Majoritarian preference, utilitarian welfare and public information in Cournot oligopoly*

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Abstract

Can individual preferences for public information among heterogeneous consumers be aggregated into a meaningful social preference that does not suffer from Condorcet cycles? In a Cournot model where homogeneous producers observe a public signal about an uncertain cost of production prior to taking quantity decisions, we show that the majoritarian preference of consumers for the precision of public information is fairly well behaved so that a Condorcet winner always exists. Under a monotonicity condition on the demand function, we characterize the Condorcet-winning precision in terms of the demand function and the number of firms under which the Condorcet-winning precision (i) hurts consumers' surplus and profits or (ii) remains conflict-free. These results have interesting implications on 'collective' Bayesian persuasion by agencies representing consumers, showing that when full transparency maximizes expected consumers' surplus, collective Bayesian persuasion can lead to full opacity, and vice versa.

Keywords: Cournot oligopoly, uncertain costs, precision of public information, majoritarian preference, Condorcet winner, conflict, collective Bayesian persuasion.

JEL Codes: D02, D11, D43, D60, D70, D80.

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1 Introduction

Policy announcements, publication of economic data, or expert commentary in the media are some of the main sources of public information about economic fundamentals that impact economy-wide investment, output and prices. As put succinctly in Morris and Shin (2002), "[i]ndeed, for policy makers in a variety of contexts, it is the 'fundamentals' information conveyed by public disclosures that receives all the emphasis." Understanding the impact of public information on utilitarian welfare has therefore been the primary concern of normative economics (see a recent summary of the literature on the social value of information in general by Pavan and Vives (2015)). The purpose of the present paper is to look at markets with homogeneous producers but heterogeneous consumers in order to study majoritarian preferences of consumers for public information and contrast it with that which maximizes consumers' surplus, profits, or utilitarian welfare. Taking a positive approach to understanding what quality of information public institutions would provide to industries is important if one wants to ask what kind of public information one would expect to obtain when policy makers in a democracy give more importance to the likes and dislikes of the majority. This is more so since political constraints, for instance, can compel governments into instructing policy makers and heads of other economic institutions to discern not necessarily how public information affects utilitarian welfare, but what impact it may have on the majority.

Studying preference aggregation of information for a group of heterogeneous consumers also provides a better understanding of the social goals and ideologies of consumers as a whole, thereby allowing us to study Bayesian persuasion undertaken by agencies that represent these consumers to persuade the producers to serve the consumers' collective objectives. In this regard, one needs to identify common persuasion schemes for consumers without running into the usual difficulties of preference aggregation. In particular, as majoritarian preferences often run into Condorcet cycles, our primary task is to ask if the majoritarian preferences of consumers for public information in markets are always well defined. And, if they are, then to identify the Condorcet winner for public information, as a function of the prevailing market conditions – aggregate demand and the number of competing producers – and to investigate conditions under which such information hurts utilitarian welfare or initiates conflict between producers and consumers. Identifying the conflict about public information between consumers and producers that we portray in the paper also helps us understand better the fight over the control of an agency devoted to providing information about economy-wide shocks.

In order to address these questions, we study a Cournot model of competition with one-sided heterogeneity. On the supply side of the market, a homogeneous group of profit-maximizing producers faces a common shock to the cost of production about which they obtain noisy public information prior to making individual quantity decisions. The precision of such information affects the noise in the producers' forecasts about the common and constant marginal cost of production (the fundamentals), which in turn affects the uncertainty over the realization of the resulting equilibrium outcome – both in the spread and the mean of prices and quantities.[1] Some of the main sources

[1] We keep the information structure as simple as possible (viz. binary states of uncertainty and binary signals) without destroying the main message of the paper. We discuss in Section 6 the robustness of our main results to the number of states and signals.

of 'cost-affecting' public information under direct or indirect government control include trade associations and forecasting markets. For instance, the FTC (Federal Trade Commission) in the US, and its counterparts in many other countries, explicitly include, among their criteria for permitting trade associations to collect and share information, the nature of the information exchanged, the existing or likely future market power of the producers sharing the information, and whether anti-competitive effects are outweighed by efficiency gains. Forecasting markets about inherent natural volatilities (stock markets, weather, or climate), and domestic, international, or geopolitical events (tax and trade policy, particularly from changes in governing parties or leaders, oil and energy prices, or war) are other sources that affect cost uncertainty at the time of production decisions. The government can influence these forecasting markets by defining the amount and type of data that producers can amass, determining the level of investment in scientific and economic research or even tacitly affecting the extent of press freedom through controlled communication with the media. Moreover, the government at times finds itself uniquely positioned to generate forecasts – for example about the economy-wide interest rates, to amass and propagate data regarding the effectiveness of its policies, to implement policies that achieve one aim but also affect uncertainties about industry-wide costs, and to provide public goods that facilitate data acquisition for predicting natural accidents.[2]

On the demand side of the market, there is a continuum of risk-neutral consumers, each with a unit demand for a homogeneous good. The consumers are heterogeneous in terms of the valuation of the good, and the distribution of the individual valuations gives rise to a market demand. Since the precision of public information determines the realizations of equilibrium market price, consumers with different valuations may have conflicting preferences.[3] For example, consider two information structures where the first one leads to a larger minimum realizable price but a smaller expected price than the second. A consumer with valuation greater than the maximum realizable price under the first information structure prefers it over the second, as it gives the consumer a lower expected price. On the other hand, a consumer with valuation less than the lowest realizable price under the first information structure (so he does not purchase in this case) but greater than that under the

[2] Interest rates are core prices in an economy, affecting cost, especially in the finance, insurance, real estate, and any capital or export-intensive sector. Central banks set a base interest rate but cannot directly control the shape of the yield curve. Rather, through the opacity or transparency of their announcements, they influence this shape. For example, the Federal Reserve Bank (FED) and European Central Bank are significantly opaque, in contrast to the Bank of England, which is quite transparent. In this sense, the central banks control the precision of public information about events that they observe regarding future interest rates; committing to opacity or transparency has implications for future interest rates and thereby for costs. While the FED and other central banks are independent agencies that are supposedly isolated from political pressure, their goal is to serve the public interest; besides, government objectives also impact their operations. As published openly on the FED website, "[t]he Federal Reserve [...] is ultimately accountable to the public and the Congress. ... [I]n a democratic society it is appropriate that operational independence is paired with central bank accountability to the public and its elected representatives. Clear communications about the goals and strategy of monetary policy are also essential for enhancing the effectiveness of systematic monetary policy because when the public and investors understand the goals of monetary policy, inflation expectations are more likely to remain well anchored."

[3] It is important at this stage to note that in our framework, the information need only be public amongst producers. As it is a production shock, consumers do not react to information, but are affected by it only from the supply side of the market, thus generating their individual preference for precision of such information.

second information structure, prefers the second information structure.[4]

We find that the individual consumer preferences for public information satisfy a weaker version of the 'single-crossing property' that also shares some features of 'intermediate preferences' á la Grandmont (1978). Moreover, when these preferences are aggregated to determine the preferences of agencies that represent consumers, the resulting majoritarian preference for such information turns out to violate transitivity and continuity. Yet we find that it always guarantees a Condorcet winner so that collective persuasion undertaken by such agencies remains well defined. We prove the existence of a Condorcet-winning precision by first showing that the majoritarian preference for it is complete, quasi-transitive (that is, its strict component is transitive), and satisfies a weak continuity property, called Transfer-lower-continuity, first introduced in Mehta (1989) and later weakened by Alcantud (2002) to obtain maximal elements of acyclic relations on compact sets. We then prove that under a mild monotonicity restriction on the elasticity of the slope of the inverse demand function, the Condorcet-winning precision is unique, being either complete or null, depending on the properties of the demand function and the number of firms. We also characterize the precision that maximizes the producers' profits. In particular, we show that these are determined by: (i) the degree of competition (i.e., number of producers in the market) and (ii) the elasticity of the slope of the inverse demand. Among other things, we provide conditions under which the Condorcet-winning precision for the consumers does not maximize consumers' surplus and at the same time remains in conflict with the producers' objective of maximizing individual profits.

When it comes to price competition à la Bertrand, we find that producers remain indifferent between different precisions and the majoritarian preference of consumers always selects complete precision as the unique maximizer. However, irrespective of the demand and cost conditions, while fully precise public information is found to be unanimously (weakly) preferred by all agents in the market under Bertrand competition, either the consumers or the producers benefit from more precise information under Cournot competition.

Some modelling features of our study need to be emphasized. We assume that the cost uncertainty is faced publicly and all information about it is common. While producers do engage in research with regards to demands and costs, there are aspects of the economy (such as oil shocks and unforeseen interest-rate changes) about which the government or other organizations who implement such policies, and who care about what the majority of consumers want, have considerable information control. Our focus is on these 'big-events' as after all such events must be 'politically important,' that is, affect the masses. If it comes to smaller events, perhaps the producers will be better informed, but such information will be accessible to all producers. Thus, no individual producer would obtain information that yields a more precise belief than what the other (identical) producers would. One may ask why producers are not allowed to pay for the information. Our

[4] More precise information generates a spread of the realized prices, but the expected price may rise or fall and need not be monotonic. Consumers with sufficiently high valuations who will purchase regardless of the realized price care only about the expected price, whereas, those with somewhat low valuations care only about the minimum realizable price. The disconnect between expected price and minimum realizable price can generate conflict among consumers; moreover, because the realized prices also depend upon the mode of competition, cost uncertainty, and demand, it is possible that some consumers' preferences for information precision change with the environment, even though their valuation for the good remains the same.

results on producers' preference over precision of public information gives us this willingness to pay. As alluded to earlier, if there is a conflict of interest between producers and the majoritarian consumer choice, the producer lobby will be willing to pay to choose the precision that maximizes profits unless it is overwhelmed by the consumer lobby opposing it.

1.1 Related literature

We have explored an area that, to the best of our knowledge, is heretofore unexplored: namely, how the interplay of market structure and mode of competition generates a well-defined majoritarian preference of market participants for public information, possibly leading to conflict within and across groups.[5] There is of course a large literature that studies the impact of public information on the utilitarian social welfare in environments such as investment games, product markets and contests. Morris and Shin (2002) study the utilitarian social value of public information in an investment game with quadratic-payoff, Gaussian information, and strategic complementarity. Agents observe noisy private and public signals on the underlying fundamentals before making an investment decision. They show that if agents have access to very precise private information, then greater precision of public information is detrimental to welfare, as the coordination motive induces agents to 'overreact' to public information. In our framework, there is no private information, or to put it differently, the precision of private information is minimal, and yet we show that the utilitarian value of public information can be negative, both for the consumers and for the producers, though, as mentioned earlier, at least one side of the market always benefits, in the aggregate, from more precise information. Cornand and Heinemann (2008) decompose public information into two components: precision of information and degree of publicity, and then show that information should always be provided with maximum precision but, under certain conditions, to only some, instead of all, market participants. Angeletos and Pavan (2007) develop a generalized investment game of Morris and Shin to assess the value of information for a class of economies with network externalities and strategic complementarity or substitutability. The information structure is decomposed into accuracy and commonality to provide insightful explanations on how the precision of information affects welfare. In particular, they show that public information can reduce welfare, but only when coordination is socially undesirable. All these models feature ex ante identical agents, and hence are not suitable for studying social preferences or conflicts in any interesting way.

There is an influential and extant body of work on the value of information in markets, although it is silent about social preferences. Hirshleifer (1971) shows that the value of information in a market can be negative when more precise information precludes mutually beneficial transactions from occurring.[6] Vives (1984) studies duopolies with uncertain linear demands and shows that only

[5] If it comes to public information controlled by some central authority, one can use the results obtained in this paper to understand why economic players know what they know when the information source cares about majoritarian preferences. A recent book by Veldkamp (2011) is also based on a similar theme about what information economic agents expect to receive (and covers a wide range of applications), though the social choice aspect is missing.

[6] Our model is a particular branch of this tree, in that information affects the welfare of one group of agents who transact with another group of agents, wherein the outcomes are identical whether or not the former group observes the signal. Levine and Ponssard (1977) extend this to non-zero sum games, such as ours.

in the case of substitutes in Cournot competition, producers' strategic incentives to share private information get reduced below what is socially optimal.[7] Sakai and Yamato (1989) show that with differentiated products, information sharing can also increase consumers' surplus and, like us, relate this to the number of producers. They show that both under perfect substitutes and under perfect complements, and when producer-specific costs are non-positively correlated, information sharing harms consumers; otherwise, the number of producers must be sufficiently large for information sharing to benefit consumers.[8]

Others have demonstrated theoretical plausibilities of conflict between consumers and producers in the aggregate. Shapiro (1986) studies Cournot oligopolies with homogeneous goods and producer-specific uncertainty about costs. Under linear demand, he shows that sharing information increases unconditional expected profits and welfare but reduces expected consumers' surplus. Schlee (1996) studies a monopoly where the consumers and the monopolist are uncertain about a good's quality. He shows, like us, that while the monopolist always prefers more precise public information about quality, the consumers may or may not. Our results demonstrate that conflict in the aggregate between producers and consumers can arise without either product differentiation or producer-specific costs. Einy et al. (2002) show that even if the technology is common, private information is beneficial only under constant returns to scale. Angeletos and Pavan (2007) report, as an application of their general study, that in a Cournot duopoly with linear demands and quadratic costs, expected industry profits increase with the precision of *private* information but can indeed decrease with the precision of *public* information. We show that absent private information, public information can only improve profits with linear demand and cost functions.

