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The importance of memory for price discovery in decentralized markets [☆]

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ABSTRACT

We study the dynamics of price discovery in decentralized two-sided markets. We show that there exist memoryless dynamics that converge to the core of the underlying assignment game in which agents' actions depend only on their current payoff. However, we show that for any such dynamic the convergence time can grow exponentially in relation to the population size. We present a natural dynamic in which a player's reservation value provides a summary of his past information and show that this dynamic converges to the core in polynomial time in homogeneous markets.

1. Introduction

We consider the classical assignment game, where workers and firms seek to match to a single partner (Koopmans and Beckmann, 1957; Shapley and Shubik, 1972). The outcome of the assignment game is given by a matching of workers to firms along with prices specifying the transfer from each firm to its matched worker. The core (Gillies, 1959) is used to define the equilibrium outcomes of the market. An outcome is in the core if no coalition of workers and firms can strictly improve their payoffs by rematching among themselves. Koopmans and Beckmann (1957) and Shapley and Shubik (1972) showed that the core of the assignment game is non-empty, and that the set of core outcomes can be easily calculated in centralized markets.

In this paper we study whether uncoupled, decentralized, natural market dynamics can lead to core outcomes. Each agent arrives at the market with private information regarding his valuations, but is unaware of other agents' valuations. Agents reach the core outcome through a sequence of random meetings with potential partners. Specifically, upon meeting a potential partner, agents can decide whether to keep their tentative match or rematch at a new price. After sufficient time, the matching and associated transfers may constitute a core outcome. We formally define such price discovery dynamics as a stochastic process. A dynamic converges to the core if it reaches an absorbing matching and prices are in the core with probability one.

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We say that a dynamic is *effectively convergent* if the required time for it to reach a core outcome grows at most polynomially in the population size. It is known that there are simple uncoupled dynamics that converge to the core (Bayati et al., 2015; Nax and Pradelski, 2015; Klaus and Payot, 2015), but the time required for convergence may be so large that the dynamic is unlikely to converge in a reasonable amount of time.[1]

Our first result shows that an effectively convergent dynamic must require agents to keep some additional memory beyond their current matching. We consider a class of *memoryless dynamics* in which agents make their decision based only on their current tentative matching. Formally, the transitions of the stochastic process depend only on the current tentative matching and prices. We show that there are simple markets where the expected time of convergence for every memoryless dynamic grows exponentially with the population size. Loosely speaking, when the tentative matching is close to a core outcome, the dynamic is more likely to move away from any core outcome since it is more and more unlikely to encounter an unmatched agent. Reaching the core requires an almost consecutive sequence of lucky draws, which are increasingly unlikely as the population size grows.

Nonetheless, we show that there is an effectively convergent dynamic that requires little additional memory. In each period of the proposed dynamic, a randomly selected agent acts as an auctioneer. Agents from the other side of the market then participate in the auction, bidding the value they need to receive in order to break their current tentative match. A new match is formed between the auctioneer and the highest bidder at the second highest price. The additional memory takes the form of a non-zero reservation value for unmatched agents. This reservation value remembers the value the agent received when they were last matched and decreases over time while agents remain unmatched. We show that this natural dynamic is effectively convergent for markets where firms differ in their value for a worker, but otherwise value all workers equally (and vice versa). We also present simulation experiments for more general markets suggesting that this dynamic is effectively convergent for generic markets. Intuitively, the reservation value makes the dynamic smooth, allowing agents to effectively 'learn' their 'value' over time.

2. Related literature

Early empirical work found ambiguous evidence for whether assignment markets are effectively convergent. Chamberlin (1948) was pessimistic, motivating Smith (1962) to conduct experiments for markets with homogeneous goods. After as little as three trading periods, prices were close to the market clearing price. The effectiveness of the Walrasian double auction (Walras, 1883) has since been confirmed in numerous experimental studies (Davis and Holt, 1994).

Theoretically, the constructive proofs showing the non-emptiness of the core for assignment games already provide centralized mechanisms to find core outcomes (Koopmans and Beckmann, 1957; Shapley and Shubik, 1972), and these are known to be effectively convergent (Edmonds and Karp, 1972). Subsequent auction mechanisms propose one-sided incentive compatible mechanisms guaranteeing convergence (Crawford and Knoer, 1981; Kelso and Crawford, 1982; Demange et al., 1986; Bertsekas, 1988; Bertsekas and Castanon, 1993).[2]

A more recent strand of the literature studies price discovery dynamics in the absence of a central market authority (Bayati et al., 2005, 2008; Nax and Pradelski, 2015; Newton and Sawa, 2015; Klaus and Newton, 2016; Hamza and Shamma, 2017).[3] In particular, Nax and Pradelski (2015) use a learning rule based on aspiration adaptation (Sauermann and Selten, 1962; Selten, 1998) which found extensive support in laboratory experiments (Tietz and Weber, 1972; Tietz et al., 1978; Scholz et al., 1983). Aspiration-based learning rules were subsequently used by Nax (2015), Pradelski (2015), and Hamza and Shamma (2017) and form the basis for our proposed effectively convergent dynamic.

There is also a related literature on one-to-one matching with non-transferable utility (Gale and Shapley, 1962; Roth and Sotomayor, 1992). Roth and Vande Vate (1990) propose a decentralized dynamic and show that it converges in finite time. Ackermann et al. (2011) show that the latter dynamic is not effectively convergent. Biró and Norman (2013) provide an empirical study on time to convergence (next to analytical results). While generally there is no dynamic that is incentive compatible for both sides of the market, Kanoria et al. (2018) and Hassidim and Romm (2015) show that for naturally arising markets the core is generally small, thus allowing for approximately incentive compatible dynamics.[4]

Returning to the transferable utility assignment games we study, a series of papers propose effectively convergent dynamics for specific classes of games and operating under varying information requirements (Bayati et al., 2015; Pradelski, 2015; Assadi et al., 2015; Pradelski and Nax, 2020). In particular, Bayati et al. (2015) propose a dynamic where players know the outside options of the other players. Their dynamic is not uncoupled and also not incentive compatible. Pradelski (2015) shows that the introduction of a correlation device gives rise to an effectively convergent dynamic, which makes firms more powerful at some times and workers at others. Finally, Assadi et al. (2015) study the reduced market with homo-

[1] See Foster and Vohra (1997); Hart and Mas-Colell (2000, 2003) for the introduction and early results of uncoupled dynamics.

[2] A related literature studies search markets with sequential bargaining (Rubinstein and Wolinsky, 1985; Gale, 1987; Lauermann, 2013; Lauermann et al., 2018). The main difference to the markets we consider is that agents leave the market once they match. This strand of work is thus concerned with analyzing the trade-off between matching in the current period and foregoing potentially better matches in the future versus the search cost incurred by remaining in the market (usually represented by discounting future profits).

[3] A series of related papers studies similar processes for one-sided markets (Andersson et al., 2014; Biró et al., 2014) and markets where firms can hire multiple workers (Nax and Pradelski, 2016; Fujishige and Yang, 2017).

[4] See Ashlagi et al. (2017) for an analogue result for non-transferable utility markets.

geneous goods and propose an effectively convergent dynamic. Importantly, their dynamic gives preference to unmatched agents when rematches occur. While these results shed light on specific dynamics, it is not yet understood whether the decentralized assignment game is generally hard to solve; this is the central question that we address.

3. Model

We consider the model of the two-sided assignment game as introduced by Koopmans and Beckmann (1957) and Shapley and Shubik (1972).

