CORVINUS UNIVERSITY OF BUDAPEST

Doctoral School of Economics, Business and Informatics

THESIS BOOK

to the Ph.D. Thesis titled

Applications of Compter Vision: Skyline Extraction and Congressional Districting

written by

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Supervisor: Attila Tasnádi, D.Sc.

CORVINUS UNIVERSITY of BUDAPEST

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Institute of Mathematics and Statistical Modelling

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Contents

1 Introduction

We discuss problems from the fields of computer vision and congressional districting. The connection between the two seemingly distant subjects is image processing, which can be applied for both skyline extraction and circularity measurement.

Hiking applications have a serious problem with the sensor accuracy of mobile devices. With the help of the mountainous skyline and a 3D map, the precision of orientation can be significantly increased. Redistricting has to be carried out to resolve geographic malapportionment caused by the different district population growth rates and migration. This process can be manipulated for an electoral advantage of a party, but achieving optimal partisan districting is not easy at all. In most states of the USA, redistricting is made by non-independent actors and often causes debates about gerrymandering. The highest possible circularity is a natural requirement for a fair legislative district. Thus, shape analysis can be a powerful tool to detect potential manipulation.

First, we present an algorithm for skyline extraction and orientation in mountainous terrain, and we also verify the method in a relevant environment. Then, we prove that optimal partisan districting and majority securing districting are NP-complete problems, and demonstrate why finding optimal districting in real-life is challenging, as well. Finally, we introduce a novel, parameter-free circularity measure that can be used to detect gerrymandering and apply it to congressional districts.

2 Orientation in Mountainous Terrian

The accuracy of mobile sensors is not suitable for high precision augmented reality applications, see, e.g., Fedorov et. al (2016). The compass is biased by metal and electric instruments nearby, although frequent calibration, so measuring the magnetic and thus the true north is not reliable, the error of digital magnetic compass could be as high as $10-30°$, see details, e.g., Blum et. al (2013). We propose a method that consists of three main phases. Firstly, we determine the panoramic skyline from an elevation map by a geometric transformation based on the idea that Zhu et. al (2012) suggested. After that, we extract the skyline from the image by a novel edge-based algorithm that uses connected component labeling. Finally, for the matching phase, we seek the largest correlation between the two skyline vectors. Publication related to Section 2 is Nagy (2020).

2.1 Method

2.1.1 Panoramic Skyline Determination

Panoramic skyline is a vector obtained from the 3D model of the terrain. We used publicly available digital elevation models: SRTM and ASTER, sampled at a spatial resolution between 30m and 90m. Depending on the distance of the viewpoint from the target and characteristic of the terrain in the corresponding geographical area that could be a bit coarse, but in most cases, this resolution was enough. The 360° panoramic skyline was calculated from a given point by a coordinate transformation, where

- $C(X_0, Y_0, Z_0)$ is the position of the camera,
- $D(X, Y, Z)$ is an arbitrary point of the DEM,

- $D'(x', y', z')$ is the projection of point D. [1]

Hereby, each point can be described by the azimuth angle:

$$\varphi = \begin{cases} 0 & \text{if } X = X_0 \text{ and } Z = Z_0 \\ \arcsin\left(\frac{z' - Z_0}{\rho}\right) & \text{if } X \geq X_0 \\ -\arcsin\left(\frac{z' - Z_0}{\rho}\right) + \pi & \text{if } X < X_0 \end{cases}$$

and the elevation angle:

$$\theta = \arcsin\left(\frac{Y - y'}{r}\right)$$

where

$$\rho = \sqrt{(x' - X_0)^2 + (z' - Z_0)^2}$$

is the distance between C and D' and

$$r = \sqrt{(X - X_0)^2 + (Y - Y_0)^2 + (Z - Z_0)^2}$$

is the distance between C and D. Azimuth angle ϕ and the elevation angle θ describe any point D in the DEM. Finally, the largest θ value determines the demanded point of the skyline for each φ.

2.1.2 Skyline Extraction

The skyline sharply demarcates terrain from the sky on a landscape photo. Our novel and automatic skyline extraction method is presented in the following. The main idea is based on the experience that large and wide connected components in the upper region of the image usually belong to the skyline. The following algorithm selects the skyline from skyline candidates in multiple steps. The candidates were sorted by the function

$$S(C) = \mu(C) + 2\rho(C),$$

[1] $y' = Y_0$

where C is a skyline candidate, μ measures the number of pixels in the candidate and ρ is the span of the candidate.

The main steps are listed below.

1. Preprocessing

   (a) The first step is to resize the original image to 640×480 pixels and adjust the contrast.

