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Material property calibration is more important than element size and number of different materials on the finite element modelling of vertebral bodies. A Taguchi study.

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Abstract

Finite element (FE) modelling of a vertebral body (VB) is considered challenging due to the many parameters involved such as element size and type, and material properties. Previous studies have reported how these parameters affect the mechanical behaviour of a VB model; however, most studies just compared results without any specific statistical tool to quantify their influence. The Taguchi Method (TM) has been successfully used in manufacturing and biomechanics to evaluate process parameters and to determine optimum set-up conditions. This study aimed to evaluate the influence of the main finite element modelling parameters on the mechanical behaviour of a VB model using the Taguchi Method. A FE model was developed based on a C2 juvenile porcine vertebral body and three of the most commonly used modelling parameters were evaluated using TM in terms of change in the predicted stiffness in comparison to experimental values: element size, number of different material properties for VB (based on grey-scale bins) and calibration factor for grey-scale to density to Young's Modulus equation. The influence of the combined factors was also assessed. The Taguchi analysis showed that the three factors are independent. The calibration factor is the main contributor, accounting for 97% of the predicted stiffness, with the value of 0.03 most closely aligning the numerical and experimental results. element size accounted for 2% of the predicted stiffness, with 0.75 mm being the optimal, while the number of grey-scale bins influenced the results by less than 1%. Our findings indicate that the calibration factor is the main modelling parameter, with the element size and number of bins accounting for less than 3% of the predicted stiffness. Therefore, calibration of material properties should be done based on a large number of samples to ensure reliable results.

Keywords: Taguchi Method, Finite Element method, vertebral bodies.

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1 1. Introduction

Finite element (FE) models have been widely applied in orthopaedic research to evaluate spine 2 injuries and to characterise the mechanical behaviour of vertebral bodies and intervertebral 3 discs (Brown, 2004; Wijayathunga et al., 2008; Cronin, 2014). However, their accuracy is highly 4 dependant on several factors, such as material properties, boundary conditions, load applic-5 ation and element size (Kopperdahl et al., 2002; Jones and Wilcox, 2007; Wijayathunga et al., 6 2008). As a consequence, several studies have explored the influence of these modelling factors 7 and how they individually affect model's prediction capabilities (Keyak and Skinner, 1992; 8 Crawford et al., 2003b; Brown, 2004; Jones and Wilcox, 2007; Zander et al., 2016). 9

The most frequent analysed factor is the element size (Keyak and Skinner, 1992; Crawford et al., 10 2003b; Yeni et al., 2005; Jones and Wilcox, 2007; Guldberg et al., 2008). According to Jones and 11 Wilcox (2007), an ideal element size is a compromise between accuracy, in the description of 12 geometrical and material features, and computational costs. Nevertheless, there is still uncer-13 tainty to what the ideal element size is. For example, a study conducted by Crawford et al. 14 (2003b) evaluated how the element size and image resolution affects the prediction of vertebral 15 stiffness. They found that element size does not affect the model stiffness, and it has a similar 16 influence as specimens anatomy variability. Another study, on the other hand, highlighted that 17 there is a significant difference in predicted vertebral stiffness for larger element sizes (more 18 than 3 mm), especially for specimen-specific models (Jones and Wilcox, 2007). 19

Another factor that has not been fully evaluated is the relationship between material properties 20 and density. The use of specimen-specific properties based on grey-scale to Young's Modulus 21 equations have increased the accuracy of the models and allowed an element-based material 22 definition (Wilcox, 2007). In other words, a body would have several groups (or bins) of ma-23 terial according to its density distribution. However, the precise number of different materials 24 required to describe the trabecullar structure is still unknown. In one of the few studies cover-25 ing this issue, Giambini et al. (2015) evaluated the influence of the number of different materials 26 on the predicted stiffness using QCT/FE models of vertebral bodies. They found that for 8, 18 27 and 50 different materials, the difference relative to the experimental results were 21%, 6% and 28 1%, respectively. 29

³⁰ An additional problem that has risen with specimen-specific FE models, and it still has not

³¹ been addressed, is the influence of the calibration factor of grey-scale to Young's Modulus
³² equations. The literature is populated with several different equations, and they widely vary
³³ in terms of density range, experimental technique and formulation type (i.e. linear or power
³⁴ laws) (Helgason et al., 2008). In order to overcome these issues, some studies make the use of a
³⁵ coefficient, which re-calibrates these equations for their sample density and testing conditions
³⁶ (Wijayathunga et al., 2008; Mengoni et al., 2016).