In this context, a market $\mathcal{E} = (F, W, \mathbf{v}, \mathbf{c})$ consists of *firms* $i \in F$ and *workers* $j \in W$. Let $N = F \cup W$ denote the set of all agents and set $f = |F|$, $w = |W|$, $n = f + w$. Each firm can be matched to a single worker, and each worker can be matched to a single firm. The value of matching the i-th firm with the j-th worker is $v_i(j) \in \mathbb{N}$, and the cost of worker j matching to firm i is $c_j(i) \in \mathbb{N}$. All agents have an outside option of remaining unmatched, which results in a value of 0.

It will be useful to denote the match value of firm i and worker j by

$$\alpha_{ij} = \begin{cases} v_i(j) - c_j(i) & \text{if } v_i(j) - c_j(i) > 0 \\ \emptyset & \text{else} \end{cases}$$

Let $\boldsymbol{\alpha} = (\alpha_{ij})_{i \in F, j \in W}$ and $\alpha^* = \max_{i,j:\, \alpha_{ij} \neq \emptyset} \alpha_{ij}$.

We say that a market $\mathcal{E}$ is a *homogeneous goods market* if for all i, j we have that $v_i(j) = v_i$, and $c_j(i) = c_j$. In other words, each firm offers an identical job and each worker provides the same labor. Otherwise, the market is a *heterogeneous goods market.*

The matching between workers and firms $\mu : F \cup W \to F \cup W$ satisfies $\mu(i) \in W \cup \{i\}$, $\mu(j) \in F \cup \{j\}$, and $\mu(\mu(k)) = k$ for all $k \in F \cup W$. We denote that k is unmatched by $\mu(k) = k$. Payoffs are given by $\Phi = (\phi_k)_{k \in F \cup W}$ where $\phi_i + \phi_j = v_i(j) - c_j(i)$ for all i, j such that $\mu(i) = j \in W$, and $\phi_k = 0$ for all $k \in F \cup W$ such that $\mu(k) = k$. An outcome of the market is then given by (μ, Φ).

In this context, a matching μ is *optimal* if for all matchings μ', $\sum_{i,j} \alpha_{ij} \cdot \mu_{ij} \geq \sum_{i,j} \alpha_{ij} \cdot \mu'_{ij}$. Similarly, we say that the payoff profile Φ is ε*-stable* if $\phi_k \geq 0$ for all $k \in F \cup W$ and for any $i \in F$, $j \in W$:

$$\phi_i + \phi_j > v_i(j) - c_j(i) - 2\varepsilon \tag{3.1}$$

Note that $\phi_k \geq 0$ implies that the assignment is individually rational, that is, every player prefers his assignment over being unmatched.

An outcome (μ, Φ) is in the ε-core of the assignment game if μ is optimal and Φ^t is ε-stable.

3.1. Memoryless dynamics

We first study uncoupled memoryless dynamics where agents only know their current payoffs and their costs or values. We describe such dynamics as a Markov process. The state of the dynamics at time $t \in \mathbb{N}_0$ is given by a tentative outcome $[\mu^t, \Phi^t]$. We denote transition probabilities by

$$\mathcal{P}([\mu^{t+1}, \Phi^{t+1}]; [\mu^t, \Phi^t], \mathcal{E}).$$

Definition 1. Fix some $\varepsilon > 0$. We say that a dynamic is a *memoryless dynamic* if it satisfies the following properties:

I: Strict blocking. A player is assigned a new match only if his payoff increases by at least ε. That is, if $\mathcal{P}([\mu^{t+1}, \Phi^{t+1}]; [\mu^t, \Phi^t], \mathcal{E}) > 0$, $\mu^{t+1}(k) \neq \mu^t(k)$, and $\mu^{t+1}(k) \neq k$ (that is, k is matched), then, $\phi_k^{t+1} > \phi_k^t + \varepsilon$.

II: Single pair transitions. Any transition that has positive probability under $\mathcal{P}$ involves at most one newly matched pair. The outcome changed only for the newly matched pair, and possibly for the two players who were previously matched to the players in the new pair, and have now become unmatched. That is, if $\mathcal{P}([\mu^{t+1}, \Phi^{t+1}]; [\mu^t, \Phi^t], \mathcal{E}) > 0$, then there exists $i \in F$, $j \in W$ such that $\mu^{t+1}(i) = j$, $\mu^t(i) \neq j$ and $(\mu_k^{t+1} = \mu_k^t, \phi_k^{t+1} = \phi_k^t)$ for all $k \in W \cup F \setminus \{i, \mu^t(i), j, \mu^t(j)\}$.

III: ε-core-absorbing. An outcome $[\mu, \Phi]$ is a fixed point of $\mathcal{P}$ if and only if $[\mu, \Phi]$ is ε-stable. That is, $\mathcal{P}([\mu, \Phi]; [\mu, \Phi], \mathcal{E}) = 1$ if and only if $[\mu, \Phi]$ is ε-stable. In addition, starting from every tentative outcome $[\mu^0, \phi^0]$ the expected time to reach a fixed point is finite.

IV: Random selection. The stochastic transitions can be decomposed into the following procedure. First, select an agent $k \in N$ uniformly at random. Then agent k selects a new matching in accordance to I-III. Formally,

$$\mathcal{P}([\mu^{t+1}, \Phi^{t+1}]; [\mu^t, \Phi^t], \mathcal{E}) = \sum_k p_k \mathcal{P}_k([\mu^{t+1}, \Phi^{t+1}]; [\mu^t, \Phi^t], \mathcal{E})$$

where $p_k = 1/|F \cup W|$. The transitions in the support of $\mathcal{P}_k$ are generated by allowing k to make a proposal based on his available information. That is, the only agents whose assignment changed are $\{k, \mu^t(k), \mu^{t+1}(k), \mu^t(\mu^{t+1}(k))\}$ and

$\mathcal{P}_k([\mu^{t+1}, \Phi^{t+1}]; [\mu^t, \Phi^t], \mathcal{E}) = 0$ if $\mu^t(l) \neq \mu^{t+1}(l)$ or $\phi_j^{t+1} \neq \phi_j^t$ for $l \notin \{k, \mu^t(k), \mu^{t+1}(k), \mu^t(\mu^{t+1}(k))\}$. On the support of $\mathcal{P}_i$ we have that transition probabilities depend only on the tentative payoffs of k and the potential new match $\mu^{t+1}(k)$.

As discussed in the introduction, for a dynamic to represent a realistic description of a market, the dynamic should converge in a reasonable time window. We thus define:

Definition 2. A dynamic is *effectively convergent* if for any economy $\mathcal{E}$ with $n = |F \cup W|$ agents, the expected convergence time to the ε-core from any initial state is $O(n^r)$ for some $r > 0$.

In other words, a dynamic is not effectively convergent if there exists a sequence of markets for which the expected convergence time grows superpolynomially in the size of the market.

4. Memoryless dynamics are not effectively convergent

We can now state our main result:

Theorem 1. *Any memoryless dynamic is not effectively convergent. That is, there exists a sequence of markets $\mathcal{E}_\ell$ with $n_\ell \to \infty$ such that, starting from the majority of states, for any memoryless dynamic the expected (with respect to the stochastic mapping $\mathcal{F}$) convergence time to the ε-core grows exponentially in the number of players.*

An informal intuition for the result is that random selection causes the market to oscillate for a long time before converging. When certain agents are selected they will rematch with a partner that is already matched correctly, thus moving the allocation away from a core allocation. We show that when the current state is *far* away from a core outcome such *bad* transitions are unlikely. On the other hand, when the current state is *close* to a core outcome *bad* transitions are likely. Thus there is a tendency for the market to oscillate around states which are not very far nor very close to the core. A dual to the latter process is a biased random walk on a finite line, biased towards the origin. Finally, note that a simple extension of our analysis can be used to show that the convergence to the core can be slow if only agents on one side are selected.