A few notable papers have found, like us, that the elasticity of the slope of the inverse demand affects the value of public information. Einy et al. (2003) study Cournot duopoly with general demands, constant returns to scale costs, and public information structures on demand and cost uncertainties. They show that if Novshek's (1985) equilibrium existence condition holds, then under some additional mild restrictions on the curvature of the demand function, the value of public information for the producers in a duopoly is positive. Their work is, however, silent on consumers and not applicable to issues around the number of producers. Einy et al. (2017) study n-player symmetric common-value Tullock contests with incomplete information in which a player's cost of effort is the product of a random variable and a deterministic real function of effort. They too find an important role played by the elasticity of the slope of the inverse demand. In particular, they find that if this curvature is increasing, then the value of public information for the contestants is nonnegative. While contests have an association with Cournot games, the consumer side of the framework is absent and so informational conflict across groups is not applicable.

As mentioned earlier, our framework also contributes to the literature on Bayesian persuasion. Kamenica and Gentzkow (2011) characterize optimal persuasion where a single sender of informa-

[7] See, for example, Novshek and Sonnenschein (1982), Basar and Ho (1974), Clarke (1983), Gal-Or (1985) and Sakai (1985) for more on producers' incentives to share private information; see Vives (1990) on how these incentives are affected by public disclosures of trade associations. Li et al. (1987) show that when producers are uncertain about the intercepts of linear demands and undertake private research to obtain information before choosing quantities, inefficiencies persist.

[8] See also Raith (1996) for a survey on the value of private information in markets.

tion persuades a single receiver to take an action closer to the sender's ideal point. Since then, this literature has evolved in two main directions. Kamenica and Gentzkow (2017) and Li and Norman (2018) look at multiple competing senders while Schnakenberg (2015), Alonso and Camara (2017) and Ghosh, Postl and Roy (2018) study multiple receivers of information. A relatively open question is whether collective decision-making processes that determine the functioning of representative agencies can deliver a sophisticated information design that an independent individual sender can deliver or would such processes make a well-defined information design difficult to achieve. In this sense, we identify a natural realm in which to study the question: collective decisions by heterogeneous consumers (through representative agencies) affecting the public information available to a group of competing producers. Since such information determines consumer welfare, we may expect consumers, who constitute powerful lobbying bodies, to choose how to deliver this information strategically, but first by resolving any conflict that may exist among themselves. We show that the existence of a natural focal point for such collective persuasion is non-trivial as we find that the majoritarian preferences for information for consumers may neither be transitive nor continuous, but that for sufficiently general market conditions, they yield a maximizer that gives us the Condorcet-winning persuasion mechanism. Moreover, we find that this representative persuasion mechanism does not necessarily benefit the consumers in the aggregate. Therefore, collective persuasion through a representative agency is somewhat tricky.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 studies the majoritarian preference of consumers and Section 4 determines the precision of information that maximizes profits. We provide a summary of our results in Section 5 with implications for conflict over public information where we also briefly mention our observations for price competition á la Bertrand. The paper concludes in Section 6. All proofs are relegated to the Appendix.

2 Model

We study a market with a continuum of consumers who have heterogeneous valuations for a homogeneous good. The good is supplied á la Cournot by a finite number of producers who face a common shock to their cost of production. Information about this uncertainty is disseminated publicly and prior to the individual quantity choices of the producers. We now describe this environment formally.

Consumers: A market for homogeneous goods comprises a continuum of consumers with valuations $v \in \mathbb{R}_+$ for a unit of the good. Let I be the set of consumers indexed by i, where each consumer buys at most one unit of the good if the price does not exceed his valuation. Consumers are risk neutral, their utility from purchase equals their valuation net of price, and they obtain zero payoff from no purchase. The cumulative distribution of consumer valuations is a mapping $F : \mathbb{R}_+ \to [0, 1]$ that yields the inverse demand function for the good $p : \mathbb{R}_+ \to \mathbb{R}_+$ with $1 - F(p(x)) = x$, the total quantity demanded in the market. We assume $p(\cdot) \in \mathbb{C}^3$. Formally, let $\mathcal{B}_{\mathbb{R}_+}$ be the Borel σ-algebra on $\mathbb{R}_+$ and μ the Lebesgue measure on $\mathcal{B}_{\mathbb{R}_+}$. Then the measure of consumers with valuations in the set $A \in \mathcal{B}_{\mathbb{R}_+}$, given the demand function $p(\cdot)$ is

$$m(A) = -\int_A p^{-1\prime} d\mu.$$

Quantity-setting producers: There are $n \geq 1$ producers indexed by j, who supply the good by independently and simultaneously setting individual quantities $q_j \geq 0$. Each producer has zero fixed cost and a common, constant, and ex-ante unknown marginal cost $c \in \{c_l, c_h\}$, with $0 < c_l < c_h$. For any demand function $p(x)$, let

$$r(x) = -\frac{xp''(x)}{p'(x)}$$

denote the elasticity of the slope of the inverse demand (or the Arrow-Pratt curvature of the demand function). We restrict attention to demand functions where the marginal revenue is strictly decreasing: $2p'(x) + xp''(x) < 0$, i.e. $r(x) < 2$, so that with constant marginal costs, existence (and uniqueness) of (symmetric) Nash equilibrium in the oligopoly is guaranteed for an arbitrary number of producers n (see for example, Vives (1999, page 99, Eq. (4.2)) and Seade (1980)).

Public information: Before each producer makes its production decision, all producers observe a binary public signal $s \in \{s_l, s_h\}$ about c. We let $\lambda \in \Lambda = [1/2, 1]$ denote the precision of these signals, that is, $\lambda = prob(s_l|c_l) = prob(s_h|c_h)$. We assume that the entropy of prior information about c is maximum – i.e., c_l and c_h are equally likely – so that differences in our metrics for the value of a more precise signal arise solely due to the precision of the signal, and not by how likely a particular signal is expected to be observed.

Quantities and payoffs: Let $Q = \sum_{j=1}^{n} q_j$ be the total output in the market, cleared at price $p(Q) \geq 0$. We write $Q = q_j + Q_{-j}$ to single out producer j in this aggregation, q_j denoting producer j's individual output, where for a monopoly (viz. $n = 1$), we set $Q_{-j} = 0$. The profit of producer j in state $c \in \{c_l, c_h\}$ is

$$\pi_j(q_j, Q_{-j}|c) = (p(q_j + Q_{-j}) - c)q_j.$$

The net returns to the consumers depend solely on $p(Q)$ once each producer has determined its q_j, and not upon the realized c. Hence, the utility of a consumer with valuation v is $\max\{0, v - p(Q)\}$, as such a consumer purchases the good if and only if $v \geq p(Q)$.

The above environment gives rise to a Bayesian Cournot game. We study symmetric Bayes-Nash equilibrium properties of the game for a given $\lambda \in \Lambda$ and ask if the majoritarian preference of the consumers over the set Λ is well defined. We then characterize the Condorcet winner for the consumers and profit-maximizing choice of the (identical) producers over Λ.

2.1 Public information and market equilibrium

Upon observing the public signal s, each producer chooses its individual output. This leads to a binary gamble over realized profits since, once the outputs are chosen, each producer realizes the actual marginal cost c that determines its profit. In this sense, before observing s, each producer faces a compound lottery, where the first uncertainty comes from the signal the producers observe jointly. We will say then that the binary gamble yields conditional expected profit, (certain) consumers' payoffs, and (certain) price; the compound lottery then yields unconditional expected profit, expected consumers' surplus, and expected price.

We will throughout use $E(y)$ to denote the mathematical expectation of the random variable y and $E(y|Z)$ to denote this expectation conditional on an event Z. Since a priori information is null, we have $prob(c_l|s_l) = \lambda$ and $prob(c_l|s_h) = 1-\lambda$. These yield $E(c|s_l) = \lambda c_l + (1-\lambda)c_h$ and $E(c|s_h) = (1-\lambda)c_l + \lambda c_h$. Thus, $\frac{\partial E(c|s_l)}{\partial \lambda} = -\frac{\partial E(c|s_h)}{\partial \lambda} = c_l - c_h < 0$. Let $q_j(s_h;\lambda)$ and $q_j(s_l;\lambda)$ be the quantities produced by producer j upon observing s_h and s_l, respectively – for brevity, when λ is clear, we write these as $q_j(s_h)$ and $q_j(s_l)$. Then, market quantities are $Q(s_h)$ and $Q(s_l)$, with market-clearing prices $p(Q(s_h))$ and $p(Q(s_l))$. Upon observing the signal $s \in \{s_l, s_h\}$, producer j's optimal output $q_j^*(s)$ solves

$$p'(q_j^*(s) + Q_{-j})q_j^*(s) + p(q_j^*(s) + Q_{-j}) \equiv E(c|s).$$

In the unique symmetric equilibrium, $q_1^*(s) = q_2^*(s) = \ldots = q_n^*(s) = q^*(s)$, yielding

$$p'(nq^*(s))q^*(s) + p(nq^*(s)) \equiv E(c|s). \tag{1}$$

The following lemma shows that more precise public information leads to a larger spread in equilibrium prices. The basic intuition here is that a low signal s_l induces a common belief that the cost will be low hence yielding a smaller expected cost as the precision of the signal is greater. This increases the equilibrium response in output from each firm, thus lowering the market price. The opposite happens when the signal received is s_h.

Lemma 1 *An increase in the precision of public signals λ increases (decreases) equilibrium output and decreases (increases) equilibrium price when $s = s_l$ ($s = s_h$).*

3 Majoritarian preference of consumers

For each $i \in I$, let $\succsim_i$ be consumer i's preference ordering over Λ under the assumption that for each $\lambda \in \Lambda$, the consequent market is in equilibrium defined by (1). As each λ is then associated with a unique equilibrium price-gamble, $\succsim_i$ is represented by consumer i's expected utility in the following way. Recall that the probability of obtaining the signal s_l is $\frac{1}{2}$. Given $\lambda, \lambda' \in \Lambda$, $\lambda \succsim_i \lambda'$ if and only if

$$\begin{aligned} Eu(\lambda|v_i) &= \frac{1}{2}(\max\{0, v_i - p(nq^*(s_l;\lambda))\} + \max\{0, v_i - p(nq^*(s_h;\lambda))\}) \\ &\geq Eu(\lambda'|v_i) = \frac{1}{2}(\max\{0, v_i - p(nq^*(s_l;\lambda'))\} + \max\{0, v_i - p(nq^*(s_h;\lambda'))\}). \end{aligned}$$

The precision of public information about industry-wide uncertain costs affects equilibrium quantities supplied by the individual producers, thereby affecting the spread in the ex-ante distribution of market prices (as demonstrated in Lemma 1), as well as the ex-ante expected price, given by

$$E(p^*;\lambda) \equiv \frac{p(nq^*(s_l;\lambda)) + p(nq^*(s_h;\lambda))}{2}.$$

As consumers have heterogeneous valuations of the good, their preferences over the precision of the signal are non-trivial: a consumer with sufficiently high valuation who purchases the product irrespective of which price prevails will prefer to lower the expected price; a consumer with low valuation will prefer a greater spread in prices without caring about the expected price, so that she may have a chance to buy the good in at least one realization. The preference relations $\succsim_i$ are not necessarily single-peaked for general demand functions. However, they satisfy the following property that is similar to, but weaker than, the 'Single-Crossing Property': for any pair of precisions $\lambda < \lambda'$ and any pair of consumers with valuations $v_j > v_i$, if $\lambda \succ_i \lambda'$ and $\lambda \succ_j \lambda'$ then $\lambda \succ_k \lambda'$ for every consumer with valuation $v_k \in (v_i, v_j)$.[9]

Given this preference-heterogeneity amongst consumers over Λ, we move to the analysis of preference aggregation. In doing so, we focus on the majoritarian preference of consumers and interpret it as the preferences of the representative agencies for the consumers. Let $\succsim_{\text{maj}}$ be the majoritarian preference of consumers over Λ. Then, for any $\lambda, \lambda' \in \Lambda$, we have $\lambda \succsim_{\text{maj}} \lambda'$ if and only if $m(\{i \in I | \lambda \succ_i \lambda'\}) \geq m(\{i \in I | \lambda' \succ_i \lambda\})$. We say that $\succsim_{maj}$ is *complete* if for any $\lambda, \lambda' \in \Lambda$, either $\lambda \succsim_{maj} \lambda'$ or $\lambda' \succsim_{maj} \lambda$. We say that $\succsim_{maj}$ is *quasi-transitive* if its strict component $\succ_{maj}$ is transitive, that is, given $\lambda, \lambda', \lambda'' \in \Lambda$, $\lambda \succ_{maj} \lambda'$ and $\lambda' \succ_{maj} \lambda''$ together imply $\lambda \succ_{maj} \lambda''$. For each $\lambda \in \Lambda$, denote the Strict Lower Contour set of λ under $\succsim_{maj}$ as $\mathcal{L}_{\succsim_{maj}}(\lambda) = \{\lambda' \in \Lambda | \lambda \succ_{maj} \lambda'\}$ and by $\mathring{\mathcal{L}}_{\succsim_{maj}}(\lambda)$ its interior. We say that $\succ_{maj}$ is *transfer-lower-continuous* if for each $\lambda \in \Lambda$ such that $\lambda' \succ_{maj} \lambda$ for some $\lambda' \in \Lambda$, there exists $\lambda'' \in \Lambda$ such that $\lambda \in \mathring{\mathcal{L}}_{\succsim_{maj}}(\lambda'')$. Finally, a Condorcet-winning precision for the consumers is a maximum of $\succsim_{\text{maj}}$ over Λ. Thus, $\lambda \in \Lambda$ is a *Condorcet winner* if for each $\lambda' \in \Lambda \setminus \{\lambda\}$, we have $\lambda \succsim_{\text{maj}} \lambda'$.

