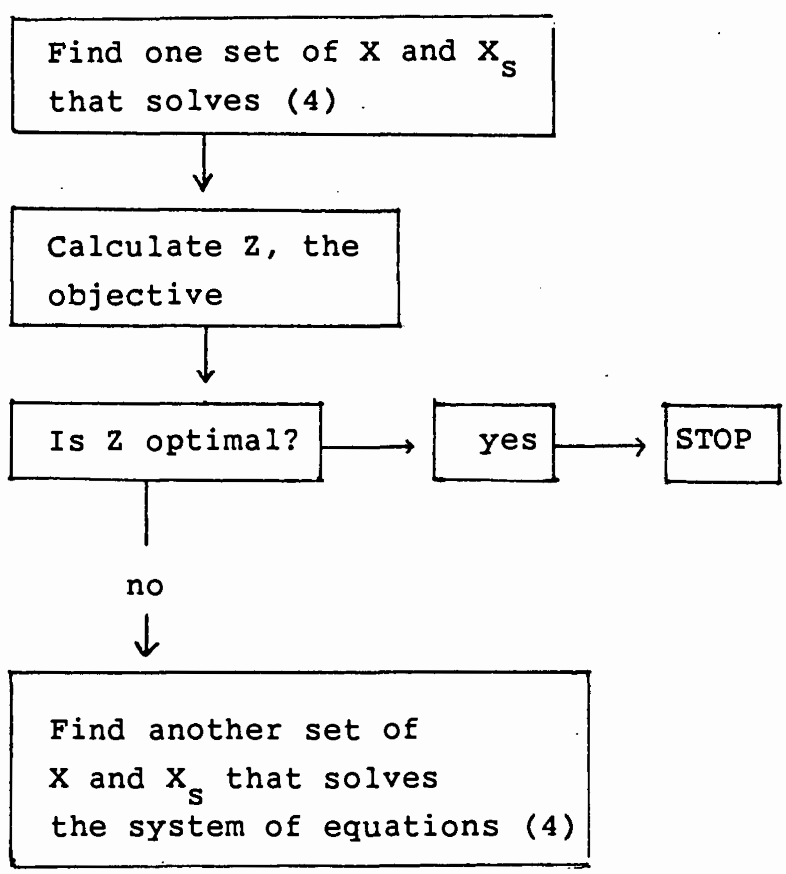
X_S is a vector of length m.

The system of equations (3) can be rewritten as follows:

$$(4) \qquad (A \;\vdots\; I) \cdot \begin{pmatrix} X \\ \cdots \\ X_S \end{pmatrix} = B$$

<u>This is the set of equations that has to be solved</u>. With a set of X and X_S that solves this system, one can calculate the value of Z, the objective. One can then see if this Z is optimal. If not, one has to find another set of X and X_S that solves (4) and calculate Z again. This procedure is continued until the value of Z cannot be improved by a different set of X and X_S.

This search process is equivalent to the following iterative algorithm:



In geometrical terms this search method means that the linear restrictions define and enclose a space for the variable Z = sum of c(i). x(i). In the process of searching for a solution, the contours of this space are explored.

The larger the number of X variables, i.e. the larger n is, the greater the dimensions of the space that has to be explored. Therefore, the larger n+ m is, the number of variables, the larger the number of iterations that can be expected to be necessary. On the other hand, the larger m is, the number of restrictions that have to be met at each iteration, the longer the calculations take during each iteration to find a solution that satisfies.

In algebraic terms, the solving of a system of equations is equivalent to inverting a (in this case) mxm matrix and requires approximately m^3 additions and multiplications. This is how the computations are done in the Simplex method. In our model we use the Revised Simplex method. In the Revised Simplex method the calculations are shortened and simplified as much as possible as compared to the Simplex method. In the Revised method this is done by defining and distinguishing between non-basic variables (the ones that are equal to zero) and the basic variables (the ones that are not equal to zero). This simplifies the calculations because we will not include all n+m (X and XS) variables in the calculations but only the m basic ones. We define:

$$BAS = (A \mid I)$$ but only with the m columns corresponding to the basic variables.