Proof. We construct a specific market with homogeneous goods to give an example for which convergence is exponential for the majority of starting states. Suppose that there are more firms than workers ($f > w$) and $f = w + c$, where c is a constant. In particular we shall choose the market defined by matrix $\boldsymbol{\alpha}$ which has a unique price supported in an ε-core allocation ($\forall i \in W : \phi_i = 10 - \varepsilon$).[5] Note that, in this market, the matching does not matter, and convergence to the ϵ-core reduces to finding the correct price.

$$\boldsymbol{\alpha} = \underbrace{\begin{pmatrix} 10 & 10 & \dots & 10 \\ 10 & 10 & \dots & 10 \\ 10 & 10 & \dots & 10 \\ 10 & 10 & \dots & 10 \\ \dots & \dots & \dots & 10 \\ 10 & 10 & 10 & 10 \end{pmatrix}}_{w} \left.\vphantom{\begin{matrix}1\\1\\1\\1\\1\\1\end{matrix}}\right\} f \tag{4.1}$$

We say that the payoff of worker $i \in W$'s is *correct* if $\phi_i^t = 10 - \varepsilon$ and firm $j \in W$'s payoff correct if $\phi_i^t = \varepsilon$. A worker (or firm) with correct payoff is said to be priced correctly. Let $k^t = |\left\{i \in W : \phi_i^t = 10 - \varepsilon\right\}|$ be the number of matched workers who are holding the correct payoff. Once all workers are holding the correct payoff it follows that all matched firms do so too, since for matched i, j, we have $\phi_i^t + \phi_j^t = 10$, implying that the ε-core has been reached. Consequently, to prove the theorem it suffices to show that the time until $k^t = w$ is exponential in w.

Given the restriction by Property II (single pair transitions) we have $k^{t+1} \in \{k^t - 1, k^t, k^t + 1\}$. If $k^t \leq w - 1$ there exists at least one firm and one worker that are not correctly priced. (If $k^t = w$ the ε-core has been reached.) Let l_W^t be the number of under-priced workers ($\phi_i^t < 10 - \varepsilon$) and l_F^t the number of under-priced firms ($\phi_j^t < \varepsilon$). Note that workers can never be overpaid, that is, $\phi_i^t \leq 10 - \varepsilon \ \forall i \in W, \forall t$. Therefore:

$$l_W^t + k^t = w \tag{4.2}$$

$$l_F^t + k^t \leq f \tag{4.3}$$

[5] Note that a payoff of 10 is not achievable by Property I (strict blocking) unless it is at that level in the starting state. We assume that this is not the case, which is consistent with the consideration of the majority of possible starting states.

Since $f > w$ there exist unmatched firms with payoff 0. But as the correct payoff is ε, we have $l^t_F > 0$ for all t. To summarize, we have for $k^t < w$:

$$l^t_W, l^t_F > 0 \tag{4.4}$$

To proceed, we first consider the probability that the random variable k^t increases. By Property I (strict blocking), in order to rematch at the correct price, an under-priced worker (firm) needs to be rematched with an under-priced firm (worker). This requires that the randomly selected agent i is either one of the l^t_W under-priced workers or one of the l^t_F under-priced firms. By Property IV (random selection) this implies the following upper bound:

$$\begin{aligned}\mathbb{P}(k^{t+1} = k^t + 1) &\leq \mathbb{P}(k^{t+1} = k^t + 1 | k^{t+1} \neq k^t) && (4.5)\\ &\leq \frac{l^t_W}{n} + \frac{l^t_F}{n} && (4.6)\\ &\leq \left(\frac{w - k^t}{n} + \frac{f - k^t}{n}\right) && (4.7)\\ &\leq \frac{n - 2k^t}{n} && (4.8)\end{aligned}$$

Next, consider the probability that k^t decreases. This occurs when a correctly priced firm leaves its current partner. By Property I (strict blocking) the firm must strictly increase its payoff and the new match must have a price that is at least 2ε. The probability that a correctly priced firm is selected is $\frac{k^t}{n}$, and for such a firm the only potential partners that satisfy Property I (strict blocking) must be under-priced workers. Such a worker always exists if $k^t < w$. Therefore, given $k^{t+1} \neq k^t$, the selection of any correctly priced firm will give a transition that decreases k^t with probability 1 ($k^t > 0$). By Property IV (random selection) this implies the following lower bound:

$$\mathbb{P}(k^{t+1} = k^t - 1 | k^{t+1} \neq k^t) \geq \frac{k^t}{n} \cdot 1 \tag{4.9}$$

Thus conditional on $k^{t+1} \neq k^t$ we find:

$$\begin{aligned}\mathbb{P}(k^{t+1} = k^t + 1 | k^{t+1} \neq k^t) &= \frac{\mathbb{P}(k^{t+1} = k^t + 1 | k^{t+1} \neq k^t)}{\mathbb{P}(k^{t+1} = k^t + 1 | k^{t+1} \neq k^t) + \mathbb{P}(k^{t+1} = k^t - 1 | k^{t+1} \neq k^t)}\\ &\leq \frac{1 - \frac{2k^t}{n}}{1 - \frac{2k^t}{n} + \frac{k^t}{n}} = \frac{(n - k^t) - k^t}{n - k^t} = 1 - \frac{k^t}{n - k^t}\end{aligned} \tag{4.10}$$

and, likewise:

$$\begin{aligned}\mathbb{P}(k^{t+1} = k^t - 1 | k^{t+1} \neq k^t) &= 1 - \mathbb{P}(k^{t+1} = k^t + 1 | k^{t+1} \neq k^t)\\ &\geq \frac{\frac{k^t}{n}}{1 - \frac{2k^t}{n} + \frac{k^t}{n}} = \frac{k^t}{n - k^t}\end{aligned} \tag{4.11}$$

Consider the latter probability for $k > \frac{5}{6}w$ and suppose that $c < \frac{1}{6}w$. The latter assumption is permissible as we assume that c is a constant and we are interested in the limit as w increases. We then have:

$$\mathbb{P}(k^{t+1} = k^t - 1 | k^{t+1} \neq k^t, k > \frac{5}{6}w) > \frac{\frac{5}{6}w}{n - \frac{5}{6}w} = \frac{\frac{5}{6}w}{\frac{7}{6}w + c} > \frac{\frac{5}{6}w}{\frac{8}{6}w} = \frac{5}{8} \tag{4.12}$$