Proposition 1 proves that the majoritarian preference of the consumers is complete, quasi-transitive and transfer-lower-continuous, and consequently has a maximum.

Proposition 1 *The ordering $\succsim_{maj}$ is complete, quasi-transitive, and transfer-lower-continuous over Λ. Consequently, a Condorcet-winning precision for the consumers exists.*

When aggregating individual preferences, we find that the majoritarian preference need not be transitive (see Remark 2 in the Appendix). This surprising result is due to the following. Consider three precisions $\frac{1}{2} \leq \lambda_1 < \lambda_2 < \lambda_3 \leq 1$ and suppose that when making a pairwise comparison, λ_1 and λ_3 obtain the same support among consumers and so do λ_2 and λ_3. This means, $\lambda_1 \sim_{\text{maj}} \lambda_3$ and $\lambda_2 \sim_{\text{maj}} \lambda_3$. It follows then that λ_1 and λ_2 must generate the same expected price, that is,

[9] Grandmont (1978) proved that in general, Condorcet cycles are avoided if preferences of individuals satisfy continuity and 'Intermediateness'. In the language of the present model, intermediateness of consumers' preferences holds if, in addition to the above condition (that holds in our model), one has the following: for any pair of precisions $\lambda < \lambda'$ and any pair of consumers with valuations $v_j > v_i$ if (a) $\lambda \succsim_i \lambda'$ and $\lambda \succsim_j \lambda'$ then $\lambda \succsim_k \lambda'$ and if (b) $\lambda \sim_i \lambda'$ and $\lambda \succ_j \lambda'$ (or $\lambda \succ_i \lambda'$ and $\lambda \sim_j \lambda'$) then $\lambda \succ_k \lambda'$ for every consumer with valuation $v_k \in (v_i, v_j)$. These two additional conditions are not satisfied for our consumers. To see this, consider two information structures: $\lambda = 1/2$ and $\lambda = 1$. Suppose $E(p^*; \lambda = 1/2) < E(p^*; \lambda = 1)$. Consider three consumers: Consumer i whose valuation is below $p_l(\lambda = 1)$, Consumer j whose valuation is above $p_h(\lambda = 1)$, and Consumer k whose valuation is in the interval $(p_l(\lambda = 1), p_l(\lambda = 1/2))$. Then $\frac{1}{2} \sim_i 1$ (as i is always out of the market), $\frac{1}{2} \succ_j 1$ (as j is always in the market but since $E(p^*; \lambda = 1/2) < E(p^*; \lambda = 1)$, she prefers null information). However, $1 \succ_k \frac{1}{2}$ since she is always out of the market when $\lambda = 1/2$ but makes a purchase when $\lambda = 1$ and $s = s_l$.

$E(p^*;\lambda_1) = E(p^*;\lambda_2)$. By Claim 1 (see Appendix) however, we conclude that $\lambda_2 \succ_{\text{maj}} \lambda_1$, as the two precisions generate the same expected price, but λ_2 induces a lower equilibrium price when s_l is observed. Transitivity fails here as two distinct precisions may each lead to majoritarian indifference with a third, greater, precision, as only the expected price induced by each of the smaller precisions determines its relative support among consumers – the spread of equilibrium prices plays no role in either comparison. But when comparing the two smaller precisions that induce the same expected price, the more precise of the two strictly majority-dominates. This is because the high-valuation consumers are indifferent between the two precisions, while the low-valuation consumers strictly prefer the more precise one as it leads to a lower equilibrium price when the low signal is observed.[10]

In addition, the majoritarian preference may fail to be continuous (see Remark 3 in the Appendix) as demonstrated below. Start from a situation with two precisions $\frac{1}{2} \leq \lambda_1 < \lambda_2 \leq 1$ such that $E(p^*;\lambda_1) = E(p^*;\lambda_2)$. In this case, a majority strictly prefers the greater precision λ_2, as high-valuation consumers (those with $v_i \geq p(nq^*(s_h);\lambda_2))$ are indifferent between the two precisions, and consumers with $v_i \in (p(nq^*(s_l);\lambda_2), p(nq^*(s_h);\lambda_2))$ strictly prefer λ_2 over λ_1. Suppose a small perturbation of λ_1 leads to a lower expected price: $E(p^*;\lambda_1+\epsilon) < E(p^*;\lambda_2)$. While $\lambda_1+\epsilon$ is still less precise than λ_2, we could potentially see a reversal in the majoritarian preference, as high-valuation consumers, who were indifferent between λ_1 and λ_2, now strictly prefer $\lambda_1+\epsilon$ over λ_2. Accordingly, even though the expected price $E(p^*;\lambda)$ is continuous in the precision of public information λ, high-valuation consumers' strict preference over lower expected prices could lead to a discontinuity in the majoritarian preference. However, we show that despite this possibility, the majoritarian preference always satisfies transfer-lower continuity as in Mehta (1989) where whenever such discontinuity appears, one can find another precision that is majority-preferred to all precisions close to $\lambda_1+\epsilon$. With this property – along with completeness and quasi-transitivity of the majoritarian preference – we invoke Theorem 4 in Mehta (1989) to show that the majoritarian preference of consumers for the precision of public information always has a maximum. Of course, the maximum is then the Condorcet winner.

3.1 The Condorcet winner

Proposition 1 establishes that a Condorcet-winning precision for consumers exists. However, obtaining a clear characterization of the set of Condorcet winners for arbitrary demand functions is generally hard. In the rest of the paper, we will assume that the elasticity of the slope of the inverse demand $r(x)$ is *monotonic*, that is, either for each $x \in \mathbb{R}_+$, $r'(x) \geq 0$ or for each $x \in \mathbb{R}_+$, $r'(x) < 0$. Using monotonicity of $r(\cdot)$, we next show (viz. Proposition 2) that the Condorcet winner is unique, and we identify it. The following Lemma is useful as it relates the Arrow-Pratt curvature $r(\cdot)$ of the demand function with the expected equilibrium price that plays a central role in Proposition 2.

[10] Sen (1969) shows that quasi-transitivity of preference relations implies the existence of maximal elements over finite sets of alternatives. Hence, if we restrict attention to a finite set of admissible public precisions, then we can readily establish the existence of Condorcet winners without resorting to any continuity condition as we show that the majority preference is complete and quasi-transitive.

LEMMA **2** *The expected equilibrium price $E(p^*;\lambda)$ is decreasing w.r.t. λ if $r'>0$, increasing w.r.t. λ if $r'<0$ and constant w.r.t. λ if $r'=0$.*

The intuition behind the above lemma is as follows. We know that an increase in λ induces a mean-preserving spread in $E(c)$. Whether this in turn induces a mean-preserving spread in equilibrium price p^* depends upon how r changes. Specifically, the spread so induced in p^* is mean-preserving if and only if $r'=0$. Otherwise, equilibrium price is more (less) sensitive at higher marginal cost than at lower marginal cost when $r'>0$ ($r'<0$). This then generates the movement of the expected price.

Let us now fix the number of producers n in the market. Consider the two extreme information structures: $\lambda=1/2$ and $\lambda=1$. Let $v_{\frac{1}{2}1}(n)\in[p(nq^*(s_h;\lambda=1/2)),p(nq^*(s_h;\lambda=1))]$ be the valuation at which a consumer is indifferent between these two precisions. Then

$$v_{\frac{1}{2}1}(n)-E(p^*;\lambda=1/2)=\frac{1}{2}\big(v_{\frac{1}{2}1}(n)-p(nq^*(s_l;\lambda=1))\big)$$

from which we get

$$v_{\frac{1}{2}1}(n)=2E(p^*;\lambda=1/2)-p(nq^*(s_l;\lambda=1)).$$

Thus, $v_{\frac{1}{2}1}(n)$ partitions the consumers into two groups, and we denote the difference in the measures of these two groups by $L(n)$:

$$L(n)\equiv\int_{p(nq^*(s_l;\lambda=1))}^{v_{\frac{1}{2}1}(n)}-p^{-1\prime}d\mu-\int_{v_{\frac{1}{2}1}(n)}^{\infty}-p^{-1\prime}d\mu.$$

We are now in a position to characterize the consumers' Condorcet-winning precision of public information. Proposition 2 shows that the monotonicity of r yields a unique Condorcet-winning precision. In particular, when $r'>0$, then the Condorcet winner is unambiguously the full-precision ($\lambda=1$); however when $r'<0$, its identity is nuanced and depends on whether the number of consumers who prefer $\lambda=1/2$ is greater or less than those who prefer $\lambda=1$, reflected by the sign of $L(n)$.

PROPOSITION **2** *(i) If $r'\geq 0$, then the Condorcet winner is $\lambda=1$; (ii) If $r'<0$, then the Condorcet winner is $\lambda=1$ if $L(n)>0$ and $\lambda=\frac{1}{2}$ if $L(n)<0$.*

In the following two subsections, we discuss the conditions on the demand that warrant a unique Condorcet winner, relate this characterization with the Median Voter Theorem, and ask if and how the identity of the winner changes with the number of producers.

3.1.1 Demand conditions, unique Condorcet winner and the median voter theorem

An individual consumer's preference for public information depends not only on his valuation of the good, but also on the distribution of valuations of others and on the number of producers. If these change, then some consumers will have different preferences, including those whose valuation did not change in the process. As noted earlier, the precision of public information determines the spread of equilibrium prices, which in turn determines consumer participation (purchase or not). When the public signal is completely uninformative ($\lambda = 1/2$), each producer's output decision is independent of the signal it receives; thus, there is no price uncertainty. As the public signal becomes more precise, the production decision is more responsive to the observed signal, and hence the spread of equilibrium prices increases.

With $r' > 0$, the expected equilibrium price decreases when information becomes more precise (Lemma 2). Consumers weakly benefit from the improved precision of the information. Those with high valuations always make a purchase, regardless of the precision and signal realization, so they strictly benefit from the decrease in the expected price; their preference for precision is single-peaked. Those with low valuations either never make a purchase, or make a purchase only when the low-cost signal is realized. They are strictly better off with the improved precision of the information, as the equilibrium price with a low-cost signal decreases with the informativeness of the signal, unless $v_i \leq p^*(s_l; \lambda = 1)$ as such consumers are always out of the market. So, barring those with $v_i \leq p^*(s_l; \lambda = 1)$, such consumers also have single-peaked preferences. Finally, more precise information increases the equilibrium price with the high-cost signal. Those with valuations between the two high-cost equilibrium prices, one for each precision, switch from purchase to not purchase when the high-cost signal is realized under greater precision. However, given that the expected price decreases, the net utility gain from a lower equilibrium price when the low-cost signal is realized outweighs the loss from not purchasing when the high-cost signal is realized. Accordingly, they strictly benefit from the improved precision of information, and again their preferences are single-peaked. This means that there is unanimity among consumers: *more precise information is strictly preferred by everyone* other than those with very low valuations as they never purchase and hence are indifferent about λ. As a consequence, $\lambda = 1$ (complete information) is the Condorcet winner. Also, ignoring (a) consumers with valuations $v_i \leq p^*(s_l; \lambda = 1)$ and (b) consumers, constituting a set of measure zero, who are indifferent between $\lambda = 1/2$ and $\lambda = 1$, each $\succsim_i$ is single-peaked. Therefore, the median consumer is always well defined: the location of the indifferent consumers under (b) determines the strict preference of the median consumer. We conclude that the Condorcet winner is the median consumer's ideal precision.

Unanimity among consumers breaks down when $r' < 0$. In this case, as information becomes more precise, both the spread of equilibrium prices and the expected equilibrium price increase. Consumers with high valuations now strictly prefer less precise information, as the expected price is increasing in λ. On the other hand, consumers with low valuations, except for whom $v_i \leq p^*(s_l; \lambda = 1)$, strictly prefer more precise information, as it gives them a chance to purchase when the low-cost signal is realized or to purchase at a lower price; they are indifferent to the increase in the price when the high-cost signal is observed since the resulting price continues to exceed their valuation. For those with valuations in between, we show that they prefer more precise information when the valuation is below $v_{\frac{1}{2}1}(n)$, but less precise information when the valuation is above it. A consumer with valuation $v_{\frac{1}{2}1}(n)$ is indifferent between complete information ($\lambda = 1$) and null information ($\lambda = 1/2$), as the expected utilities under both precisions are the same for him.[11] Thus, the

[11]Since such a consumer would not purchase under complete information if s_h is observed, complete information leaves him better off from the resulting lower equilibrium price when s_h is observed, which occurs half as often as

Condorcet winner is either 1 or 1/2 depending on the relative mass between the high-valuation and low-valuation consumers as evinced by the sign of the expression $L(n)$ which is the lead in support for $\lambda = 1$ against $\lambda = 1/2$. As before, ignoring those consumers with valuations $v_i \le p^*(s_l; \lambda = 1)$, consumer preferences are single-peaked and the Condorcet winner is the median consumer's ideal precision.