Finally, the majority of studies available in the literature only compared the results from a re-37 latively small set of simulations, without using any specific statistical tool to quantify their 38 influence, and used the simplistic approach of testing one factor at time. Also, the interactions 39 between factors, i.e. if one factor affects the other, have remained unexplored. The Taguchi 40 Method has been successfully used in engineering to estimate the effect of factors and their in-41 teractions on a desired outcome (Taguchi, 1986; Belavendram, 1995; Dar et al., 2002). Instead of 42 investigating all possible combinations to analyse the influence of a specific set of parameters, 43 which can be time-consuming, Taguchi uses orthogonal arrays, and a relatively small and spe-44 cific combination of parameters to achieve the same results, reducing time costs and increas-45 ing productivity. The aim of this study was, therefore, to quantify the influence of the main 46 well-defined modelling factors, element size, calibration factor and number of bands (bins) of 47 materials, on the prediction of the stiffness of a vertebral body FE model using the Taguchi 48 Method. 49

50 2. Materials and Methods

51 2.1. Taguchi Experiments

In order to set a Taguchi analysis, it is firstly necessary to understand the basic concepts of it. 52 Any studied parameter is called factor and any value assigned to it is named level (Taguchi, 53 1986; Belavendram, 1995). For example, if a factor has two levels, it means that the parameter 54 has two possible values. For this study, three factors were initially chosen based on literature: 55 grey-scale calibration factor, element size and number of grey-scale bins (or materials). In order 56 to explore the full potential of the method, three extra factors were added to account for the 57 interactions between the three primary factors, i.e. if the change in one affects the other. A total 58 of six factors were then analysed, and there were labelled as A, B and C for grey-scale factor, 59

element size and number of grey-scale bins, respectively, and as AxB, AxC and BxC for the
 interactions. Each level was assumed to be linear and it was labelled as one or two, Table 1.

After setting the factors and their levels, it is necessary to select an orthogonal array with 62 enough iteration spaces. An orthogonal array is a table that contains all necessary combin-63 ations of the factors. Its size varies according to the number of factors and levels, and they 64 are available elsewhere (Taguchi, 1986; Belavendram, 1995). Each combination of factors is la-65 belled as an experiment. For example, if four experiments are run, it means that four different 66 combinations of factors were tested. For the current study, an orthogonal array $L_8(2^7)$ was se-67 lected, which consists of eight experiments (or combinations), and this can analyse up to seven 68 factors, with two levels each, Table 2 (Taguchi, 1986). The results were evaluated in terms of 69 the stiffness of the vertebral body, i.e. how much the factors changed the predicted stiffness 70 of the finite element model compared to the experimentally measured stiffness. In order to 71 complement Taguchi analyses, an Analysis of Variance (ANOVA) was also conducted. 72

Table 1: Analysed factors and their levels.

Factor	1	Levels			
FactGS	Α	0.1	0.03		
Mesh	В	1.25	0.75		
Bins	С	50	20		
		1	2		

Table 2: $L_8(2^7)$ Orthogonal Array for Taguchi Experiments.

	Factors $L_8(2^7)$						
Evenorimont	1	2	3	4	5	6	7
Experiment	Α	В	AxB	С	AxC	BxC	e
1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2
4	1	2	2	2	2	1	1
5	2	1	2	1	2	1	2
6	2	1	2	2	1	2	1
7	2	2	1	1	2	2	1
8	2	2	1	2	1	1	2

73 2.2. Experimental Procedure for comparison

A compression-load experiment was performed in order to acquire data for comparison. A
juvenile porcine cervical spine (ageing between 8 and 12 months) was acquired from a local
abattoir, dissected and a C2 vertebral body was potted separately in polymethyl methacrylate

⁷⁷ (PMMA) bone cement (Simplex, Stryker Corporation, USA), Figure 1a. This specimen was ⁷⁸ μ CT scanned (Nikon XTH225ST CT Scanner - Nikon Metrology UK, Hertfordshire, UK) and an ⁷⁹ image file with a voxel size of 0.10 mm was obtained.

The sample was then positioned on a material testing machine (Instron 5967, High Wycombe, UK), and a compressive vertical load up to 10 kN at 1000 N min⁻¹ was applied to the sample's top surface. In order to avoid any local deformation on the cement and to certify that a uniform load would be applied, an aluminium plate was placed between the cement and the actuator of the test machine, Figure 1b. The experimental stiffness was measured in the most linear part of the load *versus* displacement curve generated from the materials testing machine data and processed by Matlab (v.R2016b, MathWorks Inc, MA, USA).