   (b) The sky is in the sharpest contrast to the terrain in the blue color channel in RGB color space. Thus we use the blue channel as a grayscale picture.

   (c) Morphological closing and opening operations are applied for smoothing the outlines, reducing noise, and thereby ignoring the useless details.

   (d) The edge detection results in a bitmap that contains the most distinctive edges on the image.

2. Connected components labeling detects the connected pixels on the edge map determining the skyline candidates. The top three skyline candidates are chosen by the function S.

3. A top-down search selects the first edge pixels from the most probable candidates in each column because the skyline should be on the upper region of the image.

4. In case of low resolution, the top-down search might make a one-pixel gap in the skyline. A so-called bridge operation repairs this problem by filling the holes.

5. The second connected component analysis eliminates the left-over pieces from the edge map and selects the largest one as the presumed skyline.

6. Finally, the skyline is vectorized for the matching phase.

2.1.3 Skyline Matching

The last phase of the proposed method is matching the panoramic skyline and the recognized fragment of the skyline from the image. We look for the point from where the skyline vectors interlock. The φ could be obtained from here.

Normalized cross-correlation ($a \star b$) is used, which is commonly used in signal processing as a measure of similarity between a vector a (panoramic skyline) and shifted (lagged) copies of a vector b (extracted skyline) as a function of the lag k. After calculating the cross-correlation between the two vectors, the maximum of the cross-correlation function indicates the point K where the signals are best aligned:

$$K = \underset{0^\circ \leq k < 360^\circ}{\operatorname{argmax}}((a \star b)(k)).$$

From K the azimuth φ can be determined, and the estimated horizontal orientation can be acquired.

2.2 Results

2.2.1 Results of Skyline Extraction

The outputs were classified into four classes according to the quality (%) of the result. The evaluation was done manually because an objective measure is hard to create.

- Perfect: the whole skyline [95 – 100%] is detected, no interfering fragments found.
- Good: the better part of the skyline [50 – 95%) is detected, false pixels do not affect the analyses.
- Poor: only a small part of the skyline [5 – 50%) is detected, false pixels might affect the analyses.

- Bad: skyline cannot be found or the detected edges do not belong to the skyline $[0 - 5\%)$.

Table 1 shows that the extracted skylines are assigned to Perfect or Good classes in more than 89% of the samples. In these cases, the extracted features can be used for matching in the next phase.

Class	**Rate**
Perfect	56.67%
Good	32.67%
Poor	8.00%
Bad	2.67%

Table 1: Results of automatic skyline extraction method.

2.2.2 Results of Field Tests

We also made field tests to measure the performance of our algorithm in a real-world environment. The experiments aimed to determine the orientation using only a geotagged photo and the DEM.

Table 2 presents the experimental results of the field tests. Only Good and Perfect skylines were accepted for the tests, and the correlation is almost 95% on average. These tests showed that the azimuth angles provided by the algorithm were $1.04°$ on an average from the ground truth azimuth.

Image	Viewpoint			Target			Results			
ID	Lat (°N)	Lon (°E)	Height (m)	Lat (°N)	Lon (°E)	Height (m)	Corr.	φ (°)	$\hat{\varphi}$ (°)	$\hat{\varphi} - \varphi$ (°)
FT01	47.51552	18.96866	330	47.55016	19.00178	436	0.92	31.58	32.60	**1.02**
FT02	47.51552	18.96866	330	47.53371	18.95588	429	0.96	334.62	334.61	**-0.01**
FT03	47.55555	18.99883	483	47.51827	18.95922	508	0.95	214.83	215.61	**0.78**
FT04	47.53154	18.98611	219	47.49178	18.97895	458	0.99	185.89	186.95	**1.06**
FT05	47.99865	18.86120	188	47.99564	18.86353	195	0.92	151.35	152.47	**1.12**
FT06	47.99948	18.86173	201	47.99564	18.86353	195	0.98	161.22	162.92	**1.70**
FT07	47.51827	18.95922	508	47.55016	19.00178	436	0.97	44.12	41.85	**-2.27**
FT08	47.98355	18.80440	124	47.95780	18.87714	723	0.88	118.98	118.58	**-0.40**
FT09	47.99865	18.86120	188	47.99564	18.86353	195	0.94	151.52	152.47	**0.95**
FT10	47.99948	18.86173	201	47.99564	18.86353	195	0.98	161.81	162.92	**1.11**

Table 2: Experimental results of the field tests.