3.1.2 Condorcet Winner and the Number of Producers

The Condorcet-winning precision of public information can change with the degree of competitiveness in the market. To see this, notice that the change in the first term of $L(n)$ w.r.t. n is the mass of consumers whose valuations are low enough to enter the market when s_l is observed and there are more producers, and the change in the second term w.r.t. n is (twice) the mass of consumers whose preferences switch from complete to null information. In other words, as n increases, some consumers' preferences for λ change. Thus, $L'(n)$ need not equal 0 because some consumers who were priced out of the market no longer are, and other consumers' preferences switch since the expected price is greater with $\lambda = 1$.

As we have noted above, when $r' \ge 0$, the expected price is weakly decreasing in λ and the Condorcet winner is $\lambda = 1$. In this case the consumers are also unanimous in their preferences. So conflict among consumers can only arise when $r' < 0$. In that case, a full characterization of the identity of the Condorcet winner, as the number of producers n changes, is again a difficult task in a model as general as ours since the identity of $v_{\frac{1}{2}1}(n)$ depends on equilibrium values that in turn depend on n. In what follows, we will provide some partial characterization.

Proposition **3** *Suppose $r' < 0$. (i) There exists N^* such that for all $n > N^*$, $p^{-1}(c_l) > 2p^{-1}(c_h)$ implies that the Condorcet winner is $\lambda = 1$, whereas $p^{-1}(c_l) < 2p^{-1}(c_h)$ implies that the Condorcet winner is $\lambda = 1/2$. (ii) If, in addition, we have $p''(x) > 0$, $r > 1$ and $p^{-1}(c_l) > 2p^{-1}(c_h)$, then the Condorcet winner is $\lambda = 1$ for all $n \ge 1$.*

The first part of Proposition 3 shows that when a sufficiently large number of producers compete in markets with demand functions satisfying $r' < 0$, the Condorcet winner is $\lambda = 1$ when the impact of the unknown state on the dispersion between the low and the high costs is large while it is $\lambda = 1/2$ when this impact is small. But under what conditions would one expect majoritarian consumer preference to keep the industry completely informed (viz. $\lambda = 1$) irrespective of the number of competitors? This is addressed in the second part. It shows that in addition, if the demand is convex and the elasticity of the slope of the inverse demand is larger than one, the Condorcet winner is $\lambda = 1$ for any number of producers provided the impact of the unknown state on the dispersion between the low and the high costs is sufficiently large.

purchasing under null information.

3.2 Maximizing Consumers' Surplus

The consumers' surplus $\mathbf{S}$ at aggregate output Q is

$$\mathbf{S}(Q) = \lim_{a \to 0} \int_a^Q (p(x) - p(Q))\mathrm{d}x,$$

so that the expected consumers' surplus is given by

$$E(\mathbf{S}|\lambda) = \frac{1}{2}\Bigg(\lim_{a \to 0} \left(\int_a^{Q^*(s_l)} p(x)dx + \int_a^{Q^*(s_h)} p(x)dx \right) - p(Q^*(s_l))Q^*(s_l) - p(Q^*(s_h))Q^*(s_h) \Bigg)$$

where $Q^*(\cdot) = nq^*(\cdot)$. Define

$$\underline{\rho}(x,n) = \frac{r(x) - n - 1}{x}$$

and note that $\underline{\rho} < 0$ and $\frac{\partial \rho}{\partial n} < 0$. Lemma 3 shows that the expected Consumers' Surplus is monotonic in the precision λ although the direction of this monotonicity is dependent on the whether r' is smaller or larger than $\underline{\rho}(x,n)$. Denote by Q_{duo} the total equilibrium output in a duopoly when $\lambda = 1$ and s_h is observed – this is the minimum quantity produced, given the demand and cost structure, in an oligopoly (viz. $Q^*(\cdot) = nq^*(\cdot) \geq Q_{duo}$).

Lemma **3** *In a market with $n \geq 1$ producers, (a) if for each $x \in [Q_{duo}, p^{-1}(c_l)]$ we have $r'(x) < \underline{\rho}(x,n)$ then, $E(\mathbf{S}|\lambda)$ is decreasing in λ so that the expected Consumers' Surplus-maximizing precision is $\lambda = 1/2$ and (b) if for each $x \in [Q_{duo}, p^{-1}(c_l)]$ we have $r'(x) > \underline{\rho}(x,n)$ then, $E(\mathbf{S}|\lambda)$ is increasing in λ so that the expected Consumers' Surplus-maximizing precision is $\lambda = 1$.*

Although more precise information generates a mean-preserving spread in expected marginal cost that manifests in a mean-preserving spread in prices, since $q(s_l) > q(s_h)$, the positive effect on consumers' surplus from the decrease in $p(s_l)$ outweighs the negative effect on consumers' surplus from the equal increase in $p(s_h)$. Thus, expected consumers' surplus decreases with λ only if r' is sufficiently negative so that the latter effect dominates.

4 Public information and profits

In a model with homogeneous goods and common shocks, producer preferences over the set of precisions Λ are identical. So denote by $E(\pi|\lambda)$ the unconditional expected profit (or simply, profit) of an individual producer in the symmetric equilibrium. Then

$$\begin{aligned} E(\pi|\lambda) &= prob(s_l)q^*(s_l)[p(Q^*(s_l)) - E(c|s_l)] + prob(s_h)q^*(s_h)[p(Q^*(s_h)) - E(c|s_h)] \\ &= \frac{1}{2}\left\{q^*(s_l)[p(Q^*(s_l)) - E(c|s_l)] + q^*(s_h)[p(Q^*(s_h)) - E(c|s_h)]\right\} \end{aligned}$$

For $n \geq 2$, define

$$\bar{\rho}(x,n) = \frac{(2 - r(x))(n + 1 - r(x))}{(n-1)x}$$

and note that $\bar{\rho}(x,n) > 0$.

Proposition 4 *(i) Monopoly profit increases unambiguously in the precision of public information λ so that the profit-maximizing precision is $\lambda = 1$; (ii) In an oligopoly with $n \geq 2$ producers, (a) if for each $x \in [Q_{duo}, p^{-1}(c_l)]$ we have $r'(x) < \bar{\rho}(x, n)$ then, $E(\pi|\lambda)$ is increasing in λ so that the profit maximizing-precision is $\lambda = 1$ while (b) if for each $x \in [Q_{duo}, p^{-1}(c_l)]$ we have $r'(x) > \bar{\rho}(x, n)$ then, $E(\pi|\lambda)$ is decreasing in λ so that the profit-maximizing precision is $\lambda = 1/2$.*

Part (i) is intuitive. The monopolist is the sole producer and therefore the effect of more precise information on profit is solely due to facilitating the producer to more closely align its output with the state of the world. This is seen clearly by looking at the first derivative of the unconditional expected profit w.r.t. λ. As proved in the Appendix, this derivative, for a fixed $n \geq 1$, reduces to:

$$\frac{\partial E(\pi)}{\partial \lambda} = \underbrace{-\frac{(n-1)(c_h - c_l)}{2}\left(\frac{q^*(s_l)}{n+1-r(nq^*(s_l))} - \frac{q^*(s_h)}{n+1-r(nq^*(s_h))}\right)}_{\text{Competition effect}} + \underbrace{\left(\left(\frac{c_h - c_l}{2}\right)(q^*(s_l) - q^*(s_h))\right)}_{\text{Alignment effect}}. \quad (2)$$

In a monopoly ($n = 1$), the first term vanishes and what remains is what we call the *alignment effect*. Note that it is always positive – an increase in λ induces a mean-preserving spread in the expected marginal cost, which implies that, as the value function is convex in c, the realization of the value function is increasing in λ. However, in an oligopoly ($n \geq 2$), the first term comes into play. As the signal is public, information has an additional effect on each producer's profit in that more precise information not only allows the producer to better align its production to the state of the world, but also results in all other producers doing likewise. Thus, when producers observe s_l, a larger λ results in each producer increasing its $q(s_l)$ more than it would with a smaller λ, thereby resulting in a greater decrease in each producer's residual demand. Similarly, when producers observe s_h, their output choices are smaller the greater is λ, yielding a greater residual demand compared with s_h being observed with a smaller λ. This combination of the smaller and larger residual demands when s_l and s_h are observed with a greater λ, constitutes the competition effect.[12] This competition effect counteracts, or possibly augments, the alignment effect. Thus, the characterization of the expected profit-maximizing λ is qualified. There is a critical value of $r' < 0$ at which the competition effect vanishes – which is to say that the effects of more precise information on ex ante expected profit through the residual demand that the producer faces, are offset. For r' smaller than this, the competition effect is actually positive and augments the alignment effect. However, for r' sufficiently positive, the competition effect is large enough to negate the alignment effect, yielding zero value of more precise information for producers – this occurs at $\bar{\rho}(x, n)$. For r' greater, the competition effect outweighs the alignment effect. Regardless, except for $r' = \bar{\rho}(x, n)$, either the sum of the two effects is positive, or negative, for all λ, yielding extreme preferences

[12]If $r' = 0$, it follows that $\frac{q^*(s_l)}{n+1-r(nq^*(s_l))} - \frac{q^*(s_h)}{n+1-r(nq^*(s_h))}$ is necessarily positive, yielding a negative competition effect (see (2)). Though the prices undergo a mean-preserving spread with greater λ, and so also the difference with expected cost is mean-preserving, conditional expected profit undergoes a mean-decreasing spread.

for producers. Lastly, since the equilibrium prices and quantities also depend upon the number of producers, the critical threshold $\bar{\rho}(x, n)$ itself depends on n – that is, much like consumers, there are some distributions of valuations such that, the value of more precise information is positive or negative depending upon the number of producers.

5 Conflict over public precision

The Cournot model we have analyzed is fairly general on the demand side, though admittedly elementary when it comes to costs. This simplicity has enabled us to derive a number of clean results on majoritarian preferences for public information that now place us in a position to look at conflict among consumers, as well as across consumers and producers. As the heterogeneity of consumer valuations is key to our analysis, we will represent conflict in terms of the distribution of these valuations, that is, in the space of demand functions, characterized by the responsiveness of Arrow-Pratt curvature r to aggregate quantity.

When does the Condorcet-winning precision of information for consumers disagree with that which maximizes utilitarian welfare of the consumers? It is possible that the maximization of consumers' surplus leaves some, or even a majority of, consumers worse off either because of (i) the absence of non-distortionary transfers, (ii) a missing market where consumers could trade based upon the signal realization, or (iii) that such markets exist in principle but the public signal is not observed by the consumers. Hence, the Condorcet-winning precision of information may be detrimental to expected consumer surplus. Proposition 5 reports conditions under which the majoritarian precision and that determined by maximizing expected consumers' surplus are in conflict. It shows that this conflict does not exist for demands where r increases in aggregate quantity but arises otherwise. Importantly, it highlights the fact that the disagreement can occur both when the Condorcet winner is 1/2 or 1; in other words, when full transparency maximizes expected consumers' surplus, collective Bayesian persuasion through a representative consumer agency can lead to full opacity with regards to cost uncertainty, and vice versa.

Proposition **5** *Disagreement between the Condorcet winner and the expected consumers' surplus maximizer occurs only if $r' < 0$. In particular, (i) if for each $x \in [Q_{duo}, p^{-1}(c_l)]$ we have $0 > r' > \underline{\rho}(x, n)$ and $L(n) < 0$, then the Condorcet winner is $\lambda = 1/2$ and the expected consumers' surplus maximizer is $\lambda = 1$; whereas (ii) if for each $x \in [Q_{duo}, p^{-1}(c_l)]$ we have $r' < \underline{\rho}(x, n)$ and $L(n) > 0$, then the Condorcet winner is $\lambda = 1$ and the expected consumers' surplus maximizer is $\lambda = 1/2$.*

Majoritarian social choice does not capture the strength of preferences, and so the notion of a Gorman representative agent does not apply. Nevertheless, many regulatory agencies have a criterion to maximize expected consumers' surplus. That the Condorcet-winning and the expected consumer surplus-maximizing precisions of public information may conflict then is not surprising. A full picture of conflict (or lack of it) between consumers' surplus, the Condorcet winner and producers' objectives is represented in the following summarizing table (values of respective λ's are in red).

	Conflict over Precision of Public Information		
Demand Conditions	Condorcet-winning precision	Expected Consumers' Surplus Maximizer	Expected Profit Maximizer
$r' < \underline{\rho}$	$1/2$ if $L(n) < 0$; 1 if $L(n) > 0$	$1/2$	1
$\underline{\rho} < r' < 0$	$1/2$ if $L(n) < 0$; 1 if $L(n) > 0$	1	1
$0 < r' < \bar{\rho}$	1	1	1
$\bar{\rho} < r'$	1	1	$1/2$

The following key points come out from our analysis on quantity competition. For linear demands, the Condorcet winner for the consumers is the unanimous choice of $\lambda = 1$ that therefore maximizes expected consumers' surplus; it also maximizes expected industry profits. If $r' \geq 0$ then, not only is the Condorcet winner for the consumers $\lambda = 1$, it is unanimous among consumers and maximizes expected consumers' surplus. Moreover, there is no conflict with the producers provided that $r' < \bar{\rho}$; otherwise, the producers prefer null information. Although this conflict across consumers and producers does not exist in a monopoly, $\bar{\rho}$ decreases as n increases, expanding the range of r' for which conflict arises in oligopolies. If $r' < 0$, then there is always conflict among the consumers (that is, there is always a positive measure of consumers who oppose the Condorcet winner) and there is always a situation when the Condorcet winner does not maximize expected consumers' surplus: (a) if $r' < \underline{\rho}$, then null information maximizes expected consumers' surplus, and this is also the Condorcet winner for the consumers if and only if $L(n) < 0$; (b)if $r' > \underline{\rho}$ then complete information maximizes expected consumers' surplus, and this is also the Condorcet winner for the consumers if and only if $L(n) > 0$; and (c) conflict in the aggregate between producers and consumers emerges only when either the Condorcet winner or the expected consumers' surplus maximizer is null information. Only if $r' \geq 0$ does there exist a precision level λ that is unanimously preferred by the consumers, and it is that of complete information; only if $0 \leq r' < \bar{\rho}$ does there exist a λ that is unanimously preferred by all agents – consumers and producers together, and it is that of complete information ($\lambda = 1$); there is *no other* possibility for the Condorcet-winning public information to be unanimously preferred by *all* agents in the market. And finally, for any distribution of valuations (that is, demand function) and cost conditions, at least one side of the market (i.e., either the producers or the consumers) always benefits from more precise public information.