Figure 1: (a) Potted C2 vertebral body. (b) Testing set-up. Load was applied at the centre of the vertebral body.

87 2.3. Numerical Model

⁸⁸ A numerical model was created from the μ CT images (v.2017, Simpleware ScanIP, Synopsys ⁸⁹ Inc, California, USA), Figure 2a. This model comprised the cranial and caudal cement pots, the ⁹⁰ C2 vertebral body, cartilage (remaining from dissection) and the aluminium plate. The element ⁹¹ types chosen for this study were a mixture of hexahedrons, to represent the internal trabecular ⁹² structure, and tetrahedrons, to represent the external surface (Jones and Wilcox, 2007; Chevalier ⁹³ et al., 2008; Wijayathunga et al., 2008; Robson Brown et al., 2014; Pahr et al., 2014).

⁹⁴ Two different mesh sizes were created, one with element length of 1.25 mm and the other with
⁹⁵ 0.75 mm. The first value is commonly found in literature as default element size (Jones and
⁹⁶ Wilcox, 2007; Crawford et al., 2003b; Jones and Wilcox, 2007; Zeinali et al., 2010; Unnikrishnan

and Morgan, 2011; Robson Brown et al., 2014). The latter number was the minimum element
size for which simulations could be generated within an acceptable time frame. The original
model, with a resolution of 0.10 *mm*, was then resampled to the required sizes. A total of eight
models were created, according to the required combination of parameters, Table 2.



Figure 2: Numerical model of a C2 vertebral body. (a) At Simpleware ScanIP software; (b) Boundary conditions at ANSYS Mechanical APDL v18.1.

The material properties for the bone cement and the aluminium plate were set as isotropic and linear. The cartilage was set as hyper-elastic (Rohlmann et al., 2007). The properties for the cartilage and plate were based on literature data and the cement on a custom materials test, Table 3.

Body	Туре	Elastic Parameter [MPa]	Poisson	Reference	
Cartilage	Hyperelastic Neo-Hookean	$C_{10} = 0.3448, D_1 = 0.3$	-	(Rohlmann et al., 2007)	
Cement	Isotropic	E = 1177	0.3	Custom testing	
Aluminum Plate	Isotropic	E = 70000	0.35	(McCormack et al., 1999)	

Table 3: Material properties applied to the FE model.

The number of bins (or the number of different materials) for the vertebral body was based on the common values found in literature: 50 or 20 groups of different materials (Jones and Wilcox, 2007; Giambini et al., 2015). These material properties were based on the grey-scale information acquired using calibration phantoms and on the equation provided elsewhere (Kopperdahl et al., 2002):

$$E_{zz} = 2980.\rho_{App}^{1.05} \tag{1}$$

where ρ_{App} is the apparent density, in g cm⁻³, and E is Young's Modulus, in MPa. The above 110 equation was formulated based on elderly and human cervical vertebral bodies and, therefore, 111 it needed to be adapted, i.e. rescaled, for juvenile porcine specimens using a multiplying grey-112 scale factor previously mentioned. The values for the factor, Table 1, were 0.1 (10% of the 113 original value) and 0.03 (the lowest limit which the mean Young's modulus was in the range 114 of the acceptable values (Teo et al., 2006)). The material properties for the vertebral body were 115 considered to be orthotropic, Equations 2 (Crawford et al., 2003a,b; Zeinali et al., 2010; Ayturk 116 and Puttlitz, 2011; Unnikrishnan and Morgan, 2011; Unnikrishnan et al., 2013). 117

$$E_{xx} = 0.333 \cdot E_{zz} \tag{2}$$

 $0.333 \cdot E_{77}$

$$E_{yy} =$$

$$G_{xy} = 0.121 \cdot E_{zz}$$

$$G_{xz} = 0.157 \cdot E_{zz}$$

$$G_{yz} = 0.157 \cdot E_{zz}$$

$$\nu_{xy} = 0.381$$

$$\nu_{xz} = 0.104$$

124 $u_{yz} = 0.104$

The models were then exported from ScanIP to Ansys Mechanical APDL 18.1 (Ansys Inc., Pennsylvania, USA), where the boundary conditions, constraints and load application point were applied to replicate the *in vitro* test, Figure 2b. The predicted stiffness of each model was estimated from the calculated load *versus* displacement curve. The load was acquired from the reaction forces, and the displacement was obtained from a node at the top surface of the top cement housing, directly below the load application point; a similar location to that which the testing machine applied the load.