We now define a random walk that is coupled with the process k^t for $k > \frac{5}{6}w$. Let Y^t be a random variable taking values in $\{\lceil \frac{5}{6}w \rceil, \ldots, w\}$ with $Y^0 = \lceil \frac{5}{6}w \rceil$ if $k^0 \leq \frac{5}{6}w$ and $Y^0 = k^0$ otherwise. Then, let

$$Y^{t+1} = \begin{cases} Y^t & \text{if } k^{t+1} = k^t \text{ or } k^{t+1} \leq \frac{5}{6}w \\ Y^t + 1 & \text{if } k^{t+1} = k^t + 1 \text{ and } k^{t+1} > \frac{5}{6}w \\ Y^t - 1 & \text{w.p. } \frac{1}{\rho(k^t)} \cdot \frac{5}{8} \quad \text{if } k^{t+1} = k^t - 1 \text{ and } k^{t+1} > \frac{5}{6}w \\ Y^t + 1 & \text{w.p. } 1 - \frac{1}{\rho(k^t)} \cdot \frac{5}{8} \text{ if } k^{t+1} = k^t - 1 \text{ and } k^{t+1} > \frac{5}{6}w \end{cases} \tag{4.13}$$

where $\rho(k^t) := \mathbb{P}(k^{t+1} = k^t - 1 | k^{t+1} \neq k^t)$. By Equation (4.12) the probabilities in Equation (4.13) are well defined. By construction, $k^t \leq Y^t$ for all t. The probabilities conditional on changing are:

$$\mathbb{P}(Y^{t+1} = Y^t + 1 | Y^{t+1} \neq Y^t) = 1 - \rho(k^t) + \rho(k^t) \cdot \left[1 - \frac{1}{\rho(k^t)} \cdot \frac{5}{8}\right] = \frac{3}{8} \tag{4.14}$$

$$\mathbb{P}(Y^{t+1} = Y^t - 1 | Y^{t+1} \neq Y^t) = \rho(k^t) \cdot \frac{1}{\rho(k^t)} \cdot \frac{5}{8} = \frac{5}{8} \tag{4.15}$$

Note that conditioning on the time steps where $Y^{t+1} \neq Y^t$ the transition probabilities are independent of k^t. Y^t constitutes a *lazy* biased random walk with reflecting boundary at $\lceil \frac{5}{6} w \rceil$ and absorbing boundary at w. Lazy refers to the fact that sometimes the value doesn't change, that is, $Y^{t+1} = Y^t$. In particular, the random walk is biased towards $\lceil \frac{5}{6} w \rceil$. This model is an instance of the gambler's ruin problem. It is known that the time to reach the absorbing state w is exponential in the number of steps ($\frac{1}{6}w$) and thus is exponential in n (see Epstein 2009). Recalling that $k^t \leq Y^t$, this concludes the proof by noting that the exponential convergence time in n holds when starting from the majority of states as we only needed to consider the behavior for $k > \frac{5}{6}w$. □

5. Little information gives rise to an effectively convergent dynamic

In the previous section we showed how any memoryless dynamic that allows players to base their decision only on their current payoff is not effectively convergent. In this section, we study how the addition of little information gives rise to an effectively convergent dynamic. We allow unmatched players to keep information about their payoff when last matched.

Since for any information structure one can trivially design a dynamic that is not effectively convergent (that is, the time to convergence scales exponentially in the population size) we need to specify a matching and pricing rule. A natural candidate is the second price auction with reserve prices. Following Nax and Pradelski (2015), we introduce reserve prices that are dependent on a player's previous period reserve price rather than only on a player's previous period payoff. This gives unmatched players information about their payoff when last matched. As discussed in Section 2 this is motivated by aspiration adaptation of Sauermann and Selten (1962) and Selten (1998) and experimentally confirmed by Tietz and Weber (1972) and Roth and Erev (1995). To describe it, define a player's period-t *reserve price* by

$$d_i^t = \begin{cases} \phi_i^{t-1} & \text{if matched in period } t-1, \\ (d_i^{t-1} - \delta)_+ & \text{else} \end{cases} \tag{5.1}$$

with $\varepsilon > \delta > 0$.[6] For ease of exposition suppose that the selected player is a worker $j \in W$. He then runs a second price auction with reserve price $c_j + (d_j + \varepsilon)$. The players on the other side of the market bid $v_i - (d_i + \varepsilon)$ and the highest bidder, say i', receives the match with payoff for i: $\max\{c_j + (d_j + \varepsilon), \max_{i \neq i'} v_i - (d_i + \varepsilon\})$. When there are several highest bidders assume that one is selected uniformly at random.

I′: Reserve price strict blocking. A player is assigned a new match if and only if as a consequence his payoff is at least ε greater than his reserve price. That is, if $\mathcal{P}([\mu^{t+1}, \Phi^{t+1}]; [\mu^t, \Phi^t], \mathcal{E}) > 0$, and $\mu^{t+1}(k) \neq \mu^t(k)$, then, $\phi_k^{t+1} > d_k^t + \varepsilon$.

Remark. We note that the proposed dynamic remains uncoupled and also satisfies Properties II (single-pair transitions), III (ε-core-absorbing), IV′ (random selection, where IV′ adheres to Property I′) and Property I′.

We first show that for markets with homogeneous goods the second price auction with reserve prices is effectively convergent. Intuitively, reserve prices introduce a monotonicity (in expectation) that allows agents to 'learn' their correct payoff. We then conduct a series of computational experiments which support the conjecture that the dynamic is also effectively convergent for randomly generated markets with heterogeneous goods.[7]

5.1. Fast convergence in markets with homogeneous goods

Recall that in a market with homogeneous goods each firm is offering an identical job and each worker is offering identical work. That is, for all $i \in F$, $v_i(j) = v_i$ for all $j \in W$ and for all $j \in W$, $c_j(i) = c_j$ for all $i \in F$. Also recall that $\alpha^* = \max_{i,j:\, \alpha_{ij} \neq \emptyset} \alpha_{ij}$.

Theorem 2. *For any market with homogeneous goods with $\varepsilon/\delta \in \mathbb{N}$, $\varepsilon > \delta$, from any starting state, the expected convergence time to the ε-core of the repeated second price auction with reserve price is $O(n^{3+2\frac{\varepsilon}{\delta}} \cdot \log(n) \cdot e^{\frac{\alpha^*}{\delta}} \cdot \frac{\alpha^*}{\varepsilon})$ periods. Further, the dynamic is ε-core-absorbing.*

For readability the proof of the latter Theorem is relegated to the Appendix A.

[6] If $\delta > 0$ does not hold, the proposed dynamic does not converge to the ε-core since a player may get 'stuck' with a reserve price that is not supported in a core outcome.

[7] Notably, similar simulation results have been shown for non-transferable utility matching markets (Biró and Norman, 2013) while the negative result by Ackermann et al. (2011) shows that in general efficiency may not be guaranteed. Clearly, our simulations can not give an indication for a general result.

5.2. Fast convergence in markets with heterogeneous goods

This section complements the previous results with simulations for markets with heterogeneous goods. We consider the random assignment markets described in Kanoria et al. (2018) with different types of firms ($\mathcal{T}_F = 1, ..., \tau_F$) and workers ($\mathcal{T}_W = 1, ..., \tau_W$). Denote by $\tau(i)$ the type of an agent $i \in N$ and assume that match values are additively separable, that is, of the general form:

$$\alpha_{ij} = u(\tau(i), \tau(j)) + \beta_i^{\tau(j)} + \gamma_j^{\tau(i)} \tag{5.2}$$

In particular this means that a match value is the sum of a utility $u(\tau(i), \tau(j))$ that depends only on the agents' types and idiosyncratic components. The latter depend on the identity of one of the agents and only the type of the other agent ($\beta_i^{\tau(j)}$ and $\gamma_j^{\tau(i)}$). Note that when there is a single firm type and a single worker type, we revert to the market with homogeneous goods studied above. When there are as many firm and worker types as there are firms and workers, the above definition posits no restrictions on the match values.