To complete our study, we end this section with a short comparative look at price competition.

Remark **1 (Price competition)** *Consider price competition á la Bertrand where after observing the public signal $s \in \{s_l, s_h\}$, but before knowing the state, the producers simultaneously set prices $p_j \geq 0$ and commit to supply any demand at the announced price. As goods are homogeneous, in the* *unique symmetric equilibrium, the market price is* $p^*(s) = E(c|s), s \in \{s_l, s_h\}$*. The expected profit is always zero under price competition and therefore, the producers are indifferent toward the precision of information. This rules out conflict across consumers and producers. What about conflict among consumers? While the expected equilibrium market price* $E(p^*(s)) = (E(c \mid s_l) + E(c \mid s_h))/2 = (c_l + c_h)/2$ *is independent of the precision of public information* λ*, a higher* λ *induces a mean-preserving spread in* $p^*(s)$*. Accordingly, all consumers with valuations greater than* c_h *or less than* c_l *are indifferent toward the precision of public information, but consumers with valuations between* c_l *and* c_h *prefer* $\lambda = 1$*. Thus the Condorcet-winning precision is* $\lambda = 1$ *and there is no conflict among consumers either. So to conclude, we see that if the producers engage in price competition á la Bertrand, then there is no conflict over the precision of public information. In particular, the Condorcet-winning and the expected consumers' surplus maximizing precisions are both* $\lambda = 1$*, while the producers are indifferent toward the precision of information.*

6 Conclusion

We have studied majoritarian preference of consumers for the value of public information in a symmetric Cournot oligopoly with uncertain marginal costs, homogeneous producers, and heterogeneous consumers. Although the uncertainty generates a mean-preserving spread in the expected marginal costs, the spread in equilibrium prices and quantities – and thus in conditional expected profit and expected consumers' surplus – need not be mean-preserving. Consequently, while producers and consumers are risk neutral, the value of more precise information may be positive for producers, consumers, or both. Moreover, it may also be negative for producers or consumers, but never for both. Importantly, more precise information can generate conflicts, between producers and consumers, or even amongst consumers, and we have identified the exact nature of such conflicts. We have shown that majoritarian preference of consumers for public information about cost uncertainties is well defined, thereby showing that representative consumer bodies will have clear goals identified by the Condorcet-winning precision. We have then reported conditions on the distribution of consumers' valuations for the good such that the Condorcet-winning precision for the consumers need not maximize either the expected consumers' surplus or expected profits. In such situations, majoritarian preference maximization for consumers hurts aggregate welfare. We have also shown that the problem exists particularly under quantity competition but disappears under price competition.

The results for quantity competition on profit and consumers' surplus also extend to the case where individual demand is not necessarily unitary. In particular, the proofs of Lemma 3 and Proposition 4 do not rely upon aggregate demand comprising individuals with unit demands. For the results on Condorcet-winning public information, the extension to the case where individual consumers may purchase more than one unit (with diminishing marginal valuation) is more involved and we leave for future research.[13] Moreover, while our result for Bertrand competition is simple

[13]Non-unitary demand could pose a tough technical challenge to the problem. With unitary demand, we are able to have a nice partition over the set of consumers to determine Condorcet-winning information. Without this assumption, the set of consumers who prefer one information structure over the other could be erratic, and may not be Lebesgue measurable.

and does not exhibit conflict, allowing for differentiated products could yield quite different results as there is no reason to expect that conflict will not arise when goods are horizontally or vertically differentiated. In addition, the model becomes significantly richer, as both the distribution of valuations and a producer's identity in the spectrum of goods supplied allow for the possibility of conflict amongst producers as well. Another important question is whether the majoritarian preference for consumers about public information remain well defined in the presence of private information. Understanding that and relating it to the results obtained in Morris and Shin (2002), Angeletos and Pavan (2007) or Cornand and Heinemann (2008) would be the next important step towards our understanding of the social value of public information.

We have restricted our analysis to the simplest possible 2×2 information structure (of binary states and signals). Yet, the intuition that there is conflict over the precision of public information between high-valuation and low-valuation consumers goes through with an arbitrary number of states and signals. On the one hand, more precise information increases the variance of equilibrium prices, which benefits low-valuation consumers as they get an opportunity to purchase and enjoy a positive surplus. On the other hand, more precise information hurts high-valuation consumers if and only if it leads to a higher expected price. Accordingly, one expects to obtain a clean partition between consumers who prefer more precise information and those who prefer less. For this reason we expect that the quasi-transitivity of the majoritarian preference will continue to hold. If so, then by restricting attention to a finite set of public precisions and invoking footnote 10, one can expect that a Condorcet winner exists with a more general information structure. This should be particularly for the extension to (or perturbations around) an $n \times n, n > 2$ precision matrix with a single precision parameter λ where all diagonal elements are λ and all off-diagonal elements are $(1 - \lambda)/(n - 1)$. However, with a continuum of public precisions, one requires further research to understand whether majoritarian preference still remains transfer-lower-continuous in order to admit maximizers or some further characterizations of preferences are necessary to obtain a similar conclusion.

As mentioned earlier, studying social preferences of consumers or producers can be useful in providing micro-foundations to the ideologies of interest groups that are central in the New Regulatory Economics (see early surveys by Laffont and Tirole (1993) and Laffont (1994)). Interest groups are modelled in a rudimentary form in that literature, usually consisting of a representative consumer and a representative firm. While this simplifying assumption makes a model tractable by ignoring the politics within the interest group, it brings two weaknesses to the existing framework. First, assuming that there is a representative agent that pursues the common good for an interest group eschews Arrow's impossibility theorem of aggregating social preferences; hence it does not address several important issues such as whom an interest group represents or for that matter whether a majoritarian representative can at all have well-defined preferences. This is a legitimate concern even if there are no actual voting events to determine representatives, as preference aggregation and rightful representation can evolve through a process of repeated communication, either indirectly through the press or directly through public demonstrations. Second, when an interest group's objective is exogenously given, it does not allow for an analysis about the determinants of the type of interest a group represents with respect to the market structure. It is in these spheres that our results can be used to provide a social choice foundation to the preferences of consumer and producer representatives as a function of the prevailing market conditions. We therefore view our study to be useful for the next fundamental step towards developing a complete theory of regulatory capture.

If it comes to a more comprehensive discussion on the political economy of demand for public information in markets, we have essentially explored the foundational two-party Hotelling-Downs model of elections through our characterization of the Condorcet winner under the standard assumption that producers are countable and they do not possess additional electoral powers (so that their preferences do not determine the location of the overall 'median voter'). This sets the stage for further investigation of the issue in richer electoral models with multiple parties, where parties may have ideologies or cater to conflicting interest groups. Finally, the exercise undertaken in this paper has shown how two different market structures lead to different properties of individual preferences of the market participants over public information. It will be useful to understand this connection in more general market games. We reserve these for the future.

7 Appendix

Proof of Lemma 1

Lemma 1 is easy to establish: implicit differentiation of (1) yields the producer's best responses to a change in λ, given by

$$\begin{aligned}\frac{\partial q^*(s_l)}{\partial \lambda} &= -\frac{c_h - c_l}{(n+1)p'(nq^*(s_l)) + nq^*(s_l)p''(nq^*(s_l))} > 0 \quad \textit{and} \\ \frac{\partial q^*(s_h)}{\partial \lambda} &= \frac{c_h - c_l}{(n+1)p'(nq^*(s_h)) + nq^*(s_h)p''(nq^*(s_h))} < 0,\end{aligned} \tag{3}$$

where the signs follow from the assumption that $r(x) < 2$. To see this, note that if $p'' < 0$ then the inequalities are immediate. So suppose $p'' \geq 0$. Then, $2p'(q) + qp''(q) < 0 \Rightarrow -p'(q) > \frac{qp''(q)}{2} \Rightarrow -p'(nq) > \frac{p''(nq)nq}{2}$. Since $n \geq 1$, the right-hand side is larger than $\frac{p''(nq)nq}{n+1}$ and the result follows. Of course, the signs for $\partial p^*(s)/\partial\lambda$, $s \in \{s_l, s_h\}$, follow directly from the fact that the demand function is downward sloping.

Proof of Proposition 1

(i) $\succsim_{maj}$ *is complete*: Pick any λ, $\lambda' \in \Lambda$. Without loss of generality, assume $\lambda < \lambda'$. By Lemma 1, we have

$$p(nq^*(s_l);\lambda') < p(nq^*(s_l);\lambda) \leq p(nq^*(s_h);\lambda) < p(nq^*(s_h);\lambda').$$

Consumers with valuations $v \leq p(nq^*(s_l);\lambda')$ are indifferent between λ and λ' as the expected utility is zero in either case. Consumers with valuations $v \in (p(nq^*(s_l);\lambda'), p(nq^*(s_l);\lambda))$ strictly prefer λ' to λ as they never purchase under λ but do so under λ' whenever the signal is s_l. Consumers with valuations $v \in (p(nq^*(s_l);\lambda)$,p(nq*$(s_h);\lambda))$ strictly prefer λ' to λ as they make a purchase only when s_l is observed and $p(nq^*(s_l);\lambda') < p(nq^*(s_l);\lambda)$. Consumers with valuations $v \geq p(nq^*(s_h);\lambda')$ are indifferent between λ and λ' when $E(p^*;\lambda) = E(p^*;\lambda')$, strictly prefer λ over λ' when $E(p^*;\lambda) <$

$E(p^*;\lambda')$, and strictly prefer λ' over λ when $E(p^*;\lambda) > E(p^*;\lambda')$. Finally, for consumers with valuations $v \in [p(nq^*(s_h);\lambda), p(nq^*(s_h);\lambda'))$, their expected utility under λ equals $v - E(p^*;\lambda)$ while under λ' it equals $\frac{1}{2}(v - p(nq^*(s_l);\lambda'))$. If $E(p^*;\lambda) \geq E(p^*;\lambda')$, then $\frac{1}{2}(v - p(nq^*(s_l);\lambda')) > v - E(p^*;\lambda)$, and all consumers in this region strictly prefer λ' to λ. On the other hand, if $E(p^*;\lambda) < E(p^*;\lambda')$, it can be readily seen that all consumers with $v \in [p(nq^*(s_h);\lambda), 2E(p^*;\lambda) - p(nq^*(s_l);\lambda'))$ strictly prefer λ' to λ, and consumers with $v \in (2E(p^*;\lambda) - p(nq^*(s_l);\lambda'), p(nq^*(s_h);\lambda'))$ strictly prefer λ over λ'. Thus we have shown that

$$m(\{i \in I | \lambda' \succ_i \lambda\}) = \begin{cases} m((p(nq^*(s_l);\lambda'), 2E(p^*;\lambda) - p(nq^*(s_l);\lambda'))) & E(p^*;\lambda) < E(p^*;\lambda') \\ m((p(nq^*(s_l);\lambda'), p(nq^*(s_h);\lambda'))) & E(p^*;\lambda) = E(p^*;\lambda') \\ m((p(nq^*(s_l);\lambda'), \infty)) & E(p^*;\lambda) > E(p^*;\lambda') \end{cases}$$

and

$$m(\{i \in I | \lambda \succ_i \lambda'\}) = \begin{cases} m((2E(p^*;\lambda) - p(nq^*(s_l);\lambda'), \infty)) & E(p^*;\lambda) < E(p^*;\lambda') \\ 0 & E(p^*;\lambda) = E(p^*;\lambda') \\ 0 & E(p^*;\lambda) > E(p^*;\lambda') \end{cases} .$$

Clearly both $m(\{i \in I | \lambda'_i \succ \lambda\})$ and $m(\{i \in I | \lambda \succ_i \lambda'\})$ are well defined, as all of the sets inside $m(\cdot)$ above are Borel, i.e., they belong to $\mathcal{B}_{\mathbb{R}_+}$. Accordingly, we have either $m(\{i \in I | \lambda' \succ_i \lambda\}) \geq m(\{i \in I | \lambda \succ_i \lambda'\})$ or $m(\{i \in I | \lambda \succ_i \lambda'\}) \geq m(\{i \in I | \lambda' \succ_i \lambda\})$, and hence $\succsim_{\text{maj}}$ is complete.