132 3. Results

¹³³ 3.1. Experimental Results

The data acquired from the material testing machine were plotted in a load *versus* displacement
curve and the stiffness was measured based on the most linear part. In this case, it was between
2 kN and 4 kN of the load values, giving a stiffness value of 2854 N mm⁻¹, Figure 3.



Figure 3: Experimental results of C2 vertebral body.

137 3.2. Numerical and Taguchi Results

Each model generated a load *versus* displacement curve from which stiffness also was estimated 138 between load values of 2 kN and 4 kN, Figure 4 and Table 4. The models with the greater 139 grey-scale factor presented the highest stiffness values, with the model with a element size of 140 1.25 mm and 50 bins of materials, Experiment 2, having the highest value, $6726 \,\mathrm{N}\,\mathrm{mm}^{-1}$. In 141 contrast, the models with a grey-scale factor of 0.03 had the lowest values of stiffness, with the 142 model with a element size of 0.75 mm and 20 material bins, Experiment 7, having the lowest, 143 $2521 \,\mathrm{N}\,\mathrm{mm}^{-1}$. The closest values of predicted stiffness to the experimental were from models 5 144 and 6, $2751 \,\mathrm{N}\,\mathrm{mm}^{-1}$ and $2823 \,\mathrm{N}\,\mathrm{mm}^{-1}$, respectively, with both having grey-scale factor of 0.03 145 and element size of 1.25 mm, but 50 and 20 material bins, respectively. 146

The Analyse of Variance (ANOVA) confirmed what was indicated in the Taguchi experiments (Table 5). Grey-scale factor was the main contributor of the predicted stiffness, accounting for 97% of it, with the value of 0.03 most closely aligning numerical and experimental results. element size accounted for 2% of the predicted stiffness. Due to the low influence on the prediction



Table 4: Results for the Taguchi Experiments

Figure 4: Stiffness predictions from the eight numerical models.

of stiffness, factors C, AxC and BxC were excluded from the analysis after a preliminary analysis with ANOVA.

The response graph, Figure 5, also indicates that the grey-scale factor was the main variable, as the gradient of the curve between levels one and two was the highest among the variables, followed by element size. The results also showed that these three factors are independent. In other words, one factor does not have an affect on the other, as the interaction between them were excluded from ANOVA, and the contribution of the interaction between A and B was only 0.5%. This is also confirmed by the response graph, as the curves AxB did not cross each other (Taguchi, 1986).

Table 5: Analysis of Variance (ANOVA) of the main contributors for numerical stiffness, where Sq is the Sum of Squares, ν is DoF of the variable, Mq is the Mean Sum of Squares, F-Ratio is a hypothesis test, Sq' is the Corrected Sum of Squares after pooling, and ρ is the contribution percentage. St is the total sum.

Source	Pool	Sq	ν	Mq	F-Ratio	Sq'	ρ%
Α		26231248	1	26231248	6236	26227043	97
В		507427	1	507427	121	503222	1.9
AxB		132716	1	132716	32	128510	0.5
С	Y	14740	1	14740	3.50	10535	0.039
AxC	Y	1818.04	1	1818	0.43	-2388	-0.009
BxC	Y	180.50	1	180.50	0.04	-4025	-0.015
e		84.50	0	-	-	-	-
Error	Y	84.50	1	84.50	1	84.50	0.0003
Pooled Error		16823.49	4	4205.87	1	29441.10	0.11
St		26888215.20	7	3841173.60		26888215.20	100



Figure 5: Response graph from ANOVA. As the gradient of the curve between levels one and two was the highest among the variables, grey-scale factor is the main variable, followed by element size. The results also showed that these three factors are independent.

160 4. Discussion

Finite element modelling of biomechanical structures is a challenging process due to the many factors that can affect the results (Jones and Wilcox, 2008). This study aimed to evaluate the main variables commonly presented in FE modelling of vertebral bodies - grey-scale factor, element size and number of material bands, and how they would affect the predicted stiffness. A better understanding of the modelling process and its variables will save time and reduce overall computational costs as fewer simulations are necessary to build and to calibrate a model. However, differently from other studies, this work did not focus solely on a direct comparison ¹⁶⁸ between results, but also on quantifying the influence of variables using a statistical tool.