3 Optimal Partisan Districting

In electoral systems with single-member districts or even with at least two multi-member districts, redistricting has to be carried out to resolve geographic malapportionment caused by migration and different district population growth rates. An inherent difficulty associated with redistricting is that it may favor a party. The problem becomes even worse if redistricting is manipulated for an electoral advantage, which is referred to as gerrymandering. A formal proof establishing that a simplified versions of the optimal gerrymandering problem is NP-complete were given by Puppe and Tasnádi (2009) and Lewenberg et. al (2017). Publication related to Section 3 is Fleiner et al. (2017).

3.1 The Framework

We assume that parties A and B compete in an electoral system consisting only of single member districts. In addition, voters with known party preferences are located in the plane and have to be divided into a given number of almost equally sized districts.

Definition 3.1. *A* districting problem *is given by* $\Pi = (X, N, (x_i)_{i\in N}, v, K, \mathcal{D})$*, where*

- *X is a bounded and strictly connected*[2] *subset of* $\mathbb{R}^2$*,*
- *the finite set of voters is denoted by* $N = \{1, \ldots, n\}$*,*
- *the distinct locations of voters are given by* $x_1, \ldots, x_n \in \mathrm{int}(X)$*,*
- *the voters' party preferences are given* $v : N \to \{A, B\}$*,*
- *the set of district labels is denoted by* $K = \{1, \ldots, k\}$*, where* $\lfloor n/k \rfloor \geq 3$*, and*
- *$\mathcal{D}$ denotes the finite set of admissible districts consisting of bounded and strictly connected subsets of X and each of them containing the location of $\lfloor n/k \rfloor$ or $\lceil n/k \rceil$ voters,and furthermore,*
- *we shall assume that based on their locations the n voters can be partitioned into k districts* $\{D_1, \ldots, D_k\} \subseteq \mathcal{D}$*.*

Definition 3.2. *An* $f : N \to \mathcal{D}$ *is a* districting *for problem* Π *if there exists a set of districts* $D_1, \ldots, D_k \in \mathcal{D}$ *such that*

- $f(N) = \{D_1, \ldots, D_k\}$*,*
- $\mathrm{int}(D_i) \cap \mathrm{int}(D_j) = \emptyset$ *if* $i \neq j$ *and* $i, j \in K$*,*
- $\{x_i \mid i \in f^{-1}(D_j)\} \subset \mathrm{int}(D_j)$ *for any* $j \in K$*.*

3.2 Results

We establish that even the decision problem associated with the optimization problem of determining an optimal partisan districting, i.e., deciding for a given districting problem Π whether there exists a districting with at least m winning districts for a party, say party A, is an NP-complete problem. We call this WINNING DISTRICTS problem.

[2]We call a bounded subset A of $\mathbb{R}^2$ *strictly connected* if its boundary ∂A is a closed Jordan curve.

Theorem 3.1. *WINNING DISTRICTS is NP-complete.*

The following easy consequence of Theorem 3.1 has practical importance.

Theorem 3.2. *The decision problem whether a districting problem* Π *has a districting in which party A gains majority is NP-complete.*

3.2.1 A Positive Result

The problem becomes tractable if we replace $\mathbb{R}^2$ with $\mathbb{R}$ in Definition 3.1, i.e., if we restrict the two-dimensional problem to a one-dimensional one. Based on the dynamic programming technique, we develop a polynomial time algorithm that finds a so-called party A optimal districting for the one-dimensional districting problem.

3.2.2 A Practical Approach

We consider the Hungarian Electoral System in which since 2011, Budapest has to be subdivided into 18 electoral districts from a total of 1472 electoral wards, each serving 600-1500 voters. Thus, an average district consists of approximately 82 wards. For simplicity, we model the election map by a 2-dimensional square grid, where every cell represents a ward with a given party preference A or B. In this model, two cells are connected if they share a common edge, so this defines a 4-neighborhood relation on the set of cells.

Even in this simplified structure, there is no known formula for the number of possible figures. It means, we do not know how many districts can be formed out of a given number of connected cells, so-called polyominoes. If even orientation matters, they are called fixed polyominoes. Jensen (2003) enumerated fixed n-cell polyominoes up to $n = 56$, which resulted in 6.9×10^{31} polyominoes for the last case. This result shows that it is unfeasible to examine all possible cases, even for 82 wards on a Budapest scale problem. Considering possible district shapes is just the first step in arriving to a districting.

Another starting point to obtain a heuristic for gerrymandering, i.e., an algorithm which is not optimal but quick, would be the pack and crack principle. We showed examples that the pack and crack principle does not always result in a party A optimal districting.