(ii) $\succsim_{maj}$ is quasi-transitive: From the proof of part (i), we observe that the following property holds:

Claim **1** *Let $\lambda, \lambda' \in \Lambda$ be such that $\lambda < \lambda'$. If $\lambda \succsim_{maj} \lambda'$ then $E(p^*;\lambda) < E(p^*;\lambda')$.*

Let λ_1, λ_2 and λ_3 be three distinct precisions from Λ, with $\frac{1}{2} \leq \lambda_1 < \lambda_2 < \lambda_3 \leq 1$. To prove the proposition, there are six possible cases to be considered:

Case I: $\lambda_1 \succ_{\text{maj}} \lambda_2$ and $\lambda_2 \succ_{\text{maj}} \lambda_3$ imply $\lambda_1 \succ_{\text{maj}} \lambda_3$: We establish the following slightly stronger property: $\lambda_1 \succsim_{\text{maj}} \lambda_2$ and $\lambda_2 \succsim_{\text{maj}} \lambda_3$ imply $\lambda_1 \succ_{\text{maj}} \lambda_3$. From Claim 1, $\lambda_1 \succsim_{\text{maj}} \lambda_2$ and $\lambda_2 \succsim_{\text{maj}} \lambda_3$ imply $E(p^*;\lambda_1) < E(p^*;\lambda_2) < E(p^*;\lambda_3)$. Let $v_{13} \in (p(nq^*(s_h);\lambda_1), p(nq^*(s_h);\lambda_3))$ be the valuation at which a consumer is indifferent between λ_1 and λ_3. Then

$$v_{13} - E(p^*;\lambda_1) = \frac{1}{2}(v_{13} - p(nq^*(s_l);\lambda_3)) \Longrightarrow v_{13} = 2E(p^*;\lambda_1) - p(nq^*(s_l);\lambda_3).$$

Note that v_{13} is well defined given $E(p^*;\lambda_1) < E(p^*;\lambda_3)$. It can then be readily verified that

$$m(\{i \in I | \lambda_3 \succ_i \lambda_1\}) = m((p(nq^*(s_l);\lambda_3), v_{13}))$$

and

$$m(\{i \in I | \lambda_1 \succ_i \lambda_3\}) = m((v_{13}, \infty)).$$

By the same token, let $v_{23} \in (p(nq^*(s_h);\lambda_2), p(nq^*(s_h);\lambda_3))$ be the valuation at which a consumer is indifferent between λ_2 and λ_3. Then

$$v_{23} = 2E(p^*;\lambda_2) - p(nq^*(s_l);\lambda_3),$$

$$m(\{i \in I | \lambda_3 \succ_i \lambda_2\}) = m((p(nq^*(s_l); \lambda_3), v_{23})),$$

and

$$m(\{i \in I | \lambda_2 \succ_i \lambda_3\}) = m((v_{23}, \infty)).$$

As $E(p^*; \lambda_1) < E(p^*; \lambda_2)$, $v_{13} < v_{23}$, which gives us

$$m(\{i \in I | \lambda_2 \succ_i \lambda_3\}) < m(\{i \in I | \lambda_1 \succ_i \lambda_3\}),$$

and

$$m(\{i \in I | \lambda_3 \succ_i \lambda_2\}) > m(\{i \in I | \lambda_3 \succ_i \lambda_1\}).$$

Accordingly, $\lambda_1 \succsim_{\text{maj}} \lambda_2$ and $\lambda_2 \succsim_{\text{maj}} \lambda_3$ imply $\lambda_1 \succ_{\text{maj}} \lambda_3$.

Case II: $\lambda_1 \succ_{\text{maj}} \lambda_3$ and $\lambda_3 \succ_{\text{maj}} \lambda_2$ imply $\lambda_1 \succ_{\text{maj}} \lambda_2$: We establish the following slightly stronger property: $\lambda_1 \succsim_{\text{maj}} \lambda_3$ and $\lambda_3 \succ_{\text{maj}} \lambda_2$ imply $\lambda_1 \succ_{\text{maj}} \lambda_2$. First, we claim that $\lambda_1 \succsim_{\text{maj}} \lambda_3$ and $\lambda_3 \succ_{\text{maj}} \lambda_2$ imply $E(p^*; \lambda_1) < E(p^*; \lambda_2)$. Suppose to the contrary that $\lambda_1 \succsim_{\text{maj}} \lambda_3$ and $\lambda_3 \succ_{\text{maj}} \lambda_2$ but $E(p^*; \lambda_1) \geq E(p^*; \lambda_2)$. By Claim 1, we know that $\lambda_1 \succsim_{\text{maj}} \lambda_3$ implies $E(p^*; \lambda_1) < E(p^*; \lambda_3)$. We then have $E(p^*; \lambda_2) \leq E(p^*; \lambda_1) < E(p^*; \lambda_3)$. Let $v_{j3} \in (p(nq^*(s_h); \lambda_j), p(nq^*(s_h); \lambda_3))$ be the valuation at which a consumer is indifferent between λ_j and λ_3, $j = 1, 2$. Then

$$v_{j3} = 2E(p^*; \lambda_j) - p(nq^*(s_l); \lambda_3),\ \ j = 1, 2.$$

Note that v_{j3} is well defined given $E(p^*; \lambda_j) < E(p^*; \lambda_3)$. Thus we have

$$m(\{i \in I | \lambda_3 \succ_i \lambda_j\}) = m((p(nq^*(s_l); \lambda_3), v_{j3}))$$

and

$$m(\{i \in I | \lambda_j \succ_i \lambda_3\}) = m((v_{j3}, \infty)).$$

As $\lambda_1 \succsim_{\text{maj}} \lambda_3$, $m((v_{13}, \infty)) \geq m((p(nq^*(s_l); \lambda_3), v_{13}))$. Moreover, as we have assumed that $E(p^*; \lambda_1) \geq E(p^*; \lambda_2)$, we get $v_{13} \geq v_{23}$, which in turn implies

$$m((p(nq^*(s_l); \lambda_3), v_{23})) \leq m((p(nq^*(s_l); \lambda_3), v_{13})) \leq m((v_{13}, \infty)) \leq m((v_{23}, \infty)).$$

However, $\lambda_3 \succ_{\text{maj}} \lambda_2$ implies $m((p(nq^*(s_l); \lambda_3), v_{23})) > m((v_{23}, \infty))$, a contradiction. Therefore, $\lambda_1 \succsim_{\text{maj}} \lambda_3$ and $\lambda_3 \succ_{\text{maj}} \lambda_2$ imply $E(p^*; \lambda_1) < E(p^*; \lambda_2)$.

Now let $v_{12} \in (p(nq^*(s_h); \lambda_1), p(nq^*(s_h); \lambda_2))$ be the valuation at which a consumer is indifferent between λ_1 and λ_2. Then

$$v_{12} = 2E(p^*; \lambda_1) - p(nq^*(s_l); \lambda_2).$$

Then, v_{12} is well defined given $E(p^*; \lambda_1) < E(p^*; \lambda_2)$. Similarly, we have

$$m(\{i \in I | \lambda_2 \succ_i \lambda_1\}) = m((p(nq^*(s_l); \lambda_2), v_{12}))$$

and

$$m(\{i \in I | \lambda_1 \succ_i \lambda_2\}) = m((v_{12}, \infty)).$$

As $\lambda_2 < \lambda_3$, $p(nq^*(s_l);\lambda_2) > p(nq^*(s_l);\lambda_3)$ and hence, $v_{12} < v_{13}$. Accordingly,

$$m(\{i \in I|\lambda_3 \succ_i \lambda_1\}) > m(\{i \in I|\lambda_2 \succ_i \lambda_1\})$$

and

$$m(\{i \in I|\lambda_1 \succ_i \lambda_3\}) < m(\{i \in I|\lambda_1 \succ_i \lambda_2\}).$$

Therefore $\lambda_1 \succsim_{\mathrm{maj}} \lambda_3$ and $\lambda_3 \succ_{\mathrm{maj}} \lambda_2$ imply $\lambda_1 \succ_{\mathrm{maj}} \lambda_2$.

Case III: $\lambda_2 \succ_{\mathrm{maj}} \lambda_1$ and $\lambda_1 \succ_{\mathrm{maj}} \lambda_3$ imply $\lambda_2 \succ_{\mathrm{maj}} \lambda_3$: Suppose to the contrary that $\lambda_2 \succ_{\mathrm{maj}} \lambda_1$ and $\lambda_1 \succ_{\mathrm{maj}} \lambda_3$ but $\lambda_3 \succsim_{\mathrm{maj}} \lambda_2$. From Case I, we conclude that $\lambda_1 \succ_{\mathrm{maj}} \lambda_3$ and $\lambda_3 \succsim_{\mathrm{maj}} \lambda_2$ imply $\lambda_1 \succ_{\mathrm{maj}} \lambda_2$, a contradiction. Thus, $\lambda_2 \succ_{\mathrm{maj}} \lambda_1$ and $\lambda_1 \succ_{\mathrm{maj}} \lambda_3$ imply $\lambda_2 \succ_{\mathrm{maj}} \lambda_3$.

Case IV: $\lambda_2 \succ_{\mathrm{maj}} \lambda_3$ and $\lambda_3 \succ_{\mathrm{maj}} \lambda_1$ imply $\lambda_2 \succ_{\mathrm{maj}} \lambda_1$: Suppose to the contrary that $\lambda_2 \succ_{\mathrm{maj}} \lambda_3$ and $\lambda_3 \succ_{\mathrm{maj}} \lambda_1$ but $\lambda_1 \succsim_{\mathrm{maj}} \lambda_2$. Again we reach a contradiction as from Case I we conclude that $\lambda_1 \succsim_{\mathrm{maj}} \lambda_2$ and $\lambda_2 \succ_{\mathrm{maj}} \lambda_3$ imply $\lambda_1 \succ_{\mathrm{maj}} \lambda_3$. Therefore, $\lambda_2 \succ_{\mathrm{maj}} \lambda_3$ and $\lambda_3 \succ_{\mathrm{maj}} \lambda_1$ imply $\lambda_2 \succ_{\mathrm{maj}} \lambda_1$.

Case V: $\lambda_3 \succ_{\mathrm{maj}} \lambda_1$ and $\lambda_1 \succ_{\mathrm{maj}} \lambda_2$ imply $\lambda_3 \succ_{\mathrm{maj}} \lambda_2$: Suppose to the contrary that $\lambda_3 \succ_{\mathrm{maj}} \lambda_1$ and $\lambda_1 \succ_{\mathrm{maj}} \lambda_2$ but $\lambda_2 \succsim_{\mathrm{maj}} \lambda_3$. From Case I again, we conclude that $\lambda_1 \succ_{\mathrm{maj}} \lambda_2$ and $\lambda_2 \succsim_{\mathrm{maj}} \lambda_3$ imply $\lambda_1 \succ_{\mathrm{maj}} \lambda_3$, a contradiction. Hence $\lambda_3 \succ_{\mathrm{maj}} \lambda_1$ and $\lambda_1 \succ_{\mathrm{maj}} \lambda_2$ imply $\lambda_3 \succ_{\mathrm{maj}} \lambda_2$.

Case VI: $\lambda_3 \succ_{\mathrm{maj}} \lambda_2$ and $\lambda_2 \succ_{\mathrm{maj}} \lambda_1$ imply $\lambda_3 \succ_{\mathrm{maj}} \lambda_1$: Suppose to the contrary that $\lambda_3 \succ_{\mathrm{maj}} \lambda_2$ and $\lambda_2 \succ_{\mathrm{maj}} \lambda_1$ but $\lambda_1 \succsim_{\mathrm{maj}} \lambda_3$. From Case II, we conclude that $\lambda_1 \succsim_{\mathrm{maj}} \lambda_3$ and $\lambda_3 \succ_{\mathrm{maj}} \lambda_2$ imply $\lambda_1 \succ_{\mathrm{maj}} \lambda_2$, a contradiction. Accordingly, $\lambda_3 \succ_{\mathrm{maj}} \lambda_2$ and $\lambda_2 \succ_{\mathrm{maj}} \lambda_1$ imply $\lambda_3 \succ_{\mathrm{maj}} \lambda_1$.

REMARK **2 ($\succsim_{maj}$ is not transitive)** *To see this, consider three precisions $\frac{1}{2} \leq \lambda_1 < \lambda_2 < \lambda_3 \leq 1$ and suppose (A) $\lambda_1 \sim_{maj} \lambda_3$ and (B) $\lambda_2 \sim_{maj} \lambda_3$. Transitivity would imply that $\lambda_1 \sim_{maj} \lambda_2$. However, it turns out that $\lambda_2 \succ_{maj} \lambda_1$. This follows because (A) and (B) imply that $E(p^*;\lambda_1) = E(p^*;\lambda_2)$. But then by Claim 1, $\lambda_2 \succ_{maj} \lambda_1$.*

(iii) $\succsim_{maj}$ is transfer-lower-continuous: Pick any $\widetilde{\lambda} \in \Lambda$ such that there exists some $\widehat{\lambda} \in \Lambda$ with $\widehat{\lambda} \succ_{\mathrm{maj}} \widetilde{\lambda}$. Consider two cases:

Case I. $\widetilde{\lambda} > \widehat{\lambda}$: As $\widehat{\lambda} \succ_{\mathrm{maj}} \widetilde{\lambda}$, by Claim 1 we have $E(p^*;\widehat{\lambda}) < E(p^*;\widetilde{\lambda})$. As established in part (i), we have

$$m(\{i \in I|\widehat{\lambda} \succ_i \widetilde{\lambda}\}) = m((2E(p^*;\widehat{\lambda}) - p(nq^*(s_l);\widetilde{\lambda}), \infty))$$

and

$$m(\{i \in I|\widetilde{\lambda} \succ_i \widehat{\lambda}\}) = m((p(nq^*(s_l);\widetilde{\lambda}), 2E(p^*;\widehat{\lambda}) - p(nq^*(s_l);\widetilde{\lambda}))).$$

Hence $\widehat{\lambda} \succ_{\mathrm{maj}} \widetilde{\lambda}$ is equivalent to

$$m(\{i \in I|\widehat{\lambda} \succ_i \widetilde{\lambda}\}) = m((2E(p^*;\widehat{\lambda}) - p(nq^*(s_l);\widetilde{\lambda}), \infty)) >$$

$$m((p(nq^*(s_l);\widetilde{\lambda}), 2E(p^*;\widehat{\lambda}) - p(nq^*(s_l);\widetilde{\lambda}))) = m(\{i \in I|\widetilde{\lambda} \succ_i \widehat{\lambda}\}).$$

As both $E(p^*;\lambda)$ and $p(nq^*(s_l);\lambda)$ are continuous in λ, $\widetilde{\lambda} \in \mathring{\mathcal{L}}_{\succsim_{maj}}(\widehat{\lambda})$, establishing that $\succsim_{\mathrm{maj}}$ is transfer-lower-continuous in this case.