In the computational experiments reported in this section, assignment games with input values f, w, τ_F, τ_W are generated by randomly selecting a type in $\mathcal{T}_F$ (respectively $\mathcal{T}_W$) for each firm (respectively worker). The utility components are drawn from uniform distributions with:

$$u(\tau(i), \tau(j)) \sim U[0, 1, 2, \ldots 100] \tag{5.3}$$

$$\beta_i^{\tau(j)} \sim U[0, 1, 2, \ldots 20] \tag{5.4}$$

$$\gamma_j^{\tau(i)} \sim U[0, 1, 2, \ldots 20] \tag{5.5}$$

We shall fix $\varepsilon = 1, \delta = 0.5$ throughout. We report four computational experiments where we analyze the rate of convergence for different number of types as the population size is increasing. We chose the experiments to cover markets with the same number of firms and workers and markets with a different number of firms and workers. Further we vary whether both sides of the market have different types or only one side.

Experiment 1: $f = w$ and $\tau_F = \tau_W$ (Tables 1–4).
Experiment 2: $f = w + 4$ and $\tau_F = \tau_W$ (Tables 5–8).
Experiment 3: $f = w$ and $\tau_F = 1$ (Tables 9–12).
Experiment 4: $f = w + 4$ and $\tau_F = 1$ (Tables 13–16).

We analyze the growth of the number of steps to convergence (T) as the number of agents increases ($w = 4, 8, 12, \ldots, 48$) for different number of types ($\tau_W = 1, 2, 0.5 \cdot w, 1.0 \cdot w$).[8] We randomly sampled 100 assignment games for each of these $4 \cdot 4 \cdot 12 = 192$ cases and for each sampled assignment game we ran 100 simulations and then took the average number of steps to convergence as a proxy for the expectation.[9]

Taking the logarithm, if T grows exponentially we should see linear growth, if T is sub-exponential the growth should be logarithmic. We thus estimate the following three models where T is the time to convergence and w is the number of workers:

$$\log(T(w)) = \beta_0 + \beta_1 \cdot w + \beta_2 \cdot \log(w) + \epsilon_1 \tag{Model 1}$$

$$\log(T(w)) = \beta_0 + \beta_1 \cdot w + \epsilon_2 \tag{Model 2}$$

$$\log(T(w)) = \beta_0 + \beta_2 \cdot \log(w) + \epsilon_3 \tag{Model 3}$$

where $\epsilon_1, \epsilon_2, \epsilon_3$ are assumed to be normally distributed error terms with mean 0. In most of the 16 analyzed datasets β_1 turns out to be statistically insignificant or negative in Model 1, while β_2 is significantly positive.[10] Nevertheless, in three of the datasets β_1 is significantly positive. Using Bayesian model selection (BIC test, see Schwarz 1978; Kass and Raftery 1995; Raftery 1995) we find strong evidence that also in these cases Model 3 is the preferred model. This allows us to reject Models 1 and 2, thus selection Model 3 as our preferred model and confirming our hypothesis that the convergence rate is growing polynomially in the population size. We note that the minimal R^2 across all regressions for Model 3 is 0.748 while the maximum is 0.992, thus suggesting that our models have very strong predictive power. The regressions and the BIC tests can be found in the Appendix A.

Figs. 5.1–5.4 show the results for the four experiments. The x-axis shows the number of workers (w) and the y-axis the logarithm of the number of time steps to convergence to the ε-core ($\log(T)$). Each box-plot shows the 25th to 75th

[8] The increments are chosen in order to ensure that in each market each type is represented with the same proportion.
[9] We thus have a total of 1,920,000 simulations with an average run-time of 15 seconds (mainly varying with the population size). Simulations were run on ETH Zurich's EULER cluster.
[10] Note that, if β_1 is negative this only yields further support to the hypothesis that the growth rate is logarithmic (polynomial for the original data).

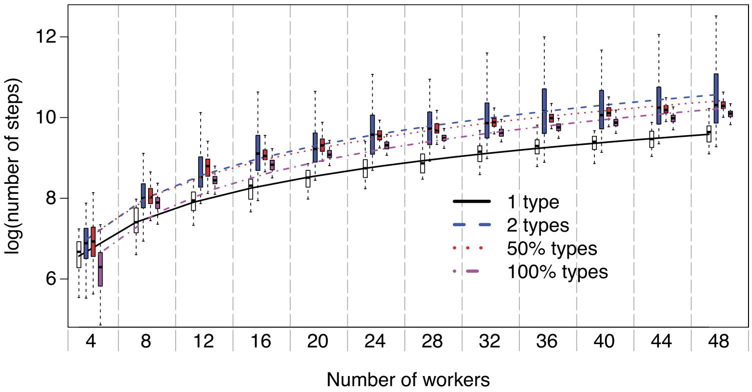

Fig. 5.1. Simulations for Experiment 1: $f = w$ and $\tau_F = \tau_W$.

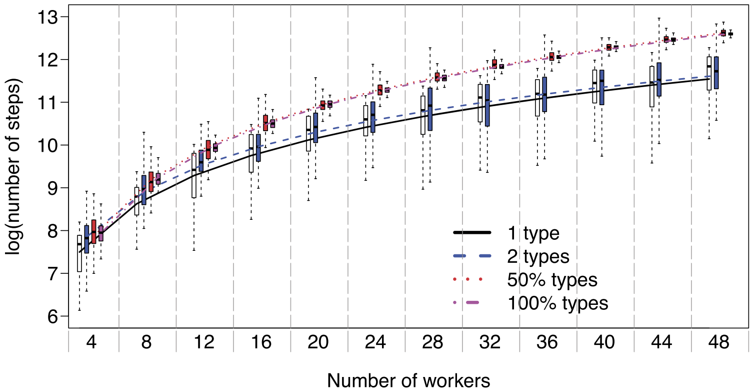

Fig. 5.2. Simulations for Experiment 2: $f = w + 4$ and $\tau_F = \tau_W$.

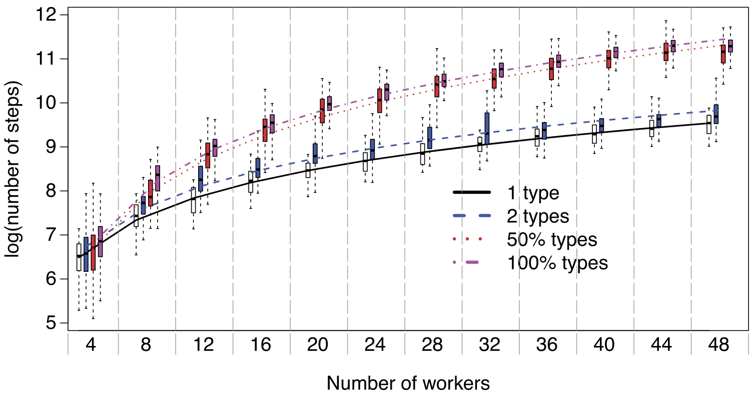

Fig. 5.3. Simulations for Experiment 3: $f = w$, $\tau_F = 1$.

percentiles and the lines show the full range of observations of the average convergence times for 100 randomly sampled assignment games. Each plot analyzes the four different number of types ($\tau_W = 1, 2, 0.5 \cdot w, 1.0 \cdot w$) for different numbers of workers (and firms). The regressions show the fitted logarithmic Model 3.

In summary, our results support the hypothesis that for most markets (randomly generated) little additional information not only suffices to design effectively convergent dynamics for markets with homogeneous goods but also for markets with heterogeneous goods. However, our simulations suggest that the expected rate of convergence does vary significantly. In Experiments 1-3 the market with homogeneous goods shows the fastest convergence times, while in Experiment 4 it is the opposite. The particularities of different markets with heterogeneous goods are not simple derivatives of a given market with homogeneous goods. In particular, our proof technique for markets with homogeneous goods does not promise to extend to this more general case.

