1-1-2013

Seismic retrofit of RC frames through beam-end weakening in conjunction with FRP strengthening

J G. Teng
Hong Kong Polytechnic University, cejgteng@polyu.edu.hk

S S. Zhang
Hong Kong Polytechnic University, shishun@uow.edu.au

D H. Jing
Southeast University

X F. Nie
Hong Kong Polytechnic University

G M. Chen
Guangdong University of Technology

Seismic retrofit of RC frames through beam-end weakening in conjunction with FRP strengthening

Abstract

The strong column/weak beam requirement is now widely accepted in the design of reinforced concrete (RC) frames to achieve good seismic performance. However, many existing RC frames violate this requirement as they were designed according to inadequate design codes (generally previous codes). In particular, RC frames designed according to the previous Chinese codes for seismic design are likely to violate this requirement as the contribution of a cast-in-place floor slab in tension is not included in assessing the moment capacity of the beam in negative bending. This paper proposes three promising beam weakening techniques in combination with FRP strengthening to achieve this strong column/weak beam hierarchy and presents the preliminary results of an ongoing study into the effectiveness of and design procedures for the proposed techniques.

Disciplines

Engineering | Science and Technology Studies

Publication Details

Teng, J. G., Zhang, S. S., Jing, D. H., Nie, X. F. & Chen, G. M. (2013). Seismic retrofit of RC frames through beam-end weakening in conjunction with FRP strengthening. In R. Al-mahaidi, S. T. Smith, Y. Bai & X. Zhao (Eds.), Proceedings of the 4th Asia-Pacific Conference on FRP in Structures (APFIS 2013) (pp. 1-8). Victoria, Australia: Swinburne University of Technology.

SEISMIC RETROFIT OF RC FRAMES THROUGH BEAM-END WEAKENING IN CONJUNCTON WITH FRP STRENGTHENING

J.G. Teng [1], S.S. Zhang [1], D.H. Jing [2], X.F. Nie [1] and G.M. Chen [3]
[1] Department of Civil and Environmental Engineering, The Hong Kong Polytechnic University, China.
[2] School of Civil Engineering, Southeast University, China.
[3]School of Civil and Transportation Engineering, Guangdong University of Technology, China.
Email: cejgteng@polyu.edu.hk

ABSTRACT

The strong column/weak beam requirement is now widely accepted in the design of reinforced concrete (RC) frames to achieve good seismic performance. However, many existing RC frames violate this requirement as they were designed according to inadequate design codes (generally previous codes). In particular, RC frames designed according to the previous Chinese codes for seismic design are likely to violate this requirement as the contribution of a cast-in-place floor slab in tension is not included in assessing the moment capacity of the beam in negative bending. This paper proposes three promising beam weakening techniques in combination with FRP strengthening to achieve this strong column/weak beam hierarchy and presents the preliminary results of an on-going study into the effectiveness of and design procedures for the proposed techniques.

KEYWORDS

FRP, seismic retrofit, RC frames, beam weakening, seismic performance

INTRODUCTION

The enforcement of a strong column/weak beam hierarchy based on the capacity design philosophy is widely accepted as an effective way to realize the beam sway mechanism (i.e. with plastic hinges at beam ends) in a reinforced concrete (RC) frame structure when subjected to seismic loading. The beam-sway mechanism is preferred to the story-sway mechanism (i.e. with plastic hinges at column ends) as the former generally leads to better seismic performance. For this reason, many current design codes specify a flexural strength ratio (ratio between the sum of the moment capacities of the columns at a joint to that of the beams framing into the joint) greater than 1 (such as 1.2 or other values) to ensure that flexural failure in the beams precedes that of columns. For example, the required flexural strength ratio is 1.2 in the current ACI Code (ACI-318 2008), 1.3 in the current European code (Eurocode-8 2005), and a variable within the range of 1.1~1.7 in the current Chinese code (GB-50011 2010).