Also:

$$XB = (X \mid Xs)$$ but only with the m basic variables,

$$CB = (C \mid 0)$$ but only with the m corresponding basic c's.

The problem can be reformulated as follows: maximize (5) under the restrictions (6) :

(5) $$Z = CB \cdot XB$$

(6) $$BAS \cdot XB = B$$

Equation (5) replaces (1) and (6) replaces (3).

Equation (6) can be rewritten as :

(7) $$XB = BAS^{-1} \cdot B$$

<u>This is the solution for the X basic variables after each iteration</u>.

Equations (5) and (7) combined :

(8) $$Z = CB \cdot BAS^{-1} \cdot B$$

<u>This is the solution for Z after each iteration</u>.

Again the above equations can be rewritten. Taking equations (7) and (8) together:

(9) $$\begin{pmatrix} Z \\ XB \end{pmatrix} = \begin{pmatrix} CB \cdot BAS^{-1} \cdot B \\ \hdashline BAS^{-1} \cdot B \end{pmatrix}$$

OR :

(10) $$\begin{pmatrix} Z \\ XB \end{pmatrix} = \left(\begin{array}{c:c} 1 & CB \cdot BAS^{-1} \\ \hdashline 0 & BAS^{-1} \end{array}\right) \cdot \begin{pmatrix} 0 \\ B \end{pmatrix}$$

Equations (1) and (3) can be put combined as:

(11) $$\begin{pmatrix} 0 \\ B \end{pmatrix} = \begin{pmatrix} 1 & -C & 0 \\ 0 & A & I \end{pmatrix} \cdot \begin{pmatrix} Z \\ X \\ X_s \end{pmatrix}$$

Replacing the (0 ¦ B) vector in (10) with its value in (11):

$$(12)\quad \begin{pmatrix} Z \\ XB \end{pmatrix} = \left(\begin{array}{c|c|c} 1 & CB.BAS^{-1}.A \;-C & CB.\;BAS^{-1} \\ \hline 0 & BAS^{-1}\;.\;A & BAS^{-1} \end{array}\right) \cdot \begin{pmatrix} Z \\ X \\ XS \end{pmatrix}$$

Using equation (9) for the left-hand side of equation (12):

$$(13)\quad \left(\begin{array}{c} CB.\;BAS^{-1}\;.\;B \\ \hline BAS^{-1}.\;B \end{array}\right) = \begin{pmatrix} \text{idem} \\ \text{as} \\ \text{RHS(12)} \end{pmatrix}$$

<u>This is the form the set of equations has after every iteration</u>. Note that this equation uses the same A and B each iteration, and CB and BAS^{-1} only change a little after each iteration. This is therefore a handy algebraic form to use in calculations. It can be shown that each new BAS^{-1} can be deduced from the former by simple computation, as after each iteration only one columm vector of this m size matrix changes. This "Revised" Simplex method is therefore more efficient than the Simplex method where each iteration involves calculations very similar in length and size to calculating the inverse of a new mxm size matrix.

Annex 4 LIST OF VARIABLES INCLUDED IN THE MODEL
(for mode of notation see 2.1)

All variables are expressed in million guilders, constant prices unless otherwise stated.

$c_{i,t}$ = consumption by households and government of goods and services produced by the domestic industry i in year t.

$e_{i,t}$ = exports of goods and services produced by industry i in year t

e_t = total exports of goods and services in year t

$i_{i,t}$ = gross investment in fixed assets produced by industry i in year t

$io^*_{i,t}$ = exogenous investment produced by industry i

$l_{i,t}$ = demand for labour in industry i in year t (thousand man-years, constant working hours)

l_t = total demand for labour in year t (thousand man-years, constant working hours)

ls^*_t = labour supply, in year t (thousand man-years, constant working hours)

l^*_t = target value for employment in year t (thousand man-years, constant working hours)

$m_{j,t}$ = imports of goods and services of sector j in year t (excl. non-competing imports of consumer goods and services)

m_t = total imports of goods and services in year t

r^*_t = interest margin of banks (not equal to zero only for sector 16: Business services)

uf^* = frictional unemployment (thousand man-years, constant working hours)