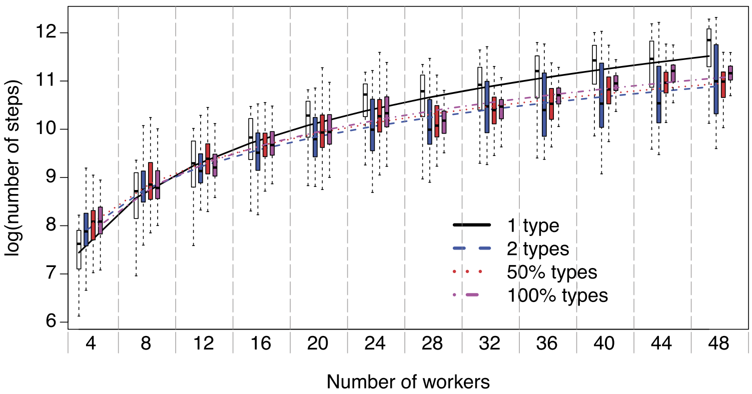

Fig. 5.4. Simulations for Experiment 4: $f = w + 4$, $\tau_F = 1$.

6. Conclusion

Price discovery (Chamberlin, 1948; Smith, 1962) and more generally the question of decentralized dynamics leading to equilibrium (Hayek, 1945) has been studied by economists for at least a century. With more and more market places moving online this topic has sparked renewed interest. In this paper, we first showed that for an important class of markets it is difficult to find core outcomes by decentralized dynamics. Any memoryless dynamic, that is, a dynamic that only relies on the primitives of the game – the current payoffs and matching – is not effectively convergent (the convergence rate grows exponentially in the population size). Next, we proposed a natural, uncoupled dynamic based on a second price auction with reserve prices and showed that through the addition of little information, an effectively convergent dynamic for price discovery exist in markets with homogeneous goods. Simulation experiments provide evidence for our conjecture that this also holds for a high proportion of markets with heterogeneous goods. Our analysis suggests that decentralized markets are not trivial, but effectively convergent dynamics can be designed with fairly little memory requirements. Further study is needed to better understand the effect of markets with heterogeneous goods and of exogenous changes such as market entry and exit.

Appendix A

A.1. Proof of Theorem 2

We prove the theorem via several steps. We assume throughout w.l.o.g. $w \le f$.

Definition 3. Say that reserve prices $(d_i)_{i\in F}, (d_j)_{j\in W}$ are (ε, δ)-pre-stable if for all $i \in F$, $j \in W$ matched (not necessarily to each other):

$$d_i + d_j > \alpha_{ij} - 2\varepsilon \tag{6.1}$$

and for all $i \in F$, $j \in W$:

$$d_i + d_j > \alpha_{ij} - 2\varepsilon - \delta \tag{6.2}$$

Note that if $\delta < \varepsilon$ is the smallest unit, the latter equation is equivalent to:

$$d_i + d_j \ge \alpha_{ij} - 2\varepsilon \tag{6.3}$$

Lemma 4. *The expected time until reserve prices are (ε, δ)-pre-stable is $O(n \log(n))$.*

Proof. If player i is activated as auctioneer in period t, by the end of the period i has

- rematched and $d_i^t + d_j^t \ge \alpha_{ij}$ for all j (since the highest bidder wins),
- remains in previous match and $d_i^t + d_j^t > \alpha_{ij} - 2\varepsilon$ for all j (since otherwise he would have rematched), or
- remains unmatched and thus in period $t-1$, $d_i^{t-1} + d_j^{t-1} > \alpha_{ij} - 2\varepsilon$ for all j (since otherwise he would have found a match).

Hence at the end of period t, $d_i^t + d_j^t > \alpha_{ij} - 2\varepsilon - \delta$ for all j and the assertion holds for the auctioneer.

We shall now show that once the assertion holds for a player it holds in all subsequent periods. Suppose that in $t-1$ either i is matched and $d_i^{t-1} + d_j^{t-1} > \alpha_{ij} - 2\varepsilon$ for all j or i is unmatched and $d_i^{t-1} + d_j^{t-1} > \alpha_{ij} - 2\varepsilon - \delta$ for all j. Suppose that j is selected as auctioneer in period t. Then, by the previous arguments, at the end of period t either j

- is matched and $d_j^{t-1} + d_{i'}^{t-1} > \alpha_{ji'} - 2\varepsilon$ for all i', or
- is unmatched and $d_j^{t-1} + d_{i'}^{t-1} > \alpha_{ji'} - 2\varepsilon - \delta$ for all i'.

In particular for i the assertion still holds since the only player with whom she could violate the condition is j because only auctioneers (potentially) reduce their reserve prices.

To summarize, after each player is selected at least once a pre-stable state is reached. The expected waiting time is $O(n \log(n))$ (by the Coupon Collectors Problem, see, for example, Feller, 1950). □

Corollary 5. *If for some $T > 0$, $(d_i^T)_{i \in F}, (d_j^T)_{j \in W}$ is (ε, δ)-pre-stable, then:*

1. *for all $t \geq T$ $(d_i^t)_{i \in F}, (d_j^t)_{j \in W}$ is (ε, δ)-pre-stable*
2. *no two matched players can rematch in the next period*
3. *the number of matchings is non-decreasing*

Definition 6. Let m^t be the number of matched workers in period t (which is the same as the number of matched firms).

Lemma 7. *Suppose the market consists of homogeneous goods. If for some $T > 0$, $(d_i^T)_{i \in F}, (d_j^T)_{j \in W}$ is (ε, δ)-pre-stable and $m^t < w$, then for all $t \geq T$ the expected time for m^t to increase is $O(n^{1+2\varepsilon/\delta} \cdot e^{\frac{\alpha^*}{\delta}})$.*

Proof. By Corollary 5(3), the number of matched agents only increases when two unmatched agents match. Hence we are interested in the following probability:

$$\mathbb{P}(\text{unmatched as auctioneer}) \cdot \mathbb{P}(\text{other unmatched wins auction}) \tag{6.4}$$

Since $m^t < w$, $\mathbb{P}(\textit{unmatched as auctioneer}) \geq \frac{1}{n}$. The expected waiting time for this event is bounded above by n.

We introduce a known decomposition of markets with homogeneous goods (see, for example, Kanoria et al. (2018)) where match values α_{ij} are composed of non-negative values β_i, γ_j such that:

$$\alpha_{ij} = \beta_i + \gamma_j \tag{6.5}$$

Given two unmatched agents i, j suppose that there exists a feasible match for each of them i', j', that is:

$$d_i + d_{j'} = \alpha_{ij'} - 2\varepsilon = \beta_i + \gamma_{j'} - 2\varepsilon \tag{6.6}$$

$$d_{i'} + d_j = \alpha_{i'j} - 2\varepsilon = \beta_{i'} + \gamma_j - 2\varepsilon \tag{6.7}$$

By adding the two equations we find:

$$d_i + d_j = \beta_i + \gamma_j - 2\varepsilon + \beta_{i'} + \gamma_{j'} - 2\varepsilon - (d_{i'} + d_{j'}) \tag{6.8}$$

$$\leq \alpha_{ij} - 2\varepsilon + \alpha_{i'j'} - 2\varepsilon - (\alpha_{i'j'} - 2\varepsilon) \tag{6.9}$$

$$\leq \alpha_{ij} - 2\varepsilon \tag{6.10}$$

Hence if i, j are feasible with some player they are also feasible with each other.