Despite the strength ratios specified in the current design codes as mentioned above, studies of failed structures after major earthquakes have shown that the beam-sway mechanism rarely occurred (ATC-40 1996) because most of the failed frames were designed according to codes (generally previous codes) which do not or do not adequately enforce the strong column/weak beam requirement. The above observation is particularly relevant to the recent magnitude (Ms) 8.0 Wenchuan earthquake in 2008 (Chinese Academy of Building Research 2008), where failure of cast-in-place RC frames commonly occurred at column ends (Figure 1); the beam-sway mechanism was normally found only in frames with no floor slabs or with precast floor slabs (Figure 2). The prevalence of column end failures has been attributed to one major deficiency in the previous versions of the Chinese code (GB-50011 2008): the code does not include the contribution of the cast-in-place slab in tension to the flexural capacity of the beam in negative bending in its specification (Lin et al. 2009). As a result, it can be expected that in many RC frames in the Chinese mainland, the beams are stronger than the columns at a joint. The new Chinese seismic design code (GB-50011 2010), which came into force in December 2010, requires the consideration of the contribution of a cast-in-place slab to the beam flexural capacity in addition to the adoption of higher flexural strength ratios. Many existing RC buildings cannot meet these new requirements. In other countries or regions, similar threats induced by the violation of the strong column/weak beam hierarchy in the older existing buildings may also exist. For example, buildings designed in accordance with ACI 318 (1983) or older versions which did not consider the contribution of the cast-in-place slab in tension to the flexural capacity of the beam in negative bending are likely to violate this hierarchy

Against the above background, this paper proposes three promising seismic retrofit techniques based on a combined use of beam weakening and FRP strengthening for non-ductile RC frames which do not satisfy the strong column/weak beam requirement. The paper also presents the preliminary results of an on-going study into the effectiveness of and design procedures for the proposed techniques.

Figure 1. Failure at column ends
(Courtesy of Dr. P. Feng,
Tsinghua University, China)

Figure 2. Failure at beam ends

SEISMIC RETROFIT TECHNIQUES

Background

The purpose of the seismic retrofit of such non-ductile RC frames is to realise a strong column/weak beam hierarchy, thereby improving the global inelastic deformation capacity of the RC frame. To achieve this purpose, two possible retrofit strategies may be considered. The first strategy is to strengthen the columns and the second strategy is to weaken the flexural capacity of T-section beams at their ends. Column strengthening is indeed an obvious option, but column strengthening alone is often insufficient to change the strength hierarchy because: (1) the extent of strength enhancement may be rather limited, particularly when the simple and cost-effective method of FRP jacketing is adopted to confine non-circular columns; (2) even if sufficient column strengthening can be achieved, such strengthening may simply shift the location of failure from column ends to beam-column joints and/or the foundation, which are both difficult to retrofit. Considering the above deficiencies of column strengthening, the more novel strategy of weakening the beam ends adjacent to a joint, particularly when the flange (i.e. the cast-in-place slab) is in tension, becomes attractive. The general concept of local weakening is not new in seismic retrofit or design of steel structures (Popov et al. 1998) and RC structures (FEMA-356 2000) but for RC structures, this has largely remained as a concept rather than a practical method so far.

Proposed seismic retrofit techniques

Three seismic retrofit techniques based on the concept of beam-end weakening in combination with FRP strengthening where necessary and/or appropriate to enforce the strong column/weak beam hierarchy are proposed herein:

1) The first technique, referred to as the beam opening (BO) technique, involves the creation of an opening on the web in each end region of a T-beam adjacent to the beam-column joint, as shown in Figure 3. The internal longitudinal steel reinforcement should be kept intact during the weakening process. If the opening is large enough, the flexural capacity of the T-beam in negative bending can be expected to reduce to a desired value. Local strengthening of regions adjacent to the opening (e.g. using FRP wraps and/or near-surface mounted FRP strips) is needed, particularly to ensure that the weakened beam still has an adequate shear resistance.
2) The second technique, referred to as the section reduction (SR) technique, involves the removal of concrete (and some of the longitudinal steel bars if necessary) from the bottom zone of the beam (i.e. the compression zone under negative bending) adjacent to the beam-column joint, as shown in Figure 4. The gap induced by the removal of material should be re-filled with a softer/weaker material and the exposed steel bars should be properly protected against corrosion. This method reduces the effective section height under negative bending and is expected to be highly effective in reducing the beam flexural capacity. The severing of some of the bottom longitudinal steel bars directly reduces the amount of longitudinal steel compression reinforcement under negative bending. Local strengthening of the region adjacent to the gap induced by the material removal can also be implemented using FRP warps and/or near-surface mounted FRP strips.
3) The third technique, referred to as the slab slit (SS) technique, involves the separation of the slab in the corner region from each supporting beam by cutting a slit (including the severing of the steel bars crossing the slit) between them, as shown in Figure 5. In this method, the path of stress transfer from the beam to the slab near the beam-column joint is weakened so that the contribution of a cast-in-place slab to the beam flexural capacity in negative bending is substantially reduced or totally eliminated. Strengthening of the

slab for its sagging moment capacity, as a result of the introduction of slits, can be easily achieved using FRP reinforcement if needed.