Taguchi's Method is a powerful statistical tool that, combined with ANOVA, allows the quan-169 tification of a parameter's influence in an outcome. Traditional methods use the direct com-170 parison approach, in which one variable is changed at a time, resulting in a large number of 171 experiments or simulations (Lee and Zhang, 2005). Taguchi, on the other hand, uses an ortho-172 gonal array approach to decrease the number of possible combinations and speeding up the 173 analysis process (Belavendram, 1995). This method was already used to analyse geometrical 174 features of dental implants (Dar et al., 2002), on monolimb design (Lee and Zhang, 2005) and 175 on intervertebral disc modelling parameters analysis (Cappetti et al., 2016), but was not previ-176 ously applied on vertebral body FE models. 177

The first variable, grey-scale factor, was found to be the main contributor for vertebral stiffness. 178 Recent FE models have used greyscale to set the material properties according to the local dens-179 ity (Tyndyk et al., 2007; Gefen, 2011; Jackman et al., 2016; Mengoni et al., 2016). This approach 180 accounts for differences in density, trabecular structure and orientation inside a vertebral body. 181 Several relationships between Young's Modulus and density are available elsewhere (Morgan 182 et al., 2003; Teo et al., 2006; Helgason et al., 2008). Such equations were developed based on eld-183 erly human vertebral bodies, which usually are characterised by low-density trabecular bone. 184 Some studies, on the other hand, tried to adapt these equations using a downgrading factor, as 185 they used porcine as testing samples (which are denser than humans vertebral bodies) (Jones 186 and Wilcox, 2007; Wilcox, 2007; Wijayathunga et al., 2008). A high dependency of stiffness on 187 the rescaling factor was expected, and it suggests that the calibration has to be done based on a 188 large number of samples, to ensure that no other external factor is affecting the results. 189

In contrast to the grey-scale factor, differences in element size and the number of material bins 190 (or bands), combined, just changed the overall stiffness by 3%. Element size effect has been 191 widely studied in recent years and highlighted as one of the main parameters in a FE model 192 (Keyak and Skinner, 1992; Yeni et al., 2005; Jones and Wilcox, 2007; Hosseini et al., 2014). The 193 majority of the studies reported good convergence for models with element size up to 1.5 mm, 194 with 1.0 mm being the most common option (Jones and Wilcox, 2007). The current study used 195 element sizes of 1.25 mm and 0.75 mm, thus within the reported range. This could explain the 196 low effect of element size in the stiffness as this study already used optimum values. 197

The influence of the number of material bands, differently from element size, has not been well 198 explored (Giambini et al., 2015). Gefen (2011) conducted a study to understand the influence of 199 the number of Young's Modulus values required to represent the vertebral bone. They found 200 that a change in the number of different materials, from two to five bands, did not affected the 201 stiffness significantly. However, another study found that between 42 and 50 different material 202 bands would be necessary to completely describe the cancellous bone structure (Giambini et al., 203 2015). In this study, 20 and 50 bands were chosen in order to explore a wider range and to 204 include the optimum value found by Giambini et al. (2015). The changing on the number of 205 material bands showed no influence on the stiffness, and it was excluded from the analysis 206 by the ANOVA. This can be also confirmed by analysing Table 4. A change of bands, from 207 Experiment 1 to 2, did not altered significantly the stiffness, from $6607 \text{ N} \text{ mm}^{-1}$ to $6726 \text{ N} \text{ mm}^{-1}$. 208

In addition to element size, another limitation of this study was the adoption of only two levels 209 and three parameters. It is widely known that several parameters can affect the numerical res-210 ults. Also, two levels should be used when the variable behaves linearly (Belavendram, 1995), 211 which might not be true for finite modelling of vertebral bodies. However, the introduction 212 of more levels and variables would increase the size and complexity of the orthogonal array 213 and this study aimed to illustrate how a statistical tool could be used to optimise the modelling 214 process of vertebral body models. Further studies are still necessary to explore the combination 215 of more factors and multiple levels in order to set a clear picture of the modelling variables. 216

217 **5.** Conclusion

This study applied the Taguchi Method to evaluate the influence of the main modelling parameters on the accuracy of finite element models of vertebral bodies. Grey-scale factor, element size and number of material bands were assessed. Grey-scale factor was the main contributor to the predicted vertebral stiffness, and Taguchi Method was shown to be an efficient statistical tool to quantify the influence of each parameter.

223 Conflict of interest

²²⁴ The authors do not have any conflict of interest.

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