$v_{k,t}$ = pre-abatement emission of pollutant k in year t (10^6 mol H^+ for k=1 : 10^3 kg for k=2; 10^6 guilders for k=3)

$w_{i,t}$ = production capacity of industry i in year t

$x_{i,t}$ = gross output produced by industry i in year t

y_0 = total gross value added in base year

$z_{k,t}$ = post-abatement emission of pollutant k in year t (units: see $v_{k,t}$ above)

Goal variables

C	=	average total consumption, domestically produced and imported (C^L and C^U lower and upper limits)
F	=	indicator for a balanced consumption pattern (unitary dimension; F^L and F^U lower and upper limits)
L	=	average demand for labour (thousand man-years; L^L and L^U lower and upper limits)
W	=	largest deviation in any year from a predetermined target trajectory of employment (thousand man-years; W^L and W^U lower and upper limits)
E	=	average exports (E^L and E^U lower and upper limits)
P	=	indicator for rise in labour productivity during the projection period (unitary dimension; P^L and P^U lower and upper limits)
Z	=	average unabated emission of potentially acidifying substances (10^6mol H^+; Z^L and Z^U lower and upper limits)
H	=	average unabated emission of hydrocarbons (10^3kg; H^L and H^U lower and upper limits)
G	=	average unabated emission of "other pollution" (G^L and G^U lower and upper limits)

Annex 5 LIST OF COEFFICIENTS INCLUDED IN THE MODEL (for mode of notation see 2.1)

$a_{i,j,t}$ = technical coefficient: sales of industry i needed to produce one unit of output by industry j in year t

$d_{i,j,t}$ = depreciation coefficient: sales of investment goods produced by industry i needed to maintain one unit of production capacity in industry j in year t

$k_{i,j,t}$ = capital coefficient: sales of investment goods produced by industry i needed to expand the production capacity in industry j by one unit in year t.

ϕ_i = minimum average degree of utilization of capital in industry i

$\omega_{i,t}$ = maximum permitted annual expansion of production capacity in sector i in year t (as % of capacity in the previous year for i=1,...18; in millions of guilders for i=19,... 23)

$\varepsilon^U_{i,t}, \varepsilon^L_{i,t}$ = upper or lower limit of change in exports of sector i in year t, expressed as a percentage of exports in the base year

δ^U_t, δ^L_t = upper or lower limit of the balance of trade in year t (millions of guilders)

δ^U, δ^L = upper or lower limit of the average balance of trade during the projection period (millions of guilders)

$\gamma^U_{i,t}, \gamma^L_{i,t}$ = upper or lower limit of change in consumption of goods and services produced in the domestic industry i in year t. Expressed as a ratio to domestically produced consumption of industry i in the base year

$\lambda^D_{i,t}, \lambda^E_{i,t}$ = disembodied or embodied technical progress in industry i in year t

$\bar{\mu}_{i,o}$ = average labour productivity in industry i in the base year

θ = number of vintage years of equipment in operation

α = annual rate of depreciation of equipment as percentage of the stock of equipment

$\pi_{i,t}$ = share of total gross emission of pollutants abateble by sector i

$q^{MK}_{j,t}$ = import coefficient of expansion investment, expressed as a ratio to total domestically produced expansion investment in the recipient industry j in year t

$q^{MD}_{j.t}$ = import coefficient of replacement investment, expressed as a ratio to total domestically produced replacement investment in the recipient industry j in year t

q^{ET} = trade margin on the exports of goods

$q^{MI}_{j,t}$ = import coefficient of raw materials and semi-manufactures as a ratio to domestic gross output in industry j in year t

$q^{MC}_{j,t}$ = import coefficient consumption goods and services (excl. non-competing consumer goods and services), expressed as a ratio to the domestically produced consumption of the competing industry j in year t

q^{MN} = import coefficient non-competing consumer goods and services, expressed as a ratio to the total yearly domestically produced consumption

q^{CT} = trade margin on consumer goods

$q^{VX}_{k,i,t}$ = emission coefficient production in industry i in year t of pollutant k (mol H^+ per guilder for k=1; grams per guilder voor k = 2; unitary dimension for k=3)