It should be noted that all the three techniques described above can be used in combination with column strengthening if necessary. While the BO method cannot be used together with the SR method, either of these two methods can be used in conjunction with the SS method to achieve a better weakening effect on the flexural capacity of the beam in negative bending.

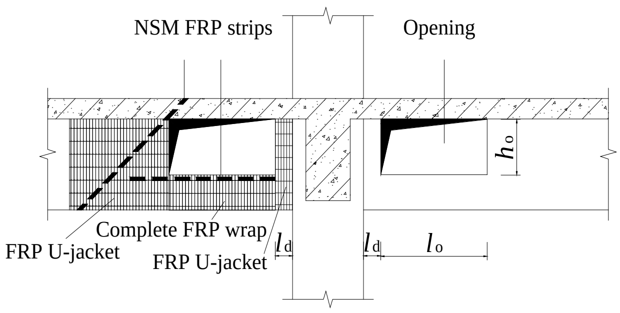


Figure 3. Beam opening technique

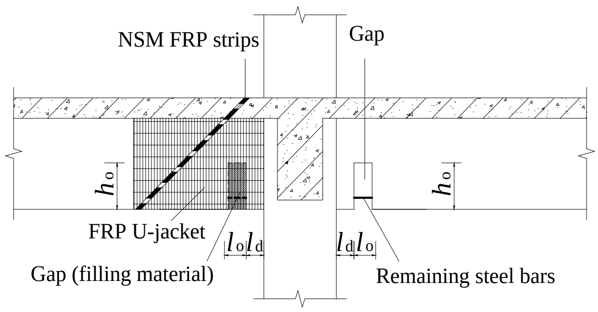


Figure 4. Section reduction technique

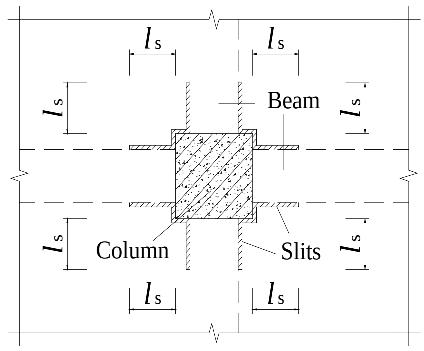


Figure 5. Slab slit technique

RESEARCH NEEDS

General

To the best of the authors' knowledge, the three techniques described above are new and no research is available on their effectiveness and the relevant design methods. For these techniques to be used in practice, research is needed on the effects of each intervention technique on the beam flexural and shear capacities as well as the seismic response of such a weakened beam.

The BO Technique

Many existing studies have examined the behaviour and design of both rectangular and T-beams with a rectangular or circular opening (e.g. Tan et al. 2001), and a few studies have addressed the effect of drilling an opening in an existing beam (e.g. Pimanmas 2010). Although these studies were motivated by the need for the passage of utility ducts and pipes (and thus the purpose was completely different from that of the present study) and no simple method exists for estimating reductions in the flexural and shear capacities of RC beams due to a web opening, the existing work does offer a useful source of information for research on the BO method. In particular, it can be concluded from these studies that an opening can significantly reduce the shear and flexural capacities of a beam. These studies also proved that the use of bonded U jackets/complete wraps (e.g. EI-Maaddawy and Sherif 2009) as well as diagonal near-surface mounted FRP bars at corners (Pimanmas 2010) are effective in controlling shear cracks emanating from the corners.

The SR Technique

The removal of the compressive concrete (and some of the steel bars) is expected to reduce the section flexural capacity significantly; this reduction can be easily estimated by a conventional section analysis, but the accuracy of such an approach does need some verification. It may be noted that severing of bottom longitudinal steel bars has recently been explored in detail as a seismic retrofit method to protect exterior beam-column joints (e.g. Kam et al. 2009), but severing the bottom bars alone cannot solve the problem associated with the contribution of a slab in tension for T-beams in negative bending. Kam et al.'s (2009) work nevertheless appears to be the closest to the present work in terms of philosophy and approach.