Since α_{ij} are multiples of δ we have for all i, j, that d_i, d_j are multiples of δ. Thus if $d_i + d_j \not> \alpha_{ij} - 2\varepsilon$ (i, j are feasible) and $d_i + d_j > \alpha_{ij} - 2\varepsilon - \delta$ (pre-stable) we must have $d_i + d_j = \alpha_{ij} - 2\varepsilon$. Consequently, if two opposite unmatched agents are feasible for each other they are also among the highest bidders for each other (and thus might match).

It then suffices to evaluate the probability that two unmatched agents i, j are feasible at the same time. We shall write a protocol in pseudo-code:

1. The probability that any given player is selected in a period is $1/n$. Hence the expected waiting time until one unmatched agent is feasible is easily seen to be $O(n \cdot \frac{\alpha^*}{\delta})$. Let i be the first unmatched agent becoming feasible.

2a. If i re-matches with another unmatched agent we are done.

2b. Otherwise she re-matches with a matched player j' who in turn leaves her previous partner unmatched, i'. Denote the player who last turned unmatched by i^*.

3. The new unmatched agent i^* has to reduce her reserve price by at most 2ε to become feasible again (since the newly matched players haven't increased their reserve price by more than ε each). Note that, as long as m^t does not increase, in

any subsequent period there is always at least one unmatched agent, namely i^*, who needs to reduce her reserve price by at most 2ε.

Now, regarding 1, we evaluate the probability that an unmatched agent from the other side, j, becomes feasible before i^* does. j has to reduce his reserve price at most α^*/δ times which takes in expectation $n \cdot \alpha^*/\delta$ steps. Hence:

$$\mathbb{P}(j\, becomes\, feasible\, before\, i^*) \geq \left(\frac{n-1}{n}\right)^{n\alpha^*/\delta} \tag{6.11}$$

$$= \left(\frac{n}{n-1}\right)^{\alpha^*/\delta} \left(\frac{n-1}{n}\right)^{(n+1)\alpha^*/\delta} \tag{6.12}$$

$$\geq \left(\frac{n}{n-1}\right)^{\alpha^*/\delta} \left(\frac{1}{e}\right)^{\alpha^*/\delta} \tag{6.13}$$

the last inequality holds because we know that $\left(1+\frac{1}{n}\right)^n \leq e \leq \left(1+\frac{1}{n}\right)^{n+1}$ and thus $\left(1-\frac{1}{n}\right)^n \geq e \geq \left(1-\frac{1}{n}\right)^{n+1}$ ("monotonicity from below"). Thus the expected time for the event 'j becomes feasible before i^*' is upper bounded by $e^{\alpha^*/\delta}$.

Now if j is feasible we have:

$$\mathbb{P}(i^* \text{ becomes feasible before j matches} | j \text{ is feasible}) \geq \left(\frac{1}{n}\right)^{2\varepsilon/\delta}, \tag{6.14}$$

where the lower bound constitutes the probability of selecting i^* $2\varepsilon/\delta$ consecutive times. Thus, the expected time is upper bounded by $n^{2\varepsilon/\delta}$. Combining the two bounds we find that the expected time before two unmatched agents match is given by:

$$n^{2\varepsilon/\delta} \cdot e^{\alpha^*/\delta} \tag{6.15}$$

Thus in total we have:

$$n^{1+2\varepsilon/\delta} \cdot e^{\alpha^*/\delta} \quad \square \tag{6.16}$$

Corollary 8. *The expected time until $m^t = w$ is $O(n^{2+2\varepsilon/\delta} \cdot e^{\frac{\alpha^*}{\delta}})$.*

Proof. This follows by applying Lemma 7 for all n players. □

Lemma 9. *Suppose that for $T > 0$, $m^T = w$. The expected time until an ε-stable state is reached is $O(n^2 \cdot \frac{\alpha^*}{\varepsilon})$.*

Proof. If $w = f$ this is immediately the case once all players are matched. Otherwise, suppose that $w < f$. Note that by Corollary 5(3) the number of matchings is non-decreasing. Thus all workers (the short side of the market) remain matched in all periods $t > T$. But matched players do not decrease their reserve prices and only re-match if their subsequent reserve price increases by at least ε. If the current state is not ε-stable there exists at least one (matched) worker and one (unmatched) firm who are feasible. The expected time until one of them is selected as auctioneer is $O(n)$. We thus have that the time until an ε-stable state is reached is:

$$O(n) \cdot n \cdot \frac{\alpha^*}{\varepsilon} = O(n^2 \cdot \frac{\alpha^*}{\varepsilon}) \quad \square \tag{6.17}$$

Combining the bounds from Lemmas 4, 7, 9, and Corollary 8 it follows that overall the expected time to reach the ε-core is given by:

$$O(n^{3+2\frac{\varepsilon}{\delta}} \cdot \log(n) \cdot e^{\frac{\alpha^*}{\delta}} \cdot \frac{\alpha^*}{\varepsilon}) \tag{6.18}$$

thus concluding the proof of Theorem 2.

A.2. Details of OLS regressions and BIC tests for Section 5.2

This section contains the detailed results of the OLS regressions and Bayesian model selections (BIC test, see Schwarz 1978; Kass and Raftery 1995; Raftery 1995) for Models 1-3 from Section 5.2. For each experiment there are four regressions for the different number of types τ_W (and τ_F unless it is kept constant at 1). As described in the main part all regressions show that Model 3 has very high R^2 for all cases. Further, when Model 1 shows a significant value for the linear term the Bayesian model selection again points strongly to Model 3 (using the classification by Kass and Raftery 1995 for strength of evidence).

A.3. Experiment 1: $f = w$, $\tau_F = \tau_W$

Table 1
$\tau_W = 1$.

	Model 1	Model 2	Model 3
w	0.002	0.060***	
	(0.002)	(0.001)	
log(w)	1.185***		1.215***
	(0.036)		(0.012)
Constant	4.930***	7.019***	4.882***
	(0.067)	(0.025)	(0.037)
Observations	1,200	1,200	1,200
R^2	0.899	0.809	0.899
BIC	−8382.372	−8296.923	−8389.396

* $p < 0.1$.
** $p < 0.05$.
*** $p < 0.01$.

Table 2
$\tau_W = 2$.

	Model 1	Model 2	Model 3
w	−0.014***	0.070***	
	(0.004)	(0.002)	
log(w)	1.690***		1.439***
	(0.074)		(0.024)
Constant	4.593***	7.573***	4.999***
	(0.136)	(0.044)	(0.076)
Observations	1,200	1,200	1,200
R^2	0.750	0.642	0.748
BIC	−8052.257	−7870.996	−8054.687

* $p < 0.1$.
** $p < 0.05$.
*** $p < 0.01$.

Table 3
$\tau_W = 0.5 \cdot w$.

	Model 1	Model 2	Model 3
w	−0.022***	0.064***	
	(0.002)	(0.001)	
log(w)	1.730***		1.328***
	(0.032)		(0.011)
Constant	4.633***	7.681***	5.280***
	(0.058)	(0.030)	(0.035)
Observations	1,200	1,200	1,200
R^2	0.934	0.769	0.924
BIC	−8407.188	−8217.127	−8402.415

* $p < 0.1$.
** $p < 0.05$.
*** $p < 0.01$.