Case II. $\widehat{\lambda} > \widetilde{\lambda}$: Analogous to Case I, by invoking continuity of $E(p^*;\lambda)$ and $p(nq^*(s_l);\lambda),\infty)$ in λ, we can establish that $\widetilde{\lambda} \in \mathring{\mathcal{L}}_{\succsim_{maj}}(\widehat{\lambda})$ when either $E(p^*;\widehat{\lambda}) < E(p^*;\widetilde{\lambda})$ or $E(p^*;\widehat{\lambda}) > E(p^*;\widetilde{\lambda})$. So consider the case when $E(p^*;\widehat{\lambda}) = E(p^*;\widetilde{\lambda})$. Fix $\epsilon \in (0, \widehat{\lambda} - \widetilde{\lambda})$ sufficiently small and consider a neighborhood of $\widetilde{\lambda}$, $N(\widetilde{\lambda}) = (\widetilde{\lambda} - \epsilon, \widetilde{\lambda} + \epsilon) \cap \Lambda$. If $\widehat{\lambda} \succ_{\text{maj}} \widetilde{\widetilde{\lambda}}$ for all $\widetilde{\widetilde{\lambda}} \in N(\widetilde{\lambda})$, then $\widetilde{\lambda} \in \mathring{\mathcal{L}}_{\succsim_{\text{maj}}}(\widehat{\lambda})$. Suppose for some $\widetilde{\widetilde{\lambda}} \in N(\widetilde{\lambda})$, $\widetilde{\widetilde{\lambda}} \succsim_{\text{maj}} \widehat{\lambda}$. From Claim 1 we know that $E(p^*;\widetilde{\widetilde{\lambda}}) < E(p^*;\widehat{\lambda}) = E(p^*;\widetilde{\lambda})$. Moreover, as $\widetilde{\widetilde{\lambda}} \succsim_{\text{maj}} \widehat{\lambda}$ and $\widehat{\lambda} \succ_{\text{maj}} \widetilde{\lambda}$, (strong) quasi-transitivity implies $\widetilde{\widetilde{\lambda}} \succ_{\text{maj}} \widetilde{\lambda}$. Consider two subcases:

(a) $\widetilde{\widetilde{\lambda}} > \widetilde{\lambda}$. As $E(p^*;\widetilde{\widetilde{\lambda}}) < E(p^*;\widetilde{\lambda})$ and $E(p^*;\widetilde{\lambda})$ is continuous in $\widetilde{\lambda}$, there exists a neighborhood of $\widetilde{\lambda}$, $O(\widetilde{\lambda})$, such that $E(p^*;\lambda) > E(p^*;\widetilde{\widetilde{\lambda}})$ for all $\lambda \in O(\widetilde{\lambda})$. By Claim 1, $\widetilde{\widetilde{\lambda}} \succ_{\text{maj}} \lambda$ for all $\lambda \in O(\widetilde{\lambda})$. Thus, $\widetilde{\lambda} \in \mathring{\mathcal{L}}_{\succsim_{\text{maj}}}(\widetilde{\widetilde{\lambda}})$.

(b) $\widetilde{\widetilde{\lambda}} < \widetilde{\lambda}$. Since $E(p^*;\widetilde{\widetilde{\lambda}}) < E(p^*;\widetilde{\lambda})$ and $\widetilde{\widetilde{\lambda}} \succ_{\text{maj}} \widetilde{\lambda}$, analogous to Case I, both $E(p^*;\widetilde{\lambda})$ and $p(nq^*(s_l);\widetilde{\lambda})$ are continuous in $\widetilde{\lambda}$ implies $\widetilde{\lambda} \in \mathring{\mathcal{L}}_{\succsim_{\text{maj}}}(\widetilde{\widetilde{\lambda}})$.

We have shown in both subcases $\widetilde{\lambda} \in \mathring{\mathcal{L}}_{\succsim_{\text{maj}}}(\widetilde{\widetilde{\lambda}})$. Therefore, $\succsim_{\text{maj}}$ is transfer-lower-continuous. The proof is completed now by noting the following. Mehta (Theorem 4, 1989) shows that an acyclic and transfer-lower-continuous (but not necessarily complete) binary relation $\succ$ on a compact set X has a maximal element $x \in X$ with respect to $\succ$, i.e., there exists no $y \in X$ with $y \succ x$. We have so far shown that $\succsim_{\text{maj}}$ is complete, quasi-transitive, and transfer-lower-continuous. Since quasi-transitivity implies acyclicity, there exists a maximal element $\lambda^* \in \Lambda$. As $\succsim_{\text{maj}}$ is complete, $\lambda \not\succ_{\text{maj}} \lambda^*$ is equivalent to $\lambda^* \succsim_{\text{maj}} \lambda$ for all $\lambda \in \Lambda$. Thus, λ^* is a Condorcet-winning precision for the consumers. This completes the proof of the Proposition. We end with the following remark.

Remark **3 ($\succsim_{maj}$ need not be continuous)** *To see this, consider $\lambda_1 < \lambda_2$ with $E(p;\lambda_1) = E(p;\lambda_2)$. Then $\lambda_2 \succ_{maj} \lambda_1$. Consider now a small perturbation of λ_1 to $\lambda_1 + \epsilon$. Suppose after the perturbation we have $E(p;\lambda_1 + \epsilon) < E(p;\lambda_2)$. Then it is possible that $\lambda_1 + \epsilon \succ_{maj} \lambda_2$ provided that (see the proof of part (i) above)*

$$m((2E(p^*;\lambda_1+\epsilon) - p(nq^*(s_l);\lambda_2),\infty)) > m((p(nq^*(s_l);\lambda_2), 2E(p^*;\lambda_1+\epsilon) - p(nq^*(s_l);\lambda_2))).$$

Hence, the strict lower contour set is not open due to the discontinuity of m at $E(p;\lambda_1) = E(p;\lambda_2)$. In other words, discontinuity of m leads to the failure of lower continuity of $\succsim_{maj}$.

□

Proof of Lemma 2

To prove Lemma 2, note that

$$\frac{\partial E(p^*;\lambda)}{\partial \lambda} = \frac{1}{2}\left(np'(nq^*(s_l))\frac{\partial q^*(s_l)}{\partial \lambda} + np'(nq^*(s_h))\frac{\partial q^*(s_h)}{\partial \lambda}\right). \tag{4}$$

Substituting from (3) and simplifying yields:

$$\frac{\partial E(p^*;\lambda)}{\partial \lambda} = \Big(\frac{np'(nq^*(s_l))}{(n+1)p'(nq^*(s_l)) + nq^*(s_l)p''(nq^*(s_l))} - \frac{np'(nq^*(s_h))}{(n+1)p'(nq^*(s_h)) + nq^*(s_h)p''(nq^*(s_h))}\Big)\frac{c_l - c_h}{2} = n\frac{c_l - c_h}{2}\Big(\frac{1}{n+1-r(nq^*(s_l))} - \frac{1}{n+1-r(nq^*(s_h))}\Big).$$

The relation between $E(p^*;\lambda)$ and r' can now be easily verified. $\square$

Proof of Proposition 2

(i) Let $r'(\cdot) \geq 0$. Pick any $\tilde{\lambda} \in [\frac{1}{2}, 1)$. By Lemma 2, $E(p^*;\lambda = 1) \leq E(p^*;\lambda = \tilde{\lambda})$. By Claim 1, $\lambda = 1 \succ_{maj} \lambda = \tilde{\lambda}$. Accordingly, the Condorcet-winning λ is $\lambda = 1$.

(ii) Let $r'(\cdot) < 0$. Consider two cases:

Case I: $L(n) > 0$. For any $\tilde{\lambda} \in [\frac{1}{2}, 1)$ denote by $v_{\tilde{\lambda}1}(n)$ the individual who is indifferent between $\lambda = \tilde{\lambda}$ and $\lambda = 1$ given the number of producers n. Clearly, $v_{\tilde{\lambda}1}(n) \in \big(p(nq^*(s_h;\lambda = \tilde{\lambda})), p(nq^*(s_h;\lambda = 1))\big)$ since otherwise, the individual would either only purchase upon s_l being realized and so strictly prefer $\lambda = 1$, or would purchase regardless of the signal and so prefer whichever yields the lower expected price. Thus, $v_{\tilde{\lambda}1}(n)$ is determined by:

$$v_{\tilde{\lambda}1}(n) - E(p^*;\lambda = \tilde{\lambda}) = \frac{1}{2}\big(v_{\tilde{\lambda}1}(n) - p(nq^*(s_l;\lambda = 1))\big).$$

Notice that individuals i for whom $v_i < v_{\tilde{\lambda}1}(n)$, the left-hand side is strictly less than the right-hand side so that $(\lambda = 1) \succ_i (\lambda = \tilde{\lambda})$, and, individuals j for whom $v_j > v_{\tilde{\lambda}1}(n)$, the left-hand side is strictly greater than the right-hand side so that $(\lambda = \tilde{\lambda}) \succ_j (\lambda = 1)$.

The masses of individuals in these two groups are, respectively,

$$\int_{p(nq^*(s_l;\lambda=1))}^{v_{\tilde{\lambda}1}(n)} -p^{-1\prime} d\mu \quad \text{and} \int_{v_{\tilde{\lambda}1}(n)}^{\infty} -p^{-1\prime} d\mu.$$

If the former exceeds the latter, then $(\lambda = 1) \succ_{maj} (\lambda = \tilde{\lambda})$; whereas, if the latter exceeds the former, then $(\lambda = 1) \succ_{maj} (\lambda = \tilde{\lambda})$.

We observe that $v_{\tilde{\lambda}1}(n)$ is strictly increasing in $\tilde{\lambda}$ by Lemma 2. Hence, for every $\tilde{\lambda} \in [\frac{1}{2}, 1)$, $m\big((p(nq^*(s_l;\lambda = 1)), v_{\tilde{\lambda}1}(n)\big) \geq m\big((p(nq^*(s_l;\lambda = 1)), v_{\frac{1}{2}1}(n)\big)$ and $m\big((v_{\tilde{\lambda}1}(n), \infty)\big) \leq m\big((v_{\frac{1}{2}1}(n), \infty)\big)$, or equivalently

$$\int_{p(nq^*(s_l;\lambda=1))}^{v_{\tilde{\lambda}1}(n)} -p^{-1\prime} d\mu \geq \int_{p(nq^*(s_l;\lambda=1))}^{v_{\frac{1}{2}1}(n)} -p^{-1\prime} d\mu, \text{and}$$

$$\int_{v_{\tilde{\lambda}1}(n)}^{\infty} -p^{-1\prime} d\mu \leq \int_{v_{\frac{1}{2}1}(n)}^{\infty} -p^{-1\prime} d\mu.$$

$L(n) > 0$ implies that the right-hand side of the first inequality is greater than the right-hand side of the second inequality; but then, the left-hand side of the first inequality is necessarily greater than the left-hand side of the second inequality, which implies that the Condorcet winner is $\lambda = 1$.

Case II: $L(n) < 0$. For any $\tilde{\lambda} \in (\frac{1}{2}, 1]$ denote by $v_{\frac{1}{2}\tilde{\lambda}}(n)$ the individual who is indifferent between $\lambda = \frac{1}{2}$ and $\lambda = \tilde{\lambda}$ given the number of producers n. Clearly, $v_{\frac{1}{2}\tilde{\lambda}} \in \big(p(nq^*(s_h; \lambda = \frac{1}{2})), p(nq^*(s_h; \lambda = \tilde{\lambda}))\big)$ since otherwise, the individual would either only purchase upon s_l being realized and so strictly prefer $\lambda = \tilde{\lambda}$, or would purchase regardless of the signal and so prefer whichever yields the lower expected price. Thus, $v_{\frac{1}{2}\tilde{\lambda}}(n)$ is determined by:

$$v_{\frac{1}{2}\tilde{\lambda}}(n) - E(p^*; \lambda = \frac{1}{2}) = \frac{1}{2}\big(v_{\frac{1}{2}\tilde{\lambda}}(n) - p(nq^*(s_l; \lambda = \tilde{\lambda}))\big).$$

Notice that individuals i for whom $v_i < v_{\frac{1}{2}\tilde{\lambda}}(n)$, the left-hand side is strictly less than the right-hand side so that $(\lambda = \tilde{\lambda}) \succ_i (\lambda = \frac{1}{2})$, and, individuals j for whom $v_j > v_{\tilde{\lambda}1}(n)$, the left-hand side is strictly greater than the right-hand side so that $(\lambda = \frac{1}{2}) \succ_j (\lambda = \tilde{\lambda})$.