The shear capacity of the beam is also expected to be significantly affected by the SR method, and this effect needs to be quantified; shear strengthening using bonded FRP reinforcement also needs to be developed. The inelastic deformation capacity of the beam is a key issue in terms of seismic performance and needs to be qualified and modelled for inclusion in the seismic performance simulation for the frame; an important issue here is the response of the remaining steel bars when subjected to cyclic stresses.

The SS Technique

The SS method may appear to have a simple and clear effect on the seismic performance of the beam, but the real effect may be quite complicated. The introduction of slits modifies the stress transfer path from the beam to the slab: due to the presence of slits, the stress transfer path may now pass around the slits and spread to a wider region of the slab. This effect needs to be understood and quantified in research. In addition, the slits modify the support condition of the slab, and leads to greater sagging moments in the slab under gravity loading. As a result, the sagging moment capacity of the slab may need to be enhanced with appropriate strengthening measures.

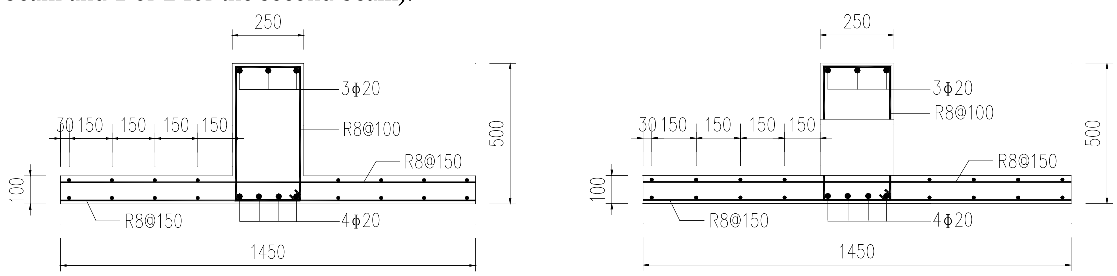
CURRENT RESEARCH

General

The authors are currently engaged in a major research programme to assess the feasibility and relative advantages of the three techniques presented above and to develop design methods for them once their feasibility is established. Of the three techniques, the reduction caused by the SR and the SS techniques can be estimated relatively easily, but the same is not true about the BO method. The first stage of the research programme has thus been focussed on the effect of the BO method on the flexural capacity of the beam. Preliminary results on this effect are presented below.

Experimental Study on RC T-Beams Weakened by a Web Opening

Two full-scale beams were initially tested under three-point bending with a mid-span load. The two test beams had the same dimensions (3,300 mm in span) and reinforcement details (Figure 6); the main differences between them were the web opening details, including the opening height, and the strengthening details. The beams were tested in the inverted position for ease of experimental execution. For different purposes, the first beam was tested four times and the second beam was tested two times, leading to six test cases in total. Each case is named as Beam-X-Y, with X representing the beam number (1 or 2) and Y representing the test number (1 to 4 for the first beam and 1 or 2 for the second beam).



(a) Outside the opening region (b) Within the opening region

Figure 6 Cross-sections of the test beams

Beam-1-1 had a web opening of 500 mm ×150 mm, without any FRP strengthening of the opening. Based on the test results of Beam-1-1, Beam-1-2 was strengthened in flexure and shear with CFRP sheets and then tested for the second time to investigate the effect of such strengthening. The flexural strengthening of Beam-1-2, achieved by bonding CFRP sheets to the top surface of the flange slab (noting that the flange was below the beam in the test configuration), was implemented to study shear failure by debonding of the CFRP U-jackets employed to enhance the shear resistance of Beam-1-2. After the testing of Beam-1-2, a new web opening of 500 mm×200 mm was added to Beam-1-2 on the other side of the mid-span at the symmetrical location to form Beam-1-3, and the new opening was then only strengthened using CFRP U-jackets for shear resistance near the load point (Figure 7). Following the testing of Beam-1-3, the beam was turned upside-down to form Beam-1-4 and tested to understand the behaviour of the beam with the flange slab in compression. Beam-2-1 had a web opening of

500mm ×220 mm, and the opening chord was strengthened with complete CFRP wraps for enhanced shear resistance and for confining the concrete; the region near each side of the opening was strengthened with CFRP U-jackets for enhanced shear resistance (Figure 8). Furthermore, a CFRP pultruded plate (50 mm wide x 1.2 mm thick) was bonded on each side of the chord as the additional longitudinal reinforcement for flexural strengthening of chord. After the testing of Beam-2-1, a new web opening with the same dimensions and the same strengthening scheme was created in Beam-2-1 at the symmetrical location on the other side. The beam was then turned upside down to form Beam-2-2 and tested to understand its behaviour with the flange slab in compression. More details of the tests are available in Jing et al. (2013).