Table 4
$\tau_W = 1.0 \cdot w$.

	Model 1	Model 2	Model 3
w	−0.035***	0.069***	
	(0.002)	(0.001)	
log(w)	2.081***		1.456***
	(0.031)		(0.012)
Constant	3.598***	7.265***	4.605***
	(0.056)	(0.034)	(0.037)

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Table 4 (*continued*)

	Model 1	Model 2	Model 3
Observations	1,200	1,200	1,200
R^2	0.948	0.749	0.928
BIC	−8412.572	−8134.343	−8390.906

* $p < 0.1$.
** $p < 0.05$.
*** $p < 0.01$.

A.4. Experiment 2: $f = w + 4$, $\tau_F = \tau_W$

Table 5
$\tau_W = 1$.

	Model 1	Model 2	Model 3
w	−0.002	0.080***	
	(0.004)	(0.002)	
log(w)	1.670***		1.628***
	(0.075)		(0.024)
Constant	5.173***	8.117***	5.241***
	(0.138)	(0.045)	(0.076)
Observations	1,200	1,200	1,200
R^2	0.789	0.702	0.789
BIC	−8040.924	−7864.102	−8047.88

* $p < 0.1$.
** $p < 0.05$.
*** $p < 0.01$.

Table 6
$\tau_W = 2$.

	Model 1	Model 2	Model 3
w	−0.007*	0.074***	
	(0.004)	(0.001)	
log(w)	1.637***		1.508***
	(0.070)		(0.023)
Constant	5.581***	8.466***	5.788***
	(0.128)	(0.042)	(0.071)
Observations	1,200	1,200	1,200
R^2	0.788	0.691	0.787
BIC	−8101.308	−7931.802	−8107.183

* $p < 0.1$.
** $p < 0.05$.
*** $p < 0.01$.

Table 7
$\tau_W = 0.5 \cdot w$.

	Model 1	Model 2	Model 3
w	0.006***	0.095***	
	(0.002)	(0.001)	
log(w)	1.808***		1.911***
	(0.030)		(0.010)
Constant	5.389***	8.576***	5.223***
	(0.055)	(0.030)	(0.031)
Observations	1,200	1,200	1,200
R^2	0.970	0.879	0.969
BIC	−8414.732	−8206.331	−8421.043

* $p < 0.1$.
** $p < 0.05$.
*** $p < 0.01$.

Table 8
$\tau_W = 1.0 \cdot w$.

	Model 1	Model 2	Model 3
w	0.035***	0.095***	
	(0.001)	(0.001)	
log(w)	1.828***		1.891***
	(0.015)		(0.005)
Constant	5.388***	8.576***	5.287***
	(0.028)	(0.030)	(0.016)
Observations	1,200	1,200	1,200
R^2	0.992	0.879	0.992
BIC	−8468.319	−8255.281	−8475.116

* $p < 0.1$.
** $p < 0.05$.
*** $p < 0.01$.

A.5. Experiment 3: $f = w$, $\tau_F = 1$

Table 9
$\tau_W = 1$.

	Model 1	Model 2	Model 3
w	−0.001	0.061***	
	(0.002)	(0.001)	
log(w)	1.246***		1.228***
	(0.035)		(0.011)
Constant	4.755***	6.952***	4.785***
	(0.065)	(0.025)	(0.036)
Observations	1,200	1,200	1,200
R^2	0.905	0.807	0.905
BIC	−8387.447	−8292.166	−8394.512

* $p < 0.1$.
** $p < 0.05$.
*** $p < 0.01$.

Table 10
$\tau_W = 2$.

	Model 1	Model 2	Model 3
w	−0.017***	0.059***	
	(0.002)	(0.001)	
log(w)	1.545***		1.236***
	(0.047)		(0.015)
Constant	4.544***	7.268***	5.043***
	(0.085)	(0.032)	(0.048)
Observations	1,200	1,200	1,200
R^2	0.849	0.711	0.843
BIC	−8315.307	−8164.99	−8315.349

* $p < 0.1$.
** $p < 0.05$.
*** $p < 0.01$.

Table 11
$\tau_W = 0.5 \cdot w$.

	Model 1	Model 2	Model 3
w	−0.015***	0.090***	
	(0.002)	(0.001)	
log(w)	2.118***		1.853***
	(0.047)		(0.016)
Constant	3.708***	7.442***	4.135***
	(0.087)	(0.039)	(0.049)

(continued on next page)

Table 11 (*continued*)

	Model 1	Model 2	Model 3
Observations	1,200	1,200	1,200
R^2	0.924	0.798	0.922
BIC	−8309.875	−8021.313	−8311.795

* $p < 0.1$.
** $p < 0.05$.
*** $p < 0.01$.

Table 12
$\tau_W = 1.0 \cdot w$.

	Model 1	Model 2	Model 3
w	−0.013***	0.089***	
	(0.002)	(0.001)	
log(w)	2.056***		1.816***
	(0.036)		(0.012)
Constant	4.044***	7.668***	4.431***
	(0.066)	(0.035)	(0.037)
Observations	1,200	1,200	1,200
R^2	0.953	0.825	0.951
BIC	−8384.385	−8112.93	−8387.24

* $p < 0.1$.
** $p < 0.05$.
*** $p < 0.01$.

A.6. Experiment 4: $f = w + 4$, $\tau_F = 1$

Table 13
$\tau_W = 1$.

	Model 1	Model 2	Model 3
w	0.002	0.082***	
	(0.004)	(0.002)	
log(w)	1.609***		1.640***
	(0.078)		(0.025)
Constant	5.219***	8.054***	5.168***
	(0.142)	(0.045)	(0.079)
Observations	1,200	1,200	1,200
R^2	0.780	0.702	0.780
BIC	−8010.803	−7847.339	−8017.82

* $p < 0.1$.
** $p < 0.05$.
*** $p < 0.01$.

Table 14
$\tau_W = 2$.

	Model 1	Model 2	Model 3
w	0.005	0.060***	
	(0.004)	(0.001)	
log(w)	1.098***		1.195***
	(0.078)		(0.025)
Constant	6.420***	8.356***	6.263***
	(0.143)	(0.042)	(0.079)
Observations	1,200	1,200	1,200
R^2	0.653	0.595	0.653
BIC	−8009.301	−7936.876	−8015.697

* $p < 0.1$.
** $p < 0.05$.
*** $p < 0.01$.

Table 15
$\tau_W = 0.5 \cdot w$.

	Model 1	Model 2	Model 3
w	−0.002	0.057***	
	(0.003)	(0.001)	
log(w)	1.188***		1.161***
	(0.056)		(0.018)
Constant	6.417***	8.511***	6.461***
	(0.102)	(0.033)	(0.056)
Observations	1,200	1,200	1,200
R^2	0.776	0.691	0.776
BIC	−8242.977	−8156.995	−8250.013

* $p < 0.1$.
** $p < 0.05$.
*** $p < 0.01$.

Table 16
$\tau_W = 1.0 \cdot w$.

	Model 1	Model 2	Model 3
w	0.013***	0.063***	
	(0.002)	(0.001)	
log(w)	1.013***		1.246***
	(0.045)		(0.015)
Constant	6.639***	8.424***	6.263***
	(0.083)	(0.027)	(0.047)
Observations	1,200	1,200	1,200
R^2	0.857	0.798	0.854
BIC	−8323.708	−8263.219	−8326.783

* $p < 0.1$.
** $p < 0.05$.
*** $p < 0.01$.

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