4 Circularity of Congressional Districts

Shape analysis has special importance in the detection of gerrymandering, the manipulated redistricting. Circularity is widely used as a measure of compactness, since it is a natural requirement for a district to be as circular as possible. We propose a novel circularity measure M based on Hu moment invariants. This parameter-free circularity measure provides a powerful tool to detect districts with abnormal shapes. We also analyze the districts of Arkansas, Iowa, Kansas, and Utah over several consecutive periods and redistricting plans, and also compared the results with some classical circularity indexes (Reock (1961), Polsby and Popper (1991), and Lee and Sallee (1970)). Publications related to Section 4 are Nagy and Szakál (2019), Nagy and Szakál (2020).

4.1 Circularity Measures

Let us assume that all the examined shapes are compact in the topological sense. The following requirements hold for a circularity measure C:

1. $C(D) \in (0, 1]$ for any planar shape D;
2. $C(D) = 1$ if and only if D is a circle;
3. $C(D)$ is invariant with respect to similarity transformations (translations, rotations and scaling);
4. For each $\delta > 0$ there is a shape D such that $0 < C(D) < \delta$, i.e., there are shapes whose measured circularity are arbitrarily close to 0.

The following Proposition 4.1 and Definition 4.1 are from Žunić et. al (2010).

Proposition 4.1. *Let D be a compact planar shape. Then*

$$\phi_1(D) = \eta_{2,0}(D) + \eta_{0,2}(D) = \frac{\mu_{2,0}(D) + \mu_{0,2}(D)}{\mu_{0,0}(D)^2} \geq \frac{1}{2\pi}$$

$$\phi_1(D) = \eta_{2,0}(D) + \eta_{0,2}(D) = \frac{\mu_{2,0}(D) + \mu_{0,2}(D)}{\mu_{0,0}(D)^2} = \frac{1}{2\pi} \Longleftrightarrow \textit{ if } D \textit{ is a circle.}$$

Based on Proposition 4.1 a circularity measure C_1 can be constructed as follows.

Definition 4.1. *Let D be a compact planar shape and the area of circle O equals to the area of D. Then* $C_1(D)$ *is a circularity measure*

$$C_1(D) = \frac{\phi_1(O)}{\phi_1(D)} = \frac{1}{2\pi} \cdot \frac{\mu_{0,0}(D)^2}{\mu_{2,0}(D) + \mu_{0,2}(D)}.$$

The following circularity measure C_β is a generalization of C_1, and it is applicable in special cases when we want to set the sensitivity manually for a specific purpose.

Definition 4.2. *Let D be a planar shape whose centroid coincides with the origin and let* β *be a real number greater than* -1 *and* $\beta \neq 0$. *Then* $C_\beta(D)$ *is the generalized moment-based circularity measure*

$$C_\beta(D) = \begin{cases} \dfrac{\mu_{0,0}(D)^{\beta+1}}{\pi^\beta(\beta+1)\displaystyle\int\int_D \left(x^2+y^2\right)^\beta dxdy} & \textit{if } \beta > 0 \\[2ex] \dfrac{\pi^\beta(\beta+1)\displaystyle\int\int_D \left(x^2+y^2\right)^\beta dxdy}{\mu_{0,0}(D)^{\beta+1}} & \textit{if } \beta \in (-1, 0). \end{cases}$$

We revealed an undesired feature of this measure, which emerged from the examined data. The circularity order can change when we apply different β parameters to dissimilar shapes. Therefore, in the next definition, we propose the normalized measure of the area under the curve of C_β for $\beta \in (-1, 0) \cup (0, \infty)$ as a novel circularity measure and denote it by M.

Definition 4.3. *Let $C_\beta(D)$ be the generalized moment-based circularity measure. Then M is a circularity measure*

$$M(D) = \lim_{b\to\infty} \frac{1}{b+1} \int_{-1}^{b} C_\beta(D)\, d\beta.$$

4.2 Results

We consider the average circularity of a state through successive Congresses and seek significant anomalies for gerrymandering detection. Thus, we can track the changes and reduce the impact of external conditions, e.g., geographical constraints. We have analyzed four states in the period of the 107^{th}, 108^{th} and 113^{th} US Congress.

All circularity indexes of Utah decreased in stages from the 107^{th} to the 113^{th} Congress. In Iowa, the examined indexes behaved similarly in these periods, the 107^{th} showed the best, while 108^{th} worst results. In Arkansas, Lee-Sallee Index and Polsby-Popper Test decreased monotonically while Reock Test and M had a peak at 108^{th}. Remarkably, M was more sensitive to the change than Reock Test. The most interesting state was Kansas, where the indexes gave completely different orders, and M was the only one with a falling trend.

An example of presumable gerrymandering is the third district of Arkansas through the 107^{th}, 108^{th} and the 113^{th} Congress. We can see an almost unambiguous improvement in the circularity values from the 107^{th} to the 108^{th} period, then a significant fall from the 108^{th} to the 113^{th} Congress.

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