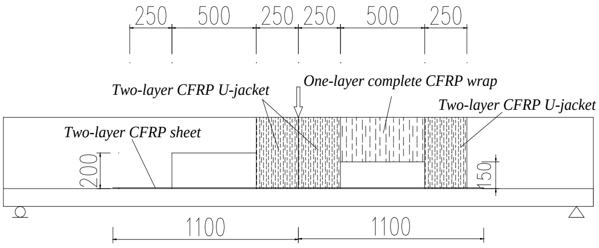


Figure 7 Schematic of Beam-1-3

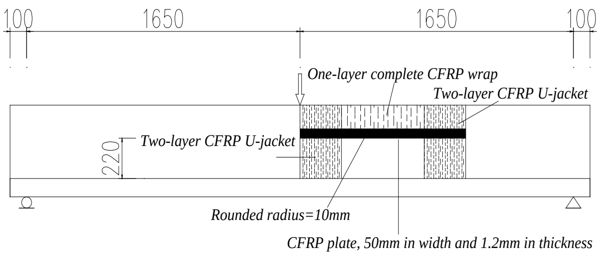


Figure 8 Schematic of Beam-2-1

The test results can be briefly summarised as follows. First, as expected, the load-carrying capacity of the beam decreases as the web opening size increases (Figure 9); Beam-1-1 which had a web opening of 500 mm×150 mm without any FRP strengthening had a larger load-carrying capacity (by about 10%) than that of Beam-2-1 which had a web opening of 500 mm×220 mm, even though Beam-2-1 received FRP strengthening around the opening. Second, nearly all the longitudinal steel bars in the slab yielded at mid-span of the beam at the ultimate state, implying that the contribution of the slab to the flexural capacity of the beam is significant. Third, bonded FRP reinforcement can provide effective strengthening to avoid undesirable failures associated with the creation of a web opening, although debonding, local bucking or tearing of the FRP reinforcement happened during the tests.

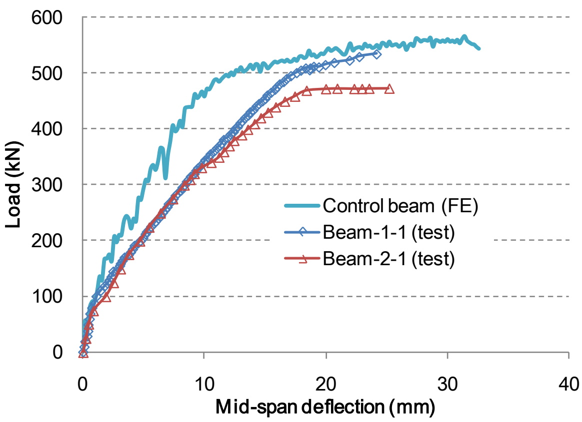
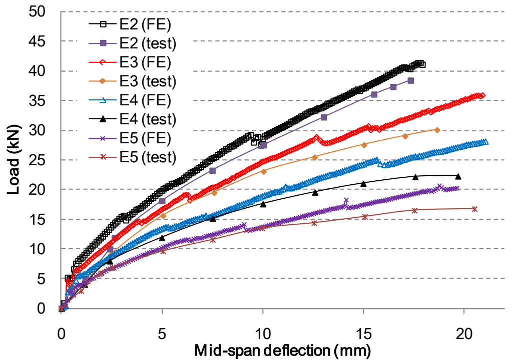