$q^{VC}_{k,i,t}$ = emission coefficient consumption of products produced by industry i in year t of pollutant k (units: see q^{vx} above)

$q^{Z}_{i,t}$ = abatement of pollution per guilder gross output of sector i in year t (mol H^+ per guilder for i = 19,...21; grams per guilder for i=22; unitary dimension for i=23)

$q^{ZH}_{i,t}$ = abatement of hydrocarbons per guilder output of acid-abating sector i (grams per guilder)

Annex 6 SEQUENCE OF EQUATIONS IN THE COMPUTER LISTING (23-6-87)

Label	Number of equat.	Corresponding rank no. in documentation	Type of equat.	Description
AAAL#..	7	-	E	Definition of goal variables
AAAO#..	7	(32b), (40c), (34b), (35c), (36b), (37b), (38c)	LE, GE	Lower and upper limits of goal variables
AGRE#..	7	(32c), (40b), (34c), (35b), (36c), (37c), (38b)	LE, GE	Lower and upper limits of goal variables
X...#..	180	(1)	E	Leontief equations
W...#..	230	(2)	LE	Production cannot exceed capacity
J...#..	230	(4), (5), (6)	LE, E	Maximum expansion of capacity
I...#..	200	(7)	LE	Maximum contraction of capacity
I...#99	23	(3)	GE	Minimum utilization of capital
GAAA#99	1	(8)	GE	Investment in last year
E...#..	170	(9)	LE	Upper limit for exports
EALL#..	10	(12)	E	Total annual exports
EALL#99	1	-	E	Total exports
F...#..	160	(10)	GE	Lower limit for exports
FALL#..	10	-	E	Total annual consumption
FALL#99	1	-	E	Total consumption
EHAN#..	10	(11)	E	Export trade margins
M...#..	230	(13)	E	Imports
MALL#..	10	(14)	E	Total annual imports
MALL#99	1	-	E	Total imports
PAAA#99	1	(17)	LE	Upper limit for average balance of trade
PBBB#99	1	(18)	GE	Lower limit for average balance of trade
HAAA#..	10	(15)	LE	Upper limit for annual balance of trade
HBBB#..	10	(16)	GE	Lower limit for annual balance of trade
C...#..	170	(19)	LE	Upper limit for consumption
D...#..	170	(20)	GE	Lower limit for consumption
A000#99	1	-	E	Scaling of goal variable F
FHAN#..	10	(21)	E	Consumption trade margins
L...#..	230	(22)	E	Demand for labour
LALL#..	10	(23)	E	Total annual demand for labour
LALL#99	1	-	E	Total demand for labour
OAAA#..	10	(24)	LE	Employment cannot exceed labour supply
V...#..	30	(25)	E	Pre-abatement pollution

Z...#..	30	(26), (28), (30), (31)	E	Post-abatement pollution
ZAAA#..	9	(27)	LE	
ZBBB#..	9	(27)	LE	Upper limit acid-abatement
ZCCC#..	9	(27)	LE	
COOO#99	1	(32a)	E	Goal variable consumption
LOOO#99	1	(34a)	E	Goal variable employment
WOOO#..	10	(35a)	GE	Goal variable unemployment
EOOO#99	1	(36a)	E	Goal variable exports
ZDDD#..	9	(29)	LE	Upper limit for hydrocarbon abatement
ZEEE#..	9	(29)	LE	Upper limit for "other-poll." abatement
HALL#99	1	-	E	Unabated hydrocarbons
HOOO#99	1	(39a)	E	Goal variable hydrocarbons
HAAA#99	1	(39c)	LE	Upper limit goal var. hydrocarbons
GALL#99	1	-	E	Unabated "other pollution"
GOOO#99	1	(40a)	E	Goal variable "other pollution"
GBBB#99	1	-	LE	Upper limit goal "other poll."
TALL#..	10	-	E	Total annual value added
TALL#99	1	-	E	Total value added
POOO#99	1	(37a)	E	Goal variable productivity
ZALL#99	1	-	E	Unabated acid
ZOOO#99	1	(38a)	E	Goal variable acid
	____ +			
	2250			