The masses of individuals in these two groups are, respectively,

$$\int_{p(nq^*(s_l;\lambda=\tilde{\lambda}))}^{v_{\frac{1}{2}\tilde{\lambda}}(n)} -p^{-1\prime} d\mu \quad \text{and} \int_{v_{\frac{1}{2}\tilde{\lambda}}(n)}^{\infty} -p^{-1\prime} d\mu.$$

If the former exceeds the latter, then $(\lambda = \tilde{\lambda}) \succ_{maj} (\lambda = \frac{1}{2})$; whereas, if the latter exceeds the former, then $(\lambda = \frac{1}{2}) \succ_{maj} (\lambda = \tilde{\lambda})$.

Given $r' < 0$, $v_{\frac{1}{2}\tilde{\lambda}}(n)$ is strictly increasing in $\tilde{\lambda}$ since $p(nq^*(s_l; \lambda = \tilde{\lambda}))$ is strictly decreasing in $\tilde{\lambda}$. Hence, for every $\tilde{\lambda} \in (\frac{1}{2}, 1]$, $m\big((p(nq^*(s_l; \lambda = 1)), v_{\frac{1}{2}1}(n)\big) \geq m\big(p(nq^*(s_l; \lambda = \tilde{\lambda})), v_{\frac{1}{2}\tilde{\lambda}}(n)\big)$ and $m\big((v_{\frac{1}{2}1}(n), \infty)\big) \leq m\big((v_{\frac{1}{2}\tilde{\lambda}}(n), \infty)\big)$, or equivalently

$$\int_{p(nq^*(s_l;\lambda=1))}^{v_{\frac{1}{2}1}(n)} -p^{-1\prime} d\mu \geq \int_{p(nq^*(s_l;\lambda=\tilde{\lambda}))}^{v_{\frac{1}{2}\tilde{\lambda}}(n)} -p^{-1\prime} d\mu$$

and

$$\int_{v_{\frac{1}{2}1}(n)}^{\infty} -p^{-1\prime} d\mu \leq \int_{v_{\frac{1}{2}\tilde{\lambda}}(n)}^{\infty} -p^{-1\prime} d\mu.$$

$L(n) < 0$ implies that the left-hand side of the first inequality is less than the left-hand side of the second inequality; but then, the right-hand side of the first inequality is necessarily less than the right-hand side of the second inequality, which implies that the Condorcet winner is $\lambda = \frac{1}{2}$. $\square$

Proof of Proposition 3

Part (i): Since $\lim_{n\to\infty} p(nq^*(s_l; \lambda = 1)) = c_l$ and $\lim_{n\to\infty} v_{\frac{1}{2}1}(n) = c_h$, we have

$$\lim_{n\to\infty} L(n) = p^{-1}(c_l) - 2p^{-1}(c_h).$$

Note that $p^{-1}(c)$ is the perfectly competitive output with marginal cost c. Using continuity properties of the demand function, the conclusion on the identity of the Condorcet winner with a large number of producers can be readily obtained.

Part (ii): Note that if for some n', the Condorcet winner is $\lambda = 1/2$, then $L'(n) \leq 0$ for all $n > n'$ implies that the Condorcet winner remains at $\lambda = 1/2$ for all $n'' > n'$. But the requirement $L'(n) \leq 0$ is satisfied if and only if

$$p^{-1\prime}(v_{\frac{1}{2}1}(n))\left(2\frac{\partial p(s_l|n,\lambda=1/2)}{\partial n} - \frac{\partial p(s_l|n,\lambda=1)}{\partial n}\right) - 1/2\left(p^{-1\prime}(\underline{v}(n))\frac{\partial p(s_l|n,\lambda=1)}{\partial n}\right) \geq 0,$$

where $\underline{v}(n) = p(s_l|\lambda = 1, n)$. This condition simplifies to

$$2\left(\frac{\frac{\partial p(s_l|n,\lambda=1/2)}{\partial n}}{\frac{\partial p(s_l|n,\lambda=1)}{\partial n}}\right) \geq 1 + \frac{1}{2}\left(\frac{p^{-1\prime}(\underline{v}(n))}{p^{-1\prime}(v_{\frac{1}{2}1}(n))}\right). \tag{5}$$

Suppose demand is convex, that is $p''(x) > 0$. Then the RHS of inequality (5) is strictly less than 2. Using this fact, it can be easily established that

$$\left(\frac{\frac{\partial p(s_l|n,\lambda=1/2)}{\partial n}}{\frac{\partial p(s_l|n,\lambda=1)}{\partial n}}\right) \geq 1 \text{ if } r'(x;n) < \frac{r(x;n)-1}{n(nq^*(n)+q^*(n))}.$$

Since $r' < 0$ and $nq^*(n) + q^*(n) > 0$, if $r > 1$ then this condition is always true. But if $p^{-1}(c_l) > 2p^{-1}(c_h)$ then Part (i) shows that for sufficiently large n, the Condorcet winner is $\lambda = 1$. This means under these conditions, there can never be an n for which $L(n) < 0$.

□

Proof of Lemma 3

Using the Fundamental Theorem of Calculus and our previous result of equilibrium quantity responses to changes in λ, the first derivative of $E(\mathbf{S}|\lambda)$ w.r.t. λ is:

$$\begin{aligned}\frac{\partial E(\mathbf{S}|\lambda)}{\partial\lambda} &= \frac{1}{2}\left(-p'(nq^*(s_l))n\frac{\partial q^*(s_l)}{\partial\lambda}nq^*(s_l) - p'(nq^*(s_h))n\frac{\partial q^*(s_h)}{\partial\lambda}nq^*(s_h)\right) = \\ \frac{n^2}{2}&\Big(\frac{-p'(nq^*(s_l))q^*(s_l)(c_l-c_h)}{(n+1)p'(nq^*(s_l)) + nq^*(s_l)p''(nq^*(s_l))} + \frac{p'(nq^*(s_h))q^*(s_h)(c_l-c_h)}{(n+1)p'(nq^*(s_h)) + nq^*(s_h)p''(nq^*(s_h))}\Big) = \\ &\qquad -\frac{n^2(c_l-c_h)}{2}\Big(\frac{q^*(s_l)}{n+1-r(nq^*(s_l))} - \frac{q^*(s_h)}{n+1-r(nq^*(s_h))}\Big).\end{aligned} \tag{6}$$

As $q^*(s_l) > q^*(s_h)$, $\frac{\partial E(\mathbf{S}|\lambda)}{\partial\lambda}$ is positive or negative as $\frac{q}{n+1-r(nq)}$ is increasing or decreasing in q. Observe that

$$\frac{\partial}{\partial q}\left(\frac{q}{n+1-r(nq)}\right) = \frac{(n+1-r(nq)) + nqr'(nq)}{(n+1-r(nq))^2}.$$

Since $r(nq) < 2$ and $n \geq 1$, the first term in the numerator is positive. If $r'(x) \geq 0$, then the second term is also positive. If $0 > r'(x) > \frac{r(x)-n-1}{x}$, then, although the second term is negative, the numerator is positive, making this derivative positive. If $r'(x) < \frac{r(x)-n-1}{x}$, then the numerator is negative, making this derivative negative. □

Proof of Proposition 4

Part (i): Upon observing s_l, the monopolist produces $q^*(s_l)$ and has conditional expected profit $(p(q^*(s_l)) - E(c|s_l))q^*(s_l)$. The first derivative of the monopolist's conditional expected profit w.r.t. λ is:

$$\frac{\partial E(\pi(q|s_l, n = 1))}{\partial \lambda} = [p(q^*(s_l)) + p'(q^*(s_l))q^*(s_l) - E(c|s_l)]\frac{\partial q^*(s_l)}{\partial \lambda} - \frac{\partial E(c|s_l)}{\partial \lambda}q^*(s_l).$$

The first term is zero since its first factor vanishes by the first-order condition. Upon observing s_h, the first derivative of conditional expected profit w.r.t. λ is analogous and equals $-\frac{\partial E(c|s_h)}{\partial \lambda}q^*(s_h)$. Using the fact that $\frac{\partial E(c|s_l)}{\partial \lambda} = -\frac{\partial E(c|s_h)}{\partial \lambda} = c_l - c_h$, the first derivative of the monopolist's unconditional expected profit w.r.t. λ becomes

$$\left(\frac{c_h - c_l}{2}\right)(q^*(s_l) - q^*(s_h)) > 0, \tag{7}$$

where the sign follows since the monopolist produces more when its expected marginal cost is smaller, that is, $q^*(s_l) > q^*(s_h)$.

Part (ii): Upon observing s_l, in equilibrium, each producer produces $q^*(s_l)$ and has conditional expected profit $(p(nq^*(s_l)) - E(c|s_l))q^*(s_l)$. Then, the first derivative of an oligopolist's conditional expected profit w.r.t. λ is:

$$\frac{\partial E(\pi(q|s_l))}{\partial \lambda} = [p(nq^*(s_l)) + p'(nq^*(s_l))nq^*(s_l) - E(c|s_l)]\frac{\partial q^*(s_l)}{\partial \lambda} - \frac{\partial E(c|s_l)}{\partial \lambda}q^*(s_l) =$$
$$[p(nq^*(s_l)) + p'(nq^*(s_l))q^*(s_l) - E(c|s_l)]\frac{\partial q^*(s_l)}{\partial \lambda} + (n-1)p'(nq^*(s_l))\frac{\partial q^*(s_l)}{\partial \lambda}q^*(s_l) - \frac{\partial E(c|s_l)}{\partial \lambda}q^*(s_l).$$

The first term is zero since its first factor vanishes by the first-order condition, leaving:

$$(n-1)p'(nq^*(s_l))\frac{\partial q^*(s_l)}{\partial \lambda}q^*(s_l) - \frac{\partial E(c|s_l)}{\partial \lambda}q^*(s_l).$$

Upon observing s_h, the first derivative of conditional expected profit w.r.t. λ is analogous and given by:

$$(n-1)p'(nq^*(s_h))\frac{\partial q^*(s_h)}{\partial \lambda}q^*(s_h) - \frac{\partial E(c|s_h)}{\partial \lambda}q^*(s_h).$$

Since the a priori probability of c_l equals $\frac{1}{2}$, the first derivative of each producer's unconditional expected profit w.r.t. λ is:

$$\frac{(n-1)}{2}\left(p'(nq^*(s_l))\frac{\partial q^*(s_l)}{\partial \lambda}q^*(s_l) + p'(nq^*(s_h))\frac{\partial q^*(s_h)}{\partial \lambda}q^*(s_h)\right) - \frac{(c_l - c_h)}{2}(q^*(s_l) - q^*(s_h)). \tag{8}$$

Substituting from (3), (8) becomes:

$$\frac{(n-1)}{2}\frac{p'(nq^*(s_l))(c_l-c_h)}{(n+1)p'(nq^*(s_l))+nq^*(s_l)p''(nq^*(s_l))}q^*(s_l)-$$
$$\frac{(n-1)}{2}\frac{p'(nq^*(s_h))(c_l-c_h)}{(n+1)p'(nq^*(s_h))+nq^*(s_h)p''(nq^*(s_h))}q^*(s_h)-\frac{(c_l-c_h)}{2}(q^*(s_l)-q^*(s_h)).$$

Dividing the numerator and denominator of the first term by $p'(nq^*(s_l))$, and similarly for the second term, we can rewrite the derivative of unconditional expected profit as:

$$\frac{\partial E(\pi)}{\partial \lambda}=\frac{(c_l-c_h)}{2}(n-1)\left(\frac{q^*(s_l)}{n+1-r(nq^*(s_l))}-\frac{q^*(s_h)}{n+1-r(nq^*(s_h))}\right)$$
$$-\frac{(c_l-c_h)}{2}(q^*(s_l)-q^*(s_h)). \quad (9)$$

Let $f'(x)=\frac{n+1-r(x)+r'(x)x}{(n+1-r(x))^2}$. Multiplying the first term of (9) by $\frac{n}{n}$, we can use the Mean-Value Theorem (on the interval $[nq^*(s_h),nq^*(s_l)]$) and guarantee the existence of some $\xi\in[nq^*(s_h),nq^*(s_l)]$ such that (9) can be rewritten as:

$$\frac{\partial E(\pi)}{\partial \lambda}=\frac{(c_l-c_h)}{2}\frac{n-1}{n}f'(\xi)n(q^*(s_l)-q^*(s_h))-\frac{(c_l-c_h)}{2}(q^*(s_l)-q^*(s_h))=$$
$$\frac{(c_l-c_h)}{2}(q^*(s_l)-q^*(s_h))\left((n-1)f'(\xi)-1\right). \quad (10)$$

The sign of $\frac{\partial E(\pi)}{\partial \lambda}$ is opposite that of the last factor $(n-1)f'(\xi)-1$ which is negative if $r'(x)<\bar{\rho}(x,n)$ but positive if $r'(x)>\bar{\rho}(x,n)$. Thus, $\frac{\partial E(\pi)}{\partial \lambda}>0$ if $r'(x)<\bar{\rho}(x,n)$ and $\frac{\partial E(\pi)}{\partial \lambda}<0$ if $r'(x)>\bar{\rho}(x,n)$. □

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