Figure 9 Load-deflection curves of test beams

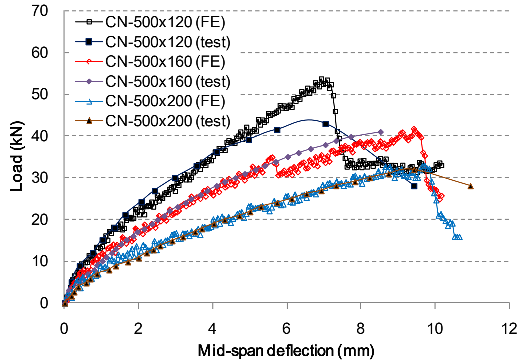
As no control beam without a web opening was tested for comparison with the beams with an opening, a nonlinear finite element model was created to predict the load-deflection curve of the test beam without an opening (i.e. the control beam); the predicted load-displacement curve is shown in Figure 9. The details of the finite element model for the control beam are the same as those described in the next sub-section for RC beams with an opening. The predicted ultimate load of the control beam is 567 kN, and the test ultimate loads of Beam-1-1 and Beam-2-1 are 534 kN and 473 kN respectively. Therefore, if the finite element prediction is taken to be accurate, the openings in these beams led to reductions in the ultimate load of 5.8% and 16.6% respectively. The percentage of reduction of Beam 2-1 is significant, although it may not be sufficient in practice to achieve the required strong column/weak beam strength hierarchy. The determination of the opening size and position to achieve a sufficient reduction in the beam ultimate load is an obvious research issue.

Numerical Study on RC T-Beams Weakened by a Web Opening

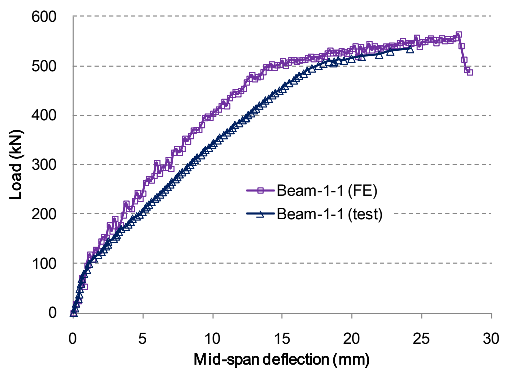
Finite element (FE) modelling is a cost-effective alternative to laboratory testing in studying the behaviour of concrete structures as the latter usually incur heavy time and financial costs. In the present study, an FE model for RC beams with an un-strengthened or FRP-strengthened web opening was developed using the general purpose FE program ABAQUS (ABAQUS 6.10) to study the behaviour of such beams. In the FE model, the concrete was modelled using 4-node plane stress elements (element CPS4R), the steel bars and the FRP were modelled using 2-node truss elements (element T2D2), and the bond behaviour between steel bars and concrete as well as that between FRP and concrete was modelled using interfacial elements (element COH2D4). The steel bars (including the tension steel bars, the compression steel bars and the stirrups) were modelled as an elastic-perfectly plastic material, and the FRP was modelled as an elastic material.



(a) Beams tested by Madkour (2009)

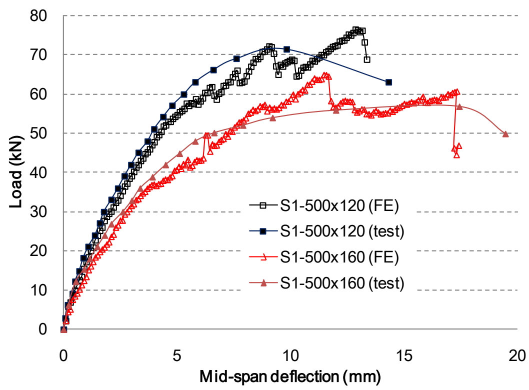


(b) Beams tested by EI-Maaddawy and EI-Ariss (2012)

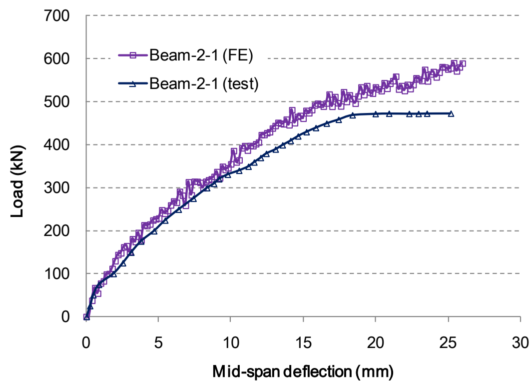


(c) Beams tested by Jing et al. (2013)

Figure 10. Load-deflection curves of beams with an un-strengthened web opening



(a) Beams tested by EI-Maaddawy and EI-Ariss (2012)



(b) Beams tested by Jing et al. (2013)

Figure 11. Load-deflection curves of beams with an FRP-strengthened web opening

As the final failure was expected to be due usually to the local tension/shear failure of concrete near the corners of the web opening, the brittle cracking model provided by ABAQUS (ABAQUS 6.10) was adopted for modelling the concrete; the concrete was assigned the uniaxial compressive stress-strain curve proposed by Saenz (1964) and the uniaxial tensile stress-strain curve proposed by Hordijk (1991). The shear retention model of concrete was that proposed by Rots (1988) with the exponent in this model assigned a value of 2; this exponent value was found to be appropriate through a preliminary finite element study. The bond-slip relationship between steel bars and concrete was the one proposed by CEB-FIP (1993) and that between FRP and concrete was the one proposed by Lu et al. (2005). The elastic modulus of the concrete was assumed to be the tensile secant modulus at the peak stress point in order to account for the damage of concrete near the corners of the web opening induced by the non-uniform shrinkage of concrete during the curing period; this secant modulus was assigned the value of half of the initial elastic modulus (Ye 2005).

Eight RC beams with an un-strengthened web opening [four beams tested by Madkour (2009), three by EI-Maaddawy and EI-Ariss (2012), and one by Jing et al. (2013)] as well as three RC beams with an FRP-strengthened web opening [two beams tested by EI-Maaddawy and EI-Ariss (2012) and one by Jing et al. (2013)] were used to verify the proposed FE model. The load-deflection curves of the eight RC beams with an un-strengthened web opening obtained from FE analyses and tests are compared in Figure 10; the corresponding comparisons for the three RC beams with an FRP-strengthened web opening are shown in Figure 11. The predicted load-displacement curves are seen to agree well with the test results. For more information, please refer to Nie et al. (2013a, b).

CONCLUSIONS

RC frames designed without due consideration of the contribution of cast-in-place slabs to the flexural capacity of RC beams are likely to fail by the undesirable story-sway mechanism when subjected to seismic loading. An obvious method to retrofit such RC frames for enhanced seismic performance is to strengthen the columns using FRP confinement which can improve both the strength and ductility of the columns, but column strengthening alone is often insufficient to achieve a strong column/weak beam strength hierarchy. A more effective seismic retrofit approach is to combine column strengthening with appropriate beam weakening.

Three beam weakening techniques to achieve a strong column/weak beam hierarchy have been proposed in this paper. These three techniques all require a reduction of the beam flexural capacity adjacent to the beam-column joint, and should be used with appropriate local strengthening of the beam/slab which can be achieved with bonded FRP reinforcement. The three techniques include: (a) the beam opening (BO) technique which requires the creation of a sizable opening in the web of a T-section beam; (b) the section reduction (SR) technique which requires the removal of concrete and severing of some steel bears near the bottom of the beam (i.e. in the compression zone when the beam is under negative bending); and (c) the creation of slits in the slab at the beam ends. The third technique may be used in conjunction with either the first or the second technique to achieve greater reductions in the flexural capacity of the beam.

Of the three techniques, the effectiveness of the second and the third techniques in reducing the flexural capacity of the beam can be easily estimated; this is not true for the first technique. A research programme has been launched at The Hong Kong Polytechnic University to confirm/demonstrate the effectiveness of these techniques and to develop relevant design guidance once their feasibility is conclusively established. Preliminary results from this research programme, focussed on the effect of a web opening on the behaviour of the beam, have been presented in the paper.

ACKNOWLEDGMENTS

The authors are grateful for the financial support received from the Research Grants Council of the Hong Kong Special Administrative Region (Project No.: PolyU 5273/11E) and from the National Basic Research Program of China (973 Program) (Project No.: 2012CB026201).

REFERENCES

ABAQUS 6.10. *ABAQUS Analysis User's Manual.* ABAQUS, Providence.

ACI-318 (1983). *Building Code Requirements for Structural Concrete and Commentary* (*ACI 318-83),* ACI Committee 318, American Concrete Institute, Farmington Hills, MI.

ACI-318 (2008). *Building Code Requirements for Structural Concrete and Commentary* (*ACI 318-08),* ACI Committee 318, American Concrete Institute, Farmington Hills, MI.

ATC-40 (1996). *Seismic Evaluation and Retrofit of Concrete Buildings*, Applied Technology Council, Redwood City, California.

CEB-FIP (1993). *Model Code 90*, Lausanne, Switzerland.

Chinese Academy of Building Research (2008). *Photo Collection of 2008 Wenchuan Earthquake Damage to Buildings*, edited by China Academy of Building Research, China Architectural & Building Press, Beijing, China (in Chinese).

EI-Maaddawy, T. and Sherif, S. (2009). "FRP composite for shear strengthening of reinforced concrete deep beams with openings", *Composite Structures*, Vol. 89, No. 1, pp. 60-69.

El-Maaddawy, T. and El-Ariss, B. (2012). "Behavior of concrete beams with short shear span and web opening strengthened in shear with CFRP composites", *Journal of Composites for Construction, ASCE*, Vol. 16, No. 1, pp. 47-59.

Eurocode-8 (2005). *Design of Structures for Earthquake Resistance – Part 1: General Rules, Seismic Actions and Rules for Buildings*", European Committee for Standardization.

FEMA-356 (2000). *Pre-Standard and Commentary for the Seismic Rehabilitation of Buildings*. Federal Emergency Management Agency, Washington, D.C.

GB-50011 (2008). *Code for Seismic Design of Buildings*, Architectural & Building Press, Beijing, China (in Chinese).

GB-50011 (2010). *Code for Seismic Design of Buildings*, Architectural & Building Press, Beijing, China (in Chinese).

Hordijk, D. A. (1991). *Local Approach to Fatigue of Concrete*, PhD thesis, Delft University of Technology, 1991.

Jing, D.H., Teng, J.G., Zhang, S.S. and X.F. Nie (2013). "RC T-beams in negative bending weakened by a web opening", in preparation.

Kam, W.Y., Pampanin, S. and Bull, D. (2009). "Experimental validation of selective weakening approach for the seismic retrofit of exterior beam-column joints", *Proceedings, 2009 New Zealand Society of Earthquake Engineering (NZSEE) Conference*, April 3-5, 2009, Christchurch, New Zealand (Paper No: 32).

Lin, X., Pan, P., Ye, L., Lu, X., and Zhao, S. (2009). "Analysis of the damage mechanism of a typical RC frame in Wenchuan earthquake", *China Civil Engineering Journal*, Vol. 42, No. 5, pp. 13-20.

Lu, X.Z., Teng, J.G., Ye, L.P., and Jiang, J.J. (2005). "Bond-slip models for FRP sheets/plates bonded to concrete", *Engineering Structures*, Vol. 27, No. 6, pp. 920-937.

Madkour, H. (2009). "Non-linear analysis of strengthened RC beams with web openings", *Proceedings of the Institution of Civil Engineers-Structures and Buildings*, Vol. 162, No. 2, pp. 115-128.

Nie, X.F., Teng, J.G., Chen, G.M. and Zhang, S.S. (2013a). "Finite element modeling of RC beams with a web opening", in preparation.

Nie, X.F., Zhang, S.S, Teng, J.G. and Chen, G.M. (2013b). "Finite element modeling of RC beams with an FRP-strengthened web opening", in preparation.

Pimanmas, A. (2010). "Strengthening R/C beams with opening by externally installed FRP rods: Behavior and analysis", *Composite Structures*, Vol. 92, No. 8, pp. 1957-1976.

Popov, E.P., Yang, T.S. and Chang, S.P. (1998). "Design of steel MRF connections before and after 1994 Northridge earthquake", *Engineering Structures*, Vol. 20, No. 12, pp. 1030-1038.

Rots, J. G. (1988). *Computational Modelling of Concrete Fracture*, PhD Thesis, Delft University of Technology, The Netherlands.

Saenz, L. P. (1964). "Discussion of equation for the stress-strain curve of concrete, by Desayi and Krishnan." *Journal of the American Concrete Institute*, Vol. 61, pp. 1229-1235.

Tan, K.H., Mansur, M.A. and Wei, W. (2001). "Design of reinforced concrete beams with circular openings", *ACI Structural Journal*, Vol. 98, No. 3, pp. 407-415.

Ye, L.P. (2005). *Concrete Structures*, 2nd edition, Tsinghua University Press, Beijing, China (in Chinese)